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MARKET VOLATILITY ASYMMETRIES: THE EFFECTS OF STOCK MARKET RETURNS ON REALIZED AND IMPLIED VOLATILITIES

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Abstract

Volatility is an integral and inescapable variable of financial engineering, modeling, and finance theory itself. Classical financial economics proxies volatility for risk itself, as it becomes difficult to predict future price realizations of a given asset when that asset exhibits significant price volatility over a given time. However, the nature of volatility as it is explained by classical financial economics has been extensively questioned in the previous three decades, since it is characterized as a function of uncertainty and aggregate market psychology—that is, as a function of fear, greed, exuberance, and other fundamental human instincts and emotions.

While previous research has primarily focused on the asymmetries between stock market returns and realized volatility, this paper examines the extent to which implied volatility is asymmetrical with regards to the nature (positive or negative) of stock market returns in simultaneous periods. Analyses indicated that negative stock market returns create uniquely positive innovations to implied volatility not created during periods of positive stock market return. Additionally, this paper attempts to reconcile the asymmetry in implied volatility back into a cogent behavioral theory. Finally, the analyses described here explore how this asymmetry causes systemic error in predicting innovations to implied volatility and suggests a simple systemic error adjusted VIX model can be utilized with great efficacy to predict future innovations to realized volatility.

1. Introduction

Volatility is an integral and inescapable variable of financial engineering, modeling, and finance theory itself. Indeed, it is a formative building block of option pricing models and sundry portfolio allocation models. Classical financial economics proxies volatility for risk itself—that is, an asset that demonstrates large volatilities of price realizations over a given time series makes predicting future price realizations with certainty very difficult. Classical efficient market paradigm, which posits markets as acting freely and appropriately to all relevant information pertaining to an asset or security’s price function, explains volatility as a perfect reflection of market uncertainty. The risk of a given security is captured within volatility in that the security’s price volatility reflects the aggregate uncertainty pertaining to that security’s future prospects (Kurz and Motolesse, 1999). The nature of volatility as it is characterized by classical financial economics has been questioned extensively in the previous three decades. Research suggests that the various price changes that the U.S. equity markets have experienced since the 1920s cannot be explained merely by the potential variability and uncertainty surrounding the discounted cash flows of future dividend payments (Shiller, 1981). Rather, realized volatility is systematically greater than could be explained by the efficient market hypothesis. Now, many financial economists have posited volatility exists not only as a function of uncertainty, but also as a function of aggregate market psychology—that is, as a function of fear, of greed and exuberance, and other fundamental human instincts and emotions. Perhaps most pronounced among these base emotions and instincts is the human inclination towards loss aversion (Kahneman and Tversky, 1978) and its relationship to volatility innovations (Dennis, Mayhew, and Stivers, 2006). Prospect theory1 posits individuals as engaging in asymmetric loss aversion. All things held constant, economic losses affect individuals emotionally and psychologically to a greater degree than gains. Individuals thus overcompensate for small probabilities of negative occurrences and assign larger weights to them in their decision criteria than they logically should2. Research also indicates that loss-aversion is not static; rather, an investor’s loss-aversion function changes during periods of stock return shocks. Thus, the dynamic nature of an investor’s loss aversion function can cause volatility feedback loops (Barberis, Huang, and Santos, 2001).

Volatility, however, has not been relegated exclusively to the confines of academia. In 1993 the Chicago Board Option Exchange (CBOE) introduced a volatility index that identified the volatility implied by a perfectly at-the-money option of the S&P 100 for the next 30 calendar days. This would later be named the VXO, as a new volatility index, named the VIX, would be introduced in 2003. In the next year, on March 26, 2004, the CBOE launched a futures exchange where investors and speculators could trade futures on volatility itself, making

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1 Prospect theory is an explication of how individuals make decisions under risk. It is descriptive in nature, as opposed to optimal (like expected utility theory). The primary difference between the two theories is that expected utility theory posits utility curves as exclusively convex and static. On the other hand, prospect theory posits that, from a given reference point, the utility function of gains is concave, whereas the utility function of losses is convex. Additionally, prospect theory posits utility curves as dynamic—that is, they shift based on the starting point of the individual making the decision (i.e., How wealthy am I? What do I have to lose?) and how the decision and risk is framed in the mind of decision maker.

2 What do I have to lose? and how the decision and risk is framed in the mind of decision maker.
volatility an asset class in and of itself.

In the seven months since October 2008, the 30-day implied volatility of the S&P 100 (see Figure 1) had reached levels never before seen since the index’s creation. Indeed, the VXO has breached levels not observed since Black Monday.

Given these levels, discussions of volatility with regards to both its nature and its relationships to other market functions are both important and timely. While realized volatility has been the topic of numerous academic inquiries, this paper further explores the functioning of realized and implied volatility in financial markets, its relation with other market variables, and the accuracy of implied volatility indexes in predicting future realized volatility. Specifically, this paper addresses the following:

a) Qualification and quantification of the asymmetries in volatilities that are realized historically and predicted for the future based on the nature of the market returns (positive or negative) that exist contemporaneous to them.

b) Investigation of whether the VXO systematically overestimates future volatility, and whether any bias is determined by the nature of contemporaneous market returns.

c) Isolation of systematic bias as from the VXO to determine if a superior model for predicting future volatility can be created.

The creation of a unique model for predicting future volatility is of paramount importance in finance, since creating models with greater explanatory power than the market is exceedingly difficult. The ability to identify volatility mispricings, in conjunction with the accurate estimation of future volatility in financial markets, would allow for the development of new, potentially lucrative volatility trading strategies.

2. Hypothesis Development

In order to develop research hypotheses related to the issues of volatility described above, it is necessary to examine more closely previous academic research that has postulated asymmetric correlations of volatility with different types of stock market returns. There have been two dominant, though not mutually exclusive, explanations of this presumed phenomenon: volatility feedback effect and the leverage hypothesis. Proponents of the volatility feedback effect explanation rely on the Capital Asset Pricing Model in Modern Portfolio Theory. This model is used to quantify the rate of return a specific asset should have in order to justify its existence in a well-diversified portfolio. All things being equal, an increase in the market risk premium (i.e., how much additional return should be generated by the market in order to justify holding risky assets relative to risk-free assets) requires an increase in firm specific expected return in order to justify an individual security’s possession in a portfolio.

Research indicates that the market risk premium is positively correlated with market volatility (Kim, Morley, and Nelson, 2008). Therefore increases in market volatility, and thus the equity risk premium, require an increase in firm specific expected return in order to justify an individual security’s possession. Volatility feedback posits that as a firm’s and market’s volatility increases over a period of time, its value should decrease if its expected future cash flows are held constant. Therefore, increases in volatility typically cause stock price depreciation, which in turn raises volatility even further, creating a volatility feedback loop (Bae, Kim, and Nelson, 2007). In contrast, the leverage hypothesis asserts that asymmetric volatility can be explained by the inherently increasing leverage of a firm (debt-to-market capitalization) as its share price decreases. The volatility of a security’s price increases as the firm’s leverage also increases. However, asymmetric volatility persists even if the firm is financially unlevered; that is, if the firm has no debt on its balance sheet (Daouk, Hazem, and Ng, 2004).

Few studies have extended the potential asymmetry of returns to innovations in implied volatility. Determining if asymmetric exists in implied volatility innovations relative to stock market returns would allow for a better understanding of the degree to which the aggregate market’s loss aversion function changes with stock market returns (both with regard to nature and magnitude). Such analyses would allow for greater clarity regarding both future volatility innovations and the extent to which implied volatility assessments might be systematically biased. The study is grouped into three hypothesis groups. The first, containing four parts, explores whether asymmetry exists between implied volatility, realized volatility, and previous 90-day returns. The existence of asymmetry will be tested where:

\[ \text{Refers to fair actuarial estimates of statistical probabilities of various gains and losses} \]

\[ \text{Black Monday refers to Monday, October 19, 1987. On said date, the S&P 100 fell 21.16\% at its closing from the previous Friday's closing.} \]
H_{\text{HGO}}: The relationship between previously realized 30-day volatility is consistent and symmetrical between previous 90-day returns of similar absolute varieties, but of different natures (positive or negative).

H_{\text{HIO}}: The relationship between previously realized 30-day volatility is inconsistent and asymmetrical between previous 90-day returns of similar absolute varieties, but of different natures (positive or negative).

H_{\text{HO}}: The relationship between implied volatility and previous 90-day returns of similar absolute varieties, but of different natures (positive or negative) is consistent and symmetrical.

H_{\text{HI}}: The relationship between implied volatility and previous 90-day returns of similar absolute varieties, but of different natures (positive or negative) is inconsistent and asymmetrical.

The second and third hypothesis groups question whether the VXO overestimates future realized volatility, if the overestimation is symmetrical with regard to the nature and magnitude of returns of previous 90-day periods, and if any systematic overestimation can be factored out to produce a superior model for estimating future volatility, where:

H_{\text{HGO}}: The VXO does not have a systematic error residual.

H_{\text{HIO}}: The VXO has a systematic error residual that is substantially affected by the nature and magnitude of the return of the previous 90-day period. The error that occurs during periods where previous 90-day returns were positive are statistically different than the error that occurs during periods where previous 90-day returns were negative.

H_{\text{HOG}}: Adjusting for systemic errors of each variety (systematic error of returns of a positive nature for time periods where previous 90-day return was positive, systematic error of returns of a negative nature for time periods where previous 90-day return was negative) as it is perceived does not render a superior model for predicting future stock market volatility.

H_{\text{HIO}}: Adjusting for systematic errors of each variety (systematic error of returns of a positive nature for time periods where previous 90-day return was positive, systematic error of returns of a negative nature for time periods where previous 90-day return was negative) as it is perceived renders a superior model for predicting future stock market volatility than other simple models.

H_{\text{HIO}}: is of particular importance. Predicting future volatility significantly better than the market’s prediction is very difficult but could also be exceedingly profitable. The models constructed hereafter, and their concomitant analysis, could be used to identify and take advantage of future volatility mispricings in financial markets.

3. Methodology
This paper uses reverse-engineered VXO values dating back to January, 1986, and the 30-day realized volatility of the S&P 100 dating back to 1982 in order to explore three specific topics. First, the analysis tests the extent of correlation between stock market returns and volatility of both a realized and implied nature. Within the scope of this topic, the extent of correlation between 90-day positive returns and 30-day realized and implied volatility is analyzed against correlations between 90-day negative returns and 30-day realized and implied volatility to determine if volatilities are symmetrical between returns of a similar absolute degree, but of a different nature. This is accomplished via a multivariable regression where previous 30-day realized volatility, 90-day S&P 100 absolute returns, and the sign of the return of previous 90-day S&P 100 returns are utilized to explain the variability of VXO innovations. This regression utilizes a format where Vol is the previous 30-day realized volatility, c_{fad} is the coefficient of realized volatility, AbsRet is the absolute return of the S&P 100 over the previous ninety days, c_{abs} is the coefficient of the absolute return, Return Sign is the nature of the return (positive or negative) of the S&P 100 over the previous ninety days, c_{Return Sign} is the coefficient of the return nature and c is the error term.

\[ \text{VXO} = c_{fad} \text{Vol} + c_{abs} \text{AbsRet} + c_{Return \ Sign} \text{Return \ Sign} + \varepsilon \]

Both the absolute return and the sign of the return over the previous 90 days are included to account for innovations to implied volatility that might be derived from the magnitude of the return alone. Therefore, the Return Sign, coefficient is used to demonstrate the degree of asymmetry derived by the nature of the return itself, positive or negative.

Second, this paper utilizes a Mann-Whitney U test to determine if the error residuals of VXO in predicting future volatilities during periods of negative stock market returns come from the same distribution as future volatilities during periods of positive stock market returns. A rejection of the null hypothesis indicates that the VXO is systematically biased in its estimation of future volatility based on the nature of the previous 90-day returns.

Last, five models (the VXO, an error-adjusted VXO, realized 30-day volatility, an error-adjusted 30-day volatility, and an equally weighted index composed of VXO and realized 30-day volatility) are analyzed to determine their respective efficacy in predicting future volatility. The error-adjusted VXO, \( \text{VXO}_{\text{adj}} \), is constructed by adjusting each VXO reading at time, by the average error term experienced up to that time period by error type and return nature. Therefore, \( \varepsilon_{\text{adj}} \), is the mean error term up to that period for all VXO readings that are contemporaneous with positive previous 90-day returns, \( \text{90-return}_{\text{pos}} \), \( \varepsilon_{\text{adj}} \), is the mean error term up to that period for all VXO readings that are contemporaneous with negative previous 90-day returns, \( \text{90-return}_{\text{neg}} \).

\[ \text{If 90-return}_{\text{neg}}: \text{VXO}_{\text{adj}} = \text{VXO} - \varepsilon_{\text{adj}} \]

\footnote{Realized Volatility is determined by calculating the standard deviation of daily lognormal returns over the previous 30 calendar days, or 21 trading days, and then annualizing the return. Thus, Realized Volatility = \( \sqrt{\frac{1}{n-1} \sum (\ln r_i - \ln r_y)^2} \times 252 \).}
If \( 90\text{-return}_{t} \) : 

\[ VXO_{adj} = VXO - \varepsilon_{t} \]

The five models are evaluated by a number of criteria, including their explanatory power (represented by \( R^2 \)), their respective mean squared error, the kurtosis of each model, the mean error of each model in order to determine systematic bias, and the range of each model’s errors, among others.

4. Results

4.1. Asymmetric correlation between realized volatilities and returns of different natures (positive or negative) of previous 90-day periods.

The degree of correlation between returns of a positive variety and realized volatility, and returns of a negative variety and realized volatility, is significantly asymmetric. However, returns and correlation are not inversely related, as popularly pronounced by stock market pundits. Indeed, on average, volatility increases as absolute stock market returns go up (in either a positive or negative manner). Additionally, substantial asymmetry exists between volatility and the type of return (positive or negative) experienced over previous 90-day periods (see Figure 2). Volatility is mildly positively correlated to the degree of positive stock market returns over previous 90-day periods \((r = 0.17899)\). Reciprocally, volatility is much more strongly negatively correlated to the degree of negative stock market returns over previous 90-day periods \((r = -0.71553)\).

4.2. Asymmetry between implied volatility innovations and periods with previous 90-day returns of different natures (positive or negative).

Realized volatilities are highly correlated with implied volatilities of contemporaneous periods \((r = 0.8792)\). Because of this, a multivariate regression is used to determine innovations of implied volatility, represented by the VXO as the dependent variable. In this model realized volatility, absolute previous 90-day return, and the sign of previous 90-day return are independent variables. The regression shows that 79.8% of the variability of implied volatility can be explained by changes to previous 30-day volatility, the absolute value of previous 90-day volatility, and the sign (positive or negative) of previous 90-day returns (see Table 1 below). The coefficient of the sign of previous 90-day returns in the regression is -0.0163. Thus, returns of a negative variety in previous 90-day returns causes an increase in implied volatility of 1.63 percentage points, all things held equal, compared to returns of a positive variety in previous 90-day returns. The asymmetry of implied volatility innovations to periods of varying natures and magnitudes of previous 90-day returns is also visually represented in Figure 3 below, which plots previous 90-day returns against average VXO readings for that return interval.

4.3. The error term of the VXO is systematically biased, and is asymmetric with regards to the nature of the previous 90-day return.

The descriptive statistics (see Table 2) of the VXO error terms shows that the VXO consistently overestimates future volatility. On average, it predicts volatility being 4.65% higher than is realized over the next thirty days. The systematic overestimation of future volatility is further determined by an analysis of the kurtosis of the error terms. Indeed, the kurtosis of the sample indicates that most of the variance of the error term is due to extreme observations in the data set. The results of the Mann-Whitney U Test (see Table 2) indicate that the error terms resulting from VXO readings contemporaneous with previous 90-day returns of a negative nature have a statistically different distribution than error terms resulting from VXO readings contemporaneous with previous 90-day returns of a positive nature. The differences in distribution of VXO error terms derived from periods exhibiting positive and negative stock market returns is further augmented by analysis of the descriptive statistics of the VXO error terms disaggregated by previous 90-day return nature (see Table 3). These findings are consistent with the findings of 4.2.
above. Since previous 90-day returns of a negative nature are associated with unique positive implied volatility innovations that are not shared by previous 90-day returns of a positive nature, it could be expected that any systematic bias of implied volatility could be further exacerbated during periods of previous 90-day negative returns.

4.4. Error adjusted VXO model predicts future volatility innovations with greatest accuracy.

The predictive power of five different models is of value in determining whether effective future predictions of implied volatility can be made, as predicting future volatility is of great importance in financial modeling. The systematic error adjusted VXO model is superior to the four competing models over the time period analyzed, as it has better explanatory power and less squared error in predicting future volatility than the other four models utilized (see Table 3). Additionally, the error adjusted VXO model has the greatest kurtosis of all models analyzed, implying that a greater amount of the variance of the error can be attributed to more extreme observations compared to the other models utilized. The error adjusted VXO has the smallest range of errors, which means that the scope of its accuracy is greatest compared to the other models. Finally, all calculations of the error adjusted VXO model were performed out-of-sample. This fact grants further legitimacy to the use of this model as a mechanism for predicting future volatility.

As explained previously, the VXO systematically overestimates future volatility. So does the equally weighted index composed of the VXO and previous 30-day volatilities, but by a lesser degree. Reciprocally, no systematic error appears to occur when utilizing previous 30-day volatilities to predict future volatility. However, this model has less explanatory power than either the stand-alone VXO model or the weighted model of the VXO and previous 30-day realized volatilities.

5. Interpretations and Conclusions

This paper analyzes the asymmetries of volatilities (both realized and implied) that occur simultaneously with returns of different natures. Based on these analyses, it appears that innovations to volatility are asymmetrically related to the nature of the return existing contemporaneous with it. With respect to implied and realized volatilities, negative returns create excess positive innovations relative to positive stock market returns. Thus, realized volatility increases during periods of negative stock market returns. In this study, the market’s aggregate expectation of future volatility is seen to increase in excess of what should be expected from contemporaneous realized volatilities during periods of negative stock market returns, as well. Further, the error terms of volatility predictions via the VXO are statistically different between volatility predictions made during periods of positive stock market returns and periods of negative stock market returns. Specifically, though implied volatility estimates are consistently positively biased, implied volatility bias during periods of negative stock market returns is substantially larger than implied volatility bias during periods of positive stock market returns.

The analyses of volatility in this paper were performed to clarify whether utility curves in capital markets follow those hypothesized by classical expected utility theory, or if they indeed instead follow an asymmetric and dynamic model proposed by prospect theory and behavioral finance. The results presented here indicate that systemic overestimation of future volatility occurs during periods of positive and negative stock market return. A systematic overestimation of volatility, in and of itself, would seem to be consistent with a typical concave utility function. Due to the concavity of the function, a given dollar loss affects an individual’s utility greater than a corresponding dollar gain. Since volatility is asymmetrically correlated to negative returns, an overpayment for protection from the impact of volatility on an investor’s portfolio should make up for the asymmetry of utility change differentials between equal gains and losses and would be consistent with classical expected utility theory. However, the results show that innovations to implied volatility are not static, but dynamic.
This is inconsistent with classical expected utility theory, since utility functions should be static. Thus, the degree to which one overpays for the right to speculate towards or protect his portfolio from future volatility per any degree of previously realized volatility should be similar, regardless of the nature of returns that exist contemporaneously.

This asymmetry in innovations to implied volatility, however, is consistent with prospect theory. Since investors will pay more for protection from future volatility during periods of previous negative stock market returns per any given unit of historical volatility, investors’ utility functions should change dynamically depending on numerous circumstances affecting myriad unknown personal considerations (health, wealth, happiness, etc.). Indeed, this asymmetry means investors gain more value for protection (or speculation) from volatility during periods of negative stock market return than they gain during periods of positive stock market return.

Finally, the analyses reported here suggest that utilizing the VIX to predict future volatilities, after adjusting for systemic error, provides a much more accurate assessment of future realized volatilities compared to the other five naive models utilized. Though simplistic, the model shows that systematic overestimations can be isolated from the VIX to render a more accurate assessment of future volatility innovations than other naive methods utilized. This finding is of considerable importance—both in academics and in the participation of financial markets. The ability to build a model with better explanatory power than the market is very challenging, given the sheer aggregate energy spent attempting to predict market variables. The ability to predict and identify volatility mispricings in capital markets would allow for the development of unique and lucrative trading methods, in concomitance to a better understanding of how volatility is uniquely innovated during different periods of stock market returns.

References


Mentor Comments:

Research mentor Alexey Malakhov describes the independent work of Matthew Chesnut in pursuing a challenging research topic and generating data that has immediate applications to understanding prediction of market risk. He notes: Matt’s thesis addresses the asymmetric nature of investor’s perception of risk as quantified by implied volatility. His research rigorously documents the difference in investors’ risk perception during good and bad times as measured by previous 90-day market return. This phenomenon can hardly be explained by the classical finance theory, which is the core of our undergraduate curriculum at the Walton College, but it is consistent with psychological and behavioral explanations suggested by Matt. Undoubtedly, the most innovative part of Matt’s thesis is his approach that quantifies behavioral biases that are present in the implied volatility estimates of future risk. Furthermore, Matt convincingly proposes a model of predicting future risk that accounts for the different nature of behavioral bias in good and bad times. This is a really innovative and exciting contribution to the current academic finance literature.

It is important to emphasize that although Matt worked under my supervision, he came up with all the research ideas in his thesis, as well as with the ways of implementing them. My involvement was limited to providing feedback on his numerous research ideas, and helping him to concentrate on the more promising areas, as well as exposing him to the existing literature and methodology that was not covered in our Finance curriculum. In the course of his research, Matt pushed the envelope of existing academic knowledge by exploring previously unexplained financial phenomena, and trying to come up with rigorous interpretations of the results. As usual in scientific inquiry, not all of Matt’s conjectures bore

*Implied volatility is also a proxy for the expensiveness of an option. At any given strike, as the premium to utilize the option increases in price, the implied volatility must also necessarily increase as well. Therefore, the implied volatility of a specific option is the volatility that is required to be experienced by the underlying asset in order to match the price of the strike.
fruit, and a good deal of projects turned out to be “dry holes”. I personally consider that a testament to the breadth and scope of Matt’s research, as the final version of his thesis represents only a fraction of overall research that he has conducted over the past few months.

Matthew has produced an honors thesis of the highest quality that may be directly applied to making predictions of future market risk, while correcting for biases caused by investor psychology. It is a fascinating result, especially in light of current events in financial markets. I believe that Matt also has a great potential, and I will not be surprised to see him produce high quality research in finance in the future.