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## The Beauty of Mathematics and the Mathematics of Beauty: Continued Fractions and the Golden Ratio

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**THE BEAUTY OF MATHEMATICS AND THE MATHEMATICS OF BEAUTY:  
CONTINUED FRACTIONS AND THE GOLDEN RATIO**

By Jessica Tush  
Department of Music

Faculty Mentors: Loredana Lanzani and Kim Sexton  
Departments of Music and Architecture

**Abstract**

*This project begins with a look at the history of simple continued fractions and how we have arrived where we are today. We then move through a study of simple continued fractions, beginning first with rational numbers and moving to irrational numbers. Continuing further in the pursuit of joining mathematics and art, we define the specific continued fraction that gives rise to the Fibonacci sequence and the Golden Ratio  $\phi$  (phi, pronounced "fai"). These two notions form a direct link to art and the properties that we hope to examine.*

*I have taken an analytic approach to showing that the Golden Ratio has been a constant presence in art history, probably as an indicator of aesthetic appeal. I collected, measured, and analyzed works of art from various periods to investigate the extent to which  $\phi$  is hidden throughout these works. I hope to affirm the hypotheses put forth by others throughout history that yes indeed, those works of art that best exemplify Platonic beauty have instances of the Golden Ratio [Hu] [Li] [Ru] [Ma].*

**Jessica Tush won one of three Undergraduate Research Awards in 2009. The full text of her paper can be found at <http://inquiry.uark.edu/>**

**Because this work was interdisciplinary in content and perspective, feedback from two of her advisors is provided below:**

**Mentor Comments:**

Kim Sexton describes the work as superb, writing:

*Ms. Tush's provocative thesis is truly interdisciplinary and based on original investigations not attempted by other scholars. While the candidate is a double-major in math and anthropology, her thesis explores the intersection of math and art history, namely, the art of representing human physiognomy and the influence a mathematical rationalization of beauty (the golden section) might have played in depictions of the human figure over the course of 5000 years (i.e., from ancient Egypt to the present). Art history was a subject in which she had no previous training. Ms. Tush took both the math and art history sections of her research equally seriously. I would like to highlight four qualities of Ms. Tush's work which place it among the best research produced by undergraduates (and by graduate students for that matter):*

- *Ms. Tush's account of the evolution of the golden section in art over 5000 years is utterly original research. No scholar has yet considered the issue over a long duration, including a photograph of Barack Obama and glossy portraits of Hollywood celebrities.*
- *The double-major's agile handling of sophisticated theoretical paradigms – mathematical proofs and proportional ideality in art – structures the analyses throughout her thesis. Here, theory is no mere introductory distraction or flourish.*
- *In the formal analysis of a wide-range of portraiture – still the key interpretative tool of art historians – this thesis shows the candidate to be not only competent, but progressive and original.*
- *Finally, what truly sets this thesis apart is Ms. Williams' avoidance of the pitfalls of essentialist reasoning that ensnares many undergraduate researchers. She works deftly with a sliding spectrum of traits rather than giving undue credence to artificially constructed norms of beauty. Ms. Tush's submission is a model of undergraduate research on a topic of fundamental importance – the relationship between science and art – both for the respective disciplines and for society at large.*

And Honors thesis advisor Loredana Lanzani says:

*Jessica's interdisciplinary thesis is a fascinating examination of "The Beauty of Mathematics and the Mathematics of Beauty." It consists of two interconnected parts. In the first part, she has explored the topic of continued fractions, which departs from and significantly expands the course material she covered in my class (Math 2103); as an application of continued fractions, she has then studied the geometric and analytical properties of a special number, the golden ratio, (also known as the divine proportion) which since Classic Greece has been considered the golden paragon in the assessment of beauty. In the second part of her thesis, mentored by Dr. Kim Sexton (School of Architecture) Jessica has been "looking for the golden ratio" throughout a large number of artworks and iconic pop-culture images that most scholars acknowledge to be good representatives of the social perception of beauty throughout different historical periods. She has then compiled various statistics with the goal to quantify the extent to which the social perception of beauty is indeed affected (consciously or subconsciously) by the presence, in some form or other, of the golden ratio.*

*Jessica was by all means the primary investigator throughout all the many aspects of this project. She selected the topics herself, and she developed all the sophisticated mathematical skills that were needed in order to master the subject of continued fractions. Under Dr. Sexton's guidance, Jessica (who is not an arts major, nor an arts minor) selected over 30 artworks and images, found the most suitable software, and learned how to use it to analyze the selected art works and images: based on these data she developed her own set of comparison criteria to evaluate the various artworks and images.*

*The interdisciplinary nature of this project makes this work quite unique and appealing to a wide array of researchers: the mathematician is excited to learn about "real-life" applications of the quite abstract but beautiful tool of continued fractions. The art historian appreciates how a generally acknowledged concept (namely, the idea that the golden ratio has some kind of an effect on the perception and the representation of beauty) is in fact precisely quantifiable and rationally explainable by means of rigorous mathematical analysis. The novelty of this work is the systematic (and highly successful) effort to present a new perspective by bringing the two subjects together in a very in-depth analysis both of the pertinent mathematical techniques and of the artworks.*

*Jessica plans to become a high school teacher. This interdisciplinary project has great potential to be developed into a successful tool for interdisciplinary teaching at the high school level. Throughout the engaging use of computer technology, high school students can be brought to learn and master sophisticated mathematical theorems and at the same time bring "life" into these techniques by using them to investigate their own favorite images and artworks. The potential rewards that this approach can bring to the high school classroom cannot be understated, for instance: a new appreciation for, and a deeper understanding of, an often intimidating subject such as Mathematics. Jessica also has all the personality traits that make a great teacher – talent, passion, great communication skills, enthusiasm for her subject matter, but also patience, the ability to listen, and a sense of humor. I know she will be able to turn this project into a highly effective and at the same time very enjoyable pedagogical tool.*