5-2011

Identifying feasible schedules for elementary school gifted and talented education programs

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Identifying Feasible Schedules for

Elementary School Gifted and Talented Education Programs

An undergraduate honors thesis submitted to the

University of Arkansas

College of Engineering

Department of Industrial Engineering

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Summer 2010

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Abstract

Elementary school Gifted and Talented (GT) education programs enrich the quality and effectiveness of public school institutions by providing qualitatively different approaches to traditional classroom education and a more aggressively paced curriculum to children who exhibit high academic and creative ability. Typically, this is provided through enrichment classes which take place during the regularly scheduled class day. Due to this, schools which provide these services often encounter a series of complex scheduling challenges as students come from different classes and different grades to attend GT enrichment classes, each having varying schedules and varying availabilities. Using *ad hoc* techniques to address these challenges proves to be extremely time consuming and inefficient due to the tightly constrained nature of elementary school class scheduling. In this study, we model the problem as an integer-linear program in order to provide an accurate means of efficiently identifying feasible schedules for GT instructors.

1 Background/Motivation

Gifted and Talented (GT) education programs play an important role in the education system by nurturing the academic ability of children and youth who exhibit exceptional potential. Similar to Special Needs Education wherein students with lower IQ levels are provided with academic programs to match their individual needs, GT programs strive to meet the needs of children who show high potential in academics, creativity, and leadership. This is accomplished by providing a qualitatively different approach to traditional classroom education and a more aggressively paced curriculum to challenge and encourage students to develop their skills further. Prospective GT candidates go through rigorous testing and are identified by their performance throughout this testing process.

While GT education programs greatly enrich the quality and effectiveness of public school institutions, they also tend to create difficult scheduling challenges within the schools and school districts that provide these services. A common structure for GT programs is to conduct GT enrichment courses during the course of the students' regularly scheduled school day. A GT student is pulled out of their normal classes to attend an enrichment class with other GT students, taught by a certified GT instructor. It becomes challenging to schedule these enrichment classes, as GT instructors typically teach students from several different classes and grades, and students have widely varied availability based on their individual scheduled activities.

2 Literature Review

Course scheduling has been a subject of research for several decades. In the 1970s, Tillet [1] and Bristle [2] formulated the university course timetabling problem as a transportation model. Harwood and Lawless [3] created a linear model that integrates goal programming with mixed-integer programming to solve the same type of problem. Abboud *et al.* [4] formulated this problem with a mathematical model and solved it heuristically. Fizzano and Swanson [5] used a combination of a "greedy" algorithm and non-bipartite matching and then applied a bipartite matching in the second stage to analyze the problem. Later, Asratian and Werra [6] also applied bipartite matching for timetabling problems in the basic training programs of some universities. Finally, Lewis and Paechter [9] explored the applicability of a generic grouping algorithm to the course scheduling problem.

While extensive studies have been conducted focusing on high school and university course timetabling, there exists very little literature documenting studies done on course scheduling methodologies at the elementary school level, and no literature which directly relates quantitative scheduling methodologies to GT programs. University course scheduling is primarily concerned with allocating courses to professors and assigning them to various classrooms. However, the GT scheduling problem is focused on assigning multiple events (class meetings) to limited resources (instructors) under extremely rigid time and capacity constraints, due to the nature of an elementary class setting. Although there are basic similarities between course timetabling in universities and the GT scheduling problem discussed, the constraints involved are significantly different in nature due to the elementary school setting, and require a new and distinct approach.

3 Problem Statement

Typically, GT scheduling decisions are made using imprecise ad hoc techniques which prove to be extremely time consuming and tedious due to the tightly constrained nature of the problem. Furthermore, this drawn out process must be repeated each school year as the schedule and availability constraints change. In this study, the scheduling problem is addressed using mathematical modeling in an integer linear program (ILP), with the aim to provide an efficient and exact scheduling tool for GT educators, which can be used to address a variety of similar scheduling challenges.

4 Model Formulation

The model deals with four data sets: the set of instructors, indexed by *i*; the set of classes, indexed by *c*; the set of days, indexed by *d*; and the set of time periods, indexed by *t*. A set of assumptions is made in the base model formulation to represent a typical situation. These assumptions can be adjusted to accommodate deviations from the typical, but are necessary to the model. First, the school day is partitioned into an appropriate number of time periods. This number is arbitrary and can be set as appropriate for each school's schedule. As classes and activities at many schools occur on the quarter hour, we have partitioned a standard 8:00 am to 3:00 pm school day into 28 15-minute time periods. Thus, $1 \leq t \leq 28$. Second, we assume that GT enrichment courses take place twice a week for each student, for a 90 minute period (six consecutive 15-minute time blocks) each time

Decision Variable and Data Parameters

There are two binary decision variables defined as

$$x_{i,c,d,t} \triangleq \begin{cases} 1 \ if \ instructor \ i \ \boldsymbol{starts} \ to \ teach \ class \ c \ at \ time \ t \ on \ day \ d \\ 0 \ otherwise \end{cases}$$

$$y_{i,c,d,t} \triangleq \begin{cases} 1 \ if \ instructor \ i \ \boldsymbol{continues} \ to \ teach \ class \ c \ at \ time \ t \ on \ day \ d \\ 0 \ otherwise \end{cases}$$

Dependant on the school, there could be any set of constraints which would hinder student availability. We have defined three common events which tend to cause student unavailability: lunch, recess and activity based classes. We represent these using 3 data parameters which are binary availability matrices, defined as:

$L_{c,d,t}$ – scheduled lunch $\triangleq \begin{cases} 1 \; if \; class \; c \; is \; not \; at \; lunch \; at \; time \; t \; on \; day \; d \\ 0 \; otherwise \end{cases}$

$R_{c,d,t}$ - scheduled recess $\triangleq \begin{cases} 1 \; if \; class \; c \; is \; not \; at \; recess \; at \; time \; t \; on \; day \; d \\ 0 \; otherwise \end{cases}$

$A_{c,d,t}$ –activity class $\triangleq \begin{cases} 1 \; if \; class \; c \; is \; not \; in \; an \; activity \; class \; at \; time \; t \; on \; day \; d \\ 0 \; otherwise \end{cases}$

Objective Function

Options for objective functions are ample, but are largely dependent on specific schools' and/or teachers' needs. For example, teachers may want to maximize the number of consecutive free periods between teaching classes. Some schools may want to minimize the overlapping time of day that a student's GT courses take place, in order ensure that the student does not miss the identical regularly scheduled class periods more than once per week. These preferences and policies will vary widely from school to school. In this work, we focus strictly on identifying feasibility in our base model, utilizing a "dummy objective" (1) which is controlled by the equality constraint (6) in the model:

$$Max \; \sum_{i,c,d,t} x_{i,c,d,t} \tag{1}$$

Constraints

The model constraints are as follows:

$$\sum_c y_{i,c,d,t} + \sum_c x_{i,c,d,t} \leq 1 \qquad \forall i \in \text{instructors}, d \in \text{days}, t \in \text{times} \tag{2}$$

$$x_{i,c,d,t} \leq L_{c,d,t} * R_{c,d,t} * A_{c,d,t} \quad (3)$$
$$\forall\, i \in \text{instructors}, d \in \text{days}, t \in \text{times}, \text{c} \in \text{classes}$$

$$y_{i,c,d,t} \leq L_{c,d,t} * R_{c,d,t} * A_{c,d,t} \quad (4)$$
$$\forall i \in \text{instructors}, d \in \text{days}, t \in \text{times}, \text{c} \in \text{classes}$$

$$x_{i,c,d,t} + y_{i,c,d,t+1} + y_{i,c,d,t+2} + y_{i,c,d,t+3} + y_{i,c,d,t+4} + y_{i,c,d,t+5} \geq 6 - M * (1 - x_{i,c,d,t})$$
$$\forall\, i \in \text{instructors}, d \in \text{days}, t \in \text{times}, \text{c} \in \text{classes} \quad (5)$$

$$\sum_{i,d,t} x_{i,c,d,t} = 2 \qquad \forall \text{ c} \in \text{classes} \quad (6)$$

$$\sum_{i,d,t} y_{i,c,d,t} = 10 \qquad \forall \text{ c} \in \text{classes} \quad (7)$$

$$\sum_{t=24}^{t=28} x_{i,c,d,t} = 0 \qquad \forall i \in \text{instructors}, d \in \text{days}, \text{c} \in \text{classes} \quad (8)$$

$$\sum_i \sum_t x_{i,c,d,t} \leq 1 \qquad \forall\, d \in \text{days}, \text{c} \in \text{classes} \quad (9)$$

Constraint (2) is an overlapping prevention constraint, which ensures that at any given time period of a day, only one class is being taught by each instructor. Constraints (3) and (4) ensure that the students are available during their assigned GT class time. Constraint (5) is necessary due to the nature of the decision variables, and ensures that a GT class which starts at period *t* continues for five more time periods, in order to make up a 90 minutes class. Constraints (6) and (7) guarantee that each class takes place exactly twice per week. Constraint (8) limits the times during which a class can start during the day in order to ensure that the class is finished by 3:00 pm each day. Finally, constraint (9) ensures that the same class is not taught more than once in one day.

4.1 Model Verification

To illustrate the model, we use Vandergriff Elementary School in Fayetteville, Arkansas as a case study. At this school, GT students attend enrichment classes twice a week for 90 minutes each time. Vandergriff has students to fill seven GT classes, one class of second graders, and two classes each of 3rd, 4th, and 5th grade. Vandergriff has a single GT instructor, who has set aside the Friday of each week to conduct parent meetings, curriculum development, testing, and teaching Kindergarten GT classes. As we do not have a data parameter representing the teacher's availability, we add the following constraint to our base model to address the instructor's unavailability on Fridays:

$$\sum_{t,c} x_{i,c,Fri,t} = 0 \qquad \forall i \in \text{instructors} \qquad (10)$$

Constraint (10) prevents any classes from being scheduled at any time on Friday.

Vandergriff has set time blocks during which students cannot take GT enrichment classes: lunch, recess, and a set of "activity"-based classes comprised of Art, Music, Library, and Physical Education. These time blocks take place at different times of the day for each grade. The challenge then is to schedule all seven classes for two 90 minute periods each, from Monday to Thursday, during the standard 8:00am-3:00pm school day, without violating each class's individual schedule constraints.

4.2 Initial Results

The model was formulated in AMPL and analyzed using CPLEX 11.0. Data supplied by Ms. Mitzi Delap at Vandergriff Elementary was used to produce the results shown in the table below. We differentiate two classes within the same grade with the letters "A" and "B" (e.g. "4thA" is a separate class from "4thB"). Table 1 shows us that each class is assigned two groups of six time period meetings with the GT instructor—this is a feasible schedule.

Table 1: Initial Results

	Mon	Tues	Wed	Thurs	Fri
8:00				4thA	
8:15				4thA	
8:30				4thA	
8:45	4thB			4thA	
9:00	4thB			4thA	
9:15	4thB	5thA	5thA	4thA	
9:30	4thB	5thA	5thA		
9:45	4thB	5thA	5thA		
10:00	4thB	5thA	5thA		
10:15	4thA	5thA	5thA	4thB	
10:30	4thA	5thA	5thA	4thB	
10:45	4thA	5thB	5thB	4thB	
11:00	4thA	5thB	5thB	4thB	
11:15	4thA	5thB	5thB	4thB	
11:30	4thA	5thB	5thB	4thB	
11:45	2nd	5thB	5thB	2nd	
12:00	2nd	5thB	5thB	2nd	
12:15	2nd			2nd	
12:30	2nd			2nd	
12:45	2nd			2nd	
1:00	2nd		3rdA	2nd	
1:15	3rdA	3rdB	3rdA	3rdB	
1:30	3rdA	3rdB	3rdA	3rdB	
1:45	3rdA	3rdB	3rdA	3rdB	
2:00	3rdA	3rdB	3rdA	3rdB	
2:15	3rdA	3rdB	3rdA	3rdB	
2:30	3rdA	3rdB		3rdB	
2:45					

5 Extending the Model for Practice

This base model can be further customized to meet the needs of specific programs. Here we discuss examples of added constraints to meet the needs of the Vandergriff GT program. Although the schedule initially generated by the model is feasible, further examination shows that it is not ideal. It can be seen that both 5th grade classes are scheduled to have their GT enrichment classes first on Tuesday and then again on Wednesday. Having GT two days in a row, followed by 5 days without is not conducive to the learning process. To address this, we incorporate a new set of constraints which forces the model to schedule classes on an every-other-day basis.

$$\sum_{i,t}[x_{i,c,d,t} + x_{i,c,d',t}] = 1 \qquad \forall\ c \in \text{classes}, d = Mon, d' = Tues \qquad (11)$$

$$\sum_{i,t}[x_{i,c,d,t} + x_{i,c,d',t}] = 1 \qquad \forall\ c \in \text{classes}, d = Tues, d' = Wed \qquad (12)$$

$$\sum_{i,t}[x_{i,c,d,t} + x_{i,c,d',t}] = 1 \qquad \forall\ c \in \text{classes}, d = Wed, d' = \ Thurs \qquad (13)$$

$$\sum_{i,t}[x_{i,c,d,t} + x_{i,c,d',t}] = 1 \qquad \forall\ c \in \text{classes}, d = Thurs, d' = \ Mon \qquad (14)$$

These constraints force classes to take place on a Monday and Wednesday or Tuesday and Thursday schedule. Incorporating these constraints and running the model again yields the schedule shown below in Table 2.

Table 2: Monday/Wednesday,Tuesday/Thursday Results

	Mon	Tues	Wed	Thurs	Fri
8:00	4thB		4thB	4thA	
8:15	4thB	4thA	4thB	4thA	
8:30	4thB	4thA	4thB	4thA	
8:45	4thB	4thA	4thB	4thA	
9:00	4thB	4thA	4thB	4thA	
9:15	4thB	4thA	4thB	4thA	
9:30		4thA			
9:45		5thA	5thB	5thA	
10:00		5thA	5thB	5thA	
10:15	5thB	5thA	5thB	5thA	
10:30	5thB	5thA	5thB	5thA	
10:45	5thB	5thA	5thB	5thA	
11:00	5thB	5thA	5thB	5thA	
11:15	5thB				
11:30	5thB				
11:45	2nd		2nd		
12:00	2nd		2nd		
12:15	2nd		2nd		
12:30	2nd		2nd		
12:45	2nd	3rdA	2nd	3rdA	
1:00	2nd	3rdA	2nd	3rdA	
1:15	3rdB	3rdA	3rdB	3rdA	
1:30	3rdB	3rdA	3rdB	3rdA	
1:45	3rdB	3rdA	3rdB	3rdA	
2:00	3rdB	3rdA	3rdB	3rdA	
2:15	3rdB		3rdB		
2:30	3rdB		3rdB		
2:45					

We can see from the generated schedule that the classes are all distributed throughout the week on a Monday/Wednesday, Tuesday/Thursday schedule.

Upon closer examination of the newly generated schedule we see that the instructor would be required to teach four classes consecutively each Monday, for 4.5 hours straight. This schedule would be exhausting for a single instructor. It is clear that the solution could be improved upon. To address this problem we introduce an additional constraint to force an idle time in the instructors schedule in order to provide a short break.

$$\sum_{i,c} \sum_{15}^{22} [x_{i,c,d,t} + y_{i,c,d,t}] \leq 7 \qquad \forall\, d \in \text{days} \tag{15}$$

Constraint (14) forces a 15-minute idle time period between the hours of 11:30am and 1:00pm (corresponding to t = 15 and t = 22 respectively). Incorporating constraint (14) and running the model again yielded the results shown in Table 3.

Table 3: With Forced Idle Time

	Mon	Tues	Wed	Thurs	Fri
8:00			4thA	4thB	
8:15	4thA		4thA	4thB	
8:30	4thA	4thB	4thA	4thB	
8:45	4thA	4thB	4thA	4thB	
9:00	4thA	4thB	4thA	4thB	
9:15	4thA	4thB	4thA	4thB	
9:30	4thA	4thB	5thA	5thB	
9:45		4thB	5thA	5thB	
10:00	5thA		5thA	5thB	
10:15	5thA		5thA	5thB	
10:30	5thA		5thA	5thB	
10:45	5thA	5thB	5thA	5thB	
11:00	5thA	5thB			
11:15	5thA	5thB			
11:30		5thB			
11:45	2nd	5thB	2nd		
12:00	2nd	5thB	2nd		
12:15	2nd		2nd		
12:30	2nd		2nd		
12:45	2nd		2nd		
1:00	2nd		2nd	3rdA	
1:15	3rdB	3rdA	3rdB	3rdA	
1:30	3rdB	3rdA	3rdB	3rdA	
1:45	3rdB	3rdA	3rdB	3rdA	
2:00	3rdB	3rdA	3rdB	3rdA	
2:15	3rdB	3rdA	3rdB	3rdA	
2:30	3rdB	3rdA	3rdB		
2:45					

We can see in this schedule that an idle time has been forced at 11:30am on Monday.

6 Conclusions and Future Work

The aim of this study was to develop a scheduling tool which could efficiently produce feasible schedules given student availability data. We present a general modeling approach and formulation that addresses this objective. Furthermore, we discussed examples of additional constraints which have the potential to improve the feasible solution.

Future work for this study is primarily concerned with sensitivity analysis and examining the model's performance with a variety of similar GT programs through testing in other schools. Another possibility is to expand the model to a county scale and examine the option of scheduling GT classes county-wide, and coordinating individual school schedules such that GT instructors are able to travel between schools to serve schools which either have too many GT students for one teacher, or too few GT students to warrant an onsite instructor.

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