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Free Convection Heat Transfer from Plates

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I. Summary

The purpose of this honors thesis is to create an experiment for the CHEG Lab I course. This report explains the motivation for creating this heat convection experiment, the results of performing the experiment, and provides recommendations for future work on this experiment. Multiple experiments were performed to assess materials and parameters to be investigated. It was determined that a 0.5” plate has smaller percent error and accommodates for the desired timeframe for a Lab I experiment compared to the first plate used (1.5” thickness). Recommendations for expanding on this project include adding experiments using vertical geometry for heat convection and/or forced convection.

II. Introduction

Undergraduate students pursuing a chemical engineering degree must successfully complete the chemical engineering Lab I and Lab II courses. The purpose of this honors thesis is to create an experiment for the Lab I course.

Students will be tasked to study free convective heat transfer by monitoring temperature changes over time for an aluminum plate that has been heated and allowed to cool in an insulated stand with the top face being exposed to atmospheric temperature and pressure. The students will record experimental data and determine a best fit experimental heat transfer coefficient by using MATLAB to solve a differential equation for the heat balance. In addition, a theoretical heat transfer coefficient will be determined from empirical correlations (Cengal 2007, Table 9-1, p. 511. The experimental and theoretical heat transfer coefficients will be compared and discussed.

III. Experimental Approach

The aluminum plate was painted black on the face that is to be exposed to air because the emissivity coefficient of black paint is known. The emissivity coefficient is used in the calculations for the experimental heat transfer coefficient. An insulated stand was built to prevent heat loss on all other sides of the plate aside from the face painted black. The stand was made out of PVC material and built with enough room to have thermal insulation underneath the plate and around the sides of the plate. A hole was drilled in the stand, insulation, and plate large enough for a thermocouple wire to fit to monitor temperature changes over time. An oven, already owned by the chemical engineering department, was used to heat the aluminum plate.

The first task was to determine what size plate needed to be used. The first plate used was 1.5” thick. This plate did not heat and cool in a reasonable timeframe for a Lab I experiment. Therefore, a new plate, 0.5” thick, was used to conduct the rest of the experimental runs. The desired temperature range to monitor was 75°C to 45°C. Multiple runs were executed to ensure that the data extracted would prove beneficial for students to replicate and discuss. The total time for the experiment takes 60-75 minutes for the plate to heat and cool long enough to see the entire temperature range desired.

I. Calculations Approach

Once sufficient data was collected, MATLAB's ode45 function was used to model the experiment. To determine the experimental heat transfer coefficient, the heat transfer coefficient was manually changed to determine the value at which the integrated differential equation best matched the experimental data plotted. This is a brute force method and most certainly not the most efficient way to find the coefficient. However, Lab I students have limited coding experience, so this method is a good approach for a beginner to try. In the future, more advanced, iteration-loop-based coding along with defining a metric to determine how well two lines match each other could be applied to find the exact number for the experimental heat transfer coefficient.

II. Results

The total time in the oven, final temperature reached, and percent error for the 0.5" plate for nine experimental runs is presented in Table 1.

Table 1. Experimental Data Summary for 0.5" Aluminum Plate

0.5" plate			
Run	Temp reached (°C)	Total time in oven (min)	Percent Error
1	85	50	32.65
2	100.3	40	32.86
1	105.5	50	13.91
1	69.9	30	11.40
1	64	30	19.21
2	115.9	30	4.93
3	132.3	30	1.19
1	85.3	40	0.70
2	103.5	30	0.38
Average			13.03

Figure 1 shows the curve for the experimental run with the smallest percent error for the 0.5" plate.

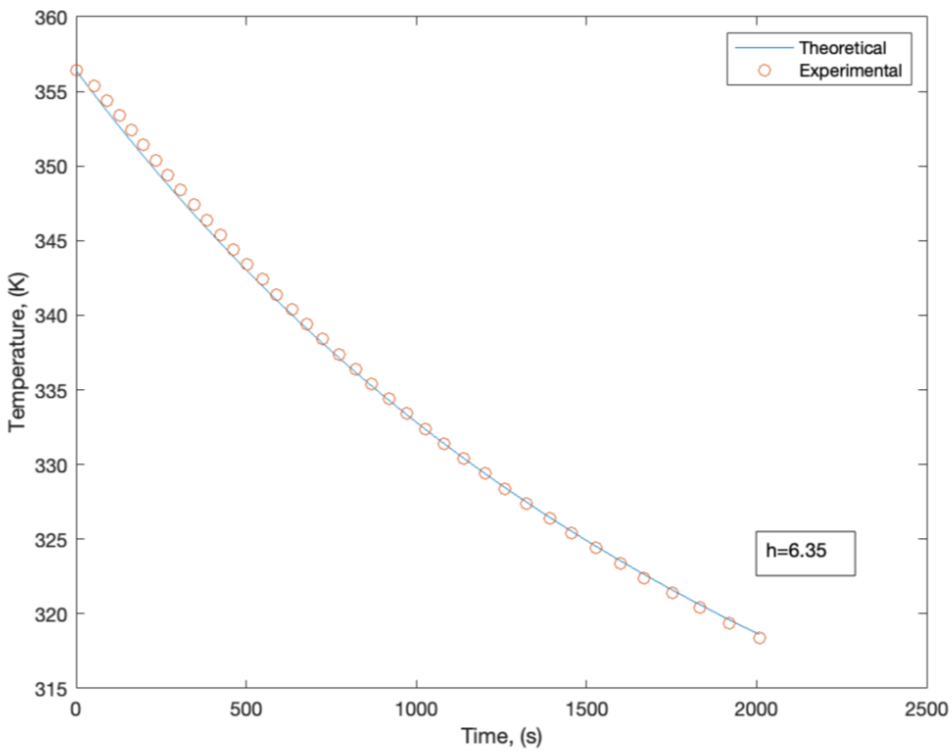

Figure 1. Experimental Run, 03/15, for 0.5" Aluminum Plate

A summary of relevant calculated values and inputs are displayed in Table 2 used to determine percent error and integrity of the experiment for the 0.5" plate.

Table 2. Summary of Inputs and Calculated Values for Experimental Run, 03/15, for 0.5" Plate

	Value	**Units**
h_{CORR}	6.39	
h_{EXP}	6.35	
Tplate	83.4	°C
Tatm	21	°C

III. Discussion

The 0.5" plate requires less time in the oven; therefore, it should be used instead of the 1.5" plate. The usage of MATLAB allows for solving the relevant differential equation and capabilities to produce a figure all in the same place. Therefore, MATLAB is a useful and effective tool for displaying results and making calculations for this experiment.

Future Work

Another important geometry for heat convection is the vertical geometry. This experiment focuses on the horizontal geometry. It would be beneficial for students to compare the two geometries to further their understanding of heat convection. In addition, a similar experiment using forced convection could be created. The MATLAB method used in this report is very inefficient as the changing variable must be changed manually and how well the lines match one another is based on visual judgement. It would be beneficial to have more experienced students do this experiment with code that defines a metric to determine how well the lines match to one another and have code that can change the variable based on that metric to eliminate any human errors.

IV. Appendix

Data Reduction

A heat balance on the center plate, with no heat generation, yields Equation 1:

$$-q_{OUT} = q_{ACC} \tag{1}$$

The plate is cooled by free convection and radiation, as is shown in Equation 2:

$$q_{OUT} = q_{CONV} + q_{RAD} = hA_S(T_{PLATE} - T_{ATM}) + \varepsilon\sigma A_S(T_{PLATE}^4 - T_{ATM}^4) \tag{2}$$

The plate accumulates heat with an inverse relationship to time as it cools back to room-temperature, noted in Equation 3:

$$q_{ACC} = m(C_p)\frac{dT}{dt} = \rho V(C_p)\frac{dT}{dt} \tag{3}$$

Thus, the heat balance of Equation 1 yields Equation 4:

$$-\left(hA_S(T_{PLATE} - T_{ATM}) + \varepsilon\sigma A_S(T_{PLATE}^4 - T_{ATM}^4)\right) = \rho V(C_p)\frac{dT}{dt} \tag{4}$$

Experimental data of temperature vs. time were used to determine the "best fit" experimental heat transfer coefficient by integrating Equation 4 using MATLAB's ode45 function.

The heat transfer coefficient from the literature was determined using the correlation for free convection from a horizontal heated, upward-facing plate (Cengal 2007, Table 9-1, p. 511), shown in Equations 5a and 5b:

$$Nu = 0.54Ra^{1/4} \qquad 10^4 < Ra < 10^7 \tag{5a}$$

$$Nu = 0.15Ra^{1/4} \qquad 10^7 < Ra < 10^{11} \tag{5b}$$

where the Rayleigh number is calculated as in Equation 6:

$$Ra = \frac{g\beta(T_{PLATE} - T_{ATM})L^3}{\nu^2}\boldsymbol{Pr} \tag{6}$$

In Equation 6, the length of the plate is the characteristic length in free convection and, for a horizontal flat plate, L = $\frac{A_S}{P}$. Assuming that the surrounding air is an ideal gas, the volumetric expansion coefficient may be calculated in Equation 7:

$$\beta = \frac{1}{T} \tag{7}$$

Finally, h_{CORR} may be calculated from the Nusselt number as shown in Equation 8:

$$h_{CORR} = \frac{kNu}{L} \tag{8}$$

The experimental coefficient will be higher than the coefficient calculated from a literature correlation since it is impossible to remove all forced convection influences and achieve only free convection.

Sample Calculation

A sample calculation will be made using the experimental data from the March 15th run which displayed a low percent error from the 0.5" plate heated to an internal temperature of 356.4K.

The theoretical heat transfer coefficient, h_{CORR}, is calculated using the equation below:

$$h_{CORR} = \frac{kNu}{L}$$

Calculation of L is through As, the surface area of the plate and P, the perimeter of the plate.

$$L = \frac{As}{P} = \frac{0.204\,m^2}{1.89\,m} = 0.108\,m$$

Nu is calculated by finding the Rayleigh number, where $\beta = \frac{1}{T}$, with T being the internal temperature of the plate at 356.4 K, g equal to 9.81 m/s^2, ν equal to 0.00002085 m^2/s, and Prandtl number of air equal to 0.7157.

$$Ra = \frac{g\beta(T_{PLATE} - T_{ATM})L^3}{\nu^2}\boldsymbol{Pr} = \frac{9.81\frac{m}{s^2} * \frac{1}{356.4K} * (356.4\,K - 294\,K) * 0.108m^3}{\left(0.00002085\frac{m^2}{s}\right)^2} * 0.7157 = 3.58 \times 10^6$$

Because the Rayleigh number is in magnitude of 10^6, equation 5a is used to calculate Nu.

$$Nu = 0.54Ra^{\frac{1}{4}} = 0.5 * 3.58 \times 10^6 = 23.48$$

The fluid thermal conductivity of air, k, is 0.02945 W/mK. Therefore,

$$h_{CORR} = \frac{0.02945W/mK * 23.48}{0.108\,m} = 6.39\frac{W}{m^2K}$$

The experimental heat transfer coefficient, h_{EXP}, was found to be 6.35. The percent error can be calculated using the following equation

$$\frac{(experimental\ value - theoretical\ value)}{theoretical\ value} \times 100\%$$

Applying this formula, the percent error for this experimental run would be

$$\frac{(6.35 - 6.39)}{6.39} \times 100\% = 0.7\%$$

Sample Code

The following code was used to find the theoretical values for the run shown in the sample calculation along with the figure produced.

```
%Theoretical Data ----------------------------------------------

%Constants

l=0.6096;                 %Length of aluminum plate, m
w=0.33528;                %Width of aluminum plate, m
As=l * w;                 %Surface area of plate, m^2
Tatm=294;                 %Temperature of the room in K
e=0.98;                   %Emissivity for black painted aluminum plate
o=5.67e-8;                %Stefan-Boltzmann constant, W/m^2*K^4
ht=0.00635;               %Thickness of plate, m
V=As*ht;                  %Volume of plate, m^3
m=6.51;                   %Mass of aluminum plate, kg
p=m/V;                    %Density of aluminum plate, kg/m^3
Cp=900;                   %Specific heat capacity of aluminum, kJ/kg*K
h=6.35;                   %Heat transfer coefficient, W/m^2*K

%Calling Experimental Data from Excel -----------------------------------

texp = HeatS10.texp;      %Time in s
Texp = HeatS10.Texp;      %Temperature in K

%tspan and function definition:

tspan = [0,2008];
odefun = @(t,Tplate) (-(h*As*(Tplate-Tatm)+e*o*As*(Tplate^4-Tatm^4)))/(p*V*Cp);

%Using ode45

[t,Tplate]=ode45(odefun, tspan, 356.4);

%Plotting

plot (t,Tplate)
hold on
scatter(texp, Texp)
hold on
xlabel('Time, (s)')
ylabel('Temperature, (K)')
```

```
legend ('Theoretical', 'Experimental')
hold off
```

The above code produces Figure A.1.

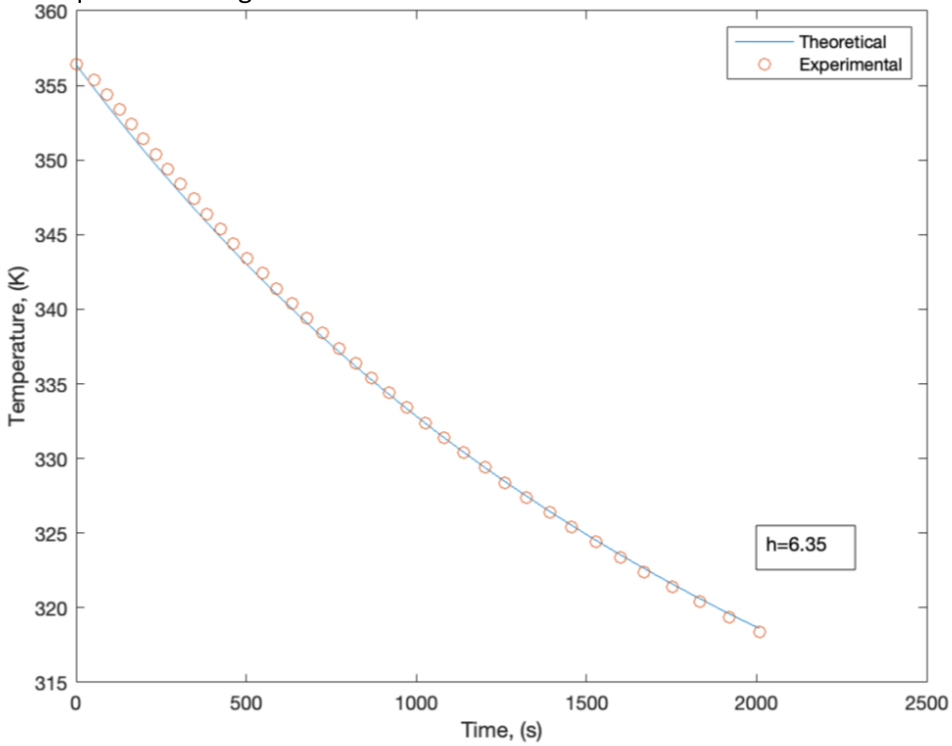

Figure A.1. Experimental and Theoretical Data, 03/15, for 0.5” Plate

Figures

Below are all figures produced throughout the collection of data for validating this lab experiment.

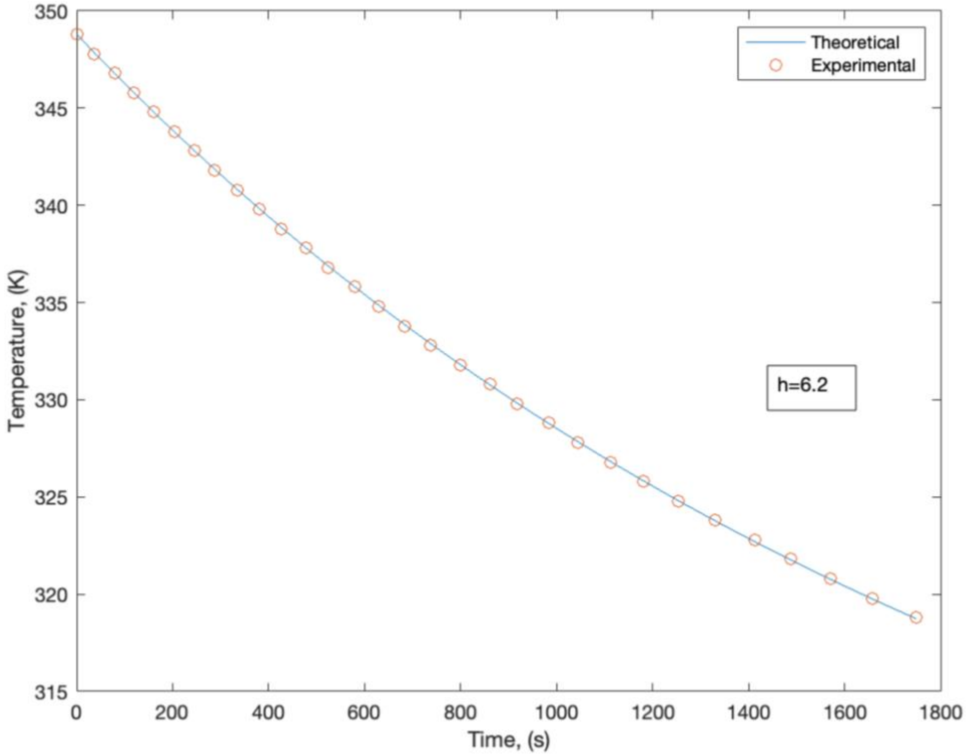

Figure A.2 Experimental and Theoretical Plot, 3/17, for 0.5" Plate

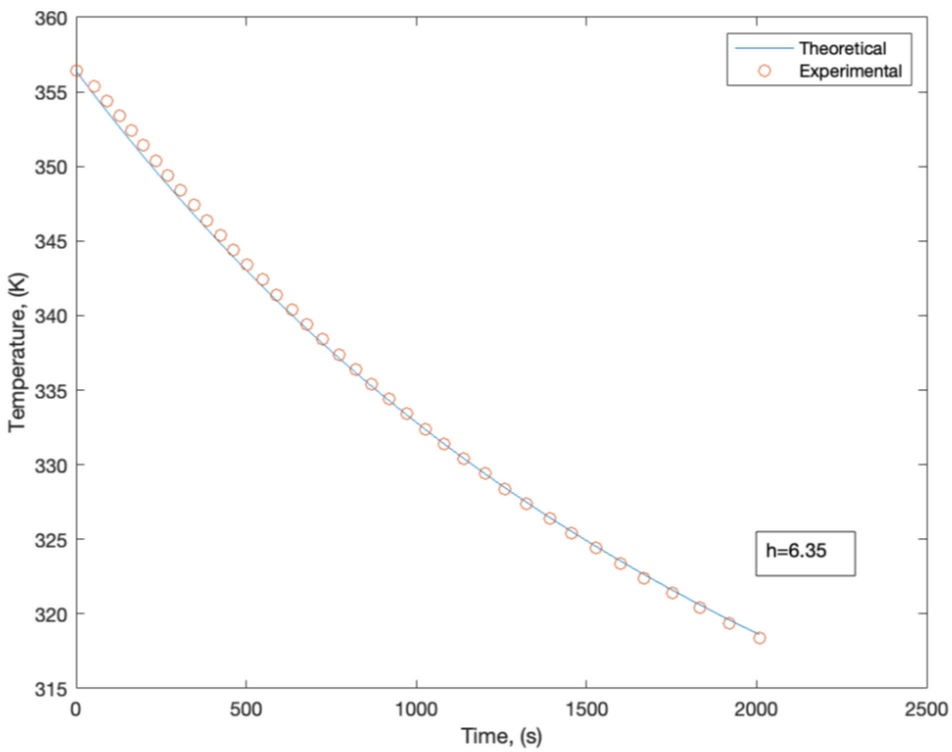

Figure A.3 Experimental and Theoretical Plot, 3/15, for 0.5” Plate

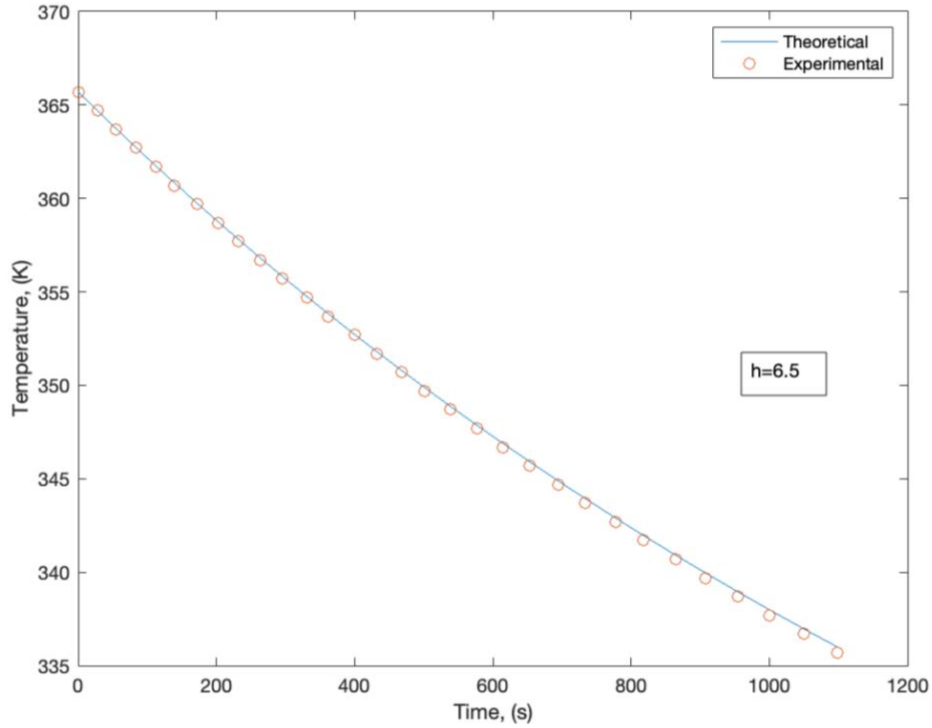

Figure A.4 Experimental and Theoretical Plot, 03/08 Run 3, for 0.5” Plate

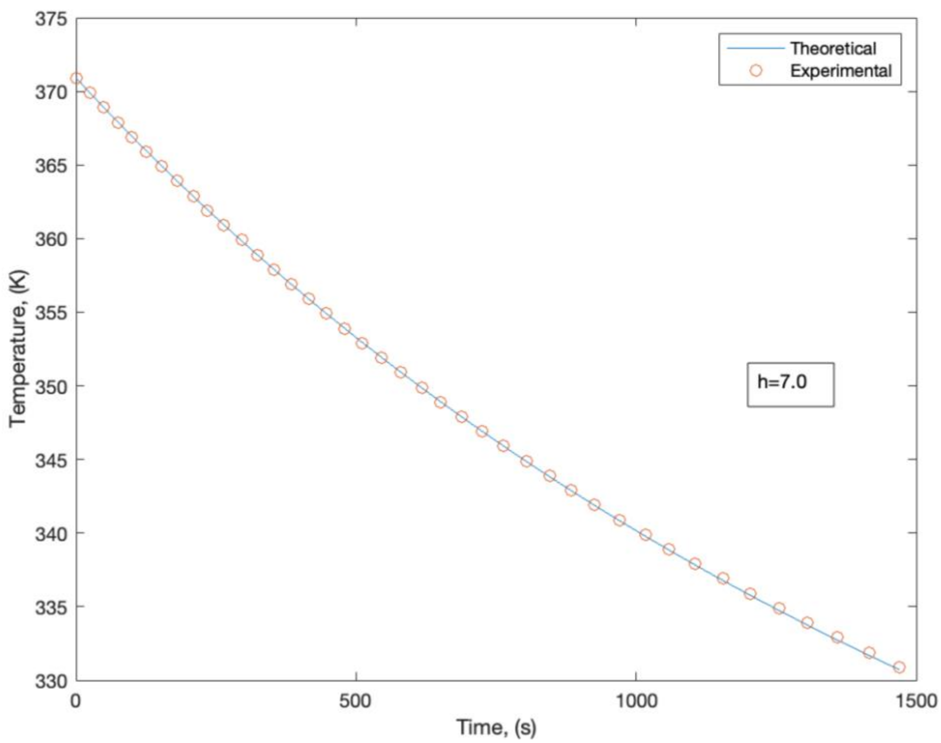

Figure A.5 Experimental and Theoretical Plot, 03/08 Run 2, for 0.5” Plate

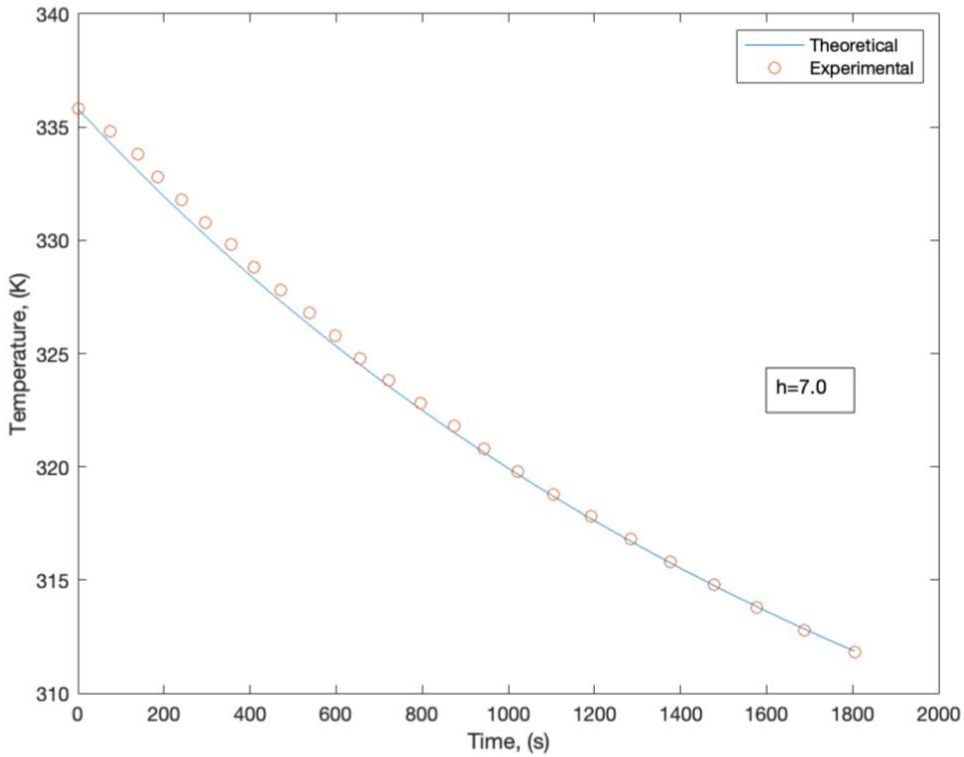

Figure A.6 Experimental and Theoretical Plot, 03/08 Run 1, for 0.5” Plate

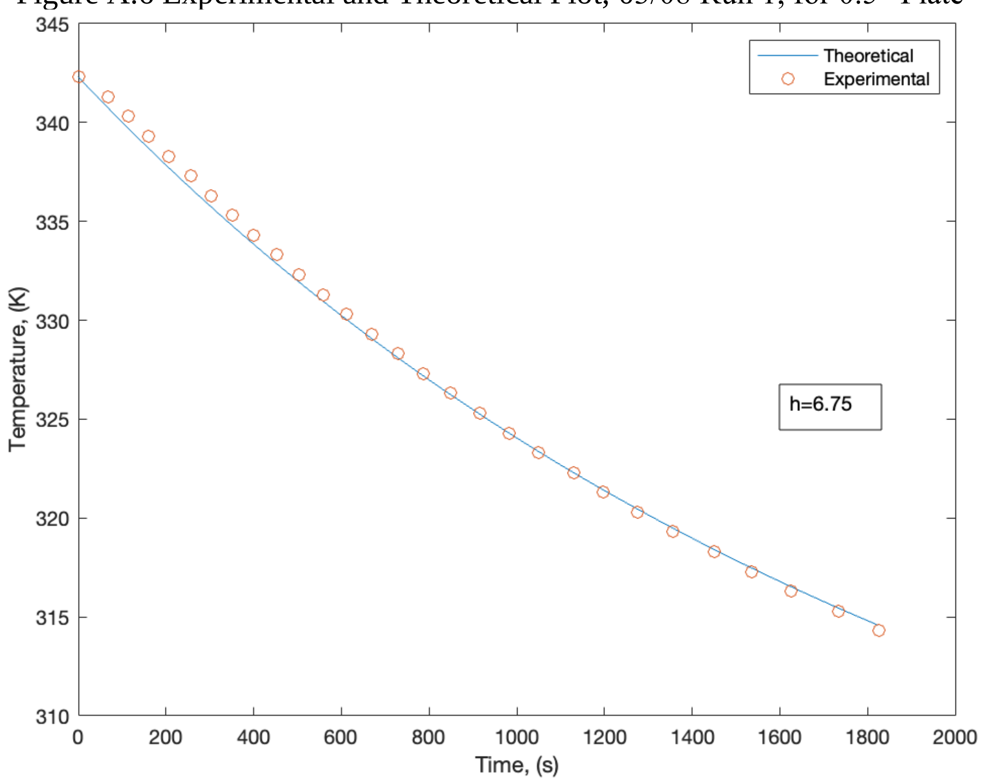

Figure A.7 Experimental and Theoretical Plot, 03/04, for 0.5” Plate

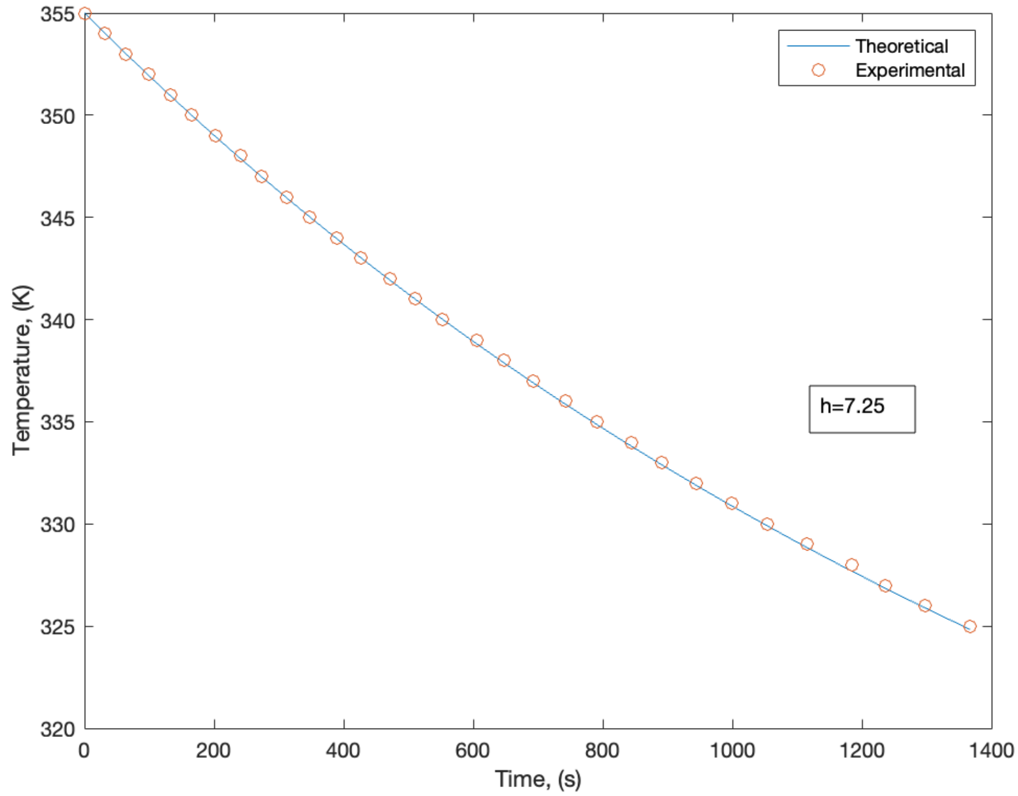

Figure A.8 Experimental and Theoretical Plot, 03/01, for 0.5” Plate

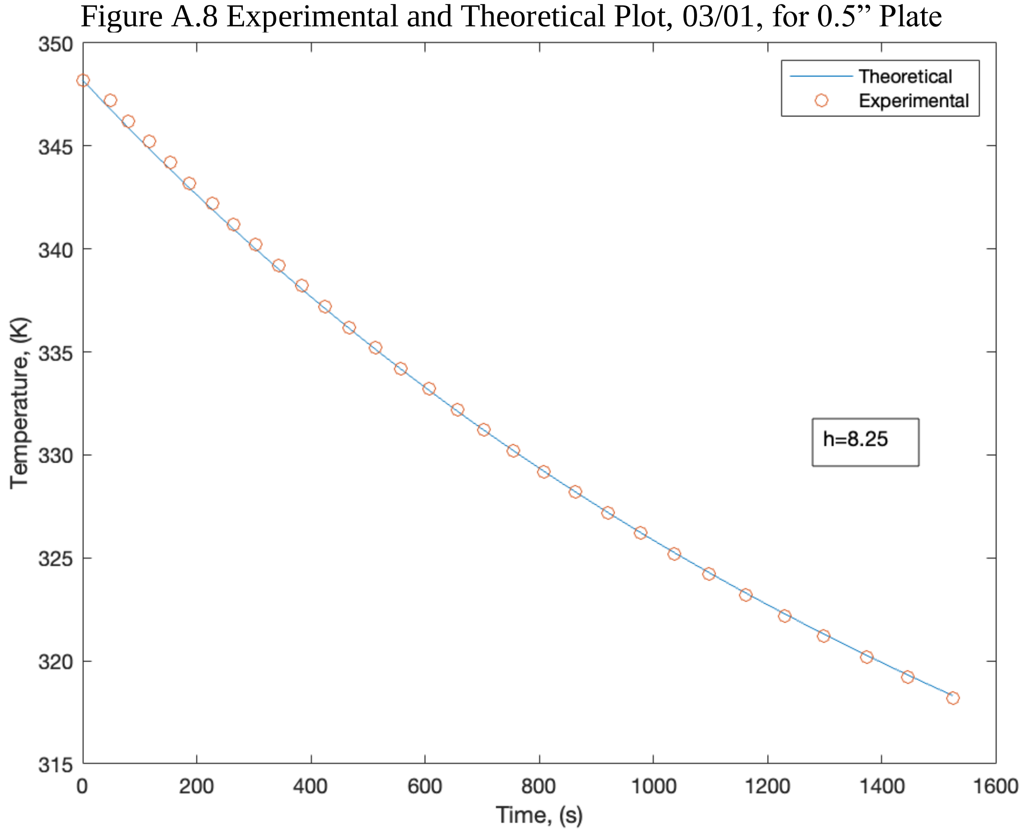

Figure A.9 Experimental and Theoretical Plot, 02/22 Run 2, for 0.5” Plate

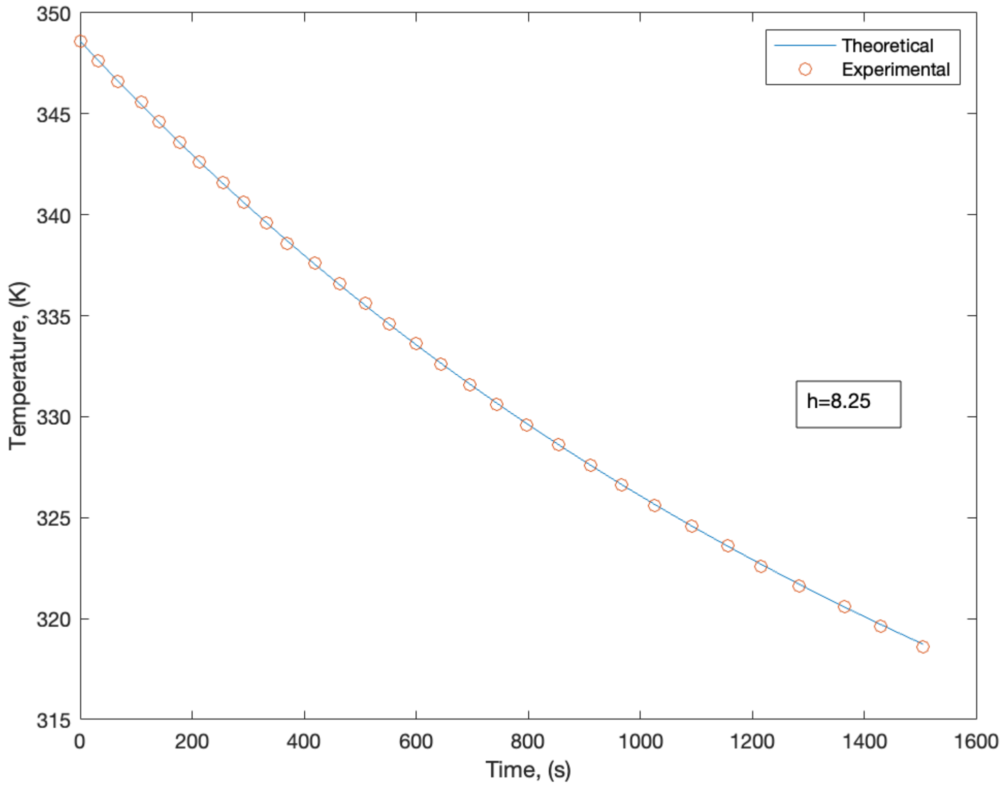

Figure A.10 Experimental and Theoretical Plot, 02/22 Run 1, for 0.5” Plate

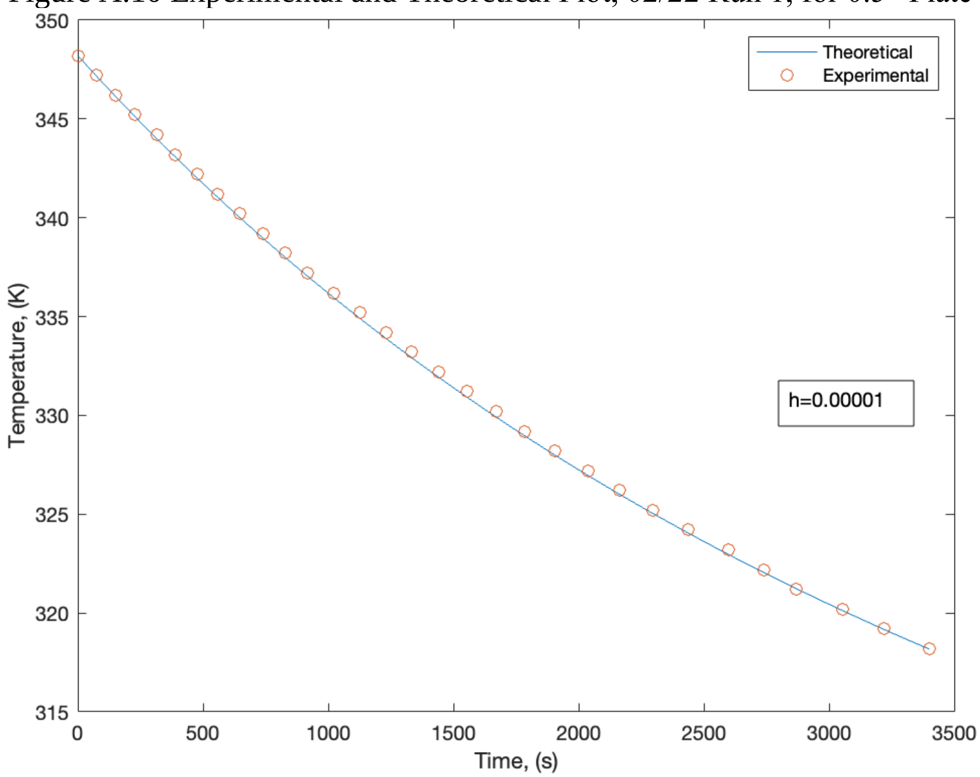

Figure A.11 Experimental and Theoretical Plot, 02/17, for 1.5” Plate

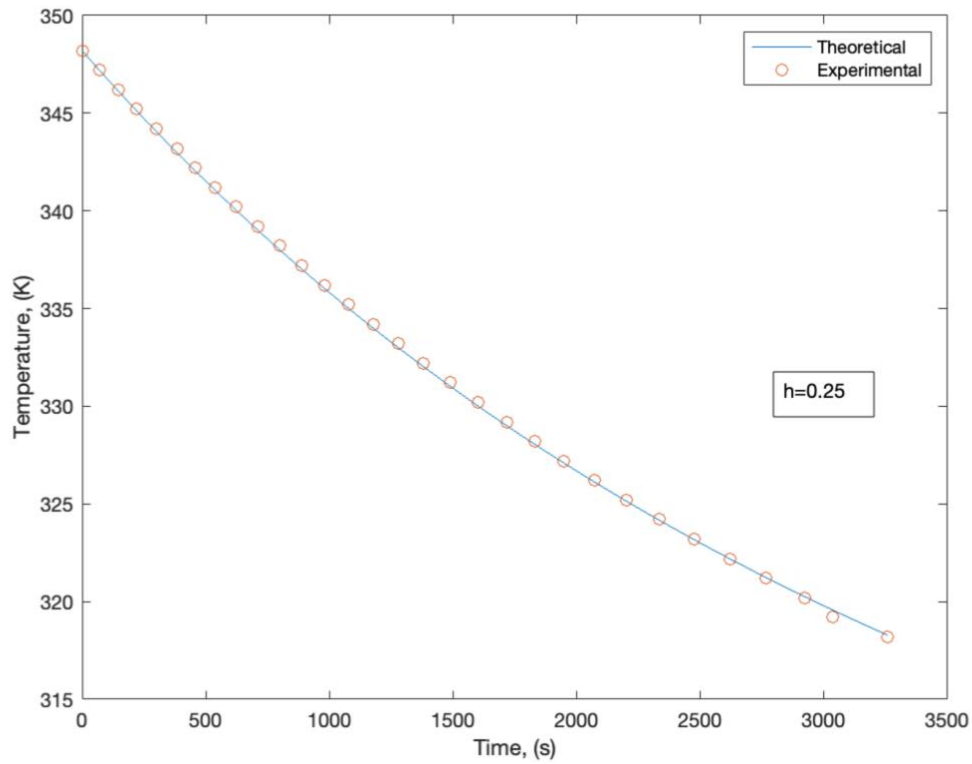

Figure A.12 Experimental and Theoretical Plot, 02/15, for 1.5” Plate

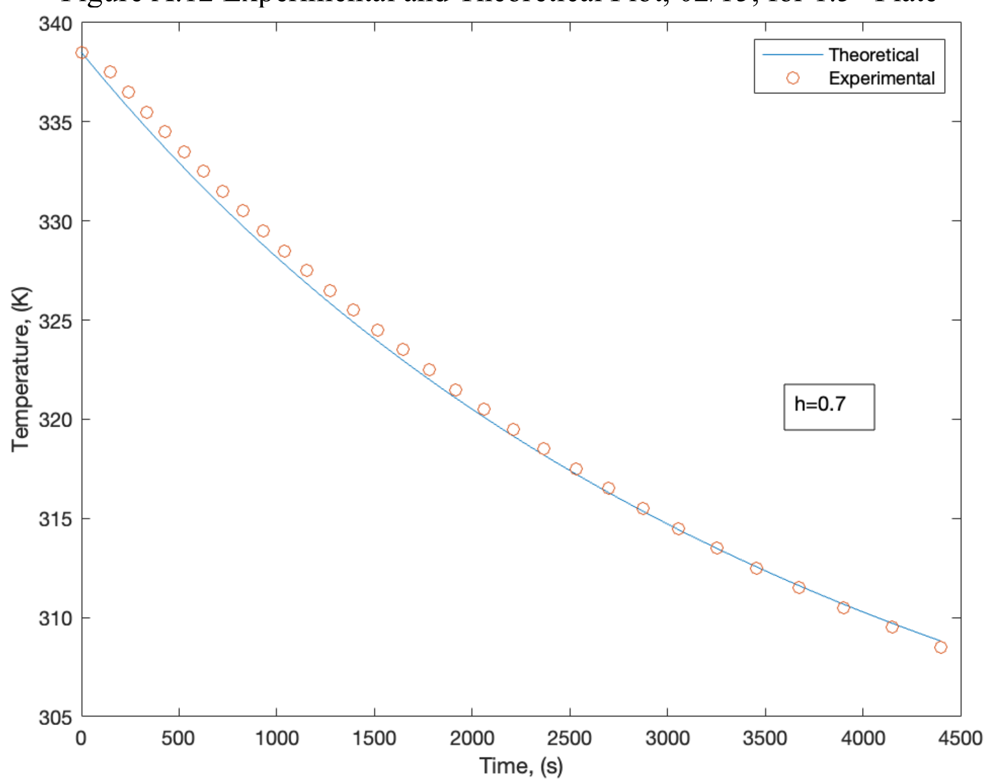

Figure A.13 Experimental and Theoretical Plot, 02/08, for 1.5” Plate

Nomenclature

A_S	area for convection, m^2
C_p	specific heat of the aluminum plate or cylinder, J/kg K
g	gravitational constant, m/s^2
h	convection heat transfer coefficient, W/m^2 K
h_{CORR}	correlated heat transfer coefficient, W/m^2 K
h_{EXP}	experimental heat transfer coefficient, W/m^2 K
k	fluid thermal conductivity, W/mK
L	length of the plate or cylinder, m
m	mass of the plate or cylinder, kg
Nu	Nusselt number
P	Perimeter of rectangular plate, m
Pr	Prandtl number of the fluid
q_{OUT}	heat transfer out of the system, W
q_{ACC}	heat accumulated in the system, W
q_{CONV}	heat transfer by convection, W
q_{RAD}	heat transfer by radiation, W
Ra	Rayleigh number of the fluid
T_{ATM}	temperature of the surroundings (atmospheric), K
T_{PLATE}	temperature at the center of the plate, K
V	volume of the plate or cylinder, m^3
β	volumetric expansion coefficient, K^{-1}
ε	emissivity of the surface
μ	dynamic viscosity of air, Ns/m^2
ν	kinematic viscosity of air, m^2/s
ρ	density of the aluminum plate or cylinder, kg/m^3
σ	Stefan-Boltzmann constant, W/m^2K^4

References

Cengel, Y.A. 2007. *Heat and Mass Transfer: A Practical Approach, Chapter 9: Natural Convection.* 3rd edition. Boston: McGraw-Hill.

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