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Development of an Equation to Correlate Brite Tank Temperature and Pressure to Beverage Carbonation Concentration

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Abstract

The Carbo Rock-It™ is an invention created to carbonate beverages at craft breweries. One of the Carbo Rock-It's main features is that it does not add a significant volume of undissolved gas bubbles to the brite tank in which the beverage is carbonated. This may allow temperature and pressure readings from the brite tank to be used to determine the concentration of dissolved carbon dioxide in the beverage continuously in real time. This eliminates the need for manual samplings, which could save breweries time and allow for more controlled, accurate carbonation. The goal of this project was to develop an equation that correlates the brite tank temperature and pressure to volumes of dissolved carbon dioxide so that the Carbo Rock-It can be completely automated. This will eliminate the need for trained workers to continually monitor the carbonation process. Data for beverage temperature, gas pressure from the brite tank, and beverage temperature, gas pressure, and volumes of dissolved carbon dioxide from a Zahm and Nagel meter were collected at thirteen separate trials carbonating a spiked seltzer beverage, Scarlet Letter, from May 2021- July 2021 at Core Brewing in Springdale, AR. The temperature and pressure readings from the brite tank were entered into an equation based on Henry's Law that predicted dissolved volumes of carbon dioxide and compared to measured values for dissolved carbon dioxide from the Zahm and Nagel meter at similar times. These values were linearly regressed and the regression equation was used to calibrate the prediction equations to match the measured data. The overall calibrated predicted values were accurate to ±0.164 vol/vol, which was 5.46% of the full scale volumes reading of 3.0 vol/vol. For the individual trials, the minimum error was ±0.0385 vol/vol, or 1.283% of the full scale, and the maximum error was ±0.2282 vol/vol, or 9.606% of the full scale.

Introduction

The Carbo Rock-It is an invention created by Osborn (2018, 2021) as a means of improving the carbonation process in the craft brewing industry. The current method for carbonating beverages in the craft brewing industry is to bubble carbon dioxide gas through a carbonation stone placed at the bottom of a pressurized carbonation tank known as a brite tank; this process is called forced carbonation. This gas dissolves into the beverage as bubbles rise. Any undissolved gas is collected and vented through a headspace at the top of the carbonation tank. This undissolved carbon dioxide contributes volume to the carbonation tank, and if not vented, pressure will build up inside the tank. Because of this, the carbonation tank must be vented to maintain the maximum pressure allowed for a brite tank. Once the maximum pressure is reached, a pressure release valve opens, releasing carbon dioxide and maintaining pressure inside the brite tank at a set point value. Approximately half of the carbon dioxide gas is not dissolved into the beverage and is vented to the outside air. Undissolved carbon dioxide contributes to wasted materials costs, increased carbon dioxide in the air outside of and around the carbonation tank, and increased carbon footprint of the brewery itself (Osborn, 2018).

The Carbo Rock-It aims to solve the issues presented by forced carbonation; the main goals are reducing carbon dioxide emissions, time, and cost. Figure 1 illustrates the process of carbonating with the Carbo Rock-It. The Carbo Rock-It operates by circulating uncarbonated beverage through a saturation chamber and injecting it with carbon dioxide. This creates a stream of supersaturated beverage that is constantly being added to the brite tank until the desired dissolved carbon dioxide concentration in the beverage is reached (3.0 vol/vol for spiked seltzer). Volumes can be defined as the volume of dissolved carbon dioxide in the beverage per volume of beverage at 0°C. The Carbo Rock-It can carbonate a 120 barrel (1 barrel (bbl) = 31 gallons) tank of spiked seltzer from 0 to 3.0 vol/vol in approximately six hours compared to 72 hours for forced carbonation. Brite tank pressure and temperature can be used to determine volumes of dissolved carbon dioxide with the Carbo Rock-It because it does not add any volume to the brite tank because carbon dioxide gas is dissolved with no bubbles. The brite tank is mixed well when carbonating with the Carbo Rock-It, so offgassing of carbon dioxide will create a pressure in the brite tank corresponding to Henry's Law. Additionally, the Carbo Rock-It dissolves all carbon dioxide into the beverage, reducing both cost and carbon dioxide waste. US Patents 10,077,418, 10,961,488, and Patent Application 20210179983, detail the mechanisms by which this device operates.

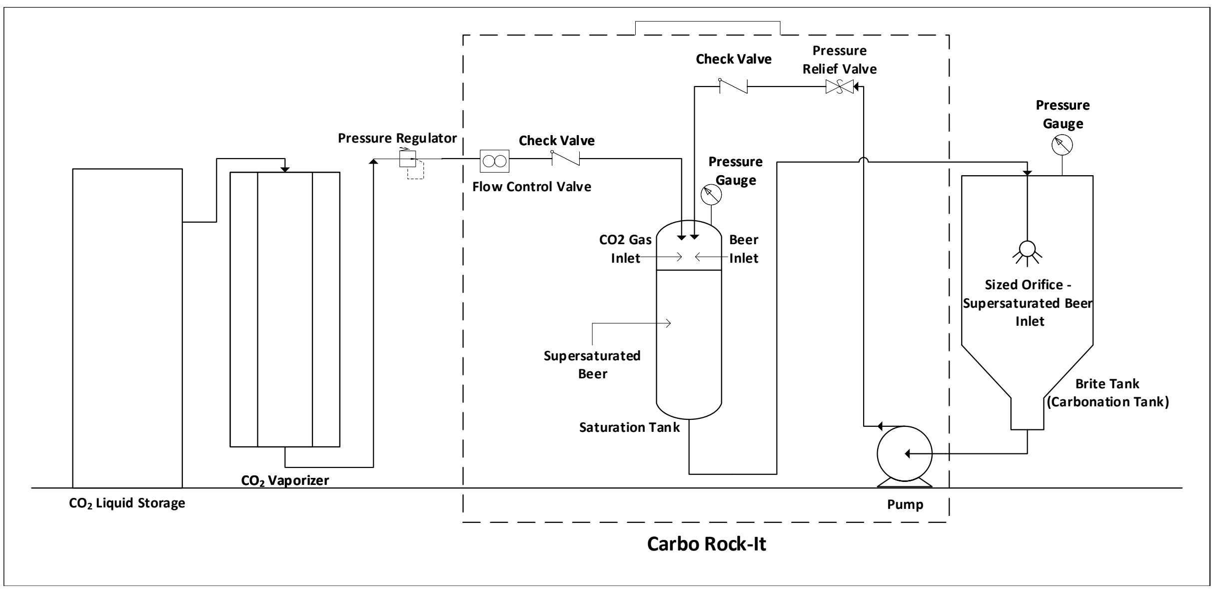

Figure 1. Carbo Rock-It schematics.

The goal of this research is to develop an equation that relates temperature of the beverage and pressure in the brite tank to real-time measurement of the volumes of dissolved carbon dioxide in the beverage. This equation would allow for the Carbo Rock-It to be operated using a programmable logic controller (PLC) for controls to use sensor input from temperature and pressure to indicate the current volumes of dissolved carbon dioxide of the beverage being carbonated and automatically shut off carbonation once a predetermined set point is reached.

Literature Review

Overview of the Brewing Process

Beer brewing involves the following processes: malting, milling, mashing, lautering, sparging, boiling, clarifying, cooling, primary fermentation, secondary fermentation, filtration, carbonation, and packaging (Aroh, 2019). Malting prepares grain for brewing by soaking and then drying grains to stop further growth. These grains are then sent for milling, where they are ground into a powder for mashing. This powder is combined with hot water in a mash tun, where it is agitated to release starches, which become fermentable sugars. This sugary liquid is known as wort. Lautering and sparging separate the wort from the malt and extract any remaining sugars before being sent to a brew kettle to be boiled. Boiling both pasteurizes the beer and imparts bitterness and other flavors from any added hops. The clarifying process separates the solid hop residue from the liquid beer, which is then sent through a heat exchanger for cooling. The temperature to which the beer is cooled varies based on the type of beer being brewed. Once the desired temperature is reached, the beer is sent to a fermenter, where yeast converts sugars into alcohol and carbon dioxide. At the later part of this stage (secondary fermentation), other flavors can be added; this process is referred to as dry-hopping. Once fermentation is complete, the beer is sent to a brite tank and is usually filtered to remove yeast. Once in the brite tank, the beer can be carbonated to the desired level by dissolving carbon dioxide into the beverage, then it is packaged and distributed.

Carbonation

Carbonation is a highly important characteristic of beer. Carbonation imparts mouthfeel and sharpness, and contributes major flavors and aromas (Briggs, 2004). A study conducted by Aquilana et al. (2015) found that “commercial and craft beer” consumers tend to attribute a high significance to aroma, foam, perceived quality, and carbonation, and that Heineken consumers placed the highest level of importance on carbonation, and Peroni consumers ranked both carbonation and foam most important. Level of carbonation also increases the release of flavor volatiles after swallowing (Clark, et al., 2011). Carbonation is particularly important for seltzer products.

Scarlet Letter, the spiked seltzer beverage used in this study, is made by boiling flavor ingredients in water in the beer boil kettle, then cooling and mixing with filtered water and ethyl alcohol in a brite tank. The spiked seltzer is then cooled and carbonated to 3.0 vol/vol and packaged (Core Brewing, 2021, personal communication).

Carbonation Fundamentals

Dissolution of a gas, such as carbon dioxide, in a liquid, such as Scarlet Letter, is described by Henry’s Law, which states that the saturated amount of gas that is dissolved in a liquid solution is directly proportional to the partial pressure of the gas. Henry’s Law is defined as:

$$K_H = \frac{P_v}{X_{gas}} \tag{1}$$

$$K_H = Henry's\ constant, \frac{bar * mol_{liquid}}{mol_{gas}}$$

$$P_v = partial\ pressure\ (absolute) of\ gas, bar$$

$$X_{gas} = mole\ fraction\ of\ gas\ in\ liquid, \frac{mol_{gas}}{mol_{liquid}}$$

Henry's constant, K_H, changes as a function of temperature. Understanding this relationship is essential for controlling carbonation, as it provides a consistent benchmark for brewers to objectively and consistently determine dissolved carbon dioxide concentration, ensuring quality of a beverage. Given a pressure and temperature (which defines K_H), the final volumes of dissolved carbon dioxide can be calculated, and the carbonation process can be stopped when the desired final volumes are reached. Volume meters, such as the Zahm & Nagel and Haffman's gehaltemeter, are typically used to manually determine the volumes of dissolved carbon dioxide in a liquid in a brite tank. The process by which the volume meters measures volumes is described in more detail later in this report.

Dissolved Carbon Dioxide Volumes

The carbonation process ends when the desired volumes of dissolved carbon dioxide is reached. For the beverage carbonated in the tests conducted with the Carbo Rock-It, the target carbonation was 3.0 vol/vol. This differs based on the beverage being carbonated; For example, for beer, an IPA is typically 2.0-2.5 vol/vol and a lager is typically 2.5 vol/vol (CraftBeer.com, 2018). The pressure and temperature of a beverage sample can be determined using a Zahm and Nagel volume meter or a Gehaltemeter. The Zahm and Nagel volume meter process involves filling the device with a sample of the carbonated beverage under the same pressure as the brite tank to avoid degassing, removing any air bubbles, sealing a chamber, and shaking the sample to release dissolved carbon dioxide bubbles while sealed, until an equilibrium pressure and temperature are reached as indicated by gauges connected to the sealed chamber. The sample's dissolved carbon dioxide volumes are read from a chart (Appendix B) based on temperature and pressure readings (Zahm & Nagel, 2019). Using a volume meter chart for beer at 12 Plato, an equation was developed by Osborn (2018) to relate the device's pressure and temperature readings to the dissolved carbon dioxide volumes of the beverage.

$$vol = \frac{(P_{Zahm} + P_{atm})}{0.0013493T^2 + 0.094214T - 4.81904} \tag{2}$$

$$vol = volumes\ of\ dissolved\ CO_2\ in\ sample, \frac{vol\ CO_2}{vol\ liquid}$$

$$P_{Zahm} = pressure\ reading\ from\ Zahm\ and\ Nagel, psi$$

$$P_{atm} = atmospheric\ pressure\ at\ location, psi$$

$$T = temperature\ reading\ from\ Zahm\ and\ Nagel, °F$$

The Haffman's gehaltemeter operates similarly to the Zahm and Nagel volume meter, but it is a more automated process. The gehaltemeter is attached to a sampling port and liquid is allowed to flow into the measuring chamber at the same pressure as the brite tank, allowing no gas bubbles to escape. A handle is slowly pressed which seals the chamber, and a button is pushed, starting the process by sending an electric current through the beverage and degassing for a certain period of time to create pressure. The gehaltemeter then measures pressure and temperature and inputs these values into an equation, which displays a measurement of vol/vol. The equation to relate measured temperature and pressure to volumes of dissolved carbon dioxide is defined in the Haffman's manual as follows (Pentair Haffmans, 2016):

$$vol = \left(A\left(P_g + P_{atm}\right) \times \exp\left(B + \frac{C}{T + 273.25}\right)\right) / \ 100) \times \frac{\rho}{1.977} \tag{3}$$

$$vol = volumes\ of\ dissolved\ CO_2, \frac{vol\ CO_2}{vol\ liquid}$$

$$P_g = gauge\ pressure, bar$$

$$T = liquid\ temperature, C$$

$$\rho = liquid\ density, kg/m^3$$

$$A, B, C = constants$$

For beer:

$$A = 0.967, B = -10.74, C = 2617.25$$

For water:

$$A = 0.967, B = -10.70, C = 2617.25$$

Equations 2 and 3 can be used to calculate the dissolved carbon dioxide concentration in beverages separate from the devices used to determine pressure and temperature. All volume readings from this study were measured using the Zahm & Nagel volume meter.

Carbo Rock-It

Carbo Rock-It operations, illustrated in Figure 1, start with a full brite tank. The Carbo Rock-It pumps beverage from the brite tank into a saturation tank. Highly pressurized (between 80 and 120 psig) carbon dioxide is injected into the top of the saturation tank, where it is mixed with the beverage. The saturation tank pressure operates at an average pressure of 70 psig, which is much higher than the brite tank pressure, which ranges from 0-14 psi throughout the carbonation process. Carbon dioxide gas bubbles dissolve into the beverage as they are vigorously mixed in the saturation tank under high pressure. The concentration of dissolved gas in the beverage will increase with increasing pressure, resulting in a high dissolved concentration of carbon dioxide in the beverage of around 5-7 vol/vols. The beverage then exits the saturation tank through a hose fitted with a nozzle. The nozzle allows the saturation tank to maintain pressure with continuous flow conditions. As the beverage enters the brite tank through the nozzle, beverage with high concentrations of dissolved carbon dioxide is mixed with the beverage with low concentrations of dissolved carbon dioxide resulting in no significant bubbles and no wasted gas. Because the beverage that has passed through the saturation tank has a higher concentration of dissolved carbon dioxide than the desired final concentration (3 vol/vol), the entire volume of the brite tank does not have to pass through the saturation tank. The Carbo Rock-It is finished operating when the final desired concentration of dissolved carbon dioxide is reached (Osborn, 2018).

One of the design features of the Carbo Rock-It is that it recirculates the beverage through the brite tank and dissolves carbon dioxide into the beverage outside of the brite tank. If the gas is dissolved into the beverage in the Carbo Rock-It such that no significant bubbles are added to the brite tank, then the volumentric flow rate into and out of the brite tank is equal and the pressure inside the brite tank will not change. As more gas is dissolved into the beverage, it will eventually become saturated and any additional gas added will exit solution. This will increase the pressure inside the brite tank, thereby allowing the liquid to hold more gas. This allows the pressure in the brite tank to correlate to Henry's Law, or Equations 2 and 3, so that along with temperature, the real time volumes measurement can be known based on brite tank temperature and pressure readings. This could remove the necessity of making manual measurements while carbonating and allow automation.

Importance of Developing Carbo Rock-It Equation

The Carbo Rock-It solves some of the problems posed by forced carbonation. The Carbo Rock-It dissolves almost 100% of the carbon dioxide that is injected into the system and carbonation can be achieved much more rapidly. Because of this, the Carbo Rock-It has the potential to be beneficial to breweries that are currently using forced carbonation withcarbonation stones to carbonate their beverages.

Research Objectives

The Zahm and Nagel and Haffman's methods both determine volumes of dissolved carbon dioxide under controlled ideal conditions created with each unit. The goal of this research is to determine if this principle can be applied to use the brite tank pressure to sufficiently determine volumes of dissolved carbon dioxide under production conditions, and if so, establish an accurate equation to predict the level of dissolved carbon dioxide of the desired liquid at any given time. This equation would be needed for automation of the Carbo Rock-It. The equation would be programmed into a PLC on the Carbo Rock-It, which will monitor the pressure of either the brite tank or the saturation tank and the temperature of the liquid and will stop the device once the desired volumes of dissolved carbon dioxide is reached. The research aims to see if the Carbo Rock-It can make the brite tank function as the sealed chamber in other volume meters.

While data for the Carbo Rock-It has only been collected using Scarlet Letter as the test beverage, the ultimate goal is to be able to apply an equation to many different beverages. This equation should work at any brewery and for any beverage by just using the temperature of the liquid and pressures created within the brite tank.

Materials and Methods

For the past two years, data has been collected at Core Brewing in Springdale, AR throughout carbonation using the Carbo Rock-It of several of their products. However, a majority of the tests were conducted using their spiked seltzer product, Scarlet Letter, and this will be the reference beverage for the development of the equation. The Carbo Rock-It was connected to a 120 bbl brite tank and temperature, pressure, and volumes were monitored throughout the carbonation process. Data from these tests will be analyzed to relate beverage temperature and brite tank pressure to volumes. This relationship will be compared to two other widely accepted relationships: the Zahm & Nagel (Zahm & Nagel, 2019) tables converted to an equation, and the Gehaltemeter (Pentair, 2016) equation. Because the Zahm & Nagel volume meter will be used for manual measurement of the volumes of dissolved carbon dioxide, the equation developed should accurately predict volumes based on temperature and pressure when compared to the volumes value given by the volume meter.

Thirteen Carbo Rock-It carbonation tests were conducted at Core Brewing. A 15/64” orifice brite tank quill was installed in the brite tank before filling it with the beverage to be carbonated. The Carbo Rock-It and brite tanks were cleaned according to Core Brewing’s procedures before filling the brite tank and operating the Carbo Rock-It. Once the brite tank was filled, the beverage was cooled to a desired temperature, of 1.7°C; this temperature was not always reached because of strain on the cooling system in the summer months. Once the beverage reached the desired temperature, the Carbo Rock-It was connected to the brite tank and a carbon dioxide gas line was attached to the Carbo Rock-It. The gas pressure regulator was set to the highest possible pressure under 655 kPa (Huck, 2022). The Carbo Rock-It was operated until the desired volumes of 3.0 vol/vol were reached or the pressure relief valve on the brite tank released, meaning no further carbonation could be done.

While the Carbo Rock-It was operating, data was collected in fifteen minute intervals for the duration of the test. Data was collected for pump outlet pressure, saturation tank pressure, brite tank pressure, gas flow rate, liquid temperature, and ambient air temperature. Electrical current and voltage feeding the pump were also measured intermittently throughout each trial. When the brite tank pressure gauge registered a reading over 0 psig (indicating the beverage in the brite tank was saturated with carbon dioxide), the Zahm & Nagel was used approximately twice each hour to manually determine the volumes of dissolved carbon dioxide in the brite tank. Pressure and temperature from the Zahm & Nagel were recorded so any equation could be used to determine volumes, and the Zahm & Nagel chart was used to determine the corresponding volumes for beer. Each Zahm & Nagel measurement was completed at least twice to ensure accuracy of each reading.

The Zahm & Nagel meter measures volumes of dissolved carbon dioxide in a beverage by removing dissolved gas bubbles from a sample of carbonated beverage by physically agitating the beverage until an equilibrium pressure and temperature are reached. The volume meter contains a pressure and temperature sensor whose readings are entered on a chart that determines volumes. The chart is specific to the Zahm & Nagel meter and beer at 12 plato. The Zahm & Nagel meter costs about $1300 and requires about 10 minutes to obtain a single reading. Proper use of the meter requires training and practice to obtain consistent readings. Typically, two or three readings are collected to ensure accuracy and consistency. There is also a level of human error associated with the Zahm & Nagel volume meter because physical agitation can occur for different lengths of time or intensities, based on the person taking the measurement. The Zahm &

Nagel was also developed specifically for beer, so it could be inaccurately predicted the volumes of Scarlet Letter, since the composition of the two beverages are very different. Because the Zahm & Nagel volume meter will be used for manual measurement of the volumes of dissolved carbon dioxide, the equation developed should accurately predict volumes based on brite tank temperature and pressure when compared to the volumes value given by the properly-used volume meter.

Analysis Method

The beverage temperature and brite tank pressures collected at the carbonation tests will be input to the Zahm & Nagel equation (2) and a derived Scarlet Letter equation to determine volumes. Because the existing equations to predict volumes from temperature and pressure were developed specifically for beer, an equation that reflects the properties of Scarlet Letter must be derived to more accurately represent the beverage. The properties of Henry's Law based on the composition of Scarlet Letter (water and ethyl alcohol) will be used to develop an equation that accurately predicts volumes of dissolved carbon dioxide in Scarlet Letter. The volumes measured with the Zahm & Nagel volume meter will then be plotted against the volumes predicted using the Zahm & Nagel and Scarlet Letter equations based on brite tank pressures. A regression equation from these plots will be used to calibrate the Zahm and Scarlet Letter equations to the manual Zahm & Nagel measurements.

Results and Analysis

Volumes Using Brite Tank Pressure

Before thorough analysis could be conducted, a relationship between brite tank pressure, beverage temperature, and volumes needed to be confirmed. Brite tank pressure and beverage temperature were entered into Equation (2) and these predicted values were compared to the measured values using the Zahm & Nagel volume meter. The brite tank pressures produced a consistent linear relationship, but the brite tank pressures consistently underpredicted the measured values from the Zahm & Nagel meter. Figures 2, 3, and 4 illustrate the relationship between predicted volumes levels using brite tank pressures and the measured volumes levels using the Zahm & Nagel volume meter from several different carbonation tests. From these tests, it appeared there was a relationship between brite tank pressure and volumes, so further analysis with an equation derived specifically for Scarlet Letter was conducted.

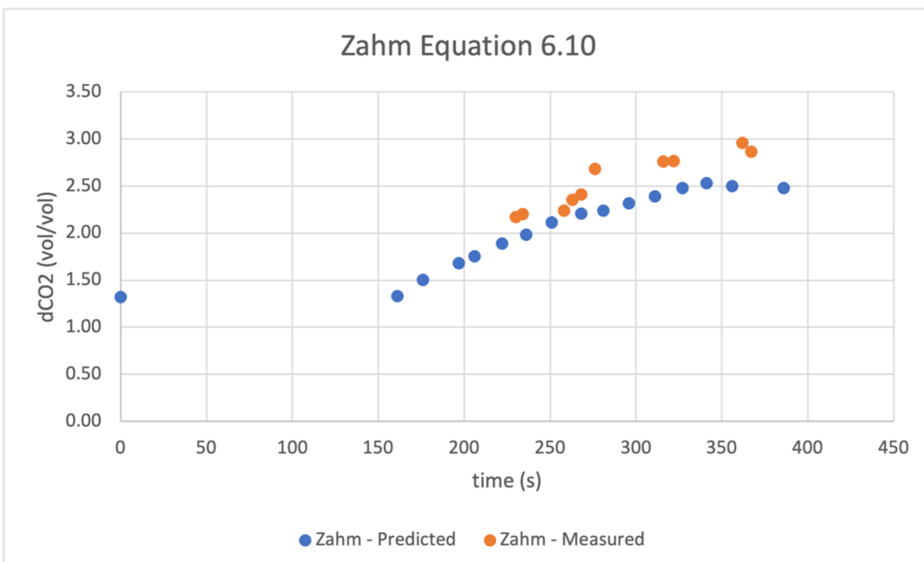

Figure 2. Measured volumes using Zahm & Nagel meter and predicted volumes using brite tank pressures in the Zahm Equation from carbonation test on 6/10/2021

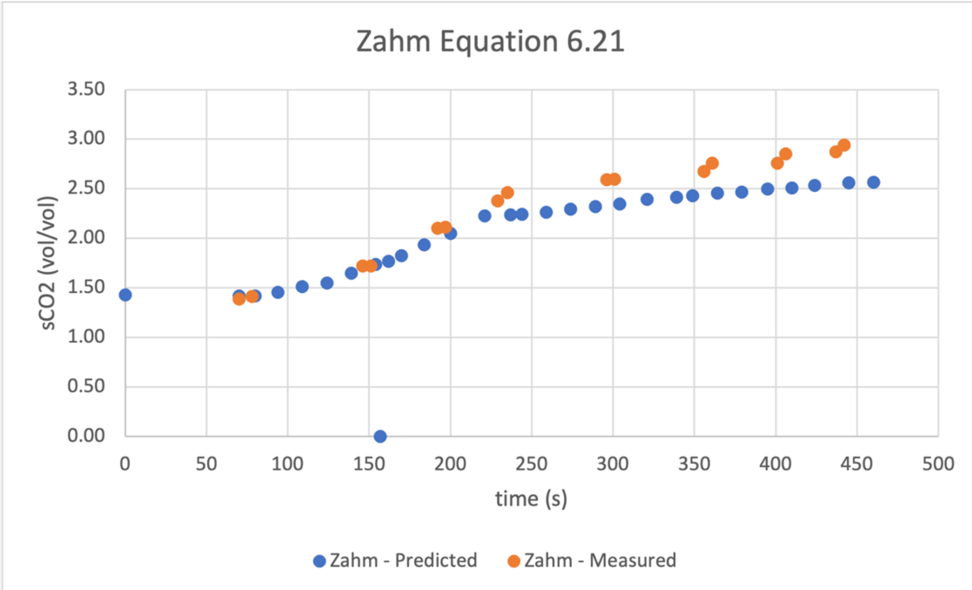

Figure 3. Measured volumes using Zahm & Nagel meter and predicted volumes using brite tank pressures in the Zahm Equation from carbonation test on 6/21/2021

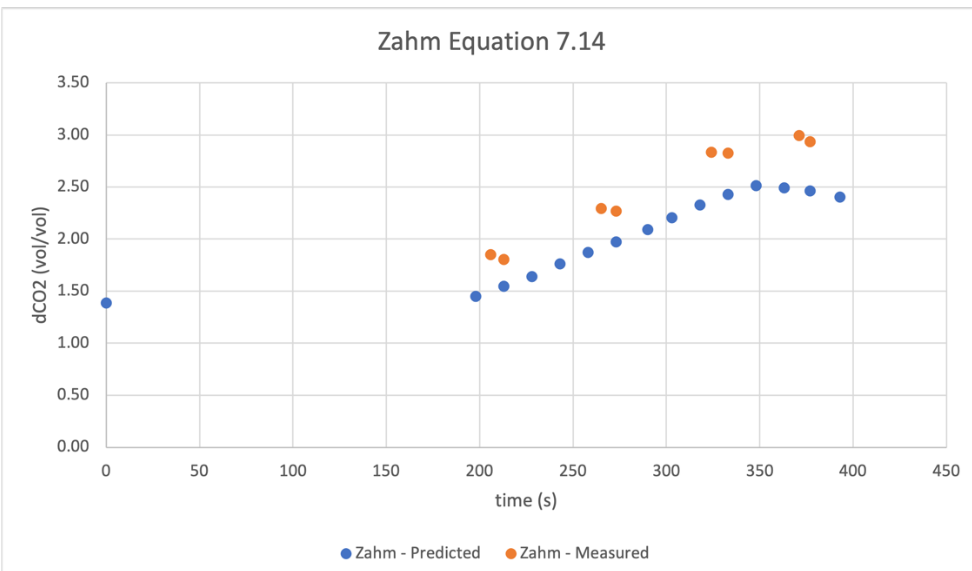

Figure 4. Measured volumes using Zahm & Nagel meter and predicted volumes using brite tank pressures in the Zahm Equation from carbonation test on 7/14/2021

Scarlet Letter Equation

Before an equation can be developed for the Carbo Rock-It, an equation must be developed to measure the level of dissolved carbon dioxide in Scarlet Letter, the test beverage.

This equation must be developed to ensure accurate analysis and compare the actual volumes of dissolved carbon dioxide to the expected volumes of dissolved carbon dioxide. It will also account for the differences in properties between beverages; the Zahm and Nagel and Haffman's equations were developed for beer, but Scarlet Letter differs from beer. The effects of the flavor ingredients on the saturated dissolved carbon dioxide concentration in Scarlet Letter are not known, so for this study, Scarlet Letter is assumed to be composed of ethyl alcohol and water, only with regard to Henry's Law behavior. This equation was developed by relating Henry's constants for carbon dioxide dissolved in water and ethanol at different temperatures and combining the equations using the known alcohol by volume (ABV) content of Scarlet Letter. This equation was developed for temperatures ranging between 0 and 10°C because Scarlet Letter is typically carbonated at these temperatures.

For carbon dioxide in water, data from the National Institute of Science and Technology (NIST) was used to relate Henry's constants to temperature. The NIST defined the following equation to relate Henry's constant to temperature (Henry's Law Data, 2021):

$$k_H = k_H^{\circ} \times \exp\left(\frac{d(\ln(k_H))}{d\left(\frac{1}{T}\right)}\right) \tag{4}$$

Using 0.034 mol/(kg*bar) for k_H° and 2400 K for $d(\ln(k_H))/d(\frac{1}{T})$, Henry's constant was graphed at various temperatures between 0 and 10°C to determine a relationship between temperature in Celsius and solubility in $\frac{bar \times mol\ CO_2}{mol\ H_2O}$.

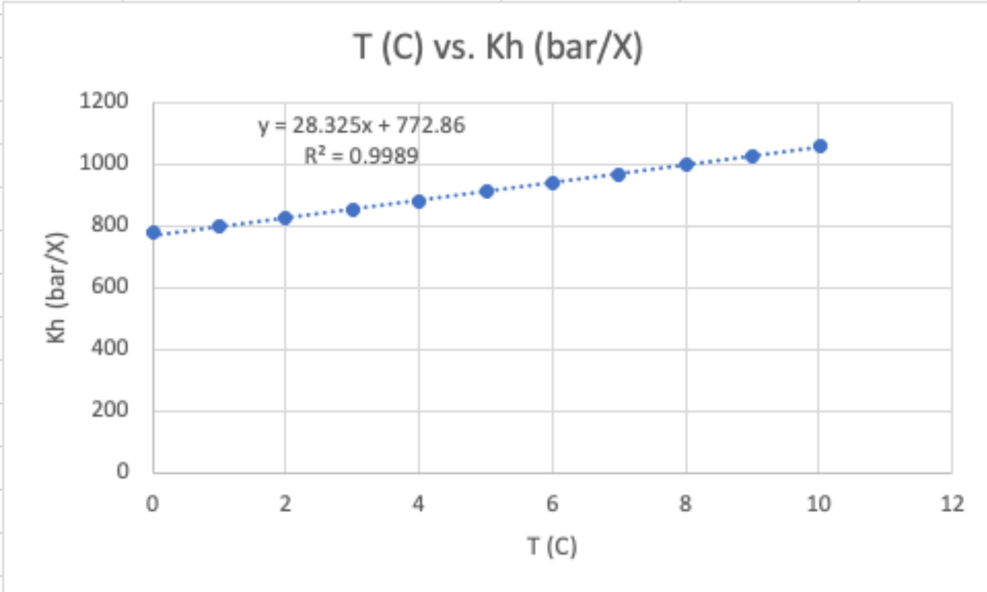

Figure 5. Graph of temperature vs Henry's constant for water.

The relationship between Henry's constant and temperature is defined as:

$$K_{H,W} = 28.325T + 772.86 \tag{5}$$

$$K_{H,W} = Henry's\ constant\ for\ CO_2\ in\ water, \frac{bar}{mole\ fraction\ CO_2}$$

$$T = liquid\ temperature, °C$$

Equation (5) was then converted to the units of $vol\ CO_2/vol\ water$. This equation is defined as:

$$vol_w = (P_{BT} + 14.018) * \frac{1\ bar}{14.5\ psi} * \frac{1}{K_{H,W}} * MW_{CO_2} * \frac{1}{MW_W} * \frac{1}{\rho_{CO_2}} * \rho_W \tag{6}$$

$$vol_w = volumes\ of\ dissolved\ carbon\ dioxide\ in\ water, \frac{vol\ CO_2}{vol\ water}$$

$$P_{BT} = brite\ tank\ pressure, psig$$

$$K_{H,W} = Henry's\ constant\ for\ CO_2\ in\ water, \frac{bar}{mole\ fraction\ CO_2}$$

$$MW_{CO_2} = 44.01\ \frac{g}{mol}\ , MW_W = 18.01\ \frac{g}{mol}$$

$$\rho_{CO_2}(0\,°C) = 1.977\ \frac{kg}{m^3}\ , \rho_W(0\,°C) = 1000\ \frac{kg}{m^3}$$

For ethanol, a study was conducted by Decultot et. al (2019) to determine the solubility of carbon dioxide in alcohols from 283.15K to 373.15K and up to 6 MPa. An equation was found to correlate Henry's law constant for carbon dioxide dissolved in ethanol to temperature. This equation was extrapolated from 283.15K down to 273.15 K and converted to the units of bar/mole fraction. The following equation was defined to relate mole fraction of dissolved carbon dioxide to temperature in degrees C:

$$K_{H,E} = 2.27T + 81.9 \tag{7}$$

$$K_{H,E} = Henry's\ constant\ for\ CO_2\ in\ ethanol, \frac{bar}{mole\ fraction\ of\ CO_2}$$

$$T = liquid\ temperature, °C$$

$K_{H,E}$ was then converted to the units of $vol\ CO_2/vol\ ethanol$. This equation is defined as:

$$vol_e = (P_{BT} + 14.108) * \frac{1\ bar}{14.5\ psi} * \frac{1}{K_{H,E}} * MW_{CO_2} * \frac{1}{MW_e} * \frac{1}{\rho_{CO_2}} * \rho_e \tag{8}$$

$$vol_e = volumes\ of\ dissolved\ carbon\ dioxide\ in\ ethanol, \frac{vol\ CO_2}{vol\ ethanol}$$

$$P_{BT} = brite\ tank\ pressure, psig$$

$$K_{H,E} = Henry's\ constant\ for\ CO_2\ in\ ethanol, \frac{bar}{mole\ fraction\ of\ CO_2}$$

$$MW_{CO_2} = 44.01\ \frac{g}{mol}\ , MW_e = 46.07\ \frac{g}{mol}$$

$$\rho_{CO_2}(0\,°C) = 1.977\ \frac{kg}{m^3}\ , \rho_e(0\,°C) = 806\ \frac{kg}{m^3}$$

Scarlet Letter contains 4.3% alcohol by volume, meaning that 4.3% of the volume of Scarlet Letter is ethanol and 95.7% is water. Using this ratio, the two equations for water and ethanol were combined to create an equation for Scarlet Letter. This equation is defined as the following:

$$vol_{SL} = 0.043 vol_e + (1 - 0.043) vol_w \tag{9}$$

Entering the equations for vol_e and vol_w, the Scarlet Letter equation becomes:

$$vol_{SL} = 0.043(\frac{26.859(P_{BT} + 14.108)}{K_{H,E}}) + (0.957)\frac{85.244(P_{BT} + 14.108)}{K_{H,W}}) \tag{10}$$

Equation (10) will be used to determine volumes from temperature and pressure data from both the brite tank and Zahm meter. This will serve as a means of comparison for the accuracy of the Scarlet Letter equation and will also be used as a standard for developing the Carbo Rock-It equation. The temperature and pressure data from both the brite tank and Zahm & Nagel meter will also be used to determine volumes levels based on the Equation (2) because the carbonation of Scarlet Letter and many other non-standard beer beverages are measured using the standard beer equation or the chart from the Zahm & Nagel meter.

Application of Equations to Data

The 13 carbonation tests conducted with the Carbo Rock-It resulted in brite tank pressure and liquid temperature at fifteen-minute intervals. Using these values and four different volume equations, the predicted volumes of dissolved carbon dioxide were recorded at each time step. The predicted equations are as follows:

Name:	Equation:	Units:
Zahm & Nagel	$vol = \frac{(P_{Zahm} + P_{atm})}{0.0013493T^2 + 0.094214T - 4.81904}$	P=psi T=°F

Haffman's Gehaltemeter	$vol = \left((0.9863\left(P_g + P_{atm}\right) e^{\left(-9.9394+\frac{2401.9}{T+273.25}\right)}\right) / 100) \times \frac{\rho}{1.977}$	P=psi T=°C $\rho = \frac{kg}{m^3}$
Henry's Law – Water	$vol = \frac{85.244(P_{BT} + 14.108)}{K_{H,W}})$	P=psi $K_H = \frac{bar}{x_{CO_2}}$
Scarlet Letter	$vol = 0.043(\frac{26.859(P_{BT} + 14.108)}{K_{H,E}}) + (0.957)\frac{85.244(P_{BT} + 14.108)}{K_{H,W}})$	$vol = \frac{vol\ CO_2}{vol\ liq.}$

Table 1. Summary of equations used to assess volumes of dissolved carbon dioxide.

The volume level was also manually measured with a Zahm & Nagel volume meter consistently throughout every test. Two measurements were taken each time the Zahm & Nagel was used. The Zahm & Nagel measures pressure and temperature and these values were input into the above equations to find the measured volumes of dissolved carbon dioxide. This process was repeated for all thirteen tests and the results were compiled into a single spreadsheet. There did not appear to be any difference between the equations, except for offset values, so it was decided that there was no advantage for further analysis of the Gehaltemeter or water equations. The Scarlet Letter equation was formulated specifically for the test beverage and the Zahm & Nagel is the volume meter standardly used for measurement, and is often used for beverages that are not beer, so these equations were the most relevant to the data set.

To find the relationship between the predicted and measured values for volumes of dissolved carbon dioxide, these values were graphed against each other with predicted values on the x-axis and measured values on the y-axis, and a linear regression performed, as shown in Figures 6 and 7.

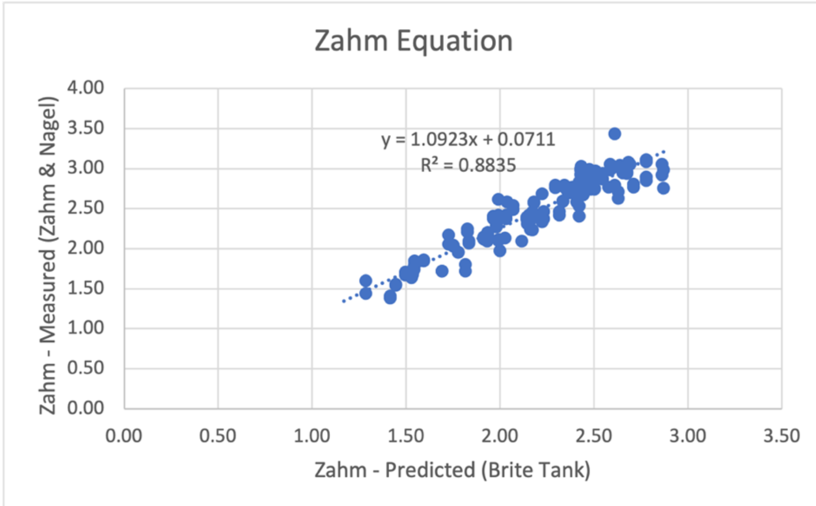

Figure 6. Predicted vs. measured volumes of dissolved carbon dioxide using the Zahm & Nagel equation.

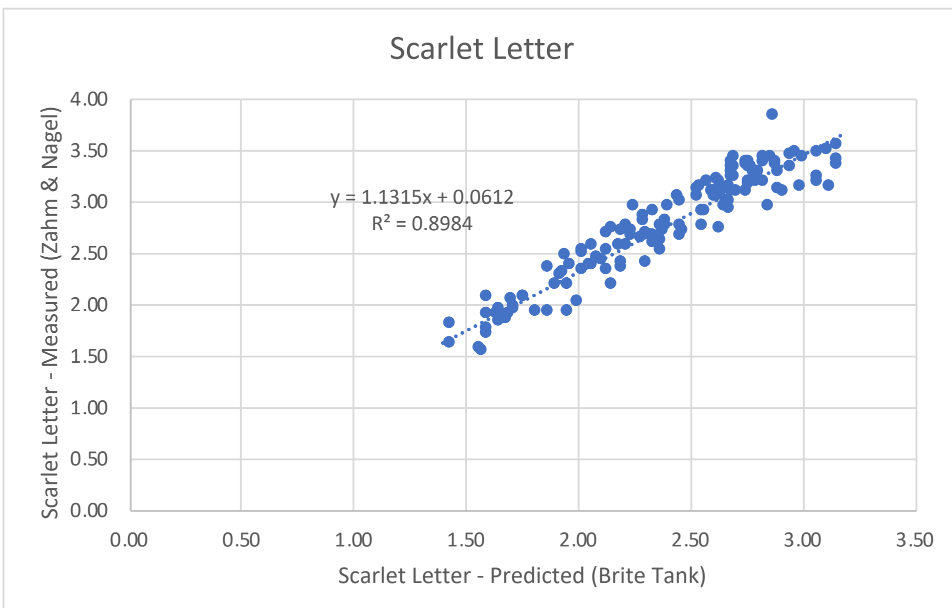

Figure 7. Predicted vs. measured volumes of dissolved carbon dioxide using the Scarlet Letter equation.

The regression equations from Figures 6 and 7 were used to calibrate the existing Zahm & Nagel and Scarlet Letter equations for predicting volumes from brite tank data to the measured

Zahm & Nagel data. These calibration equations were then tested for each individual carbonation trial. The final calibration equations are as follows:

$$vol_{SL\ eqn} = 1.1315 \times (\frac{103.76(P_{BT} + P_{atm})}{(206.75T + 7459.48)}) + \frac{81.53(P_{BT} + P_{atm})}{(28.325T + 772.86)}) + .0612 \tag{11}$$

$$P_{BT} = brite\ tank\ pressure, psig$$

$$T = liquid\ temperature, °C$$

$$vol_{Zahm\ eqn} = 1.0923\left(\frac{P_{BT} + P_{atm}}{.0013493T^2 + .094214T + 4.81904}\right) + .07011 \tag{12}$$

$$P_{BT} = brite\ tank\ pressure, psig$$

$$T = liquid\ temperature, °F$$

Using the measured and calibrated prediction values, error was calculated using the following equations:

$$Squared\ Error = (measured - predicted)^2 \tag{13}$$

$$Standard\ Error = \sqrt{\frac{\sum Squared\ Error}{number\ of\ error\ measurements}} \tag{14}$$

$$\%\ Full\ Scale\ Error = \frac{Standard\ Error\ (\frac{vol}{vol})}{3\ (\frac{vol}{vol})} \tag{15}$$

The following error values were obtained:

	Zahm Equation		Scarlet Letter Equation	
Test	Standard Error (vol/vol)	Error - % Full Scale	Standard Error (vol/vol)	Error - % Full Scale
7.28	0.0669	2.231	0.0570	1.901
7.26	0.1657	5.523	0.1630	5.434
7.19	0.0920	3.067	0.0866	2.886
7.14	0.1212	4.041	0.1547	5.158
7.12	0.1297	4.324	0.1243	4.144
6.26	0.2382	7.941	0.2882	9.606
6.24	0.0516	1.719	0.0385	1.283
6.21	0.1259	4.197	0.1537	5.123
6.17	0.1124	3.747	0.1351	4.503
6.15	0.2045	6.816	0.2099	6.997
6.10	0.1084	3.614	0.1271	4.237
5.24	0.2119	7.062	0.2118	7.061
5.17	0.2460	8.199	0.2467	8.222
OVERALL	**0.1539**	**5.131**	**0.1640**	**5.462**

Table 2. Summary of error using calibrated volume equations. The lowest, highest, and average values were highlighted for each equation.

The volumes of dissolved carbon dioxide were graphed over time using both the measured values and the predicted values with the calibrated equations for each test. The graphs from the tests with the highest standard error, lowest standard error, and nearest to average standard error with the Scarlet Letter equation are shown below. The results from the rest of the tests can be found in the appendix under "Volumes of Dissolved Carbon Dioxide for Each Trial".

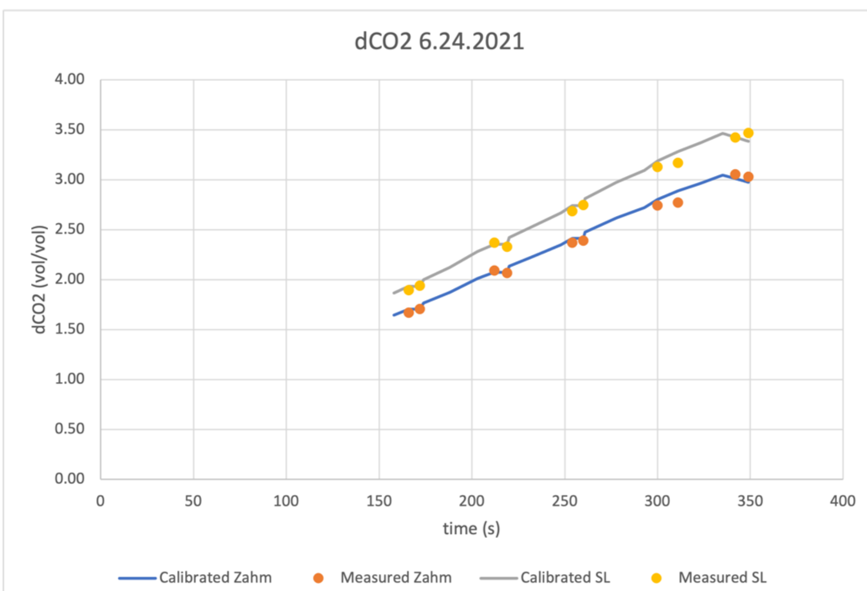

Figure 8. Volumes of dissolved carbon dioxide with calibrated predicted equations. This test had the lowest error.

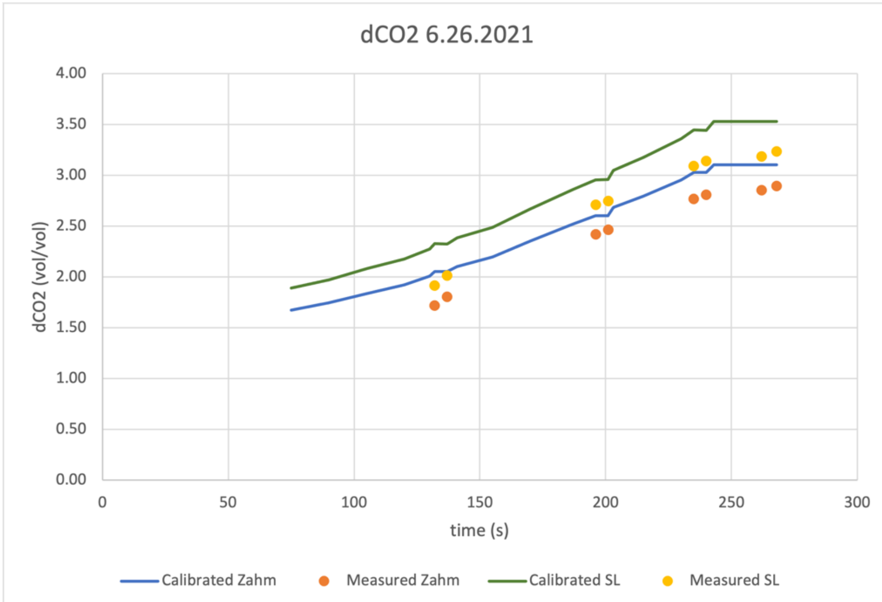

Figure 9. Volumes of dissolved carbon dioxide with calibrated predicted equations. The test had the highest error for Scarlet Letter.

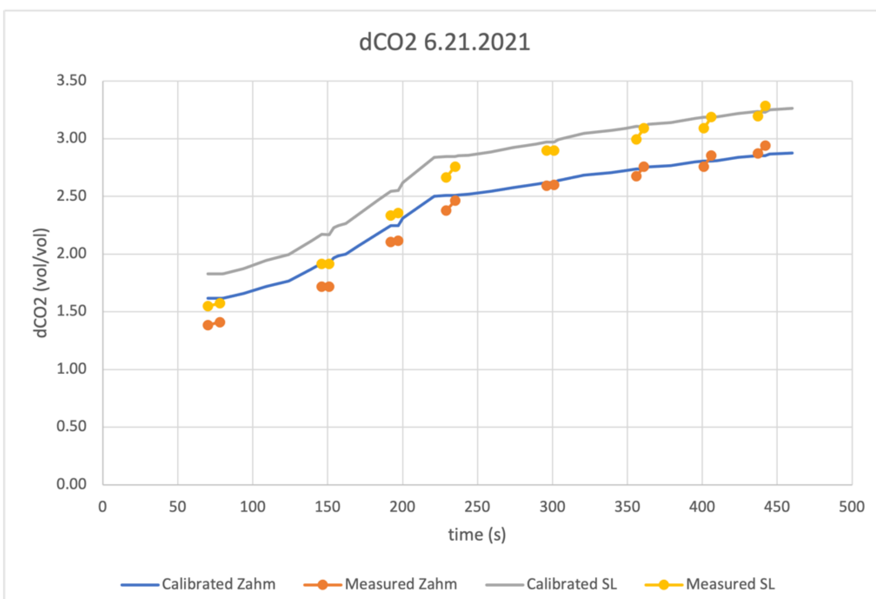

Figure 10. Volumes of dissolved carbon dioxide with calibrated predicted equations. The test had the average standard error when using the Scarlet Letter equation.

Discussion of Results

The linear repression for the Scarlet Letter equation produced a slope of 1.1315 and an offset value of 0.0612. A slope greater than one and an offset greater than zero indicates that, on average, the predicted volumes using brite tank pressure tended to underpredict the measured volumes using the Zahm & Nagel volume meter.

The calibrated equations appear to accurately predict the actual volumes of dissolved carbon dioxide based on the measured volumes using the Zahm & Nagel volume meter. Based on the graphs (Figures 8, 9, 10**,** Appendix), there does not appear to be a consistent problem of over- or underpredicting vol/vol, as would be expected after calibration. The predicted volumes level is usually close to the measured volumes level, but can be both lower and higher than the measured value. The percent full scale error ranged from 1.28% to 9.61%. The average error was around 5%, which correlates to a ±0.15 vol/vol fluctuation. This value is higher than ideal, so a manual Zahm & Nagel measurement is still recommended at the end of the carbonation process. Because each measurement device has an error associated with its readings, the derived equation cannot be expected to have a higher accuracy than the devices that produce the inputs to the equation. The pressure gauge used is accurate to ±3% in the first and last quarter of the gauge range and ±2% in the middle half of the gauge range. The thermocouple used is accurate to ±.05°F. To determine the error in measurement, a sample point at the end of carbonation (35°F and 14 psi) was varied to reflect the maximum and minimum errors in each of these measurements (see Table 3). From this process, the error range was found to be ± .0425 vol/vol (±.1.42% full scale).

Pressure Gauge Error (%)	Brite Tank Pressure (psig)	Thermocouple Error (%)	PLC Liquid Temp (deg F)	Zahm (C) dCO2 (vol/vol)	Squared Error - SE (%)	SL (C) dCO2 (vol/vol)	Squared Error - SE (%)
3	14.42	0.05	35.0175	3.25	0.00031	3.69	0.00274
3	14.42	0	35	3.25	0.00000	3.69	0.00287
3	14.42	-0.05	34.9825	3.25	0.00031	3.69	0.00300
0	14	0.05	35.0175	3.20	0.00031	3.63	0.00000
0	14	0	35	3.20		3.64	
0	14	-0.05	34.9825	3.21	0.00031	3.64	0.00000
-3	13.58	0.05	35.0175	3.16	0.00031	3.58	0.00299
-3	13.58	0	35	3.16	0.00000	3.58	0.00287
-3	13.58	-0.05	34.9825	3.16	0.00031	3.58	0.00275
				SUM SE:	0.0018	**SUM SE:**	0.0145
				Std Error:	0.0152	**Std Error:**	0.0425

Table 3. Summary of instrument error calculations.

The error from the Zahm & Nagel is difficult to quantify. One major issue with the Zahm & Nagel is that all bubbles are supposed to be removed from the beverage sample when taking a measurement with the volume meter. This is nearly impossible to do when sampling Scarlet Letter because carbon dioxide degasses from water much more easily than beer, since there is no component in water to slow bubble formation. Other sources of error with manual measurement include differing agitation lengths and intensities, and amount of liquid dispelled from the sampling port prior to attaching the Zahm & Nagel. Dispelling liquid helps to ensure the sample of beverage in the volume meter is representative of the brite tank volumes level, but the amount of liquid that was dispelled before sampling was not standardized among measurements. All of these factors are difficult to quantify, and therefore, it is hard to know the accuracy of the measured volumes. However, because the predicted values were calibrated to a large number of Zahm measurements, the equation should predict the actual volumes of dissolved carbon dioxide accurately enough for the applications of this project.

The calibrated Scarlet Letter equation predicted volumes slightly more accurately than the calibrated Zahm equation; this is expected as the Zahm equation was developed for beer, and the Scarlet Letter equation directly reflects the constituents of the test beverage. However, both equations can be used to predict the volumes in the brite tank at any given time. The calibrated Zahm equation will become more important when the Carbo Rock-It is used to carbonate other beverages, such as beer, instead of Scarlet Letter.

Discussions

One important fact to note is that the Carbo Rock-It was developed on the assumption that no bubbles will enter the brite tank. Since development and testing, it has been found that this is not true, but the bubbles do not have any effect on the carbonation rate or total carbonation time. This is because the bubbles are either dissolving before they add pressure to the tank or because the bubbles do not have enough volume to significantly affect pressure within the brite tank. If the bubbles affected the pressure reading within the brite tank, pressure would have read higher on the gauge than was actually represented in the tank. This would lead the dissolved volumes equation to overpredict the actual volumes of dissolved carbon dioxide, but it consistently underpredicted throughout each test.

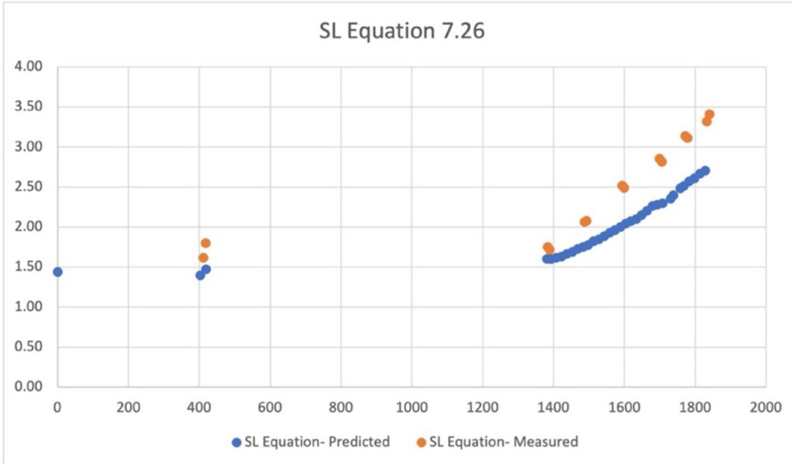

Figure 11. Predicted volumes of dissolved CO2 using Scarlet Letter equation vs. measured dissolved volumes of CO2 using Zahm equation from test on 7.26

At the completion of each carbonation test, the head brewer consistently noted that the Scarlet Letter did not taste as carbonated as he wanted it to be, despite the Zahm & Nagel measurements indicated that the Scarlet Letter was carbonated to 3.0 vol/vol. Though the exact reason for this is not known, it was speculated that water degasses much more easily than beer. For each sample from the brite tank, the beverage must pass through several fittings before it can be poured for tasting. The carbonated water is likely to degas while it is being poured, causing the sample to poorly represent the actual carbonation level of the brite tank.

Summary and Conclusion

The Carbo Rock-It is a device that carbonates beverages in a manner that could be beneficial to craft breweries. Before this product can be sold commercially, an equation must be developed to allow automation of system operations based on measured volumes of beverages from temperature and pressure readings from the brite tank. Analysis of data collected over thirteen carbonation trials at Core Brewing revealed that there is a direct relationship between pressure, temperature, and volumes of dissolved carbon dioxide. This relationship can be expressed by Equation (11). This equation can be programmed into a PLC, which could determine volumes readings in real time and automatically turn off the Carbo Rock-It when the desired concentration of dissolved carbon dioxide is reached (i.e. 3.0 vol/vol for Scarlet Letter). Equation (11) produced an error range of 1.28%-9.61%**,** which is reasonable, but still higher than wanted for application in an operating craft brewery. This error is comparable to the error in measurement of the parameters of Equation (11) using a pressure gauge and thermocouple (±.0425 vol/vol, 1.45% full scale). While this equation appears to be a good fit for the data, additional tests should be run, both under the same conditions as previous tests and at a new brewery to validate that the equation works outside of the test conditions.

The calibrated Scarlet Letter equation (Equation (10)) makes it possible to accurately monitor volumes in a brite tank throughout the carbonation process. This provides more data than manual volumes measurements using a volume meter, which is important if rapid changes were to occur that differ from typical carbonation.

Recommendations and Future Work

A senior design team is programming a PLC to use the Scarlet Letter equation to display volumes of dissolved carbon dioxide continuously in real time and turn off Carbo Rock-It operations when a user-defined setpoint for volumes is reached. While the Scarlet Letter equation appears to fit the data well, new carbonation tests should be run at Core Brewing to check the equation against a new set of data. This same equation derivation process could be repeated with the saturation tank pressures instead of the brite tank pressures. This would enable the Carbo Rock-It to be a self-contained unit (i.e. a gauge would not have to connect from the brite tank to the Carbo Rock-It), which would allow greater ease of use.

Currently, the Carbo Rock-It is mainly used for Scarlet Lettter at Core Brewing. To check the accuracy of the Zahm equation used on brite tank data, more beer carbonation tests could be conducted. Tests at other breweries would also help validate the equation, as operations at another location could change the results of the readings. For example, a gehaltemeter or a pressure gauge could be calibrated differently than at another brewery, which would skew this equation to better match Core Brewing's operations. Unintentional biases such as this should be double-checked before the Carbo Rock-It is ready for commercialization as a completely automated device.

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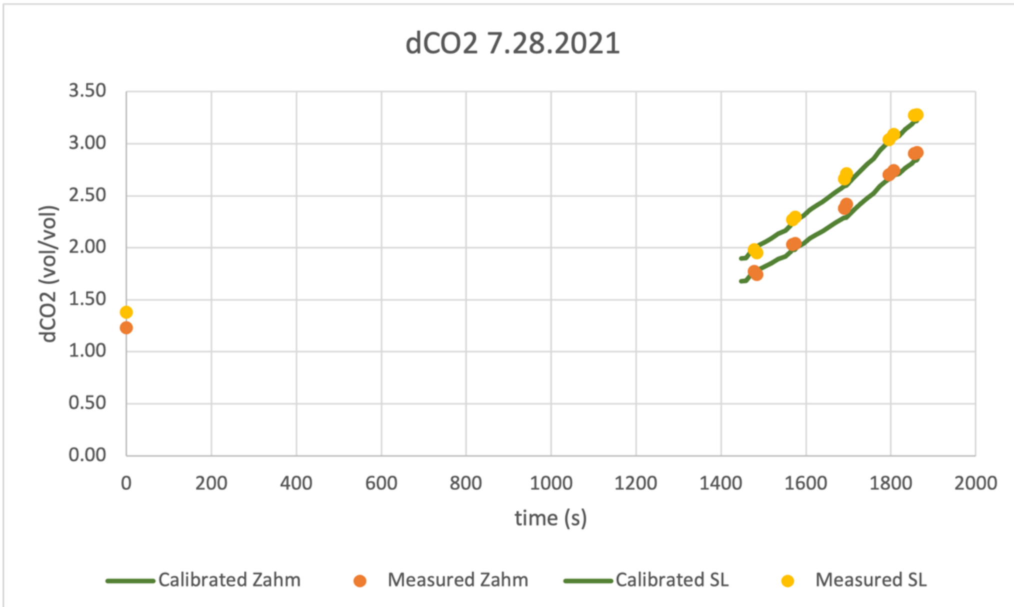
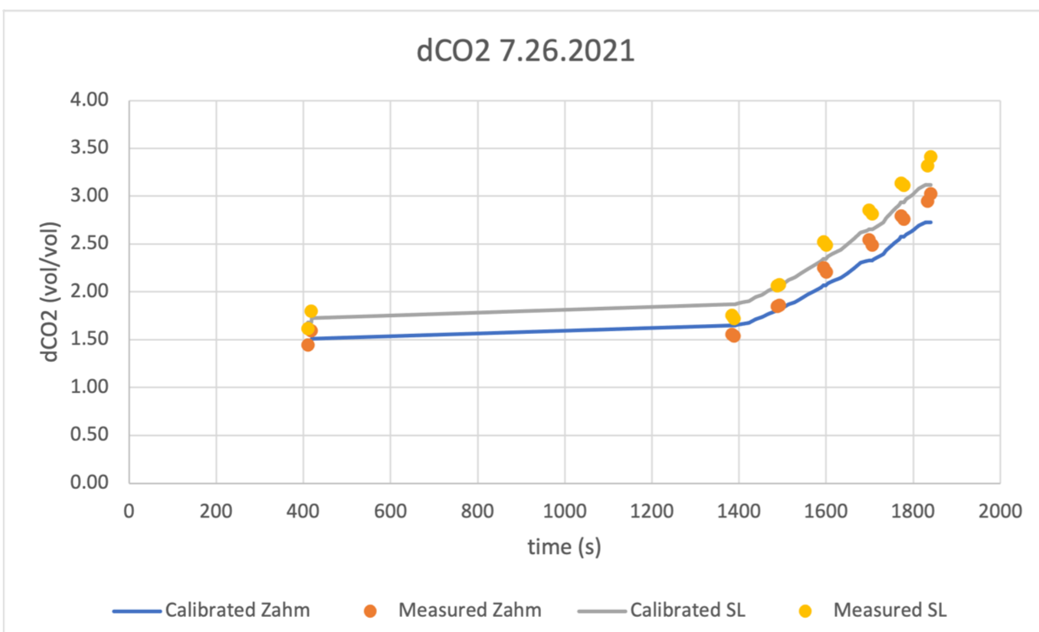
Appendix

A- Volumes of Dissolved Carbon Dioxide for Each Trial

7.28.21



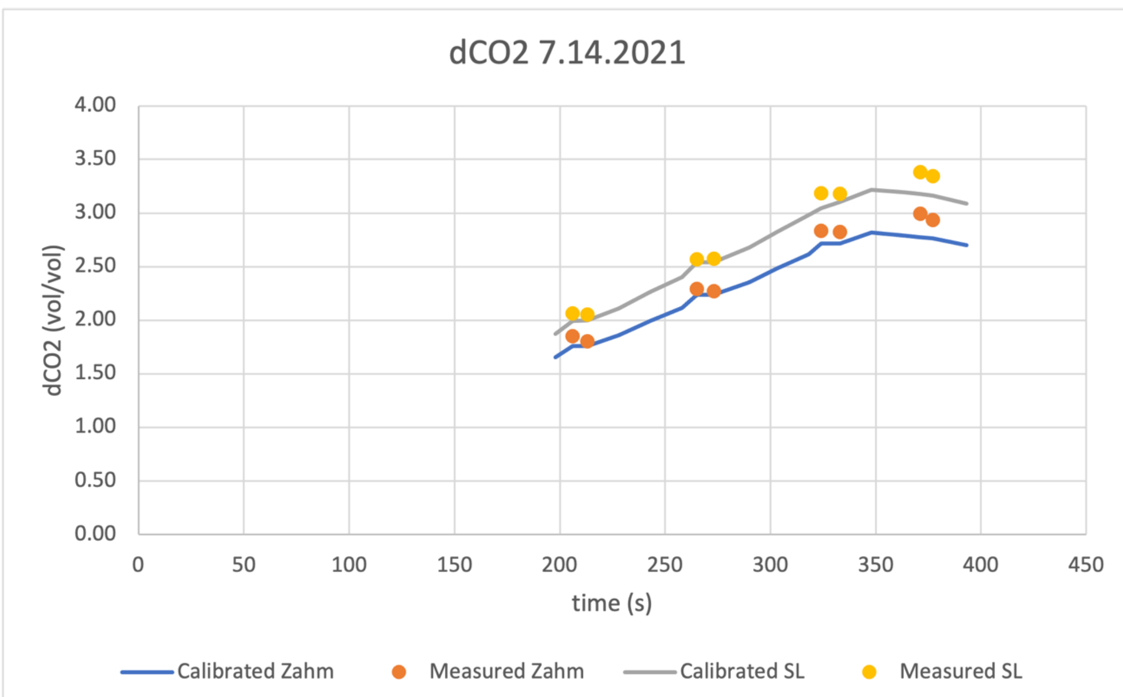
7.26.21



7.19.21


dCO2 7.19.2021
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
dCO2 (vol/vol)
0
50
100
150
200
250
300
350
400
450
time (s)
Calibrated Zahm
Measured Zahm
Calibrated SL
Measured SL

7.14.21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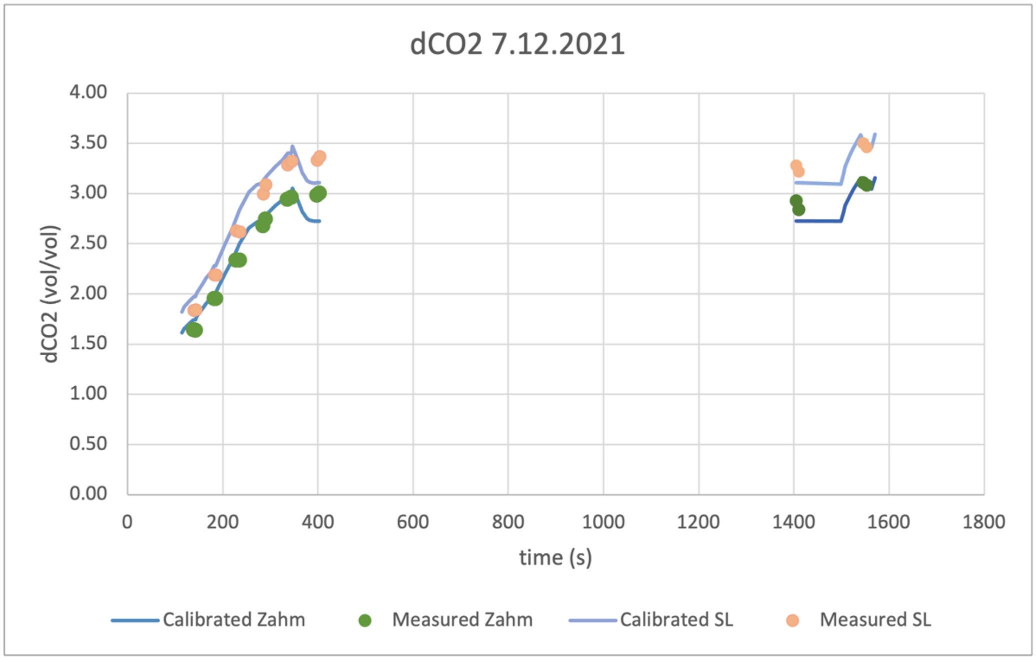
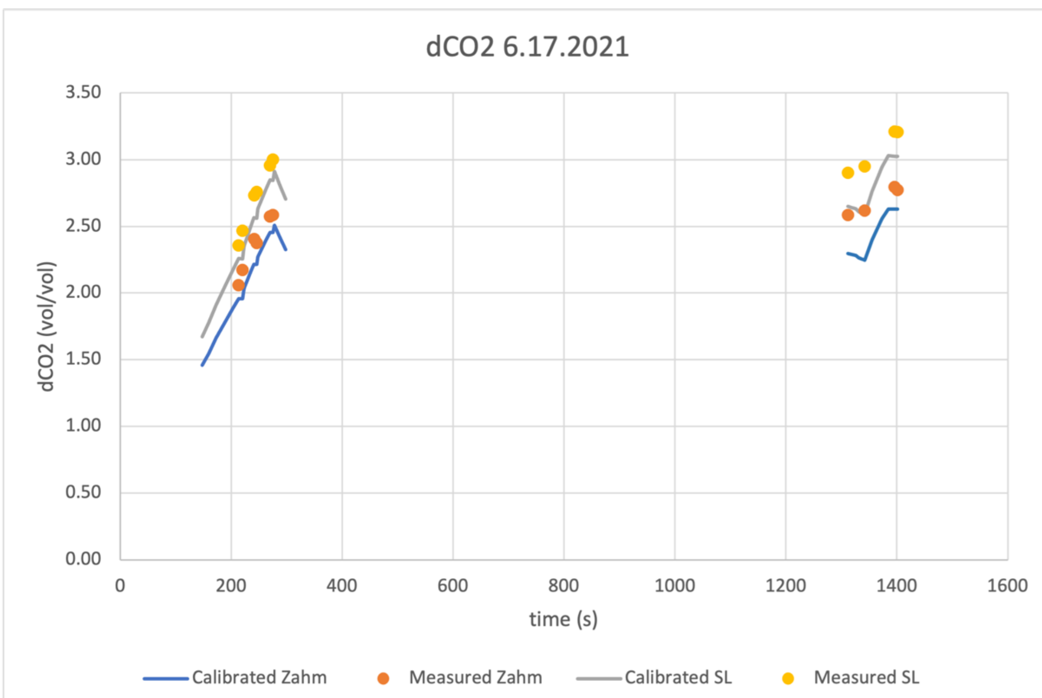
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
dCO2 (vol/vol)
0
50
100
150
200
250
300
350
400
450
time (s)
Calibrated Zahm
Measured Zahm
Calibrated SL
Measured SL

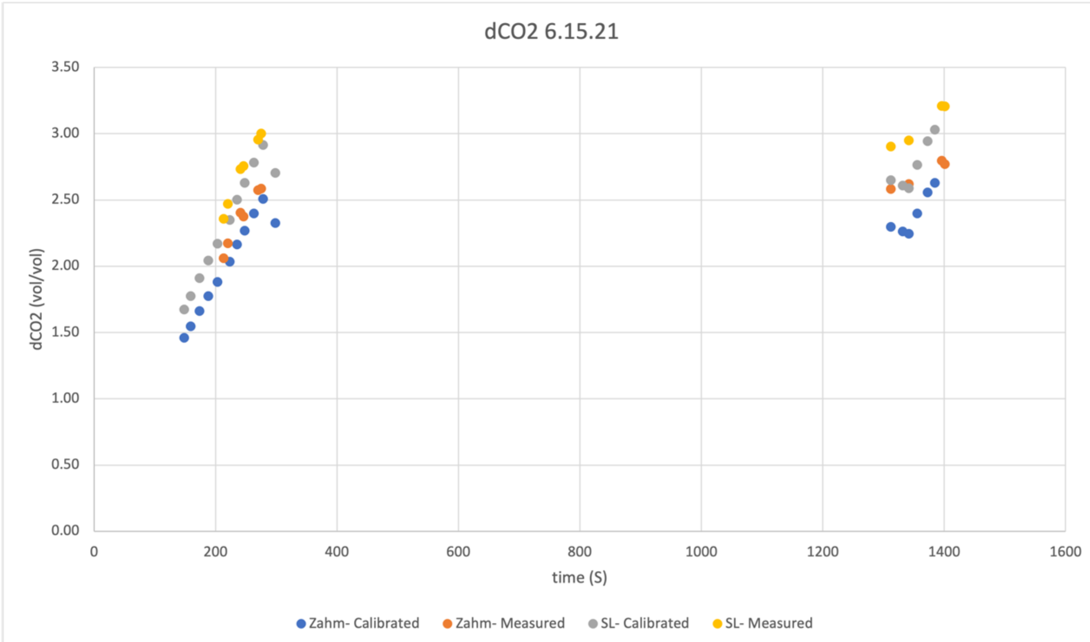
7.12.21


dCO2 7.12.2021
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
dCO2 (vol/vol)
0
200
400
600
800
1000
1200
1400
1600
1800
time (s)
Calibrated Zahm
Measured Zahm
Calibrated SL
Measured SL

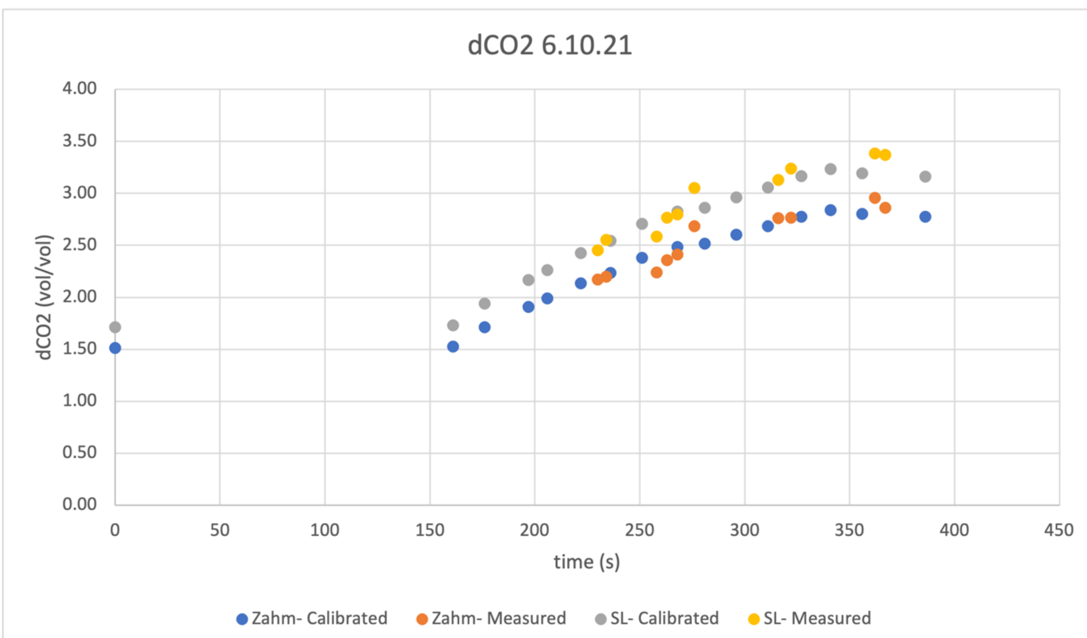
6.17.21


dCO2 6.17.2021
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
dCO2 (vol/vol)
0
200
400
600
800
1000
1200
1400
1600
time (s)
Calibrated Zahm
Measured Zahm
Calibrated SL
Measured SL

6.15.21


dCO2 6.15.21
dCO2 (vol/vol)
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0
200
400
600
800
1000
1200
1400
1600
time (S)
Zahm- Calibrated
Zahm- Measured
SL- Calibrated
SL- Measured

6.10.21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dCO2 (vol/vol)
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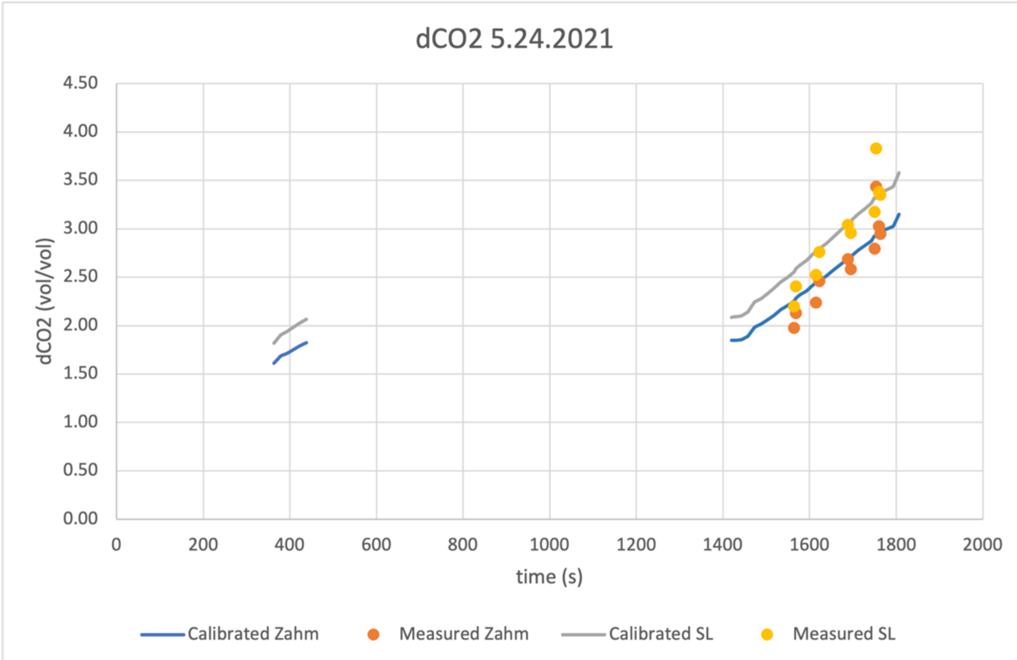
3.00
2.50
2.00
1.50
1.00
0.50
0.00
0
50
100
150
200
250
300
350
400
450
time (s)
Zahm- Calibrated
Zahm- Measured
SL- Calibrated
SL- Measured

5.24.21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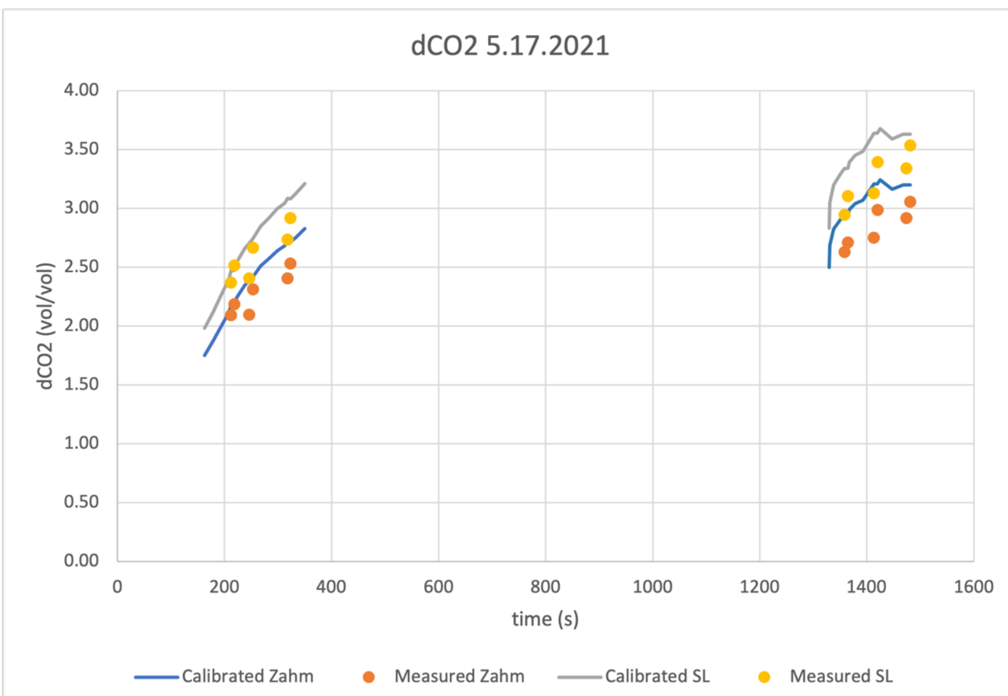
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
dCO2 (vol/vol)
0
200
400
600
800
1000
1200
1400
1600
1800
2000
time (s)
Calibrated Zahm
Measured Zahm
Calibrated SL
Measured SL

5.17.21


dCO2 5.17.2021
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.00
dCO2 (vol/vol)
0
200
400
600
800
1000
1200
1400
1600
time (s)
Calibrated Zahm
Measured Zahm
Calibrated SL
Measured SL

B- Zahm & Nagel Carbon Dioxide Chart

SOLUBILITY OF CARBON DIOXIDE IN BEER
PRESSURE–TEMPERATURE RELATIONSHIPS

Results Expressed as Volumes CO_2 (0° C.–760mm.) Per Volume Beer

CONVERSION SCALE
(Specific Gravity Beer–1.015)

Volumes	1.5	2.0	2.5	3.0

% by wgt.	0.30	0.35	0.40	0.45	0.50	0.55	0.60

DATA TAKEN FROM
"Methods of Analysis"
AMERICAN SOCIETY OF BREWING CHEMISTS
5th Edition–1949

1 Volume of CO_2 = 1.969 g/L

COPIES AVALIABLE FROM
ZAHM & NAGEL CO., INC.
210 VERMONT STREET
HOLLAND, NY 14080 USA

GAUGE PRESSURE – POUNDS PER SQUARE INCH

°C	°F	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
0	32	2.15	2.27	2.38	2.48	2.59	2.70	2.80	2.90	3.00	3.11	3.21																																			
.55	33	2.10	2.23	2.33	2.43	2.53	2.63	2.74	2.84	2.96	3.06	3.15	3.25																																		
1.11	34	2.06	2.18	2.28	2.38	2.48	2.58	2.69	2.79	2.90	3.00	3.09	3.19																																		
1.66	35	2.02	2.14	2.24	2.34	2.43	2.52	2.63	2.73	2.83	2.93	3.02	3.12	3.22																																	
2.22	36	1.98	2.09	2.19	2.29	2.38	2.47	2.57	2.67	2.77	2.86	2.96	3.05	3.15	3.24																																
2.77	37	1.94	2.04	2.14	2.24	2.33	2.42	2.52	2.62	2.71	2.80	2.90	3.00	3.09	3.18	3.27																															
3.33	38	1.90	2.00	2.10	2.20	2.29	2.38	2.48	2.57	2.66	2.75	2.85	2.94	3.03	3.12	3.21																															
3.88	39	1.86	1.96	2.06	2.15	2.25	2.34	2.43	2.52	2.61	2.70	2.80	2.89	2.98	3.07	3.16	3.25																														
4.44	40	1.83	1.92	2.01	2.10	2.20	2.30	2.39	2.47	2.56	2.65	2.75	2.84	2.93	3.01	3.10	3.19	3.28																													
5.00	41	1.79	1.88	1.97	2.06	2.16	2.25	2.34	2.43	2.52	2.60	2.70	2.79	2.88	2.96	3.05	3.14	3.23																													
5.55	42	1.75	1.85	1.94	2.02	2.12	2.21	2.30	2.39	2.48	2.56	2.65	2.74	2.83	2.91	3.00	3.09	3.18	3.26																												
6.11	43	1.72	1.81	1.90	1.99	2.08	2.17	2.26	2.34	2.43	2.52	2.61	2.69	2.78	2.86	2.95	3.04	3.13	3.21																												
6.66	44	1.69	1.78	1.87	1.95	2.04	2.13	2.22	2.30	2.39	2.47	2.56	2.64	2.73	2.81	2.90	2.99	3.07	3.16	3.24																											
7.22	45	1.66	1.75	1.84	1.91	2.00	2.08	2.17	2.26	2.34	2.42	2.51	2.60	2.69	2.77	2.86	2.94	3.02	3.11	3.19																											
7.77	46	1.62	1.71	1.80	1.88	1.96	2.04	2.13	2.22	2.30	2.38	2.47	2.55	2.64	2.72	2.81	2.89	2.98	3.06	3.15	3.23																										
8.33	47	1.59	1.68	1.76	1.84	1.92	2.00	2.09	2.18	2.26	2.34	2.42	2.50	2.59	2.67	2.76	2.84	2.93	3.02	3.09	3.18																										
8.88	48	1.56	1.65	1.73	1.81	1.89	1.96	2.05	2.14	2.22	2.30	2.38	2.46	2.54	2.62	2.71	2.79	2.88	2.96	3.04	3.13																										
9.17	49	1.53	1.62	1.70	1.79	1.86	1.93	2.01	2.10	2.18	2.25	2.34	2.42	2.50	2.58	2.67	2.75	2.83	2.91	3.00	3.07	3.15																									
10.0	50	1.50	1.59	1.66	1.74	1.82	1.90	1.98	2.06	2.14	2.21	2.30	2.38	2.46	2.54	2.62	2.70	2.78	2.86	2.94	3.02	3.10	3.17																								
10.55	51		1.57	1.64	1.71	1.79	1.87	1.95	2.02	2.10	2.18	2.26	2.34	2.42	2.49	2.57	2.65	2.74	2.82	2.90	2.97	3.05	3.13	3.19																							
11.11	52		1.54	1.61	1.68	1.76	1.84	1.92	1.99	2.06	2.14	2.22	2.30	2.38	2.45	2.53	2.61	2.68	2.76	2.84	2.92	3.00	3.06	3.13	3.22																						
11.66	53		1.51	1.59	1.66	1.74	1.81	1.89	1.96	2.03	2.10	2.18	2.26	2.34	2.41	2.49	2.57	2.64	2.71	2.79	2.86	2.94	3.01	3.09	3.16																						
12.22	54			1.56	1.63	1.71	1.78	1.86	1.93	2.00	2.07	2.15	2.22	2.30	2.37	2.45	2.52	2.59	2.66	2.74	2.81	2.89	2.96	3.04	3.10	3.17																					
12.77	55			1.53	1.60	1.68	1.75	1.82	1.89	1.97	2.04	2.12	2.19	2.26	2.33	2.40	2.47	2.54	2.62	2.69	2.76	2.83	2.89	2.97	3.04	3.11	3.18																				
13.33	56			1.50	1.57	1.65	1.72	1.79	1.86	1.93	2.00	2.08	2.15	2.22	2.29	2.36	2.43	2.50	2.57	2.64	2.71	2.78	2.85	2.92	2.99	3.06	3.13	3.20																			
13.88	57				1.54	1.62	1.70	1.77	1.83	1.90	1.97	2.04	2.11	2.18	2.25	2.32	2.39	2.46	2.53	2.60	2.66	2.73	2.80	2.87	2.94	3.00	3.08	3.15	3.22																		
14.44	58				1.51	1.59	1.67	1.74	1.80	1.87	1.94	2.01	2.08	2.15	2.21	2.28	2.35	2.42	2.48	2.55	2.62	2.69	2.75	2.82	2.88	2.95	3.02	3.09	3.16																		
15.0	59					1.56	1.64	1.71	1.77	1.84	1.91	1.98	2.04	2.11	2.17	2.24	2.31	2.38	2.43	2.50	2.57	2.64	2.70	2.77	2.84	2.91	2.97	3.03	3.10	3.17																	
15.55	60					1.54	1.62	1.69	1.75	1.82	1.88	1.95	2.01	2.08	2.14	2.21	2.27	2.34	2.40	2.47	2.53	2.60	2.66	2.73	2.79	2.86	2.92	2.99	3.05	3.11	3.18																
16.11	61					1.51	1.59	1.66	1.72	1.79	1.85	1.91	1.97	2.04	2.10	2.17	2.23	2.30	2.36	2.43	2.49	2.56	2.62	2.69	2.75	2.81	2.87	2.94	3.00	3.07	3.14	3.19															
16.66	62						1.56	1.63	1.69	1.76	1.82	1.88	1.94	2.01	2.07	2.14	2.20	2.26	2.32	2.39	2.45	2.52	2.58	2.64	2.70	2.77	2.83	2.90	2.96	3.02	3.08	3.14	3.20														
17.22	63						1.54	1.60	1.66	1.72	1.78	1.85	1.91	1.98	2.04	2.10	2.16	2.22	2.28	2.35	2.41	2.48	2.53	2.60	2.66	2.72	2.78	2.85	2.90	2.97	3.03	3.09	3.16														
17.77	64						1.51	1.58	1.64	1.70	1.75	1.82	1.88	1.94	2.00	2.06	2.12	2.18	2.24	2.30	2.36	2.43	2.49	2.55	2.61	2.68	2.74	2.80	2.86	2.92	2.98	3.04	3.10	3.17													
18.33	65							1.55	1.61	1.67	1.73	1.79	1.85	1.91	1.96	2.03	2.09	2.15	2.21	2.27	2.33	2.39	2.45	2.51	2.57	2.63	2.69	2.75	2.81	2.87	2.93	3.00	3.05	3.11	3.18												
18.88	66							1.52	1.58	1.64	1.70	1.76	1.82	1.88	1.93	1.99	2.05	2.11	2.17	2.23	2.29	2.35	2.41	2.47	2.53	2.59	2.65	2.71	2.76	2.82	2.88	2.94	3.00	3.06	3.12	3.18											
19.44	67							1.50	1.55	1.61	1.67	1.73	1.79	1.85	1.90	1.96	2.02	2.08	2.14	2.20	2.25	2.31	2.37	2.43	2.49	2.55	2.61	2.67	2.72	2.78	2.84	2.90	2.96	3.02	3.07	3.13	3.19										
20.0	68								1.53	1.59	1.64	1.70	1.76	1.83	1.87	1.93	1.98	2.04	2.10	2.16	2.22	2.38	2.33	2.39	2.44	2.50	2.56	2.62	2.67	2.73	2.78	2.85	2.90	2.96	3.02	3.07	3.13										
20.55	69								1.51	1.57	1.62	1.68	1.73	1.79	1.84	1.90	1.96	2.01	2.07	2.13	2.18	2.24	2.30	2.36	2.41	2.47	2.52	2.58	2.63	2.69	2.74	2.80	2.86	2.92	2.97	3.03	3.08	3.14									
21.11	70									1.54	1.59	1.65	1.70	1.76	1.81	1.87	1.92	1.98	2.05	2.10	2.15	2.21	2.26	2.31	2.36	2.42	2.48	2.54	2.59	2.65	2.70	2.76	2.81	2.87	2.92	2.98	3.03	3.09	3.14								
21.66	71									1.52	1.57	1.63	1.68	1.73	1.78	1.84	1.89	1.95	2.01	2.06	2.11	2.17	2.22	2.28	2.33	2.39	2.44	2.50	2.55	2.61	2.66	2.72	2.77	2.83	2.88	2.94	2.99	3.04	3.09	3.15							
22.22	72										1.54	1.60	1.65	1.71	1.76	1.81	1.86	1.92	1.97	2.03	2.08	2.14	2.19	2.24	2.29	2.35	2.40	2.46	2.51	2.57	2.62	2.67	2.72	2.78	2.83	2.89	2.94	3.00	3.05	3.10	3.15						
22.77	73										1.52	1.57	1.62	1.68	1.73	1.78	1.83	1.89	1.94	2.00	2.05	2.11	2.16	2.21	2.26	2.31	2.36	2.42	2.47	2.53	2.58	2.63	2.68	2.73	2.79	2.84	2.89	2.95	3.00	3.05	3.10	3.16					
23.33	74										1.50	1.55	1.60	1.65	1.70	1.76	1.81	1.86	1.91	1.97	2.02	2.07	2.12	2.18	2.23	2.28	2.33	2.38	2.43	2.48	2.53	2.58	2.64	2.69	2.74	2.79	2.84	2.90	2.95	3.00	3.06	3.11	3.16				
23.88	75											1.52	1.57	1.63	1.68	1.73	1.78	1.83	1.88	1.94	1.99	2.04	2.09	2.14	2.19	2.24	2.29	2.35	2.40	2.45	2.50	2.55	2.60	2.65	2.70	2.75	2.80	2.86	2.91	2.96	3.01	3.06	3.11	3.16			
24.44	76											1.50	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.01	2.06	2.11	2.16	2.21	2.26	2.31	2.36	2.41	2.46	2.51	2.56	2.61	2.66	2.71	2.76	2.81	2.86	2.91	2.96	3.01	3.06	3.12	3.16		
25.0	77												1.53	1.58	1.62	1.67	1.72	1.77	1.82	1.87	1.92	1.97	2.02	2.07	2.12	2.17	2.22	2.27	2.32	2.37	2.42	2.47	2.52	2.57	2.62	2.67	2.72	2.77	2.82	2.87	2.92	2.97	3.02	3.07	3.12	3.17	3.22
25.55	78												1.50	1.55	1.60	1.65	1.70	1.75	1.79	1.84	1.89	1.94	1.99	2.04	2.09	2.14	2.18	2.23	2.28	2.34	2.38	2.43	2.48	2.53	2.58	2.63	2.67	2.72	2.78	2.83	2.88	2.93	2.97	3.02	3.07	3.12	3.16
26.11	79													1.53	1.58	1.63	1.67	1.72	1.77	1.82	1.87	1.92	1.96	2.01	2.06	2.11	2.16	2.21	2.25	2.30	2.35	2.40	2.45	2.50	2.54	2.59	2.64	2.69	2.74	2.79	2.83	2.88	2.93	2.98	3.03	3.08	3.12
26.66	80													1.50	1.55	1.60	1.65	1.70	1.74	1.79	1.84	1.89	1.93	1.98	2.03	2.08	2.13	2.18	2.22	2.27	2.32	2.37	2.41	2.46	2.50	2.55	2.60	2.65	2.70	2.75	2.79	2.84	2.89	2.94	2.98	3.03	3.07
27.22	81														1.53	1.58	1.63	1.68	1.72	1.77	1.82	1.87	1.91	1.96	2.00	2.05	2.10	2.15	2.20	2.25	2.29	2.34	2.38	2.43	2.48	2.53	2.57	2.62	2.66	2.71	2.76	2.81	2.85	2.90	2.94	2.99	3.04
27.77	82														1.52	1.56	1.60	1.65	1.70	1.75	1.79	1.84	1.89	1.94	1.98	2.03	2.07	2.12	2.16	2.21	2.26	2.31	2.36	2.40	2.44	2.49	2.54	2.59	2.63	2.68	2.72	2.77	2.82	2.86	2.90	2.95	3.00
28.33	83														1.50	1.54	1.58	1.63	1.68	1.73	1.77	1.82	1.86	1.91	1.95	2.00	2.04	2.09	2.13	2.18	2.22	2.27	2.32	2.37	2.41	2.46	2.50	2.55	2.59	2.64	2.68	2.73	2.78	2.82	2.87	2.91	2.96
28.88	84															1.52	1.56	1.61	1.65	1.70	1.74	1.79	1.83	1.88	1.92	1.97	2.02	2.06	2.10	2.15	1.19	2.24	2.28	2.33	2.37	2.42	2.46	2.51	2.56	2.60	2.64	2.69	2.74	2.78	2.82	2.87	2.91
29.44	85															1.50	1.54	1.58	1.62	1.67	1.71	1.76	1.80	1.85	1.89	1.94	1.98	2.03	2.07	2.12	2.16	2.21	2.25	2.29	2.33	2.38	2.43	2.47	2.51	2.56	2.60	2.65	2.69	2.74	2.78	2.82	2.87
30.0	86																1.52	1.56	1.60	1.65	1.69	1.74	1.78	1.83	1.87	1.91	1.95	2.00	2.04	2.09	2.13	2.18	2.22	2.26	2.30	2.35	2.39	2.43	2.48	2.53	2.57	2.61	2.65	2.69	2.73	2.78	2.83
kg/cm²		.351	.421	.492	.562	.632	.703	.773	.843	.913	.984	1.05	1.12	1.19	1.26	1.33	1.40	1.47	1.54	1.61	1.68	1.75	1.82	1.89	1.96	2.03	2.10	2.17	2.24	2.32	2.39	2.46	2.53	2.60	2.67	2.74	2.81	2.88	2.95	3.02	3.09	3.16	3.23	3.30	3.37	3.44	3.51