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Electric Discharge: Boundary Conditions

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Abstract

The electron gas in electric discharge can be described by a set of one-dimensional fluid dynamical equations. The fundamental equations are those of a three-component (electrons, ions, and neutral particles) fluid, different from the treatment of the problem in plasma physics, a fully ionized two-component case. The leading edge of the wave is treated as a shock front driven mainly by the electron gas pressure. Integrating the one-dimensional global differential equations for mass balance, conservation of momentum and energy, and evaluating the constant of integration at the wave front permits derivation of boundary conditions on electron temperature and electron velocity. Using the boundary conditions on electron temperature and electron velocity we have been able to calculate the initial boundary condition on energy terms due to the electron random and directed motions. Using the initial boundary conditions we have been able to integrate the set of electron fluid dynamical equations through the dynamical transition region of the wave. We will present the derivation of the boundary conditions as well as the wave profile for the electric field, electron velocity, electron temperature, electron number density, and ionization rate within the dynamical transition region of the wave for a fast moving wave.

Introduction

Breakdown waves are the propagating processes which convert ion-less gas into neutral plasma. Lightning is probably the best example of breakdown waves. Although people have been searching for thousands of years for the cause of this phenomenon, it has only been in somewhat recent decades that it has been scientifically studied.

The first person to study these breakdown waves was Hauksbee (1706), who studied luminous pulses in evacuated chambers. Thompson (1893) made measurements on the velocity of breakdown waves and concluded that velocities of these waves may reach up to one half the speed of light. These early experimental data, however, were not reliable due to limited equipment capabilities in making accurate measurements of wave velocities.

An important finding of these early experiments was the lack of Doppler shift in emitted radiation from breakdown waves reported by Von Zahn (1879). This means that there is negligible mass motion, and thus heavy particles cannot account for the movement of the waves. This indicates that the electrons are the main element driving the wave.

Following Thompson's (1893) experiments and after extensive experimental investigations, Snoody et al. (1937) concluded that the ionization process must be of the Townsend type. Through their experimental data in both an 18 mm and a 5 mm diameter tube, they showed that if a constant potential is supplied across each tube, then the change in pressure would be of the same ratio (3.6 in their case). The change in pressure as a function of diameter demonstrates the fundamental principle of similarity. In other words, this experiment showed that the pressure has a linear relationship with tube diameter in an equally applied potential. Their experiments also showed that the waves traveled at a speed of approximately 10^{10} cm/sec and their speed did not vary with tube diameter. These measurements on breakdown wave speeds confirmed Thompson's findings.

Paxton and Fowler (1962) employed a hydrodynamical model for theoretical explanation of breakdown waves. They used a one-dimensional, steady-state model that included the equations of conservation of mass, momentum, and energy and considered these waves to be electron shock waves (discontinuous or shock solutions). Based on this model many investigators (Shelton, 1968) continued research by adding relevant terms particularly to the equations of conservation of momentum and energy, terms which were neglected in Paxton and Fowler's (1962) investigation.

Due to negligible Doppler shift in emitted radiation, Shelton and Fowler (1968) considered the electrons to be the main element in the propagation of the wave and gave the appropriate name of Electron Fluid Dynamical Waves to breakdown waves. This title is appropriately given considering that these waves, as stated above, are fluid in nature and are shown to be shock waves. Shelton advanced the equations of conservation of mass, momentum, and energy for electrons, ions, and neutral particles and also added Poisson's equation to the set of equations. His equations take into account both electrons and heavy particles. He formulated equations for calculating the boundary values for electron temperature and velocity at the leading edge of the wave.

Fowler et al. (1984) introduced additional relevant terms to the equation of conservation of energy, the most important of which was the heat conduction term. They also introduced a discontinuity condition in the electron temperature derivative at the shock front. The addition of new terms in the equation of conservation of energy and the acceptance of a temperature derivative discontinuity at the shock front resulted in a different set of boundary condition equations.

A year later, Hemmati and Fowler (1985) were able to solve the general set of equations for both proforce and antiforce waves. Proforce and antiforce refer to breakdown waves in which the electric field force on electrons is in the same direction and opposite direction of wave propagation, respectively. Proforce waves correspond to dart leaders in lightning, while antiforce waves correspond to return strokes of lightning. They introduced a computer program in their numerical integration of the set of equations through the dynamical transition region of the wave. Their solutions conformed with the expected conditions at the trailing edge of the wave and also with the experimental results.

Analysis

In Fowler and Shelton's (1973) attempt to solve the electron fluid dynamical equations using approximate methods, they considered the energy losses by the electrons in their random and directed motion to be negligible and also neglected the heat conduction term in the equation of conservation of energy. Fowler et al. (1984) introduced a set of electron fluid dynamical equations for a multi-fluid system consisting of neutral atoms, positive ions, and electrons subjected to an electric field (the applied field plus the space charge field) applied in the negative x direction. Their set consisted of the equations of conservation of mass, momentum, and energy coupled with Poisson's equation. Their (Fowler et al., 1984) set of equations (conservation of mass, momentum, energy, and Poisson's equation, respectively), in its complete, non-dimensional form is

$$\frac{d(\nu\psi)}{d\xi} = \kappa\mu\nu, \tag{1}$$

$$\frac{d}{d\xi}[\nu\psi(\psi-1)+\alpha\nu\theta] = -\nu\eta - \kappa\nu(\psi-1), \tag{2}$$

$$\frac{d}{d\xi}[\nu\psi(\psi-1)^2 + \alpha\nu\theta(5\psi-2) + \alpha\nu\psi + \alpha\eta^2 - \frac{5\alpha^2\nu\theta}{\kappa}\frac{d\theta}{d\xi}] = = -\omega\kappa\nu(3\alpha\theta + (\psi-1)^2), \tag{3}$$

$$\frac{d\eta}{d\xi} = \frac{\nu}{\alpha}(\psi-1). \tag{4}$$

The non-dimensional symbols used in the above equations are as follows

$$\eta = \frac{E}{E_0},\ \nu = \frac{2e\phi}{\varepsilon_0 E_0^{\,2}}n,\ \psi = \frac{v}{V},\ \theta = \frac{T_e k}{2e\phi},\ \xi = \frac{xeE_0}{mV^2},\ \alpha = \frac{2e\phi}{mV^2},$$

$$\kappa = \frac{mV}{eE_0}K,\ \mu = \frac{\beta}{K},\ \omega = \frac{2m}{M},$$

where η, ν, ψ, θ, ξ, μ, and w are non-dimensional net electric field, electron number density, electron velocity, electron temperature, electron position, ionization rate, and mass ratio of electrons to heavy particles, respectively. α is ratio of [eφ, energy required for each electron during the ionization process] to the kinetic energy of electron traveling at wave speed. κ relates wave velocity to the electric field at the wave front. The dimensional variables used in the above equations are as follows: *m* is the electron mass, *M* is the neutral particle mass, e is the electron charge, E_0 is the electric field at the shock front, *E* is the electric field inside the sheath region, *n* is the electron number density, v is the electron velocity, T_e is the electron temperature, κ is Boltzman's constant, *K* is the elastic collision frequency, *x* is the position in the wave profile, β is the ionization frequency, φ is the ionization potential, and *V* is the wave velocity.

Assuming that the electron gas pressure is much larger than the partial pressures of the other species, Fowler and Shelton (1973) proposed that the breakdown waves consisted of a shock front, followed by a transition region, and a quasi-neutral region. A transition region, in which the electric field is reduced to zero ($E \to 0$; $\eta \to 0$) and the electrons come to rest relative to the ions and neutral particles ($v \to V$; $\psi \to 1$), follows the shock front. This thin region is called the sheath region. The sheath region is followed by a relatively thick thermal layer, in which the electron and heavy particle velocities are equal ($\psi \approx 1$) and the electric field is zero ($\eta = 0$). In this layer, ionizing additional neutral particles, the high temperature electron gas will cool to approximately room temperature. This region is called the quasi-neutral region.

The equation representing electron temperature at the shock front employed by Fowler and Shelton (1973) in terms of our non-dimensional variables is

$$\theta_1 = \frac{\psi_1}{\alpha}(1-\psi_1). \tag{5}$$

In non-dimensional form the heat conduction term is as follows (Fowler, 1984)

$$\frac{-5\alpha^2\nu\theta}{\kappa}\frac{d\theta}{d\xi}. \tag{6}$$

Including the heat conduction term in the equation of conservation of energy (equation 3), integrating the resultant equation and calculating the constant of integration at the shock front, and using equation 5 for electron temperature at the shock front results in the following equation

$$v_1\psi_1(\psi_1-1)^2+v_1\psi_1(1-\psi_1)(5\psi_1-2)+\alpha v_1\psi_1+\alpha(\eta_1{}^2-1) - \frac{5\alpha v_1\psi_1}{\kappa}(1-\psi_1)\,\theta_1{}'=0, \tag{7}$$

where $\theta_1{}'$ and ψ_1 are the electron temperature derivative and electron velocity at the shock front, respectively. Using the requirement that at the wave front [$E = E_0$, $\eta_1 = 1$], the above equation reduces to

$$-4\psi_1{}^2+5\psi_1-1+\frac{5\alpha\psi_1}{\kappa}\theta_1{}'+\alpha+\frac{5\alpha\theta_1{}'}{\kappa}=0. \tag{8}$$

Solving this quadratic equation for ψ_1 results in the initial condition on electron velocity.

$$\psi_1=\frac{5\left(1+\frac{\alpha\theta_1{}'}{\kappa}\right)\pm\sqrt{\left(3-5\frac{\alpha\theta_1{}'}{\kappa}\right)^2+16\alpha}}{8}. \tag{9}$$

Since $16\alpha > 0$ and ψ_1 has to be less than one ($|v| < |V|$), the negative sign in the above equation is accepted, which results in

$$\psi_1=\frac{5\left(1+\frac{\alpha\theta_1{}'}{\kappa}\right)-\sqrt{\left(3-5\frac{\alpha\theta_1{}'}{\kappa}\right)^2+16\alpha}}{8}. \tag{10}$$

Solving for $\theta_1{}'$ from equation 8 results in

$$\theta_1{}'=\frac{4\psi_1{}^2-5\psi_1+1-\alpha}{\frac{5\alpha}{\kappa}(\psi_1-1)}. \tag{11}$$

Using the substitution

$$\frac{dW}{d\xi}=-\omega\kappa v\left(3\alpha\theta+(\psi-1)^2\right)$$

in the energy equation (equation 3) and integrating the resultant equation with respect to position results in

$$v\psi(\psi-1)^2+\alpha v\theta(5\psi-2)+\alpha v\psi+\alpha\eta^2-\frac{5\alpha^2 v\theta}{\kappa}\frac{d\theta}{d\xi}=W+c, \tag{12}$$

where c is the constant of integration. At the shock front this equation becomes

$$v_1\psi_1(\psi_1-1)^2+\alpha v_1\theta_1(5_1\psi-2)+\alpha v_1\psi_1+\alpha\eta_1{}^2-\frac{5\alpha^2 v_1\theta_1}{\kappa}\theta_1{}'=W_{11}+c. \tag{13}$$

Using the requirement that $\eta_1 = 1$ at the wave front and using equations 5 and 11 to simplify equation 13 reduces it to

$$v_1\psi_1(\psi_1-1)^2+v_1\psi_1(1-\psi_1)(5_1\psi-2)+\alpha v_1\psi_1+\alpha - \frac{5\alpha v_1\psi_1(1-\psi_1)}{\kappa}\left(\frac{4\psi_1{}^2-5\psi_1+1-\alpha}{\frac{5\alpha}{\kappa}(\psi_1-1)}\right)=W_{11}+c. \tag{14}$$

which simplifies to

$$W_{11}+c=\alpha. \tag{15}$$

W_{11} is a constant; therefore, the constant c can be absorbed into W_{11} resulting in the value of W at the shock front of

$$W_1=\alpha. \tag{16}$$

Substituting α for the constant of integration in equation 12 and solving the resultant equation for

$\frac{d\theta}{d\xi}$ results in

$$\frac{d\theta}{d\xi}=\frac{\kappa}{5\alpha^2 v\theta}\left[v\psi(\psi-1)^2+\alpha v\theta(5\psi-2)+\alpha v\psi+\alpha(\eta^2-1)-W\right] \tag{17}$$

Results

Expanding the equation of conservation of momentum and substituting for $\frac{dv}{d\xi}$ from the equation of conservation of mass results in

$$\frac{d\psi}{d\xi}=\frac{\kappa\psi(1-\psi)(1+\mu)-\kappa\mu\theta-\eta\psi-\alpha\psi\frac{d\theta}{d\xi}}{\psi^2-\alpha\theta}. \tag{18}$$

Replacing the equations of conservation of momentum (equation 2) and energy (equation 3) with equations 18 and 17 respectively, using equations 5 and 10 to determine electron temperature and velocity at the shock front, and using $W_1 = \alpha$ as the wave front value for energy losses due to random and directed motion of electrons combined, we were able to successfully integrate the set of electron fluid dynamical equations through the dynamical transition region of the wave. For antiforce waves our exact numerical solutions of the set of equations resulted in the expected conditions at the trailing edge of the wave. For a fast moving wave $\left(\alpha = .01, V = 3\times10^{7}\ ^{m}\!/_{s}\right)$ the successful integration of the set of equations required the following initial conditions for electron number density and velocity.

α	υ_1	ψ_1	κ
.01	.04	.65	.379750013

The following graphs show the wave profile for electric field, η, electron velocity, ψ, electron temperature, θ, electron concentration, ν, and ionization rate, μ, within the dynamical transition region of the wave.

Figure 1 shows a graph of electric field versus electron velocity within the sheath region. This graph shows that the required conditions at the trailing edge of the wave have been met. The electron velocity goes to one, and the net electric field reduces to zero at the end of the sheath.

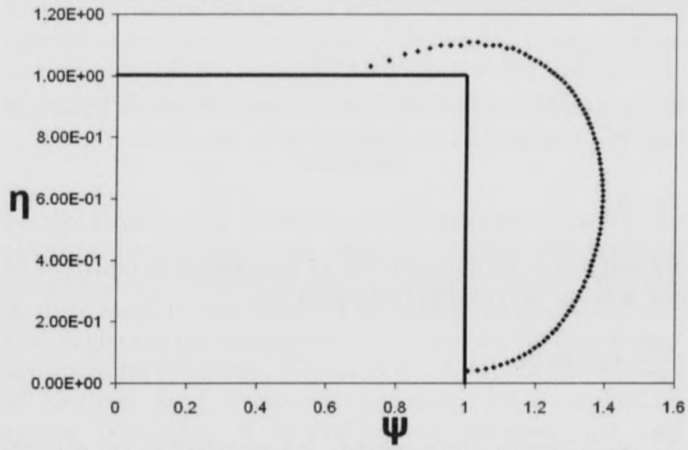

Fig. 1. Electric field, η, as a function of electron velocity, ψ, inside the sheath region.

Figure 2 is a graph of the net electric field, η, as a function of position within the sheath region of the wave. As expected, the net electric field reduces to zero at the trailing edge of the wave.

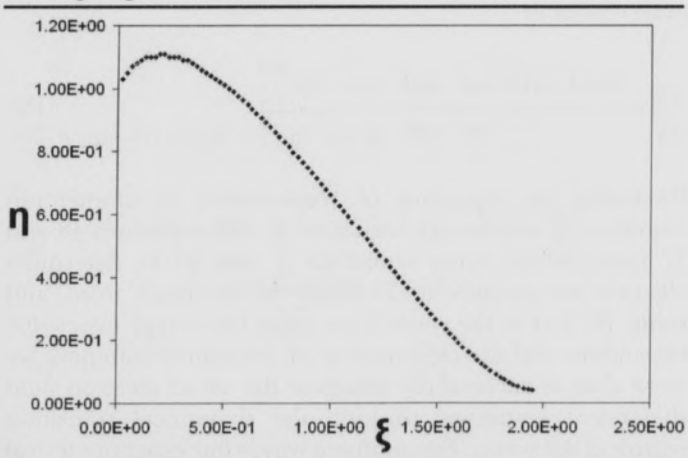

Fig. 2. Net electric field, η, as a function of position, ξ, inside the sheath region.

Figure 3 is a graph of the electron velocity, ψ, as a function of position inside the sheath region. This graph shows that, at the end of the sheath region, as expected, the electron non-dimensional velocity approaches one $(\psi \to 1)$, indicating that the electrons come to rest relative to ions and neutral particles.

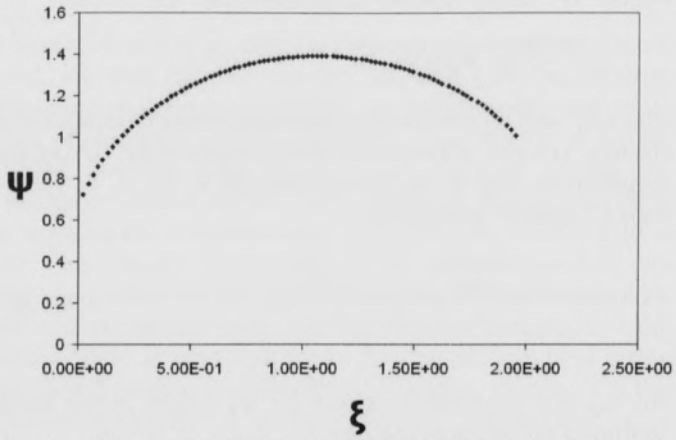

Fig. 3. Electron velocity, ψ, as a function of position, ξ, inside the sheath region.

Figure 4 is a graph of electron temperature, θ, as a function of position, ξ, inside the sheath region. From this graph it can be noted that the temperature continues to increase throughout the sheath region.

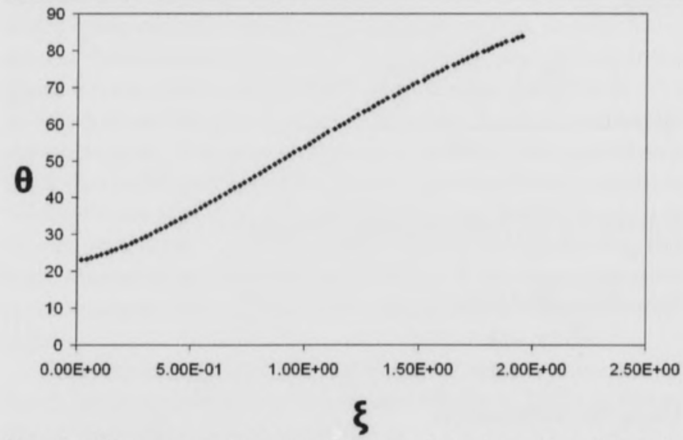

Fig. 4. Electron temperature, θ, as a function of position, ξ, inside the sheath region.

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Figure 5 shows a graph of the electron number density, ν, plotted versus position in the sheath region. From this plot one can note that the electron number density varies throughout the sheath region.

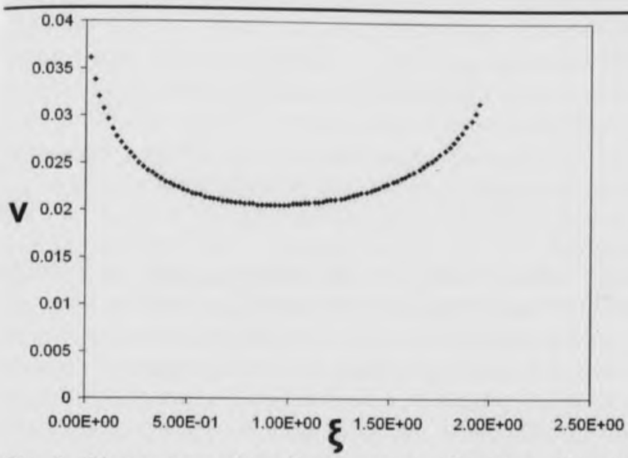

Fig. 5. Electron number density, ν, as a function of position, ξ, inside the sheath region.

Figure 6 shows the ionization rate, μ, plotted versus position inside the sheath region. The ionization rate has been calculated from a double integral (Fowler, 1983), based on free trajectory theory. From this graph it can be seen that the ionization rate continues to increase and varies throughout the sheath region.

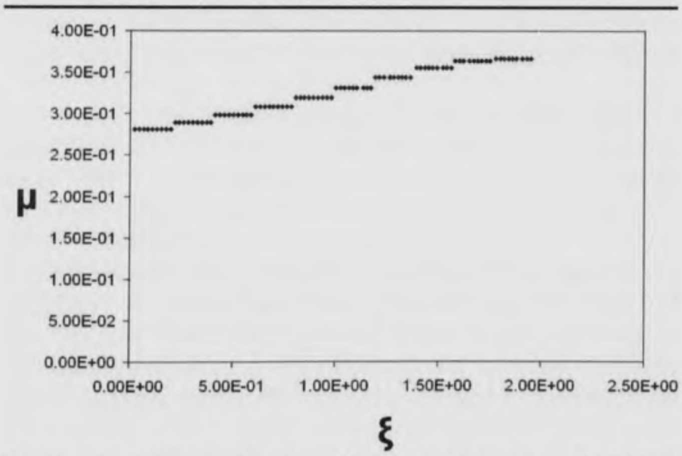

Fig. 6. Ionization rate, μ, as function of position, ξ, inside the sheath region.

Conclusions

The initial condition for the energy loss terms due to random and directed motion of electrons in the equation of conservation of energy was successfully found. Antiforce wave propagation can in fact be modeled by the electron fluid dynamical equations. Numerical integration of the electron fluid dynamical equations through the sheath region has been successful, meeting the required conditions at the end of the sheath region. The ionization rate was calculated from a double integral based on free trajectory theory which takes ionization due to random and directed motion of electrons into consideration. The set of electron fluid dynamical equations were successfully integrated for an antiforce wave moving into a non-ionized medium. Our results conform to the experimental results reported by Rakov et al. (1998). This is another confirmation of the validity of the application of fluid equations to breakdown waves.

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