Development and Analysis of a Statics and Kinematics Demonstration as a Learning Tool in the Biomechanics Classroom

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Development and Analysis of a Statics and Kinematics Demonstration as a Learning Tool in the Biomechanics Classroom
by Bethany Knight

Introduction

Engineering, as a discipline, is generally defined as applied science and mathematics to real world problems. Biomedical Engineering focuses on the healthcare and medical engineering problems. At the University of Arkansas, many Biomedical Engineering classes have laboratory components; however, Biomechanics does not. Biomechanics (BMEG 2813) is a sophomore level course at the University of Arkansas within the Biomedical Engineering department. As the second or third biomedical class in the four-year sequence, giving students the opportunity for hands-on experience at this stage would be invaluable. Classes within engineering without an applied learning experience are shown to have less student excitement about the subject, less understanding, and less retention [3]. Demonstrations are shown to double the amount of information students retain [3]. Biomechanics covers a range of topics, including stress-strain curves, force analysis, Casson/Poiseuille Blood Flow, and Mechanobiology/Cell Biomechanics. At this stage in students’ studies, they have had the basic math and science courses as prerequisites, but have not begun to apply them to problems until BMEG 2813, which most students find challenging. The fact that biomechanics does not have a lab component does not aid in making the topics covered more tangible. In a preliminary survey conducted to assess the needs for an in class demonstration, 31 students who took BMEG 2813 agreed 100% that visual demonstrations help them understand topics, shown in figure 1A. Also, as seen in figure 1B, when asked what the hardest topic was within the course, 41% said kinematics was the most difficult, and the test over this material generally has the lowest average.

Figure 1. Pie charts displaying results from survey of students who took Biomechanics prior to Spring 2016 [1].

a.)

Do visual demonstrations help you understand a topic?

<table>
<thead>
<tr>
<th></th>
<th>yes</th>
<th>neutral</th>
<th>no</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>32</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

b.)

Which topic was most challenging?

<table>
<thead>
<tr>
<th></th>
<th>Count</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Force Analysis (load bearing/magnitude)</td>
<td>7</td>
<td>21.9%</td>
</tr>
<tr>
<td>Kinematics (Linear/Angular displacement, acceleration, velocity, torque)</td>
<td>13</td>
<td>40.6%</td>
</tr>
<tr>
<td>Stress-Strain Curves/Viscoelasticity</td>
<td>2</td>
<td>6.3%</td>
</tr>
<tr>
<td>Casson/Poiseuille Blood Flow Principles</td>
<td>10</td>
<td>31.3%</td>
</tr>
<tr>
<td>Mechanobiology/Cell Biomechanics</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>Other</td>
<td>0</td>
<td>0%</td>
</tr>
</tbody>
</table>
While demonstrations and lab components are becoming more common in college courses, the equipment available to obtain data and utilize it for homework problems can be expensive. The same is true for demonstrations used to collect biomechanical data. Some biomechanics systems can cost upwards of a few million dollars, depending on the cameras and other technology used to capture gait and force analysis. Most biomechanics systems used currently are from BioPac, whose introductory kit costs approximately $4,000. Thorough research of biomechanic testing equipment lead to the discovery of Pasco, a company that offers goniometers, force load cells, and other biomedically applicable devices for a fraction of the price of BioPac’s. Pasco also utilizes a software that plots data in real time and is available for free to students for use on laptops and personal computers through a university wide license purchased through the University of Arkansas’ physics department.

This research aimed to develop a supplemental classroom model that demonstrates biomechanics topics in a hands-on way through measuring and analyzing force vectors and angles, angular momentum, angular and linear velocity and acceleration. By creating a model that can be easily and affordably integrated into the classroom, students will be able to see first hand the breadth of biomechanics and how this relates to the bigger picture of Biomedical Engineering. Applications of these biomechanical principles include gait analysis for prosthesis and orthotics, identification in crime scene investigations, weight/load bearing applied to injuries and braces, angular and linear acceleration of objects being thrown, and the impact of the weight of these objects on joints such as elbows and shoulders.

Materials and Methods
Arm Model Equipment, Set Up, and Presentation
The Human Structures set from PASCO (ME-7001) was used for the first demonstration. This kit is comprised of truss set members and screws that can be disassembled and reassembled into three different human model pieces including a leg, back, and arm. The arm was constructed per the user manual, and is shown in figure 2. String was chosen for all connection points to provide uniformity within the model and force measurements. A load cell (PASCO PS-2199) was attached at a location on the model to simulate the bicep muscle and show approximate stress the bicep in a human arm would experience. This load cell was connected to a load cell amplifier, which connected to a USB link (PASCO PS-2100A) and could be plugged into any computer with a USB port. Utilizing the PASCO Capstone software, the USB link would display the force being exerted on the connected load cell. To display the force exerted on the load cell, when Capstone is opened, the “Large Digits Display” button is chosen. This number changes with the amount of force exerted on the load cell, and stabilizes quickly after the force stabilizes. Using a protractor, the forearm was placed at an angle of 90 degrees from the upper arm segment of the model. A roll of tape weighing 129.3 grams was used as a mass and counter weights were attached to the back of the shoulder to counterbalance the weight of the tape and arm, as demonstrated in figure 2.
For the actual demonstration, the arm was transported to the Biomechanical Engineering classroom. The demonstration’s purpose was to have the students think about how as the angle of the forearm to the upper arm changed, the force exerted on the bicep muscle also changed, and then demonstrate this principle. Secondly, the demonstration showed that as the attachment point of the string, or muscle changed, the forces needed to hold the forearm in place also changed. For almost all of the demonstrations, one load cell was used in the bicep position and one attachment point. The last row in table 1 shows the exception. For the last demonstration, two load cells were attached, as seen in figure 2, and two attachment points were used to show the distribution of the load when divided across two muscles. A powerpoint presentation accompanied the demonstration to explain the setup of the arm, as well as show how the model related to the homework assignment. The slides to this powerpoint are located in the appendix. Data gathered during the demonstration is located in table 1. The class discussed limitations to the model in comparison to a human arm as well as how these limitations could be addressed in the future.

A homework assignment related to the first presentation was assigned, and the students were asked to compare the theoretical numbers they calculated to the numbers collected from the model in class. This exercise allowed the students to think critically not only about the limitations of the model, but also the limitations of theoretical calculations.
Table 1. Angles and resulting forces gathered in class demonstration.

<table>
<thead>
<tr>
<th>Angle (deg)</th>
<th>Distance from Joint (cm)</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>6.7</td>
<td>9.5</td>
</tr>
<tr>
<td>70</td>
<td>6.7</td>
<td>7.2</td>
</tr>
<tr>
<td>110</td>
<td>6.7</td>
<td>10.4</td>
</tr>
<tr>
<td>90</td>
<td>27.9</td>
<td>3</td>
</tr>
<tr>
<td>90</td>
<td>8.7</td>
<td>5.4</td>
</tr>
<tr>
<td>90</td>
<td>6.7 &amp; 27.9</td>
<td>.8 &amp; 4.4</td>
</tr>
</tbody>
</table>

Angular Kinematics Equipment, Set Up, and Presentation

The second experiment done utilized a goniometer and angle sensor (PASCO PS-2137). Goniometers measure the change in angle or position over time. This is used to measure and calculate the angular velocity and angular acceleration. The goniometer can be strapped to a student’s hip joint, arm joint, or knee joint. The set-ups used in class are shown in figure 3.

Figure 3. (a) Goniometer attached to elbow joint. (b) Goniometer attached to knee joint [2].

The frequency or period of a person’s movement can also be deducted from the measurements obtained. Before demonstrating, the concept of deriving the angular velocity from the change in angle over time was discussed, as well as the derivation of the angular acceleration from angular velocity. For the demonstration, two velcro straps
were placed on the arm and the goniometer's hinge ends were placed in a similar orientation to the orientation shown in figure 3. The cord from the goniometer was plugged into the PASCO angle sensor. This angle sensor was then plugged into the USB link and a laptop computer equipped with PASCO's Capstone. Figure 4 shows the output from the angle sensor during the arm demonstration, with the top left graph being the change in angle, the top right being change in velocity, and the bottom left being change in angular acceleration over time. The hard data collected from the angle sensor was exported into a csv file that the students uploaded into MatLab. The data from the angle change was used in MatLab to derive the velocity and acceleration. The students then graphed the derived data and compared this with the observed data and explained in the homework why there might be differences in the observed and derived graphs. The demonstration done in class also included utilizing the goniometer on the knee joint; however, the data from the knee was collected for the purpose of the demonstration. The arm data was much more periodic.

![Figure 4. Goniometer arm data collected by Bethany Knight. Top left is change in angle, top right is change in velocity, bottom left is change in acceleration, and the bottom right is a gauge that changes with angle change.](image)

A sample MatLab code was typed to compare to the student's code. This code graphed the calculated or derived values versus the collected data and outputs of the range of motion of the arm during the demonstration. The MatLab code can be found in the appendix. The plots generated include the original csv file data all plotted on one graph in figure 5a, a graph of all the derived and observed measurements on one comparative graph in figure 5b, the derived velocity compared with the observed velocity in figure 5c, and figure 5d displays the observed acceleration plotted with the derived acceleration. The sampling frequency was set to 50 hertz and the samples were taken over a span of 10 seconds.
Results

A main focus of this study was to determine if adding a more interactive component relating to biomechanics would help students retain or enjoy the subject matter more. Students enrolled in the Spring 2016 biomechanics class took both a pre-semester and post semester survey. The pre-semester survey was used to show the need for a demonstration or laboratory component within the class structure. The pre-survey results indicated that visual demonstrations do help students, while also showing that the students polled prefer topics presented either with supplemental visual aides or models and classroom activities. These results are seen in figure 6.
Do visual demonstrations help you understand a topic? (56 responses)

- yes: 92.9%
- neutral: 14.3%
- no: 44.6%

Which statement do you agree with the most concerning how you process and learn information?
(56 responses)

- I can remember best about a subject by listening to a lecture that includes information, explanations and discussions. 41.1%
- I prefer to see information written on a chalkboard and supplemented by visual aids and assigned readings. 14.3%
- I prefer to use posters, models, or actual practice and other activities in class. 44.6%

**Figure 6.** Results from the pre-semester survey of Spring 2016 biomechanics students [1].

The post-semester survey was done of the 54 students enrolled in the Spring 2016 biomechanics class after both demonstrations, homework, and tests had been completed. Figure 7 shows results from the survey that related to both demonstrations. Figure 8 displays the survey results based on the different demonstrations. The left hand side is the muscle forces demonstration survey data, and the right hand side is the angular kinematics demonstration survey data.
Figure 7. (a) Pie chart from Post Spring 2016 survey showing that both topics were equally liked by the students. (b) Bar chart expressing that over a majority of the students felt that demonstrations and utilizing software made them feel well equipped to proceed in their field. (c) Bar chart displaying that well over a majority of the students believe demonstrations help to motivate students within the Science Technology Engineering and Math (STEM) fields[1].
Figure 8. (a) and (b) display how difficult students felt the topics were overall. (c) and (d) show helpfulness of the demonstrations assessed by the students. (e) and (f) demonstrate the interest levels in the demonstrations. (g) and (h) assess how the students felt the demonstrations helped them understand real life applications of the topics [1].
Table 2. Results from Force Demonstration and Homework Calculations Compared.

<table>
<thead>
<tr>
<th>Angle (degrees)</th>
<th>Distance from Joint (cm)</th>
<th>Measured/Observed Force (N)</th>
<th>Calculated Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>6.7</td>
<td>7.2</td>
<td>11.34</td>
</tr>
<tr>
<td>90</td>
<td>6.7</td>
<td>9.5</td>
<td>10.89</td>
</tr>
<tr>
<td>90</td>
<td>6.7 &amp; 27.9</td>
<td>0.8 &amp; 4.4</td>
<td>2.37 &amp; 5.57</td>
</tr>
<tr>
<td>110</td>
<td>6.7</td>
<td>10.4</td>
<td>12.33</td>
</tr>
</tbody>
</table>

The equations used to get the calculated muscle forces in table 2 are shown below. Equation 3 is equation 1 solved for the force exerted by the biceps. The force experienced by the joint was then calculated using equations 4 and 5 utilizing the fact that the model was a static model therefore the sum of the forces was equal to zero. Figure 9 also shows the calculations used to obtain the last column of data in table 2.

\[
\Sigma \text{Moments} = 0 \quad \text{Equation 1}
\]
\[
0 = -L_1(W_1) - L_2(W_2) + L_3(F_b) \quad \text{Equation 2}
\]
\[
F_b = \frac{L_1(W_1) + L_2(W_2)}{L_3} \quad \text{Equation 3}
\]

\[
\Sigma \text{Forces} = 0 \quad \text{Equation 4}
\]
\[
F_b = W_1 + W_2 + F_j \quad \text{Equation 5}
\]
\[
F_b = W_1 + W_2 + F_j - F_2 \quad \text{Equation 6}
\]
\[
F_j = F_b - W_1 - W_2 \quad \text{Equation 7 (manipulation of Equation 5)}
\]

Variables Defined/Explained:
- \(F_b\) = Force Exerted by Biceps
- \(F_j\) = Force experienced by elbow joint
- \(W_1\) = Force of forearm (for the demo was 1.13N)
- \(W_2\) = Force of weight in hand (for demo was 1.26N)
- \(L_1\) = Length to middle of forearm (for demo was 20 cm)
- \(L_2\) = Length to end of forearm (for demo was 40 cm)
- \(L_3\) = Length from joint to string attachment point (shown in column 2 of table 2)
- \(F_2\) = Force experienced by second load cell attachment for two muscle demo
Conclusions

Both the model of the arm and the goniometer models were successfully constructed and utilized to measure forces applied to muscles under different anatomical set-ups and measure angular kinematics involved in multiple joint movements respectively. The measured kinematics data was imported and analyzed in MatLab to yield derivations based on the principles relating change in angle, angular velocity, and angular acceleration as well as range of motion calculations.

From a learning standpoint, the students expressed through survey results that demonstrations did help improve their learning experiences in multiple different ways. Figure 6 shows that visual demonstrations do help students within the classroom, even if it is just seeing the material reiterated one more time. The classroom demonstrations enhanced students’ understanding of force load bearing, the relationships between angular kinematic principles, and MatLab coding software.

Table 2 shows that in the theoretical calculations/homework, the 70 degree position caused the bicep to exert more force to keep the model static; however, the model readings showed that as the angle increased, the force exerted by the bicep increased. This demonstrates the flaws of only using calculated data versus using real physiological or demonstration data. In humans, so many muscles and joints have influence on each individual movement of the body that strict calculations, no matter how careful, can never include all potential variables.

Figure 5 shows the limitations of deriving instead of utilizing a software that automatically plots the output graphs. Due to the limitation of the time sampling step, with each derivation, the derived plot became less accurate. Increasing the sampling rate would increase derivation accuracy, but also increase the already large export file created by the program. In the future, demonstrations could be done with higher sampling rates to determine the ideal sampling size to file size ratios for exportation.
In the exit survey, students had the option to add additional comments on the demonstrations. Several students mentioned that the demonstrations helped given them a mental picture of what was happening. Students expressed a desire to have more interaction with the models and the opportunity to use them, which will hopefully become a reality in the near future. Another comment mentioned creating a YouTube video, which would definitely be a consideration for future demonstrations.

Overall, the kinematics portion of the course ranked the most difficult based off of figures 8a and 8b. The arm model demonstration proved to be more helpful according to the survey results. From these observations, if only one demonstration was to be done in class in the future, the arm model of forces demonstration may prove to be more beneficial and interesting to students. The objectives of this study included engaging students in a new, more active way, while encouraging their interest in the STEM fields, and allowing them to see data collected that could be applied in the homework. All of these objectives were met based on the survey results and homework assigned.

Future Directions
In the future, biomechanics has lots of potential for expanding and modifying the above demonstrations to further enhance the educational experience of the students. One demonstration that has potential includes combining both the goniometer and the load cells to create a pendulum set up. This would allow students to view the changes in the force exerted as the pendulum position moves.

The human structures set is very versatile in that it can be constructed into multiple different body parts. For this study, only the arm setup was used, but in the future all three models could potentially be built and used in multiple demonstrations.

The goniometer has endless potential possibilities due to its ability to be attached at the hip/back joint, knee joint, or elbow joint. One real world application includes measuring a student’s gait and then comparing this control measurement to a measurement of their gait as they use different types of crutches. The comparison between these three measurements will give students insight into some of the ways that crutches inhibit movement. It will also allow them to see which types of crutches are more physiologically similar to actual human gait. Purchasing a second goniometer would allow students to monitor two joints at once, such as the knee joint and hip/back joint giving it many applications left to be explored.

Acknowledgements
This thesis study was performed under the guidance of Dr. Michelle Kim. Special thanks to the Biomedical Engineering Department of the University of Arkansas and Dr. Kim and for supplying the means to obtain the equipment needed. A special thank you to Dr. Kevin Hall from Civil Engineering for suggesting the PASCO system. I acknowledge the Physics Department for purchasing the University of Arkansas site-wide license for the CAPSTONE software. I would like to acknowledge my study abroad trip to Jonkoping, Sweden for sparking my interest in biomechanics demonstrations after visiting a prosthetics laboratory at Jonkoping University and seeing their cutting edge technology.
References


Appendix

MatLab Code Table of Contents

Load Data into MatLab ................................................................. 1
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Accelerations Compared ............................................................ 4
Overall Comparison ................................................................. 5

%Pasco Arm Demo
%Written by Bethany Knight
close all
clear all
clc

Load Data into MatLab

arm=csvread('arm_biomech_hw.csv',2,0);
time=arm(:,1); %assigns first column of arm data as time
angle=arm(:,2)-1; % the minus one centers data around 0
h=0.05; %time step determined by sampling frequency
%sampling frequency was 20Hz

figure (1)
title('Orig Data')
hold on
plot(time,angle)
plot(time,arm(:,3)) %uses velocity data collected by demo material (column 3)
plot(time,arm(:,4)) %uses acceleration data collected by demo materials (column 4)
legend('angle','vel','acc')
xlabel('Time (s)')
hold off

vel=diff(angle)/h; %divide by time step
vel(end+1)=0;

figure (2)
subplot(2,2,1)
plot(time,angle)
title('Angle vs Time')
xlabel('Time (s)')
ylabel('Angle (rad)')

subplot(2,2,2)
plot(time,vel)
title('Velocity vs Time Derived')
xlabel('Time (s)')
ylabel('Velocity (rad/s)')

acc = diff(vel) / (3*h);
acc(end+1) = -10; % allows matrices to agree

subplot(2,2,3)
plot(time,acc)
title('Acceleration vs Time Derived')
xlabel('Time (s)')
ylabel('Acceleration (rad/s^2)')

subplot(2,2,4)
hold on
plot(time,angle)
plot(time,vel)
plot(time,acc)
legend('angle', 'vel', 'acc')
title('Angle, Velocity, Acceleration vs Time Derived')
xlabel('Time (s)')
hold off

legend

Orig Data

Time (s)
Calcs

\[
\text{maxang} = \max(\text{angle}) \times 180/\pi \\
\text{minang} = \min(\text{angle}) \times 180/\pi \\
\text{ROM} = 2 \times (\text{maxang} - \text{minang}) \quad \text{range of motion in degrees. Equation from} \\
\text{Lecture 5 Biomechanics}
\]

\[
\text{maxang} = 68.7549
\]

\[
\text{minang} = -17.1887
\]

\[
\text{ROM} = 171.8873
\]
Velocities Compared

```matlab
figure (3)
plot(time,vel,time,arm(:,3))
legend('derived','measured')
title('Velocities Compared')
xlabel('Time (s)')
ylabel('rad/s')
```

![Velocities Compared](image)

Accelerations Compared

```matlab
figure (4)
plot(time,acc,time,arm(:,4))
legend('derived','measured')
title('Accelerations Compared')
xlabel('Time (s)')
ylabel('rad/s^2')
```
Overall Comparison

```matlab
figure (5)
hold on
plot(time, acc)
plot(time, arm(:,4), '-*')
plot(time, vel)
plot(time, arm(:,3), '-*')
plot(time, angle)
title('Derived and Measured')
legend('der acc','measured acc','der vel','measured vel','angle')
xlabel('Time (s)')
hold off
```
Published with MATLAB® R2015b
Biomechanics of the Arm

Joint and Muscle Forces
For the purposes of this demo, we are going to assume the bicep tendon is directly attached to the radius bone, which as you can tell from this image, is an appropriate assumption.
For the demo, our “control” is the muscle attached at 6.7 cm from the elbow joint with the weight being held at a 90 degree angle. The attachment points remain the same, but the angle of the arm relative to the joint changes. How will this end up impacting the forces?
How do you think changing the angle will affect the amount of force on the bicep muscle and the joint?
By not having a solely perpendicular force, the overall force of the muscle must be greater. The force the muscle exerts in parallel does not help keep the arm static.
How do you think the forces would change if the attachment point of the muscle was moved instead of changing the angle?
How does changing the point of attachment to the radius affect the amount of force the muscle must use to maintain static equilibrium?
Arm Muscles

The major flexor muscles of the elbow.

- Humerus
- Brachialis
- Radii
- Ulna
- Brachioradialis
- Recap: brachi
Two Muscle Attachment

How would adding another muscle change the force distribution of the arm? Would this be more or less beneficial for the joint?
Limitations:

• Why would the data obtained from the load cells potentially be different from theoretical calculations using the model’s parameters?
• What are some differences/limitations when comparing the forces the model experiences and forces experienced by a human elbow and bicep?
Lecture 5. Dynamics—Kinematics

Dr. Michelle Kim
January 24, 2014

Example: elbow

Elbow is flexed to a right angle and an object is held in hand. Determine the magnitudes of the muscle tension and the joint reaction force at the elbow.

Assumptions:
1. Biceps is the major flexor
2. Line of action of the tension in the biceps is vertical
3. O is axis of rotation of joint, assumed to be fixed
4. A is the attaching point of the biceps muscle
5. Joint reaction force points downward

Given:
- a = 4 cm
- b = 15 cm
- c = 35 cm
- W = 20 N
- W_o = 80 N

Elbow Problem

Things to consider
1. The joint reaction force is pretty large. What helps reduce the force?
2. The force exerted by the biceps muscle is also pretty large. What helps reduce the force?

Another Problem

Consider an athlete wearing a weight boot, and from a sitting position, doing lower leg flexion/extension exercises to strengthen quadriceps muscles.

- \( W_1 \) = weight of lower leg = 50 N
- \( W_2 \) = weight of the boot = 100 N

O: the knee joint
A: center of gravity of lower leg, located a = 20 cm from O
B: center of gravity of boot, located b=50 cm from boot.

Determine the net moment generated about the knee joint when the lower leg is at positions 1, 2, and 3, 0°, 30°, 60°, and 90° from the horizontal.
**Torque (Moment)**

- Tendency of a force to rotate about an axis
- Vector quantity
- \( \vec{\tau} = \vec{r} \times \vec{F} \)
- Magnitude \( \tau = rF \sin \theta \) or \( \tau = rF_\perp \)
- Cross product of vectors result in a vector in perpendicular direction to the plane of the other two vectors

![Torque Diagram]

**Dynamics**

- **Study of bodies in motion**
- **Kinematics**: Study of motion with geometry and time-dependency without regard to the forces that cause that motion
  - Analysis based on displacement, velocity, acceleration
- **Kinetics**: Incorporates effect of forces and torques that cause motion
  - Analysis based on Newton’s 2\(^{nd}\) law

**Dynamics Terms**

- **Distance** \( d \): the total length of the path of movement (scalar)
- **Displacement** \( \vec{x} \): line segment with direction between initial and final position of a moving body (vector)
- **Velocity** \( \vec{v} \): time rate of change of position (vector)
- **Speed**: magnitude of velocity vector (scalar)
- **Acceleration** \( \vec{a} \): time rate of change of velocity (vector)
Linear Kinematics

\[
\begin{align*}
  v &= \frac{dx}{dt} \\
a &= \frac{dv}{dt} = \frac{d^2x}{dt^2}
\end{align*}
\]

**Constant Acceleration**

- \(a = a_0\)
- \(v = v_0 + a_0 t\)
- \(x = x_0 + v_0 t + \frac{1}{2} a_0 t^2\)

**Constant Velocity**

- \(a = 0\)
- \(v = v_0\)
- \(x = x_0 + v_0 t\)

Examples: Linear Kinematics

A skier is descending a slope with constant acceleration of \(a_0 = 2 \text{ m/s}^2\) and the speed of skater at position 0 is observed to be 10 m/s. Calculate the speed \(v\), of skier when skier is at position 1, at a distance 100m from position 0. Also, calculate the **time it took** for skier to cover the distance between positions 0 and 1.

\[\text{Time} = \int_{t_0}^{t_f} \frac{1}{v} \, dt\]

Polar Coordinates

- 2D angular motions are commonly described in polar coordinates
  - \(r\) = radius or radial coordinate
  - \(\theta\) = polar angle, angular coordinate

Conversion to cartesian (rectangular coordinate)

- \(x = r \cos \theta\)
- \(y = r \sin \theta\)

Conversion from cartesian

- \(r = \sqrt{x^2 + y^2}\)
- \(\theta = \arctan \left( \frac{y}{x} \right)\)

Angular Kinematics

- **Angular displacement** \(\theta = \theta_2 - \theta_1\)
- **Arc length** \(s = r\theta \quad (\theta \text{ in radians})\)
- **Angular velocity** \(\omega = \frac{d\theta}{dt}\)
- **Average angular velocity** \(\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}\)
- **Angular acceleration** \(\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}\)
- **Average angular acceleration** \(\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}\)
- **Units?**
  - **Displacement:** unit-less
  - **Angular Velocity:** Radians/Second : 1/s : Hz
  - **Angular Acceleration:** Radians/Second^2 : 1/s^2

Appendix 17
**Joint angles** exhibited by seven collegiate softball players during fast-pitch strikes

**Joint velocities** exhibited by seven collegiate softball players during fast-pitch strikes

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**Example: Simple Harmonic Motion**

- Consider a simple pendulum (mass attached to string)
- Ignore friction and air resistance for now

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**Example: Simple Harmonic Motion**

- **Period** of harmonic motion $T$: time taken to complete one cycle
- **Range of Motion (ROM) $\theta$**: total angle covered by the pendulum ($= 2\theta_0$ here)
- Angular position of pendulum $\theta_0 = \theta_0 \cos(\omega t)$
- Angular frequency $\omega = \frac{2\pi}{T}$ (rad/s)
- Frequency $f = \frac{1}{T} = \frac{\omega}{2\pi}$
- Determine the angular velocity $\omega$ and angular acceleration $\alpha$ of the pendulum
Example: Shoulder Abduction

- Assume symmetric motion with respect to line OA
- Derive the expressions for the angular displacement, velocity, and acceleration of the arm. Take the period of angular motion of the arm to be 3s and the angle $\theta_0$ to be $80^\circ$

Rotation about a fixed axis: Angular to Linear

- Tangential or linear velocity
  \[ v = \frac{ds}{dt} = r\omega \]
  - In direction perpendicular to axis of rotation

- Tangential acceleration $a_t = \frac{dv}{dt}$
  - In direction perpendicular to axis of rotation

- Normal (radial) acceleration $a_n = \frac{v^2}{r}$
  - Or centripetal acceleration: in direction toward the axis of rotation
  - Takes into account the direction change (i.e. there is normal acceleration even when $v$ is constant)
**Resultant Linear Acceleration**

Since the tangential acceleration and the normal acceleration are **orthogonal** (perpendicular), the magnitude of the **resultant linear acceleration** can be found using the Pythagorean Theorem:

\[
\vec{a} = \vec{a}_t + \vec{a}_n
\]

Magnitude \(a = \sqrt{a_t^2 + a_n^2}\)

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**Example:**

- A windmill-style softball pitcher executes a pitch in 0.65s. If her pitching arm is 0.7m long:
  - What are the magnitudes of the **tangential** and **radial accelerations** on the ball just before the ball release, when tangential ball speed is 20 m/s?
  - What is the magnitude of the **total acceleration** on the ball at this point?