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Abstract

Considering the electrons as the main element in breakdown wave propagation and using a one-dimensional, steady-state, three-fluid, hydrodynamical model, previous investigations have resulted in the completion of a set of equations for conservation of mass, momentum, and energy. We will use the terms proforce and antiforce waves, depending on whether the applied electric field force on electrons is with or against the direction of wave propagation. In the case of antiforce waves, the electron gas temperature and therefore the electron fluid pressure is assumed to be large enough to sustain the wave propagation down the discharge tube.

For strong discontinuity and based on the conditions existent at the leading edge of the wave, previous investigations have concluded a minimum wave velocity condition for breakdown waves. However, allowing for a temperature derivative discontinuity at the shock front, we have been able to derive a new set of conditions at the shock front and therefore a lower range of electron drift velocity. This conforms with the experimentally observed wave speeds. The solution to the set of electron fluid-dynamical equations involves a previously discovered method of integration of the equations through the sheath (dynamical transition) region. For a wide range of wave speeds, the appropriate set of electron fluid-dynamical equations has been integrated through the sheath region.

Introduction

Shelton and Fowler (1968) argued that a fluid phenomenon involving no mass motion must be due to electron-fluid action. Therefore, they thought that the name "Electron Fluid-Dynamical Wave" represented a better description of the basic nature of the event. Owing to the smallness of the velocity changes of the heavy particles as the wave passes over them, a set of conservation equations applying only to the electron fluid could be derived. Using the principle of frame invariance, Shelton and Fowler (1968) found analytical forms for both the elastic and inelastic collision terms in the equations of conservation of momentum and energy. From the conservation of energy equation, they derived the conditions existent at the leading edge of the wave. Based on these conditions and for strong discontinuity, they found a minimum wave velocity condition ($\frac{1}{2}mV_0^2 \geq e\phi_i$), where ϕ_i, V_0, m, and e are ionization potential, wave velocity, and electron mass and charge respectively. To achieve a solution to the electron fluid dynamical equations, Shelton and Fowler had to resort to approximation methods.

For successful integration of the set of equations through the dynamical region, Fowler et al. (1984) had to modify Shelton's three component fluid equations for conservation of mass flux, momentum, and energy. With Poisson's equation included and for proforce waves, their set of equations which have proved to be successful are

$$\frac{dE}{dx} = \frac{e}{\epsilon_0} n\left(\frac{v}{V} - 1\right), \tag{1}$$

$$\frac{d(nv)}{dx} = \beta n, \tag{2}$$

$$\frac{d}{dx}\left\{ mnv(v - V) + nkT_e \right\} = -enE - Kmn(v - V), \tag{3}$$

$$\frac{d}{dx}\left\{ mnv(v - V)^2 + nkT_e(5v - 2V) + 2e\phi_i nv - \frac{5nk^2T_e}{mK}\frac{dT_e}{dx} \right\} = -3\left(\frac{m}{M}\right)nkKT_e - \left(\frac{m}{M}\right)Kmn(v - V)^2. \tag{4}$$

Where the variables are electric field E, electron temperature T_e, electron concentration n, electron velocity v, and position in the wave profile x. Also, β and K are the ionization frequency and elastic collision frequency, respectively.

In the case of proforce waves, to reduce equations (1) through (4) to nondimensional form, Shelton and Fowler (1968) introduced a set of dimensionless variables. We have slightly modified the dimensionless variables and they are

$$\omega = \frac{2m}{M},\ \kappa = \frac{mVK}{eE_0},\ \mu = \frac{\beta}{K},\ \alpha = \frac{2e\phi}{mV^2},\ v = V\psi,\ n = \frac{\epsilon_0 E_0^2}{2e\phi}\nu,\ T_e = \frac{2e\phi}{k}\theta,\ E = \eta E_0,\ x = \frac{mV^2}{eE_0}\xi.$$

In the above equations, υ, ψ, θ, μ, κ, η, and ξ are the dimensionless electron concentration, electron velocity, electron temperature, ionization rate, elastic collision frequency, electric field, and position inside the wave, respectively. The symbols n and T_e represent electron number density and temperature inside the sheath, and β, φ, V, M, E_0 are ionization frequency, ionization potential, wave velocity, neutral particle mass, and electric field at the wave front. In terms of dimensionless variables the electron fluid dynamical equations become

$$\frac{d\eta}{d\xi} = \frac{\nu}{\alpha}(\psi - 1), \tag{5}$$

$$\frac{d(\nu\psi)}{d\xi} = \kappa\mu\nu, \tag{6}$$

$$\frac{d}{d\xi}\{\nu\psi(\psi - 1) + \alpha\nu\theta\} = -\nu\eta - \kappa\nu(\psi - 1), \tag{7}$$

$$\frac{d}{d\xi}\{\nu\psi(\psi - 1)^2 + \alpha\nu\theta(5\psi - 2) + \alpha\nu\psi + \alpha\eta^2 - \frac{5\alpha^2\nu\theta}{\kappa}\frac{d\theta}{d\xi}\} = -\omega\kappa\nu\{3\alpha\theta + (\psi - 1)^2\}. \tag{8}$$

Solution of the Equations

For strong discontinuity, Shelton and Fowler (1968) derived the following conditions for electron velocity and temperature at the leading edge of the wave.

$$\psi_1 = \frac{5 - \sqrt{9 + 16\alpha}}{8}, \tag{9}$$

$$\theta_1 = \frac{\psi_1(1 - \psi_1)}{\alpha}. \tag{10}$$

Based on these conditions, he concluded a minimum wave velocity condition $(v_o \geq \sqrt{\frac{2e\phi_i}{m}})$. However, allowing for a temperature derivative discontinuity at the shock front, introducing the initial condition on electron temperature from equation (10), and substituting the values of the other variables at the leading edge of the wave ($\xi_1 = 0, \eta_1 = 1$) into the energy equation with the heat conduction term included results in

$$\nu_1\psi_1(\psi_1 - 1)^2 + \nu_1\psi_1(1 - \psi_1)(5\psi_1 - 2) + \alpha\nu_1\psi_1 + \alpha(\eta_1^2 - 1) - \frac{5\alpha\nu_1\psi_1}{\kappa}(1 - \psi_1)\theta_1' = 0. \tag{11}$$

This equation reduces to

$$-4\psi_1^2 + 5\psi_1 - 1 + \frac{5\alpha\psi_1}{\kappa}\theta_1' + \alpha - \frac{5\alpha\theta_1'}{\kappa} = 0. \tag{12}$$

Solving this quadratic equation for ψ_1, one can find the initial condition on electron velocity to be

$$\psi_1 = \frac{5(1 + \frac{\alpha\theta_1'}{\kappa}) - \sqrt{(3 - \frac{5\alpha\theta_1'}{\kappa})^2 + 16\alpha}}{8}. \tag{13}$$

From Poisson's equation and for waves moving into an unionized medium, one can conclude $e(N_iV - nv) = 0$. This is called the zero current condition, and it requires that V and v be of the same sign. Therefore, $\psi_1 = \frac{v_1}{V}$ has to be positive. From equation (13) one can conclude

$$5(1 + \frac{\alpha\theta_1'}{\kappa}) - \sqrt{(3 - \frac{5\alpha\theta_1'}{\kappa})^2 + 16\alpha} > 0 \tag{14}$$

or

$$0 < \alpha < (1 - \frac{5\theta_1'}{2\kappa})^{-1} \tag{15}$$

With positive values of θ_1', α can have values larger than one, and we have been able to find solutions for α as large as 2. This conforms with experimentally observed wave speeds (Uman, 1993).

To integrate the equation set through the sheath region, one has to place the singularity inherent in the equation set in the denominator of the momentum integral. If we solve the momentum equation (7) for $\frac{d\psi}{d\xi}$ it will become:

$$\frac{d\psi}{d\xi} = \frac{\kappa\psi(1 - \psi)(1 + \mu) - \alpha\theta'\psi + \eta\psi - \alpha\kappa\mu\theta}{\psi^2 - \alpha\theta}. \tag{18}$$

A zero denominator in the momentum integral represents an infinite value for the derivative of electron velocity with respect to the position inside the sheath. This condition requires the existence of a shock inside the sheath region, which is not allowed. The numerator in the momentum integral, therefore, has to become zero at the same time that the denominator becomes zero. In the process of integration of the equations through the sheath region, comparing the numerator and denominator values will allow one to choose the required initial parameters by trial and error. A successful solution has to allow passage through the singularity and satisfy the physically acceptable conditions at the trailing edge of the sheath. The expected conditions at the end of the sheath are a) the electrons have to come to rest relative to neutral particles ($\psi \rightarrow 1$), and b) the net electric field has to reduce to a negligible value ($\eta \rightarrow 0$). The method of integration of the set of electron fluid dynamical equations is identical to the one adopted previously (Hemmati et al., 1998).

For a wide range of wave speeds and for proforce waves moving into a nonionized medium, we have integrated the set of electron fluid-dynamical equations through the sheath region. The solutions meet the expected conditions at the trailing edge of the wave ($\psi \rightarrow 1, \eta \rightarrow 0$). Successful integration of the set of equations through the sheath region required the use of the following values for κ, ψ_1, and υ_1.

$\alpha = 0.01, \kappa = 1.239718, \psi_1 = 0.327, \upsilon_1 = 0.023$
$\alpha = 0.1, \kappa = 1.071818, \psi_1 = 0.29125, \upsilon_1 = 0.235$
$\alpha = 0.25, \kappa = 0.959363, \psi_1 = 0.26016, \upsilon_1 = 0.666$
$\alpha = 0.5, \kappa = 1.08576, \psi_1 = 0.25375, \upsilon_1 = 1.05$
$\alpha = 1, \kappa = 1.0426635, \psi_1 = 0.2071, \upsilon_1 = 2.1$

Figure 1 is a plot of the electric field, η, as a function of electron velocity, ψ, inside the sheath for five values of α. α = 0.01 represents a fast wave speed ($V = 3 \times 10^7$ m/s) and α = 1 represents a slow wave speed ($V = 3 \times 10^6$ m/s). The graph shows that for all five values of α the solutions to the electron fluid dynamical equations conform to the expected physical conditions at the trailing edge of the wave.

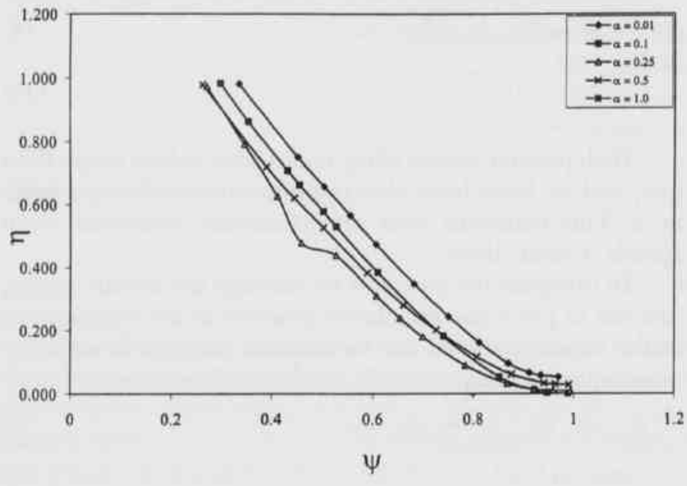

Fig. 1. Electric field, η, as a function of electron velocity, ψ, for five values of α. α = 0.01, 0.1, 0.25, 0.5, and 1. α represents wave speed.

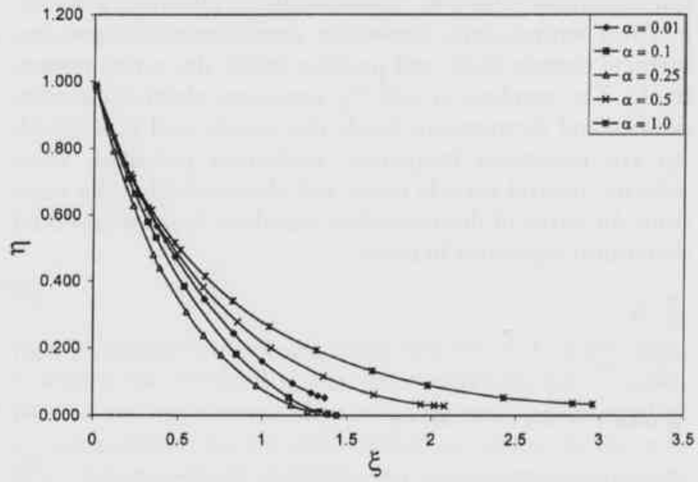

Fig. 2. Electric field, η, as a function of position inside the sheath, ξ, for five values of α. α = 0.01, 0.1, 0.25, 0.5, and 1. α represents wave speed.

Figure 2 shows graphs of electric field, η, as a function of position inside the sheath, ξ, for five values of wave speed. As the value of the wave speed decreases (α increases), the integration of the equations becomes more difficult and time consuming. $\alpha = 4$ seems to be the cut-off point, representing a minimum wave speed. For $\alpha > 4$, the integration of the set of equations through the sheath region becomes impossible. The present limit on the values of wave speed conforms with the experimentally measured values (Uman, 1993). For $\alpha = 0.01$, the nondimensional sheath thickness is $\xi = 1.4$. To connect to the physical world, this represents a sheath thickness of $x = 0.0007$ m.

Conclusions

For proforce waves, the electron fluid dynamical equations have successfully been integrated for five different values of wave speeds. Allowing for a temperature derivative discontinuity at the shock front results in a lower limit on breakdown wave speeds. Our newly derived limit on wave velocity conforms with the experimentally observed wave speeds.

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Literature Cited

Fowler, R. G., M. Hemmati, R. P. Scott, and **Parsenajadh.** 1984. Electric breakdown waves: Exact numerical solutions. Part 1., Phys. Fluids. 27:1521-1526.

Hemmati, M., M. S. Doss, and **E. L. George.** 1998. Speed cut-off point for antiforce waves. J. Arkansas Acad. Sci. 52:63-66.

Shelton, G. A., and **R. G. Fowler.** 1968. Nature of electron-fluid-dynamical waves. Phys. Fluids. 11:740-746.

Uman, M. A. 1993. Natural Lightning. Proc. IEEE Industrial and Commercial Power Systems Technical Conference. Vol. 93ch3255-7 pp. 1-7.