Developing equivalent second order system models and robust controls for servo-drive related systems

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DEVELOPING EQUIVALENT SECOND ORDER SYSTEM MODELS AND ROBUST CONTROLS FOR SERVO-DRIVE RELATED SYSTEMS
DEVELOPING EQUIVALENT SECOND ORDER SYSTEM MODELS AND ROBUST CONTROLS FOR SERVO-DRIVE RELATED SYSTEMS

A thesis submitted to the Honors College in partial fulfillment of the requirements for the degree of Honors Bachelor of Science in Electrical Engineering

By

Christopher Hoyt

April 2009
University of Arkansas
Abstract

Numerous industrial and other plant processes today represent systems in which a motor supplies torque to a drive disk, which in turn through a pulley system drives a larger load. Developing models for these complex systems can become time consuming and expensive. A simple second order system approximation can be developed for these systems under certain system conditions which can greatly reduce the complexity of controlling and modeling the system. A second order feed-forward notch controller can then be introduced to the system which greatly improves performance.

The results of this proposed method of approximating complex servo-drive systems show that such reductions in complexity can be made with positive results. Further implementation of the proposed second order notch filter improved system responses. In order to demonstrate these methods, Simulink in conjunction with MATLAB was used to first run simulations of the proposed method. The ECP Model 220 Servo Drive system was then used to physically test the approximations.

The successful results obtained have the potential to lead to cheaper control system solutions with better responses in industrial processes.
This thesis is approved for recommendation to the Graduate Council.

Thesis Director:

[Signature]

Dr. Roy McCann
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ACKNOWLEDGEMENTS

I thank my thesis advisor, Dr. McCann, for his support throughout the past year in his courses as well as with the thesis process. He’s continually pointed me in the right direction when I’ve run into a road block. I also thank Dr McCann and his respective graduate students for providing me with the system parameters used in running simulations as well as numerous other pieces of information which saved me from countless hours of arduous work.

I also thank my family and fellow electrical engineering classmates. They are my true support structure and have always been there to back me up. I would not have been able to make it through these four years without them.
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1. INTRODUCTION

1.1 Problem: Complex Industrial System Models

Numerous industrial processes and other mechanical systems involve dynamics in which a motor torque is applied to a drive disk which in turn is connected to a load through a pulley system. These processes are often in need of control systems in order to produce desired and stable outputs. The problem that arises is that the systems often contain numerous state variables, thus making them complex systems to model and to control. Often the more complex a system and therefore the more complex its control system is, the more expensive it is to model the process as well as implement the control system.

To fix the problem of modeling complex systems and developing control systems, approximations need to be made to simplify the system and thus decreasing time in developing controls for the systems and costs in implementation. Also with a simpler system, the ability to design a more robust control is possible. Robust controls are needed in order for a process to withstand variability in the system and well as unforeseen disturbances in the system.

1.2 Thesis Statement

It is the goal of this research to develop a second order approximation of a complex servo-drive system as well as develop a feed-forward notch filter control in order to produce an accurate output to a given input. The equivalent system and notch filter will be a robust model capable of withstanding a variety of parameter and sample time changes.

1.3 Approach

In developing the system approximation for the servo-drive simulation device, state-space equations were developed for the fourth order system and the proposed second order
system approximation. These fourth order and second order models were simulated using Simulink and MATLAB to compare compatibility. After extensive testing, a notch filter was then developed. To do so the natural frequency of the system and damping coefficient were found, and the appropriate notch filter was designed.

After extensive research had been done on the system using MATLAB, the control system parameters were implemented on the ECP Model 220 plant model. The robustness of the controls were tested further physically by varying system parameters and sample times for the controller.

Once both avenues of testing the proposed approximation and control system had been visited, the results were checked for consistency.

1.4 Potential Impact

In developing simpler system models and controls for the proposed system setup (being a torque applied to a drive disk which is in turn applied to a load through a pulley system), benefits in numerous industrial applications are present. Approximating the complex systems to second order allow for easier system analysis and control development, thus resulting in time and cost savings. The robustness of the controls also allows for a greater range of variability from device to device, changes in system parameters, and disturbances introduced to the system.

1.5 Organization of Thesis

This thesis is organized into five chapters. The first is an introductory chapter including basic information regarding the reasoning for the proposed thesis, the approach in researching the proposition, and its potential impact. The second chapter provides background information regarding the theory behind the second order approximation method implemented, as well as the notch filter design and the servo drive system modeled. The third chapter involves simulations of
the approximation and notch filter control along with the robustness of the system through varying system parameters and sample rates. The fourth chapter carries these simulations over to the physical system which inspired the thesis, and the analysis associated with the success of the approximation and design. The last chapter draws conclusion based on the research.
2. BACKGROUND

2.1 ECP Model 220 Servo-Drive System

The system used to conduct the research for this thesis was the ECP Model 220 Servo-Drive System. The system is used to simulate a variety of device and plant processes. A graphical representation of the system can be seen below in Figure 2.1.

![ECP Model 220 servo-drive system diagram](image)

Figure 3.1: ECP Model 220 servo-drive system.

The system allows for a wide range in variability through the capability of adding weights to the drive and load disk, and through changing the pulley ratios. The setup used for the purpose of this thesis involved using four 500 kg weights at maximum radius on the load disk, no weights on the drive disk, and a flexible pulley between the load disk and the speed reduction assembly. This allowed for the simulation of a spring compliance present in many plant processes.

In order to develop the state-space equations for the above system, the system was put into another graphical representation which was needed in order to view the state variables
present in the system. Figure 2.2 shows the representation with state variables and system parameters labeled.

![Diagram of a servo-drive system with state variables.](image)

Figure 2.2: Servo-drive system with state variables.

The variables present in the above figure represent the following: \( \tau_m \) – input motor torque, \( \Theta_{1,2} \) – the drive and load disk positions respectively, \( J_{1,2} \) – the drive and load inertias respectively, \( B_{1,2} \) – the friction coefficients associated with the drive and load disks respectively, \( B_{12} \) – the friction coefficient associated with the band between the disks, and \( K_{12} \) – the spring compliance from the band between the two disks. The resultant differential equations from Figure 2.2 are shown below.

\[
J_1 \Theta_1'' = -B_1 \Theta_1' + K_{12} (\Theta_2 - \Theta_1) + B_{12} (\Theta_2' - \Theta_1') + \tau_m \quad (2.1)
\]

\[
J_2 \Theta_2'' = -B_2 \Theta_2' + K_{12} (\Theta_1 - \Theta_2) + B_{12} (\Theta_1' - \Theta_2') \quad (2.2)
\]

Which when placed into state-space form, yield a 4x4 system matrix, with state variables being

\[
x_1 = \Theta_1, x_2 = \Theta_1', x_3 = \Theta_2, x_4 = \Theta_2' \quad (2.3)
\]

This system is complex for analysis purposes, and poses problems in designing controls.
2.2 Second Order Approximation of System

For analysis purposes, the above system can be approximated as a second order system by considering a few things. Assuming that the drive disk inertia, $J_1$, is much smaller than the load disk inertia, $J_2$, a feedback loop can be inserted into the system which measures the drive disk position, $\Theta_i$. Then the system can be multiplied by a proportional gain constant, $K$. In considering the drive disk inertia is negligible, and in feeding back the drive disk position and multiplying the system by a proportional gain constant, a new equivalent system can be developed. The following graphic is the new modified version of Figure 2.2.

![Figure 2.3: Approximated second order servo-drive system.](image)

This new system is a much simpler second order system, shown by the new differential equation for the system as follows.

$$J_2 \Theta_2'' = -B_2 \Theta_2' + B_{12} (\Theta_1' - \Theta_2') + K_{12} (\Theta_1 - \Theta_2)$$  \hspace{1cm} (2.4)

The input for the system is now the drive disk position, which is approximately equivalent to the motor torque in this approximation. This new differential equation yields the new $2 \times 2$ system matrix in state space form.

$$\frac{d}{dt} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{J_2} \\ -\frac{K_{12}}{J_2} & -\frac{B_{12} + B_2}{J_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_{12}}{J_2} \end{bmatrix} \Theta_1$$  \hspace{1cm} (2.5)
This transfer function for the newly approximated system has the general form of a standard second order system as shown in the following equation.

\[
\frac{\theta_2(s)}{\theta_r(s)} = \frac{B_{12}s^2 + K_{12}}{s^2 + \frac{B_{12} + B_{2} + K_{12}}{J_2}} \approx \frac{\omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \tag{2.6}
\]

The transfer function can be equivalent as the standard second order equation due to the s-term coefficient on the numerator being much smaller than that value on the denominator.

### 2.3 Notch Filter Utilization

As can be seen in Equation 2.6, the second order form of a system includes a natural frequency term, \(\omega_n\), in which the system naturally resonates at this frequency when disturbed. In order to compensate for this natural frequency, a notch filter can be implemented to filter that resonating frequency response out of the system. This will create a simple feed-forward control to the approximated second order system, which greatly improves the response. The standard form for the notch controller is below.

\[
N(s) = \frac{s^2 + 2\zeta_z \omega_n s + \omega_n^2}{s^2 + 2\zeta_p \omega_n s + \omega_n^2} \tag{2.7}
\]

The frequency at which the notch controller filters is designated by the \(\omega_n\) term, and is easily found in the standard second order system in Equation 2.6 as \(\frac{K_{12}}{J_2}\). The attenuation the filter provides at \(\omega_n\) is equal to the ratio between the zero zeta and the pole zeta as \((\zeta_z/\zeta_p)\). This value can be chosen as needed.
3. **MATLAB/SIMULINK SIMULATIONS**

### 3.1 Fourth Order to Second Order Approximation

In order to demonstrate the method discussed earlier in approximating the original system as a second order system, MATLAB and Simulink models were produced. State-space equations were produced for both the 4x4 matrix system and the 2x2 matrix system and the following Simulink model was developed for testing the similarity between the two. Figure 3.1 shows the Simulink model used.

![Simulink model](image)

**Figure 3.1: Fourth order to second order Simulink comparison model.**

As can be seen, by feeding back the drive disk position through a proportional gain constant, the system can be approximated as second order. In order to find the optimal gain value to best estimated the system, several values were tested. To adhere to consistency, a value of $K=1$ was first chosen. Running the system produced the following response.
Figure 3.2: Simulation of load disk position with $K=1$.

As can be seen, with a unity gain value, the actual system exhibits much slower behavior than the obtained second order system. In order to increase response time, the inner loop gain was increased. The next value chosen was $K=50$, which produced the below plot.

Figure 3.3: Simulation of load disk position with $K=50$. 
With the increasing inner loop gain values, the actual system beings to closely resemble the equivalent second order system. In order to proceed with simulations and to ensure an accurate approximation, an inner loop gain value of $K=1000$ was used. As can be seen in the below waveform, the two systems were equivalent with these parameters.

![System Approximation with K=1000](image)

**Figure 3.4:** Simulation of load disk position with $K=1000$.

### 3.2 Implementation of Notch Filter

In order to design the notch filter for the system, the natural frequency of the system needed to be found. In order to do so, the drive disk was locked in place as shown in Figure 3.5 and the load disk was displaced roughly 1000 counts and then released. This simulated a step response of the equivalent system shown in the previous Figure 2.3.
After displacing the load disk and then releasing, the system returned to equilibrium through its natural frequency. Below is the data obtained from the servo-drive system plotted in MATLAB.

![Figure 3.5: Physical system with locked drive disk setup.](image)

![Figure 3.6: Locked drive disk system response from displaced load disk.](image)
From viewing the frequency at which the system returned to equilibrium, the natural frequency of the system is approximately 13 rad/s. The attenuation chosen for the filter at this frequency was $1/15$ or $\zeta_z=0.1$, $\zeta_p=1.5$. These values chosen for the notch controller produced the transfer function in relation to Equation 2.7 as follows.

$$N(s) = \frac{s^2 + 2.6s + 169}{s^2 + 39s + 169}$$

(3.1)

In order to better understand the function of the notch filter in the approximated second order system, see Figure 3.7 below. This figure represents the Bode plots for both the filter transfer function as well as the approximated second order transfer function from Equation 2.6. As can be seen on lower plot (the Bode plot of the second order system), a resonant frequency exists at roughly 13 rad/s. The magnitude response of the notch filter (the upper Bode plot) attenuates at roughly 13 rad/s. This eliminates the resonate frequency of the system, and produces an overall better response. This can be seen in Figure 3.8 in which a step response excites the system with the notch filter, and then without the notch filter respectively.
Figure 3.7: Bode plots of notch filter (top) and servo-drive system (bottom).

Figure 3.8: Step response of system with notch controller present (top) and without (bottom).
The instantaneous change of the step input though is not a good simulation of the response of the actual mechanical system due to the high torque transient at the step moment. A ramp style input was used when physically implementing the controller, so a ramp input was used in a Simulink model. In order to test the system using Simulink, the following setup was used.

Simulations were then run and both responses were viewed from the system with the notch controller and then the system without the notch controller. The following two figures show the response with the notch filter, and without the notch filter respectively. As can be seen, the response with the notch filter present, though slightly slower, does not contain the resonant frequency the second plotted response contains. The filter therefore produces a better response.
3.3 Discrete Time Control

Due to most modern controllers being micro-processor based, the controlling effort is in discrete time as opposed to continuous time as previously modeled. Most control schemes can be
modeled as continuous time due to the speed at which current processors run. For consistency and later robust testing purposes, the notch filter was converted to discrete time form. After converting the notch controller to discrete time using the standard sample time of the ECP Model 220 Servo-Drive \( T_s = 0.002652 \) seconds, the transfer function took the new form:

\[
N(z) = \frac{z^2-1.992z+0.9934}{z^2-1.901z+0.9017}
\]  \( (3.2) \)

Using the new discrete form of the notch controller, a new Simulink model was produced in order display consistency between the continuous and discrete forms of the notch filter and for later testing in robustness of the design. Figure 3.12 shows the Simulink model used and Figure 3.13 displays the output from the discrete time version of the controller. In comparing the responses shown in Figure 3.13, the continuous and discrete responses are identical.

![Simulink model for testing system robustness.](image-url)
3.4 Robustness of Approximation Model and Controls

In first testing the robustness of the equivalent system, the inertia of the drive disk, $J_1$, was varied. As mentioned earlier, the principle behind the ability to approximate the system to a second order system was the fact that the drive disk inertia was relatively negligible as compared to the load disk inertia, $J_2$. Considering that the loop gain, $K$, was kept high, the drive disk inertia could be increased significantly with little to no effect on the output. As a test, the drive inertia coefficient was increased 400%. Figure 3.14 represents output between the approximated system and the original system with the increased drive disk inertia, which shows little change.

![Figure 3.13: Discrete and continuous model system outputs.](image-url)
The next process in testing the robustness of the control device was in varying the sample rate for that of the discrete notch filter. The discrete filter coefficients were kept the same as shown in Equation 3.2, but the sample time was varied to values of \( T_s = 0.000884s, 0.001768s, 0.003536s, \) and \( 0.004420s \) from the original \( 0.002652s \) value. These values are intervals available on the ECP Model 220 Servo-Drive system. The following four figures represent the response of the system with the variable sample times. The outputs show the reference input, continuous input, and discrete input in order to compare the effect of the variable sample time.
Figure 3.15: Discrete time system output with $T_s=0.000884\text{s}$.

Figure 3.16: Discrete time system output with $T_s=0.001768\text{s}$.
As can be seen in these plots, as the sample time went from the low end to the high end, the response of the system began to gradually slow. This is to be expected. What can be seen in Figure 3.15, is with the small sample time, there is evidence of a resonant frequency still present.
in the system. This will be shown further in the physical testing of the servo drive in the next chapter. Overall from these simulations, the system appears to be fairly robust through a range of ±200% of the standard sample time of 0.002652s, as well as through a significant increase in the drive disk inertia.
4. PHYSICAL IMPLEMENTATION ANALYSIS

4.1 Physical System with Second Order Approximation and Notch Filter

In finalizing the research involved with approximating the discussed servo-drive model as a second order system and increasing its response accuracy, tests were performed using the values found from MATLAB physically on the ECP Model 220 Servo-Drive system. The software associated with the servo-drive allows for a general form of control algorithms to be implemented in the system. Therefore employing the designed feed-forward notch filter along with the inner loop gain measuring the drive disk inertia was easily done. Figure 4.1 displays the outputs from data obtained by the servo-drive and compiled with MATLAB. The top plot displays the output of the system without the notch filter, with the bottom plot showing the output of the system with the filter. This plot correlates directly with Figure 3.11 and 3.10 respectively, considering the natural frequency of the system is present when the notch controller is not present. Revisiting the top plot, it has the general response of a second order system. This being the case, modeling the system as a second order system by creating an inner loop gain from measuring the drive disk position was successful. Also in revisiting the bottom plot showing the notch filter implemented using the coefficients obtained from previous calculations, the improved response shows the feasibility of implementing a simple feed-forward notch controller to improve system responses.
4.2 Robustness of Model and Controls Through Variability

In order to verify the robustness of the system physically, the same variability as performed in MATLAB/Simulink was applied to the servo-drive system. Initially, the inertia of the drive disk was varied to a variety of weights and configurations. Figure 4.2, 4.3, and 4.4 represent the different configurations used. Initially, 200g weights were placed at the center of the drive disk to increase $J_1$, and gradually the weights were moved out and increased to 500g at a maximum radius from the center as can be seen in Figure 4.4. For each configuration, the same control and inner loop gain parameters were used to view the robustness of the output when increasing drive disk inertia. As can be seen from the outputs in Figure 4.5, the system response remained stable. Only at the configuration of 500g weights at maximum radius on the drive disk
did the system begin to show signs of approaching instability. Therefore like the simulations previously, the system shows substantial robustness when greatly varying the drive disk inertia. The second order approximation of the original system therefore is validated.

![Figure 4.2: Servo-drive system with 200g weights on drive disk at minimum radius.](image)

![Figure 4.3: Servo-drive system with 200g weights on drive disk at maximum radius.](image)
Figure 4.4: Servo-drive system with 500g weights on drive disk at maximum radius.

Figure 4.5: Load disk position of system with varying drive disk inertias.
After showing robustness from variability in the drive disk inertia, the discrete time implementation of the notch controller was tested for robustness by varying the sample rates for the system. As in the previous MATLAB/Simulink simulations, the normal sample time for the discrete notch controller was $T_s=0.002652 \text{s}$, while values of $T_s= 0.000884 \text{s}$, $0.001768 \text{s}$, $0.003536 \text{s}$, and $0.004420 \text{s}$ were used to test for robustness. Figure 4.6 displays the output of the system given these varied sample rates. In using the bottom plot of Figure 4.1 as a reference for the standard operation of the system with $T_s=0.002652 \text{s}$, it can be seen that with the faster sample rates from the norm, the system showed more overshoot and oscillation. This is the same result as shown in Figure 3.15. With the slower sample rates, the system naturally responded slower as can be compared with the simulations in Figures 3.17 and 3.18. Though the sample rate was varied ±200% (as with the simulations), the system still responded in a stable manner. The ability for the equivalent system and notch controller to perform under a variety of conditions further displays its robustness.
Figure 4.6: Servo-drive load disk position using different discrete time sample rates.
5. CONCLUSION

5.1 Summary

In this thesis, a method was used to simplify many industrial and mechanical processes in which a motor torque is applied to a drive disk which in turn drives a load disk through a pulley system. It was shown that the complexity of this type of system could be simplified to a standard second order system by assuming the drive disk inertia was considerably lower than that of the load disk. Then by measuring the drive disk positioning and feeding it back into the system through a proportional gain constant, the system could be modeled as second order. It was also found that by increasing the proportional gain constant, the system could be modeled more accurately. The accuracy of the response of the system could then be increased dramatically by eliminating the second order resonating frequency through employing a simple feed-forward notch controller.

The robustness of the approximation and controller was then thoroughly researched through varying system parameters and sample times for the notch controller in both MATLAB/Simulink and on the ECP Model 220 Servo-Drive system, the inspiration and center point for this thesis. It was found that the system maintained a stable and accurate response through a variety of tests in both the simulations and the physical implementation.

The success of the approximation and control to reduce the order of the system and improve its response has a number of applications in numerous mechanical systems and plant processes. The ability to reduce the complexity of the system and provide simple feed-forward controls to better the response allows for application in industry to provide both savings in costs and time.
APPENDIX

A. References


B. MATLAB Source Code

%Christopher Hoyt
%Honors Thesis - April 2009
%Advisor - Dr. Roy McCann
clc
clear

%Servo Drive System Values
J1=(8)*0.0011; J2=0.0206;
B1=0.002; B2=0.054;
K12=8.45; B12=0.017;
K=1000; %Inner Loop Gain

%Original 4x4 System Matrix before Approximation
Ao=[0 1 0 0; -K12/J1 -(B12+B1)/J1 K12/J1 B12/J1; 0 0 0 1; K12/J2 B12/J2 -
  K12/J2 -(B12+B2)/J2];
Bo=[0; 1/J1; 0; 0];
Co=eye(4);
D=[0];
Go=ss(Ao,Bo,Co,D);

%Approximated 2nd order 2x2 System Matrix After Inner Loop Approximation
Aa=[0 1; -K12/J2 -(B12+B2)/J2];
Ba=[0; K12/J2];
Ca=eye(2);
Ga=ss(Aa,Ba,Ca,D);

%Approximated System Transfer Function
F=tf([B12/J2 K12/J2], [1 (B2+B12)/J2 K12/J2]);

%Notch Filter Design
wn=12; %Natural Frequency of System.
zetaa=0.1;
zetaap=1.5; %Magnitude of attenuation is ratio of zetaz/zetap. Set at 1/15.
N=tf([1 2*zetaa*wn wn^2], [1 2*zetaap*wn wn^2])
figure(1)
subplot(2,1,1)
bode(N)
subplot(2,1,2)
bode(F)
figure(2)
subplot(2,1,1)
step(N*F)
subplot(2,1,2)
step(F)

%Discrete Time Notch Filter
Ts=0.002652;
Nz=c2d(N,Ts) %Used to determine coefficients with standard sample time.
Nz=tf([1 -1.993 0.9939], [1 -1.908 0.9089],-1) %TF used to test variable sample times
%Ts=0.000884; %Low end sample time
%Ts=0.001768;
%Ts=0.003536;
%Ts=0.004420; %High end sample time

clc
clear

%Loading different k values for system approximation.
load kis1
figure(5)
plot(ans(1,:),ans(2,:),ans(1,:),ans(3,:))
title('System Approximation with K=1')
xlabel('Time(s)');ylabel('Position(radians)')
grid on
load kis50
figure(6)
plot(ans(1,:),ans(2,:),ans(1,:),ans(3,:))
title('System Approximation with K=50')
xlabel('Time(s)');ylabel('Position(radians)')
grid on
load kis1000
figure(7)
plot(ans(1,:),ans(2,:),ans(1,:),ans(3,:))
title('System Approximation with K=1000')
xlabel('Time(s)');ylabel('Position(radians)')
grid on

%Loading data from simulink on notch filter performance
load notch
figure(8)
plot(ans(1,:),ans(2,:),ans(1,:),ans(3,:))
title('System Response With Notch Filter')
xlabel('Time(s)');ylabel('Position(radians)')
grid on
load nonotch
figure(9)
plot(ans(1,:),ans(2,:),ans(1,:),ans(3,:))
title('System Response Without Notch Filter')
xlabel('Time(s)');ylabel('Position(radians)')
grid on

%Loading discrete simulink model for comparison to continuous model
load discrete
figure(10)
plot(ans(1,:),ans(2,:),ans(1,:),ans(3,:),ans(1,:),ans(4,:))
title('Discrete Notch Filter System Response')
xlabel('Time(s)');ylabel('Position(radians)')
grid on

%Loading drive disk inertia increase of 400% to test robustness - simulink
load driveinertiaincrease
figure(11)
plot(ans(1,:),ans(2,:),ans(1,:),ans(3,:))
title('System Response with Increased Drive Disk Inertia')
xlabel('Time(s)');ylabel('Position(radians)')
grid on

%Loading simulink variable sample time robustness tests
load ts0884
figure(12)
plot(ans(1,:),ans(2,:),ans(1,:),ans(3,:),ans(1,:),ans(4,:))
title('Discrete Time Response with Ts=0.884')
xlabel('Time(s)');ylabel('Position(radians)')
grid on
load ts1768
figure(13)
plot(ans(1,:),ans(2,:),ans(1,:),ans(3,:),ans(1,:),ans(4,:))
title('Discrete Time Response with Ts=1.768')
xlabel('Time(s)');ylabel('Position(radians)')
grid on
load ts3536
figure(14)
plot(ans(1,:),ans(2,:),ans(1,:),ans(3,:),ans(1,:),ans(4,:))
title('Discrete Time Response with Ts=3.3536')
xlabel('Time(s)');ylabel('Position(radians)')
grid on
load ts4420
figure(15)
plot(ans(1,:),ans(2,:),ans(1,:),ans(3,:),ans(1,:),ans(4,:))
title('Discrete Time Response with Ts=4.420')
xlabel('Time(s)');ylabel('Position(radians)')
grid on

%Loading and displaying displaced disk to find natural frequency
load locked_drivedisk_positivedisplacement.txt;
data = locked_drivedisk_positivedisplacement;
figure(1)
plot(data(:,2),data(:,3)/16000)
title('Displaced Disk Response')
xlabel('Time(s)');ylabel('Revolutions')
grid on

%Loading and displaying inner loop gain system without and with notch controller
load rampinput_1p5pulleyratiogain_nofilter.txt;
nofil = rampinput_1p5pulleyratiogain_nofilter;
load rampinput_1p5pulleyratiogain_withfilter.txt;
yesfil = rampinput_1p5pulleyratiogain_withfilter;
figure(2)
subplot(2,1,1)
plot(nofil(:,2),nofil(:,3)/16000,nofil(:,2),nofil(:,4)/16000)
title('System Response Without Filter')
xlabel('Time(s)');ylabel('Revolutions')
grid on
subplot(2,1,2)
plot(yesfil(:,2),yesfil(:,3)/16000,yesfil(:,2),yesfil(:,4)/16000)
title('System Response With Filter')
xlabel('Time(s)');ylabel('Revolutions')
grid on

%Loading and displaying different inertia configurations on drive disk
load rampinput_robusttest_200gdrivedisk_allin.txt;
config1 = rampinput_robusttest_200gdrivedisk_allin;
load rampinput_robusttest_200gdrivedisk_allout.txt;
config2 = rampinput_robusttest_200gdrivedisk_allout;
load rampinput_robusttest_500gdrivedisk_allout.txt;
config3 = rampinput_robusttest_500gdrivedisk_allout;
figure(3)
subplot(3,1,1)
plot(config1(:,2),config1(:,3)/16000,config1(:,2),config1(:,4)/16000)
title('System Response with 200g Weights at Minimum Radius')
xlabel('Time(s)');ylabel('Revolutions');
grid on
subplot(3,1,2)
plot(config2(:,2),config2(:,3)/16000,config2(:,2),config2(:,4)/16000)
title('System Response with 200g Weights at Maximum Radius')
xlabel('Time(s)');ylabel('Revolutions');
grid on
subplot(3,1,3)
plot(config3(:,2),config3(:,3)/16000,config3(:,2),config3(:,4)/16000)
title('System Response with 500g Weights at Maximum Radius')
xlabel('Time(s)');ylabel('Revolutions');
grid on

%Loading and displaying different discrete time sample rates
load rampinput_discrete_000884samplerate.txt;
sample1 = rampinput_discrete_000884samplerate;
load rampinput_discrete_001768samplerate.txt;
sample2 = rampinput_discrete_001768samplerate;
load rampinput_discrete_003536samplerate.txt;
sample3 = rampinput_discrete_003536samplerate;
load rampinput_discrete_004420samplerate.txt;
sample4 = rampinput_discrete_004420samplerate;
figure(4)
subplot(4,1,1)
plot(sample1(:,2),sample1(:,3)/16000,sample1(:,2),sample1(:,4)/16000)
title('System Response with Sample Rate of 0.000884s')
xlabel('Time(s)');ylabel('Revolutions');
grid on
subplot(4,1,2)
plot(sample2(:,2),sample2(:,3)/16000,sample2(:,2),sample2(:,4)/16000)
title('System Response with Sample Rate of 0.001768s')
xlabel('Time(s)');ylabel('Revolutions');
grid on
subplot(4,1,3)
plot(sample3(:,2),sample3(:,3)/16000,sample3(:,2),sample3(:,4)/16000)
title('System Response with Sample Rate of 0.003536s')
xlabel('Time(s)');ylabel('Revolutions');
grid on
subplot(4,1,4)
plot(sample4(:,2),sample4(:,3)/16000,sample4(:,2),sample4(:,4)/16000)
title('System Response with Sample Rate of 0.004420s')
xlabel('Time(s)');ylabel('Revolutions');
grid on