12-2012

An Economic Alternative to the c Chart

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A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Industrial Engineering

By

Ryan Black
University of Arkansas
Bachelor of Science in Industrial Engineering, 2011

December 2012
University of Arkansas

ABSTRACT

Because the probability of Type I error is not evenly distributed beyond upper and lower three-sigma limits the *c* chart is theoretically inappropriate for a monitor of Poisson distributed phenomena. Furthermore, the normal approximation to the Poisson is of little use when *c* is small. These practical and theoretical concerns should motivate the computation of true error rates associated with individuals control assuming the Poisson distribution. An economic alternative to the c chart is described as a statistical model of upward shift from c_0 to c_1 and the two charts are compared in theory. For a range of c chart costs the savings associated with economic design increase linearly.

This thesis is approved for recommendation
to the Graduate Council.

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INTRODUCTION

Control charts are used as an effective tool in many fields to monitor both manufacturing and non-manufacturing processes. A control chart illustrates a process's behavior and allows the user to control for variation. However, choosing the most effective control chart design method is a critical aspect of integrating statistical quality control into a process or system. A balance of economic cost and statistical quality must be achieved. In practice, economic models consider the cost of poor quality to include the costs of sampling, repair, defective items, customer dissatisfaction, lost sales, and liability claims. Most of these cost and risk parameters are rarely available and in many cases cannot be estimated accurately. Thus, an alternative may be to simply consider the cost of Type I and Type II error rates.

The Poisson distribution is positively skewed. Therefore, the probability of Type I error is not evenly distributed beyond upper and lower statistical quality control limits of traditional charts for nonconformities, given Poisson distributed defects. Also, the normal approximation to the Poisson is of little use when the expected number of nonconformities is small and using it when inappropriate may give way to negative lower control limits. Given these practical and theoretical concerns we are motivated to design low-cost, theoretically appropriate control charts that assume the Poisson distribution. We do this by finding minimum cost control limits assuming equal cost errors with respect to alpha and beta for upward shifts from c_0 to c_1.

Results are used to develop a statistical regression model that estimates the minimum cost upper control limit for an upward shift from c_0 to c_1. Total error costs for both the economic alternative and the traditional c chart are calculated and compared for a wide range of upward shifts. The first paper presents the methodological details behind the new alternative and the minimum cost calculations while the second paper compares the economic alternative to the c chart based on a large number of upward shifts from c_0 to c_1.

A Theoretically Appropriate Poisson Process Monitor

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(Received on January 25, 2012)

Abstract – Because the probability of Type I error is not evenly distributed beyond upper and lower three-sigma limits the *c* chart is theoretically inappropriate for a monitor of Poisson distributed phenomena. Furthermore the normal approximation to the Poisson is of little use when *c* is small. These practical and theoretical concerns should motivate the computation of true error rates associated with individuals control assuming the Poisson distribution.

Keywords: ***attributes, control charts, economic design, Poisson***

1. Introduction

The probability of Type I error is not evenly distributed beyond upper and lower statistical quality control limits of traditional charts for nonconformities, given Poisson distributed defects. Also the normal approximation to the Poisson is of little use when the expected number of nonconformities is small. Therefore we are motivated to design low-cost, theoretically appropriate control charts that assume the Poisson distribution. Such economic design requires computation of true error rates and assumptions about the relative costs of errors Type I and II.

Suppose that nonconformities or defects occur in an inspection unit according to the Poisson distribution: $p(x) = e^{-c} / x!$, $x = 0, 1, 2, \ldots$, where x is the number of defects, and $c > 0$ defines the Poisson distribution (mean and variance). Assuming a standard value for c, the traditional c chart for nonconformities is defined as follows: Upper control limit = c + 3 SQRT (c), Centerline = c, and lower control limit = c – 3 SQRT (c).

Because the c chart effectively assumes the normal distribution for a counting process calculations can yield a negative value for the lower control limit (LCL) in which case it is suggested that we set LCL = 0. For example this is one practical consequence of an ill advised normal approximation. It should motivate economic design of theoretically appropriate quality control for Poisson distributed defects.

2. Relevant Literature

The relevant design literature can be divided among three areas: statistical quality control charts, economic quality control charts, and economic-statistical quality control charts. Kaminsky, *et al.* noted that in some instances using a shifted geometric distribution may be more appropriate for Poisson distributed defects, because traditional c charts tend to underestimate process variability [1]. Results from their study showed that compared to more traditional charts false alarm rates were reduced by assuming the geometric distribution. Later a method which dealt explicitly with the number of observations between defects was introduced by Nelson and found to be particularly good for the case of near-zero defects [2]. Chang and Gan further extended these ideas by proposing a scheme for the cumulative count [3], and with every technological advance charts became more cumbersome and perhaps difficult to justify as departures from the c chart.

Straightforward moving averages have been used to monitor nonconformities and compared to the *c* chart [4], and similar improvements were found when exponentially weighted moving average (EWMA) control charts were designed and analyzed [5]. Woodall provides an extensive literature review of control charts designed for observations including the EWMA and cumulative sums (CUSUM) [6]. More recently authors have focused on enhancements of the original *c* chart [7].

Developments in the economic design of quality control charts seek to reduce the cost of process control. Traditionally the four main components of cost are sampling, the false alarm, finding and correcting an assignable cause, and the cost of a defective item. These components are used to determine an economic combination of sample size, control limits, and inter-sample interval. Authors have compared economic designs for CUSUM and geometric moving averages to find that *X-bar* is better to detect large shifts [8]. However many economic models can be prohibitively intricate. Taken separately each of the four cost components can be difficult to estimate accurately. For this reason we favor a simplified approach to monitoring individual observations of a counting process, where the only costs to consider are those associated with errors Type I and II.

The trouble with economic quality control has been that minimum cost solutions can actually run counter to business constraints. For example Williams, *et al.* displayed an optimal solution to produce 64% defectives [9]. The design might have been optimal, but the results would not have conformed to the company's objectives with respect to customer satisfaction. According to Ho and Case in economic-statistical design the loss function of a process is minimized subject to three main constraints: minimum power, maximum Type I error rate, and average time to detect a shift [10]. An excellent example of this proposed an optimization model for the joint design of

X-bar and *R* charts [11]. In the words of the author, “The actual users of control charts are interested in designs that are simple to understand and use.” We have found this ultimate goal to be entirely compatible with theoretically appropriate methods for economic quality control of a Poisson process.

Demerit control limits for Poisson-distributed defects have already been presented and discussed in the context of economic design [12, 13]. The work described here is also related to demerit systems assuming the binomial distribution that were recently introduced and applied to medication error severity data [14, 15]. The particular emphasis we place on economic design might have been most recently featured in a diversity monitor with known errors for process variability observed in categorical data [16].

3. Methods and Results

We examined the concept of a theoretically appropriate monitor for the Poisson process by first arbitrarily choosing some values for c, and computing the associated Type I error rates for combinations of reasonable upper control limits (UCL) and lower control limits (LCL). An observation greater than the UCL or less than the LCL is considered to be out of control. Under the assumption that no real shift has occurred an out of control signal is a Type I error. Assuming a value for c we can find the probabilities associated with observing any number of defects and so the Type I error rate. See Table 1 for an example when $c = 2$.

Next for every combination of shift from c to c_1 we computed the probabilities of Type II errors. An observation between or equal to control limits is considered to be in control. Under the assumption that a shift has actually occurred an in control signal is a Type II error. It is convenient that the Type II error associated with a shift from c to c_1 complements the Type I error associated with $c = c_1$. For example the Type II error rates associated with a shift from any

c to $c_1 = 2$ are equal to "one minus" the values in Table I.

Table 1: Example Type I Error Rates (when $c = 2$)

	UCL = 1	2	3	4	5	6	7
LCL = 5							0.9880
4						0.9639	0.9519
3					0.9098	0.8737	0.8616
2				0.8196	0.7293	0.6932	0.6812
1			0.7293	0.5489	0.4586	0.4226	0.4105
0		0.7293	0.4586	0.2782	0.1880	0.1519	0.1399
None	0.8647	0.5940	0.3233	0.1429	0.0526	0.0166	0.0045

Table 2: Total Costs Assuming Equal Cost Errors (shift from $c_0 = 2$ to $c_1 = 6$)

	UCL = 1	2	3	4	5	6	7
LCL = 5							1.1486
4						1.1245	1.2731
3					1.0436	1.1682	1.3168
2				0.9088	0.9524	1.0770	1.2255
1			0.7739	0.6827	0.7264	0.8509	0.9995
0		0.7442	0.5181	0.4269	0.4706	0.5951	0.7437
None	0.8647	0.6113	0.3853	0.2941	0.3377	0.4622	0.6108

Table 3: Minimum Costs (assuming equal cost errors)

c_0	Type I error rate	c_1	LCL	UCL	Type II error rate	Minimum cost
0.5	0.3935	1	None	1	0.3679	0.7613
0.5	0.0902	2	None	2	0.4060	0.4962
0.5	0.0143	6	None	3	0.0620	0.0764
1	0.2642	2	None	2	0.4060	0.6702
1	0.0803	6	None	3	0.0620	0.1423
2	0.1429	6	None	4	0.1512	0.2941
6	0.1512	2	3	19	0.1429	0.2941

6	0.0620	1	2	19	0.0803	0.1423
6	0.0620	0.5	2	19	0.0143	0.0764
2	0.4060	1	1	13	0.2642	0.6702
2	0.4060	0.5	1	13	0.0902	0.4962
1	0.3679	0.5	0	10	0.3935	0.7613

Assuming equal cost errors we summed the error rates Type I and II for every combination of control limits and shift to discover the minimum cost and associated control limits. See Table 2 for an example of the shift from $c = 2$ to $c_1 = 6$. Obviously the economic design among those in Table 2 has the minimum cost of 0.2941: LCL is None, and UCL is 4. Table 3 shows what are the minimum cost control limits for combinations of shifts from c to c_1.

4. Conclusions and Future Work

We have presented the concept of an economically designed, theoretically appropriate monitor for the Poisson process. Future work should include additional values for *c;* upward and downward shifts to and from each parameter would be evaluated. Another idea is to look for a good meta model of results like the ones appearing in Table 3. For example it would be useful to know if the variation in minimum cost can be understood as a smooth function of c and c_1. One might also like to know if results change in a simple way according to different error costs. Finally the work here would be strengthened by showing an application where interesting data conform well to Poisson distributions like the ones we consider, and theoretically appropriate monitoring decisions can be made more intelligently, according to economic design.

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An Economic Alternative to the *c* Chart

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Abstract – An economic alternative to the *c* chart is described as a statistical model of upward shift from c_0 to c_1, and costs of the two charts are compared in theory. For a range of *c* chart costs the savings associated with economic design increase linearly.

Keywords: *attributes,* c *chart, economic design, Poisson distribution*

Introduction and literature

Suppose that defects occur in an inspection unit of a product according to the Poisson distribution. To clarify *x* is the number of defects, and *c* > 0 is the Poisson distribution parameter:

$$p\,(x) = e^{-c}c^{x} / x! \qquad 1$$

The conventional control chart for defects is the *c* chart with three-sigma upper control limit *(UCL)* and lower control limit *(LCL):*

$$UCL = c + 3\ \mathrm{SQRT}\ (c) \qquad 2$$

$$\text{Centerline} = c \qquad 3$$

$$LCL = c - 3\ \mathrm{SQRT}\ (c) \qquad 4$$

If no standard value for *c* is available, it may be estimated as the average number of defects in a preliminary sample (Montgomery 2013). Because the Poisson distribution is asymmetric, and probability of a Type I error is not equally allocated beyond the *c* chart control limits, most alternatives use probability limits (Grant and Leavenworth 1996). Still control charts for *c* based on three-sigma limits or probability limits that seek to equally allocate Type I error are designed to ignore Type II error. Therefore we are motivated to provide a more economic monitor for the Poisson process.

Studies of the *c* chart include Suich (1988), Khoo (2004), and Kittlitz (2006). Control chart research more generally devoted to the Poisson distribution includes Mhatre, *et al.* (1981); Borror, *et al.* (1998); and Chan, *et al.* (2007). Other control charts for defects include procedures with variable sample size such as the *u* chart (Gardiner and Montgomery 1987, Rocke 1990), and demerit systems (Jones, *et al.* 1999; Chimka and Cabrera 2006; Chimka and Cabrera 2007; Chang, *et al.* 2008). Nelson (1994) and Kittlitz (1999) dealt specifically with low defect levels.

As Jackson (1972) pointed out, “All count distributions are not alike,” and situations lead to distributions other than Poisson, where for example defects occur in clusters or result from multiple underlying causes. For the univariate case Johnson, *et al.* (1993) described such generalized distributions as mixtures. Scheaffer and Leavenworth (1976) considered counts in units of varying size and assumed a negative binomial distribution. Kaminsky, *et al.* (1992) developed statistical control charts when the geometric model is appropriate.

Black and Chimka (2012) introduced a theoretically appropriate Poisson process monitor which allows minimum cost control limits for anticipated shifts from c_0 to c_1. The authors showed example Type I error rates for combinations of control limits (when $c = 2$), their total costs assuming equal cost errors (for a shift from $c_0 = 2$ to $c_1 = 6$), and minimum cost control limits for token combinations c_0 and c_1.

The research extension described in this manuscript had two objectives: 1) to expand the space of upward shifts in hopes of estimating a useful function for the minimum cost *UCL*, and 2) to understand practical cost differences between the *c* chart and the economically designed alternative. In section 2 we present methods and results; in section 3 are conclusions and limitations.

Methods and results

In order to estimate a function for minimum cost upper control limit *(UCL)* we first considered Poisson processes given by every integer $c = 1$ to 100, and every conceivable upward shift from c_0 to c_1. Therefore the number of shifts considered was $1 + 2 + \ldots + 99 = 4950$. For every Poisson process *c* and combination of control limits there is a Type I error rate, and for every shift from c_0 to c_1 there is a Type II error rate. Therefore every Poisson process and conceivable shift has an economic combination of control limits assuming equal (or any other) cost errors. Since we are anticipating upward shifts there should be no *LCL*, no Type I error rate associated with rejecting the hypothesis of control due to a relatively small individual observation. (For the same reason alternative hypotheses are one-sided versus two-sided.) Processes are considered out of control if individual observations are greater than or equal to the upper control limit. With no *LCL* as the *UCL* increases Type I error decreases, and Type II error increases for any upward shift. Assuming equal cost errors we sought to find the minimum cost *UCL* for each of 4950 upward shifts between integer c = 1 to 100.

For every Poisson process given by c we can find conventional control limits of the c chart. It comes with a Type I error rate, and for every shift from c_0 to c_1, a Type II error rate. Assuming equal cost errors we can compute total cost for every c chart associated with the same 4950 upward shifts between integer c = 1 to 100. Table 1 is an example of shifts from c_0 to c_1; associated control limits, error rates, total costs, and total cost differences between c chart and the economically designed alternative:

2. LCL_c is the lower control limit of c chart
3. UCL_c is the upper control limit of c chart
4. a_c is the Type I error rate of c chart
5. b_c is the Type II error rate of c chart
6. $Cost = a_c + b_c$, cost of the c chart
7. UCL^* is minimum cost upper control limit of the economically designed alternative
8. a is the Type I error rate of economically designed alternative
9. b is the Type II error rate of economically designed alternative
10. $Cost^* = a + b$, cost of the economically designed alternative
11. $Delta = Cost – Cost^*$, difference between control charts

Table 1. Example results

c_0	c_1	LCL_c	UCL_c	a_c	b_c	*Cost*	*UCL**	*a*	*b*	*Cost**	*Delta*
1	2	None	4	0.0190	0.8571	0.8761	2	0.2640	0.4060	0.6702	0.2059
1	6	None	4	0.0190	0.1512	0.1702	3	0.0803	0.0620	0.1423	0.0279
2	6	None	6	0.0166	0.4457	0.4622	4	0.1429	0.1512	0.2941	0.1682
2	10	None	6	0.0166	0.0671	0.0836	5	0.0526	0.0292	0.0819	0.0017
4	10	None	10	0.0081	0.4579	0.4661	7	0.1107	0.1301	0.2408	0.2252
10	20	1	19	0.0072	0.3814	0.3886	15	0.0836	0.1049	0.1883	0.2003
20	30	7	33	0.0050	0.6845	0.6895	25	0.1568	0.1572	0.3140	0.3755
20	60	7	33	0.0050	0.0000	0.0050	37	0.0004	0.0006	0.0010	0.0040
40	60	21	59	0.0051	0.3808	0.3860	49	0.0703	0.0844	0.1547	0.2312
60	80	37	83	0.0034	0.6166	0.6200	69	0.1118	0.1186	0.2304	0.3896

Results of the following relatively large shifts were discarded for a lack of precision to computing Type I error: $c_0 = 1$ to $c_1 > 74$, $c_0 = 2$ to $c_1 > 83$, and $c_0 = 3$ to $c_1 > 91$. This eliminated 51 observations leaving 4899 for estimating *UCL** as a function of c_0 and c_1. Approximately 99% of variability in the minimum cost UCL is accounted for by the following multiple linear regression model, a viable alternative to cumbersome lookup table of *UCL** for combinations of c_0 and c_1.

$$UCL^* = 0.5068 + 0.6696c_0 + 0.3560c_1 \qquad 5$$

Conclusions and future work

To help us understand practical cost differences between the *c* chart and economically designed alternative we illustrate *Delta* versus *Cost* in the scatterplot Figure 1, where *Delta* is difference between control charts, and *Cost* is that of *c* chart.

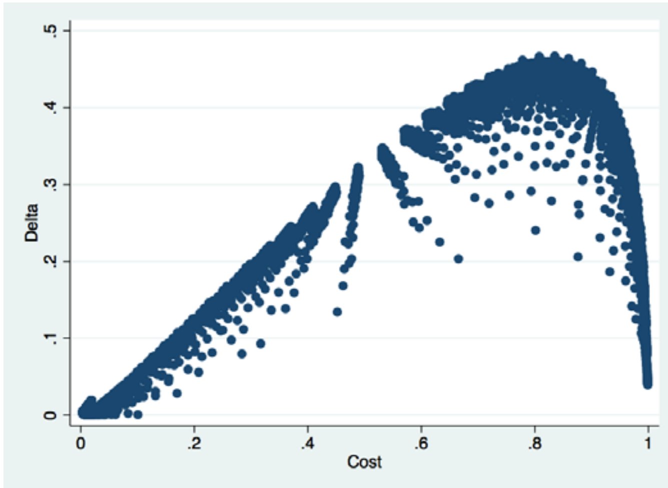

Figure 1. Total cost difference between control charts versus total cost of *c* chart

As the *c* chart becomes more expensive savings associated with the economically designed alternative increase linearly up to a point where they begin to decrease sharply. Future work may include a more objective understanding of this curve and variability around it. We have tried and failed to understand the relationship between total cost difference and nature of the shift c_0 to c_1. And even though we have fit a strong model of *UCL** we do not have a mechanistic understanding of it. Computational obstacles that led to eliminating 51 observations made analysis of downward shifts even more difficult. Though downward shift results could lead to an even more useful model of minimum cost control limits. There would be no upper control limits for downward shifts, so a single minimum cost control limit may be estimated as a function of c_0 and c_1, and perhaps a new binary variable to describe direction of shift. Finally an approach like the one taken here may be extended to design economic control charts assuming other distributions.

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CONCLUSION

While there have been many developments in economic and statistical quality control charts over the years, few of these ideas have been implemented in practice. There are several reasons for this, with one critical aspect being that economic and statistical models are often complex, difficult to understand, and challenging to implement. The theory presented in this study is simple, easy to understand, and flexible to fit different situations. This new economic alternative to the c chart balances the statistical integrity of the quality control model and the economic costs associated with Type I and Type II error probabilities. This allows optimal upper control limits to be determined that minimize total error costs for upward shifts from c_0 to c_1.

The regression model developed in this study provides an accurate estimation of a minimum cost upper control limit for an upward shift as a function of c_0 and c_1. In summary, as the c chart becomes more expensive savings associated with the economically designed alternative increase linearly up to a point where they begin to decrease sharply. This economic and theoretically appropriate alternative to the c chart provides a simple, low-cost methodology for calculating control limits for processes where the number of conformities can be represented by the Poisson distribution.

APPENDIX

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