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Compton Scattering of γ-Rays from Electrons in Advanced Laboratory

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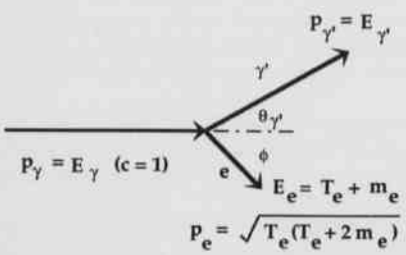
Abstract

A kinematically-complete 2-body final state measurement of Compton scattering of 662-keV photons is presened, where both scattered photon energy and electron recoil energy are measured versus photon scattering angle, $\theta\gamma'$. Passive collimation of the photon beam is avoided; each recoiling electron triggers a photon-scattering event providing active beam collimation. Recoiling electrons have low energies at small $\theta\gamma'$, impairing electron detection efficiency. Examining the recoiling-electron energy spectra in coincidence with high-resolution gammas indicates a 1"x1"NaI detector is superior to a 1"x1" NE-102 plastic scintillator as the active scattering material, for efficient recoil-electron detection. Electron efficiencies versus $\theta\gamma'$ are measured by comparing e-γ' coincident yield with the relativistically-correct Klein-Nishina predictions, indicating the detection-efficiency for recoil-electrons is near 100% at $\theta\gamma' \geq 30$ degrees. Scattered-photon energy pulses and recoil-electron energy pulses are summed electronically to produce an invariant peak at 662 keV, reducing systematic errors in coincident-yield extraction, In addition, $E\gamma'$ spectra are taken at several $\theta\gamma'$ to provide an experimental value for electron mass; an easier measurement than the Millikan oil drop experiment, but with similar predictive consequences.

Introduction

Students in the physical sciences are introduced to 4-momentum conservation (or momentum-energy conservation) with special relativity, usually in first-year physics. Compton scattering of energetic photons from electrons (Compton, 1923) provides a graphic example of this principle, while establishing the photon with tangible particle properties in the mind of each student (possibly supporting laboratory work by the student on the photoelectric effect which also shows photons have particle properties).

As seen below, both scattered photon energy $E\gamma'$ and recoiling electron kinetic energy T_e (and electron-recoil angle ϕ) are predicted as a function of outgoing photon angle $\theta\gamma'$ in this two-body final state. Little energy variation is predicted for soft X-ray photon scattering in contrast to the strong $\theta\gamma'$ dependence seen for incident photon energies ≈511 keV (m_ec^2). In the present work 662-keV photons from a ^{137}Cs source were scattered from electrons.



Energy Conservation

$$E_\gamma = E_{\gamma'} + T_e = 662\ \text{keV}$$

Momentum Conservation

$$E_\gamma = E_{\gamma'} \cos\theta_{\gamma'} + p_e \cos\phi$$

$$0 = E_{\gamma'} \sin\theta_{\gamma'} + p_e \sin\phi$$

Eliminating p_e and ϕ from equations above gives: $\frac{1}{E\gamma'} - \frac{1}{E\gamma} = \frac{1}{me}(1 - \cos\theta\gamma')$, where m_e is the electron mass (in energy units). This expression predicts a plot of the photon scattering data as $\frac{1}{E\gamma'}$ versus $(1 - \cos\theta\gamma')$ will result in a straight line whose slope is $\frac{1}{m_e}$ (Melissinos, 1973). Thus, the mass of the electron may be extracted from a least-squares fit to this straight line, providing an experimental value for m_e.

Measuring the mass of the electron by Compton Scattering is easier than measuring the electronic charge in the Millikan Oil Drop Experiment. Either measurement is historically interesting, even at the few percent level, as electromagnetic measurements only provide e/m_e and m_e/m_p ratios. With m_e (or e) measured, the charge of the electron (or its mass m_e) and the mass of the proton m_p may be extracted, as well as Avogadro's Number ($1/m_p$ in grams).

Laboratory measurements of $E\gamma'$ versus $\theta\gamma'$ are complicated by the finite geometry of the detectors. Students are introduced to "kinematic line broadening," an experimental condition seen in particle scattering, in the rapid variation of $E\gamma'$ with $\theta\gamma'$. In developing student experimental design skills, predictions of line widths at several detector positions may be carried out and tested.

The TELTRON Company offers a laboratory series on X-rays, with the Compton Effect listed as Physics Experiment D.21 (TELTRON, 1994). The development of this experiment has not been completed as yet. A soft X-ray experiment could serve as an introductory experiment where scattering yield measurements should compare fairly well with predictions from classical electromagnetic theory (Thompson, 1907; Evans, 1955b).

The 662-keV gamma rays in the present experiment bombard electrons at energies comparable to the electron rest mass, recoiling them in rough analogy to billiard balls, providing results in agreement with relativistic momentum-energy conservation, and allowing the extraction of m_e. For students who completed a soft-X-ray scattering experiment, this value of m_e may be compared with the m_e value extracted from soft-X-ray scattering using a curved crystal spectrometer. Since these students found the Thompson yield predictions satisfactory for soft-X-ray scattering, and since all photons move a the speed of light, students are likely to expect Thompson predictions to correctly provide scattering yields for gamma-ray photons as well.

The failure of the Thompson yield predictions for photon scattering in the energy regime $E\gamma \approx m_e c^2 = 511$ keV may induce student interest in measuring the yield versus scattering angle at fairly high precision to investigate this conundrum. Klein and Nishina (1929) applied Dirac Theory to the relativistic scattering of electrons. Agreement with data provided early confirmation of Dirac Theory, initially suspect because of its prediction of negative energy states for the electron (the "positron").

In this experiment students may be introduced to the relativistic Klein-Nishina formulation of photon scattering probability as a function of $\theta\gamma'$ (Evans, 1955). Comparing measured photon scattering probability versus $\theta\gamma'$ allows Klein-Nishina predictions to be used to extract the recoiling-electron efficiency as a function of $\theta\gamma'$. This establishes a region (e.g., $\theta\gamma' \geq 30°$) where the electron-recoil detection efficiency is flat (near 100%), allowing centroid to be extracted reliably from the $E\gamma'$ peaks, in order to measure the mass of the electron.

Materials and Methods

A 3" X 3" NaI detector was used to measure the energy of the scattered γ-ray photon and a vertically-mounted cylindrical 1" by 1" NaI detector was used as the active scattering material, taken in coincidence with each scattered photon, allowing the simultaneous measurement of each scattered photon energy ($E\gamma'$) with each recoil-electron kinetic energy (T_e). Figure 1 is an electronics diagram, showing the electron-gamma coincidence scheme using these two NaI detectors. Summing the two energy pulses, $E\gamma' + T_e$, provides an invariant peak in the Multi-Channel Analyzer (MCA) spectrum equal to the incident photon energy. Figure 2 shows this sum peak at 662 keV at 35 different scattering angles. Gain matching between the two NaI detectors was accomplished by triggering the linear gate, in turn, with the output of each timing SCA. A similar circuit is discussed in some detail in an earlier publication (Braithwaite, 1990).

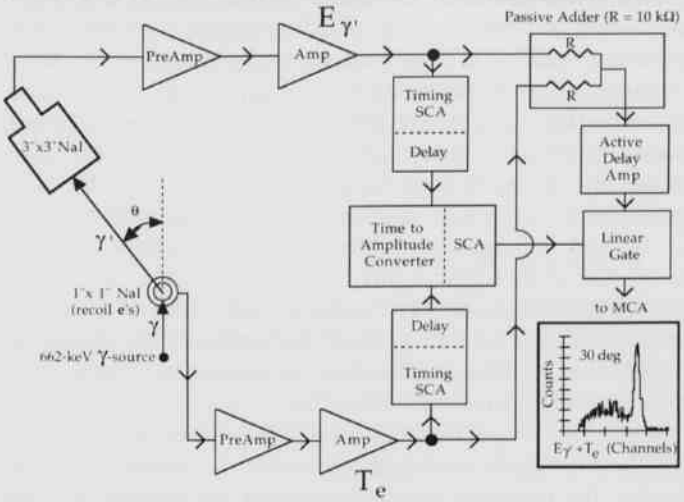


Fig. 1. Electronics diagram for electron-gamma coincidence using small NaI detectors. SCA (single channel analyzer) MCA (multi channel analyzer).

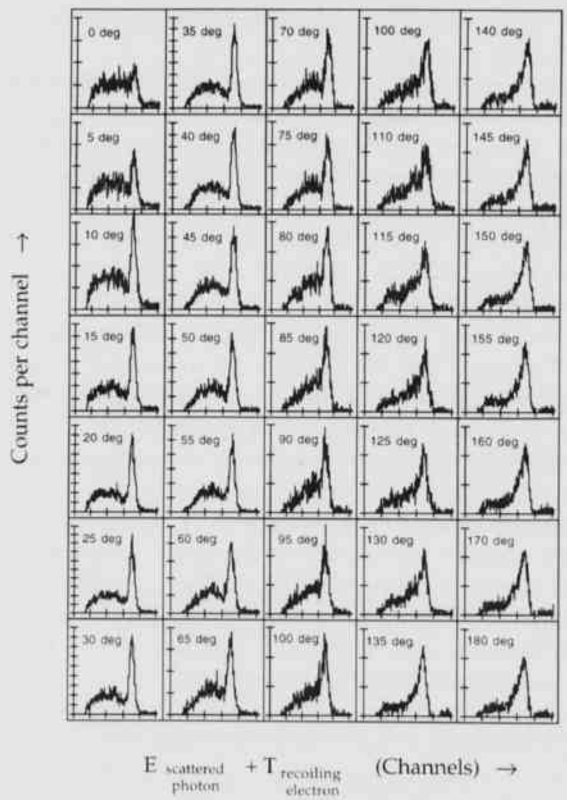


Fig. 2. Coincident photon yields at different photon scattering angles, versus total deposited energy.

Finite geometrical acceptance in both the 3" by 3" NaI detector and the 1" by 1" NaI detector results in broad peaking in both photon and electron distributions, each varying as a function of photon scattering angle θ. The precision of yield extraction is reduced by systematic errors, when taken at a variety of different energy positions and different peak widths, under variable background conditions. However, increased precision in yield extraction may be obtained by adding energy signals from γ and e [$E\gamma + T_e$ = 662 keV] to provide a spectral peak whose position is independent of θγ and narrower than either the individual gamma-ray or electron peaks. The peak width in the sum spectrum is fairly insensitive to θγ, as the NaI detector have comparable resolutions (with 3" by 3" NaI resolution slightly better than the 1" by 1" NaI resolution). Even so, Fig. 2 suggests some difficulty in peak extraction due to the varying background conditions at different scattering angles, despite constancy in peak position and near-constancy in width.

Detection of each scattered photon is triggered in coincidence by each recoiling electron. At forward photon angles, the recoiling electron has very low energy, and the student must examine its detection efficiency. A high-resolution Intrinsic Germanium detector was used to detect each scattered photon at 14.4 degrees in the laboratory, triggered by its recoiling electron in a 1" by 1" NaI detector, for three different angular acceptances: Δθ = ± 2.1, ± 3.2 and ± 6.4 degrees.

Figure 3 shows three sets of paired coincidence spectra at these three angular acceptances. The left spectrum in each of the three paired spectra is a Germanium detector gamma ray spectrum taken in coincidence with the recoil-electron spectrum (from a 1" by 1" NaI detector), and vice versa. For the 1" by 1" NaI spectra with the smallest angular acceptance of ± 2.1 degrees, the coincident electron yield drops into the noise above the lower-discriminator level, indicating detection efficiency is approximately 100%. For the 1" by 1" NaI spectra with the ± 3.2 degree acceptance, some loss to the lower-discriminator is seen, with even greater losses seen at the ± 6.4 degree acceptance. The electron-recoil spectrum shows significant losses (roughly 1/3) at the ± 6.4 degree acceptance. The geometrical acceptance (± 6.4 degrees) is about the same as the total angular acceptance of the detector geometry for the NaI detectors shown in Fig. 1. The 1/3 efficiency loss which may be estimated roughly from Fig. 3 is approximately the same size as the 28% efficiency drop from the Klein Nishina prediction seen in Fig. 4.

The greatest efficiency loss seen on the right-hand side of Fig. 3 is for the 1" by 1" NE-102 plastic scintillator spectrum at the smallest angular acceptance of ± 2.1 degrees, shown at the top right. This means the plastic scintillator is unreliable for either centroid or yield extraction for a wider range of angles.

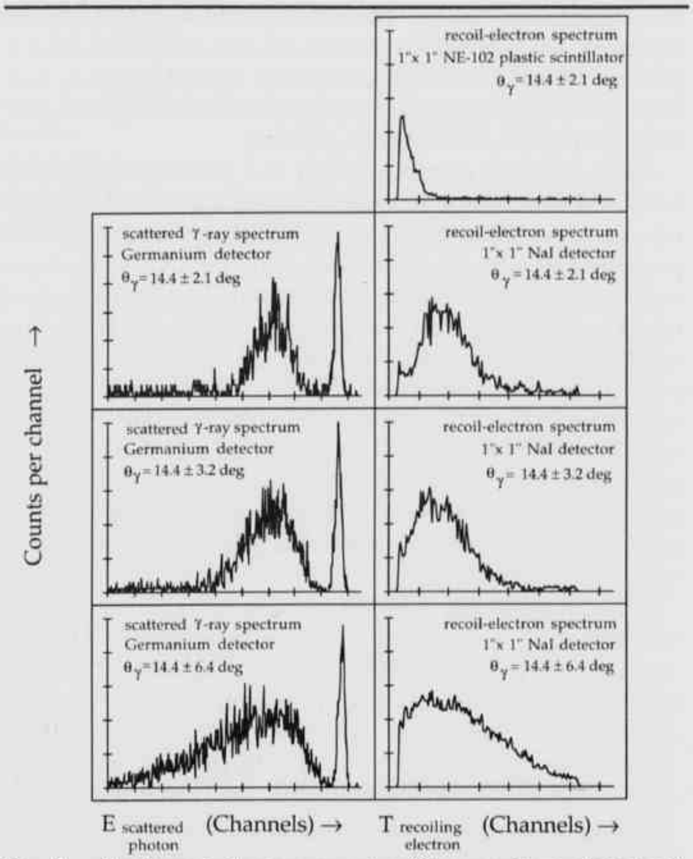


Fig. 3. Scattered-photon spectra (Germanium detector) are compared to corresponding recoil-electron spectra (1" by 1" NaI detector): Δθ = ± 2.1, ± 3.2 and ± 6.4 deg.

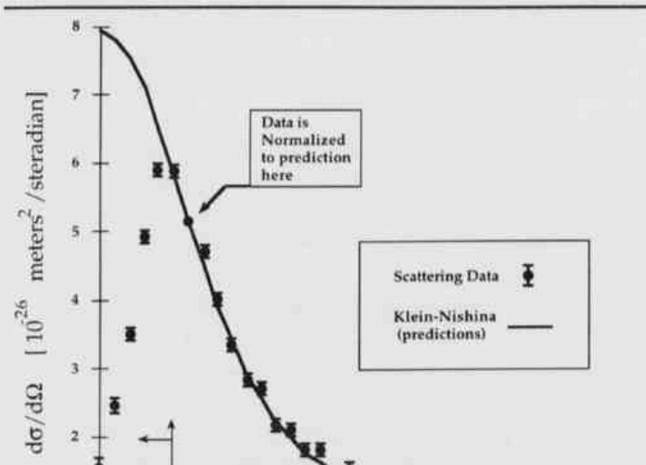


Fig. 4. Yield versus photon angle for photon scattering from electrons: Data and Klein-Nishina predictions.

A 1" by 1" NE-102 plastic scintillator will have a smaller multiple-photon scattering probability than will a 1" by 1" NaI detector, a possible concern for the lower photon energies at the backward scattering angles. However, the fair agreement seen in Fig. 4 between the coincident scattering data and the Klein-Nishina predictions indicates multiple-photon scattering is negligible for 662-keV photon scattering by electrons in a 1" by 1" NaI detector. Also, estimates of recoiling-electron ranges associated with back-angle photon scattering within the 1" by 1" NaI indicates ≥97% of the recoil electron energy is deposited within the NaI (averaged over scattering events). γ-detection efficiency in the 3" by 3" NaI detector is determined for each scattering energy as a product of detection efficiency times photopeak fraction (Marian and Young, 1968).

Figure 4 compares data and prediction for photon yield versus photon scattering angle, for an incident photon energy of 662 keV. Fair agreement is obtained except at the most forward angles, where the data is significantly lower than the Klein-Nishina predictions.

These data points are lower than prediction, due to a reduced detection efficiency for recoil electrons at the forward angles where electron kinetic energies are quite low. This attribution may be tested by changing the effective electron discriminator, by changing either the amplifier gain or the discriminator level. Lowering the effective discriminator level results in obtaining 100% efficiency at the smaller scattering angles, whereas increasing this level results in an efficiency reduction at even larger angles than 30 degrees.

Results and Discussion

The present work presents a kinematically-complete 2-body final state measurement of Compton scattering of 662-keV γ-ray, avoiding passive collimation of the incident γ-rays, as each recoiling electron triggers a γ-ray-scattering event. Systematic errors are reduced in coincident-yield extraction by summing electronic pulses associated with each scattered γ-ray energy and electron recoil energy, at each scattering angel, $\theta\gamma$, producing spectrally invariant peaks at 662 keV, as seen in Fig. 2.

Detection efficiency was examined at small $\theta\gamma$ for recoiling electrons, where they have low energies. Examining the recoiling-electron energy spectra, taken in coincidence with high-resolution gammas, indicates a 1" by 1" NaI detector is superior to a 1" by 1" NE-102 plastic scintillator as the active scattering material, for efficient recoil-electron detection.

A second method examined recoil-electron efficiency versus $\theta\gamma$ by comparing the e-γ coincident yield with the relativistically-correct Klein-Nishina predictions. This work indicated the detection-efficiency for recoil-electrons is essentially 100% for $\theta\gamma$ ≥30 degrees. Thus a region (e.g., $\theta\gamma$ ≥30°) is established where the electron-recoil detection efficiency is essentially 100%, allowing centroid to be extracted reliably from the Eγ spectral peaks, in order to measure the mass of the electron.

Centroid measurements are less sensitive than coincident-yield measurements to varying peak widths and varying background conditions, which is fortunate, as the centroid of the gamma-ray peak $E\gamma(\theta)$ versus $\theta\gamma$ was measured in order to test momentum-energy conservation, and to provide the basic data for extracting a value for the electron mass, m_e. $E\gamma$ spectra, taken at several $\theta\gamma$ are shown in Fig. 5, provide experimental values needed to extract electron mass. Figure 6 is a plot of photon scattering data as $\frac{1}{E\gamma}$ versus $(1 - \cos\theta\gamma)$. The straight line in Fig. 6 is a linear fit to this data, where $\frac{1}{m_e}$ is obtained from the slope of this straight line. Measuring electron mass to a few percent is an easier than carrying out the Millikan oil drop experiment, but either allows an unlocking of the charge/mass ratios from electromagnetic measurements, providing e, m_e, proton mass m_p (and Avogadro's number = $1/m_p$).

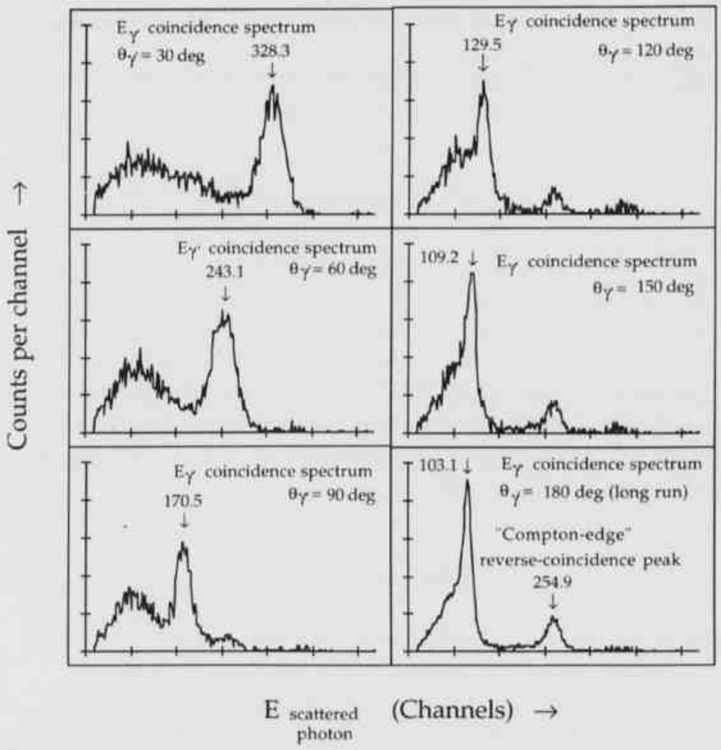


Fig. 5. $E\gamma$ peak position versus $\theta\gamma$ for photon scattering from electrons.

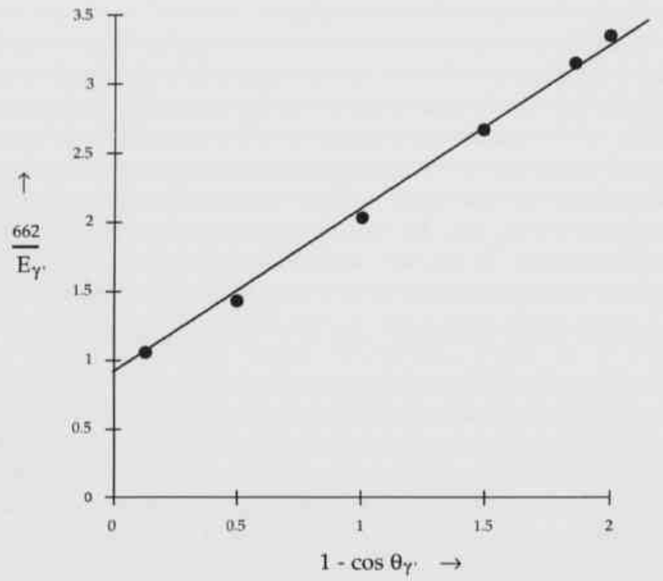


Fig. 6. Plot of photon scattering data as $\frac{1}{E\gamma}$ versus (1 - cos $\theta\gamma$). The straight line is a linear fit to this data, where $\frac{1}{me}$ is obtained from the slope of this straight line.

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Literature Cited

Braithwaite, W. J. 1990. Rotational symmetries of nuclear states: spin determinations in advanced laboratory. Proc. Arkansas Acad. of Sci. 44:19-22.

Compton, A.H. 1923. Physical Review 21:483 and Physical Review 21:715.

Evans, R.D. 1955. The atomic nucleus. McGraw-Hill Book Co., New York, 972 pp.

Evans, R.D. 1955b. The atomic nucleus. McGraw-Hill Book Co., New York, Chapter 1 (Section 2) and Appendix A.

Klein, O. and **Y. Nishina.** 1929. Z. Physik 52:853.

Marian, J.B. and **F.C. Young.** 1968. Nuclear Reaction Analysis Graphs, and Tables. (North Holland, Amsterdam) p. 10 and pp. 48-53.

Melissinos, Adrian C. 1973. Experiments in modern physics. (Academic Press, New York) 252-265 pp.

TELTRON. 1994. Student Enquiry Series D the production properties and uses of X-rays. Part 2: Experimental Manual. PHYSICS D.21 The Compton Effect, A Quantitative Measurement.

Thompson, J.J. 1907. The corpuscular theory of matter. Constable and Company, Ltd., London.