Retrieval of Atmospheric Turbidity Coefficient and Water Column Density from Solar Irradiance Data

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Abstract

Ground-based measurement of solar irradiance has been made using a spectroradiometer at wavelengths in \( \sigma \)-water absorption band. An algorithm is formulated to retrieve simultaneously the atmospheric water column density and Ångström's turbidity coefficient. The transmission models used to account for the contribution from diverse atmospheric absorbing and scattering elements are presented.

Introduction

In optical modeling for the development of solar energy devices, the knowledge of the spectral distribution of solar radiation reaching the earth's surface is essential. When passing through the atmosphere, solar radiation is attenuated due to scattering by aerosols and absorption by various atmospheric components, mainly water vapor and gases such as ozone, carbon dioxide and oxygen. Some of these attenuators are permanent while others vary as a function of time and geographical location. To characterize the atmosphere the minimum parameters required are the turbidity and the amount of water vapor.

Solar radiation is affected by atmospheric water vapor due to absorption bands in the solar spectrum. Also, with increasing humidity water vapor condenses onto aerosols, changing their size and refractive index and consequently their radiation scattering ability. The experimental determination of atmospheric water column density may be made by measurement of the direct normal irradiance in a water vapor absorption band in the solar spectrum, typically at 942 nm.

Atmospheric turbidity is a measure of the attenuation of the direct beam solar radiation by atmospheric aerosols. It is affected by the amount, kind and size distribution of aerosols as well as the amount and distribution of atmospheric water vapor. It is commonly represented as an index. The three indices in use are Linke's turbidity factor, \( T_l \), Ångström's turbidity coefficient, \( \beta \), and Schüepp's turbidity coefficient, \( B \).

Linke's turbidity factor (Linke, 1922) is a measure of the number of clean dry atmospheres that would be necessary to produce the attenuation of extraterrestrial radiation equivalent to that produced by the real atmosphere. \( T_l \), is the attenuation due to the presence of aerosols and gaseous water vapor spectrally integrated over the solar spectrum. Although a useful parameter, its drawback is its dependence on air mass. Attempts have been made to relate it to Ångström's turbidity coefficient (Grenier et al., 1994).

Ångström's turbidity coefficient is a dimensionless index that represents the amount of aerosols in the vertical column. It appears in Ångström's formula (Ångström, 1961; and Ångström, 1964), namely,

\[ \tau^\alpha = \beta \lambda^{-\alpha} \]  

(1)

where \( \beta \), the turbidity coefficient, is a constant parameter defining the amount of aerosol in the air; \( \tau \) is the optical thickness due to aerosol scattering; \( \lambda \) is the wavelength in micrometers; and \( \alpha \) is the wavelength exponent which is representative of the average aerosol size distribution. A number of techniques are currently employed for measuring \( \beta \). In one method, \( \beta \) is obtained by measuring the optical depth at two wavelengths where molecular absorption is negligible. This method yields simultaneously values for both \( \alpha \) and \( \beta \). In another method, \( \beta \) may be measured with a single wavelength instrument at \( \lambda = 1 \mu m \). Under this condition the wavelength effect of \( \alpha \) disappears as can be seen from equation (1). The representation of atmospheric turbidity by Ångström's turbidity coefficient is very common. Its determination has been the subject of a number of projects, such as Katz et al., (1982); Prodi et al., (1984); Ideriah, (1985); and Louche et al., (1987). It can vary from 0.0 for absolutely clean atmosphere to about 0.40 for very high aerosol amounts.

The third index, Schüepp's turbidity index (Schüepp, 1949) is obtained by the measurement of the direct spectral irradiance at 500 nm. It is related to Ångström's turbidity coefficient by

\[ B = \beta 2^{a \log e} \]  

(2)

Many atmospheric aerosols are hygroscopic and
change their size according to the humidity. Inspite of this fact, water vapor amount and turbidity are often measured separately. In this paper a method is described to determine the two parameters simultaneously from solar irradiance data measured at wavelengths in opt-water vapor absorption band extending approximately from 890 to 990 nm. Turbidity will be specified by Ångström's turbidity coefficient.

**Methods**

The direct solar irradiance on a surface element normal to the sun's rays at the ground may be written as

\[ I(\lambda) = I_0 (\lambda) T_R (\lambda) T_w (\lambda) T_g (\lambda) T_a (\lambda) \]  
(3)

where \( I_0 \) is the extraterrestrial spectral irradiance at mean solar distance; \( T_R, T_w, T_g, T_a \) are respectively the spectral transmittance functions of Raleigh (molecular) scattering, ozone absorption, absorption by uniformly mixed gases (CO\(_2\), O\(_2\), CH\(_4\), NO\(_2\), etc), water vapor absorption and aerosol attenuation.

An expression for molecular scattering transmittance based on Penndorf's data (Penndorf, 1957) and used in LOWTRAN 5 (Kneizys et al., 1980), is

\[ T_R = \exp \left[ -m / \left( \lambda^4 \left( 115.6406 - 1.355 / \lambda^2 \right) \right) \right] \]  
(4)

where \( \lambda \) is the wavelength in \( \mu \)m, \( m \) is the pressure-corrected air mass, \( m = m_r P / P_0 \), \( P_0 = 1013 \) mb, \( P \) is the surface pressure, and \( m_r \) is the relative air mass which may be computed as (Kasten, 1966):

\[ m_r = \left[ \cos (Z) + 0.15 \left( 93.885 - Z \right) ^ {1.253} \right] ^ {-1} \]  
(5)

where \( Z \) is the solar zenith angle.

The transmittance through ozone may be expressed as

\[ T_{O2} (\lambda) = \exp \left[ -k(\lambda) \cdot T_{mO2} \right] \]  
(6)

where \( k(\lambda) \) are ozone absorption coefficients (Vigroux, 1953), \( T_{mO2} \) is the air mass expression for ozone given by Paltridge and Platt (1976): 

\[ T_{mO2} = \frac{35.0}{(1224.0 \cos^2(Z) + 1)^{0.5}} \]  
(7)

The transmittance function for a path at air mass \( m \) through uniformly mixed gases is given by (Leckner, 1978):

\[ T_G = \exp \left[ \frac{-1.41 k_g m}{(1 + 118.3 k_g m)^{0.45}} \right] \]  
(8)

where \( k_g \) are the effective absorption coefficients for uniformly mixed gases.

Aerosol attenuation is adequately represented by Ångström's power law, thus the transmittance may be expressed as

\[ T_a (\lambda) = \exp \left[ -\beta \cdot \delta_m \right] \]  
(9)

where \( m, \alpha \) and \( \beta \) have been defined earlier.

In the treatment of water vapor absorption transmittance, several functional expressions are found in the literature, for example, Moskealenko (1969), Koepke and Quenzel (1978), Leckner (1978), Bird (1984) and Pierluissi et al., (1989). The spectral transmittance model of water vapor may be expressed as (Leckner, 1978):

\[ T_{w} = \exp \left[ -\frac{0.2385 k_{w} \mu m}{(1 + 20.07 k_{w} \mu m)^{0.45}} \right] \]  
(10)

where \( k_w \) is the effective absorption coefficient of water vapor for a given wavelength interval \( i \), and \( w \), the column density (gm/cm\(^2\)), is the integrated amount of water in a vertical column defined as:

\[ w = \int_0^{z_o} \rho_w(W) \, dz \]  
(11)

where \( \rho_w \) is the water vapor density (gm/cm\(^3\)), \( z \) is the altitude (cm) and \( z_o \) is the elevation above ground at which \( \rho_w \) may be considered to reach zero.

In opt-water absorption region absorption by ozone and uniformly mixed gases is known to be negligible, thus equation (3) may be written as

\[ I(\lambda) = I_0 (\lambda) \exp \left[ -m \tau_i \right] \]  
(12)

where \( \tau_i = \tau_R + \tau_W + \tau_a \) is the total optical depth; \( \tau_R, \tau_W, \) and \( \tau_a \) are the optical depths due to molecular scattering, water vapor and aerosols respectively.

In the following, a method is described that utilizes the total optical depth, \( \tau_i \), determined from radiometric measurements to obtain both atmospheric water column density, \( w \), and Ångström's turbidity coefficient, \( \beta \). Water column density, \( w \), is set up as the main variable while aerosol optical depth is calculated, for \( m = 1 \), using

\[ \tau_a = \tau_i - \tau_R - \tau_W (W) \]  
(13)

where, from equation (4)
\[
\tau_R = 1/ \left[ \lambda^4 \left( 115.6406 - 11.335 / \lambda^2 \right) \right] \\
\text{(14)}
\]

and from equation (10)
\[
\tau_w = 0.2385 k_w W / \left( 1 + 20.07 k_w W \right)^{0.45} \\
\text{(15)}
\]

For values of \( w \) varying from zero to a maximum, \( \tau_w \) calculated from (13)-(15) are least-squares fitted to (1). The value of \( \alpha \), the wavelength exponent, depends on the location of the experiment. It is found to be in the range \( 1.3 \pm 0.5 \) for real atmospheres whereas \( \alpha = 1.30 \) is representative of a standard aerosol size distribution.

The following are calculated at the convergence of the fit:
\[
\chi^2 (w) = \sum_{i}^{1} \sigma_i^2 \left[ \tau_i - \beta \lambda_i \alpha \right]^2 \\
\text{(16)}
\]

and
\[
\frac{d \chi^2 (w)}{dw} = -2 \sum_{i}^{1} \sigma_i^2 \left[ \tau_i - \beta \lambda_i \alpha \right] \frac{d \tau_w}{dw} \\
\text{(17)}
\]

where the summation is made over the wavelengths of measurement. When \( \chi^2 (w) \) attains a minimum and \( \frac{d \chi^2 (w)}{dw} \) is zero, its partial derivative with respect to \( w \) is zero, the corresponding values of \( w \) and \( \beta \) represent water column density and turbidity coefficient. The above minimization technique has been demonstrated in Tendeku (1994).

**Results and Discussion**

Measurements of the direct solar irradiance were made using a spectroradiometer (Tendeku, 1994) with five interference filters centered at 903, 941, 951, 974 and 1000 nm, respectively. The bandwidths of the filters are 10.5, 8.0, 8.2, 11.5 and 8.8 nm, respectively. Four of the filters are in the optical-water absorption band. Figure 1 shows a typical variation of atmospheric transmission in the optical-water absorption band.

The filter centered at 1000 nm plays two roles. First, it is used with the other filters to determine the total optical depth; second, it simulates a single wavelength measurement of the turbidity coefficient. From equation (1), at \( \lambda = 1 \mu m \), \( \tau_\lambda = \beta \). The measured total optical depth, \( \tau_o \), is the sum of the molecular scattering optical depth, \( \tau_R \), and the aerosol optical depth, \( \tau_w \), since the absorption by other elements is negligible. Thus, \( \beta = \tau_o - \tau_R \).

Figure 2 shows a typical variation of the direct solar irradiance with solar zenith angle. The irradiance increases from sunrise to a maximum at the sun's highest elevation. A plot of the natural logarithm of irradiance readings versus air mass is made for each of the five filters.

On a sunny day when the atmosphere is homogeneous, the plot produces straight lines whose slopes give the total optical depth of the atmosphere for each wavelength. These values form the input to the minimization algorithm described above. The absorption coefficients, \( k_w \), used in equation (15) are taken from Leckner's table (Leckner, 1978).

Table 1 shows results for days in the first trimester of 1995 when the atmosphere was found to be fairly stable. The mean turbidity coefficient, \( \beta \), for the period is 0.14.
The value is lower for a comparatively drier atmosphere and higher for increasing water vapor amounts. In 1961-66 a network program was established to make routine measurements of turbidity over the United States. Results from this program published by Flowers et al., (1969) show mean turbidities for the eastern parts in the range of 0.10 - 0.20. These results show seasonal variations with low values in winter and high values in summer. Results obtained in this work, shown in Table 1, are considerably higher than those obtained some three decades ago. Increased pollution due to industrial and agricultural activities may be the cause. Results also show a positive correlation between turbidity and atmospheric water vapor content.

Table 1. Results of experimental measurement of atmospheric water vapor and Ångström's turbidity coefficient. \( \beta^1 \) is single wavelength measurement. Location is Pine Bluff (34.13°N, 92.0°W).

<table>
<thead>
<tr>
<th>Date</th>
<th>( w ) (gm/cm²)</th>
<th>( \beta )</th>
<th>( \beta^1 )</th>
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<td>1/20/95</td>
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<td>0.12</td>
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<td>0.15</td>
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<td>1.6</td>
<td>0.16</td>
<td>0.17</td>
</tr>
</tbody>
</table>

**Literature Cited**


Katz, M., A. Baille and M. Mermier. 1982. Atmospheric turbidity in a semi-rural site - I: Evaluation and comparison of different atmosphere turbidity coeffi-