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# Analyzing Vulnerabilities in the Northwest Arkansas Highway Network Using Mathematical Optimization

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# Analyzing Vulnerabilities in the Northwest Arkansas Highway Network Using Mathematical Optimization

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**Abstract**

The highway and bridge network is a critical infrastructure that allows for the free transportation of citizens and enables truck-borne freight transportation. Disruption of this system could be caused by a terrorist attack, natural disaster, growth of population, required repairs and upgrades, or collapse caused by old age or malfunction. In the event of a disruption cities and regions can experience increased traffic and supply chain shortages, thus causing cascading effects throughout surrounding areas. With this motivation, we develop a network interdiction optimization model to identify a limited subset of roads that, if disrupted, causes the greatest increase in the weighted sum of shortest path distances associated with a collection of origin-destination pairs.  We apply the model to perform a vulnerability analysis on the network consisting of interstate highways, U.S. highways, and state highways in Northwest Arkansas.

**Section 1 - Introduction**

The highway and bridge system is one of the largest and most critical infrastructures in the United States containing over 4 million miles of roads and 600 thousand bridges [1, 2]. Disruptions within this network can slow critical transportation of travelers and supply chains and create expensive repairs or upgrades that are funded by taxpayers. Disruptions can be improvements, repairs, or nonrecurring traffic incidents which include car wrecks, construction zones, and inclement weather [3]. In the event of a disruption cities and regions can experience increased traffic, supply chain shortages, and loss of life which can have cascading effects throughout surrounding areas, as displayed by the I-35W bridge collapse in Minneapolis. This bridge fell during rush hour killing 13 people, injuring 145 more, and requiring over 230 million dollars and a full year to be reinvested in building a new bridge.

In the US funding for the highway and bridge network's maintenance and upgrades comes from a combination of federal, state, and local government spending. In 2019 around $203 billion dollars was spent on the road transportation network with state and local governments funding 76% and the federal government funding the remaining 24% [4]. It is estimated that over the next 20 years the US will spend $41 billion dollars per year on road repair but required funding for repair and operation is estimated to be $53 billion dollars per year.

One way of identifying an effective use of funding is through a vulnerability analysis on the road and bridge network. Vulnerability in road networks is defined by Berdica [5] as the "susceptibility to incidents that can result in considerable reductions in road network serviceability" [5]. A common method to identify critical parts of transportation networks is to use a full scan approach in which each arc between nodes is iteratively eliminated and the subsequent cost of this removal are measured in a reduction of network performance [6]. To increase the realism of the full scan approach it is common to equate arc travel times to the amount of congestion along an arc using traffic assignment models [7]. In this form traffic assignment models allow a researcher to include behavioral responses from travelers which Nicholson and Dalziell [8] describe as canceling trips, postponing trips, choosing alternate destinations, choosing a different mode of travel, or choosing a different route [8]. To include all five behavioral factors a combined travel demand model (CTDM) that considers each behavioral factor as a probability is used. The CTDM model quickly becomes difficult to solve due to the complexity of the nested behavioral probabilities and has only been numerically tested on small experimental networks. Additionally, the CTDM model does not take into consideration capacity constraints on arcs and does not consider combinations of road disruptions simultaneously.

One way to consider combinations of road disruptions in a directed network is to use network interdiction modeling. The basic structure of a network interdiction model has a follower and a leader. The follower runs a network with the goal of maximizing or minimizing some function while the leader strives to inhibit the objective from occurring by interdicting (i.e., damaging or removing) arcs in the followers' network. The goal of these models is to then find the most disruptive combination of arcs to remove [9]. Interdiction modelling has previously been used for many applications, including hospital infection modeling [10], distribution of hazardous materials [11], and military and security efforts to disrupt enemy supply lines [12].

To model a road network the network users play the role of followers who aim to identify a shortest origin-destination route, and the leader is a malicious entity allowing for the testing of network vulnerability. Our model is an extension of the shortest path network interdiction problem [9], in which the interdictor removes arcs between nodes in a network to maximize the shortest path length between an origin and destination. Since the original paper on shortest path network interdiction, research has considered variants of this problem including a study of network interdiction with asymmetric information where an interdictor and evader have different levels of information [13]. Borrero et al. [14] expand upon asymmetric information through a study of sequential interdiction in which an interdictor does not have initial information but over time, through the decisions of the evader, learns the structure and arc cost of the network [14]. Most recently, Nguyen and Smith [15] studied a similar interdiction scenario except the interdictor knows initial information is uniformly distributed between an upper and lower bound and the interdictor is tasked with maximizing the expected shortest path an evader can take [15].

This thesis contributes an extension of the shortest path network interdiction model that can be used to run a vulnerability analysis over road networks while considering a combination of road disruptions with multiple origins and destinations. We apply the network interdiction model to perform a vulnerability analysis of the Northwest Arkansas highway network and attempt to detail priority road sections for future funding.

We present the mathematical model in Section 2 and summarize the road network data used in the model in Section 3. We then experiment with changing the allowable budget and modifying the model to allow arcs to be interdicted more than once in Section 4 and give conclusions and proposals for future research in Section 5.

**Section 2 - Methodology**

In this section the mathematical formulation and description of the interdiction problem is described based upon an extension of Israeli and Wood's shortest path network interdiction model Israeli and Wood [9] to incorporate multiple origins and destinations. This problem maximizes the weighted sum of shortest path distances between multiple origin and destination pairs by choosing a subset of arcs to lengthen.

To define the interdiction model we first define $A \subseteq N \times N$ as a set of directed arcs where $N$ defines the set of nodes. Let $c_{ij}$ denote the length of arc $(i, j) \in A$, and define a nonnegative integer variable $z_{ij}$ to indicate the number of times arc $(i, j)$ is interdicted. We assume each interdiction adds $d_{ij} > 0$ units of length to arc $(i, j)$, i.e., the interdicted length is $c_{ij} + d_{ij} z_{ij}$.

To develop a mathematical model of the interdiction problem, we begin by formulating a linear program to represent the weighted sum of shortest path distances given assigned values to $z_{ij}$ that indicate which arcs have been interdicted. Let $x_{ij}^s$ express the amount of flow on arc $(i, j)$ $\in A$ that originates at node $s \in N$ with $w_j^s$ units flowing from source nodes and $w_i^s$ units consumed by sink nodes. We can then describe the following optimization Model (1) – (3).

$$\text{Min}_x \quad \sum_{s \in N} \sum_{(i,j) \in A} (c_{ij} + d_{ij} z_{ij}) x_{ij}^s \tag{1}$$

$$\text{s.t.} \quad \sum_{j \in N:(i,j) \in A} x_{ij}^s - \sum_{j \in N:(j,i) \in A} x_{ji}^s = \begin{cases} \sum_{j \in N \setminus \{s\}} w_j^s & if\ i = s, \\ -w_i^s & if\ i \neq s, \end{cases} \quad \forall s, i \in N \tag{2}$$

$$x_{ij}^s \geq 0, \forall s \in N, \forall (i, j) \in A \tag{3}$$

Given values for $z_{ij}$, Equation (1) minimizes the weighted sum of interdicted shortest path distances across all origin-destination pairs subject to flow balance Constraints (2) and nonnegativity Constraints (3). The model is an uncapacitated multi-commodity flow model in which a different commodity is used to represent flow originating at each source node $s \in N$, and $w_i^s$ units of this commodity must be sent from $s$ to each node $i \in N \setminus \{s\}$. The cost per unit flow on arc $(i,j) \in A$ is described by the interdicted length $c_{ij} + d_{ij}z_{ij}$ for arcs $(i,j) \in A$. Constraints (2) require each node $i \neq s$ to consume $w_i^s$ units of commodity $s \in N$ while also implying that a total of $\sum_{j \in N \setminus \{s\}} w_j^s$ units of commodity $s \in N$ must leave node $s$.

To convert the minimization Model (1) – (3) to a maximization model we can take the dual of the model, letting $u_i^s$ define the dual variable for constraint (2). The dual of the model (1) – (2) is then given as

$$\text{Max}_u \quad -\sum_{s \in N} \sum_{i \in N \setminus \{s\}} w_i^s u_i^s \tag{4}$$

$$\text{s.t.} \quad u_i^s - u_j^s \leq c_{ij} + d_{ij}z_{ij}, \forall s \in N, \forall (i,j) \in A \tag{5}$$

Due to strong duality, the optimal objective value of Model (4) – (5) is equal to the weighted sum of interdicted shortest path distances across all origin-destination pairs.

To formulate the interdiction model, we introduce the interdiction variables $z_{ij}, (i,j) \in A$, to Model (4) – (5) and include the constraint

$$\sum_{(i,j) \in A} z_{ij} \leq K \tag{6}$$

to impose that at most $K$ arcs can be interdicted.

We then created a parameter $l_{ij}$, used in subsequent tests of the model, that defines the maximum number of times an arc $(i, j)$ can be interdicted and introduced this parameter to Model (4) – (5) by including the constraint

$$z_{ij} \leq l_{ij}, \forall (i,j) \in A \tag{7}$$

**Section 3 - Data**

This section describes where we obtained the data used in the interdiction model, how this data was cleaned and filtered, and finally how the data was formatted and input in AMPL solver.

We used data from the Northwest Arkansas highway network to test and extend the network interdiction model. Northwest Arkansas is a metropolitan region that hosts four of the largest cities in Arkansas: Fayetteville, Springdale, Bentonville, and Rogers and is home to the headquarters for Walmart, Tyson, JB Hunt, and Arc Best, making it a regional hub for transportation and commerce. This region is growing rapidly as census reports show population growth is 3.6x higher in Northwest Arkansas than the United States and 8.6x higher than the state of Arkansas [16]. In the 2030 Northwest Arkansas Regional Transportation Plan there was an estimated need for $1.9 billion dollars in road construction and improvements, but only an estimated $411 million dollars in total funding [17]. The increased transportation importance and high population growth combined with a constrained budget for road improvements in Northwest Arkansas raises the need to understand where resources should go to effectively improve the transportation network. We obtained data for interstate and state highways in the Northwest Arkansas region and summarize below how this data was used to create the sets and parameters for the network interdiction model.

The main dataset used to create the network graph containing arcs $A \subseteq N \times N$ came from transportation data in the US Census Bureau's TIGER/Line Shapefile. A shapefile stores geospatial data and information on roads, buildings, water features, and other areas useful for research. To read and manipulate the Arkansas shapefile we created a data frame, using the Geopandas Python package, containing network information on Washington and Benton counties' road network. Utilizing a separate Python package, Folium, we created our first visualization of the Arkansas road network as detailed in Figure 1.

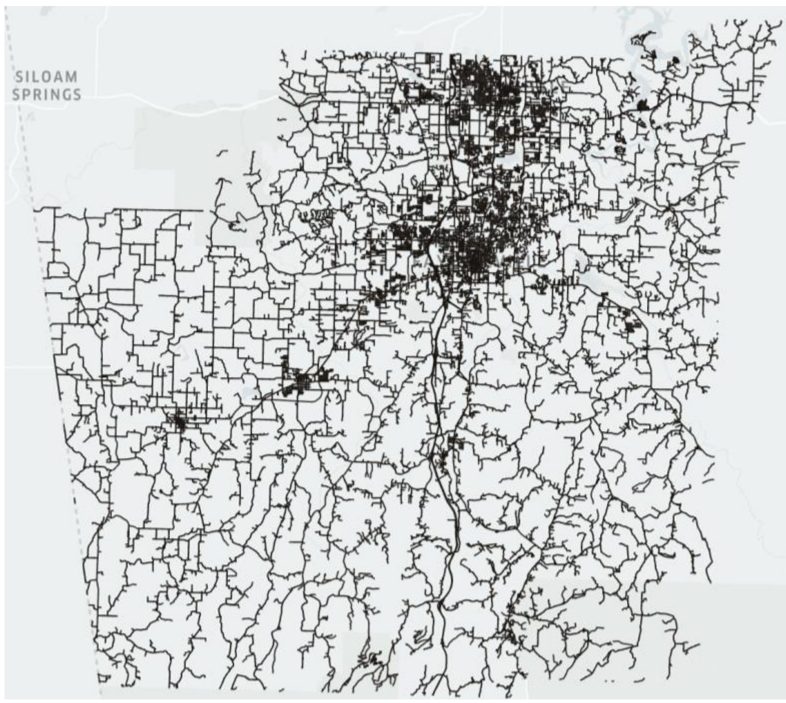


*Figure 1: Visualization of Washington and Benton counties' road transportation network*

The dataset created from the shapefile contained 1,313 nodes with multiple road types and summary information. To decrease the complexity of the road network municipal, county, and dead-end roads were filtered from the map, leaving interstate highways, state highways, and U.S. highways. To further reduce the complexity of the transportation network we utilized the Folium package to drop pins at intersections of multiple roads. These 34 pins were used to construct the node set $N$ used in the interdiction model.

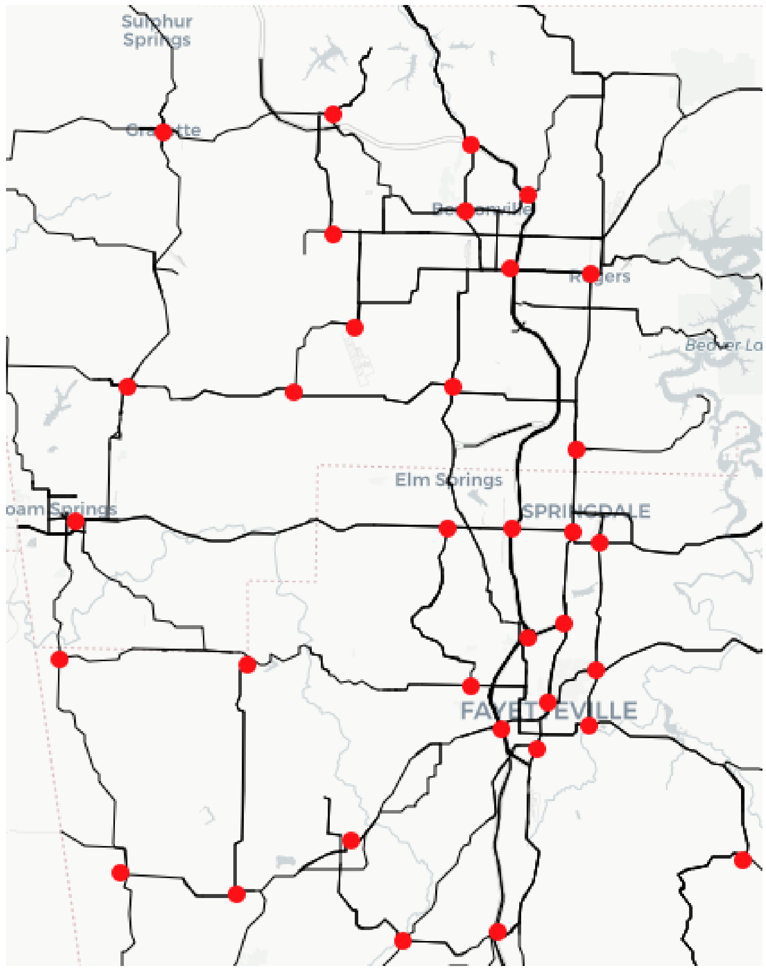


*Figure 2: Simplified transportation network with node's visible*

The arc set $A \subseteq N \times N$ was created by connecting each node to neighboring nodes. In creating the arc set $A$, which contains 110 arcs, the directed arcs $(i, j)$ and $(j, i)$ are both created if nodes $i$ and $j$ are connected by a road, thus allowing for bi-directional flow on the directed graph.

The cost $c_{ij}$ for flow between arcs is equal to the distance between $(N, N)$ node pairs in miles. These distances were calculated using the latitude and longitude of each node $N$ and plugging these values into the Geopandas geodesic function, which outputs the distance between the two points based on the shortest path considering the curve of the Earth. Performing the distance calculations in this manner was done to simplify distance calculations since we created custom nodes that did not have distance values already populated within the Shapefile dataset.

The integer budget $K$ that constrains the number of interdictions was tested with different values less than 40, which we viewed as representing a substantial disruption of the 110-arc network

We assigned weights to the each of the $|N|(|N|-1)$ origin-destination pairs by giving each pair $(s,i)$ a weight $w_i^s$ that defines the number of units of flow from node $s$ to node $i$; thus, the higher the value of $w_i^s$ the more important the node pair. We created these weights using population projections for all cities within Washington and Benton counties based upon the 2025 NWA Regional Transportation Plan [18]. The weight associated with node pair $(s,i)$ was computed as

$$w_i^s = \alpha_s \alpha_i, \tag{8}$$

where $\alpha_i$ is the proportion defined by the projected population of node $i$ divided by the total projected population of all nodes. In calculating the proportion $\alpha_i$ associated with each node $i$, we assumed the projected population of node $i$ was equal to the projected population of the city in which node $i$ is located divided by the number of nodes in that city.

An example $w_i^s$ calculation using the origin node 22, which is in the town of Gentry, and the destination node 21, located in Siloam Springs, is given below.

Projected population of Gentry = 3043

Projected population of Siloam Springs = 16227

Projected total population of all areas with 1+ node = 348,241

$\alpha_{22} = \frac{3{,}043}{348{,}241} = 0.0087$

$\alpha_{21} = \frac{16{,}227}{348{,}241} = 0.046$

$w_{21}^{22} = w_{22}^{21} = \alpha_{22}\alpha_{21} = 0.0087 \times 0.046 = 0.0004002$

In the case of Gentry, the value $\alpha_{22}$ can be interpreted as the percentage of people within the network that live in Gentry, which we assume is equal to the proportion of flow that begins or ends in Gentry. This probability is multiplied in the model by the negative resulting length $u_i^s$ to proportionally weight the importance of the origin destination pair in the objective. Every increase in the weight values corresponds with additional cost to travel along that arc. Table 1 summarizes the $a_i$ values obtained for each node $i \in N$.

*Table 1: $\alpha_i^s$ values given to each node*

| Node | Weight | City |
|---|---|---|
| 0 | 0.003354 | Lincoln |
| 1 | 0.003354 | Lincoln |
| 2 | 0.005367 | Prairie Grove |
| 3 | 0.008362 | West Fork |
| 4 | 0.006803 | Elkins |
| 5 | 0.000010 | N/A |
| 6 | 0.000010 | N/A |
| 7 | 0.032288 | Fayetteville |
| 8 | 0.005367 | Prairie Grove |
| 9 | 0.032288 | Fayetteville |
| 10 | 0.032288 | Fayetteville |
| 11 | 0.032288 | Fayetteville |
| 12 | 0.032288 | Fayetteville |
| 13 | 0.032288 | Fayetteville |
| 14 | 0.032288 | Fayetteville |
| 15 | 0.002736 | Tontitown |
| 16 | 0.002736 | Tontitown |
| 17 | 0.111156 | Springdale |
| 18 | 0.032288 | Fayetteville |
| 19 | 0.111156 | Springdale |
| 20 | 0.004537 | Bethel Heights |
| 21 | 0.046597 | Siloam Springs |
| 22 | 0.008738 | Gentry |
| 23 | 0.006831 | Cave Springs |
| 24 | 0.007833 | Centerton |
| 25 | 0.002783 | Highfill |
| 26 | 0.192691 | Rogers |
| 27 | 0.035106 | Bentonville |
| 28 | 0.007483 | Gravette |
| 29 | 0.045342 | Bella Vista |
| 30 | 0.007833 | Centerton |

| | | |
|---|---|---|
| 31 | 0.035106 | Bentonville |
| 32 | 0.045342 | Bella Vista |
| 33 | 0.035106 | Bentonville |
| TOTAL | 1.000104 | |

**Section 4 - Results**

This section details the computational tests and results used to investigate what sections of roads within the Northwest Arkansas transportation system are interdicted using the shortest path interdiction model.

**Initial Experimental Run**

The first test used our original $w_i^s$ values, based on the multiplication of population densities of source and sink nodes, and set the value of $l_{ij}$ equal to one to limit the number of interdictions on each arc $(i, j) \in A$. The length $d_{ij}$ added per interdiction on arc $(i, j) \in A$ was initially defined to equal $c_{ij}$; thus, a single interdiction on arc $(i, j) \in A$ initially has the effect of doubling the length of the arc. This added distance $d_{ij}$ is measured as a distance but can be interpreted to signify an obstruction increasing the time required to travel along an arc. We solved eight instances of the interdiction model corresponding to each value of interdiction budget $K$ in $\{5,10, \dots ,40\}$. In Table A (provided in the Appendix), we note that 76% of the time each arc $(i, j)$ is interdicted $(j, i)$ is interdicted. This is likely because the objective function applies identical weights to the shortest path distance from an origin node $s$ to a destination node $t$ and the shortest path distance from $t$ to $s$.

We then created a visualization, using Table A (provided in the appendix), detailing which arcs were interdicted through the eight trials and to what frequency they were interdicted, as depicted in Figure 3.

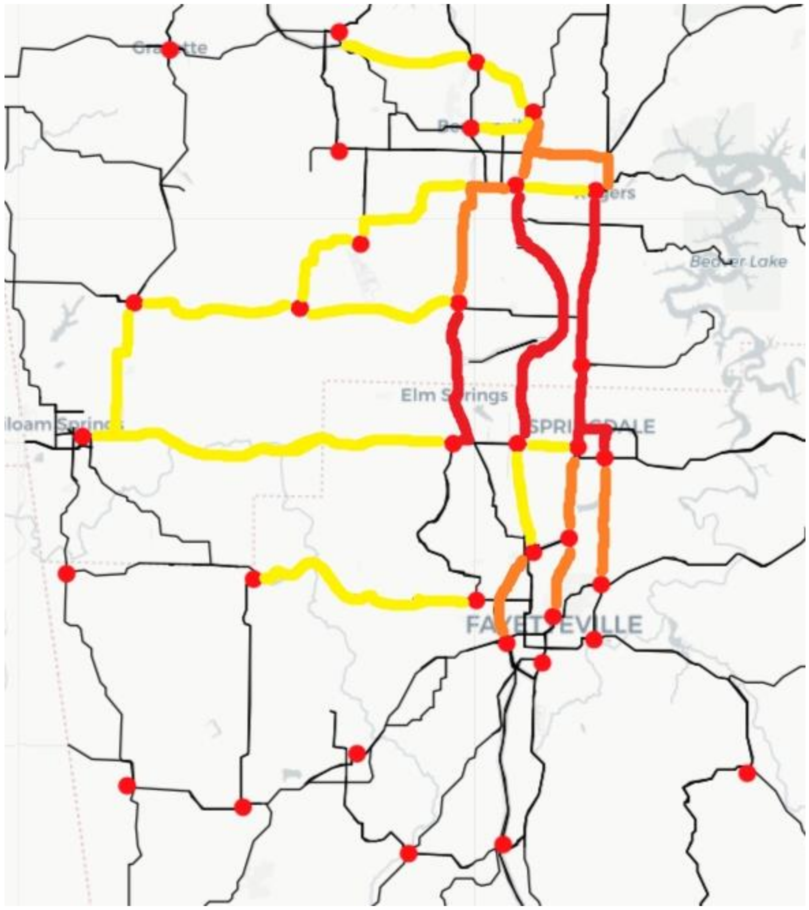

*Figure 3: Resulting map from experiment 1*

In Figure 3 any line that is colored red was interdicted in at least 7 of the 8 instances, orange lines were interdicted in at least 4 instances, and yellow lines were interdicted in at least 1 instance. Roads surrounding higher density metropolitan regions like Fayetteville, Rogers, and Springdale were interdicted the most.

Figures 4 and 5 depict the interdiction solutions from the trials with $K = 15$ and $K = 10$.

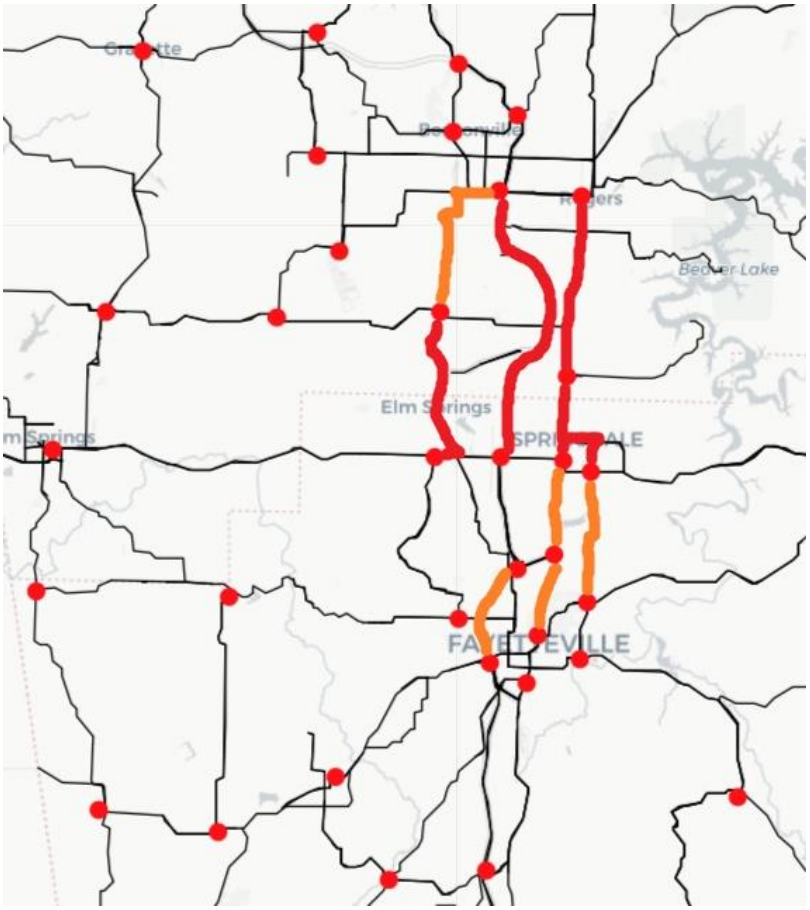


*Figure 4: Visualization of trial with K = 15*

In Figure 4 we see that as the value of $K$ decreases the model continues to interdict major highways between Rogers, Springdale, and Fayetteville and continues to interdict both the $(i, j)$ and $(j, i)$ arcs. A smaller value of $K$ means the model can make less interdictions; thus, the interdictions made for small values of $K$ can be considered as roads that are more critical to the network's performance. This trend continues in Figure 5, which depicts the visualization of a trial run with $K = 10$, in this scenario all 5 arcs were interdicted in the $(i, j)$ and $(j, i)$ direction showing that these 5 arcs have a greater impact on the objective than any other arcs.

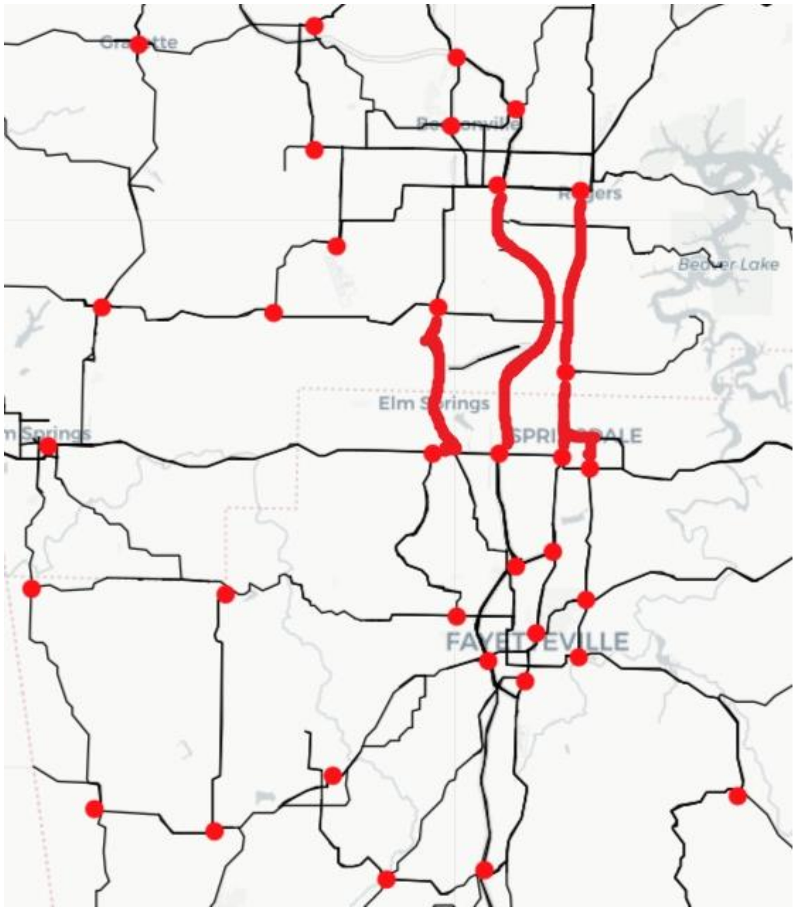

*Figure 5: Visualization of Trial with K = 10*

To investigate how the value of $K$ affects the length of routes we looked at the average distance between four routes in the Northwest Arkansas Region, depicted in Figure 6.

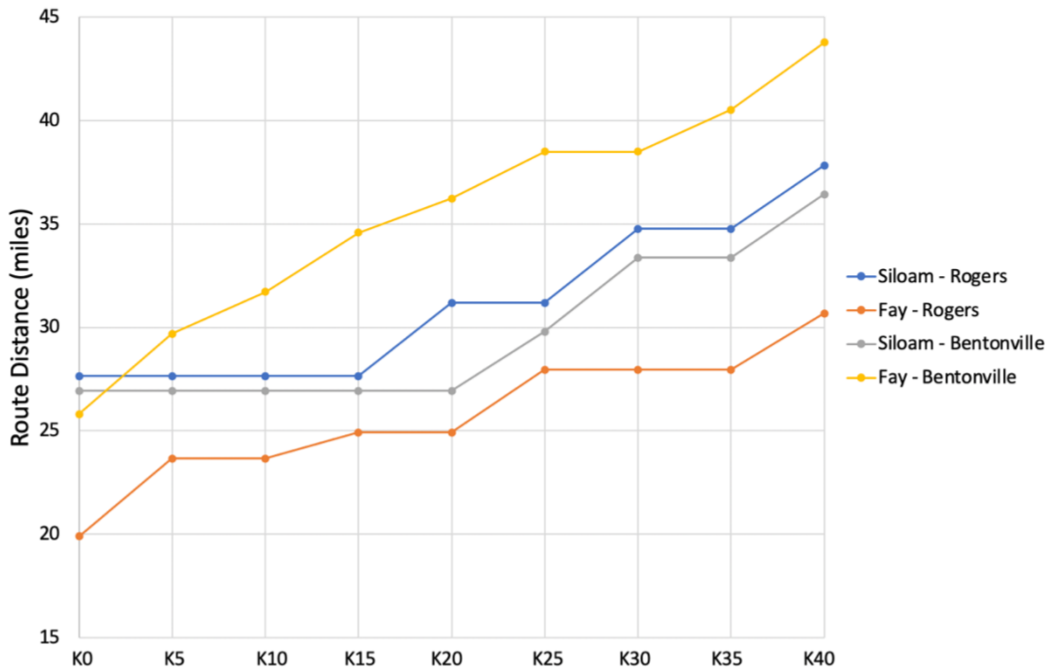


*Figure 6: Distance of four routes as K decreases in first experiment*

We can see that as the number of interdictions allowed increases so too do the distances of each route. This increase is most dramatic in the route between Fayetteville and Bentonville presumably because the model is unable to interdict as many arcs near Fayetteville or Bentonville when the budget $K$ is low, and instead must use up the $K$ budget to interdict key arcs that affect multiple routes, as shown in the five arcs interdicted in Figure 5.

**Modification to allow Multiple Arc Interdiction**

Many of the arcs interdicted in our first model setup are longer potentially signaling that because the cost values $d_{ij}$ per arc are directly related to the length $c_{ij}$ longer arcs may be getting unfairly interdicted more often. We believe these longer arcs are also getting interdicted more often because the budgetary cost $K$ is 1 no matter the length allowing the model to disproportionally affect the length of a route while not significantly affecting the $K$ number of interdictions. To create a fairer scenario we allowed the model to interdict an arc multiple times, up to a set budget for each arc, but standardized the $d_{ij}$ value by setting $d_{ij} = 1, \forall(i,j) \in A$. This then allows the model to interdict longer arcs more frequently but the cost on the budget $K$ is more significantly altered. In this set of experiments, we set $l_{ij} = \lceil c_{ij} \rceil$ for each arc $(i, j) \in A$; thus, we allow the model to interdict longer arcs more often, but each interdiction adds only 1 unit to the length of an arc. Because we now allow for multiple interdictions on an arc, we increased the maximum budget $K$ to 135, which is the sum of the previous interdicted arc lengths in Figure 3. We solve 8 instances in this set of experiments corresponding to $K \in \{15, 30, \ldots, 135\}$ which all solved to optimality in less than 20 seconds.

Figure 7 displays a summary of the results from this set of experiments which allowed the model more freedom to make interdictions by decreasing the cost associated with the $d_{ij}$ values and allowing multiple interdictions on each arc.

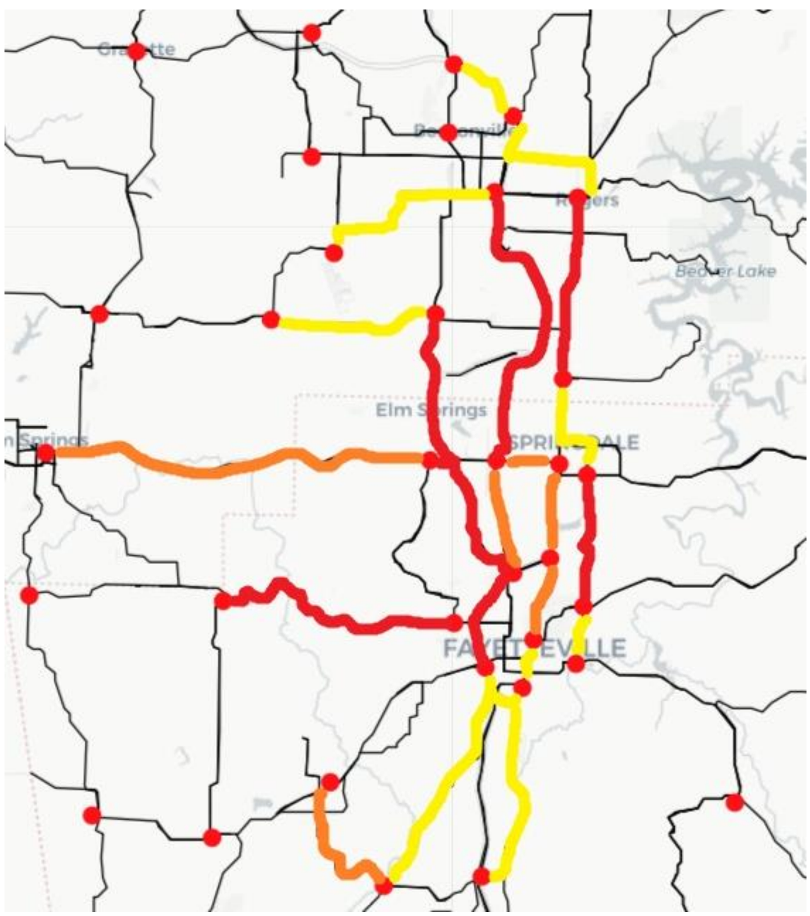

*Figure 7: Visualization of experiment with modified model (4) – (5)*

Because we allowed the model to have more freedom in choosing which routes to interdict, we again wanted to look at the same four routes chosen previously to see how the distance of routes is affected by the budget $K$.

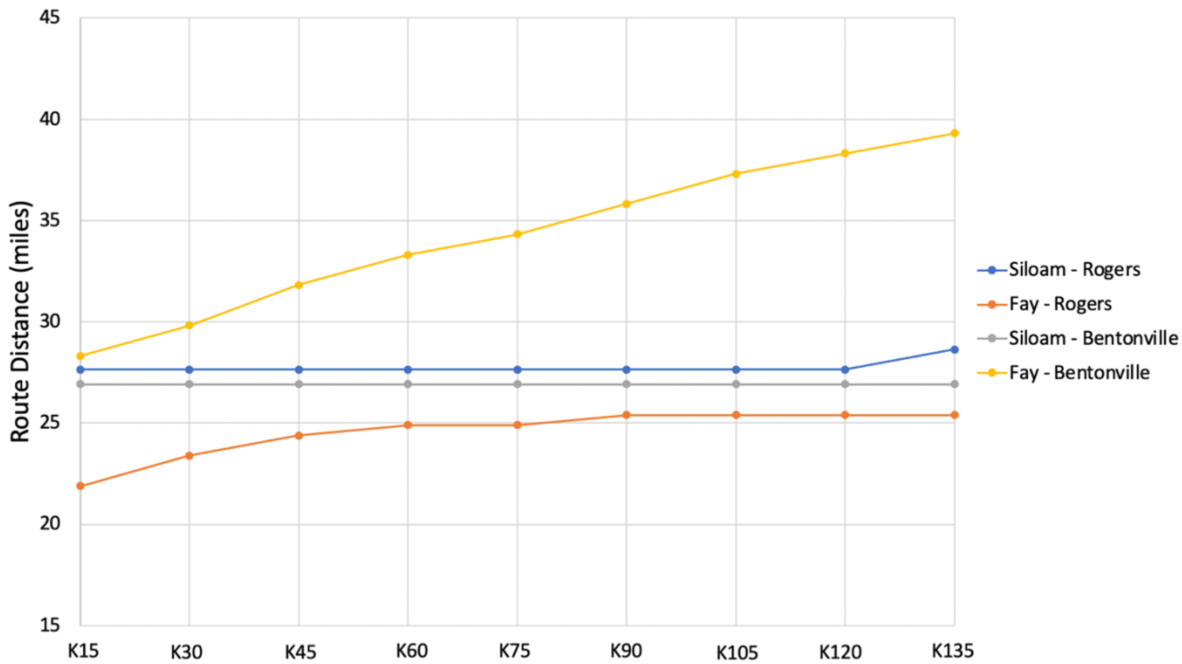


*Figure 8: Distance between routes as $K$ decreases in modified experiment*

In Figure 8 most of the route distances still increase as the budget $K$ increases, but because the new modified model had more freedom in interdiction these increases were more gradual than compared to Figure 7. We can also see that the distance between Siloam Springs and Bentonville was not altered between any of the trial runs which shows that the model no longer interdicted arcs between those two cities. The model also did not interdict arcs between Siloam Springs and Rogers until the maximum budget of $K$. Because the second model did not interdict routes to and from Siloam Springs, we can hypothesize that the arcs from Siloam Springs are not as vulnerable as those within the 3 major cities, that run along Interstate 49, of Bentonville, Springdale, and Rogers.

## Section 5 - Conclusion and Future Research

This paper details the use of a shortest path network interdiction problem to model the vulnerability of road networks by identifying combinations of simultaneous arc disruptions that maximally increase the weighted shortest path distance between multiple origins and destinations. This model was tested with a dataset using selected interstate and state highways from the Northwest Arkansas road transportation network, and was iteratively tested by changing the number of interdictions allowed.

The model was shown to solve to optimality, with a solve time less than 20 seconds, in a network with 110 arcs and 1122 origin destination pairs. In both experiments Interstate 49 between Springdale and Rogers, Arkansas Highway 265 between Springdale and Rogers and the Arkansas Highway 71B Corridor between Springdale and Rogers were interdicted in all trials. These key arcs make sense as Rogers and Springdale make up a large portion of the population in Northwest Arkansas but are only directly connected by three arcs whereas Fayetteville, the largest city in the region, has a greater number of major connections allowing for more potential routes into the city. After we allowed the model more freedom to decide which routes to interdict we continued to see similar arcs being interdicted, but did see more emphasis on interdicting arcs within the Fayetteville area when compared to the first experiment. This could show that the arcs within the Fayetteville area were unfairly discounted in the first experiment. After both experiments our model was able to generate solutions that detail what roads are important for future funding.

This model lays the groundwork for future research that can create a more complete road network by adding municipal and county roads and including the speed limits on each road. Future research could seek to identify more accurate weights for the origin and destination pairs that could be based on road congestion around the nodes, and instead of calculating distance based off straight lines these calculations could be found by using the shapefile data to find distances over roads. Furthermore, future research could use combined demand traffic models that model behavioral factors in a driver's decision making. With the addition of speed limits, more realistic weights for origin and destination pairs, distance values based on road travel, and behavioral factors the model could be tested more accurately in larger road networks allowing for a comprehensive vulnerability analysis. Similarly, because the dataset used was filtered to create more simplicity in running there is room for future testing with larger datasets to find the limits of the proposed interdiction model's solving abilities.

## Appendix

The table below contains the sum of interdictions for the arc $(N, N)$ between all eight trials using different values of $K$. For arcs with a value of 8 these arcs were interdicted in all eight trials meaning they significantly affected the objective values. We used this table to create Figure 3.

*Table A: Results from experiment 1*

| Arc | Sum of Interdictions |
|---|---|
| 23,15 | 8 |
| 27,16 | 8 |
| 26,20 | 8 |
| 20,26 | 8 |
| 16,27 | 8 |
| 20,17 | 7 |
| 20,19 | 7 |
| 19,20 | 7 |
| 17,20 | 7 |
| 15,23 | 7 |
| 19,18 | 6 |
| 27,23 | 6 |
| 9,14 | 5 |
| 14,9 | 5 |
| 13,12 | 5 |
| 12,13 | 5 |
| 17,12 | 5 |
| 12,17 | 5 |
| 18,19 | 5 |
| 23,27 | 5 |
| 26,33 | 5 |
| 33,26 | 4 |
| 33,27 | 4 |
| 27,33 | 4 |
| 6,7 | 3 |
| 7,6 | 3 |
| 21,15 | 3 |
| 15,21 | 3 |
| 14,16 | 2 |
| 22,21 | 2 |

| | |
|---|---|
| 21,22 | 2 |
| 25,23 | 2 |
| 24,25 | 2 |
| 23,25 | 2 |
| 22,25 | 2 |
| 16,14 | 1 |
| 17,16 | 1 |
| 25,22 | 1 |
| 25,24 | 1 |
| 24,27 | 1 |
| 26,27 | 1 |
| 33,31 | 1 |
| 33,32 | 1 |
| 29,32 | 1 |

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