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Ionization Rate, Temperature, and Number Density for Breakdown Waves with a Large Current Behind the Shock Front

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Abstract

A three component, one-dimensional, constant velocity, steady-state fluid model is employed to describe the breakdown waves with a current behind the wave front propagating into a neutral gas subjected to an external electric field. Electron gas partial pressure is much larger than that of the other species and therefore is considered to provide the driving force for the wave. The system of equations includes the equations of conservation of mass, momentum, and energy coupled with Poisson's equation.

Inclusion of current behind the wave front in the system of fluid equations also alters the initial boundary conditions and ionization rate. We are considering an ionization rate which changes from accelerational ionization at the front of the wave to directed velocity ionization in the intermediate stages of the wave to thermal ionization at the end of the wave. Using the modified boundary conditions, the set of electron fluid dynamical equations have been integrated through the dynamical transition region of the wave. The effect of current behind the shock front on the wave profile for electric field, electron velocity, ionization rate, electron number density, and electron temperature will be presented.

Background

Breakdown waves are propagating processes converting ionless gas into neutral plasma. Lightning is the classic example of propagating breakdown waves. Depending on the direction of the electric field force on the electrons, breakdown waves are referred to as: proforce and antiproforce waves. For proforce waves the electric field force on electrons is in the direction of wave propagation yet for antiproforce waves it is in the opposite direction of wave propagation. The study of breakdown waves first began with Hauskbee in 1705 by studying luminous pulses in evacuated chambers. In 1893, Thompson discovered that breakdown waves travel with speeds as high as half the speed of light (Thompson 1893). Beams confirmed Thompson's results and he proposed that electron gas

partial pressure provides the driving force for breakdown waves (Beams 1930). Finally, the fluid model was proposed independently by Paxton and Fowler (1962) and Haberstich (1964) to describe breakdown waves as fluid phenomena. To describe the waves, Shelton and Fowler (1968) referred to them as "Electron Fluid Dynamical" (EFD) waves since there is negligible mass motion and the phenomenon is solely caused by electron fluid action. They also derived equations for momentum and energy transfer operators along with the boundary conditions at the wave front. Fowler and Shelton (1974) then used an approximation method to solve the electron fluid dynamical equations for the sheath region of proforce waves. The EFD equations include the equations of the conservation of mass, momentum, and energy along with Poisson's equation. Major modifications to the equations, including the heat conduction term in the equation of conservation of energy and inclusion of electron gas temperature derivative at the shock front, were made by Fowler et al. (1984). Modifications calculating ionization rate were also made by Fowler (1983).

There are two main regions of breakdown waves: the sheath region and the quasi-neutral region. The sheath region is a thin dynamical region that follows the shock front. In this region, the electric field decreases to a minimal value and electrons slow down to rest due to collisions with heavy particles. The quasi-neutral region is a thicker area that follows the sheath region where the ionization of heavy particles continues, causing the electron gas to cool. When a substantial current is behind the shock front of a proforce wave, it is known as a proforce current-bearing wave. For breakdown waves with a current behind the shock front, the equations of conservation of mass, momentum, and energy remain unchanged. However, Poisson's equation must be modified. The initial boundary conditions for electric field and electron velocity remain unchanged, yet there is a necessary modification for the shock condition on electron temperature.

The modified set of electron fluid dynamical equations was integrated through the dynamical

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transition region of the wave using the modified boundary conditions and the trial and error method. The current behind the shock front effects the electron fluid dynamical equations, boundary conditions, and therefore changes the wave profile. The results from the integrations satisfy the required conditions at the end of the sheath region. The wave profiles for electron velocity, electron temperature, electron number density, electric field, and the ionization rate were determined.

Analysis

To analyze breakdown waves propagating into a neutral medium, we utilized a set of one-dimensional, steady-state, electron fluid dynamical equations developed by Fowler et al. (1984). This set of EFD equations consists of the equations of conservation of mass, momentum, and energy coupled with Poisson's equation:

$$\frac{d(nv)}{dx} = \beta n \tag{1}$$

$$\frac{d}{dx}[mnv(v-V) + nkT_e] = -enE - KmnV(v-V) \tag{2}$$

$$\begin{aligned} \frac{d}{dx}[mnv(v-V)^2 + nkT_e(5v-2V) + 2env\phi + \\ \epsilon_0VE^2 - \frac{5nk^2T_e}{mK} \frac{dT_e}{dx}] \\ = -3\left(\frac{m}{M}\right)nkKT_e - \left(\frac{m}{M}\right)Kmn(v-V)^2 \end{aligned} \tag{3}$$

$$\frac{dE}{dx} = \frac{e}{\epsilon_0} n\left(\frac{v}{V} - 1\right) \tag{4}$$

where n , v , T_e , e , and m respectively represent the electron number density, velocity, temperature, charge, and mass. Also, M represents neutral particle mass, E is electric field within the sheath region, E_0 is electric field at the wave front, V is wave velocity, k is the Boltzmann's constant, K is the elastic collision frequency, x is the position within the sheath, β is the ionization frequency, ϕ represents the ionization potential of the gas, and ϵ_0 is the permittivity constant.

We reduce the electron fluid dynamical equations to their non-dimensional form using the following dimensionless variables:

$$\begin{aligned} \eta &= \frac{E}{E_0}, v = \left(\frac{2e\phi}{\epsilon_0 E_0^2}\right)n, \psi = \frac{v}{V}, \theta = \frac{T_e k}{2e\phi}, \\ \xi &= \frac{eE_0 x}{mV^2}, \alpha = \frac{2e\phi}{mV^2}, \kappa = \frac{mV}{eE_0} K, \mu = \frac{\beta}{K}, \\ \omega &= \frac{2m}{M} \end{aligned}$$

where η is electric field strength within the sheath, v is electron number density, ψ is electron velocity, θ is the electron gas temperature, μ is ionization rate, ξ is the position within the sheath region, and ω is the ratio of electron mass over neutral particle mass. α and κ are wave parameters.

These dimensionless variables are substituted into equations 1 through 4, yielding:

$$\frac{d(v\psi)}{d\xi} = \kappa\mu v \tag{5}$$

$$\frac{d}{d\xi}[v\psi(\psi-1) + \alpha v\theta] = -v\eta - \kappa v(\psi-1) \tag{6}$$

$$\begin{aligned} \frac{d}{d\xi}[v\psi(\psi-1)^2 + \alpha v\theta(5\psi-2) + \alpha v\psi + \alpha\eta^2 \\ - \frac{5\alpha^2 v\theta}{\kappa} \frac{d\theta}{d\xi}] = -\omega\kappa v[3\alpha\theta + (\psi-1)^2] \end{aligned} \tag{7}$$

$$\frac{d\eta}{d\xi} = \frac{v}{\alpha}(\psi-1) \tag{8}$$

To calculate ionization rate, μ , we utilize an equation introduced by Fowler (1983), which is a computation based on free trajectory theory and includes ionization from random as well as directed electron motions:

$$\mu = \mu_0 \int_A^\infty \sigma_i x^2 dx \int_B^\infty \frac{e^{-(x-u)^2} - e^{-(x+u)^2}}{u} du e^{-2Cu} \tag{9}$$

where

$$A = \frac{1}{\sqrt{2\theta}}, B = \frac{(1-\psi)}{\sqrt{2\alpha\theta}}, C = k\sqrt{2\alpha\theta}$$

We expand the equation of conservation of linear momentum (equation 6) and also the equation of conservation of mass (equation 5), then substitute from the expanded form of the conservation of mass into the expanded form of the equation of conservation of linear momentum to solve for the electron velocity

derivative with respect to position, $\frac{d\psi}{d\xi}$. The

singularity inherent in the set of EFD equations appears in the denominator of the resulting equation:

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$$\frac{d\psi}{d\xi} = \frac{k(1+\mu)(1-\psi)\psi - \kappa\mu\alpha\theta - \eta\psi - \alpha\psi\theta'}{\psi^2 - \alpha\theta} \quad (10)$$

For proforce waves with a current behind the shock front, modifications must be made for the initial boundary conditions and Poisson's equation. The equation of conservation of current at the wave front is

$$eN_iV_i - env = I \quad (11)$$

where I represents the current, n is the number of electrons per unit volume, N_i is the number of ions per unit volume, and V_i is the ion velocity in the wave frame. Substituting from the equation of conservation of current into Poisson's equation results in:

$$\frac{dE}{dx} = \frac{e}{\epsilon_0} \left(\frac{I}{eV_i} + \frac{nv}{V_i} - n \right). \quad (12)$$

Considering that the change in ion velocity is negligible and introducing the dimensionless variables

along with $t = \frac{I}{\epsilon_0 E_0 K}$ as dimensionless current in

Poisson's equation (equation 12) reduces it to:

$$\frac{d\eta}{d\xi} = \frac{v}{\alpha}(\psi - 1) + \kappa t. \quad (13)$$

Integrating the all particle (electrons, ions, and neutral particles) equation of conservation of momentum, evaluating the constant of integration utilizing the variable values at the wave front, introducing all relevant dimensionless variables, and solving for electron temperature at the wave front, θ_1 , results in the following equation for electron temperature at the shock front:

$$\theta_1 = \frac{\psi_1(1-\psi_1)}{\alpha} + \frac{\kappa}{v_1} t. \quad (14)$$

Results

The singularity inherent in the set of equations that appears in the denominator of equation 10 is utilized to integrate the set of equations through the sheath region. For given α and κ values, a set of v_1 and ψ_1 values are selected to integrate the set of equations through the sheath region. While integrating the set of equations, the values of numerator and denominator in equation 10 are compared. If the denominator in equation 10 approaches zero, the electron velocity derivative with respect to position approaches infinity. This indicates the existence of a shock within the sheath, which is not allowed. Therefore, with gradual change in the value

of v_1 , we pass through the singularity where the numerator and denominator values in equation 10 are almost equal and maintaining the values of numerator and denominator constant for ten integration steps. At the conclusion of integration of the equations through the sheath region, if the required conditions at the end of the sheath are not achieved, we change the value of ψ_1 by trial and error until the conditions are achieved at the end of the sheath ($\psi_2 \rightarrow 1$ and $\eta \rightarrow 0$).

In studying lightning attachment processes, Wang et al. (1999) obtained highly resolved optical images, electric field measurements, and channel-base current readings for dart leader/return-stroke lightning flashes triggered using the rocket-and-wire technique at Camp Blanding, Florida. They reported return-stroke peak currents of 21 kA and 12 kA. Directly measuring the time derivative of the electric field of triggered lightning strokes at distances of 10 m to 30 m, Uman et al. (2000) reported peak current values of 24 kA - 5.5 kA. A current value of 5000 A represents an t value of approximately 0.1.

We have integrated the set of electron fluid dynamical equations through the sheath region for a zero current value and also current values of 0.1 and 0.2. A current behind the shock front changes the wave profile and will be further discussed.

Figure 1 depicts electric field, η , as a function of electron velocity, ψ , within the sheath region. An α value of 0.1 was selected which represents a wave speed of 9.37×10^6 m/s. To ensure successful integration through the sheath region, for current values of 0.1 and 0.2, the initial boundary values were $\psi_1 = 0.2525$, $v_1 = 0.240$, $\kappa = 0.823$; $\psi_1 = 0.260$, $v_1 = 0.280$, and $\kappa = 0.849$, respectively. The solutions met the expected physical conditions at the end of the sheath where $\psi_2 \rightarrow 1$ and $\eta_2 \rightarrow 0$.

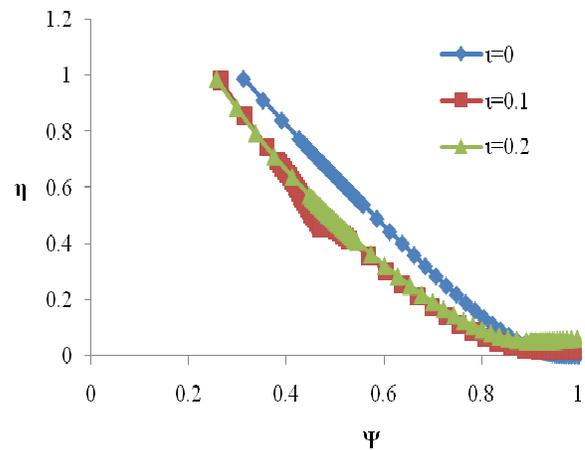


Figure 1. Electric field, η , as a function of electron velocity, Ψ , inside the sheath.

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Figure 2 contrasts electron temperature, θ , as a function of position, ξ , within the sheath region. As expected, the temperature of the electron gas decreases as the position within the sheath increases. For $\tau = 0.2$, the electron gas temperature reduces to $\theta_f = 0.316$ at the end of the sheath. This represents a gas temperature of 1.83×10^5 K. A ξ value of 1.5 at the end of the sheath represents a sheath thickness of 3.3×10^{-5} m.

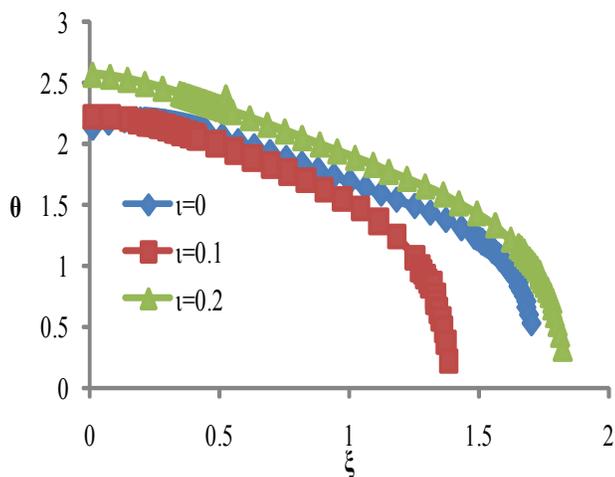


Figure 2. Electron gas temperature, θ , as a function of position, ξ , inside the sheath.

Figure 3 is a graph of electron number density, v , as a function of position, ξ , within the sheath region. For $\tau = 0.1$, there is a visible jump or kink in the electron number density values around the position where $\xi = 0.4$. This results from approximation utilized in order to pass through the singularity. For $\tau = 0.1$, the electron number density reduces to 0.1641 at the end of the sheath. This represents an electron number density of 1.82×10^{17} electrons/m³.

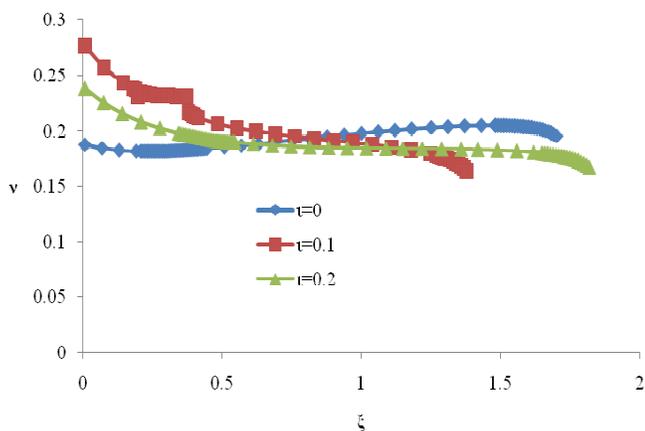


Figure 3. Electron number density, v , as a function of position, ξ , inside the sheath.

Figure 4 depicts ionization rate, μ , as a function of position, ξ , within the sheath region. Shelton (1968) assumed that the ionization rate was constant throughout the sheath region. The graph shows that for all different values of current the ionization rate remains constant at the beginning of the sheath. However, it changes considerably at the end of the sheath.

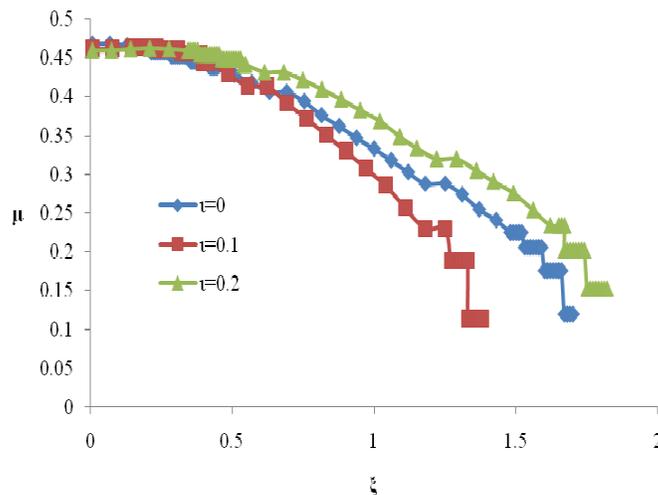


Figure 4. Ionization rate, μ , as a function of position, ξ , inside the sheath.

Conclusions

For the current values reported by the Uman et al. (2000) and Wang et al. (1999), we were able to integrate the electron fluid dynamical equations through the sheath region. Our solutions met the expected physical conditions at the end of the sheath. Calculation of ionization rate using an equation based on the free trajectory theory allows for a successful integration of the electron fluid dynamical equations for breakdown waves with a large current behind the shock front. Our results on ion number density compare well with those reported by Gogolides and Sawin (1992) and Degeling and Boswell (1997). Our ionization rate values compare well with observations made by Brok et al. (2003). This is another confirmation on the application of the fluid model to breakdown waves with a large current behind the shock front.

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