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Dark Energy Interaction Models

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Running Title: Dark Energy Interaction Models

Abstract

A specific interaction between dark energy and matter has been introduced in order to present possible solutions to the cosmic coincidence problem. We also compared the result of our differential equations with the experimental supernova data obtained by WMAP.

Introduction

The discovery of the accelerated expansion of the universe in 1998 (Riess et al. 1998) led to the resurrection of the concept of dark energy and cosmological constant. With only matter present (dark or baryonic), the dominating force in the universe would be gravity, which would result in an overall attraction between the constituents of the universe and a decelerated expansion. However, with accelerated expansion came the need for a component of the cosmic inventory that would provide the “negative pressure” necessary to create such conditions. The mysterious component with this property is called dark energy.

NASA’s Wilkinson Microwave Anisotropy Probe (WMAP) was launched in June 2001. One of the results returned was the fact that the curvature of universe is very close to zero. For any general space-time, the curvature can either be positive, negative, or zero. In order for a universe to be flat, or the curvature to be zero, the density of the universe has to be equal to a quantity called the critical density. However, the latest results from WMAP show that the density of matter is only 28.35% of the critical density. This implies that there must be another component in the cosmic inventory (dark energy) that accounts for the missing 71.65% of the critical density.

The identity of dark energy remains a mystery although there are many hypotheses to explain it. Possible candidates include a Quintessence field, Phantom energy, and even alternate theories of gravity (Copeland et. al. 2006). Another one of the candidates for dark energy is the famous cosmological constant

proposed by Einstein to maintain a static universe. The idea was abandoned upon evidence of the expanding universe, but has returned to the forefront as one of the most promising candidates for dark energy. The negative pressure property of dark energy is correctly established by the cosmological constant. For the cosmological constant

$$P_{\Lambda} = -\rho_{\Lambda},$$

where P_{Λ} and ρ_{Λ} are the pressure and density of dark energy respectively.

Current Model

WMAP has established a highly constrained Λ CDM (Lambda Cold Dark Matter) cosmological model with incredibly precise measures of parameters. The numerical findings that are of particular importance to the current studies are:

$$\Omega_{\Lambda} = 0.7165^{+0.0095}_{-0.0096} \text{ and } \Omega_m = 0.2836^{+0.0096}_{-0.0095},$$

where Ω_{Λ} and Ω_m are the dark energy density and the matter density parameter respectively (Nine year WMAP observations).

Assuming a homogeneous and isotropic universe, the geometry of space-time can be determined by the FLWR (Friedmann-Lemaître-Robertson-Walker) metric

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1-kr^2} + d\Omega^2 \right], \quad (1)$$

where $a(t)$ is the scale factor, k is the curvature parameter, and Ω is the spatial components of the metric. The components of the universe can be shown to behave according to

$$\dot{\rho}_{\Lambda} + 3H\rho_{\Lambda}(1 + w_{\Lambda}) = 0, \quad (2)$$

$$\dot{\rho}_m + 3H\rho_m(1 + w_m) = 0, \quad (3)$$

where H is the Hubble parameter and $w = P/\rho$ is the equation of state for each component of the cosmic inventory. These equations are derived from Einstein's field equations with the FLWR metric and are considered a part of the standard model of cosmology.

It can be shown from equations (2) and (3) that ρ_Λ stays constant, assuming an equation of state equal to -1 for dark energy. However, assuming pressureless dust, $\rho_m \propto a^{-3}$. This means that the density of dark energy remains constant while the density of matter will reduce with a positive change of the scale factor, which represents the size of the universe. Thus, for a universe that is expanding at an accelerated rate such as ours, the density of matter will eventually become negligible. Since it has been observed that our universe is around 29% matter, Equations (2) and (3) imply that the universe is coincidentally at its current state and that it is heading toward a dark energy dominated phase. This predicament is referred to as the "cosmic coincidence problem" (Steinhardt 1997).

Interaction

In order to resolve the cosmic coincidence problem, interaction models have been introduced to alter the equilibrium solutions of the differential equations that govern the behavior of the components of the universe (Myung 2005, Zhang 2004). Interaction between matter and dark energy is defined in the following manner:

$$\dot{\rho}_\Lambda + 3H\rho_\Lambda(1 + w_\Lambda) = -Q, \quad (4)$$

$$\dot{\rho}_m + 3H\rho_m(1 + w_m) = Q. \quad (5)$$

The interaction term, Q , is introduced in this fashion so that a positive interaction will convert dark energy into matter.

The governing equations for the evolution of the components of the universe are

$$\begin{aligned} \frac{d\Omega_\Lambda}{dx} = & -3\Omega_\Lambda(1 - \Omega_\Lambda)(w_\Lambda^{eff} - w_m^{eff}) \\ & + \Omega_\Lambda\Omega_k(1 + 3w_m^{eff}), \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{d\Omega_k}{dx} = & 3\Omega_\Lambda\Omega_k(w_\Lambda^{eff} - w_m^{eff}) \\ & + \Omega_k(1 + \Omega_k)(1 + 3w_m^{eff}). \end{aligned} \quad (7)$$

In these equations x represents time. It is defined in such a way that $x = 0$ represents the present time, positive values are future events, and negative values

are past events. The density parameters are defined as

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}}, \Omega_m = \frac{\rho_m}{\rho_{cr}}, \text{ and } \Omega_k = \frac{k}{(aH)^2},$$

where ρ_{cr} is the critical density and Ω_k is called the curvature density parameter. The three parameters are not independent; they obey the following relation:

$$\Omega_\Lambda + \Omega_m = 1 + \Omega_k. \quad (8)$$

Finally, considering a holographic condition on dark energy density (Pavon and Zimdahl 2005) and using the future event horizon (Setare 2006) the effective equations of state are defined as follows:

$$w_\Lambda^{eff} = -\frac{1}{3} - \frac{2}{3}\sqrt{\Omega_\Lambda - \Omega_k} \text{ and } w_m^{eff} = -\frac{Q}{3H\rho_m}.$$

When there is no interaction, $Q = 0$ and the differential equations yield solutions of a dark energy dominated universe. In other words, a dark energy dominated universe is a stable equilibrium point for the universe. The interaction term can be introduced in such a way that the equilibrium points for the density parameters are equal to the values observed by WMAP for the present time (Berger and Shojaei 2008).

Results and Discussion

Many forms of interaction have been explored. An interaction of particular interest is one in which it is related exponentially to the curvature density parameter: (Berger and Shojaei 2008)

$$\frac{Q}{3H\rho_\Lambda} = \frac{b^2 e^{-p\Omega_k(1+\Omega_k)}}{\sqrt{\Omega_\Lambda}}, \quad (9)$$

where p and b are parameters. Altering values for p and b^2 creates changes in the evolution of the density parameters as well as the stable equilibrium points. Interesting behavior is exhibited in Figure 1 with $p = 5$ and $b^2 = 0.301$.

The interesting part of the graph in Figure 1 is the spike in the matter density parameter curve which could resemble the behavior of matter at the end of inflation or the reheating era. The sharp decrease in the density of the matter right before the spike could represent the behavior of matter density during inflation, when it significantly decreased as a result of rapid expansion of the universe. The equilibrium

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positions of the density parameters are

$$\Omega_\Lambda = 0.7162 \text{ and } \Omega_m = 0.2838,$$

which agree with the values obtained by WMAP within the errors.

Figure 1. Evolution of the density parameters with interaction in Equation (9), $p=5$ and $b^2=0.301$.

Supernova data can be used to compare the interaction model predictions with experimental results. The luminosity distance is defined as

$$d_L(z) = (1+z) \int_0^z \frac{dz'}{H(z')}, \quad (10)$$

where z is the redshift and is related to x in the following way:

$$z = e^{-x} - 1.$$

The evolution of the Hubble parameter is given by (Berger and Shojaei 2006)

$$\frac{1}{H} \frac{dH}{dx} = -\frac{3}{2}(1 - \Omega_\Lambda) - \frac{\Omega_\Lambda}{H} \frac{\dot{\Omega}_\Lambda}{\Omega_\Lambda} + 2 \frac{Q}{3H\rho_\Lambda} \Omega_\Lambda, \quad (11)$$

where

$$\frac{1}{H} \frac{\dot{\Omega}_\Lambda}{\Omega_\Lambda} = 1 - \frac{\sqrt{\Omega_\Lambda}}{c},$$

and c is a constant set to unit value. Using the evolution of the density parameters given by Equations (6) and (7), the Hubble parameter can be found by solving Equation (11) and then used to find the luminosity distance for the model. The supernova data is expressed in terms of an apparent magnitude and

redshift. Assuming that the supernovae have the same absolute magnitude, the extinction-corrected distance moduli is given by

$$\mu(z) = 5 \log_{10}(d_L(z)) + 25. \quad (12)$$

The calculated distance moduli from Equation (12) can be compared with the supernova data. Figure 2 shows the supernova data plotted with the distance moduli corresponding to the interaction model of Figure 1. The observational data points are the currently available points in the Supernova Cosmology Project (Supernova Cosmology Project 2012).

Figure 2. Model comparison with supernova data, interaction given by Equation (9).

A χ^2 fit can be performed where

$$\chi^2 = \sum_i \frac{[\mu_{obs}(z_i) - \mu_{mod}(z_i)]^2}{\sigma_i^2},$$

and the sum runs over the supernova data points and σ_i is the experimental error associated with each observed data point. Minimizing the χ^2 value provides the best fit of the data for the model. The interaction model in Figures 1 and 2 have a minimized χ^2 value equal to 789.61 with over 580 degrees of freedom.

The best fit was obtained by altering the value of b^2 and the initial value of the Hubble parameter. The initial value of the Hubble parameter is expected to be very large compared to the current value, which is what happens for the minimized χ^2 plot. Figure 3 shows the relation between χ^2 and the varied value of b^2 . A similar fit was performed for the initial value of the Hubble parameter, H_i . The minimized χ^2 for the model occurred when $b^2 = 0.301$ and $H_i = 6.79 \times 10^{16}$.

Figure 3. χ^2 vs. b^2 for the model of Figures 1 and 2, $H_i=6.79 \times 10^{16}$.

Conclusion

Interacting dark energy models have the potential to alter the equilibrium conditions for the behavior of the components of the universe in a favorable manner, and hence they provide an explanation for the coincidence problem. It should be noted that the solution to the cosmic coincidence problem implied by the interaction of Equation (9) is not unique. Other interaction models that have been explored can be manipulated to yield the desired equilibrium conditions (Berger and Shojaei 2006). Good fits with supernova data are also not unique. Since the supernova data only extends to redshifts of about 1.4, which corresponds to a value of x that is relatively close to the present time, behavior of the density parameters that extend far into the past don't affect the experimental data fits very much. At the same time there are some issues with interacting dark energy models suffer from some issues like the stability of the models as well as the degeneracy of the solutions. We also needed to fine tune the parameters in order to find the best fit solution. This is a shortcoming of these interacting models.

These models can show interesting evolutionary behavior that could possibly describe the entire history of the universe in one concise model. Interaction models can be helpful in understanding the nature of dark energy as well. Finding an appropriate interaction which governs the behavior of the components of the cosmic inventory from the beginning could shed some light on the true nature of dark energy.

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