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Abstract

Interacting dark energy models have been employed to study the behavior of dark energy and matter in the presence of an interaction between the two. One of the successful aspects of these models is how they explain the cosmic coincidence problem. In this work we consider a specific interaction to study the behavior of dark energy and matter through the history of evolution.

Introduction

Although it was well known by cosmologists that the universe was expanding, the discovery of the accelerated expansion of the universe in 1998 was quite interesting and puzzling at the same time. Matter, ordinary or dark, cannot drive such an accelerated expansion and we need a component in cosmic inventory that effectively exerts an outward or negative pressure. The component with this peculiar property is called dark energy.

Dark energy is one of the most important mysteries of modern cosmology. Although there is no fundamental theory that explains dark energy and all of its properties in full details, one can find some candidates for dark energy in the literature, including Quintessence field, Phantom energy, etc. (Copeland et al. 2006). One of these candidates is the famous cosmological constant, which was first introduced by Einstein. Although it was abandoned by him and other cosmologists upon discovery of the expanding universe, this constant returned as one of the most plausible candidates for dark energy. It correctly creates the negative pressure required to explain the accelerated expansion. As a matter of fact for the cosmological constant

$$P_\Lambda = -\rho_\Lambda,$$

where P_Λ and ρ_Λ represent the pressure and the density of dark energy respectively. Since the density is positive, cosmological constant produces the desired negative pressure.

In another breakthrough discovery by the WMAP (Wilkinson Microwave Anisotropy Probe) survey, the universe was determined to be flat. In general any space-time can be positively curved (like a sphere), negatively curved (like a saddle) or flat (like a plane). In order to have a flat universe, the total density of universe has to be equal to the critical density. However, the most recent measurements show that the density of matter in the cosmic inventory is just 27% of the critical density. So there has to be another component present in the inventory which, despite being undetectable directly by any experiment, accounts for 73% of the total density of the universe. This is another indirect proof for the existence of dark energy.

Model with No Interaction

Assuming that the universe is homogeneous and isotropic, the geometry of space-time can be determined by the FLWR[1] metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + d\Omega^2\right], \qquad (1)$$

where $a(t)$ is the scale factor and k is the curvature parameter. Using Einstein's field equations one can show that densities for different components in the cosmic inventory evolve in the following way

$$\dot{\rho}_\Lambda + 3H\rho_\Lambda(1+w_\Lambda) = 0, \qquad (2)$$

$$\dot{\rho}_m + 3H\rho_m(1+w_m) = 0 \qquad (3)$$

Where H is the Hubble parameter and $w = \frac{P}{\rho}$ is the equation of state which is defined for any component of the cosmic inventory. As it is clear from the above differential equations, dark energy and matter evolve independently. In other words, there is no correlation

[1] Friedmann-Lemaitre-Robertson-Walker

between the dark energy and matter component of the cosmic inventory. If we solve the first equation we obtain $\rho_\Lambda = constant$. That means the expansion of the universe will not affect the density of dark energy at all. On the other hand the second equation gives rise to $\rho_m \propto a^{-3}$. This suggests that when the universe expands, matter density will decrease. This solution makes sense since density of matter is inversely proportional to the volume and volume is proportional to the size of the universe, which is determined by scale factor. In other words, no matter how big the density of matter is at the beginning, it will be diluted out if we wait enough. The fate of the universe is a dark energy dominated universe, where the density of matter will eventually become zero.

As mentioned above, at the present time the total density of the universe is somehow divided between dark energy and matter. In other words, we do not live in a dark energy dominated universe. Although dark energy density is roughly twice as large as matter density, they have the same order of magnitude. This leads to the question of why? Is it a coincidence that we see this proportion at the present time? Are we in a transition era? Will we really encounter a totally dark energy dominated universe if we wait long enough? These questions are usually referred to as the "Cosmic Coincidence Problem" in modern cosmology (Steinhardt 1997).

Models with Interaction

Interacting models were introduced to resolve the cosmic coincidence problem. In these models an interaction between dark energy and matter is introduced. This interaction actively converts dark energy into matter as the universe expands. As a result a dark energy dominated universe will be avoided. In these models the equations of the evolution of dark energy and matter will be modified as follows (Myung 2005, Berger and Shojaei 2008, Li 2004, Zhang 2004):

$$\dot{\rho}_\Lambda + 3H\rho_\Lambda(1+w_\Lambda) = -Q,$$

$$\dot{\rho}_m + 3H\rho_m(1+w_m) = Q$$

The interaction term Q, is introduced to convert dark energy into matter. However, the total density will remain conserved. Adding interaction will change the differential equations governing the evolution of the energy densities. For convenience, we use density parameters rather than densities for different components in the cosmic inventory. They are defined as:

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{cr}},\ \Omega_m = \frac{\rho_m}{\rho_{cr}} \text{ and } \Omega_k = \frac{k}{(aH)^2}.$$

For convenience a density parameter for curvature, Ω_k, is also introduced. These three parameters are not independent. In general, if we know the value for two of them, we can find the third one by

$$\Omega_\Lambda + \Omega_m = 1 + \Omega_k. \qquad (4)$$

Although we are interested in dark energy and matter, we study the differential equations governing the dark energy and curvature for convenience. At the end we can find matter density using the relation between the three parameters. Dark energy and curvature evolution is governed by this set of differential equations

$$\frac{d\Omega_\Lambda}{dx} = -3\Omega_\Lambda(1-\Omega_\Lambda)\left(w_\Lambda^{eff} - w_m^{eff}\right) + \Omega_\Lambda\Omega_k\left(1+3w_m^{eff}\right),$$

$$\frac{d\Omega_k}{dx} = 3\Omega_\Lambda\Omega_k\left(w_\Lambda^{eff} - w_m^{eff}\right) + \Omega_k(1+\Omega_k)\left(1+3w_m^{eff}\right).$$

In these equations, x represents the time and it is defined in such a way that $x = 0$ represents the present time. The effective equations of state are defined in this new scheme depends on the interaction as well as the type of horizon we consider for the dark energy. A holographic condition on dark energy density is usually desirable (Pavon and Zimdahl 2005). Considering the future event horizon, the effective equation of state for dark energy is (Setare 2006):

$$w_\Lambda^{eff} = -\frac{1}{3} - \frac{2}{3}\sqrt{\Omega_\Lambda - \Omega_k}.$$

On the other hand the effective equation of state for matter is $w_m^{eff} = -\frac{Q}{3H\rho_m}$. When there is no interaction the equation of state for matter is zero, or the pressure is zero. This is what we have in regular cosmological models for matter, or what is usually referred as dust. If there is no interaction the above equations have three equilibrium points. The stable equilibrium point for this set occurs at $\Omega_\Lambda = 1$ and

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$\Omega_k = 0$. This is equivalent to $\Omega_m = 0$. In other words a dark energy dominated universe is the ultimate fate of the universe in this model as well. By adding the interaction, one can change the behavior of the right hand side of the differential equations. We can add new equilibrium points or change the behavior of the existing ones. We can even remove some of the existing equilibrium points.

Results and Discussion

We assumed that the interaction term is proportional to dark energy density. This means that the chance of conversion of dark energy to matter increases with increasing dark energy. We employed an interaction which depends exponentially on curvature parameter as well. This interaction is:

$$\frac{Q}{3H\rho_\Lambda} = \frac{b^2 e^{-p\Omega_k}(1+\Omega_k)}{\Omega_\Lambda^{\frac{1}{2}}}. \qquad (5)$$

The flow diagram for density parameters in this case is as follows

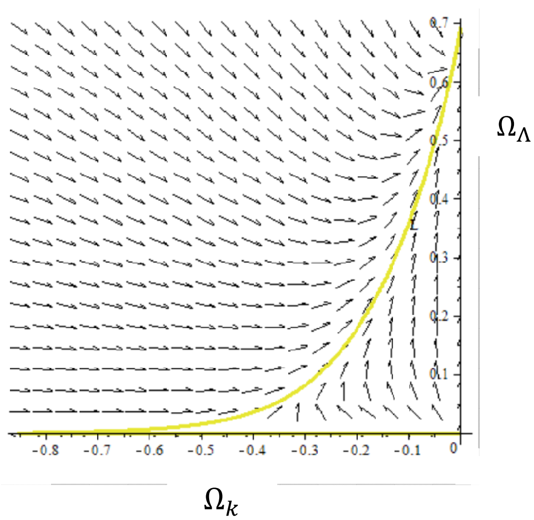

Figure 1. Flow diagram for the interaction (5) with $b^2 = 0.26$

As we can see in figure 1, there is a stable equilibrium at around $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$. This equilibrium point represents the present time and it essentially resolves the cosmic coincidence problem by replacing the stable equilibrium point at a dark energy dominated universe with the one we are interested in. We also can plot different density parameters in order to observe their behavior at any given time. Figure 2 shows the behavior of these three parameters in this model. One interesting feature of this graph is the sudden increase in matter density for a short period of time before going back to 0.3. This could resemble the behavior of matter at the end of inflation. In the current theories of inflation, at the end of the inflationary era the inflation field decays into matter through a process that is called reheating. As we can see in figure 2, matter is generated sharply in a short period of time in a similar way to reheating. That means our model could explain the inflation without requiring an inflaton field. To do so, we need to be able to have enough e-foldings in the inflationary era and this matter is under investigation at the moment.

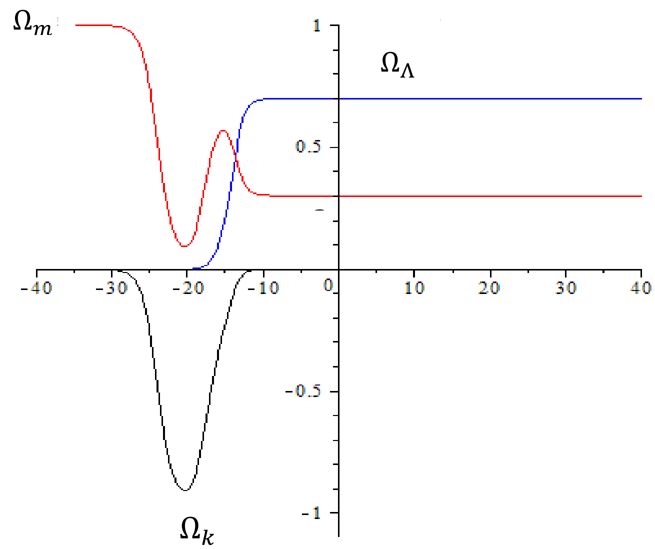

Figure 2. Behavior of density parameters using interaction (5) with $b^2 = 0.26$

Summary

Interacting dark energy models can play a significant role in understanding the nature of dark energy as well as how it interacts with matter. Yet these models suffer from instability issues and fine tuning. In this work we studied the effects of the interaction on the behavior of different components in the cosmic inventory. Although this model was able to answer the cosmic coincidence problem, we would like to point out the one might get similar results using different interactions. In other words, the interaction (5) is not a unique interaction which yields the desirable results.

At the same time, these models can be very helpful in understanding the nature of dark energy. Finding an interaction which has been active from the beginning

of the time could provide a way to describe the entire history of time in just one scheme.

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