5-2013

Load Mixing to Improve Container Utilization

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A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Industrial Engineering

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May 2013
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Abstract

The underutilization in trucking leads to nearly 5 billion gallons of wasted fuel annually. One way to recapture part of this waste is to use collaborative logistics. This research focuses on one specific aspect of collaborative logistics: *load mixing.* Load mixing is the idea of mixing two or more items of different weights in the same container to reduce the number of trucks needed.

Load mixing is similar to other packing problems such as the knapsack and container loading problems. However, traditional packing problems typically only assume a single type of capacity (e.g., weight), whereas load mixing must simultaneously the weight and spatial capacities to effectively utilize containers. We propose a mathematical model with constraints for the space and weight limits that can be used to minimize the number of trailers used. Though there are some tractability issues associated with this formulation, a more fundamental issue is the existence of multiple optima for this type of problem. While solutions that all use the same number of containers to transport to the commodities are all viewed equally by the model, these solutions vary in terms of how easily they can be implemented. For these reasons, I propose a heuristic to solve the load mixing problem. The performance of the heuristic is tested against our exact formulation to compare solutions.

First a heuristic to load two commodities with different weight and demand – one heavy and one light – on a set of containers was created. The theoretical minimum number of containers needed is calculated based on the total demand and total weight of the commodities. Once this heuristic was examined it was generalized to handle more than two commodities. Both heuristics were implemented in C++. Testing for both heuristics shows both achieve the theoretical minimum in a large amount of cases. The testing of these heuristics also led to many insights about when to use or to not use load mixing. For example, when more of the demand is composed of lighter commodities the savings are greater than when the demand is mostly composed of heavy commodities.

This thesis is approved for recommendation
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1. Introduction

Typically companies produce and therefore ship multiple types of products. The trucks used to ship these products are limited in two ways: there is both a weight and spatial (cube) limit as to the amount of freight that can be transported. Some freight is very heavy, leading to full utilization of a truck's weight capacity but an underutilization of a truck's cube. Conversely, other freight is very light, leading to high cube utilization but very low weight utilization.

Take for instance a company such as PepsiCo that produces Frito Lay potato chips and Pepsi soda. Chips are a very light product; however, soda is considerably denser. Consider the example shown in Table 1, which shows demands and approximate weights for these items.

Table 1: Example Problem Data

Product	Weight (lbs/pallet)	Demand (pallets)
Soda	1200	60
Chips	300	60

Typical 53' truck trailers can hold up to 60 standard-sized pallets and up to 45,000 pounds. Shipped separately, three trucks would be required to transport these items. One container would contain 60 pallets of the chips, which fully utilize the truck's cube capacity, but only utilize 40% of the weight. Two additional containers would be required to transport the soda. One container would contain 37 pallets of soda, which would fully utilize the container's weight capacity, but only utilize 62% of the cube. The remaining 23 remaining pallets of soda would be shipped in an additional container, leaving both the weight and cube underutilized.

However, suppose that chips and soda could be shipped together in common containers. Only two containers would be required now each containing 30 pallets each of soda and chips, fully utilizing both the weight and cube of the trailers.

While this is only a small example, it illustrates the powerful concept of shipping different products in the same container to achieve better weight and cube utilization. We refer to this as *load mixing*; this has been implemented in by some companies in industry. Kraft implemented a load mixing system, which led to 9000 fewer loads and 6.2 million less miles driven (Neufarth, Haining, & Moore, 2011). Kimberly Clark and Colgate-Palmolive have partnered together and successfully implemented a load mixing system as well. This partnership increased truckload utilization by 9%. The reduction of trucks on the road and fewer miles driven saved 28.3 carbon tons, which shows the impact load mixing can have on the environment (Degroot, Hood, & McHugh, Next Generation in Collaborative Shipping, 2012). These examples demonstrate impact that load mixing can have on transportation efficiency.

Although some individual companies have implemented load mixing solutions, many companies have not yet implemented load mixing. Furthermore, this important problem has not been studied in literature to the best of our knowledge. The work on this problem has been motivated by the collaboration with a major retailer. This retailer is looking into the effect of mixing many of their suppliers' freight on the overall trailer utilization. They are particularly interested to implement load mixing systems in the distribution center (DC) to store operations where they have to transport various types of freight. However, other opportunities for load mixing arise in shipping goods directly from the supplier to the store and in multi-stop situations where a truck visits many suppliers en route to a store or DC.

Load mixing not only can have an impact for individual companies, but on the trucking industry as a whole due to the collaborative nature of load mixing. Trucking is the primary mode of US freight transportation and accounts for 79% of freight costs (Sutherland, 2006). On average these trucks are only 60% utilized, leading to nearly 5 billion gallons of wasted fuel annually (Federal Highway Administration, 2009). The Kimberly Clark and Colgate-Palmolive example from above shows how load mixing, which is form of collaborative logistics, can help recapture part of the current waste. By the mixing of two or more types of items in the same container, Kimberly Clark and Colgate-Palmolive found an increase in overall truck utilization and therefore fewer trucks on the road, which would increase the overall truck utilization and reduce the amount of fuel wasted. Similar improvements can also be obtained when other companies collaborate on load mixing.

The objective of this research is to develop a heuristic that can be used to effectively mix freight to minimize the total number of containers required. In particular, this heuristic would determine how the freight should be loaded onto containers (i.e. how many pallets per item per truck) and the total number of containers required to transport the freight. This heuristic is then used to show when a load mixing system would be beneficial – in other words what kind of freight can or should be mixed, and what types of benefits would be realized by mixing this freight. This methodology will make load mixing accessible to industries with varying type of products

2. Literature Review

The load mixing problem is a packing problem similar to both the bin packing and a container loading problems. In both problems, a known collection of items must be assigned to containers. In bin packing problems the objective is to minimize the number of containers used, and in container loading the objective is to load the container or containers to maximize volume utilization. Bin packing and container loading problems are extensively studied in literature and known to be NP-hard; therefore different solution methodologies to solve these problems have been developed. Classifications of various packing problems are given in Dyckhoff (1990) and Wascher, Haubner, Schumann (2007). Using these packing problem typologies, the classification of the load mixing problem is 2/V/I/C. The 2 indicates that this is a 2-dimensional problem because it is constrained by the spatial and weight capacities of the container. The V indicates that all of the items – in this case pallets – have to be loaded onto a container, but not all of the containers available have to be used. The containers in this problem (i.e., trailers) are all assumed to be the same size, so they are identical figures (I). Finally, all of the pallets are the same size with varying weights; this is referred to as congruent (C). The load mixing problem will be formally introduced in Section 3.

The majority of problems previously examined in the literature consider a heterogeneous set of boxes (i.e. boxes of varying sizes) on containers to maximize the volume utilization, while meeting various side constraints. The most commonly used constraints require that each box be completely in the container, boxes do not overlap with each other, each box is only placed on the floor or other boxes that can support it, and boxes are parallel to the side walls. To solve this set of bin packing and container loading problems, Gehring and Bortfeldt (1994, 2000) propose genetic algorithm approaches, as well as tabu search approaches (Bortfeldt & Gehring, 2001; Bortfeldt, Gehring, & Mack, A Parallel Tabu Search Algorithm for Solving the Container Loading Problem, 2003). Morabito and Arenales (1994) use of AND/OR-graphs, and Cesar and Armentano (2007) use a multi-start constructive heuristic to solve the container loading problem (Morabito & Arenales, 1994; Cesar & Armentano, 2006). Pisinger (2002) presents a heuristic to solve the Knapsack Container Loading Problem. Lodi, Martello and Monaci (2002) present a survey of algorithms, heuristics, and metaheuristics used in solving Two-Dimensional Bin Packing problems and Two-Dimensional Strip Packing problems (Pisinger, 2002; Lodi, Martello, & Monaci). Lim, Rodrigues, and Yang (2005) present a survey of heuristics used to solve the three-dimensional container loading problem, and introduce their own heuristic for packing containers (Lim, Rodrigues, & Yang, 2005). The reader is referred to Dyckhoff (1990) for detailed descriptions of these problems (Dyckhoff, 1990).

The primary difference between the load mixing problem and previous research is that this research considers both weight and volume. In previous work the focus is to maximize the volume utilization, without any weight considerations. That is, previous research assumes that the weight of the objects does not further limit the capacity of the containers into which the objects are loaded. While this is true for many objects, this is untrue for very dense, heavy items in many types of containers. Although weight limitations were identified by Bischoff and Ratcliff (1995) as a real-world constraint that made existing container loading research difficult or impossible to implement in practice, to the best of our knowledge this has not been addressed in the literature (Bischoff & Ratcliff, 1995).

The remainder of the paper is organized as follows. In Section 0 the load mixing problem is formally defined. Section 4 presents how this problem can be formulated and solved as an optimization problem. Section 5 presents a heuristic for load mixing when there are just two products and describes the computational experiments used to validate the present heuristic. Section 6 extends the heuristic presented in a Section 5 to a generalized form and describes computational experiments to validate it. Finally Section 8 presents the conclusions found and possible future work.

3. Problem Description

For the load mixing problem, we are given a set of products, $P = \{1, 2, \ldots, n\}$, where each product *p* has a weight, w_p, and demand, d_p. It is assumed that pallet quantities of each product p are being shipped, and that each of these pallets has the same spatial dimensions. The load mixing problem determines an assignment of products to containers so as to minimize the total number of containers used. It is assumed that each container has the same weight capacity, W, and a volume capacity, V, and there are an infinite amount of containers available to use. The assignment of products to trailers must not violate these capacities. This problem can be solved using a mathematical model as well as a heuristic approach both of which can be seen in future sections.

4. Mathematical Formulation

To assess the tractability of the load mixing problem we use the following formulation. The sets for this problem are listed below.

$P = Set\ of\ products, indexed\ by\ p$

$T = Set\ of\ trucks,\ indexed\ by\ t$

The parameters for this problem are defined as follows.

$w_p \triangleq weight\ of\ product\ p\ in\ pounds\ per\ pallet$

$d_p \triangleq demand\ of\ product\ p\ in\ pallets$

$W \triangleq maximum\ weight\ allowed\ on\ a\ container$

$V \triangleq maximum\ number\ of\ pallets\ allowed\ on\ a\ container$

Finally the decision variables for this problem are to decide how many pallets of each product are to be shipped on each truck and whether a truck is used or not. The mathematical definition of these variables can be seen below.

$x_{p,t} \triangleq the\ number\ of\ pallets\ of\ product\ p\ in\ truck\ t$

$$y_t \triangleq \begin{cases} 1 & if\ truck\ t\ is\ used \\ 0 & otherwise \end{cases}$$

The values of y_t and $x_{p,t}$ determine the assignment of pallets to containers and therefore how many containers are needed. The mathematical model of the load mixing problem is given below.

$$\min \sum_{t \in T} y_t \qquad (1)$$

$$\sum_{p \in P} w_p x_{p,t} \leq W y_t \quad \forall\ t \in T \qquad (2)$$

$$\sum_{p \in P} x_{p,t} \leq V y_t \quad \forall\ t \in T \qquad (3)$$

$$\sum_{t \in T} x_{p,t} \geq d_p \quad \forall\ p \in P \qquad (4)$$

$$x_{p,t} \geq 0 \ and \ integer \ \forall \ p \in P, t \in T \ , \ y_t \in \{0,1\} \ \ \forall \ t \in T \qquad (5)$$

The objective function (1) seeks to minimize the total number of containers used. Constraint sets (2) and (3) ensure that the weight and spatial capacities respectively of each container are not violated. Constraint set (4) ensures that each product is assigned to containers. Finally, constraint set (5) enforces the integrality of all variables.

Because it assumed than an unlimited number of trucks are available to transport items, it is assured that this mathematical model will have a feasible solution and can therefore be solved to obtain the optimal solution. However, there are some drawbacks to optimization-based approaches. First, preliminary research showed that a non-trivial number of instances cannot be solved within an hour. For an operational level problem such as load mixing, this is problematic if these problems are going to be implemented in industry. Note that the time it takes to solve the load mixing problem significantly increased as the number of pallets increased, which would be a problem in cases with a large quantity of items. Second, since there are many different ways that items can be loaded into the same number of containers, there are typically many alternative optimal solutions to each instance. Note that while these are all the same from the perspective that they utilize the same number of containers, these solutions can be more or less desirable based on secondary objectives such as how easily they can be loaded by operators. For these reasons a heuristic was created to solve this problem. The heuristic is more efficient than optimization solvers, and results in solutions with similar loading configuration for each trailer without significantly affecting solution quality.

5. Two Product Heuristic Approach

Motivated by work with a major retailer, we begin by focusing on cases where just two types of freight must be transported: one heavy and one light. This section details the notation needed for the heuristic then walks through the heuristic used to load two products onto a set of containers. First, an assumption about the number of pallet positions and maximum allowed weight for a container needs to be made. This information would then be used to determine what products are considered to be light or heavy. Trailers loaded with only heavy products would fully utilize the weight capacity of the container, while underutilizing the space. Before presenting the heuristic, the notation is introduced in Figure 1.

Notation

F = set of freight types f: F={H, L}
W = Total weight capacity of trailer
V = Total cube capacity of trailer
w_f = Weight per pallet of the freight type f (in lbs.), f in{H,L}
d_f = Demand of freight type f (in pallets), f in{H,L}
T = number of trailers required to transport freight
$n_f[i]$ = number of pallets of freight type f assigned to truck i (in pallets), i = 1…T, f in{H,L}
a_f = number of assigned pallets of freight type f, f in{H,L }
$s_W[i]$ = weight slack of trailer i (in lbs.), i = 1…T
$s_V[i]$ = volume slack of trailer i (in lbs.), i = 1…T

Figure 1: Notation for 2-Product Heuristic

The overall approach for this heuristic was to first load the heavy product onto containers based on the demand for the product then load the light product based on the remaining space and weight of the container. But first, the theoretical minimum number of trucks needed is calculated. The theoretical minimum is the fewest number of trucks required based on the weight and demand of the products. For items that weigh less, the theoretical minimum will be based on the demand since the space of the container is the main constraint. Conversely, with heavier items the minimum number of trucks required is based on total weight since the weight capacity of the container met first. Note that this theoretical minimum is obtained by selecting the maximum of the number of containers needed based on the weight constraint and the cube constraint. This equation, used to calculate the theoretical minimum, T, is given in line 1 of the heuristic shown in Figure 2. For each of the containers the slack weight, slack cube, and number of pallets for each product are initialized (lines 4-9). The slack weight is the remaining weight left on the truck, which is initialized to 45,000 and the slack cube is the remaining pallets on the trucks, which is initialized to 60. The number of pallets for each product just refers to the number of pallets for that product that will be loaded onto an individual truck. Next the number of pallets of the heavy product to be loaded on each container is calculated based on the demand of the item and the theoretical minimum number of containers (line 10-12). When the demand is a multiple of the minimum number of trailers needed, T, then this process loads the entire demand for the product. If the demand is not a multiple T, then there are at most T-1 remaining pallets, which are loaded one at a time until the entire demand for the heavy product is assigned to trailers (lines 14-20). Once the heavy product is completely loaded the light product is loaded onto the containers based on the combination of demand of the item, the slack weight and the slack cube of the container are updated (lines 25-32). Throughout the entire heuristic the number of pallets loaded of each product is tracked to ensure the demand for each product is met. This value is initialized in lines 2 and 3 and updated in lines 13, 18, and 30. If assigned number of pallets for the light product is less than the demand and no more space is available in the current set of containers, a new container is added and the heuristic restarts (lines 33-36). The heuristic is formally given in Figure 2 below.

Two-Product Loading Heuristic

1. T ← max{ $\left\lceil \frac{d_H w_H + d_L w_L}{W} \right\rceil$, $\left\lceil \frac{d_H + d_L}{V} \right\rceil$ }
2. a_H ← 0
3. a_L ← 0
4. FOR i = 1…T
5. $n_H[i]$ ← 0
6. $n_L[i]$ ← 0
7. $s_W[i]$ ← W
8. $s_V[i]$ ← V
9. END FOR
10. FOR i = 1…T
11. $n_H[i]$ ← $\left\lfloor \frac{d_H}{T} \right\rfloor$
12. END FOR
13. a_H ← T· $n_H[1]$
14. IF $a_H < d_H$
15.   Value ← $min\left\{1, \frac{s_W[1]}{w_H}\right\}$
16.   FOR i = 1… (d_H - a_H)
17. $n_H[i]$ ← $n_H[i]$ + Value
18. a_H ← a_H + Value
19.   END FOR
20. END IF
21. FOR i=1..T
22. $s_W[i]$ ← $s_W[i]$ - $w_H \cdot n_H[i]$
23. $s_V[i]$ ← $s_V[i]$ - $n_H[i]$
24. END FOR
25. FOR i = 1… T
26.   IF $a_L < d_L$
27. $n_L[i] \leftarrow min\left\{\left\lfloor \frac{s_W[i]}{w_L} \right\rfloor, s_V[i], d_L - a_L\right\}$
28. $s_W[i] \leftarrow s_W[i] - w_L \cdot n_L[i]$
29. $s_V[i] \leftarrow s_V[i] - n_L[i]$
30. a_L ← a_L + $n_L[i]$
31.   END IF
32. END FOR
33. IF $a_L < d_L$
34.   T ← T+1
35.   RETURN to line 2
36. END IF

Figure 2: Heuristic to Load Two Products

6. Two Product Computational Experiments

6.1 Data Generation

To assess how the heuristic performs, computational experiments were performed to vary both the total demand for the products and the proportion of heavy and light products shipped. For this project, it is assumed that on the container there are 60 pallet positions and maximum allowed weight of 45,000 pounds on each container. To have a fully utilized container with respect to weight and cube, the average weight per pallet would need to be 750 pounds (i.e., 45,000/60). Knowing this, a light product is defined as a product that weighs less than 750 pounds. Trailers loaded only with light products would fully utilize the container's spatial capacity while underutilizing its weight capacity. A heavy product is then defined as a product weighing more than 750 pounds per pallet, and would fully utilize a container's weight capacity without using its full spatial capacity.

The total demand for the products refers to the combined total demand of the products; values for the total demand considered are shown in Table 2. The number of heavy and light pallets for each demand level will be based on the demand distributions. The demand distributions indicate the proportion of freight that is light and heavy, respectively. For example, if the total demand level is set at 100 pallets and the demand needs to be equally distributed between the products then there will be approximately 50 pallets that are considered light and approximately 50 pallets that are considered to be heavy. Note that the values given in Table 3 are approximate; this will be discussed further later in this section along with additional details about the data generated to test this heuristic.

Table 2: Total Demand Levels to be Examined

Total Demand Levels (pallets)
50
100
150
300
500
750
1,000
2,000
5,000
10,000

For each of the total demand levels given in Table 2, we considered several demand distributions. These are summarized in Table 3 below.

Table 3: Demand Distributions to be Examined

Demand Distributions		
Cases	Light Product	Heavy Product
Even	45%-55%	45%-55%
Skewed Light	70%-80%	20%-30%
Skewed Heavy	20%-30%	70%-80%

The heuristic was then used to solve the instances generated for each demand level-demand distribution combination. To show the quality of the heuristic, 50 instances were taken from each combination and the optimal solution was found for all of the instances that were not loaded in the theoretical minimum. For most of the instances, our heuristic obtained the optimal solution.

The total demand levels were manually input into the system. The weight for each of the products was randomly generated. Recall that the 750 pounds per pallet cutoff is based on the weight of a pallet that would fully utilize both a container's weight and spatial capacity. The weight of light products was uniformly distributed between 150 and 750 pounds per pallet; the weight of heavy products was uniformly distributed between 751 and 2200 pounds per pallet. The lower and upper bounds were based on data provided from a major retailer and were used for data generation.

The three demand distributions shown in Table 3 were considered: even, skewed heavy and skewed light. In the even case, the total demand is distributed approximately evenly for both products. For example if the total demand level was set at 50 pallets, then the demand for the light product would be approximately 25 pallets and the demand for the heavy product would be approximately 25 pallets. To model variation, the demand of each product will be between 45% and 55% of the total demand. Note that because the demand of heavy and light products are generated independently, the total demand does not always add up to the demand levels given in Table 2; this is intended to represent slight randomness in the demand for products. These three cases will show how the how the composition of the total demand (i.e., heavy-light distribution) may influence the effects of mixing.

For each demand distribution, random numbers in the intervals given above was found. For example, in the skewed light case, a number between 0.7 and 0.8 was randomly generated and a second random number between 0.2 and 0.3 was independently generated. Then each of the numbers is then multiplied by the total demand level of interest. This gives the demand for each of the products, the larger demand in this case belonging to the light product. This calculated value for the demand alternates between being round up or down to ensure variation among the instances.

For each demand level-demand distribution combination, 500 instances were created. This means that 30 total different combinations were examined with a total of 15,000 instances. The data was generated in Visual Basic for Applications in Excel and the heuristic implementation was completed in C++ in Visual Studio 2008 on a computer with an Intel® Core™ i7-2620M CPU @ 2.70Ghz processor.

6.2 Results

Two methods were used to determine the effectiveness of the heuristic. First, 50 instances from each demand level and demand distribution combination, 1500 overall were solved. The instances where the theoretical minimum was not obtained by the heuristic were solved to optimality using the mathematical model. The information used in the analysis that can be seen in the Table 4 below includes the average time it took the heuristic to find a solution, the average time it took the model took to find the optimal solutions, and the number of cases that were sent to the model to be solved, which is the number of cases where the theoretical minimum was not met. Note that the average times recorded in first columns only include the cases solved to optimality (i.e. for the even demand distributions the average time is for the 76 cases that were solved to optimality). The table also includes information about how many more trucks are used using the heuristic solution versus the optimal solution (% more trucks used with heuristic). The final piece of information given is the percentage of instances that were optimal (% Optimal) in each demand distribution.

Table 4: Optimality Results

Demand Distribution	Avg Time to Find Heuristic Solution (min)	Avg Time for Model to find Optimal (min)	# Cases solved to optimality	% more trucks used with heuristic	% Optimal
Even	0.0005	127.865	76	0.289%	90.6%
Skewed Light	0.0003	28.014	34	0.155%	95.6%
Skewed Heavy	0.0009	21.852	95	0.187%	89.4%

This analysis was performed to assess the quality of the heuristic. As can be seen in Table 4, the heuristic was able to obtain high-quality solutions for each of the demand distributions very quickly. In the cases that were the heuristic was not able to obtain the optimal solutions, there was not a large increase in the number of more trucks used. These results show that this heuristic is very effective.

The second method used to assess the heuristic compared the results obtained from the computational experiments against those obtained from the mathematical model presented in Section 4. To make a proper comparison, the time the heuristic takes to find a solution was compared to the time the mathematical model takes to find the same solution, for each instance that was not loaded to the theoretical minimum number of trucks. This analysis shows the speed of the heuristic relative to that of the mathematical model. The mathematical model was implemented in C++ linked to CPLEX 12.2 libraries. The initial experimentation revealed that it could take a very long time to solve certain instances. As a result we implemented a maximum time limit of 60 minutes for each instance to reach the heuristic solution. The results of these experiments can be seen in the table below.

Table 5: Timing Results

Demand Distribution	Avg Time to Find Heuristic Solution (Min)	Avg Time for Model to find our Solution (Min)	# Cases	# Cases that took 60 minutes	Avg Optimality Gap	# Failed
Even	0.0398	2.439	751	14	2.150%	19
Skewed Light	0.0005	8.958	332	11	2.511%	3
Skewed Heavy	0.0013	2.951	929	22	1.644%	11

The first column of Table 5 is the average time it takes the heuristic to solve an instance, which is much less than a minute. The second column shows the time it takes the mathematical model to find the heuristic solution, which in all cases is much longer to solve than the heuristic. Since the model was only solved for cases that did not obtain the theoretical minimum, this number was also recorded, which is in column 3. Column 4 shows the number of cases that took 60 minutes to find the heuristic solution and the average optimality gap for the instances is seen in column 5. The final column (# failed) refers the number of instances that could not find the heuristic solution in the 60 minute time limit. Overall this table provides support to the claim that the heuristic solves much faster than the mathematical model.

Recall that understanding how load mixing enables companies to more efficiently utilize trailers was a main interest in this research. To assess this, the number of trucks required if each product were shipped independently (i.e., containers contain only a single type of product) was compared to the number required using load mixing. Then the percentage difference (i.e., one minus the total amount of trucks used by the heuristic divided by the total number of trucks used if the products were shipped separately) was computed.

The results from the experiments can be summarized in the graph below.

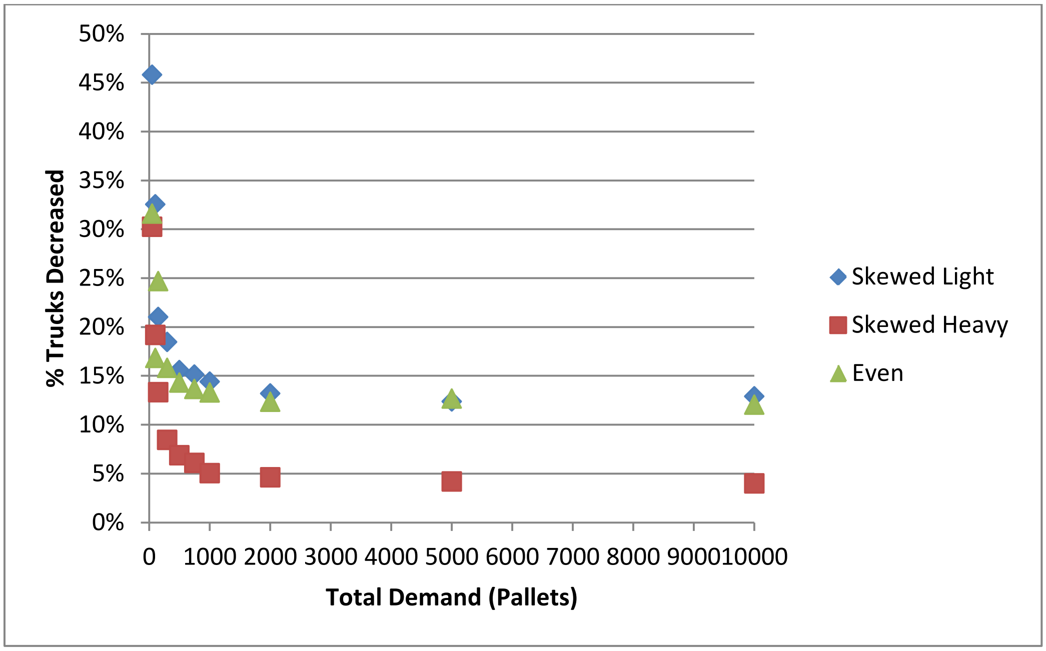

Figure 3: Summary Graph for Two Product Heuristic Experiments

Many conclusions can be drawn from Figure 3. First, it is observed that as the total demand level increases, the percentage of trucks decreased asymptotically approaches a non-zero limit beyond which further improvements are not possible. Though currently it is not fully understood how this limit can be obtained, this result will be examined further in future work. Second, it can be seen that mixing is most effective when the demand is highly composed of the lighter product and least effective when the demand is highly composed of the heavy product. This demonstrates that the demand distributions have a significant impact on the amount of savings.

In the case composed mostly of the light products, load mixing is able to decrease the number of containers used by approximately 12%. With the total demand split evenly between heavy and light products, load mixing is able to decrease the number of containers used by approximately 12%. In the case with mostly heavy pallets, the percent of trucks decreased is only around 4%. This indicates that mixing does not have a large effect when the demand is mostly the heavy product, but a significant effect when the demand is either mostly light or split evenly between the products. Intuitively, these results make sense. In the case with mostly light products, the number of containers needed is based on the total demand. There are a few heavy pallets on each container followed by many light pallets, which better utilize the weight and cube of the container than if the products were shipped separately. As a result, fewer containers are required. The same reasoning explains the reduced number of trucks required in the even case. Since the containers utilize both weight and cube instead of just one or the other, the number of trucks needed is less. In the case with mostly heavy products, the number of containers is based on the total weight. In this case, the containers will still use more utilize the weight of the container and underutilize the cube, so the number of trucks used is not as reduced to the same extent as in the other two cases.

Note also the impact of demand level on the effectiveness of load mixing. In particular, it is observed that load mixing is most effective when the demand level is low. This can be explained by the low number of containers needed in general when the demand is low. For example, in the case of the total demand level of 50, the total number of trucks required when shipping the products separately is two, but with mixing only one truck is needed. Therefore, the percent of trucks decreased is around 50%. This drives the average percentage for that case up, but as can be seen this quickly decreases to the limit value. An anomaly in the graph for the “Even” case can be seen. It actually decreases substantially for a total demand level of 100 but increases again for 150. Therefore, instances with a total demand level of 75 and 125 were also run for this demand distribution to give insight to why the graph behaved this way. The only explanation for this behavior is that at demand levels of 75 or 100 the total number of trucks needed would be at least 2 if shipped separately and shipped with mixing. Since many of the cases will have the same number of trucks using the heuristic as with shipping separately there is not as big of a percentage decrease, but as the total demand goes to 150 where at least 4 trucks will be needed to ship separately and only 3 with the heuristic then the percent decrease is higher. From this point on the curve behaves like the curves from the other demand distributions.

This research shows that for two products, load mixing has the most effect when the total demand is mostly composed of light pallets or evenly mixed between both the products. Mixing is also more effective with lower total levels of demand, but as the total demand increases the percent of trucks decreased approaches an asymptotic limit.

7. Generalization Heuristic Approach

The heuristic described in Section 5 focused on loading only two products. This section will extend the heuristic previously presented to now accommodate more than two products. The same assumptions about weight and spatial capacity of the truck will be used here as well. An additional assumption made in this case is that the products will be ordered from heavy to light, which will be known before the heuristic begins.

This heuristic has three main parts. Two of the parts account for loading the heaviest and lightest items and are the same as in the two-product heuristic. The additional part loads the second heaviest to the second lightest items. The generalized heuristic can be seen in Figure 5 and Figure 6 below.

The notation for the heuristic is similar to the notation from Section 5 can be seen in Figure 4 below.

Notation

N types of freight, f
W = Total weight capacity of trailer
V = Total cube capacity of trailer
w_f = Weight per pallet of the freight type *f* (in lbs.), *f* = 1, …, N
d_f = Demand of freight type *f* (in pallets), *f* = 1, …, N
T = number of trailers required to transport freight
$n_f[i]$ = number of pallets of freight type *f* assigned to truck *i* (in pallets), *i* = 1…T, *f* = 1, …, N
a_f = number of assigned pallets of freight type *f*, *f* = 1, …, N
$s_W[i]$ = weight slack of trailer *i* (in lbs.), *i* = 1…T
$s_V[i]$ = volume slack of trailer *i* (in lbs.), *i* = 1…T

Figure 4: Notation for Generalized Heuristic

Generalized Loading Heuristic

$T \leftarrow \max\{\left\lceil \frac{\sum_{f=1}^{N} d_f * w_f}{W} \right\rceil, \left\lceil \frac{\sum_{f=1}^{N} d_f}{V} \right\rceil\}$

FOR $i = 1 \ldots T$

FOR $f = 1 \ldots N$

$n_f[i] \leftarrow 0$

$s_W[i] \leftarrow W$

$s_v[i] \leftarrow V$

END FOR

END FOR

FOR $f = 1$

FOR $i=1 \ldots T$

$n_f[i] \leftarrow \left\lfloor \frac{d_f}{T} \right\rfloor$

$s_w[i] \leftarrow s_w[i] - w_f \cdot n_f[i]$

$s_V[i] \leftarrow s_V[i] - n_f[i]$

$a_f \leftarrow a_f + n_f[i]$

END FOR

IF $a_f < d_f$ THEN

FOR $i=1 \ldots d_f - a_f$

$n_f[i] \leftarrow n_f[i] + 1$

$s_w[i] \leftarrow s_w[i] - w_f$

$s_V[i] \leftarrow s_V[i] - 1$

$a_f \leftarrow a_f + 1$

END FOR

END IF

END FOR

FOR $f = 2 \ldots N-1$

$j \leftarrow 1$

FOR $i=j \ldots T$

$n_f[i] \leftarrow MIN\left\{\left\lfloor \frac{d_f}{T} \right\rfloor, \left\lfloor \frac{s_W[i]}{w_f} \right\rfloor, s_v[i]\right\}$

$s_w[i] \leftarrow s_w[i] - w_f \cdot n_f[i]$

$s_V[i] \leftarrow s_V[i] - n_f[i]$

$a_f \leftarrow a_f + n_f[i]$

END FOR

Figure 5: Generalized Heuristic Part 1

```
    FOR i=1…j
        n_f[i] ← MIN{ ⌊d_f / T⌋, ⌊s_W[i] / w_f⌋, s_v[i] }
        s_w[i] ← s_w[i] – w_f·n_f[i]
        s_V[i] ← s_V[i] – n_f[i]
        a_f ← a_f + n_f[i]
    END FOR
    j ← j+1
    IF j > T THEN
        j ← 1
    END IF
    IF a_f < d_f THEN
        WHILE a_f < d_f
            n_f[i] ← n_f[i]+MIN{ 1, ⌊s_W[i] / w_f⌋, s_v[i] }
            s_w[i] ← s_w[i] – w_f · MIN{ 1, ⌊s_W[i] / w_f⌋, s_v[i] }
            s_V[i] ← s_V[i] – MIN{ 1, ⌊s_W[i] / w_f⌋, s_v[i] }
            a_f ← a_f + MIN{ 1, ⌊s_W[i] / w_f⌋, s_v[i] }
        END WHILE
    END IF
END FOR
FOR f = N
    FOR i = 1… T
        IF a_f < d_f
            n_f[i] ← MIN{ ⌊s_W[i] / w_f⌋, s_v[i], d_f − a_f }
            s_w[i] ← s_w[i] – w_f·n_f[i]
            s_V[i] ← s_V[i] – n_f[i]
            a_f ← a_f + n_f[i]
        END IF
    END FOR
END FOR
IF a_f < d_f then
    T ← T+1
    RETURN TO LINE 2
END IF
```

Figure 6: Generalized Heuristic Part 2

As in the two – product heuristic line 1 calculates the minimum number of trucks needed based on the total weight and demand and lines 2 - 8 initialize the number of pallet per freight type, slack weight, and slack cube for each truck. Lines 9 - 24 load the heaviest product on to the trucks similarly to how the heavy product is loaded in the two-product heuristic. Lines 25 - 51 load all of the products from the second heaviest to the second lightest. Line 27 starts the first loop through the containers, note the starting point for this loop changes with each product. Line 28 calculates the number of pallets to be loaded on to each container based on the minimum of the slack weight and slack cube of the container and the demand of the product. Lines 29 -31 update the slack weight of the container, the slack cube of the container, and the number of assigned pallets for that product respectively. Lines 33 -38 loops through the containers that were not accessed in the loop from line 27, performing the same task as lines 28-31. Then lines 43-50 ensure that the complete demand for that product is loaded. As the loop progresses, the starting point for the first loop is increased (line 39). This means that as the product changes the container that is the first to be loaded changes. All of the containers still have items added, which is ensured by the loop in lines 33-38. Finally, the lightest product is loaded in lines 52-61. This again is similar to the two-product heuristic presented in section 5. The lightest product is loaded onto each truck based on the slack weight of the truck, the slack cube of the truck, and the remaining demand of the product. The structure of this heuristic ensures that in the case of two products it is the same as the two –product heuristic presented previously.

7.1 Generalization Computational Experiments

7.1.1 Data Generation

To assess the performance of the heuristic presented in Section 7, experiments were performed by varying many different factors. These factors include the number of products, the total demand, and how the product demand is distributed (i.e., proportion of different product weights). In the two products heuristic there were definitions for heavy and light products. To test the generalized heuristic the idea of a “medium” product is introduced. The new definitions can be seen in Table 6 below.

Table 6: Definitions of different weight classes

Weight Definitions		
Light	Medium	Heavy
150-599 lbs	600-899 lbs	900-2200 lbs

These weights were randomly generated in the ranges in Table 6. The total demand levels that were examined can be seen in Table 7 below.

Table 7: Total Demand Levels Examined

Total Demand Level
50
100
150

These demand levels were used as inputs along with the number of products examined, which can be seen in the table below.

Table 8: Number of Products Examined

Number of Products
3
4
5
6
7
8
9
10

Varying the number of products in this way gives insight to how the heuristic performs as the number of products increases. The last factor examined was how the demand was distributed.

Table 9: Demand Distribution Cases

Demand Distributions			
Cases	Light Product	Medium Product	Heavy Product
Half Light/Half Heavy	45%-55%	0	45%-55%
Skewed Heavy	20%-30%	0	70%-80%
Skewed Light	70%-80%	0	20%-30%
All Equal	27%-38%	28%-39%	27%-38%

As can be seen in Table 9, four different cases were examined. The half-light/half-heavy case only contained items that had weights between 150-600 pounds and 900-2200 pounds ant the demand was equally distributed between heavy and light items. The skewed heavy and skewed light cases only considered very light or very heavy products, with the demand consisting of more heavy items or more light items respectively. Finally, the all equal case distributes the demand relatively even over all the products. To implement this design in code values were randomly generated within the ranges given in the same manner described in Section 7.1. The ranges help model variability that may occur in practice.

All of the combinations of number of products, total demand, and demand distributions were examined, for a total of 96 different combinations. For each of these combinations, 500 instances were generated and tested using the heuristic.

7.2 Results

To assess how well the generalized heuristic performs, we compared the number of trucks used in the solutions generated by our heuristic to the theoretical minimum number of trucks. Note that when our heuristic can assign the demand to the theoretical minimum number of trucks, we are assured we have obtained an optimal solution for that instance. However recall that in many situations, the number of trucks required in the optimal solution exceeds the theoretical minimum – that is to say, that even when our solutions require more trucks than the theoretical minimum, our solutions may or may not be optimal. In the tables below we report how many instances are loaded into the theoretical minimum number of trucks (% Theo. Min.) . Additionally, for the cases where more trucks than the theoretical minimum were required, the percentage of extra trucks the solution uses than if everything was loaded in the theoretical minimum is reported (% More Trucks Used) are reported. These results are broken down by demand level and the demand distributions discussed in Section 7.1.1, which shows how the heuristic performs at different demand levels.

Table 10: Results for Half Light/Half Heavy Case

Half Light/Half Heavy		
Demand Level	% Theo Min	% More Trucks Used
50	99.550%	0.161%
100	98.975%	0.379%
150	98.675%	0.483%

Table 11: Results for Skewed Light Case

Skewed Light		
Demand Level	% Theo Min	% More Trucks Used
50	100.000%	0.000%
100	99.975%	0.012%
150	99.750%	0.118%

Table 12: Results for Skewed Heavy Case

Skewed Heavy		
Demand Level	% Theo Min	% More Trucks Used
50	99.425%	0.178%
100	99.225%	0.234%
150	99.400%	0.176%

Table 13: Results for All Equal Case

All Equal		
Demand Level	% Theo Min	% More Trucks Used
50	99.675%	0.121%
100	99.050%	0.354%
150	99.100%	0.344%

As can be seen in Table 10, Table 11, Table 12, and Table 13 for all demand distributions and demand levels, our heuristic was able to obtain solutions that loaded the freight into the theoretical minimum number of trucks for the vast majority of instances. Also note that, even when we were not able to obtain solutions that loaded the freight into the theoretical minimum number of trucks, very few additional trucks were required to move the freight. For all of the demand distributions overall all the products in the demand level (i.e., 4000 examples), less than a full percent of trucks more were used when the theoretical minimum was not met. This shows that even when the theoretical minimum was not met, our heuristic was still able to obtain high quality solutions.

Analysis was also completed to assess how the number of products affects quality of the solutions found. This analysis can be seen in the graphs below.

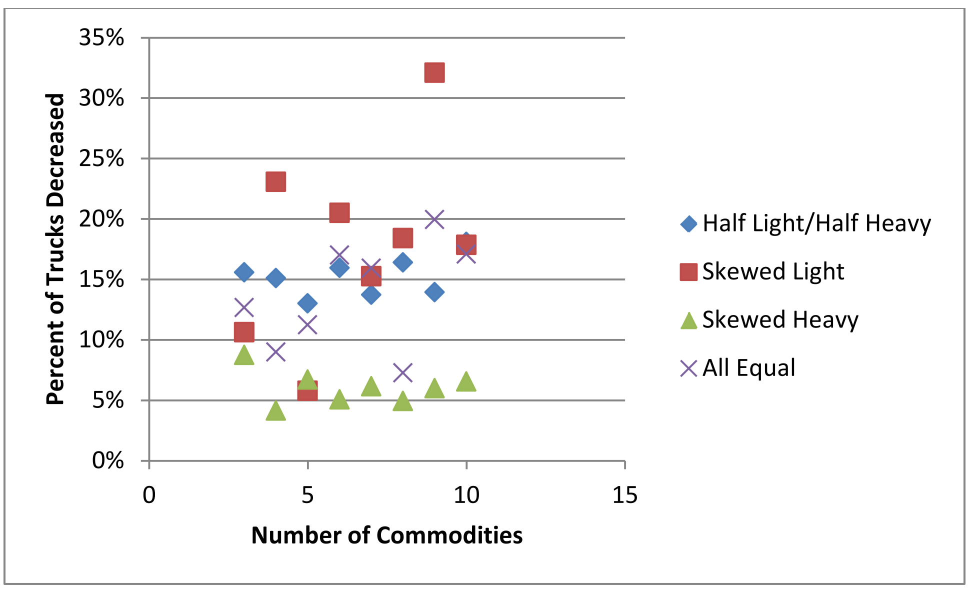

Figure 7: Graph of % Known Optimal vs. Number of Products

Figure 7 shows that for this set of number of products that we considered our heuristic is able to find the known optimal solution in the vast majority of the cases.

To assess the actual effect of the load mixing, a method of counting the number of trucks used when the products are shipped separately needed to be developed. This is not as straightforward as in the two product case. The method developed for this research first calculates the number of trucks each individual item needs to ship all of its demand. Then the number of trucks is added up over all the products and the ceiling is taken to get the total number. This process can be expressed by the equation $\# \text{ trucks} = \left\lceil \sum_{i=1}^{P} \max\left\{\frac{w_i * d_i}{W}, \frac{d_i}{V}\right\} \right\rceil$. For each case, the number of trucks that may be used if the items are shipped without a mixing system was calculated and compared to the number of trucks used by the heuristic. The results can be seen in Figure 8 below.

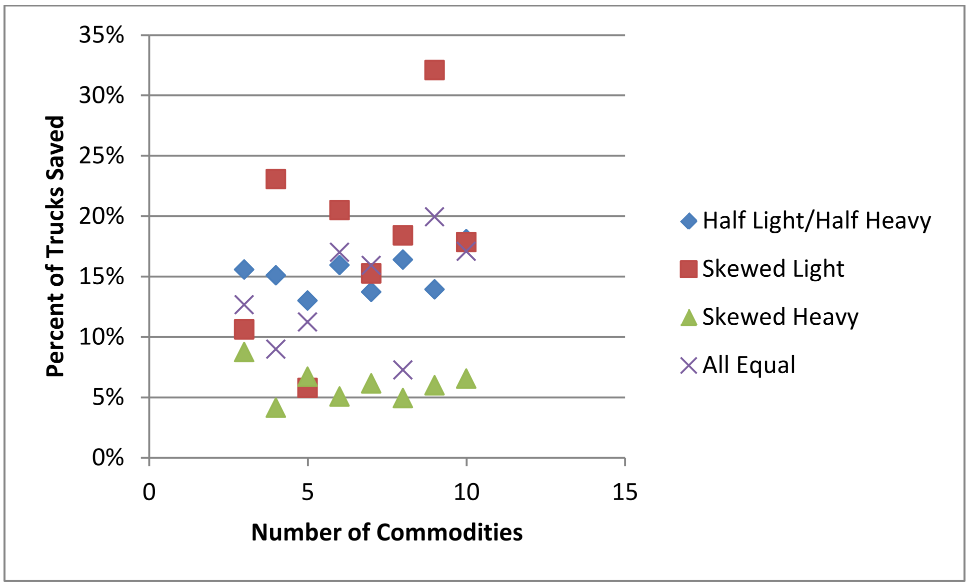

Figure 8: Savings from Generalized Heuristic below shows the graph of the change in percentage of more trucks used in the cases where the theoretical minimum is not met. These numbers are low showing that even when the theoretical minimum is not met, very few additional trucks are used overall. These results show that as the number of product increases the quality of results only slightly varies.

To assess the actual effect of the load mixing, a method of counting the number of trucks used when the products are shipped separately needed to be developed. This is not as straightforward as in the two product case. The method developed for this research first calculates the number of trucks each individual item needs to ship all of its demand. Then the number of trucks is added up over all the products and the ceiling is taken to get the total number. This process can be expressed by the equation $\# \text{ trucks} = \left\lceil \sum_{i=1}^{P} \max\left\{\frac{w_i * d_i}{W}, \frac{d_i}{V}\right\}\right\rceil$. For each case, the number of trucks that may be used if the items are shipped without a mixing system was calculated and compared to the number of trucks used by the heuristic. The results can be seen in Figure 8 below.

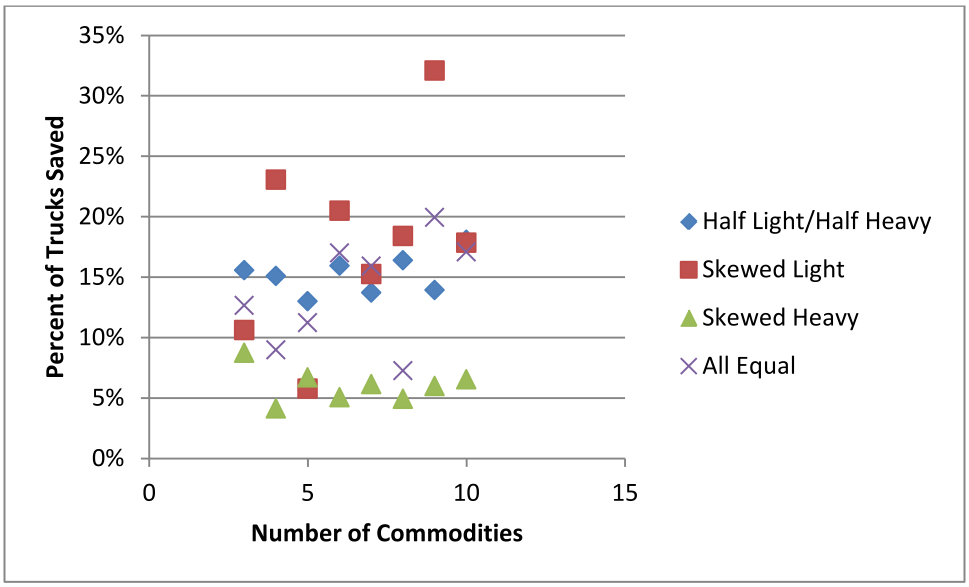

Figure 8: Savings from Generalized Heuristic

From the graph in Figure 8, substantial savings can be seen for the skewed light, all equal, and half-light/half heavy cases. The lowest savings were seen in the skewed heavy case similarly to what was seen in the two product case.

8. Conclusions and Future Work

8.1 Conclusion

This research makes several important contributions to the literature. The heuristic that was created is effective in that it usually obtains the optimal solutions to the instances we consider quickly. Even when the heuristic is not able to obtain the optimal solutions, very few additional trucks are required. Experiments were conducted to determine when the load mixing made the biggest impact.

The results show that load mixing is most effect when the total demand to be shipped is composed mostly of light freight or evenly divided among heavy and light freight. They also show that mixing is most effective for lower total demand to be shipped. As the total demand increases, the number of trucks saved approaches a limit. The results presented show that for ten or less products the heuristic performs well as indicated by our ability to obtain optimal solutions.

8.2 Future Work

There are a number of natural extensions of this work. First, higher number of products and higher demand levels should be examined using the generalized heuristic. This would lead to additional information such as how higher demand levels affect the heuristic solution. Additionally, a method for counting the trucks that are used to ship the products separately in general case should be created and then used to find the true effectiveness of load mixing in the many product case. Second, the heuristic presented can be modified to incorporate real world constraints such as stackability constraints and freight due dates. This would ensure that product is stacked in a way that nothing is damaged and that all items are delivered on time, as well as lend insight as to how these constraints impact the solutions obtained. Additionally, the scope of the problem can be expanded to consider product routing considerations. For the routing piece, consider there are many suppliers with different items located stored or produced in different locations that need to get their items to the same locations (e.g., common retailer DCs). The containers should be loaded not only based on weight and demand as before, but also with location of supplier. This piece will assess the tradeoff between the savings incurred by using load mixing verses the increased transportation cost of having to make many stops before reaching its final destination.

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