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Compatible Stem Taper and Total Tree Volume Equations for Loblolly Pine Plantations in Southeastern Arkansas

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Abstract

A system of equations was used to produce compatible outside-bark stem taper and total tree volume equations for loblolly pine (*Pinus taeda* L.) plantations in southeastern Arkansas. Paired height-diameter stem measurements were obtained from trees located in one 45-year-old unthinned plantation. After fitting and integrating the stem taper equation to total tree height, an individual tree constant form factor volume equation was obtained. The stem taper equation can also be integrated to any merchantable height to obtain merchantable volume. To see how the constant form factor volume equation predicts outside-bark volume for trees vastly different than those used in model fitting, trees were harvested from a 17-year-old loblolly pine plantation in southeastern Arkansas. Predictions from the volume equation developed during this research were compared to three other commonly used volume equations. Validation results showed the current constant form factor equation produced the best predictions.

Introduction

Taper equations provide information about how stem diameter changes as height increases. Conversely, these equations can be used to estimate height for a given stem diameter. Thus, taper equations, when integrated, can provide estimates of volume to merchantable top limits as well as for total tree height. Compatibility between taper and volume equations is defined as when the total volume obtained by summation of the sections whose volumes are defined using the taper equation is identical to the volume calculated by the volume equation, or, more precisely, when integration of the taper equation produces the same total volume as given by the volume equation (Demaerschalk 1973). Several taper models have been developed for loblolly pine plantations throughout the southeastern US (e.g. Lenhart et al. 1987, Tasissa et al. 1997, Jordan et al. 2005, Coble and Hilpp 2006) but few have been developed exclusively for southeastern Arkansas. The objectives of this study were to estimate parameters of a taper equation for loblolly pine plantations in southeastern Arkansas that was then integrated to total tree height producing a compatible individual tree total cubic meter outside-bark volume equation. Validation analyses were conducted to determine how predictions from this newly developed volume equation compare to commonly used volume equations for trees located in southeastern Arkansas. An example is given of how to estimate outside-bark merchantable cubic meter volume to a particular upper-stem height.

Materials and Methods

Model fitting data

Equations were developed using 493 paired stem diameter-height measurements obtained from 71 trees found in five permanent research plots located in a 45-year-old unthinned loblolly pine plantation near Monticello, Arkansas. An abandoned row-cropped field was machine-planted in 1958 at a spacing of 2.44 m square using seedlings obtained from a state nursery located in Arkansas. Plots were originally established at 27 yrs. See Table 1 for a summary of tree characteristics and Figure 1 for a graphic depiction of total tree height and DBH pairs. Soils in the study area were Tippah silt loams, which are moderately well drained, with slow permeability (NRCS 2008). Slopes ranged from 0 to 4 percent, mainly southward. The climate for this area is warm and humid with annual precipitation and mean annual air temperatures ranging from 46 to 63 inches and 51° to 74°; respectively. Site index (base age 25 yr) was determined to be 18.9 m.

Estimation of the taper equation parameters

Proc Model of the SAS Institute (SAS 2003) and the Gauss-Newton algorithm were used to estimate all parameters.

Development of a constant form factor total cubic meter outside-bark volume equation

A taper equation originally developed by Kozak et al. (1969) was used to model stem diameter:

$$\frac{d^2}{DBH^2} = b_0 + b_1\left(\frac{h}{H}\right) + b_2\left(\frac{h^2}{H^2}\right) \quad (1)$$

where d is the stem outside-bark diameter (cm), h is the stem height (m), DBH is outside-bark diameter breast height (cm), H is total tree height (m), and b_0, b_1, b_2 are parameters to be estimated.

Table 1. Summary statistics of trees used to obtain stem diameter and height pairs to estimate parameters of equation (2). DBH is diameter at breast height (1.37 m aboveground), H is total tree height, and Std. dev. is the standard deviation. Number of trees was 71.

	Min	Mean	Max	Std. dev.
DBH (cm)	14.0	34.0	46.7	8.37
H (m)	12.2	24.0	29.1	3.10

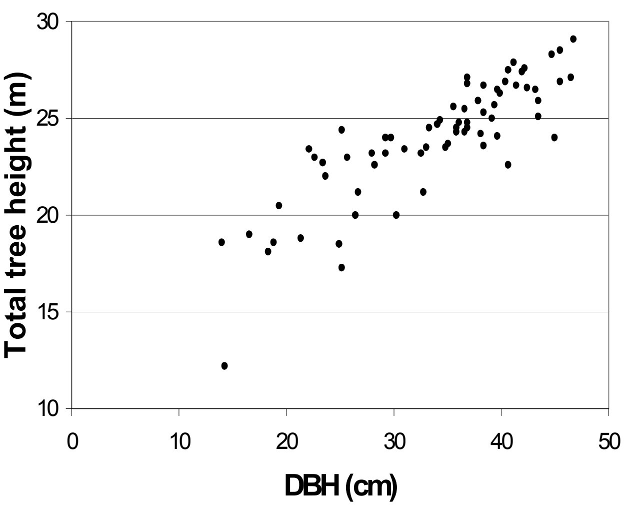

Figure 1. Scattergram of trees used to obtain stem diameter and height pairs to estimate parameters of equation (2).

Equation (1) can be rearranged:

$$d = DBH\sqrt{b_0 + b_1\left(\frac{h}{H}\right) + b_2\left(\frac{h^2}{H^2}\right)} \quad (2)$$

To obtain a total cubic meter volume equation, it must be assumed the tree bole for a particular stem diameter is circular. To get the area of the bole for that particular stem diameter, equation (3) is used:

$$\text{Area} = \frac{\pi d^2}{4(10000)} = 0.00007854\ d^2 \quad (3)$$

Where $0.00007854d^2$ derives from:

$$\text{Area} = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \frac{\pi}{4}d^2 \quad (4)$$

Equation (4) is in square cm units since d is in cm. To obtain square meters (since we want volume in cubic meters), we need to divide the right-hand side (RHS) by 100 cm squared – 10000:

$$\text{Area} = \frac{\pi}{4(10000)}d^2 = 0.00007854\ d^2 \quad (5)$$

Total tree volume is obtained by integrating the area from equation (5) at each infinitesimal stem diameter along the entire height of the tree:

$$\text{Volume} = \int_{h_1}^{h_2} 0.00007854\ d^2\,dh \quad (6)$$

where h_1 is stem height (m) and h_2 is stem height (m) that is greater than h_1.

The number 0.00007854 is a constant and thus equation (6) can be reexpressed as:

$$\text{Volume} = 0.00007854\int_{h_1}^{h_2} d^2\,dh \quad (7)$$

By replacing d^2 in equation [7] with the square of equation [2], one obtains:

Volume =

$$0.00007854\int_{h_1}^{h_2} DBH^2\left[b_0 + b_1\left(\frac{h}{H}\right) + b_2\left(\frac{h^2}{H^2}\right)\right]dh \quad (8)$$

Where, DBH^2 is a constant and thus equation (8) can be reexpressed as:

Volume =

$$0.00007854\ DBH^2\int_{h_1}^{h_2}\left[b_0 + b_1\left(\frac{h}{H}\right) + b_2\left(\frac{h^2}{H^2}\right)\right]dh \quad (9)$$

After integrating equation [9], one obtains:

Volume =

$$0.00007854DBH^2\left[b_0h + \frac{b_1}{2H}h^2 + \frac{b_2}{3H^2}h^3\right]_{h_1}^{h_2} \quad (10)$$

When h_2 is replaced by H (total tree height), and h_1 is replaced by 0, an equation to estimate total cubic meter volume is obtained:

Volume =

$$0.00007854DBH^2\left[b_0h + \frac{b_1}{2H}h^2 + \frac{b_2}{3H^2}h^3\right]_0^H \quad (11)$$

$$\text{Volume} = 0.00007854DBH^2\left[b_0H + \frac{b_1}{2H}H^2 + \frac{b_2}{3H^2}H^3\right] - 0.00007854DBH^2\left[b_0 0 + \frac{b_1}{2H}0^2 + \frac{b_2}{3H^2}0^3\right] \quad (12)$$

Where the RHS equation goes to 0, and after simplifying the LHS equation:

$$\text{Volume}= 0.00007854\text{DBH}^2\left[b_0\text{H}+\frac{b_1}{2}\text{H}+\frac{b_2}{3}\text{H}\right] \quad (13)$$

Equation (13) can be further simplified:

$$\text{Volume} = 0.00007854\text{DBH}^2\left[b_0+\frac{b_1}{2}+\frac{b_2}{3}\right]\text{H} \quad (14)$$

and finally:

$$\text{Volume} = 0.00007854\left[b_0+\frac{b_1}{2}+\frac{b_2}{3}\right]\text{DBH}^2\text{H} \quad (15)$$

After estimating parameters for equation [2] as given in Table 2:

$$\text{Volume}= 0.00007854\left[1.318172-\frac{2.09544}{2}+\frac{0.830902}{3}\right]\text{DBH}^2\text{H} \quad (16)$$

Equation (16) can then be simplified obtaining the constant form factor total cubic meter volume equation:

$$\text{Volume} = 0.00004299\,\text{DBH}^2\text{H} \quad (17)$$

Users need to be aware that equations (10) and (17) were developed using data from only one site that was limited in terms of genetic diversity and different site preparation and cultural practices and therefore predictions may not be fully representative of their population.

Table 2. Parameter estimates for equation [2]. Std. errors are the standard errors of the estimates. Number of stem diameter and height pairs equaled 493 obtained from a total of 71 individual trees.

	b_0	b_1	b_2	Adj. R^2
Estimates	1.318172	-2.09544	0.830902	0.8876
Std. errors	0.0228	0.0563	0.0341	

Validation comparison of equation (17) to other commonly used volume equations

To help determine if equation (17) provides better estimates of volume relative to other available equations for trees located in southeastern Arkansas, an independent dataset was used for model validation. For this analysis, five loblolly pine trees found in a 17-year-old plantation planted at a spacing of 1.83 m square located on the University of Arkansas at Monticello Prisoner of War Camp forest near Monticello, AR were harvested in March 2007. In addition to the planted seedlings, the stand also contains non-planted (wildling) loblolly pine trees. This is a vastly different population of trees in terms of heights and diameters than those used in fitting equation (17). Due to the small sample size of the validation dataset, results from this analysis should only be considered indicative and not definitive as to the ability of equation (17) to predict volume for trees in southeastern Arkansas. Diameter measurements were made along the stem at DBH (1.37 m above the ground) and at 0.30 m intervals up to 3.1 m of height, at 0.61 m intervals past 3.1 m and up to 6.1 m of height, and at 1.22 m intervals past 6.1 m and to the tip of the stem. To calculate total tree volume, cubic meter volume for each separate section of the stem was calculated using Smalian's formula (Tasissa et al. 1997):

$$\text{Volume}_{\text{Segment}} = \left[\frac{\text{BA}_{\text{Large}}+\text{BA}_{\text{Small}}}{2}\right]\text{L} \quad (18)$$

where $\text{Volume}_{\text{Segment}}$ is the total outside-bark volume of the stem segment, BA_{Large} is cross-sectional basal area (square m) of the large diameter end of the segment, BA_{Small} is cross-sectional basal area (square m) of the small diameter end of the segment, and L is the length (m) of the segment.

Each separate section was then summed to obtain total tree volume. For the five validation trees, total tree height ranged from 9.4 to 13.0 m, DBH ranged from 6.9 to 11.2 cm, and total outside-bark cubic meter volume ranged from 0.017 to 0.073 m^3.

Other equations used in model validation analyses were:

$$\text{Volume} = 0.21949 + 0.00238\,\text{DBH}^2\text{H} \quad (19)$$

$$\text{Volume} = 0.002103\;\text{DBH}^{1.958489}\;\text{H}^{1.062348} \quad (20)$$

$$\text{Volume} = 0.002404\;\text{DBH}^2\;\text{H} \quad (21)$$

Equation (19) (Tasissa et al. 1997), of the combined-variable equation form, was fit using data from across the southeastern United States, equation (20) (Lenhart et al. 1987) was fit using data exclusively from east Texas, and parameters of equation (21) were estimated using the same data as those used in fitting equation (20) plus additional sources of data from east Texas. For these three equations, volume estimates are in cubic feet and were converted to cubic meters (DBH is in inches and height is in feet).

Prediction errors were compared between the three equations using the validation process proposed by Arabatzis and Burkhart (1992). The difference between the observed and predicted volume for each individual tree (e_i = [$\text{Volume}_{\text{Observed}} - \text{Volume}_{\text{Predicted}}$]) was calculated for all three equations. The mean residual ($\bar{e}$) and the sample variance (v) of residuals were computed separately for each equation and considered to be estimates of bias and precision; respectively. An estimate of mean square error (MSE) was obtained combining the bias and precision measures using the following formula: $\text{MSE} = \bar{e}^2 + v$

The equation with the lowest MSE was selected as the model that best predicts volume of the trees in the validation dataset.

Results and Discussion

Only equation (17) produced a negative bias, yet the bias was smaller in magnitude than the other equations (Table 3). Based on the % Bias measure, equation (17) produced significantly better volume predictions for the trees used in model validation. It is somewhat surprising that the equations presented by Lenhart et al. (1987) and Coble and Hilpp (2006) did not perform better. The size of trees in the validation analyses was represented in their model fitting dataset and all trees used in model fitting were from east Texas.

Table 3. Validation results when predicting total outside-bark cubic meter volume for five trees located in a 17-year-old southeastern Arkansas loblolly pine plantation. The column Volume is the outside-bark cubic meter volume as determined when using Smalian's formula [equation (18)] for each segment of the stem and is considered to be the true volume.

	Volume		Tasissa et al.	Lenhart et al.	Dean and Hilpp
Tree	(m^3)	Eqn. (17)	Eqn. (19)	Eqn. (20)	Eqn. [21]
1	0.036	0.038	0.036	0.032	0.031
2	0.073	0.067	0.060	0.056	0.054
3	0.017	0.019	0.021	0.016	0.015
4	0.05	0.052	0.048	0.044	0.042
5	0.027	0.028	0.028	0.023	0.022
Average	0.041	0.041	0.039	0.034	0.033
Bias		-0.0003	0.0017	0.0063	0.0082
% Bias		-0.74%	4.19%	15.52%	19.91%
Variance		0.0000115	0.0000449	0.0000369	0.0000453

Conclusions

Based on validation analyses, equation (17) provides a reasonable alternative to other available equations when predicting total outside-bark cubic meter volume for loblolly pine plantations in southeastern Arkansas. In some cases predicting both wood and bark volume is desired because bark is often used as a fuel source. Additionally, equation (10) provides users a means to obtain outside-bark volume to any desired upper-stem merchantable height.

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Literature Cited

Arabatzis AA and **HE Burkhart**. 1992. An evaluation of sampling methods and model forms for estimating height-diameter relationships in loblolly pine plantations. Forest Science. 38:192-198.

Coble DW and **K Hilpp**. 2006. Compatible cubic-foot stem volume and upper-stem diameter equations for semi-intensive plantation grown loblolly pine trees in E. Texas. Southern Journal of Applied Forestry 30: 132-141.

Demaerschalk JP. 1973. Integrated systems for the estimation of tree taper and volume. Canadian Journal of Forest Research 3: 90-94.

Jordan L, **K Berenhaut**, **R Souter**, and **RF Daniels**. 2005. Parsimonious and completely compatible taper, total, and merchantable volume models. Forest Science 51: 578-584.

Kozak A, **DD Munro**, and **JHG Smith**. 1969. Taper functions and their application in forest inventory. Forestry Chronicle 45: 1-6.

Lenhart JD, **TL Hackett**, **CJ Laman**, **TJ Wiswell**, and **JA Blackard**. 1987. Tree content and taper functions for loblolly and slash pine trees planted on non-old-fields in east Texas. Southern Journal of Applied Forestry 11: 147-151.

SAS Institute Inc. 2003. SAS/ETS user's guide, Version 6, 1st ed. Cary, NC: SAS

Soil Survey Staff, Natural Resources Conservation Service (NRCS), USDA Department of Agriculture. Web Soil Survey. Available online at http://websoilsurvey.nrcs.usda.gov/ accessed [08/22/2008].

Tasissa G, **HE Burkhart**, and **RL Amateis**. 1997. Volume and taper equations for thinned and unthinned loblolly pine trees in cutover, site-prepared plantations. Southern Journal of Applied Forestry 21: 146-152.