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Abstract

Recently, the initial boundary conditions for proforce waves with a substantial current behind the shock front have been derived. Computer solutions of the Electron Fluid Dynamical equations meet the expected boundary conditions at the end of the sheath region. This paper will compare the wave structure for proforce waves with and without current behind the shock front.

Introduction

Electrical Breakdown waves in which electric field force on electrons is in the direction of wave propagation are referred to as proforce waves. Proforce current-bearing waves are proforce waves with a substantial current behind the shock front. Proforce current-bearing waves describe the natural phenomena "stepped leader" in lightning.

Breakdown waves consist of two distinct regions. Immediately following the front is the thin Debye layer which will be referred to as the sheath region. In this region the net electric field decreases to a nominal value and collisions with neutral particles cause the electrons to come to rest relative to heavy particles. Following the sheath region is a thicker region referred to as the quasi-neutral region. In this region, by further ionizing neutral particles, the electron gas cools. This paper is concerned with solutions of the Electron Fluid Dynamical equations within the sheath region.

A set of Electron Fluid Dynamical (EFD) equations for proforce waves has previously been formulated. Paxton and Fowler (1962) introduced a fluid approach to breakdown waves using a three-fluid, hydrodynamical model that is applied to a quasi-steady state three-component (electrons, neutral particles, and ions) system. Their set of equations consists of equations of conservation of mass,momentum, and energy. In their model they assumed that the heavy particles inside the wave only have a slight kinetic energy change during their interaction with the electron shock wave. The electron gas partial pressure was assumed to be much greater than that of other species, and the heat conduction and energy loss by electrons due to inelastic collisions were considered negligible. Later, Shelton and Fowler (1968) introduced modifications to Paxton and Fowler's equations. The use of Poisson's equation along with the introduction of dimensionless variables in the set of equations, and derivation of the initial boundary conditions allowed an approximate solution to the set of EFD equations.

For successful numerical integration of the set of EFD equations, the following major modifications were made by Fowler et al. (1984). First, they introduced the heat conduction term, which was considered negligible by Shelton and Fowler. Second, they allowed for temperature derivative discontinuity at the shock front and derived a new set of boundary conditions for variables such as electron temperature and velocity. Finally, they used an expression derived by Fowler (1983) to calculate the ionization rate throughout the zone where the electric field is present. Shelton and Fowler (1968) considered the ionization rate to be constant throughout the sheath region.

Model

The model introduced by Paxton and Fowler (1962), and later completed by Fowler et al. (1984), is a one-dimensional, steady profile, constant velocity Electron Fluid Dynamical wave. The wave propagates through a neutral gas from an electrode with a potential to the ground electrode regardless of the polarity of the applied potential. The set of EFD equations are equations of conservation of mass, momentum, and energy, coupled with Poisson's equation:

$$\frac{d(nv)}{dx} = \beta n, \tag{1}$$

$$\frac{d}{dx}\{nmv(v-V) + nkT_e\} = -enE - KmnV(v-V), \tag{2}$$

$$\frac{d}{dx}\{ nmv(v-V)^2 + nkT_e(5v-2V) + 2e\o nv + \varepsilon_o VE^2 - \frac{5nk^2T_e}{mk}\frac{dT_e}{dx} \} = -3(m/M)nKkT_e - (m/M)\, nmK(v-V)^2, \tag{3}$$

$$\frac{dE}{dx} = \frac{e}{\varepsilon o}(N_i - n). \tag{4}$$

The variables are electron concentration n, ion number density N_i, electric field E, electron velocity v, electron mass m, electron temperature T_e, and position in the wave profile x. ø is the ionization potential of the gas; V is the wave velocity; M is the neutral particle mass; e is the electron charge. The dimensionless variables used are

$$\eta=\frac{E}{E_o},\ \nu=\frac{2e\phi}{\varepsilon_o E_o^2}n,\ \psi=\frac{v}{V},\ \theta=\frac{kT_e}{2e\phi},\ \xi=\frac{eE_o}{mV^2}x,\ \alpha=\frac{2e\phi}{mV^2},\ \kappa=\frac{mV}{eE_o}K,\ \mu=\frac{\beta}{K},\ \omega=\frac{2m}{M}$$

where η is the electric field strength, υ is the electron number density, ψ is the electron velocity, θ is the electron gas temperature, ξ is the position within the sheath, α and κ are wave parameters, μ is the ionization rate, K is the elastic collision frequency, β is the ionization frequency, and E_o is the electric field magnitude at the wave front. Introducing the dimensionless variables into equations 1-4, they reduce to:

$$\frac{d(\upsilon\psi)}{d\xi}=\kappa\mu\upsilon, \tag{5}$$

$$\frac{d}{d\xi}\left\{\upsilon\psi(\psi-1)+\alpha\upsilon\theta\right\}=-\upsilon\eta-\kappa\upsilon(\psi-1), \tag{6}$$

$$\frac{d}{d\xi}\left\{\upsilon\psi(\psi-1)^2+\alpha\upsilon\theta(5\psi-2)+\alpha\upsilon\psi+\alpha\eta^2-\frac{5\alpha^2\upsilon\theta}{\kappa}\frac{d\theta}{d\xi}\right\}=-\omega\kappa\upsilon\left\{3\alpha\theta+(\psi-1)^2\right\}, \tag{7}$$

$$\frac{d\eta}{d\xi}=\frac{\upsilon}{\alpha}(\psi-1). \tag{8}$$

The expression used to calculate the ionization rate, μ, is based on free trajectory theory that includes ionization from both random and directed electron motions

$$\mu=\mu_o\int_A^\infty \sigma_i x^2\,dx\int_B^\infty \frac{e^{-(x-u)^2}-e^{-(x+u)^2}}{u}du\, e^{-2Cu}, \tag{9}$$

where $A=\frac{1}{\sqrt{2\theta}}$, $B=\frac{(1-\psi)}{\sqrt{2\alpha\theta}}$, and $C=k\sqrt{2\alpha\theta}$.

Fowler et al. (1984) expanded the Momentum balance equation (6) and used other equations in the expanded form to solve for $\frac{d\psi}{d\xi}$. The singularity inherent in the set of equations, therefore, appears in the denominator of the equation

$$\frac{d\psi}{d\xi}=\frac{\kappa(1+\mu)(1-\psi)\psi-\kappa\mu\alpha\theta-\eta\psi-\alpha\psi\theta'}{\psi^2-\alpha\theta} \tag{10}$$

Analysis

With a current, I, behind the shock front, modifications must be made on the initial boundary conditions and Poisson's equation used by Fowler et al. (1984). According to Kirchoff's current law:

$$eN_iV_i-env=I, \tag{11}$$

where V_i is the ion velocity in the wave frame. Solving equation (11) for N_i and substituting it into equation (4) reduces the Poisson's equation to:

$$\frac{dE}{dx}=\frac{e}{\varepsilon_o}\left(\frac{I}{eV_i}+\frac{nv}{V_i}-n\right). \tag{12}$$

The change in ion velocity is negligible; therefore, V can be substituted for V_i. Substituting the dimensionless variables and introducing $\iota=\frac{I}{\varepsilon_o E_o K}$ into equation (12), it becomes

$$\frac{d\eta}{d\xi}=\frac{\upsilon}{\alpha}(\psi-1)+\kappa\iota. \tag{13}$$

In order to derive the initial boundary condition, θ_1, the global momentum equation

$$\frac{d}{dx}\left\{MNV^2+M_iN_iV^2+mnv^2nkT_e+(N+N_i)kT-\frac{\varepsilon_oE_o^2}{2}\right\}=0 \tag{14}$$

must be integrated, and the integration constant has to be evaluated using the values for the variables immediately ahead of the wave ($n_o=0$, $N_{io}=0$, $V=V_o$). Equation (14) then reduces to

$$m\left\{n_1v_1^2-\frac{I_1V_o}{e}\right\}+n_1kT_e=0, \tag{15}$$

where n_1, v_1, I_1, and V_o are the electron number density, electron velocity, current at the wave front, and wave velocity, respectively. By introducing the dimensionless variables into equation (15), the electron temperature at the wave front can be isolated as

$$\theta_1=\frac{\psi_1(1-\psi_1)}{\alpha}+\frac{\kappa}{\upsilon_1}\iota. \tag{16}$$

The major task in integrating the set of EFD equations is to pass through the singularity which presents itself in the denominator of equation 10. When ($\psi^2 - \alpha\theta$) approaches zero, $\frac{d\psi}{d\xi}$ approaches infinity, indicating the presence of a shock. Since there can be no shock inside the sheath region, the denominator and numerator, therefore, must both approach zero at the same time. This allows one to choose a starting value for ψ_1, for a given value of κ, α, and ν_1, by trial and error.

Keeping the values of the numerator and denominator at the singularity constant allows one to pass through the singularity. After passing through the singularity and completing the integration, if the values of ψ and η do not satisfy the acceptable conditions at the end of the sheath, new values of ν_1 must be considered. This process must be repeated until one reaches the acceptable condition at the end of the sheath ($\psi_2 = 1$).

Results

Uman and McLain (1970) derived expressions relating the stepped leader radiation field (electric field intensity or magnetic flux density) to the leader current. By measuring the radiation field from a distance, they were able to calculate the current by using the derived expressions. For the stepped leader, they calculated peak currents in the range of 800 to 5,000 amperes. These values correspond to a range of ι of between 0.004 and 0.1. We have attempted to integrate the set of equations for a broader range of currents.

The solutions for a fast moving wave ($\alpha = 0.01$) for current values of ι = 0.001, 0.01, and 0.1 are available now. $\alpha = 0.01$ represents a wave speed of 3×10^7 meters per second. Figure 1 is a graph of electron velocity, ψ, as a function of position, ξ, with appropriate initial electron velocity ψ_1, electron number density, ν_1, and wave constant, κ, for the above mentioned values of current. (+) $\iota = 0.001$, $\kappa = 1.18424$, $\nu_1 = 0.025$, ψ_1 0.32,(*) $\iota = 0.01$, $\kappa = 1.24194$, $\nu_1 = 0.0221$, $\psi_1 = 0.3275$ and(□) $\iota = 0.1$, $\kappa = 1.010453$, $\nu_1 = 0.025$, $\psi_1 = 0.32$.

Figure 2 is a graph of electric field η as a function of electron velocity ψ inside the sheath. The initial value of the electric field is equal to that of the applied field ($\eta_1 = 1$); the net electric field (applied plus space charge field), however, approaches a minimal value at the end of the sheath.

Figure 3 contrasts the electric field η as a function of electron velocity ψ for proforce waves with $\iota = 0.1$ and $\iota = 0$. The electric field at the end of the sheath for proforce current bearing waves is not zero.

Figure 1 shows that, in general, higher currents increase the sheath thickness. With high values of current behind the shock front, the singularity becomes very sharp, making the passage through the singularity very difficult. There seems to be a cut-off point for current values greater than $\iota = 0.25$. All attempts at integrating the set of equations for a current value of $\iota = 0.5$ failed to pass through the singularity. In order to pass through the singularity at $\iota = 0.25$, we had to resort to a higher order of approximation at the singularity. This was achieved by doubling the number of integration steps for which the numerator and denominator were held constant.

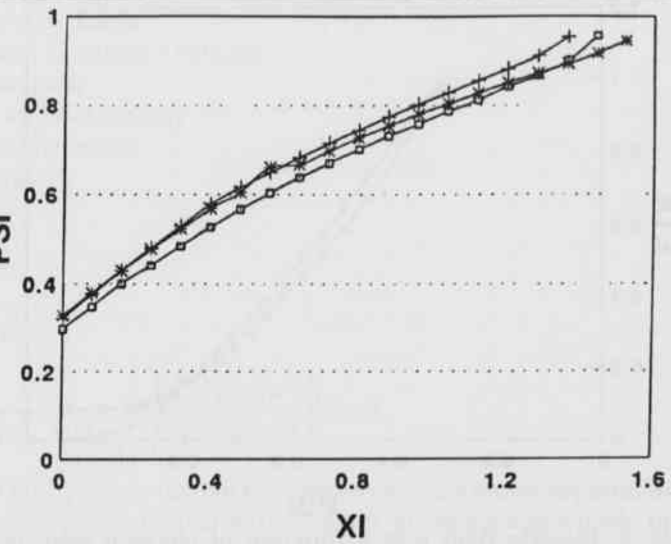

Fig.1. Electron velocity ψ as a function of position ξ inside the sheath.
+ $\iota = 0.001$ * $\iota = 0.01$ □ $\iota = 0.1$

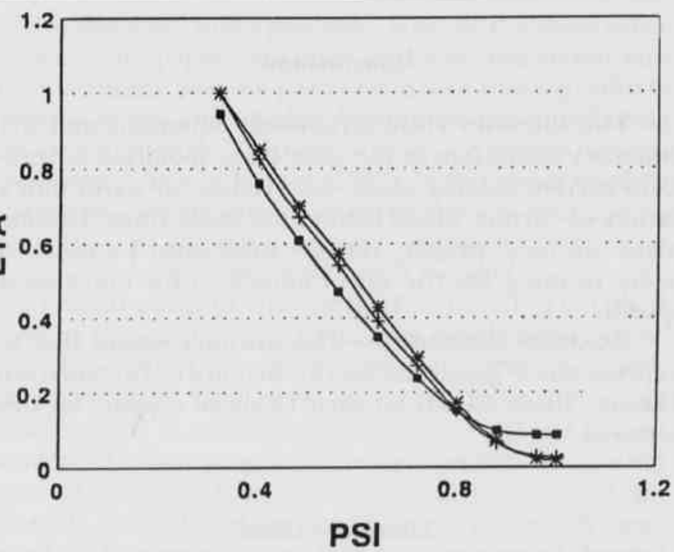

Fig. 2. Electric field η as a function of electron velocity ψ inside the sheath.
+ $\iota = 0.001$ * $\iota = 0.01$ □ $\iota = 0.1$

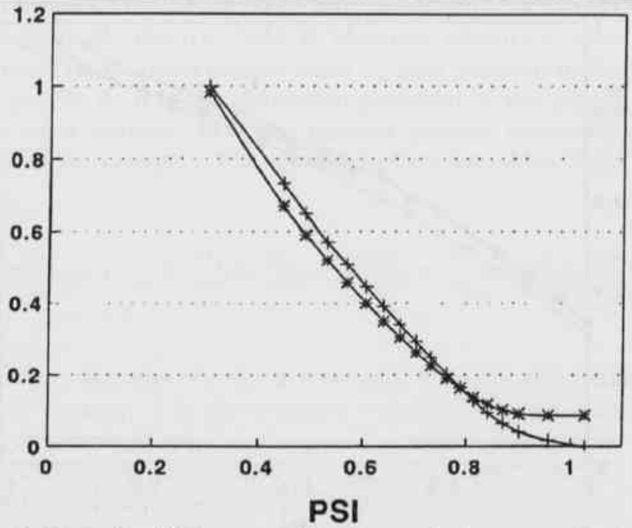

Fig. 3. Electric field η as a function of electron velocity ψ inside the sheath.
+ $\iota = 0$ * $\iota = 0.1$

Conclusions

The Electron Fluid Dynamical equations and the boundary conditions at the wave front, modified for proforce current bearing waves, yield results for waves with a variety of current values behind the shock front. To complete the wave profile, further work must be done in order to integrate the set of equations for lower wave speeds.

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