12-2014

Hospital Discharge Decision Modeling for Mental Health Inpatients

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This thesis is approved for recommendation to the Honors College.

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An Undergraduate Honors College Thesis

in the

Department of Industrial Engineering
College of Engineering
University of Arkansas
Fayetteville, AR

by

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Abstract

This research addresses the issue of unnecessary treatment and late discharge for mental health inpatients. In this thesis, a dynamic decision model is proposed to determine the optimal timing to discharge a patient from hospital that considers both the impacts of over-treatment and not enough treatment on patients' recovery. We measure the reward of making the discharge decisions in terms of hospitalization cost and readmission cost. The objective is to minimize the total expected cost for patients and provides hospital care providers with theoretical methods to assist them to make a better decision about discharge planning and management.

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1. Introduction

Mental disorders, especially depression, are severe diseases that affect a significant number of people worldwide every year. Depression is estimated to be the leading cause of disability worldwide, and the fourth largest global burden of disease, according to the World Health Organization. It is thus particularly important to be able to describe the epidemiological dynamics of mental health problems. Better descriptive models may help assist in understanding the relationship between provision of treatment and population health status (Patten and Lee, 2003). In principle, such models could inform decision-makers about the potential impact of primary prevention strategies as well; on a practical level, models capable of describing the mental health problem may prove useful for supporting rational decision-making and resource allocation within health care systems. (Patten and Lee, 2003).

Effective detection and management of mental health problems have been identified as priority areas within the national service framework for preventative mental health and reduction of suicide rate. A Markov chain model provides a method to evaluate sequences of information in various situations, and it can be applied to analyze and predict the transition probabilities from one state to another. The future emotional state can be predicted by knowing the individual's current mental status. This approach can provide hospital health care providers with suggestions about when to discharge a hospitalized patient based on his or her current state, which is the optimal timing from both the patient's and hospital's perspectives. It is expected that the proposed approach can stimulate productive research to address the prevention and treatment of mental illnesses in order to improve theoretical methods and interventions in mental health research.

There are three phases for the treatment of depression: detection, in-hospital treatment and out-patient follow-up after the patient is discharged from the hospital. For the first phase: detection, the mental health first aid (MHFA) is mostly used and most beneficial at the early stage of the mental issue. MHFA teaches individuals to be aware of early symptoms of depression, and provides early stage patients with plans on assessing help and possible medical treatment. It also teaches the patients to better access local resources and know where to seek for help. At the early stage, patients have a very high chance to self-recover by means of simple consultation and medical treatment.

If a patient's mental health situation gets worse due to late-detection or other various reasons, medicine alone at this state cannot control the patient's condition; admitting to a hospital to receive treatment would be a better choice for the patient. For inpatients, they need regular and continuous treatment, but to certain point, they can be discharged from hospital and visit their primary care physician instead or they don't need treatment any more if completely recovered. From the patient's perspective, staying too long in hospital is not beneficial for their health concern, which probably can add more mental issues such as additional stress that impedes rehabilitation. On the other hand, if the patient is discharged too early from hospital, he/she cannot receive enough required treatment, which slows recovery and may lead to readmission at a later time. In addition, early discharge can sometimes result in worse patient conditions. From the hospital's perspective, discharging patient too early can be seen as irresponsibility to patients; if too late, shortage of beds for patients in more serious conditions is a waste of resources. Discharge decisions should be made considering these aspects in order to achieve the greatest benefits for both the patient and the hospital.

For outpatients, if they are asked by doctors to see specialists, they need to receive treatment in the special clinics or hospital outpatient department. At certain point the patient may just need drug prescriptions and active treatment is no longer needed. At this stage, making regular appointments with primary care physicians would be enough for follow-up care.

In this thesis, we focus on the inpatient treatment phase. The goal is to identify an optimal timing for discharging a patient based on his/her condition at the time. We develop a Markov decision process (MDP) model that provides a way to analyze the severity of patient's health conditions and the natural development of the disease without treatment. The results from the MDP model can also provide suggestions on advising patients to receive treatment at the right time based on the reward system.

From the patients' point of view, since they pay for their treatment, they deserve a complete and effective medical care. It would be ruthless and immoral for the hospital to discharge the patient during convalescent period. The hospital also has its own concern: for example the hospital bed demands sometimes exceed capacity, leading to delays in patient admission, transfers and cancellations of surgical procedures. The current tool available is a web-based software application called "Patient Tracker" which assists care providers to manage the discharge process, minimize delays in admission and reduce surgical procedure cancellations (Maloney, 2007). This is a tool which can be applied generally for all different departments in the hospital; therefore it also can be used for mental health problems. In this thesis, a new method is introduced using an MDP model.

MDP provides a mathematical framework for modeling decision making in different situations which helps decision makers to optimize the problem solution. In this case, MDP method provides a way of building mathematical models for doctors to decide the proper action a in different state S. After the action is taken, a corresponding reward $R_a(S)$ is obtained, the patient may move to another state S'. In this study, four states of a patient's mental health conditions are considered: S1- not depressed but have the symptoms, S2- mildly depressed, S3- moderately depressed, and S4- severely depressed. For each state observed, two possible discharge actions can be made: A1- discharge the patient, or A2- do not discharge the patient. A reward is calculated based on different states and the action taken. The optimal decision can be derived based on the total expected rewards that consider possible transitions among health states in the future. Therefore, this method can help make decisions of whether or not to discharge patients at different states, and further observations will be allowed for health providers to keep track of a patient's mental health conditions and make different decisions whenever needed.

Stochastic models for this system evaluation in mental health are especially applicable due to the large amount of variability present in the psychodynamic processes of individuals. The model utilizes sequence of observations rather than fixed point observations in order to reflect patients' psychological transitions. Chassan comments on how statistical models should capture the psychological processes of individuals: "not only is the patient to be described in terms of a multidimensional or multivariate probability distribution, but the study of change in the patient-state, or the evaluation of a patient's progress, or lack of progress, in the course of time in relation to any program of therapy, is then to be performed in theory by comparisons of individual variability. Such variability is the basis of defining the patient state in terms of statistical distributions, or probabilities, estimated from the sequences of observations of a given patient, and consequently provide the framework for the application of statistical inference and experimental design to the data of the individual patient." (Shachtman and Feuer, 1980)

Besides variability, patients' interactions may violate the independence of observations assumption that would have an impact on the accuracy of evaluation. These problems are especially troublesome in hospital ward settings where patients have close and continual contacts. Applying the stochastic view of modeling, patients' interactions are part of the psychopathological process and should be included, not excluded from the study.

Although this work is preliminary and needs further investigation, it provides a modeling framework that considers new discharge standard development, explores the structural components of this model, and calculates the intended reward for discharge patients at various states. In addition, due to the limitation on the input data, the results may not apply to all regions.

2. A Markov Decision Process Approach

MDP models provide a mathematical modeling framework for optimizing the discharge decision of inpatients as these models are well suited to determining the optimal timing for decision to be made under different states and under unpredictable uncertainties. Although MDP was traditionally applied to problems in inventory control and machine maintenance, MDP can also be valuable when applied to modeling population harvesting, agriculture, finance investment, etc. (D. White). Nowadays, MDPs have increasingly been applied to modeling medical treatments. The model in this paper is built on MDPs framework to analyze discharge vs. not discharge decision making for inpatients with mental health issue. The transition probabilities of different health condition states are incorporated in this model to better analyze and model the system. Rewards in this model are used to evaluate the conditions of different patients, knowing the potential trend of how the patients' health conditions would develop;

doctors can make more reasonable and appropriate decisions of whether to keep or discharge the patient. The rewards considered in this study include the cost based on readmission probability of different health state; the cost of keeping patients in hospital instead of treating another more severe patient, and the cost of early discharge.

The model incorporates a patient perspective that the decision of discharge or not will directly affect the patient's health conditions. On the other hand, the decision maker in this system is assumed to be the care providers. In this model, both progression and steadiness are considered, as can be seen in Figure 1, at every decision epoch, a patient undergoes the examination to evaluate his or her current health condition; meanwhile, doctor is capable of making the right judgment according to which state the patient is at. If at a certain time epoch, the doctor decides not to discharge the patient, the total reward in this case would be the current reward plus the future reward; while if the doctor decides to discharge the patient at certain state, the patient leaves the model the reward for the patient will be calculated as a lump sum reward.

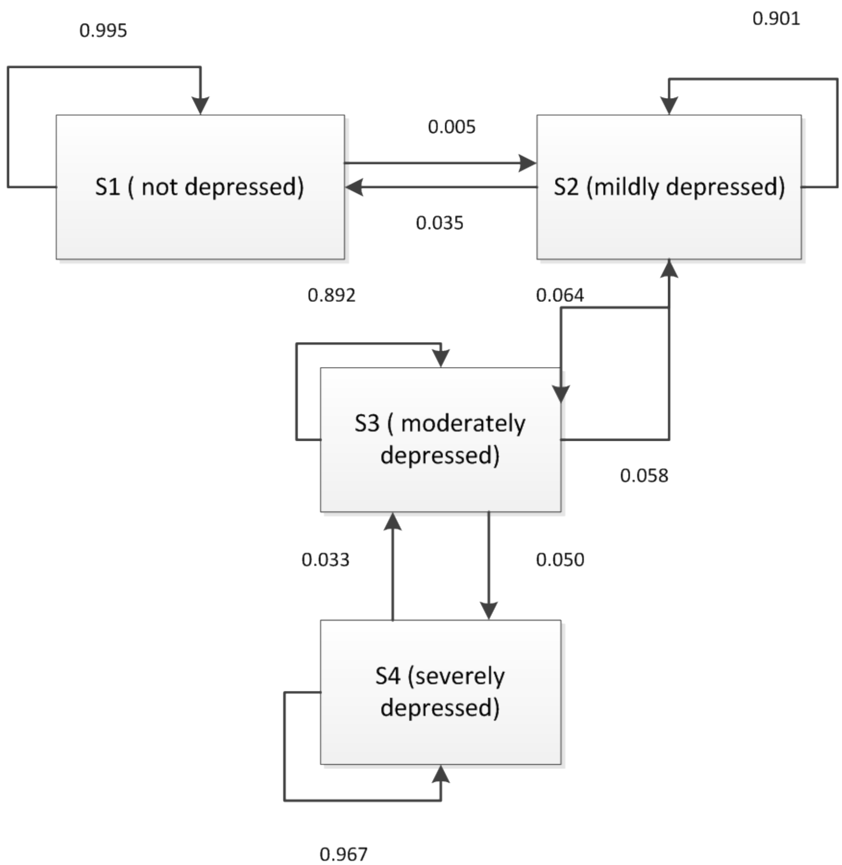

Figure 1: Four states Markov chain for mental health patient

Figure 1 gives an idea of how the Markov chain actually works in this system. In this model, a state transition from day t_k to day t_{k+1} may represent disease progression, patient recovery or health conditions unchanged.

A sensitivity analysis is performed to evaluate the relationship between input and output variables in this system; meanwhile, it can be used to test the robustness of the results of a model in the presence of uncertainty. In this case, sensitivity analysis is very useful when attempting to determine the impact the actual outcome rewards by changing the parameters.

Given all the particularities above, an MDP model can be defined with the following components:

- Set of decision epochs T = {1, 2, 3….N}, N is the longest day a mental health patient stays in hospital, after that, the patient will be either discharged or transferred to other facilities (e.g., nursing homes).
- Set of different states S= {1, 2, 3, 4}, where the mental health condition of the patient at decision epoch t is defined as S_t, $s_t \in S, \forall t \in T$. 1 represents the not depressed but have symptoms state, 2 represents the mildly depressed state, 3 represents the moderately depressed state, and 4 represents the severely depressed state.
- Set of actions A= {1, 2}. Where 1 represents discharge the patient, 2 represents not to discharge patient. In this model, for each decision epoch, no matter what health condition the patient is, the care provider always needs to make a decision of either discharge the patient or keep the patient in hospital for another day. The reward score of each decision making in different decision epochs and current state of the patient can be a reference for doctors to make an appropriate decision.
- Transition probability matrix P (a_t), $\forall t \in T$. According to (Fisher and Knesper, "Markov models and the utilization of mental health services: A study of endogenously depressed patients",1983), the transition probability matrix used in this model aims at quantitatively describe the health condition trend and how the progression works within different states. Here, a patient condition can become better or worse, but not with dramatic change, i.e., a moderately depressed patient will not become "not depressed" in one step transition. The complete transition probability matrix is shown below.

$$P(a_t) = \begin{bmatrix} p_{11}(a_t) & p_{12}(a_t) & 0 & 0 \\ p_{21}(a_t) & p_{22}(a_t) & p_{23}(a_t) & 0 \\ 0 & p_{32}(a_t) & p_{33}(a_t) & p_{34}(a_t) \\ 0 & 0 & p_{43}(a_t) & p_{44}(a_t) \end{bmatrix}$$

- Immediate rewards $r(s_t)$, $\forall s_t \in S$. At every decision epoch, if the care provider makes the decision of not discharging the patient, the immediate reward is the amount of cost for the patient to stay in hospital for one more day.
- Lump- sum reward W_s is applied when a discharge decision is made; this reward varies with the possibility of readmission at different states.
- Cumulative reward is the sum of the rewards based on the decision made at the future epochs.

$$\text{CumReward}_t(S_{t,}) = \sum_{S_{t+1} \in S} V_{(t+1)}(S_{t+1}) * P_t(S_{t+1} \mid S_t)$$

- All possible value is the variable used in Matlab to operate the sum of current reward and cumulative rewards. It is only applied when action 2 – not discharge patient at day t_k is made

$$\text{AllPossibleValue}(t, S_t) = r_t(s_{t,}) + \sum_{S_{t+1} \in S} V_{(t+1)}(S_{t+1}) * P_t(S_{t+1} \mid S_t)$$

- Chosen reward is used in the model to choose the minimum value by comparing the lump- sum reward if discharge action is made and AllPossibleValue if the action not discharge is made at time t_k. It is displayed in a matrix format to summarize appropriate decisions to make at each different time epoch and depends on the patient's health status at that specific time.

$$\text{chosenreward}(t, S_t) = \min\{[W(s), [r_t(s_t) + \sum_{S_{t+1} \in s} V_{(t+1)}(S_{t+1}) * P_t(S_{t+1} \mid S_t)]\},$$

The equation above is equal to: chosenreward (t,S_t) = min (W(s), AllPossibleValue (t,s,)).

- Inside of this equation, V (t+1) (S_{t+1}) is the chosenreward for patient on day t_{k+1} when patient is in state S_{t+1}.
- $P_t(S_{t+1} \mid S_t)$ is the probability the patient will be at state S_{t+1} on day $t_{k+1,}$ given the patient's state S_t at the current decision epoch t.

- In order to display the best action to take at each step in terms of time epoch and patient's status, OptAction is the output variable to indicate the optimal actions at different time epochs and states that would lead to the highest reward scores.

  OptAction (t,s) = 1 or 2, t$\in$ $TotalEpoch\ T$, s$\in$ S

3. Numerical Experiment

According to the OECDiLibrary (http://www.oecd-ilibrary.org), the average length of stay in hospital for patients has fallen over the past decades in nearly all countries, from 8.2 days in 2000 to 7.2 days in 2009 in general. In this model, the maximum number of days in hospital is assumed to be 30, thus T = {1,2,…,30}.

In order to find the solution to this finite-horizon MDP model, we use the backward induction algorithm. It is assumed the patient will be discharged or transferred at time N = 30 days after admission. A lump sum reward is calculated for different health status at the time when discharge decision is made, and rewards for do not discharge are calculated using AllPossibleValue variable introduced.

In this hospital discharge model, all the rewards are converted to dollar per patient; In order to find out the best action to take, the smaller amount of cost is better.

Table 1 summarizes the transition probabilities from a state on day t_k to a state on day t_{k+1}. In the table, S1 stands for not depressed but have symptoms of getting worse, S2 means mildly depressed, S3 represents moderately depressed, and S4 is for the severely depressed state. Table 1 depicts the transition probabilities before considering progression of patients' conditions as an input factor, this table is based on the action of not discharge the patient on day $t_{k+1,}$ so the doctors can follow up the health condition changes of the patient.

Table 1 Transition probability Table

		State on Day t_{k+1}			
		S1	S2	S3	S4
State on Day t_k	S1	.995	.005	0	0
	S2	.035	.901	.064	0
	S3	0	.058	.892	.050
	S4	0	0	.033	.967

According to the paper "Expenditure on specialized mental health services" (MHSA), the $1.9 billion of recurrent expenditure for public sector specialized mental health hospital services during 2011 – 2012 equate to an average cost per patient day of $887. In this hospital discharge model, the same amount of money can be applied to the cost of stay for each patient on each day. Since there are four different levels of severity in the model, it is assumed that it would cost hospital $887 to keep a mild state patient in hospital for one day; the cost is assumed $787 for severity level 1 patient, $987 for state 3 patient, and $1087 for state 4 patient.

Current reward r (1) = $787, r (2) = $887, r (3) = $987, r (4) = $1087.

There is a tradeoff between keeping a patient in the hospital for one more day and discharge a patient to give bed to another one who needs treatment. If the former action is taken, there is a chance that actually the patient has been treated well enough that is ready to be discharged, but keeping him/her in the hospital for one more day can result in a cost of one more day of stay; if the latter action is taken, there is a possibility the patient who is discharged didn't receive enough treatment that he/ she will be readmitted again in the near future.

The potentially preventable readmission is another cost considered in the model. Lindsey and Patterson discussed the idea of potentially preventable readmission (PPR). Not all hospital readmissions can be prevented, there is a growing consensus that with appropriate inpatient care, post-surgical follow- up and outpatient care, a substantial number are potentially preventable. The most frequent conditions associated with PPRs were mental health and/ or substance abuse, particularly among Medical fee-for-service (FFS) recipients.

Table 2 below displays the total and average costs associated with potentially preventable readmissions (PPR) by region, Medicaid recipient health condition, and Medicaid payment category.

Table 2 PPR cost in New York

	New York City			Rest of the State			New York State		
Fee-for-Service									
Recipient Health Condition	Total PPR Cost	%	Average Cost per PPR	Total PPR Cost	%	Average Cost per PPR	Total PPR Cost	%	Average Cost per PPR
Mental Health	$116,482,748	76.2	$16,154	$36,346,399	23.8	$8,972	$152,829,147	100.0	$13,570
Substance Abuse	$52,101,548	79.6	$11,312	$13,346,674	20.4	$9,186	$65,448,222	100.0	$10,802
Mental Health and Substance Abuse	$227,936,192	77.8	$11,740	$64,875,061	22.2	$7,557	$292,811,253	100.0	$10,457
All Others	$67,292,285	75.8	$14,336	$21,511,948	24.2	$10,071	$88,804,233	100.0	$13,002
Total	$463,812,773	77.3	$12,910	$136,080,082	22.7	$8,387	$599,892,856	100.0	$11,503
Managed Care[1]									
Recipient Health Condition	Total PPR Dollars	% of Total Dollars	Average PPR Dollars	Total PPR Dollars	%	Average Cost per PPR	Total PPR Cost	%	Average Cost per PPR
Mental Health	$40,926,371	81.8	$16,215	$9,086,600	18.2	$8,900	$50,012,971	100.0	$13,614
Substance Abuse	$19,270,436	76.3	$11,471	$5,996,332	23.7	$9,169	$25,266,767	100.0	$10,905
Mental Health and Substance Abuse	$53,466,514	69.0	$12,075	$23,994,886	31.0	$7,452	$77,461,400	100.0	$10,177
All Others	$48,728,896	80.8	$14,141	$11,583,357	19.2	$9,909	$60,312,253	100.0	$12,828
Total	$162,392,217	76.2	$13,445	$50,661,174	23.8	$8,354	$213,053,390	100.0	$11,744

[1]Managed care costs are estimated based on fee-for-service costs.

As can be seen from the table, the average cost per PPR for the managed care mental health category in New York State is $13,614, this is the amount of money would cost the hospital to treat a readmitted patient again. This amount is considered in the reward function in this work.

Selection of readmission time interval has an important effect on the PPR rate. A longer readmission time interval will identify more readmissions. Longer time intervals after the initial readmission decreases the likelihood that a readmission was related to the clinical care or discharge planning in the initial admission and increase the relative importance of outpatient management of chronic illness. In this model, the possibilities of readmission changes in terms of a patient’s state, the PPR rate increased consistently as the severity increases. For readmissions to any hospital within 15 days the PPR rate increased more than threefold for medical patients and more than fourfold for surgical patients as severity increases from severity level 1 to 4. (Goldfield et al., 2007).

Table 3 below contains the top 10 largest number of PPR chains, by severity of illness (SOI) level. Three of the top 10 medical initial admissions were related to mental health: schizophrenia, major depressive disorder and bipolar disorders. A refined table 4 can be made to extract the information about the major depressive disorder. The average readmission rate for each severity level can be seen in Table 5, PPR rate for SOI 1 is 8.3%, SOI 2 is 12.6%, SOI 3 is 16.5%, and SOI 4 is 10.8%.

Table 3 : Top 10 Medical Candidate Admission with the Largest Number of PPR chain, by SOI Level

APR DRG	Medical	Number of PPR	All Patients	SOI 1	SOI 2	SOI	SOI 4
194	Heart Failure	Chains	15,053	1,304	8,151	4,675	923
		Rate	12.5	8.9	11.7	15.0	19.4
140	Chronic	Chains	8,271	1,737	3,745	2,416	373
		Rate	9.7	7.3	9.3	12.7	17.3
750	Schizophrenia	Chains	7,592	3,382	3,931	251	28
		Rate	17.7	17.1	18.1	20.8	16.8
139	Other	Chains	7,579	393	3,295	3,394	497
		Rate	7.7	2.7	6.5	11.4	16.4
751	Major	Chains	5,608	1,814	3,391	339	64
		Rate	10.9	8.3	12.6	16.5	10.8
198	Angina	Chains	5,151	1,414	2,685	982	70
		Rate	5.6	3.7	6.2	9.9	17.3
753	Bipolar	Chains	4,830	2,366	2,260	179	25
		Rate	14.0	12.7	15.3	18.8	11.6
720	Septicemia &	Chains	4,370	46	881	1,808	1,635
		Rate	12.6	3.6	8.3	12.7	19.3
460	Renal Failure	Chains	4,288	92	471	3,250	475
		Rate	12.8	11.0	10.6	12.5	21.1
201	Cardiac	Chains	4,066	898	1,950	1,070	148
		Rate	6.3	4.0	6.4	10.2	16.0
All Other Medical APR		Chains	41,412	8,036	15,942	13,011	4,423
		Rate	2.9	1.7	2.5	5.0	9.4
Total Medical APR DRG		Chains	108,220	21,482	46,702	31,375	8,661
		Rate	5.0	3.2	4.7	7.4	11.7

Table 4: Major Depressive Disorder PPR Chains/ Rate

Medical Description	Number of PPR chains/ Rate	All patient	SOI 1	SOI 2	SOI 3	SOI 4
Major Depressive disorder	Chains/ Rate	5608 | 10.9	1814 | 8.3	3391 | 12.6	339 | 16.5	64 | 10.8

As the average readmission rates for each different severity level were given, and the average cost to treat a readmitted patient was known, therefore, cost times corresponding probability at each state gives the amount of cost to treat a patient under certain health condition. The lump-sum reward scores under different states were calculated as following: w_1 = 8.3% * $13614 = $1129.96, w_2 = 12.6%* $13614 = $1715.36, w_3 = 16.5% * $13614 = $2246.31, w_4 = 10.8% * $13614 = $1470.31.

4. Results and Discussion

After coding in Matlab, when I run the model using all initially gathered data, the results are displayed as all OptAct (t,s) = 1, which means in this scenario, no matter what decision epochs or states patients are at, the optimal action to take for hospital is to discharge patient. As we can see from Table 5 below, there are 30 rows which represents 30 decision epochs, the 4 columns stand for 4 different states (from severity level 1 to severity level 4). The numbers in each cell (A/R) represent the optimal action (A) at each decision epoch and every state and the best rewards obtained (R). If at decision epoch t_k, the decision made by the care provider is not to discharge the patient, the reward is calculated as the sum of current reward which is the cost of one more day stay in hospital and future reward that is based on the chosenreward at the next time epoch t_{k+1}.

The reason that all optimal actions are 1 is the reward scores for not discharge are always greater than the reward of discharge, here in this case, the reward is considered as cost for hospital, hence the smaller reward, the better decision. It is necessary to do sensitivity analysis to check how input values can affect output values.

Another fact that can be figured out from the result is that the readmission rate in state 4 is smaller than previous states, which is not common. The reasons behind it can be patients at a severe health state are usually kept in hospital; there are seldom cases that patients would be discharged at very severe state; even if patients are discharged at state 4, some of them are unwilling to return for treatment, while some of them have very high possibilities to commit suicide.

Input variables that may be sensitive to the outcomes are readmission treatment costs which depend on the severity condition of patients, readmission rates which also depend on patients' state, and the immediate reward which is the cost per patient per day in hospital which is related to patients' states.

Table 5: Summary of Results

Decision Epoch/ States	**1**	**2**	**3**	**4**
1	1/1130	1/1715	1/2246	1/1470
2	1/1130	1/1715	1/2246	1/1470
3	1/1130	1/1715	1/2246	1/1470
4	1/1130	1/1715	1/2246	1/1470

5	1/1130	1/1715	1/2246	1/1470
6	1/1130	1/1715	1/2246	1/1470
7	1/1130	1/1715	1/2246	1/1470
8	1/1130	1/1715	1/2246	1/1470
9	1/1130	1/1715	1/2246	1/1470
10	1/1130	1/1715	1/2246	1/1470
11	1/1130	1/1715	1/2246	1/1470
12	1/1130	1/1715	1/2246	1/1470
13	1/1130	1/1715	1/2246	1/1470
14	1/1130	1/1715	1/2246	1/1470
15	1/1130	1/1715	1/2246	1/1470
16	1/1130	1/1715	1/2246	1/1470
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25	1/1130	1/1715	1/2246	1/1470
26	1/1130	1/1715	1/2246	1/1470
27	1/1130	1/1715	1/2246	1/1470
28	1/1130	1/1715	1/2246	1/1470
29	1/1130	1/1715	1/2246	1/1470
30	1/1130	1/1715	1/2246	1/1470

It is significant to change one variable one time to check the differences in results cause by that change. Also, in the experiment part, I made a lot of assumptions based on different severity levels, it is essential to test the sensitivity of those assumptions.

The first scenario I made is by changing readmission cost related to different states. The initial assumption I used is to apply the average cost to all different health conditions. The results of lump-sum rewards will be much different, by relating the costs to patients' states. Readmitted patients with low severity level are supposed to cost less when comparing with patients with high severity level. I assumed the average cost of $13614 is the cost for mental health patient with a mild state, for patients with symptoms to be depressive again, the cost is $12614, the cost for SOI 3 is $ 15614, for SOI 4 the cost is $18614. The cost differences between two continuous states vary; they are increased along with severity level. Table 6 displays the final result.

As we can see from Table 6, the rewards changed as I changed the readmission costs. The optimal actions are still all 1s, which means rewards for not discharge are greater than rewards for discharge. Therefore, the next step of change I made is increasing the amount of readmission costs by great amounts to test whether outputs are very sensitive to readmission costs. The reason I did this is to erase the great impact of immediate reward to the AllPossibleValue. Then I set the cost for SOI 1 is $30000, for SOI 2 is $40000, for SOI 3 is $60000, and for SOI 4 is $90000, the outputs are shown in Table 7.

In Table 7, the optimal actions are still all 1s, and rewards all go with the lump sum reward when discharge decisions are made. As a result, it seems no matter how big changes I did to the readmission cost, the lump sum rewards will always be the smaller values. I can conclude that the readmission cost is not sensitive in this model.

Table 6: Scenario 1 Sensitivity Analysis

Decision Epoch/ States	**1**	**2**	**3**	**4**
1	1/1047	1/1715	1/2576	1/2010
2	1/1047	1/1715	1/2576	1/2010
3	1/1047	1/1715	1/2576	1/2010
4	1/1047	1/1715	1/2576	1/2010
5	1/1047	1/1715	1/2576	1/2010
6	1/1047	1/1715	1/2576	1/2010
7	1/1047	1/1715	1/2576	1/2010
8	1/1047	1/1715	1/2576	1/2010
9	1/1047	1/1715	1/2576	1/2010
10	1/1047	1/1715	1/2576	1/2010
11	1/1047	1/1715	1/2576	1/2010
12	1/1047	1/1715	1/2576	1/2010
13	1/1047	1/1715	1/2576	1/2010
14	1/1047	1/1715	1/2576	1/2010
15	1/1047	1/1715	1/2576	1/2010
16	1/1047	1/1715	1/2576	1/2010
17	1/1047	1/1715	1/2576	1/2010
18	1/1047	1/1715	1/2576	1/2010
19	1/1047	1/1715	1/2576	1/2010
20	1/1047	1/1715	1/2576	1/2010
21	1/1047	1/1715	1/2576	1/2010
22	1/1047	1/1715	1/2576	1/2010
23	1/1047	1/1715	1/2576	1/2010
24	1/1047	1/1715	1/2576	1/2010
25	1/1047	1/1715	1/2576	1/2010
26	1/1047	1/1715	1/2576	1/2010

27	1/1047	1/1715	1/2576	1/2010
28	1/1047	1/1715	1/2576	1/2010
29	1/1047	1/1715	1/2576	1/2010
30	1/1047	1/1715	1/2576	1/2010

Table 7: Scenario 2 Sensitivity Analysis

Decision Epoch/ States	1	2	3	4
1	1/2490	1/5040	1/9900	1/9720
2	1/2490	1/5040	1/9900	1/9720
3	1/2490	1/5040	1/9900	1/9720
4	1/2490	1/5040	1/9900	1/9720
5	1/2490	1/5040	1/9900	1/9720
6	1/2490	1/5040	1/9900	1/9720
7	1/2490	1/5040	1/9900	1/9720
8	1/2490	1/5040	1/9900	1/9720
9	1/2490	1/5040	1/9900	1/9720
10	1/2490	1/5040	1/9900	1/9720
11	1/2490	1/5040	1/9900	1/9720
12	1/2490	1/5040	1/9900	1/9720
13	1/2490	1/5040	1/9900	1/9720
14	1/2490	1/5040	1/9900	1/9720
15	1/2490	1/5040	1/9900	1/9720
16	1/2490	1/5040	1/9900	1/9720
17	1/2490	1/5040	1/9900	1/9720
18	1/2490	1/5040	1/9900	1/9720
19	1/2490	1/5040	1/9900	1/9720
20	1/2490	1/5040	1/9900	1/9720
21	1/2490	1/5040	1/9900	1/9720

22	1/2490	1/5040	1/9900	1/9720
23	1/2490	1/5040	1/9900	1/9720
24	1/2490	1/5040	1/9900	1/9720
25	1/2490	1/5040	1/9900	1/9720
26	1/2490	1/5040	1/9900	1/9720
27	1/2490	1/5040	1/9900	1/9720
28	1/2490	1/5040	1/9900	1/9720
29	1/2490	1/5040	1/9900	1/9720
30	1/2490	1/5040	1/9900	1/9720

In this model, it is not meaningful to change either the transition probability matrix or the readmission rate. The next input variable I test sensitivity on is the current reward which is the immediate reward of one day cost per patient to stay in hospital. The original data I collected has a great effect on the total reward of not discharge action; therefore I dramatically decrease values of immediate rewards that helps to cut down total reward of action 2. As the original data values are too large, it is necessary to decrease them to a certain range. Afterwards, I altered the values by small values to test the sensitivity. In the first scenario, I chose immediate reward for SOI 1 patient is $5, for SOI 2 is $10, for SOI 3 is $15, and for SOI 4 is $20. The outputs are displayed in Table 8. In the second scenario, the immediate reward for SOI 1 patient is $10, for SOI 2 patient is $12, for SOI 3 patient is $16, and for SOI 4 patient is $22. The outcomes are shown in Table 9

It is obvious to notice that in these two scenarios, the more severe the patients are, the longer time they will be kept in hospital, and this fits the situation in reality. When comparing these two scenarios, for patients at state 2, the decision epoch to make discharge decision is different. We can tell even if there are only minor changes to the value of immediate rewards, the output results have a significant change. According to this fact, I concluded that the immediate reward is a sensitive input in this model.

Based on the previous sensitivity analysis tests I conducted above, it is clear that changing the immediate reward can cause a significant difference in the results, while, if we simply change the values of lump –sum reward, changes to the output are not too obvious.

5. Conclusion and Future Expectation

To decide the appropriate time to discharge mental health patients from hospitals is a common problem faced by health care organizations. Modelling this discharge decision as an MDP is a new approach, which results in more effective decisions considering the balance between the quality of health care service and utilization of limited hospital resources.

This research have several limitations: First, in the model, most reward considerations are from health care givers' perspective, incorporating with more data from the patient's perspective will make the model more realistic; second, as for discharge reward, the lump-sum rewards consider the readmission possibility, but other discharge outcomes should be included, which will make the reward system more accurate. With more accurate parameters, this method can provide valuable suggestions to health care organizations, and it has a great potential for general applications related to discharge decision making.

Table 8: Scenario 3 Sensitivity Analysis

Decision Epoch/ States	1	2	3	4
1	1/1330	2/1662	2/1745	1/1470
2	1/1330	2/1665	2/1749	1/1470
3	1/1330	2/1669	2/1753	1/1470

4	1/1330	2/1673	2/1757	1/1470
5	1/1330	2/1680	2/1762	1/1470
6	1/1330	2/1684	2/1767	1/1470
7	1/1330	2/1688	2/1772	1/1470
8	1/1330	2/1691	2/1783	1/1470
9	1/1330	2/1695	2/1790	1/1470
10	1/1330	2/1699	2/1797	1/1470
11	1/1330	2/1702	2/1804	1/1470
12	1/1330	2/1706	2/1813	1/1470
13	1/1330	2/1709	2/1822	1/1470
14	1/1330	2/1711	2/1832	1/1470
15	1/1330	2/1713	2/1843	1/1470
16	1/1330	2/1715	2/1855	1/1470
17	1/1330	2/1715	2/1869	1/1470
18	1/1330	1/1715	2/1885	1/1470
19	1/1330	1/1715	2/1902	1/1470
20	1/1330	1/1715	2/1922	1/1470
21	1/1330	1/1715	2/1943	1/1470
22	1/1330	1/1715	2/1968	1/1470
23	1/1330	1/1715	2/1995	1/1470
24	1/1330	1/1715	2/2026	1/1470
25	1/1330	1/1715	2/2061	1/1470
26	1/1330	1/1715	2/2100	1/1470
27	1/1330	1/1715	2/2143	1/1470
28	1/1330	1/1715	2/2192	1/1470
29	1/1330	1/1715	2/2246	1/1470
30	1/1330	1/1715	1/2263	1/1470

Table 9: Scenario 4 Sensitivity Analysis

Decision Epoch/ States	1	2	3	4
1	1/1130	2/1685	2/1761	1/1470
2	1/1130	2/1687	2/1764	1/1470
3	1/1130	2/1690	2/1768	1/1470
4	1/1130	2/1693	2/1771	1/1470
5	1/1130	2/1695	2/1775	1/1470
6	1/1130	2/1698	2/1779	1/1470
7	1/1130	2/1701	2/1784	1/1470
8	1/1130	2/1703	2/1789	1/1470
9	1/1130	2/1706	2/1794	1/1470
10	1/1130	2/1708	2/1800	1/1470
11	1/1130	2/1710	2/1806	1/1470
12	1/1130	2/1712	2/1813	1/1470
13	1/1130	2/1714	2/1821	1/1470
14	1/1130	2/1715	2/1830	1/1470
15	1/1130	1/1715	2/1839	1/1470
16	1/1130	1/1715	2/1850	1/1470
17	1/1130	1/1715	2/1862	1/1470
18	1/1130	1/1715	2/1876	1/1470
19	1/1130	1/1715	2/1891	1/1470
20	1/1130	1/1715	2/1908	1/1470
21	1/1130	1/1715	2/1928	1/1470
22	1/1130	1/1715	2/1949	1/1470
23	1/1130	1/1715	2/1973	1/1470
24	1/1130	1/1715	2/2000	1/1470
25	1/1130	1/1715	2/2030	1/1470
26	1/1130	1/1715	2/2064	1/1470

27	1/1130	1/1715	2/2102	1/1470
28	1/1130	1/1715	2/2145	1/1470
29	1/1130	1/1715	2/2193	1/1470
30	1/1130	1/1715	1/22463	1/1470

6. Appendix

```
S = 4;
state = [1 2 3 4];
A = 2;
TotalEpoch = 30;
P1 = [0.995 0.005 0 0; 0.035 0.901 0.064 0; 0 0.058 0.892 0.050; 0 0 0.033 0.967];

CumReward = zeros(TotalEpoch, S);
OptAction = zeros(TotalEpoch, S);
AllPossibleValue = zeros(TotalEpoch,S);
CurrentReward = zeros(S);
chosenreward = zeros(TotalEpoch,S);
lumpsumreward = zeros(S);


for j = 1:S
    CumReward(30,j) = 0;
    OptAction(30,j) = 1;
    if j == 1
        chosenreward(30, 1) = 1129.96 ;
    end
    if j == 2
        chosenreward(30, 2) = 1715.36;
    end
    if j == 3
        chosenreward(30, 3) = 2246.31;
    end
    if j == 4
        chosenreward(30, 4) = 1470.31;
    end
end

for t= TotalEpoch-1:-1:1
    for s= 1:S
        if s == 1
       CurrentReward(1) = 787;
        end
        if s == 2
       CurrentReward(2) = 887;
        end
        if s == 3
       CurrentReward(3) = 987;
        end
        if s == 4
       CurrentReward(4) = 1087;
        end
    end

    for s = 1:S
        for a = 1: A
           if a == 1
               if s == 1
                  lumpsumreward(1) =  chosenreward(30, 1);
               end
               if s == 2
                  lumpsumreward(2) =  chosenreward(30, 2);
               end
               if s == 3
                  lumpsumreward(3) =  chosenreward(30, 3);
               end
               if s == 4
                  lumpsumreward(4) =  chosenreward(30, 4);
               end
           end
           if a == 2
               for x = 1:S
                   CumReward(t,s) = CumReward(t,s)+P1(s,x)*chosenreward(t+1,x);
               end
               AllPossibleValue(t,s) = CurrentReward(s)+ CumReward(t,s);
           end
        end
        chosenreward(t,s) = min(lumpsumreward(s), AllPossibleValue(t,s));

        if (lumpsumreward(s) > AllPossibleValue(t,s))
            OptAction(t,s) = 2;
        else
            OptAction(t,s) = 1;
        end
    end
end

    disp (OptAction);
    disp (chosenreward);
```

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