Journal of the Arkansas Academy of Science

Volume 57 Article 11

2003

Antiforce Wave Profile for Quasi-Neutral Region

Mostafa Hemmati
Arkansas Tech University

Pashupati Adhikari
Arkansas Tech University

Jeremy Eckart
Arkansas Tech University

Marcus Ilbara
Arkansas Tech University

Antiforce Wave Profile for Quasi-Neutral Region

Mostafa Hemmati*, **Pashupati Adhikari, Jeremy Eckart** and **Marcus Ilbara**
Department of Physical Science
Arkansas Tech University
Russellville, AR 72801

*Corresponding Author

Abstract

This article will present a fluid dynamical theory for breakdown waves in which the direction of electric field force on electrons is in the opposite direction of wave propagation. We will refer to such waves as antiforce waves. The set of equations describing the model will include the equation of particle mass balance, equation of conservation of momentum, and equation of conservation of energy, coupled with Poisson's equation. This model treats the potential wave front as an electron shock wave propagating forward mainly due to the electron impact ionization. The shock front is succeeded by a thin dynamical transition region (sheath region) in which the electric field reduces to zero and the electron velocity approaches that of heavy particle velocity. Following the sheath region, ionization continues as long as electrons possess sufficient thermal energy. This thermal region is referred to as the quasi-neutral region. For antiforce waves, we have been able to integrate the set of electron fluid dynamical equations through the sheath region and through the quasi-neutral region. Our results conform to the expected physical conditions at the end of the sheath region and at the end of the quasi-neutral region. For three wave propagation velocities we will present the wave profile for each of the following: the electric field as a function of electron velocity inside the sheath, the ionization rate inside the sheath region and the quasi-neutral region, and the electron temperature and number density inside the quasi-neutral region.

Introduction

Hauksbee (1706) is believed to be the first person to study luminous pulses caused by a large potential difference between two electrodes in evacuated chambers. Hauksbee's research led him to discover the conditions for producing luminous pulses in gases.

Snoddy et al. (1937) made extensive measurements on the speed of propagation of breakdown waves in long discharge tubes containing dry air at different air pressures. In their experiments, they used discharge tubes with three different diameters. They observed that at approximately constant applied potential of 1.25×10^5V and low air pressure range, the breakdown wave speed increases with increasing air pressure. Keeping their tube diameter constant, they did not detect a considerable change in wave propagation speed when they conducted their experiments in dry air, CO_2 or H_2. However, they did discover a linear relationship between breakdown wave speed and applied potential at constant pressure in dry air. They observed that immediately following the initial wave, which propagates from the high voltage end to the grounded end of the tube, a second return stroke would propagate in the opposite direction. For the return stroke, the electric field force on electrons is in the opposite direction of wave propagation. This electric field force, however, is overcome by the electron gas partial pressure, which being very large provides the driving force for wave propagation. Snoddy et al. (1937) reported a wave speed of approximately 10^8 m/s for return discharge waves.

Asinovski et al. (1994) reported that an increase in the applied voltage leads to an increase in wave velocity. They also reported that in a low-pressure region, the propagation speed of breakdown waves of positive polarity is less than those of negative polarity.

A lack of Doppler shift in the spectral lines of the radiation emitted during the gas electrical discharge indicates an absence of heavy particle (ions and neutral particles) motion. The breakdown wave propagation therefore must be due to the electron mobility only. Consequently, Shelton et al. (1968) referred to such waves as Electron Fluid Dynamical Waves. They assumed that the high temperature electron gas expands very rapidly producing an electron shock wave. Therefore, they formulated a set of one-dimensional, steady state, fluid equations to describe breakdown waves. Shelton's set of electron fluid dynamical equations consist of the equation of conservation of mass, momentum, and energy plus Poisson's equation. Using an approximation method, Shelton et al. (1968) solved the set of electron fluid dynamical equations and the results of their approximate solutions conform to experimental results reasonably well.

Fowler et al. (1984) completed Shelton's set of electron fluid dynamical equations, adding several terms to the energy equation, among which the heat conduction term proved to be the most significant one. Also, Fowler et al. (1984) allowed for temperature derivative discontinuity at the shock front. Their allowance for temperature derivative discontinuity proved to be essential in integration of the electron fluid dynamical equations through the sheath region.

Analysis

Fowler's (1984) set of electron fluid dynamical equations: equations of conservation of mass, momentum, and energy plus Poisson's equation for proforce waves respectively are

$$\frac{d(n\upsilon)}{dx} = \beta n, \tag{1}$$

$$\frac{d}{dx}\{nm\upsilon(\upsilon - V) + nkT_e\} = -enE - KmnV(\upsilon - V), \tag{2}$$

$$\frac{d}{dx}\left\{nm\upsilon(\upsilon - V)^2 + nkT_e(5\upsilon - 2V) + 2e\phi n\upsilon + \varepsilon_0 VE^2 - \frac{5nk^2T_e}{mk}\frac{dT_e}{dx}\right\}$$

$$= -3(m/M)nKkT_e - (m/M)nmK(\upsilon - V^2), \text{ and} \tag{3}$$

$$\frac{dE}{dx} = \frac{en}{\varepsilon_0}\left(\frac{\upsilon}{V} - 1\right). \tag{4}$$

Where E_0 is electric field magnitude at the wave front, m is the electron mass, e is the electron charge, n is the electron number density, E is the electric field inside the sheath, υ is the electron velocity, T_e is the electron temperature, k is the Boltzman's constant, M is the neutral particle mass, K is the elastic collision frequency, x is the position in the wave profile, β is the ionization frequency, ϕ is the ionization potential of the gas, and V is the wave velocity.

For antiforce waves, the electron fluid dynamical equations have to be modified due to the change in the electric field direction at the shock front. Also, we introduce an appropriate set of non-dimensional variables, which takes the direction of the electric field force on electrons into consideration. The non-dimensional variables employed are

$$\eta = \frac{E}{E_0}, \quad \nu = \frac{2e\phi}{\varepsilon_0 E_0^{\,2}}n, \quad \psi = \frac{\upsilon}{V}, \quad \theta = \frac{Tek}{2e\phi}, \quad \xi = -\frac{eE_0x}{mV^2}, \quad \omega = \frac{2m}{M},$$

$$\kappa = -\frac{mVK}{eE_0}, \quad \mu = \frac{\beta}{K}, \quad \alpha = \frac{2e\phi}{mV^2}.$$

Where υ is the non-dimensional electron number density, η is the net electric field (applied plus space charge field), ψ is the electron velocity, θ is the electron gas temperature, ξ is the position within the sheath, μ is the ionization rate, κ and α are wave parameters.

In non-dimensional form, the electron fluid dynamical equations representing antiforce waves become:

$$\frac{d(\nu\psi)}{d\xi} = \kappa\mu\nu, \tag{5}$$

$$\frac{d}{d\xi}\{\nu\psi(\psi - 1) + \nu\alpha\theta\} = \nu\eta - \kappa\nu(\psi - 1), \tag{6}$$

$$\frac{d}{d\xi}\{\nu\psi(\psi - 1)^2 + \alpha\nu\theta(5\psi - 2) + \alpha\nu\psi + \alpha\eta^2 - \frac{5\alpha^2\nu\theta}{\kappa}\frac{d\theta}{d\xi}\}$$

We will define the variable values at the wave front with a

$$= -\omega\kappa\nu\left\{3\alpha\theta + (\psi - 1)^2\right\}, \text{ and} \tag{7}$$

$$\frac{d\eta}{d\xi} = -\frac{\nu}{\alpha}(\psi - 1). \tag{8}$$

subscript (1), the variable values at the end of the sheath with a subscript (2), and the variable values at the end of the quasi-neutral region with a subscript (f).

At the wave front the electron velocity is less than the wave velocity ($\psi_1 < 1$); therefore, according to Equation 8, the net electric field intensity will increase $\{(\frac{d\eta}{d\xi}) > 0\}$ until electrons gain speeds in excess of ions and heavy particles ($\psi > 1$). The electric field then will start decreasing. Since a conductor cannot support an electric field, the magnitude of the electric field has to approach zero at the end of the sheath region ($\eta_2 \to 0$). Without the supporting potential of the electric field the electrons gradually slow down due to collisions with neutral particles until their speeds compare with that of the ions and heavy particles ($\psi_2 \to 1$).

Successful integration of the non-dimensional electron fluid dynamical equations makes it possible to attain values of the variables including electron number density, ionization rate, electron temperature, and position at the end of the sheath region. The variable values at the end of the sheath region will constitute the initial conditions for integration of the set of electron fluid dynamical equations through the quasi-neutral region. The electron number density, ν_f, should approach one at the end of the quasi-neutral region, because the hot electron gas following the sheath region ionizes neutral particles during its cooling. Electrostatic energy density, therefore, is converted into ionization density behind the wave, resulting in neutral plasma. Electrons at the end of the quasi-neutral region reach thermal equilibrium with surrounding heavy particles. The dimensionless electron temperature, θ_f, in this region has to approximately reduce to 0.065.

All attempts in integration of Equations (5-8) through the quasi-neutral region failed. In our final attempt in integration of the set of electron fluid dynamical equations through the quasi-neutral region we chose to utilize the original form of the equations. Expressed in dimensionless variables, these original equations are

$$\frac{d(\nu\psi)}{d\xi} = \kappa\mu\nu, \tag{9}$$

$$\frac{d}{d\xi}\{\nu\psi^2 + \alpha\nu\theta\} = -\nu\eta - \kappa\nu(\psi - 1) + \kappa\mu\nu, \tag{10}$$

$$\frac{d}{d\xi}\left\{\nu\psi^3 + 5\nu\psi\alpha\theta - \frac{5\alpha^2\nu\theta}{\kappa}\frac{d\theta}{d\xi}\right\}$$

$$= -2\nu\psi\eta - 2\kappa\nu(\psi - 1) + \kappa\mu\nu(\psi - 1) - \omega\kappa\nu\left\{3\alpha\theta + (\psi - 1)^2\right\}, \text{ and} \tag{11}$$

$$\frac{d\eta}{d\xi} = -\frac{\nu}{\alpha}(\psi - 1). \tag{12}$$

We will use Ibarra et al.'s (2002) approach to integrate the set of equations through the quasi-neutral region. Expanding the momentum balance equation (Equation 6) and using the equation of conservation of mass balance in the expanded form to solve for $(\frac{d\psi}{d\xi})$, the singularity inherent in the set of equations appears in the denominator of the equation

$$\frac{d\psi}{d\xi}=\frac{\kappa(1+\mu)(1-\psi)\psi-\kappa\mu\alpha\theta-\eta\psi-\alpha\psi\theta'}{\psi^2-\alpha\theta}.$$

When the denominator in $(\frac{d\psi}{d\xi})$ approaches zero, $(\frac{d\psi}{d\xi})$ approaches infinity. This would indicate a shock within the sheath region. Since no new shock is possible inside the sheath, therefore, the denominator and numerator must both approach zero at the same time. This condition allows one to choose a starting value for $\psi1$, for a given value of κ, α, and υ_1, by trial and error. We note that, at the end of the sheath region, the electron velocity, ψ, becomes equal to one and $(\frac{d\psi}{d\xi})$ approaches zero. Also, the electric field, ξ, and its derivative, $(\frac{d\eta}{d\xi})$, both approach zero at the end of the sheath region. The resulting formulated equations with respect to previously noted conditions are

$$(\frac{d\nu}{d\xi})_2=\kappa\mu_2\nu_2, \tag{13}$$

$$(\frac{d\theta}{d\xi})_2=-\kappa\mu_2\theta_2. \tag{14}$$

We have been successful in integrating equations (13-14) through the quasi-neutral region for antiforce waves propagating into a non-ionized medium. For three wave speeds, our solutions meet the expected conditions at the trailing edge of the wave ($\nu_f \approx 1$, and $\theta_f \approx 0.065$).

Results

For wave speeds of $\alpha = 0.05$, $\alpha = 0.1$, and $\alpha = 1.0$, we have successfully integrated the electron fluid dynamical equations through both the sheath and quasi-neutral region. After integration of Equations (5-8) throughout the sheath region we needed to integrate Equations (13) and (14) through the quasi-neutral region. The following initial values for electron number density (ν_1), electron velocity (ψ_1), and wave parameter (κ) were utilized in order to obtain solutions that met the expected physical conditions at the end of the sheath region and the quasi-neutral region.

α	ν_1	ψ_1	κ
0.05	0.09	0.95	0.3511
0.10	0.14	0.8679	0.2995
1.0	0.40	0.97501	0.175

The data indicates that sheath thickness increases as wave velocity decreases. A value of $\alpha = 0.01$ represents a wave speed of $3\text{x}10^7$ m/s and $\alpha = 2.0$ represents a moderately slow wave possessing a velocity of $2\text{x}10^6$ m/s. We were also able to integrate the set of equations through the sheath and quasi-neutral region for $\alpha = 0.005$. $\alpha = 0.005$ represents a fast wave speed of $V = 10^8$ m/s.

Figure 1 is a graph of the electric field, η, as a function of electron velocity, Ψ, inside the sheath region. Figure 2 is a graph of the ionization rate, μ, as a function of position, ξ, inside the sheath region. $\xi = 5.72$ represents a sheath thickness of 0.03 mm.

Figure 3 is a graph of the ionization rate, μ, as a function of position, ξ, inside the quasi-neutral region. Figure 4 is a log-log plot of the electron temperature, θ, as a function of position, ξ, inside the quasi-neutral region. $\theta = 20$ represents an electron temperature of approximately 10^7 K. Figure 5 is a plot of electron number density, ν, as a function of position, ξ, inside the quasi-neutral region. $\nu = 1$ represents an electron number density of approximately 10^{20} electrons/m^3.

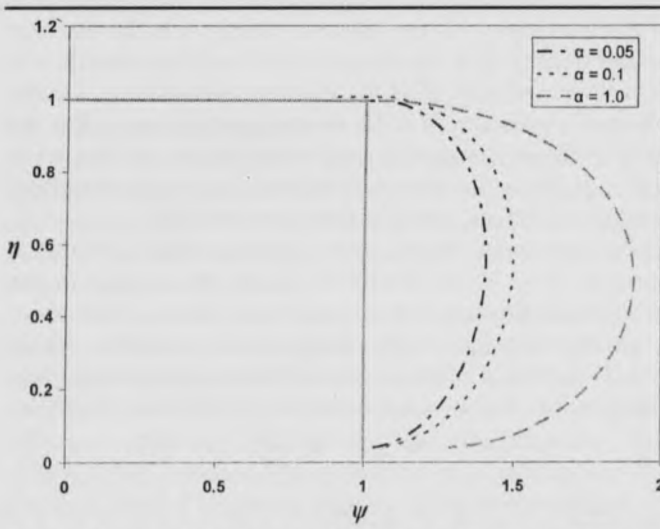

Fig. 1. Electric field, η, as a function of electron velocity, ψ, inside the sheath region for $\alpha = 0.05$, $\alpha = 0.1$, and $\alpha = 1$.

Our study of antiforce waves, or waves that propagate in the opposite direction as the electric field force on electrons, is exemplified in nature by lightning's return stroke. The range of speed for which our integration of the set of electron fluid dynamical equations has been successful, conform to the experimental results reported by Rakov et al. (1998). They employed triggered-lightning experiments to make measurements on lightning propagation speeds, temperatures, light intensity, and electric fields, and their average return stroke speeds are indeed on the order of 10^8 m/s.

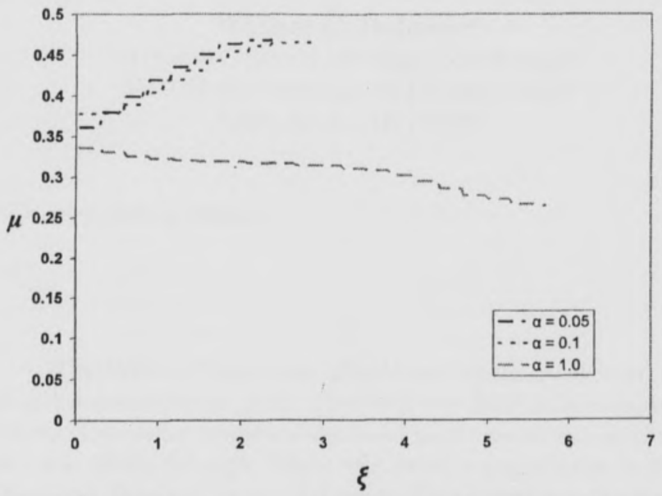

Fig. 2. Ionization rate, μ, as a function of position, ξ, inside the sheath region for α = 0.05, α = 0.1, and α = 1.

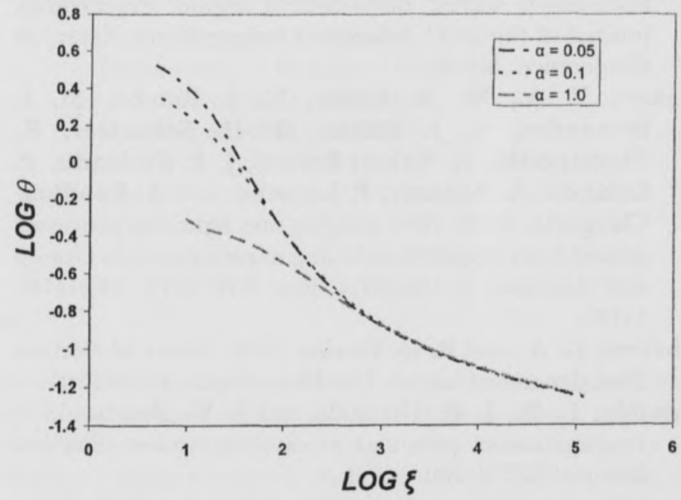

Fig. 4. Electron temperature, θ, as a function of position, ξ, inside the quasi-neutral region for α=0.05, α=0.1, and α=1.

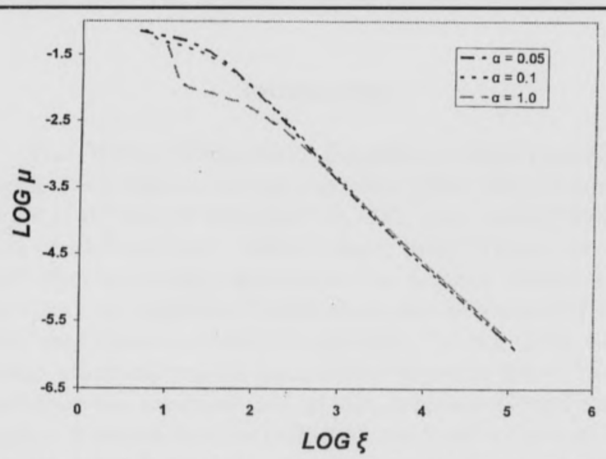

Fig. 3. Ionization rate, μ, as a function of position, ξ, inside the quasi-neutral region for α=0.05, α=0.1, and α=1.

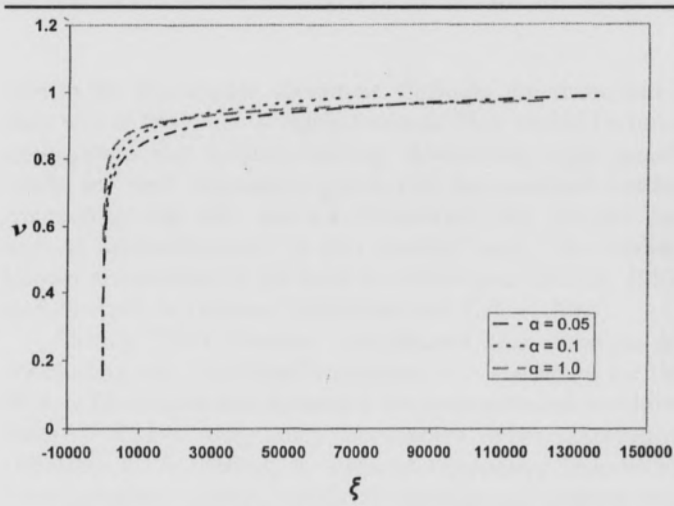

Fig. 5. Electron number density, ν, as a function of position, ξ, inside the quasi-neutral region for α=0.05, α=0.1, and α=1.

Conclusions

For antiforce waves, we have been able to integrate the set of electron fluid dynamical equations through both the sheath region and the quasi-neutral region for a range of wave speeds. The solutions meet the expected physical conditions at the end of the sheath region and the quasi-neutral region. The range of speeds for which integration of the set of electron fluid dynamical equations become possible conforms to the experimental results. This is another confirmation on the validity of the application of fluid equations to breakdown waves.

ACKNOWLEDGMENTS.—The authors would like to express their gratitude to the Arkansas Space Grant Consortium and the Scientific Information Liaison Office for their financial support of this project.

Literature Cited

Asinovsky, E. I., A. N. Lagarkov, V. V. Markovets, and **I. M. Rutkevich.** 1994. On the similarity of electric breakdown waves propagating in shielded discharge tubes. Plasma Sources Sci. Technol. 3:556-563.

Fowler, R. G., M. Hemmati, R. P. Scott, and **S. Parsenajadh.** 1984. Electric breakdown waves: Exact numerical solutions. Part I. Phys. Fluids. 27:1521-1526.

Hauksbee, F. 1706/7. An account of an experiment, confirming one lately made, touching the production of light, by the effluvia of one glass falling on another in motion. Phil. Trans. Roy. Soc. 25:2413.

Ibarra, M., R. C. Cook, and **M. Hemmati.** 2002.

Breakdown waves: quasi-neutral region. Proceedings Journal of the 2002 Arkansas Undergraduate Research Conference. 143-151.

Rakov, V. A., M. A. Uman, K. J. Rambo, M. I. Fernandez, R. J. Fisher, G. H. Schnetzer, R. Thottappillil, A. Eybert-Berard, J. P. Berlandis, P. Lalande, A. Bonamy, P. Laroche, and **A. Bondiou-Clergerie.** 1998. New insights into lightning processes gained from triggered-lightning experiements in Florida and Alabama. J. Geophys. Res. NO. D12. 100:14117-14130.

Shelton, G. A., and **R. G. Fowler.** 1968. Nature of electron fluid dynamical waves. The Phys. Fluids. 11:740-746.

Snoddy, L. B., J. R. Dietrich, and **J. W. Beams.** 1937. Propagation of potential in discharge tubes. Physical Review. 52:739-746.