Multiphoton Interaction in a System of Two Quantum Dots

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Abstract:
As the size and proximity of components on modern computer chips approaches quantum mechanical limits, various novel solutions have been proposed to ensure further increases in processing speed and reliability. Of these, small semiconductor devices called quantum dots may constitute the logic gates of future quantum computers - processors taking advantage of phenomena such as entanglement and quantum teleportation to enable ultra-fast computation speeds. Quantum dots behave much like designer atoms in that their absorption/emission energies can be adjusted to desired values. A quantum mechanical model of semiconductor quantum dots having equal size and interacting with a single-mode electric field tuned to some fraction of the transition frequency has been developed. Due to the similarities between two-level atoms and quantum dots, techniques common in quantum optics have been employed to describe the time evolution of the resulting exciton-field system. It is assumed that the quantum dots are initially prepared in a Bell entangled state and that the field is in a coherent state. Collapse and revivals in the entanglement are found when the mean photon number of the coherent state is very large.

Introduction

Ever since the development of the integrated circuit in the late 1950s, digital technology has been improving at an alarming rate. The speed at which electronic devices such as computers can process information depends directly on the number of components (primarily transistors) that can be contained on a single microchip. The rate of increase of this "transistor density," commonly referred to as Moore's Law, is quite staggering. Moore's Law states that the number of transistors that a chip can accommodate doubles once every year. While this is an approximate figure, it has held true for over thirty years, decreasing only recently to a doubling once every two years. Nevertheless, for decades scientists have wondered how many transistors can be packed onto a microchip before its data processing capabilities are sacrificed? In recent years, many obstacles to the advancement of chip technology have been overcome with development of new lithographic methods allowing for smaller-scale components on the chips themselves [1].
These new methods hold promise for the near future, but the fundamental problem of size remains. Quantum theory places a limit on the proximity of components on a microchip. At the smallest of scales, the wave nature of the electrons on the IC chip becomes more prominent, leading to catastrophic results as far as data processing is concerned.

One of the proposed solutions to the above problem makes use of the quantum nature of the electron, and it is the so-called "quantum computer (QC)." Integral to the function of the quantum computer would be the quantum dot, a structure of atomic dimensions (made of fewer than one thousand atoms) that, in some respects, behaves like a macroscopic atom. One may imagine the quantum dot as a small cell within a semiconductor material trapping electrons within it. The size of the cell so confines the electron's wave nature that only certain energies are allowed. As a result, many of the properties of relatively simple atoms can be used to model quantum dots.

Recent experimental work has demonstrated the viability of quantum dot systems as quantum bits for future QCs [2,3,4]. Such "qubits" would possess a considerable advantage over conventional systems in their implementation of quantum superposition to make many simultaneous calculations. Richard Feynman was the first to propose that entangled states between QDs in a cavity be exploited to cause changes in one qubit to affect all other qubits in the system. In fact, a topic of intense interest in quantum information research is the development of quantum algorithms for future QCs. The first of these, proposed by Peter Shor in 1994, is a method for factoring very large numbers (10\(^{200}\)) in a matter of seconds! The most obvious application of such an algorithm would be in the field of cryptography, which relies on the difficulty in factoring very large integers to encrypt data. Clearly, the possible applications of such quantum parallelism are staggering. However, the potential of quantum computation is rivaled only by the difficulties inherent in such a complex system. The ever-present quantum measurement problem requires a high degree of isolation from the outside environment for the entangled qubits to retain coherence. This problem of decoherence only worsens as the number of qubits increases. Therefore, detailed information concerning time evolution of initially coherent states is crucial to the understanding of this system.

**Multiphoton Interaction**

Let us consider the case of two coupled quantum dots within a high-Q (lossless) microcavity. In general, the Hamiltonian describing N equidistant, coupled QD's can be written as [5],

\[
\hat{H} = \omega \hat{a}^\dagger \hat{a} + \frac{\epsilon}{2} \sum_{n=1}^{N} (\hat{c}_{n}^\dagger \hat{c}_{n} - \hat{h}_{n} \hat{h}_{n}^\dagger) + g \sum_{n=1}^{N} (\hat{c}_{n} \hat{h}_{n}^\dagger \hat{a}^p + \hat{a}^\dagger \hat{h}_{n} \hat{c}_{n}) \\
+ \frac{W}{2} \sum_{n,m=1}^{N} (\hat{c}_{n} \hat{h}_{m}^\dagger \hat{c}_{m} \hat{h}_{n} + \hat{h}_{n} \hat{c}_{m}^\dagger \hat{c}_{m} \hat{h}_{n})
\]

Here \(\hat{a}\) and \(\hat{a}^\dagger\) correspond to the electromagnetic field annihilation and creation operators, respectively, and the constant \(p\) denotes the number of photons involved in the
creation of one exciton. When either $\hat{a}^p$ or $\hat{a}^{+p}$ act on the field, $p$ photons are either removed or added. The constant $g$ is the QD/field interaction strength while $\varepsilon$ represents the QD band gap. In addition, the constant $W$ describes the strength of the interdot Coulomb interaction. Also, the $\hat{e}$ and $\hat{h}$ are Fermi operators representing electron and hole annihilation, respectively. As is the case with the field operators, $\hat{e}^\dagger$ and $\hat{h}^\dagger$ are Fermi creation operators. As Fermi operators, $\hat{e}$ and $\hat{h}$ obey the following anti-commutation rules,

$$\{\hat{e}, \hat{e}^\dagger\} = \hat{e}\hat{e}^\dagger + \hat{e}^\dagger\hat{e} = 1 = \{\hat{h}, \hat{h}^\dagger\}$$

$$\{\hat{e}, \hat{h}\} = \{\hat{e}^\dagger, \hat{h}^\dagger\} = \{\hat{e}, \hat{h}^\dagger\} = \{\hat{e}^\dagger, \hat{h}\} = 0$$

(2)

To simplify manipulation, we make use of the above anti-commutation relations to define the so-called quasi-spin operators $\hat{J}_+, \hat{J}_-, \hat{J}_z$ as follows:

$$\hat{J}_+ = \sum_{n=1}^{N} \hat{e}_n^\dagger \hat{h}_n^\dagger, \quad \hat{J}_- = \sum_{n=1}^{N} \hat{h}_n \hat{e}_n, \quad \hat{J}_z = \frac{1}{2} \sum_{n=1}^{N} (\hat{e}_n^\dagger \hat{e}_n - \hat{h}_n \hat{h}_n^\dagger).$$

(3)

As is suggested by the use of symbols $\hat{J}_+, \hat{J}_-$, and $\hat{J}_z$, the quasi-spin operators obey commutation relations identical to all other angular momentum operators, which are that $[\hat{J}_+, \hat{J}_-] = 2\hat{J}_z$ and $[\hat{J}_z, \hat{J}_i] = \pm \hat{J}_i$. Making the necessary substitutions to (1) we arrive at

$$\hat{H} = \omega \hat{a}^\dagger \hat{a} + \varepsilon \hat{J}_z + W (\hat{J}_z^2 - \hat{J}_z) + g (\hat{J}_z \hat{a}^p + \hat{a}^{+p} \hat{J}_z)$$

(4)

Now, we assume a rotating frame of frequency $\omega$, and the resulting Hamiltonian is

$$\hat{H}_r = \Delta\omega \hat{J}_z + W (\hat{J}_z^2 - \hat{J}_z) + g (\hat{J}_z \hat{a}^p + \hat{a}^{+p} \hat{J}_z)$$

(5)

where $\Delta\omega = \varepsilon - \omega$ represents the field detuning from the QD band gap, $\varepsilon$ (from this point, we will assume zero detuning). To interpret Eq. 5, notice that the last term associates $\hat{a}^p$ with $\hat{J}_z$ and $\hat{a}^{+p}$ with $\hat{J}_z$, which relates loss of $p$ field photons with the gain of one unit of angular momentum and vice versa. The alternative interaction corresponding to a gain of photons and a gain in angular momentum does not preserve photon number and hence is strongly suppressed [6]. The same is true for a loss of photons with a loss in angular momentum.

The Two Dot Hamiltonian

For two coupled quantum dots with a basis of eigenstates $\hat{J}^2$ and $\hat{J}_z$, the following angular momentum states apply: $|J=1, M=-1\rangle = |0\rangle$, $|J=1, M=0\rangle = |1\rangle$, and $|J=1, M=1\rangle = |2\rangle$. The J=0 subspace is considered to be optically dark, and the number
assigned to each state represents the number of excitons present in the system. Excitons are the result of an interband transition in which an excited conduction electron binds to a valence hole. If the radiation field is in the Fock state $|n\rangle$, then in a subspace spanned by $|0\rangle \otimes |n + 2p\rangle$, $|1\rangle \otimes |n + p\rangle$, $|2\rangle \otimes |n\rangle$, $|0\rangle \otimes |n - p\rangle$, $|2\rangle \otimes |n - 2p\rangle$, the Hamiltonian matrix takes the form

$$
\begin{pmatrix}
W & \Omega_1 & 0 & 0 & 0 & 0 \\
\Omega_1 & 2W & \Omega_2 & 0 & 0 & 0 \\
0 & \Omega_2 & W & 0 & 0 & 0 \\
0 & 0 & 0 & \Omega_3 & 0 & 0 \\
0 & 0 & 0 & \Omega_3 & 2W & \Omega_4 \\
0 & 0 & 0 & 0 & \Omega_4 & W \\
\end{pmatrix}
$$

where

$$
\Omega_1 = g \sqrt{2(n + p + 1)\ldots(n + 2p)}, \quad \Omega_2 = g \sqrt{2(n + p)\ldots(n + 1)},
$$

$$
\Omega_3 = g \sqrt{2(n - p + 1)\ldots n}, \quad \Omega_4 = g \sqrt{2(n - 2p + 1)\ldots(n - p)}.
$$

In accordance with the postulates of quantum mechanics, the allowed energies and energy states of the system are precisely the eigenvalues and eigenvectors of the above Hamiltonian, respectively. The energies are

$$
E_1 = W
$$

$$
E_{2,3} = \frac{3W}{2} \pm \frac{\sqrt{W^2 + 4(\Omega_2^2 + \Omega_3^2)}}{2} = \frac{3W}{2} \pm \frac{x}{2}
$$

$$
E_4 = W
$$

$$
E_{5,6} = \frac{3W}{2} \pm \frac{\sqrt{W^2 + 4(\Omega_3^2 + \Omega_4^2)}}{2} = \frac{3W}{2} \pm \frac{y}{2}
$$

and the corresponding allowed energy states are

$$
|F_1\rangle = \frac{2}{\sqrt{x^2 - W^2}} \left( \Omega_2 |0, n + 2p\rangle - \Omega_1 |2, n\rangle \right)
$$

$$
|F_{2,3}\rangle = \frac{\Omega_2 \sqrt{2}}{\sqrt{x(x \pm W)}} \left( |0, n + 2p\rangle \pm \frac{\sqrt{x \pm W}}{2x} |1, n + p\rangle + \frac{\Omega_3 \sqrt{2}}{\sqrt{x(x \pm W)}} |2, n\rangle \right)
$$

$$
|F_4\rangle = \frac{2}{\sqrt{y^2 - W^2}} \left( \Omega_3 |0, n\rangle - \Omega_3 |2, n - 2p\rangle \right)
$$

$$
|F_{5,6}\rangle = \frac{\Omega_3 \sqrt{2}}{\sqrt{y(y \pm W)}} \left( |0, n\rangle \pm \frac{\sqrt{y \pm W}}{2y} |1, n - p\rangle + \frac{\Omega_4 \sqrt{2}}{\sqrt{y(y \pm W)}} |2, n - 2p\rangle \right).
$$
The dots are assumed to be initially prepared in a Bell state consisting of a coherent superposition of the vacuum and bi-exciton states of the form

\[ |\Psi(x,t=0)\rangle = \sum_{n=0}^{\infty} A(n)[c_0|0\rangle + e^{i\phi}c_2|2\rangle] \otimes |n\rangle. \] (8)

The sum over coefficients \( A(n) = \exp\left(-\frac{|\alpha|^2}{2}\frac{|\alpha|^n}{\sqrt{n!}}\right) \) and states \(|n\rangle\) represents an expansion of a coherent state \(|\alpha\rangle\) of the laser field in Fock states, while the coefficients \(c_0^2\) and \(c_2^2\) correspond to the probabilities for the system to initially be in the states \(|0\rangle\) and \(|2\rangle\), respectively [7]. The \(e^{i\phi}\) term simply accounts for phase differences between the two states, and contributes nothing to the initial probability distribution. The \(|\alpha|^2\) term in \(A(n)\) is the mean photon number \(\bar{n}\) of the radiation field. The transition from the single-exciton to the bi-exciton state has been found to occur at slightly lower energies than the transition from the vacuum to single-exciton state as a result of the Coulomb interaction due to the first exciton [2]. One may imagine a qubit in which the vacuum and bi-exciton states represent a 0 and 1, respectively. The time-dependent wave function is therefore

\[ |\Psi(x,t)\rangle = \sum_{n=0}^{\infty} A(n)[c_0\left(\frac{2\Omega_4}{\sqrt{y^2-W^2}}e^{-i\kappa t}\right)|E_4\rangle + \frac{\Omega_3\sqrt{2}}{\sqrt{y(y+W)}}e^{i\kappa t}|E_3\rangle + \frac{\Omega_3\sqrt{2}}{\sqrt{y(y-W)}}e^{-i\kappa t}|E_5\rangle] + c_2e^{i\phi}\left(\frac{-2\Omega_4}{\sqrt{x^2-W^2}}e^{-i\kappa t}|E_3\rangle + \frac{\Omega_2\sqrt{2}}{\sqrt{x(x+W)}}e^{i\kappa t}|E_4\rangle + \frac{\Omega_2\sqrt{2}}{\sqrt{x(x-W)}}e^{-i\kappa t}|E_5\rangle\right). \] (9)

Of course, the primary goal here is to determine the probability for the system to be in the initial coherent state \(|\Psi(x,t=0)\rangle\) at some time \(t\). For completeness and error checking, the time dependence of the following three orthonormal states has been calculated:

\[ P_1(t) = \sum_{n=0}^{\infty} |\langle 1,m|\Psi(x,t)\rangle|^2 \]

\[ P_2(t) = \sum_{m=0}^{\infty} |\langle m|\otimes(|0\rangle c_0 - |2\rangle c_2)|\Psi(x,t)\rangle|^2 \]

\[ P_3(t) = \sum_{m=0}^{\infty} |\langle \Psi(x,t=0)|\Psi(x,t)\rangle|^2 \]

\[ = \sum_{m=0}^{\infty} |\langle m|\otimes(|0\rangle c_0 + |2\rangle c_2 e^{-i\phi})|\Psi(x,t)\rangle|^2. \] (10)
Since the states shown in (10) comprise an orthonormal basis, the sums of all three probabilities at any time \( t \) must be unity, and therefore the accuracy of the calculations can easily be determined.

**Results**

Figure 1 displays the time development of the single exciton state in the cases of the one- through four-photon interaction. Notice that, aside from time scale differences, the first, third, and fourth plots show similar phenomena: an initial spike followed by alternating periods of collapse and revival of the state. In all cases, the collapse/revival phenomenon is the result of the spread in the initial photon number distribution of \( |\alpha\rangle \) [6]. The revivals occur when sufficiently many oscillatory terms are in phase with one another. However, the case of two-photon absorption is quite distinct, in that we observe relatively sharp revival peaks. The explanation for this lies in the form of \( P_1(t) \):

\[
P_1(t) = \sum_{m=0}^{\infty} 4 |c_0 A(m + p) \frac{\Omega_2}{u} + c_2 e^{i\phi} A(m - p) \frac{\Omega_3}{u}|^2 \times \sin^2 \left( \frac{u}{2} t \right)
\]

\[
u = \sqrt{W^2 + 4(\Omega_2^2 + \Omega_3^2)}.
\]

In the given figures, \( W=1 \) and \( g=0.25 \), which means for the \( p=2 \) case, when \( m \) grows much larger than 1, \( u = m \) and the sum becomes a Fourier series. As a result, the \( p=2 \) case displays much more regularity.

Figure 2 displays the basis of states for the two-photon absorption. One interesting result of the aforementioned periodicity of \( P_1(t) \) lies in the drastic revival pattern of the initial coherent state. Instead of merely decaying over time through a series of collapses, \( P_3(t) \) reaches unity with the frequency of \( P_1(t) \) (\( \approx 0.16 \) Hz).

**Conclusion**

A theoretical model of the interaction between two entangled quantum dots and a coherent laser field has been developed. In addition, the time evolution of the complete set of exciton states has been discerned for general multiphoton interactions. Two-photon absorption has proven quite interesting due to the mathematical structure of the wave function, which allows for complete periodic revival of the initial state. Future research may focus on different radiation fields (i.e. squeezed states) and varying phase values in the initial wave function \( |\Psi(x, t = 0)\rangle \). Also, larger numbers of entangled quantum dots may be considered, resulting in a larger total angular momentum \( J \).

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References

Figure 1. Time evolution of the single exciton state for the one- through four-photon interactions. Here $W=1$, $g=0.25$, $|\alpha|^2=75$, $\phi=0$, and $c_0=c_2=1/\sqrt{2}$. 
Figure 2. Time evolution of all three orthonormal states describing two-photon excitation. Constants are equivalent to those in Figure 1. Note the effect of $P_1$ on the revival pattern of the initial state $P_3$. 