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Maria Rene Arandia Jimenez
University of Arkansas, Fayetteville

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Locating Relay Nodes to Maximize Wireless Sensor Network Lifetime: A Numerical Study

An undergraduate honors thesis to fulfill

the requirement for the degree in

Bachelor of Science in Industrial Engineering with Honors

by

Maria Rene Arandia Jimenez

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University of Arkansas

Honors Thesis Advisor: Dr. Kelly M. Sullivan

Honors Thesis Reader: Dr. Chase Rainwater

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To my parents, thank you for your unconditional love and support in all my academic endeavors. To Dr. Kelly M. Sullivan, thank you for your guidance, patience, and for showing me what research is and what it means to society. Last but not least, I would like to thank the University of Arkansas Honors College for supporting me financially through the Honors College Research Grant.

Abstract

A wireless sensor network (WSN) is a group of sensors deployed over an area, which monitor changes in the environment, collects them as data and forwards it between sensors through wireless links. Data is routed, either in a single-hop or multi-hop manner, with the goal of getting this collected data to the sink nodes, which have higher computational capabilities and connects the network with a user interface. Studies have determined that multi-hop WSNs that integrate relay nodes, which function is to only receive and forward data, can maximize lifetime network. A linear programming model, created by Chang and Tassiulas in 1999, aims to maximize network lifetime by routing data from each sensor node to the sink node. This model can be adapted to incorporate the energy consumed and data transmitted by relay nodes.

This thesis contributes to the research done on WSNs with a numerical study done on the modified lifetime maximization model. The numerical study is based on sensitivity analyses that observe how changing network parameters affects relay node locations, having WSN as a performance measure. The results for this study show that relay nodes follow a ring structure around the sink node and might slightly change when parameter values are changed. This ring structure can be explained by the energy hole problem, that describes that sensor nodes around the sink nodes will fail first because of the great amount of energy spent in sensing and transmitting data. Therefore, relay nodes are placed near where sensor nodes will first run out of battery.

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1. Introduction

In the past few years, wireless sensor networks (WSNs) have been one of the most researched areas of technology, and it has been predicted that the WSN market will rise to $2 billion by 2022 [1]. A WSN is a group of sensors deployed over an area, which monitor changes in the environment while collecting and forwarding the data between sensors through wireless links [1] [2]. WSNs have applications in areas such as healthcare, industrial diagnostics, and border security [3]. WSNs can be deployed on land, underground, or underwater. WSNs can also be classified as mobile or static. Static WSNs have sensor nodes deployed to adopt a permanent location, while mobile WSNs contain sensor nodes that move freely within a specified area [4]. For the purpose of this research project, I will be focusing on static WSN.

WSNs are composed of many sensor nodes and one or more sink nodes. Sensor nodes are composed of sensors that capture signals, and an analog-to-digital converter that transforms the sensed signal into digital format. They also include a processor and a memory card that can process and record the data. Sensor nodes use a radio interface to transmit data to other nearby nodes until it reaches the sink node. Each node has an integrated power supply [1]. Generally, transmissions between sensor nodes are carried out with the goal of moving data to the sink nodes, which have more computational capabilities and connect the WSN to a bigger network or to a user interface. Each node inside the network has a transmission radius that determines the farthest distance it can transmit data to another node [5]. Data can be routed to the sink in a single-hop or multi-hop manner. If all sensors are close to a sink node, data can be routed to the sink node in a single hop. In contrast to a single-hop network, sensors in a multi-hop network may be more distant from the nearest sink node. Therefore, data is transmitted to the sink node through a sequence of other sensor nodes that forward this data until reaching a sink node [6].

There are many protocols regarding data routing, such as "shortest path routing" when data travels through the path of sensors that takes less time, and "network lifetime maximization" when transmission path might not be the shortest, but leads to a prolonged network lifetime [2] [6] [7]. Examples of these routing protocols are shown in Figure 1.

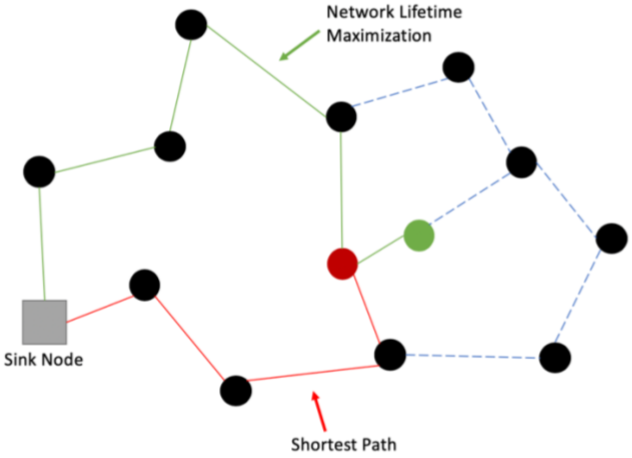


Figure 1. Illustration of a WSN transmitting data from two sensor nodes to the sink node. Each path is chosen by different routing protocols.

All sensor nodes have an initial and maximum power supply they can use, and when they consume all this energy, a network becomes unreliable [8]. Nodes can have a rechargeable battery, which are expensive and also require an energy-harvesting device, such as a solar panel. Alternatively, having a WSN with non-rechargeable sensor batteries is less expensive than having one with rechargeable batteries [1]. Every time a node senses a change in the environment, receives data or forwards data to other nodes, energy is consumed. In the modeling literature, it is common to assume that energy consumed is proportional to both the amount of data units collected and the distance over which the data is transmitted [6] [7].

Various circumstances, such as physical network damage and limited energy in sensor nodes, can shorten a network's lifetime [9]. Commonly, WSN failure is assumed to occur at the moment the first sensor node consumes all its battery, because it does not have 100% coverage of the area [8].

Studies suggest strategies to prolong network lifetime by more efficiently using limited battery. Given this, it has been studied how deploying the WSN in a multi-hop configuration reduces transmission distances and may therefore reduce energy consumed by as compared to a single-hop-only network. Second, the use of *relay* nodes—nodes without sensors or analog-to-digital converters that function only to forward data from sensors to the sink nodes—to increase WSN lifetime [6] [9]. Conceptually, relay nodes can increase the lifetime of a network by preventing sensor nodes from directing substantial energy towards data transmission, thus allowing them to focus on in monitoring the surrounding environment [7] [9]. A hybrid sensor/relay network is an example of a *heterogeneous* WSN, which broadly describes any WSN that contains more than one type of node. (For example, the nodes in a heterogeneous WSN may have different data collection, data processing, and data transmission capabilities [1].) When having relay and sensor nodes within the same WSN, relay nodes are predetermined to have higher initial energy and enhanced data retrieving and processing capabilities, as well as a higher economic cost [10]. There will be fewer relay nodes than sensor nodes, but because of their enhanced capabilities the network is expected to maximize lifetime without greatly increasing cost.

Given a fixed network design, the linear programming model of Chang and Tassiulas [6] aims to maximize network lifetime by routing data from each sensor node to the sink node. This model can be adapted to incorporate energy consumed by relay nodes, data transmitted between all nodes, and the number of relay nodes needed within the WSN [11] [12]. The adapted model is assumed to optimize relay node arrangement.

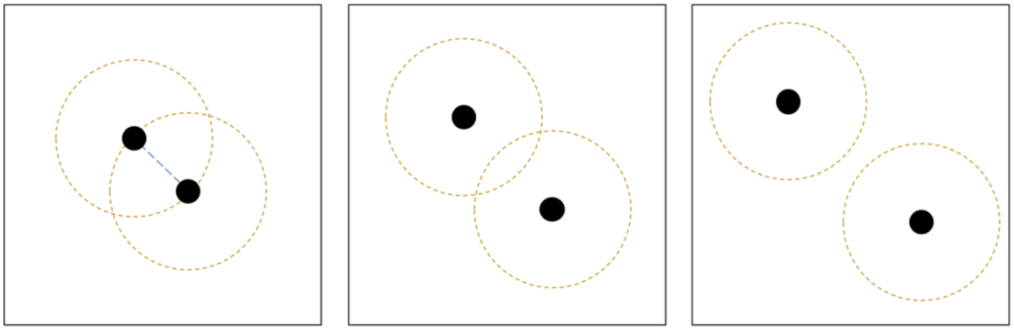
This thesis contributes to the study of maximizing the lifetime of the adapted WSN by conducting a numerical experimental study. WSN parameter values will be changed to observe how relay node placement and routing is affected, and we will use network lifetime as a performance measure.

2. Methodology

This section explains the optimization model used to optimize relay node location and data routing decisions. Section 2.1 summarizes the lifetime maximization linear programming model and Section 2.2 demonstrates how it can be adapted to locate relay nodes. Section 2.3 explains the sensor node topology testing performed on the model. Section 2.4 summarizes numerical experiments done on model parameters.

2.1 Lifetime Maximization Linear Programming Model

The linear program due to Chang and Tassiulas [6] maximizes the lifetime of a WSN by routing data from every sensor node to a sink node. The model depicted a WSN with a single sink node and multiple sensor nodes surrounding it. Every sensor node was assumed to be located at a point in space where data originated at a given rate [6]. The possible data transmission between nodes is defined as an arc. Other parameters involved in this model were the initial energy of sink, the energy consumed by a sensor node to forward data, and the energy used by the sink and sensor nodes to receive data [1] [6].The sink node is assumed to never fail, therefore, its initial energy is unlimited. We adapted the model by allowing arcs to be used only if their endpoints are located close enough to each other. Figure 2 shows when arcs can be used.



Arc Used

Arc Not Used

Arc Not Used

Figure 2. Arcs are used to transmit data if a node is within the radius of the other.

The linear program notation of sets, parameters, decision variables, and model is shown below:

<u>Sets</u>

$N = \{0, \dots n\}$ the set of all nodes were node 0 is the sink node and nodes $\{1, \dots, n\}$ are sensor nodes

$S = \{1, \dots n\}$ the set of all sensor nodes

$A = \{(i,j) : i \in N, j \in N, i \neq 0\}$ the set of arcs over which data can be routed (if the end points are close enough)

<u>Parameters</u>

P_i = initial and maximum energy used by node $i \in S$ [energy-units]

$e_{i,j}$ = energy to send one data unit through arc $(i,j) \in A$ [energy-units/data-units]

$c_{h,i}$ = energy to receive one data unit through arc $(h,i) \in A$ [energy-units/data-units]

b_i = data origin rate at sensor $i \in S$ [data-units/time-units]

r = communication radius of node [distance-units]

$posX_i$ = x-axis position for node $i \in N$ [distance-units]

$posY_i$ = y-axis position for node $i \in N$ [distance-units]

Decision Variables

$y_{i,j}$ = units of data transmitted in arc $(i,j) \in A$ [data-units]

z = lifetime of network [time-units]

Model

$$\text{Max} \quad z \tag{1}$$

$$\text{s.t.} \quad \sum_{(i,j)\,\in\,ARCS} y_{i,j} - \sum_{(h,i)\,\in\,ARCS} y_{h,i} = b_i z, \quad \forall i \in S \tag{2}$$

$$\sum_{(i,j)\,\in\,ARCS} e_{i,j} y_{i,j} + \sum_{(h,i)\,\in\,ARCS} c_{h,i} y_{h,i} \leq P_i, \quad \forall i \in S \tag{3}$$

$$y_{i,j} = 0\,, \quad \forall (i,j) \in A: \sqrt{\left(posX_i - posX_j\right)^2 + \left(posY_i - posY_j\right)^2} > r \tag{4}$$

$$y_{i,j} \geq 0, \quad \forall (i,j) \in A \tag{5}$$

Objective (1) maximizes the network lifetime. Constraint (2) imposes data flow balance for every sensor node $\forall i \in S$, i.e., the cumulative amount of data generated at node i by time z must equal the net (outgoing minus incoming) amount of data routed out of node i. Constraint (3) ensures that the energy consumed by the sensor node i when receiving and forwarding does not exceed the initial maximum energy. Constraint (4) prohibits flow between any pair of sensor nodes not within communication range. Constraint (5) prevents the data transmission to be negative, as that is impossible.

2.2 Adapting the Model to Incorporate Relay Node Location and Routing

To adapt the existing lifetime maximization model to incorporate relay node location decisions, new sets, parameters, decision variables, and constraints were added to the model [12]. It is important to remember relay nodes cannot sense changes in the environment and are only allowed to retrieve and forward data to other nodes. The following notation is added:

Sets

$R = \{n+1, n+2, \dots, n+m\}$ the set of all potential relay nodes

$N = \{0\} \cup S \cup R,$ $(\{0,1,\dots,n,n+1,\dots,n+m\})$

Parameters

$k =$ maximum number of relay nodes in the network

$r_i =$ communication radius of node $i \in S \cup R$ [distance-units]

Decision Variables

$x_i =$ variable indicating selection of relay node $i \in R$

The updated optimization model is to maximize objective (1) subject to constraints (2), (3), (5) and the following additional constraints:

s.t. $\sum_{i \in R} x_i \leq k$ (6)

$$\sum_{(i,j) \in A} y_{i,j} - \sum_{(h,i) \in A} y_{h,i} = 0, \quad \forall i \in R \tag{7}$$

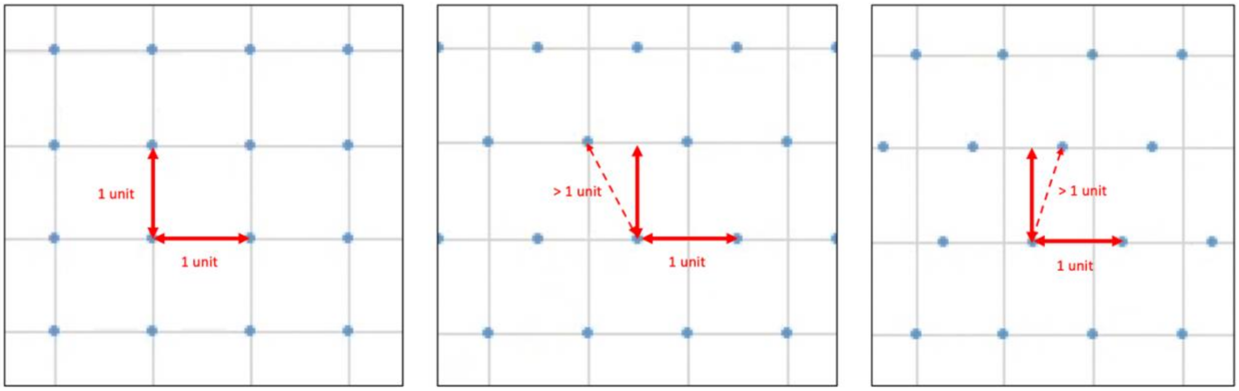
$$\sum_{(i,j) \in A} e_{i,j} y_{i,j} + \sum_{(h,i) \in A} c_{h,i} y_{h,i} \leq P_i x_i, \quad \forall i \in R, \tag{8}$$

$$y_{i,j} = 0, \quad \forall (i,j) \in A: \sqrt{\left(posX_i - posX_j\right)^2 + \left(posY_i - posY_j\right)^2} > \min\{r_i, r_j\} \tag{9}$$

The new decision variable x_i determines if relay node i is selected or not. Constraint (6) controls that the number of relay nodes does not exceed k. Constraint (7) guarantees data that is received by a relay node i is transmitted to another node i; note that relay nodes do not originate data. Constraint (8) makes sure the energy spent receiving and sending data by a relay node i does not exceed its initial energy, only if the relay node exists. Relay and sensor nodes will have different radius. In constraint (9), we are assuming communication is allowed between two nodes if each is within the other's communication radius.

2.3 Experiment Design on Sensor Node Arrangement

Several studies have examined how grid-like sensor topology affects the energy consumption and the area coverage of a WSN, depending on how sensors node communicate to their neighboring nodes [13], as the probability of exchanging data with closer nodes is greater the probability of exchanging data with farther nodes. For this study, several sensor node topologies were considered. The sensor node arrangements chosen were based on Bravais two-dimensional lattice structures. Bravais Lattices are ways of classifying crystal symmetry [14]. The lattice structures chosen to be the experimental sensor arrangement were the following: oblique, hexagonal, and square. The sensor arrangements can be seen in Figure 3.



Square Lattice Hexagonal Lattice Oblique Lattice

Figure 3. Lattice structures used for sensor node placement and the approximate distance between nodes.

Our meaning for investigating sensor node arrangements is to determine which, if any, lead to improved lifetime and how this affects relay node placement. For all the arrangements, the decision that sensor nodes are deployed at determined points was made because of the assumption they are placed where targets originate [6]. As for relay nodes, the decision of locating them at random positions was taken because the maximization model will dictate where these need to be located. The model is fed with 200 possible random locations for sensor nodes and it will select k possible and optimal relay node locations for each network arrangement.

Each sensor arrangement has its own data file to be solved in AMPL. The parameters $posX_i$ and $posY_i$ for sensor nodes will change for every lattice structure. A file was created for each lattice arrangement and included parameters P_i, $e_{i,j}$, $c_{h,i}$, r_i, and k of the base instance, which is explained in the following section.

2.4 Experiment Design on Model Parameters

We want to test how sensitive WSN lifetime and relay node placement are to changing the modified linear programming model parameters, such as transmission radius, node initial energy, energy consumption rates, and maximum number of relay nodes. The data found in this study is notional to demonstrate basic tradeoffs of the network. Future works may seek improved data sources. First, we will introduce the base instance for all parameters, and the next subsections will show how this base instance was modified for the different sensitivity analysis.

Base Instance

The base instance contains the original parameters used for each sensitivity analysis before modifying the data. For all instances, a single sink node, 100 sensor nodes and 200 possible relay nodes are assigned to the network. The area on the coordinate plane covered by the network depends on the lattice structure, so each sensor node arrangement has a rectangular shape if seen from afar. The square lattice structure is chosen as the base instance and the area covered will be a 10x10 xy-plane. The sink node is located at the center of the network. The maximum number of relay nodes to be deployed k is 25, and the data origin rate b_i is assumed to be 5 data-units/time-units. The energy consumed by a node to receive data $c_{i,j}$ is the same for all nodes, as well as the energy spent to transmit data $e_{i,j}$. An assumed low is chosen for both in the base instance. Given that relay nodes tend to have more energy and stronger transmission capabilities [10], P_i and r_i for relay nodes is higher than for sensor nodes. The initial energy value is 3 energy-units and transmission radius value is 3 distance-units. For all instances, the initial energy is 10,000 energy units, representing an "infinite" value, for the sink node, as it cannot run out of energy. The 200 possible relay nodes were randomly assigned positions within the network. Figure 4 depicts the base instance for this study. The base instance is the following:

$P_i = 1, \quad i \in S$

$P_i = 3, \quad i \in R$

$e_{i,j} = 0.05, \quad (i,j) \in A$

$c_{h,i} = 0.05, \quad (h,i) \in A$

$b_i = 5, \quad i \in S$

$k = 25$

$r_0 = 4, \quad (i = 0 = sink\ node)$

$r_i = 1, \quad i \in S$

$r_i = 3, \quad i \in R$

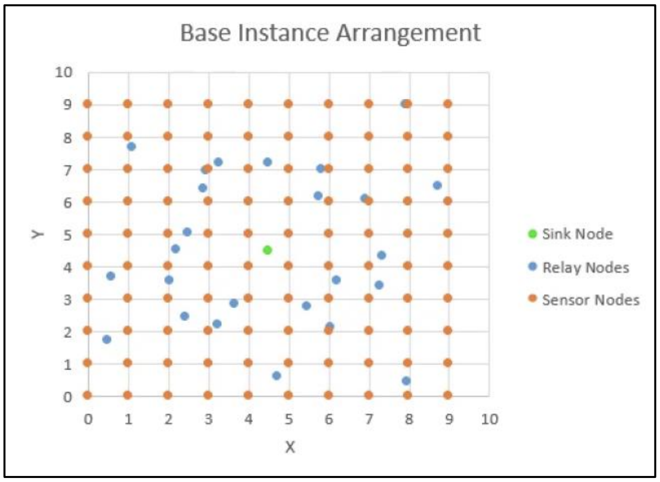


Figure 4. Solved base instance used to compare with other tests results.

Testing Sensitivity on Initial Energy

Beginning with the base instance, we then constructed and solved a sequence of instances to test how initial energy affects network lifetime and relay node locations. For all the runs, the initial energy for the relay nodes will be higher than the sensor nodes’, and the sink node will still have “infinite” initial energy. The rest of the parameters will be the same as the parameters in the base instance. The test run combinations are shown in Table 1.

Table 1. Test run combinations for sensitivity on initial energy.

Combination	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
$\boldsymbol{P_i \;\; i \in S}$	1	1	1	1	2	2	2	3	3	4
$\boldsymbol{P_i \;\; i \in R}$	2	3	4	5	3	4	5	4	5	5

Testing Sensitivity on Energy Consumption

Energy consumption rates for receiving and forwarding data were also tested. Because of differences in hardware and software, nodes spend different amounts of energy when getting and sending data. Therefore, to see if the focus has to be on energy consumption to maximize WSN lifetime, several $e_{i,j}$ and $c_{h,i}$ combinations were tested. The test run combinations are shown in Table 2.

Table 2. Test run combinations for sensitivity on energy consumption.

Combination	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
$\boldsymbol{e_{i,j} \;\; (i,j) \in A}$	0.05	0.05	0.05	0.15	0.15	0.15	0.25	0.25	0.25
$\boldsymbol{c_{i,j} \;\; (i,j) \in A}$	0.05	0.15	0.25	0.05	0.15	0.25	0.05	0.15	0.25

Testing Sensitivity on Transmission Radius

A heterogeneous network includes nodes with different capabilities, such as transmission capabilities [1], so several combinations of transmission radius will be tested to measure its sensitivity on WSN lifetime and relay node placement. Sensor nodes will have a smaller radius in all the combinations. The sink node will have a radius of 4. The test run combinations are shown in Table 3.

Table 3. Test run combinations for sensitivity on transmission radius.

Combination	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
$r_i \; i \in S$	1	1	1	1	2	2	2	3	3	4
$r_i \; i \in R$	2	3	4	5	3	4	5	4	5	5

Testing Sensitivity on Maximum Number of Relay Nodes

To test if the maximum allowed number of relay nodes affects the WSN lifetime, multiple test runs were done and are shown in Table 4. We want to see if having more or less relay nodes in the network increases its lifetime. Tradeoffs of each situation are also evaluated.

Table 4. Test run for sensitivity on maximum number of relay nodes.

Run	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
k	0	5	10	15	20	25	30	35	40	45	50

3. Results

The instances described in section 2.3 and 2.4 were solved using AMPL/CPLEX, using a time limit of ten minutes. For each instance, the resulting lifetime z and the optimality gaps are displayed and analyzed, as well as its relay node placement. Section 3.1 shows the numerical results on changing the sensor node arrangement. Section 3.2 shows the numerical results on changing the parameters of the base model.

3.1 Analysis on Sensor Node Arrangement Changes

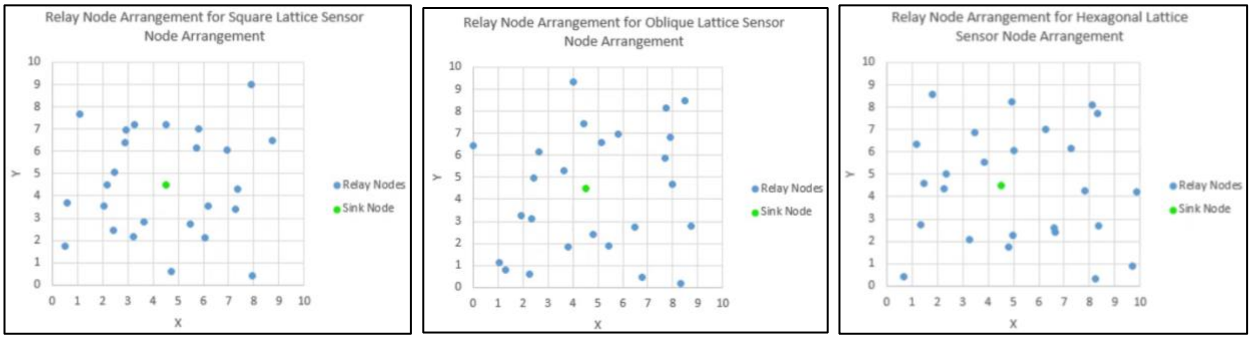
The AMPL output showed how the lifetime of a network can change if sensor node arrangements vary. The results show that WSN that had their sensor node topology on a square lattice formation, using the base instance parameters had the highest lifetime. It had 1.1837 time units. The least favorable sensor node arrangement in this testing was hexagonal lattice structure, with 0.852792. These results are shown in Table 5.

Table 5. Test run results for WSN lifetime in different topology.

Sensor Node Arrangement	Lifetime z
Square Lattice	1.18367
Hexagonal Lattice	0.852792
Oblique Lattice	0.877551

For all the network arrangements, we tried to keep all the nodes within a 10x10 area, so no arrangement had coverage advantages. Also, all network arrangements contain the same number of nodes. Nevertheless, the longer lifetime for square lattice arrangement might be caused by how data is routed. Square lattice arrangement has all sensor node separated by one distance-unit, meaning they have a slightly bigger density, making the network make better and more efficient data routing decisions, as seen in Figure 3.

As for relay nodes, in the three structures, the maximum number of relay nodes was used, 25, and Figure 5 shows how they were placed in each lattice structure. Relay nodes do not seem to be densely packed in the middle or by the edges of the area, which means they are not closely packed near the sink node. Relay nodes appear to form a ring structure around the sink node, which can be best seen when sensor nodes are placed in square lattice formation. In Table 6, the average distance from the relay nodes to the sink node was found for each lattice instance, and square lattice has a smaller average distance, as well as a smaller standard deviation. This explains why this ring structure is more visible in that sensor node arrangement and can also explain why it looks denser.

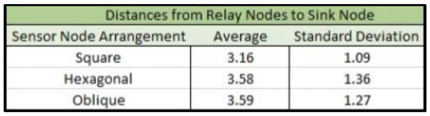

Relay Node Arrangement for Square Lattice Sensor Node Arrangement

Relay Node Arrangement for Oblique Lattice Sensor Node Arrangement

Relay Node Arrangement for Hexagonal Lattice Sensor Node Arrangement

Figure 5. Relay node arrangement for AMPL/CPLEX runs for square, oblique, and hexagonal lattices

Table 6. Statistics on relay node location from the sink node for each lattice structure



Distances from Relay Nodes to Sink Node		
Sensor Node Arrangement	Average	Standard Deviation
Square	3.16	1.09
Hexagonal	3.58	1.36
Oblique	3.59	1.27

To find an explanation as to why this ring structure happened, we graphed the sensor nodes that ran out of energy by the end of the WSN lifetime together with the relay nodes, as seen in Figure 6. Both types of nodes seems to be located by the center an surrounding the sink node. Therefore, the average distance from the failed sensor nodes to the sink node was found, and it can be seen in Table 7. There is a greater number of sensor nodes that fail and are near the center when using a square lattice, and a greater number of sensor nodes that failed.

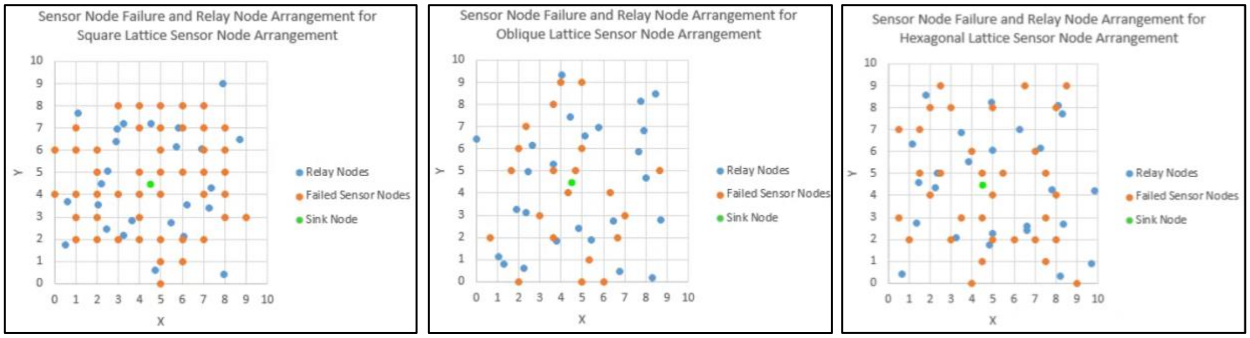
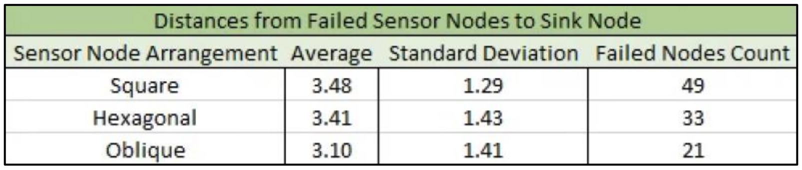


Figure 6. First sensor nodes to fail and relay node arrangement for square, oblique, and hexagonal lattices

Table 7. Statistics on sensor node failure location from the sink node for each lattice structure



Distances from Failed Sensor Nodes to Sink Node			
Sensor Node Arrangement	Average	Standard Deviation	Failed Nodes Count
Square	3.48	1.29	49
Hexagonal	3.41	1.43	33
Oblique	3.10	1.41	21

The greater amount of sensor nodes that failed can be because the longer lifetime gives more nodes the opportunity to exhaust their energy. As for why sensor nodes form a ring around the sink node, it can be explained with the energy hole problem. The energy hole problem states that sensor nodes near the sink node will exhaust their battery first because they serve as both sensor and relay nodes, gathering and transmitting data. When these sensor nodes stop working, there is not routing path from the outsides of the network to the sink, and communication is lost [15]. Therefore, to alleviate the energy consumption on sensor nodes, relay nodes are placed around the sink, which may explain the ring formation. In square lattice arrangement, since nodes are slightly more densely packed, the nodes near the corners would require more hops to send data to the sink. Sensor nodes near the center are bearing the load of many more sensors, reason why there are more sensor nodes that fail in this instance. For further comparisons the sensor node failure and relay node arrangement for square lattice will be used as the base instance.

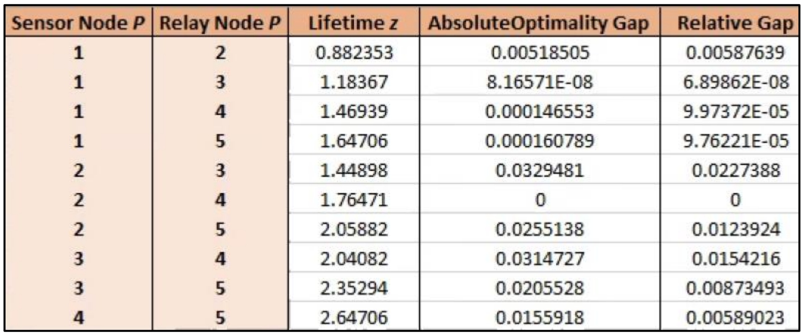
3.2 Analysis on Parameter Changes

In this subsection, we will display the results of testing the sensitivity of WSN lifetime to changing network parameters. We will show how relay node placement is affected.

Sensitivity on Initial Energy

A summary of the AMPL output, has been provided in Table 8, lifetime of a WSN network will change when the initial energy of the nodes varies. The output includes the absolute optimality gap and relative gap. The maximum WSN lifetime occurs when initial energy for sensor nodes has a value of 4 and when relay node initial energy is 5, and the shortest lifetime when initial energy for sensor nodes has a value of 1 and when relay node initial energy is 2.

Table 8. Test run results for different initial energy parameter value combinations.



Sensor Node *P*	**Relay Node *P***	**Lifetime *z***	**AbsoluteOptimality Gap**	**Relative Gap**
1	**2**	0.882353	0.00518505	0.00587639
1	**3**	1.18367	8.16571E-08	6.89862E-08
1	**4**	1.46939	0.000146553	9.97372E-05
1	**5**	1.64706	0.000160789	9.76221E-05
2	**3**	1.44898	0.0329481	0.0227388
2	**4**	1.76471	0	0
2	**5**	2.05882	0.0255138	0.0123924
3	**4**	2.04082	0.0314727	0.0154216
3	**5**	2.35294	0.0205528	0.00873493
4	**5**	2.64706	0.0155918	0.00589023

As expected, the larger the combination of initial energy, the higher is the WSN lifetime, as seen in Figure 7, because it means the nodes can send and receive data for longer periods of time. Also, WSN lifetime is slightly higher when the initial energy of relay nodes increases. The absolute optimality and relative gaps are low for all of the runs, meaning the data used for these instances did not make the linear programming model difficult to be solved.

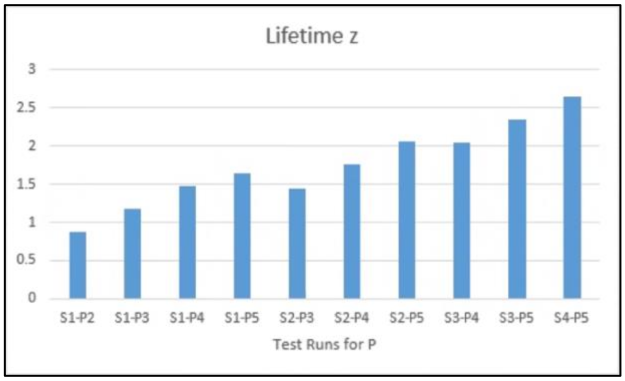


Figure 7. Bar graph showing WSN lifetime variation on different P_i combinations.

The relay node placements for the shortest and longest lifetime instances of these set of runs are seen in Figure 8 and are compared with the base case. The 25 relay nodes used seem to be placed more towards the center of the area, far from the edges, but also not including many nodes densely packed near the sink node, forming a ring structure. The average distances and standard deviations from the relay nodes to the sink node are seen in Table 9.

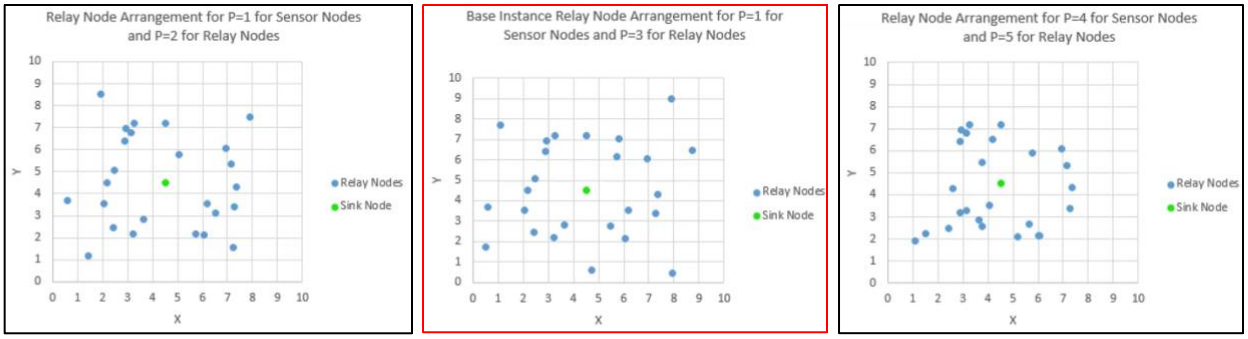
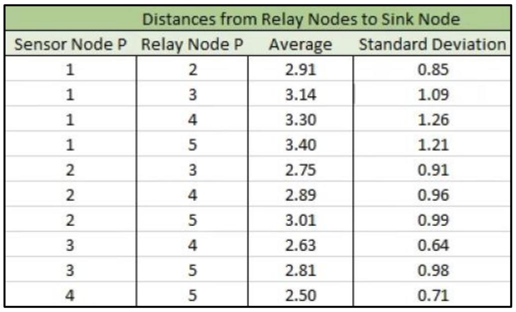


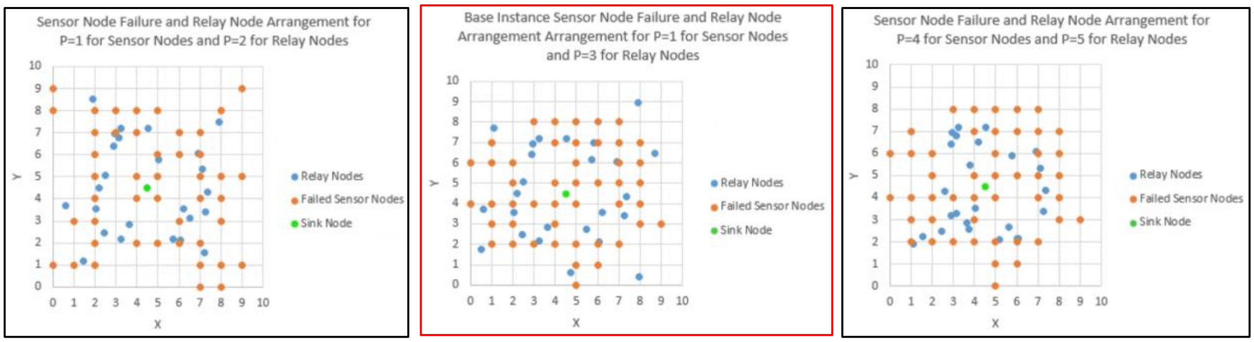
Figure 8. Relay node arrangement for AMPL/CPLEX runs for shortest and longest lifetime for initial energy instances and the base instance (red outline).

Table 9. Statistics on relay node location from the sink node for initial energy sensitivity test.



Distances from Relay Nodes to Sink Node			
Sensor Node P	Relay Node P	Average	Standard Deviation
1	2	2.91	0.85
1	3	3.14	1.09
1	4	3.30	1.26
1	5	3.40	1.21
2	3	2.75	0.91
2	4	2.89	0.96
2	5	3.01	0.99
3	4	2.63	0.64
3	5	2.81	0.98
4	5	2.50	0.71

The distance of relay nodes to the sink node is slightly increased when relay node initial energy increases, but not drastically. The failed sensor nodes for the shortest and longest lifetime instances of the maximization model were plotted, which are shown in Figure 9.



Sensor Node Failure and Relay Node Arrangement for P=1 for Sensor Nodes and P=2 for Relay Nodes

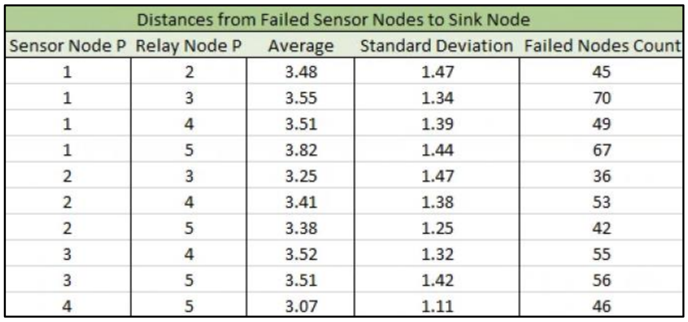
Base Instance Sensor Node Failure and Relay Node Arrangement Arrangement for P=1 for Sensor Nodes and P=3 for Relay Nodes

Sensor Node Failure and Relay Node Arrangement for P=4 for Sensor Nodes and P=5 for Relay Nodes

Figure 9. First sensor nodes to fail and relay node arrangement for shortest and longest lifetime instances and the base instance (red outline).

Again, sensor nodes around the sink node are the ones that exhaust all their battery, which supports the energy hole problem. To see if there was any relation between sensor node failure that depended on initial energy of nodes, we found the distance from failed sensor nodes to the sink node in Table 10 but found we did not find any pattern in this study. The data we had on the count of failed sensor nodes also did not provide us with any insights.

Table 10. Statistics on sensor node failure location from the sink node for initial energy sensitivity test.

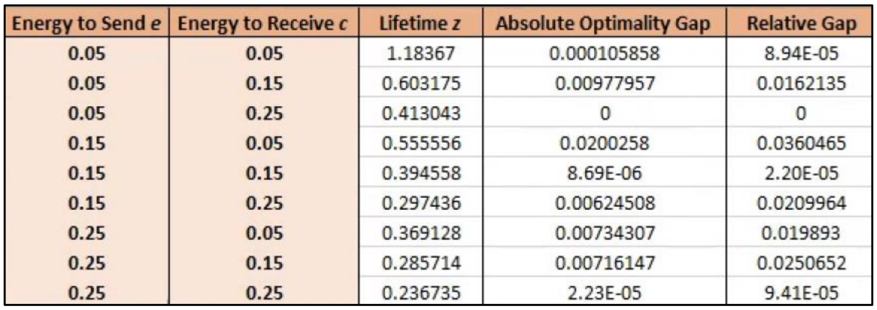


Distances from Failed Sensor Nodes to Sink Node				
Sensor Node P	Relay Node P	Average	Standard Deviation	Failed Nodes Count
1	2	3.48	1.47	45
1	3	3.55	1.34	70
1	4	3.51	1.39	49
1	5	3.82	1.44	67
2	3	3.25	1.47	36
2	4	3.41	1.38	53
2	5	3.38	1.25	42
3	4	3.52	1.32	55
3	5	3.51	1.42	56
4	5	3.07	1.11	46

Sensitivity on Energy Consumption

The AMPL output, seen in Table 11, shows how the lifetime of a WSN network does change when the changing the energy consumption rates for receiving and sending data, and it shows that WSN lifetime is maximized when energy consumption rates are smaller. In our data, this happens when both rates have a value of 0.05.

Table 11. Test run results for different energy consumption parameter value combinations.



Energy to Send *e*	Energy to Receive *c*	Lifetime *z*	Absolute Optimality Gap	Relative Gap
0.05	**0.05**	1.18367	0.000105858	8.94E-05
0.05	**0.15**	0.603175	0.00977957	0.0162135
0.05	**0.25**	0.413043	0	0
0.15	**0.05**	0.555556	0.0200258	0.0360465
0.15	**0.15**	0.394558	8.69E-06	2.20E-05
0.15	**0.25**	0.297436	0.00624508	0.0209964
0.25	**0.05**	0.369128	0.00734307	0.019893
0.25	**0.15**	0.285714	0.00716147	0.0250652
0.25	**0.25**	0.236735	2.23E-05	9.41E-05

Lifetime decreases as transmission when the energy consumption rate combination increases, and it can be seen in Figure 10. This was expected because it means the nodes are using great amounts of energy at once. The absolute optimality and relative gaps are very low for all runs meaning the data was not hard to solve.

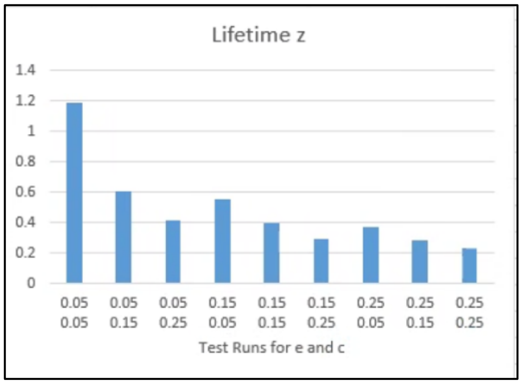


Figure 10. Bar graph showing WSN lifetime variation on different $e_{i,j}$ and $c_{h,i}$ combinations.

The relay node placement for the shortest and longest lifetime instances showed that relay nodes are forming a ring shape around the sink node. This can be seen in Figure 11. Table 12 contains the calculation done on the average distance from relay nodes to the sink node in each instance tested. No relevant observations were made from this table.

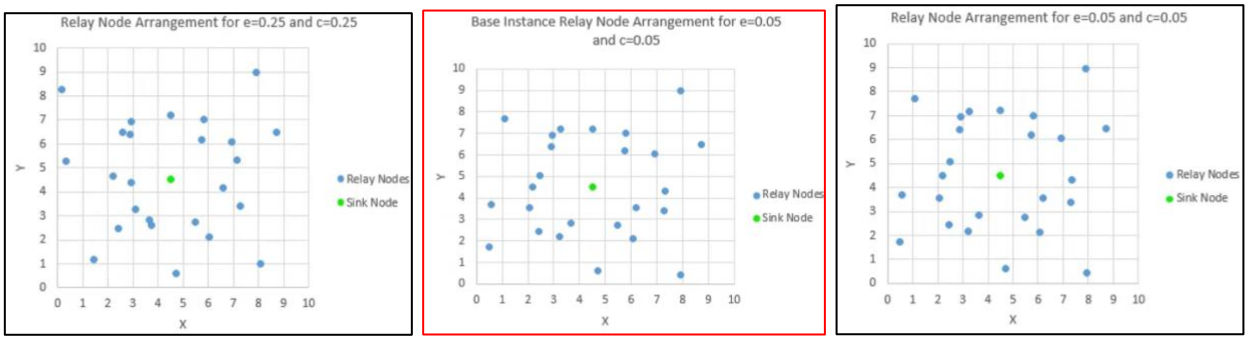
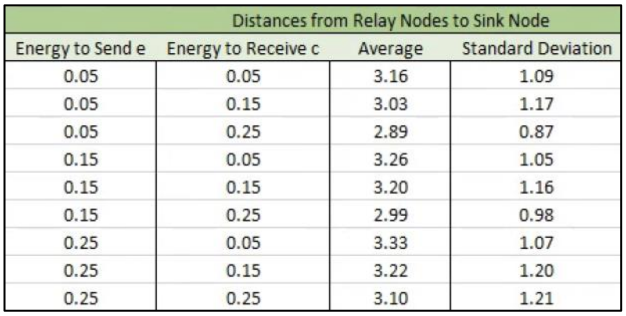


Figure 11. Relay node arrangement for AMPL/CPLEX runs for shortest and longest lifetime instances and the base instance (red outline).

Table 12. Statistics on relay node location from the sink node for energy consumption rates sensitivity test.



Distances from Relay Nodes to Sink Node			
Energy to Send e	Energy to Receive c	Average	Standard Deviation
0.05	0.05	3.16	1.09
0.05	0.15	3.03	1.17
0.05	0.25	2.89	0.87
0.15	0.05	3.26	1.05
0.15	0.15	3.20	1.16
0.15	0.25	2.99	0.98
0.25	0.05	3.33	1.07
0.25	0.15	3.22	1.20
0.25	0.25	3.10	1.21

To understand the ring structure for relay node placement, in Figure 12, we plotted the failed sensor nodes for the shortest and longest lifetime instances. We observed that they were all around the sink node. Like in previous instances, this follows the energy hole problem, which suggests using relay nodes to alleviate the burden of sensor nodes of sensing and transmitting data. In Table 13, we found some statistics on the positions of failed sensor nodes regarding the sink node but did not find new insights.

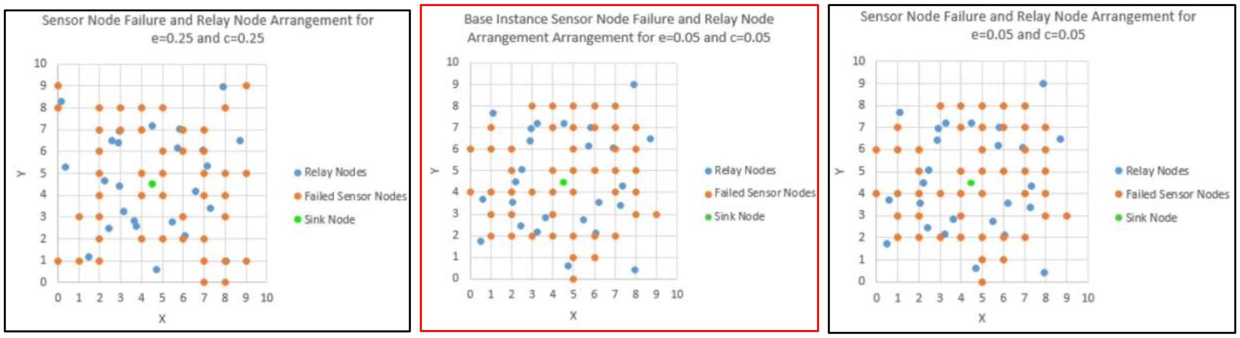
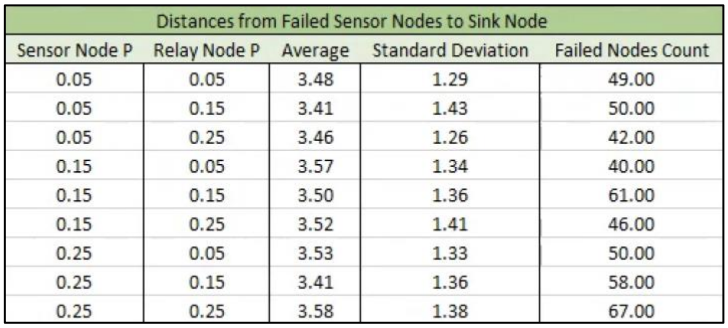


Figure 12. First sensor nodes to fail and relay node arrangement for shortest and longest lifetime instances and the base instance (red outline).

Table 13. Statistics on sensor node failure location from the sink node for energy consumption rates sensitivity test.

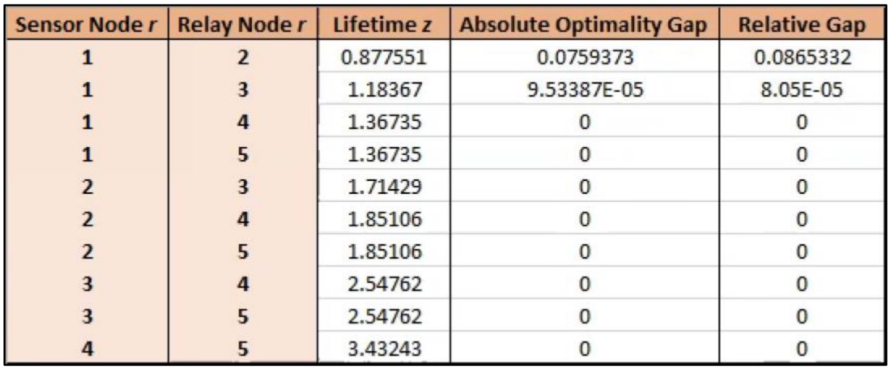


Distances from Failed Sensor Nodes to Sink Node				
Sensor Node P	Relay Node P	Average	Standard Deviation	Failed Nodes Count
0.05	0.05	3.48	1.29	49.00
0.05	0.15	3.41	1.43	50.00
0.05	0.25	3.46	1.26	42.00
0.15	0.05	3.57	1.34	40.00
0.15	0.15	3.50	1.36	61.00
0.15	0.25	3.52	1.41	46.00
0.25	0.05	3.53	1.33	50.00
0.25	0.15	3.41	1.36	58.00
0.25	0.25	3.58	1.38	67.00

Sensitivity on Transmission Radius

Table 14 provides a summary of the AMPL output of how WSN lifetime is affected when changing the radii of sensor and relay nodes. WSN lifetime is at its maximum when the sensor node radius is 4 and the relay node radius is 5, and the lowest when the sensor node radius is 1 and the relay node radius is 2.

Table 14. Test run results for different transmission radius parameter value combinations.



Sensor Node *r*	Relay Node *r*	Lifetime *z*	Absolute Optimality Gap	Relative Gap
1	2	0.877551	0.0759373	0.0865332
1	3	1.18367	9.53387E-05	8.05E-05
1	4	1.36735	0	0
1	5	1.36735	0	0
2	3	1.71429	0	0
2	4	1.85106	0	0
2	5	1.85106	0	0
3	4	2.54762	0	0
3	5	2.54762	0	0
4	5	3.43243	0	0

As we were expecting, lifetime increases as transmission radii get bigger, which can be seen in Figure 13. Lifetime is also longer if the relay node radius size is increased. This can mean that a larger transmission radius allows for more, and maybe more efficient, routing paths. The absolute optimality and relative gaps are zero for all except the first two runs. This might be because when communication radii are smaller, there are less possible arcs used, making it harder for AMPL to make routing decisions.

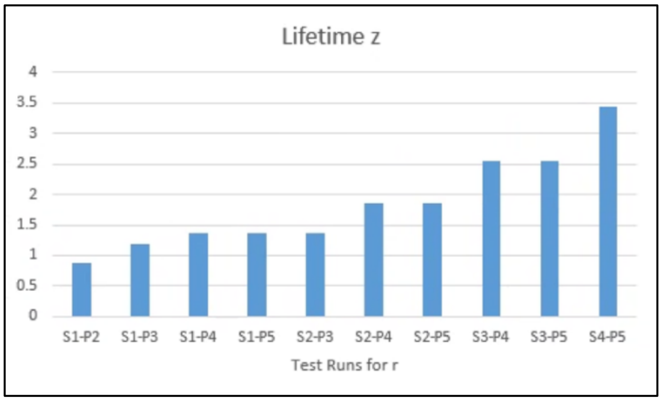


Figure 13. Bar graph showing WSN lifetime variation on different r_i combinations.

In Figure 14, we can observe the relay node arrangement for the shortest and longest lifetime instances. This also provides us a ring structure for relay nodes. The shortest lifetime instance showed to have the relay nodes closer to the sink node than the one with the longest lifetime. In Table 15, we displayed the average distance from relay nodes to the sink nodes for each instance. Here, the size of the radii affects the relay node placement slightly, because smaller radii show the relay nodes are placed nearer the sink node than in instances where radii are larger. This is so that communication arcs between nodes are used.

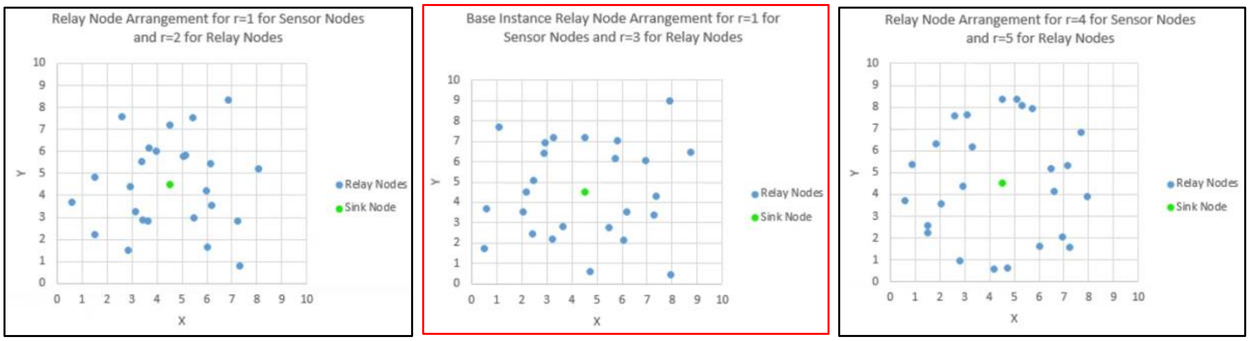
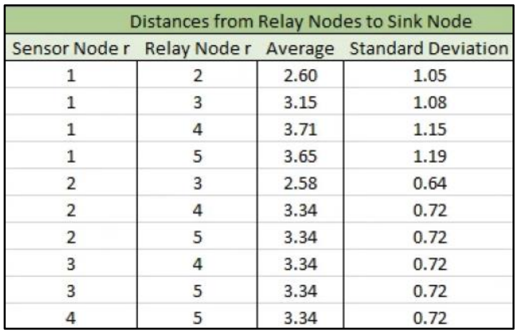


Figure 14. Relay node arrangement for AMPL/CPLEX runs for shortest and longest lifetime instances and the base instance (red outline).

Table 15. Statistics on relay node location from the sink node for transmission radius sensitivity test.



Distances from Relay Nodes to Sink Node			
Sensor Node r	Relay Node r	Average	Standard Deviation
1	2	2.60	1.05
1	3	3.15	1.08
1	4	3.71	1.15
1	5	3.65	1.19
2	3	2.58	0.64
2	4	3.34	0.72
2	5	3.34	0.72
3	4	3.34	0.72
3	5	3.34	0.72
4	5	3.34	0.72

Like in other instances, we plotted the sensor nodes that failed before the WSN lifetime ended. We could still observe the sensor nodes by the sink node exhausting their energy, which is near where the relay nodes are deployed. In Figure 15, it can be seen that there is a greater number of sensor nodes that exhausted their battery before the WSN failed completely, especially for the instance with the longest lifetime. In Table 16, we counted the number of sensor nodes that failed, and we could observe it increased as the radius of relay nodes becomes larger. This means that increasing relay node transmission radius allows for more and longer network communication before the WSN fails.

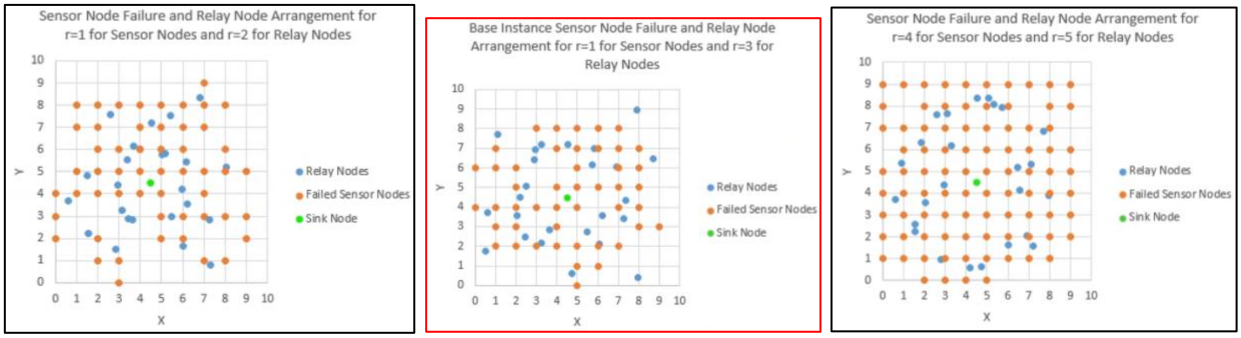
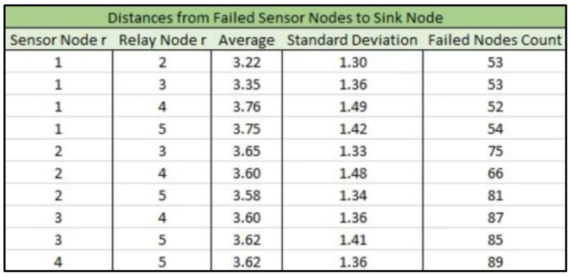


Figure 15. First sensor nodes to fail and relay node arrangement for shortest and longest lifetime instances and the base instance (red outline).

Table 16. Statistics on sensor node failure location from the sink node for transmission radius sensitivity test.

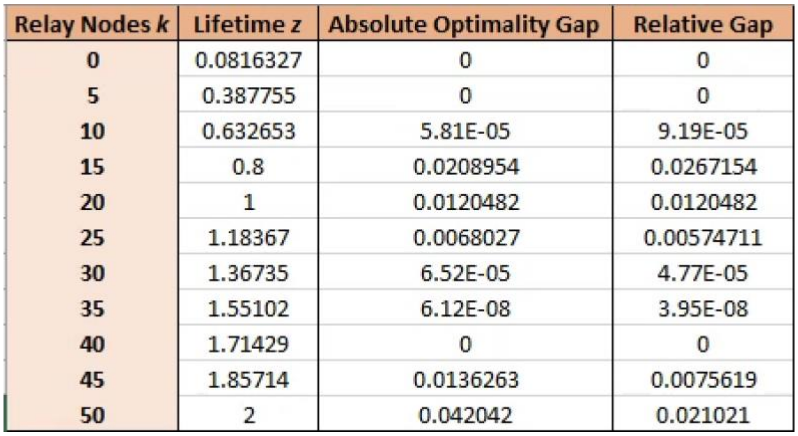


Distances from Failed Sensor Nodes to Sink Node				
Sensor Node r	Relay Node r	Average	Standard Deviation	Failed Nodes Count
1	2	3.22	1.30	53
1	3	3.35	1.36	53
1	4	3.76	1.49	52
1	5	3.75	1.42	54
2	3	3.65	1.33	75
2	4	3.60	1.48	66
2	5	3.58	1.34	81
3	4	3.60	1.36	87
3	5	3.62	1.41	85
4	5	3.62	1.36	89

<u>Sensitivity on Maximum Number of Relay Nodes</u>

Last but not least, the AMPL output for testing sensitivity of maximum number of relay nodes allowed on a network on WSN lifetime is shown in Table 17. Lifetime of a WSN network increases when the maximum number of relay nodes changes, as seen in the bar graph in Figure 16. The longest lifetime was seen when k was 50, which was what we expected.

Table 17. Test run results for different maximum relay node number parameter value.



Relay Nodes k	Lifetime z	Absolute Optimality Gap	Relative Gap
0	0.0816327	0	0
5	0.387755	0	0
10	0.632653	5.81E-05	9.19E-05
15	0.8	0.0208954	0.0267154
20	1	0.0120482	0.0120482
25	1.18367	0.0068027	0.00574711
30	1.36735	6.52E-05	4.77E-05
35	1.55102	6.12E-08	3.95E-08
40	1.71429	0	0
45	1.85714	0.0136263	0.0075619
50	2	0.042042	0.021021

The absolute optimality and relative gaps are very low for all the runs, except for when $k = 0$. This could be because our model was adapted to have relay nodes, making it impossible to solve if there aren't any.

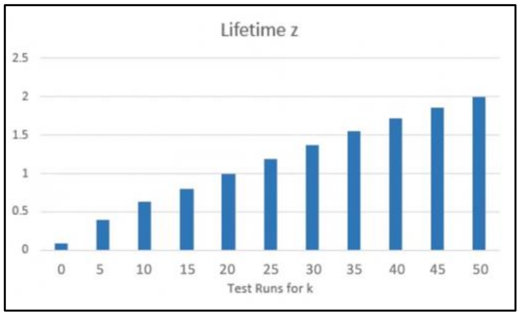


Figure 16. Bar graph showing WSN lifetime variation on different k.

Since k was 50, the integer linear programming model could select up to 50 relay nodes, and it did select 50. This relay nodes in this instance are plotted in Figure 17, and again, relay nodes are located around the sink. There obviously is a higher density of relay nodes when k is increased. Table 18 shows some statistics about the distance from relay nodes to the sink node. It shows the standard deviation grows as there are more relay nodes allowed in the network, which explains how the ring structure grows thicker in Figure 17.

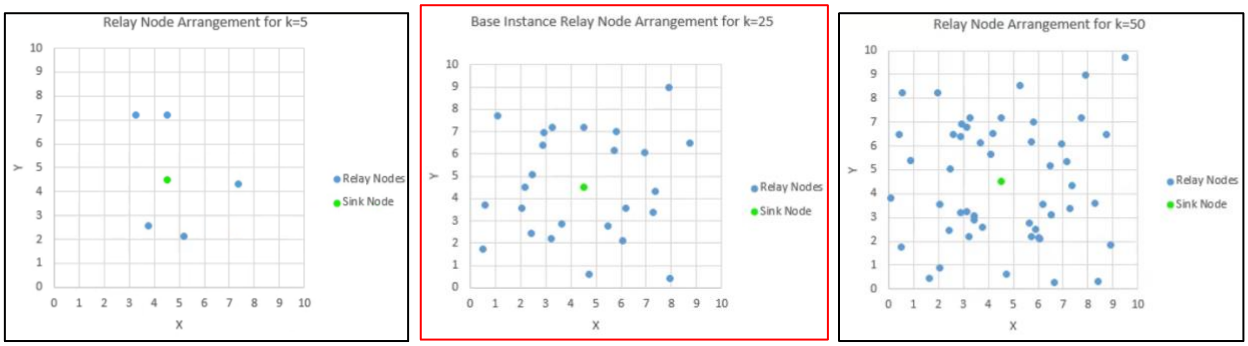
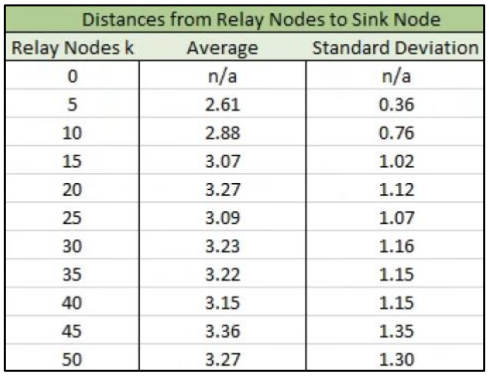


Figure 17. Relay node arrangement for AMPL/CPLEX runs for shortest and longest lifetime instances and the base instance (red outline).

Table 18. Statistics on relay node location from the sink node for maximum number of relay nodes sensitivity test.



Distances from Relay Nodes to Sink Node		
Relay Nodes k	Average	Standard Deviation
0	n/a	n/a
5	2.61	0.36
10	2.88	0.76
15	3.07	1.02
20	3.27	1.12
25	3.09	1.07
30	3.23	1.16
35	3.22	1.15
40	3.15	1.15
45	3.36	1.35
50	3.27	1.30

In Figure 18, failed sensor nodes were plotted, and for the instance with the longest lifetime, the failed sensor nodes go farther away from the sink node and not only near the ring of relay nodes, especially when compared to the base instance. The energy hole problem still applies for this problem, as relay nodes are still placed near the sink nodes, which is where sensor nodes are prone to run out of energy faster.

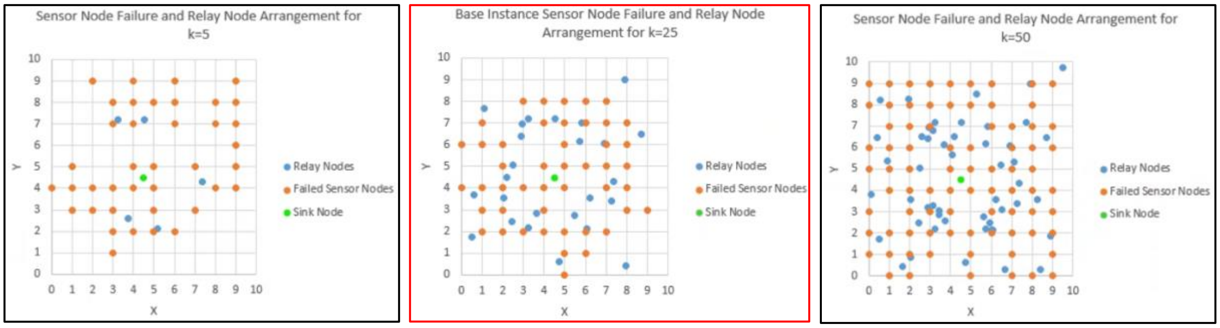
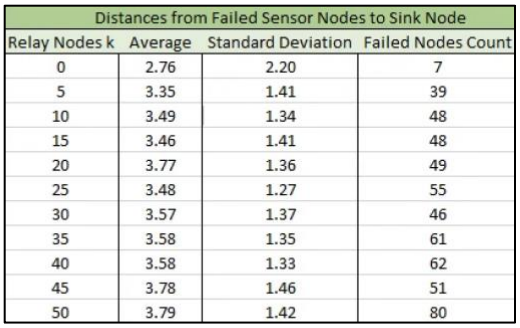


Figure 18. First sensor nodes to fail and relay node arrangement for shortest and longest lifetime instances and the base instance (red outline).

Table 19 shows the number of sensor nodes that failed, and it shows how the count increases as k increases. This is because, is the lifetime is increase, there is more chance and time for sensor nodes to use up their initial energy and transmit more data.

Table 19. Statistics on sensor node failure location from the sink node for maximum number of relay nodes sensitivity test.



Distances from Failed Sensor Nodes to Sink Node			
Relay Nodes k	Average	Standard Deviation	Failed Nodes Count
0	2.76	2.20	7
5	3.35	1.41	39
10	3.49	1.34	48
15	3.46	1.41	48
20	3.77	1.36	49
25	3.48	1.27	55
30	3.57	1.37	46
35	3.58	1.35	61
40	3.58	1.33	62
45	3.78	1.46	51
50	3.79	1.42	80

4. Conclusions

In conclusion, we formulated a relay node location problem with the objective of maximizing network lifetime, using the adapted the linear program due to Chang and Tassiulas. We used notional data to test the sensitivity of different network parameters on WSN lifetime and to observe how relay node placement was affected by parameter changes. We observed optimal relay node placement is affected by the energy hole problem and that they will form a ring structure. We also observed that changing some parameters will not have a significant impact on relay node location.

5. Future Research

To expand the scope of this project, future research can use node location and distance between nodes to assign energy consumption rates to the nodes. Also, as previously mentioned, the data used here is notional, so particular types of WSN with actual data can be analyzed. Bigger-scaled instances would be great to solve, as plots for bigger data sets could help visualize better the ring structure and more node density. The same process can also be repeated but modifying the model to consider a three-dimensional space in which nodes are deployed. This would require a lot more of data input, as three-dimensional WSN represent a more realistic approach of how WSN are used in different scenarios [16].

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