Vulnerability Analysis of Modern Electric Grids: A Mathematical Optimization Approach

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Vulnerability Analysis of Modern Electric Grids: A Mathematical Optimization Approach

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by

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Acknowledgements

First and foremost, I owe all credit to my God; without Him, I am nothing. To my parents, thank you for setting the bar high. Your standard for me is excellence, which I would never achieve without you. To Maddie, in me you instill patience and persistence, two qualities this thesis would not have been completed without. To Dr. Kelly Sullivan, thank you for helping me find a topic, for encouraging me to believe in myself and my work, for keeping me dedicated and humble, and most of all, for consistently being there. I am a better student, researcher, and man because of you. Lastly, I would like to thank the State of Arkansas for the financial support through the Student Undergraduate Research Fellowship.
Abstract

Electrical power must be transmitted through a vast and complicated network of interconnected grids to arrive at one’s fingertips. The US electric grid network and its components are rapidly advancing and adapting to the advent of smart technologies. Production of electricity is transitioning to sustainable processes derived from renewable energy sources like wind and solar power to decrease dependence on nonrenewable fossil fuels. These newly pervasive natures of smart technology and the variable power supply of renewable energy introduce previously unexamined vulnerabilities into the modern electric grid. Disruption of grid operations is not uncommon, and the effects can be economically and socially severe. Thus, a vulnerability analysis can provide decision makers with the ability to characterize points of improvement in the networks they supervise.

This thesis performs a vulnerability analysis of electric grid operations including storage. This vulnerability analysis is achieved through a set of numerical experiments on a multi-period optimal power flow model including storage and variable demand. This model resulted in an analysis indicating storage is helpful in increasing resilience in networks with excess generation, no matter how severe the disruption. Networks with constrained generation benefit little, if at all, from storage. This analysis allows us to conclude careful implementation is the best way to improve electric grid security in the face of widespread use of renewable energy and smart technology.
Table of Contents

1. Introduction .......................................................................................................................... 5
2. Research Contributions ........................................................................................................ 7
3. Methodology .......................................................................................................................... 7
3.1. Discipline Techniques ......................................................................................................... 8
3.2. Model Development and Computation ............................................................................... 8
3.3. Data Specification ............................................................................................................... 12
3.4. Summary of Numerical Experiments ................................................................................. 15
4. Results .................................................................................................................................. 16
4.1. Component Analysis .......................................................................................................... 16
4.2. Broad Outcomes ................................................................................................................. 19
5. Conclusion ............................................................................................................................. 20
6. Future Work ........................................................................................................................... 20
7. References ............................................................................................................................. 22
8. Appendix ............................................................................................................................... 24
1. Introduction

An electric grid is an interconnected network that delivers electricity from producers to consumers. The electric grid in the continental US is composed of three interconnected grids: The Eastern Interconnection, Western Interconnection, and Texas Interconnected System [1, 6]. The American electric grid is becoming more technologically advanced to meet our population’s increasing desire for an inexpensive, immediate, and environmentally friendly energy supply.

There are few infrastructures as extensive as the American electric grid, and there is a clear governmental interest in electric grid operations (largely because of the national security implications). On May 11th, 2017, President Trump issued an Executive Order in which he expressed concern about deliberate cyberattacks on critical points of infrastructure and identified steps to be taken to advance our government’s risk management techniques [16]. While we as a nation have yet to experience a widespread, deliberate attack on our electric grid by an external force, it is common for regions to experience outages from systems brought down by extreme weather, human error, and/or system error. After Hurricane Irma struck the Southeast United States in September 2017, over seven million households in the region were without power. A spokeswoman for a Florida energy provider stated approximately 15.25 million Floridians (59% of the state) were without power [9]. Additionally, Florida Power and Light had over 17,000 personnel from over 30 states on standby to aid power restoration efforts [11]. As evidenced by this and other recent events, concern for addressing system outages is relevant and current.

As is typical with advancing technology, there are benefits and detriments to every innovation. The nature of renewable energy and its decentralized production can lead to network load stability problems and requires energy storage [12], which is pivotal to successful operation of the electric grid. Further, and somewhat at odds with the goals of increasing reliability and
efficiency, renewable energy is not always able to provide a steady supply of power for consumption needs [12]. The advent of smart technology is partially motivated by the widespread adoption of renewable energy. The primary functions of smart grids are characterized by efforts to mitigate the negative effects of reduced efficiency, reliability, and safety introduced by the inherent nature of a vast network, especially one comprised of both renewable and nonrenewable sources of energy [10]. However, these improvements might also mitigate these vulnerabilities in growing networks, further compounding the need for defensible networks [2]. Due to the unreliable nature of renewable energy, safe, cost effective storage techniques are being developed to allow renewable energy sources to be integrated into established networks.

It is unclear how significantly the incorporation of variable power supply and introduction of storage influences the vulnerability of electric grid networks. Thus, the dependence of an entire nation on such a network requires deeper analysis of emerging disruptions to enable decision makers to design and operate more resilient electric grid networks. In what follows, we present a vulnerability analysis approach, based on an optimal power flow model, to analyze the impact of storage on the vulnerability of the electric grid.

Although electric grid networks operate on AC flow, the need for scalable mathematical models to address the design and operation of electric grids has prompted the use DC approximations of nonlinear AC power flow models [8]. DC flow models are often applied across multiple periods using unit commitment to determine optimal scheduling. These DC flow models are developed to evaluate grid operations, commonly with the objective of minimizing cost of power generation and delivery. In literature, it is commonly seen to model electric grid operations with DC flow, which approximates AC flow, because it is difficult to address AC
flow due to its nonlinear nature [8]. AC flow is more realistic, but it also carries significant modeling and computational challenges; we have therefore chosen to focus on the simpler DC approximation for the purposes of this research.

2. Research Contributions

This research contributes a methodology for assessing how grid vulnerability is impacted by incorporating storage in an interdicted network. We designed a mathematical model to assist decision makers with mitigating the risks of implementing storage. The mathematical model attempts to reflect the current trends of technological advancements in the modern electric grid. This thesis provides methodology to perform a vulnerability analysis of networks to external disruptions, deliberate or otherwise.

Vulnerability studies in power flow have been performed on the physical components of networks [12], the unit commitment approach [18], reliability of power supply [5], and cascading failures [13], to name a few. Our research methodology first solves a multi-period DC power flow network to determine a plan for generating, distributing, and storing power over time. Multiple energy generation and storage strategies are implemented under a variety of disruption situations. The results of this analysis were used to draw conclusions about the most effective ways to mitigate risk in constructing and securing electric grids including storage. To the best of our knowledge, our research is the first to quantify how different energy storage strategies impact the vulnerability of power flow networks.

3. Methodology

The following section provides an overview of the existing power flow analysis techniques within the discipline. Subsection 3.2 details and examines our model development and describes
3.1. Discipline Techniques

Optimal power flow (OPF) is a class of network optimization problems used to describe and/or prescribe operational characteristics of an electric grid system with defined topology, or arrangement of the constituent parts [8]. This research developed an OPF model to analyze the vulnerabilities of electric grid networks with storage over time.

OPF models have been applied widely in analysis of power grid operations. For this research, we have relied upon Frank and Rebennack’s “An introduction to optimal power flow: Theory, formulation, and example” [8] to understand the space of OPF models at a high level. Mégel, Andersson, and Mathieu’s “Reducing the Computational Effort of Stochastic Multi-Period DC Optimal Power Flow with Storage” [14] provided robust background on what complications are introduced through transforming an OPF model into a multi-period example.

3.2. Model Development and Computation

We adapted a computational model of a small, easily manipulated representation of an electric grid from the IEEE 14 Bus Test Case [15]. This Test Case represents a portion of the American Electric Power System in the Midwestern US as of February 1962. This test case is simplistic and has been widely analyzed. We integrated storage, variable demand, variable power supply, and multiple periods into the network. As demand changes, the generation and distribution plans vary over time; our model is designed to account for these rapid changes in operation. Model development began by using existing literature to verify that our model was capable of reproducing expected outputs of previously observed and analyzed electric grids. Once we
verified operation of the model, we introduced disruptions to demonstrate potential vulnerabilities.

The model was optimized according to an objective function minimizing costs of generation and transmission. The objective function was further strengthened for the purposes of this research by introducing resilience as another priority by incorporating costs of storage and load shedding (based on the certainty of eventual disruption). Under conditions of potential grid failure, the concept of load shedding can be applied [6]. Load shedding occurs when demand outweighs the network’s capacity to supply power, which leads to reallocation of power based on network priorities, or, in extremely rare cases, the cost of delivering power is simply prohibitively high. Literature often considers the concept of \((n - 1)\) failure in design of networks [18]. An \((n - 1)\) failure is when a disruption wholly removes a single component of a network, often a branch, and is able to continue functioning normally or near normally. In this research we consider a variety of \((n - 1)\) interdictions, as well as more catastrophic \((n - 2)\) disruptions. The network topology was affected by the disruptions for the entirety of the testing period with no opportunity for recovery.

Notation for the uninterrupted model is presented below.

**Sets**

\(A\): the set of branches \((i, j)\), where \(i < j\)

\(N\): the set of nodes

\(N(i), i \in N\): the set of nodes \(j \in N\) such that \((i, j) \in A\) or \((j, i) \in A\)

\(T = \{1, \ldots, |T|\}\): the set of time periods

**Parameters**

\(r_{i,j} = \text{removal of branch } (i, j) \in A\) (1 if line \((i, j)\) has been removed); 0 otherwise

\(b_{i,j} = \text{susceptance of branch } (i, j) \in A\) [Siemens]
\( c_{i,j} = \text{cost of flow on branch } (i,j) \in A \) [S/MWh]

\( g_i = \text{cost of generation on node } i \in N \) [S/MWh]

\( e_i = \text{cost of storage on node } i \in N \) [S/MWh]

\( m_i = \text{cost of shedding on node } i \in N \) [S/MWh]

\( k_i = \text{injected power on node } i \in N \text{ in time } t \in T \) [MW]

\( d_i = \text{generation limit on node } i \in N \) [MW]

\( s_i = \text{storage limit on node } i \in N \) [MW]

\( l_{i,j} = \text{flow limit on branch } (i,j) \in A \) [MW]

\( q_{i,t} = \text{initial storage limit on node } i \in N \text{ in time } t \in T \) [MW]

\( z_{i,t} = \text{load at node } i \in N \text{ in time period } t \in T \) [MW]

**Decision Variables**

\( f_{i,j,t} = \text{absolute flow on branch } (i,j) \in A \text{ in time period } t = 1, \ldots, T \) [MW]

\( p_{i,j,t} = \text{power flow on branch } (i,j) \in A \text{ in time period } t = 1, \ldots, T \) [MW]

\( y_{i,t} = \text{generation at node } i \in N \text{ in time period } t = 1, \ldots, T \) [MW]

\( w_{i,t} = \text{injection at node } i \in N \text{ in time period } t = 1, \ldots, T \) [MW]

\( u_{i,t} = \text{storage at node } i \in N \text{ in time period } t = 0, \ldots, T \) [MW]

\( h_{i,t} = \text{power shed at node } i \in N \text{ in time period } t = 1, \ldots, T \) [MW]

\( v_{i,t} = \text{voltage angle at node } i \in N \text{ in time period } t = 1, \ldots, T \) [radians]

**Model**

**Objective**

\[
\min \quad \Sigma_{(i,j) \in A} c_{i,j} f_{i,j,t} + \Sigma_{i \in N, t \in T} e_i u_{i,t} + \Sigma_{i \in N, t \in T} g_i y_{i,t} + \Sigma_{i \in N, t \in T} m_i h_{i,t} \tag{1}
\]

**Constraints**

\[
s.t. \quad p_{i,j,t} \leq f_{i,j,t}, \quad \forall (i,j) \in A, \forall t \in T, \tag{2}
\]
-p_{i,j,t} \leq f_{i,j,t}, \quad \forall (i,j) \in A, \forall t \in T, \quad (3)

\hat{f}_{i,j,t} \leq l_{i,j}, \quad \forall (i,j) \in A, \forall t \in T, \quad (4)

w_{i,t} = y_{i,t} + h_{i,t} + u_{i,t-1} - u_{i,t} - z_{i,t}, \quad \forall i \in N, \forall t \in T, \quad (5)

p_{i,j,t} = (1 - r_{i,j})b_{i,j}(v_{i,t} - v_{j,t}), \quad \forall (i,j) \in A, \forall t \in T, \quad (6)

w_{i,t} = \sum_{j \in N(i)} (1 - r_{i,j})b_{i,j}(v_{i,t} - v_{j,t}) \quad (7)

0 \leq y_{i,t} \leq d_{i}, \quad \forall i \in N, \forall t \in T, \quad (8)

0 \leq u_{i,t} \leq s_{i}, \quad \forall i \in N, \forall t \in T, \quad (9)

q_{i,0} = 0 \quad \forall i \in N. \quad (10)

Objective (1) minimizes the total cost due to power flow, power generation, power storage, and load shedding. Constraints (2) – (3) are added to enforce $f_{i,j,t} = |p_{i,j,t}|$ for the purpose of allowing the objective to assess a flow cost for flow in either direction on a given branch; (4) ensures the flow (in either direction) is less than or equal to the flow limit on a branch; (5) enforces the injection at each node by stating it is equal to the sum of generation, shed power, storage, and demand in a given period, as well as storage from the previous period; (6) states the power flow on a branch is controlled by the suscceptance of the branch, as well as the difference between the voltage angles of the origin and destination nodes; (7) enforces nodal power balance from the intersecting branches; (8) defines the generation limit of each node; (9) defines the storage limit of each node; (10) guarantees that all nodes begin with no storage.

While formulating this model, we have made some assumptions about the network. As previously explained, we assume the power to be DC flow, because AC flow is computationally much more complicated [8]. Following [18], assume transmissions are lossless. Similarly, we assume generation, storage, and consumption of power all are lossless as well. We assume there
are no ramp rates or penalties for changes in generation or transmission because we wished to allow the grid to respond to disruptions as quickly and naturally as possible. We assume storage is fixed at a single location, node 13 (i.e. $s_i = 0 \forall i \in N\backslash\{13\}$), which we determined by executing a screening set of experiments to determine where it would have the greatest impact. We assume all disruptions occur instantaneously, with no possibility for recovery. We made this assumption because it is computationally much simpler to understand disruptions over a fixed period of time and compare them to other grid operations, disrupted or not.

Consult Appendix A for contact information to retrieve the AMPL model, data file, and any results.

### 3.3. Data Specification

The mathematical model was adapted from single period solutions of a commonly studied IEEE electric grid to an OPF multiperiod analysis of an electric grid including storage and variable demand interrupted by various interdiction efforts. The network topology is provided in Figure 1 [15].
We arrived at costs of generation, transmission, storage, and load shedding of $45/MWh, $35/MWh, $110/MWh, and $1,000/MWh, respectively. These costs are reflective of true generation and transmission costs, but it should be acknowledged that different regions and electric power providers have various costs associated with their generation and transmission that can change based on grid operations [3, 7]. Similarly, these storage costs and capabilities are based on realistic values, but depend on available technology, so as technology advances, it will be valuable to continue analyzing the effect of storage on OPF [4]. Load shedding costs are variable as well, depending on grid operations and preferences of the grid operators; the intention of having a drastically high load shedding cost is to prioritize storage and satisfy demand whenever possible.

In establishing generation limits, we allowed the initial model to be completely unconstrained, or ‘free,’ in order to encounter a situation of generation limits that become
binding under duress. These values were used to determine capacity values associated with the five generation nodes. We tested three generation capacity settings: this initial setting (hereafter referred to as “Free”), an instance of 5,000 MW/h limit (hereafter referred to as “Excess”), and an instance of 2,500 MW/h (hereafter referred to as “Constrained”). Transmission limits were set 2,000 MW/h as an unconstrained limit that becomes binding under duress but is not exceeded in the uninterdicted network. The storage limit at node 13 was set at 10,000 MW, a threshold we did not arrive at or exceed in any tests. The susceptances of the grid come from given data [15]. Forecasted demand over a period of 72 hours was retrieved from ISO New England [17]. This demand was divided by 14, the total number of nodes in the network, and multiplied by a random coefficient between 90% and 110%. The loads over 72 hours are depicted in Figure 2.

![Figure 2: Loads over 72 hours](image-url)
3.4. Summary of Numerical Experiments

Through the process of experimentation, we tested 1,926 scenarios by removing, or “interdicting,” components and subsets of components of the network and then solving the resulting version of Model (1)-(10). We defined five groups of scenarios in the network: A, the uninterdicted network, of which there is just one scenario; B, single-branch interdictions, which consists of completely interdicting all 20 branches; C, two-branch interdictions, which consists of all (20*19)/2=190 scenarios that can be obtained by pairing and partially interdicting each branch (reducing the dividing the capacity by 2); D, single-branch, single-node interdictions, which consists of completely interdicting and pairing the 5 generation nodes with each of the 20 branches; and E, complete two-node interdictions, which consists of completely interdicting and pairing each of the 5 generation nodes. While there are 14 nodes in the network, we only interdict the 5 generation nodes; regardless of a disruption occurring at a node, the demand remains unaffected. Each of these five groups of scenarios were tested under the three generation profiles (free, excess, and constrained), and two storage profiles (storage and no storage).

\[ [A + B + C + D + E] \ast \text{Generation} \ast \text{Storage} = [1 + 20 + 190 + 100 + 10] \ast 3 \ast 2 = 1,926 \]

Before beginning disruption efforts, we determined the single most effective location for storage in this topology, node 13. Through these disruption experiments we were interested in determining key measures such as percent demand satisfied, absolute power shed, and percent reduction in power shed. These measures allowed us to evaluate the effects of storage on network under disruption. Through testing of various \((n - 1)\) and \((n - 2)\) scenarios, we were able to determine the most destructive interdictions. All further experimentation proceeded with
the assumption that storage was fixed at this location, and all disruptions would occur
instantaneously and with no possibility for recovery.

4. Results

Since the efforts to develop this model were based in realistic assumptions about electric grid
operations, we attempted to interdict the model with feasible disruptions. These feasible
disruptions entail complete or partial removal of various branches, or complete or partial
restriction of generation at specific nodes; further, these disruptions were selected because we
consider them to be a reasonable summation of practically possible disruption efforts. We
concluded more severe disruption efforts, of more than two branches or nodes, could not be
reasonably assumed to occur in a network of this nature. While these disruptions did strain the
network, we only prevented the generating nodes from reaching the necessary levels of power to
satisfy demand in the situations of extremely constrained generation. Thus, when load shedding
occurs, we believe it is because the changed topology prevents satisfaction of demand, not
because the cost is prohibitive.

As previously stated, the results of this research are limited by the defined topology and
components of the network. Regardless, it is possible to understand the potential effects of
storage on a network comprised of variable generation and demand under duress. Section 4.1
details the results specific to this topology. Section 4.2 analyzes more general trends behind
incorporating storage into an electric grid, applicable to other networks.

4.1. Component Analysis

Almost as a rule, interdiction of generating nodes is more impactful than interdiction of
transmission branches. Node 6 is by far the most pivotal generation node, followed by node 8,
then 1, 2, and 3, not necessarily in that order. Branches emerging from node 6 and node 9, near
the north, or upper edge, of the topology generally have a greater effect on transmission, especially under duress, than those to the south, or lower edge. This is because the northern edge of this topology is rather isolated and can only be supplied power from nodes 6 and 8. These flows and generations of the uninterdicted network are depicted in Figure 3. Figure 4 depicts situation C, with branches 6 11 and 6 12 partially interdicted under constrained generation. The distinct differences between these two situations demonstrate the interesting results that can be interpreted from various disruptions.

Figure 3: Uninterdicted Flows and Generation by Component
In Table 1, we observe some clear trends resulting from the incorporation of storage. 

**Percent Demand Satisfied** is calculated by dividing the **Total Generation with Storage** of the situation by **Total Demand**. **Absolute Power Shed** is simply the total power shed in a situation with storage. **Percent Power Shed Reduction** is calculated by subtracting total power shed without storage from total power shed with storage, then divided by total power shed with storage.
Table 1: Aggregated Results of Disruption

<table>
<thead>
<tr>
<th>Storage</th>
<th>Disruption</th>
<th>Generation</th>
<th>Percent Demand Satisfied</th>
<th>Absolute Power Shed</th>
<th>Percent Power Shed Reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>Free</td>
<td>100.0%</td>
<td>0.00</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Excess</td>
<td>90.77%</td>
<td>0.00</td>
<td>14.56%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constrained</td>
<td>90.79%</td>
<td>0.00</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>Free</td>
<td>98.71%</td>
<td>100.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Excess</td>
<td>99.76%</td>
<td>0.00</td>
<td>68.46%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constrained</td>
<td>99.79%</td>
<td>0.00</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>Free</td>
<td>95.39%</td>
<td>100.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Excess</td>
<td>96.43%</td>
<td>0.00</td>
<td>76.02%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constrained</td>
<td>96.99%</td>
<td>0.00</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>Free</td>
<td>76.68%</td>
<td>99.5%</td>
<td>465.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Excess</td>
<td>81.45%</td>
<td>99.73%</td>
<td>2698.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constrained</td>
<td>71.43%</td>
<td>72.70%</td>
<td>262773.00</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>Free</td>
<td>54.51%</td>
<td>79.03%</td>
<td>39688.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Excess</td>
<td>76.46%</td>
<td>87.79%</td>
<td>51689.71</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constrained</td>
<td>54.95%</td>
<td>54.95%</td>
<td>442773.50</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>Free</td>
<td>99.59%</td>
<td>99.86%</td>
<td>3987.16</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Excess</td>
<td>99.52%</td>
<td>1336.52</td>
<td>106805.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Constrained</td>
<td>89.13%</td>
<td>1336.52</td>
<td>106805.00</td>
</tr>
</tbody>
</table>

Through experimentation, we discovered \((n - 1)\) disruptions, whether branch or node, had little effect on the network’s ability to satisfy demand, with all occurrences resulting in a vast majority of demand being satisfied. In these instances, storage is sometimes used, and sometimes has a significant impact. Introducing storage rarely impacts cost, at most reducing the total cost by 3%.

4.2. Broad Outcomes

A network must have excess generation for storage to reduce load shedding a significant amount; when there is no excess generation, storage is nearly useless. Constrained generation significantly lowers the potential load shed reduction because the network seems to be focused on delivering power in the short term and is unable to respond to disruptions. In Table 1, it can be observed that the constrained situations have very limited spread from 5th to 95th percentile of Percent Demand Satisfied in all groups, both with and without storage. The free and excess storage can sometimes have a significant impact.
generation profiles, on the other hand, have large spreads between the 5th to 95th percentile of 
Percent Demand Satisfied in some groups, and small spreads in others. All three generation 
profiles have small differences in Percent Demand Satisfied between situations with and without 
storage.

5. Conclusion
Through the results of this research, we can conclude that under certain circumstances electric 
grid security can be significantly augmented by storage in \((n - 1)\) and \((n - 2)\) scenarios only if 
there is potential excess generation. Thus, we observed storage can be beneficial to electric grid 
security, but only to a point; the results of this research indicate storage does not reduce an 
electric grid’s vulnerability to disruption to a substantial degree with consistency. The individual 
topology of any network should be carefully considered when selecting a storage node and 
performing any network hardening activities.

6. Future Work
This research can be furthered in the future by expanding the model in scope. We considered 
instances of limited size; thus, more robust conclusions about larger, more realistic grids can be 
drawn from a model with more nodes and branches, such as the IEEE 30, 57, 118, or 300 node 
power flow topologies [15]. In larger topologies, it could be helpful to examine the potential 
effect of multiple storage nodes, even shifting ones. The model can also be made more robust by 
addressing this problem with AC power, not just DC. Similarly, some of the other assumptions 
we decided not to include, such as ramp rates or lossless transmissions, could be incorporated to 
allow for a more realistic OPF test case. As storage technology becomes more advanced, costs 
will change; we believe it could be valuable to identify a break point for when storage should be 
implemented, especially at multiple locations. Interdiction responses could be better understood
if disruptions occur at random intervals over a longer period of analysis, with operations
beginning and continuing normally before and after the disruption. Further, the disruptions could
be randomized and take place over varying periods of time, increasing the uncertainty of
operation. Conclusions drawn from this research could be further investigated within the context
of a formal network interdiction optimization model.
7. References

8. Appendix

Appendix A:
Contact Dr. Kelly Sullivan at ksulliv@uark.edu
Contact Matthew Millis at millismatt13@gmail.com