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Abstract

This paper demonstrates an application of the natural selection process to the design of structural members. Reinforced concrete beam design is used as the example to show how various chromosomes representing a design solution can be formulated. Fitter chromosomes (or better solutions) have a better chance of being selected for cross over; this in turn creates better generations. Random mutation is used to enhance the diversity of the population. The evolution progresses through several generations, and the best solution is then used in the design. The method gives reasonable results, but sometimes a local (as opposed to the global) optimized solution is obtained.

Introduction

Structural engineers traditionally design structural elements based on a trial-and-error process. An educated guess is made for a trial size of the member, then the performance is checked. Adjustments are then made for the next trial. An experienced designer normally starts with a reasonable trial size which a good design is obtained after a few iterations. For a typical new designer, this process can become tedious.

In recent years, genetic algorithms (GA) have been used in various optimization problems (Michalewicz, 1992). Structural design is another form of an optimization problem, in which the designer looks for the optimal solution (or a near-optimal solution) under a set of constraints. This paper demonstrates that GA can be applied to structural design problems by using the design of a reinforced concrete beam as an example.

Materials and Methods

The evolution process starts with a randomly created first generation. A generation consists of a constant population size, in which an individual in the population is represented by a chromosome. Each chromosome, consisting of genes, represents a design solution. A fitness value is then evaluated for each chromosome. Fitter chromosomes are assigned greater probabilities to be selected as parents for the next generation. Some of these selected chromosomes exchange genes with others during the crossover stage. Some genes are also randomly mutated. The process repeats through several generations. The fittest chromosome is then used as the design solution. The following sections will describe the details of this process in the context of reinforced concrete beam design.

Chromosome Formulation.--In designing a rectangular reinforced concrete beam for bending strength, the design solution consists of the section dimensions (width and effective depth) and the steel area, as shown in Fig. 1(a) where "b" is the section width, "d" is the section effective depth (the distance from the extreme compression fiber to the centroid of the tension steel), and "A_s" is the area of reinforcing steel.

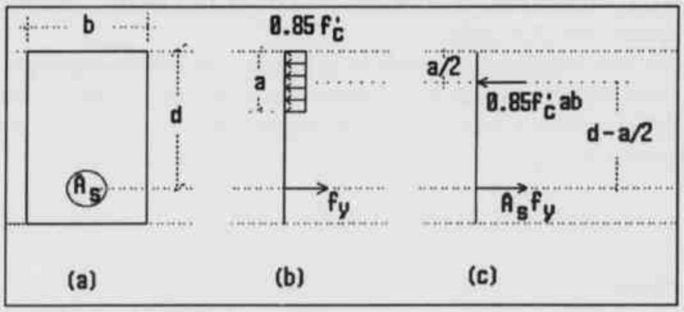

Fig. 1. Reinforced Concrete Beam: (a) Dimensions, (b) Stresses, and (c) Forces.

Thus, a chromosome must consist of three sets of genes representing these three quantities. In this particular implementation, each of these sets is represented by 12 binary digits, which gives the maximum decimal number of 4095. This maximum number is then divided by 100, so each parameter is in the range of 0 to 40.95. This range covers most of the practical problems. Fig. 2 shows a chromosome with its genes and the parameter range.

The First Generation.--Population in the first generation is created using random numbers. To avoid starting the sequence of random numbers at the same location every time the program is executed, the current minute from the computer time clock is used as the seed value for the random number generator. If "r" is the random number generated for a gene, the value of the j-th gene of the i-th chromosome ($gene_{ij}$) is determined based on the following rule:

If $r < 0.5$ then $gene_{ij} = 0$, otherwise $gene_{ij} = 1$, where $0 <= r <= 1$

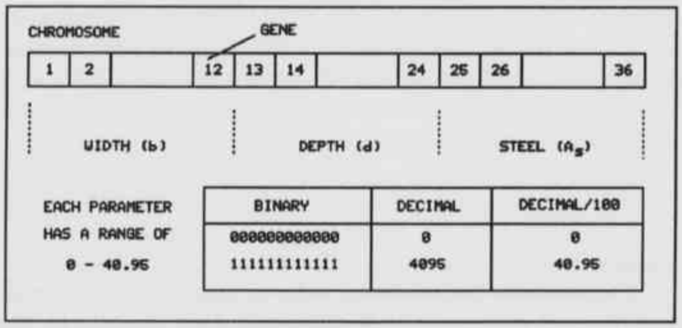

Fig. 2. Chromosome, genes and parameter range.

Fitness Evaluation.--Once the population in a generation is defined, the fitness of each chromosome can be evaluated. For the reinforced concrete beam problem, the fitness is determined based on its bending strenth (M_d), the section proportion (width/depth ratio), and the steel ratio ($A_s/(bd)$).

The bending strength is given by the following equation which was derived from engineering mechanics (Nawy, 1990) based on the stress and force diagrams shown in Fig. 1 (b) and (c).

$$M_d = (0.9)(A_s f_y)(d-a/2)$$

where

f_y = yield strength of reinforcing steel
a = $(A_s f_y)/(0.85 f'_c b)$
and f'_c = concrete strength at 28 days

This M_d is then compared with the required moment (M_u) which is specified as part of the input data. If M_d is greater than or equal to M_u, then the section is acceptable; otherwise, the section is rejected.

There are different section proportions that provide the desired strength. When "b" is too large compared with "d", the section is not economical. On the other hand, when "b" is too small compared with "d", the section is too slender and lateral buckling can occur. For a practical design, many designers keep the width/depth ratio around 0.5.

There are several combinations of section dimensions and steel reinforcement that provide sufficient bending strength. Larger sections require less steel, while smaller sections require more steel. There are minimum and maximum limits on the steel reinforcement set by the American Concrete Institute (American Concrete Institute, 1989) to avoid the sudden failure of concrete beams. Steel ratio is used in the comparison with these limits, as shown below:

$$200/f_y <= A_s/(bd) <= 0.75(0.85)\ B_1 f'_c(87000) / (f_y(87000+f_y))$$

where

$$B_1 = 0.85 - (f'_c - 4000)/1000$$

and $0.65 <= B_1 <= 0.85$

The fitness of a chromosome is then determined from the following rules:

1. The smaller the difference of M_d and M_u, the higher the fitness. When M_d is less than M_u, a penalty is applied.
2. The closer the b/d ratio is to 0.5, the higher the fitness.
3. When the steel ratio exceeds the maximum or minimum limits, a penalty is applied.

Based on these general rules, the fitness is determined by:

$$Fitness = 10^6 / (|M_d - M_u|)/(|0.5 - b/d|)/p_1/p_2$$

where

$| \ |$ = Absolute value
p_1 = Penalty factor for bending capacity If $M_d >= M_u$, then $p_1=1$, otherwise $p_1=2$ (for $M_d < M_u$)
p_2 = Penalty factor for steel reinforcement If the steel ratio is within the minimum and maximum limits, $p_2=1$, otherwise $p_2=10$
10^6 = Scaling factor to make sure that the fitness value is not too small

Population Selection.--Once the fitness for each chromosome has been evaluated, they are selected according to a probability weighing scheme as an imaginary spinner. The fitter chromosomes occupy larger areas on the spinner. In this implementation, the relative probability is used to represent these areas on the spinner. Let p_i be the probability of the i-th chromosome. Thus, p_i can be computed from the following equation:

$$p_i = Fitness_i / Fitness_{gen},$$

where

$Fitness_i$ = Fitness of the i-th chromosome,
and $Fitness_{gen}$ = Summation of all fitnesses of the generation.

Let n be the number of chromosomes in a generation (population size). The spinner is spun "n" times, during which the new population is selected. The i-th chromosome is selected from a spin if the random number, r, satisfies the following condition:

$$(p_1+p_2+...p_{i-1}) < r <= (p_1+p_2+...p_i)$$

Cross Over.--After the spinner is spun and a new pool of chromosomes is selected, a number of chromosomes (based on the probability of crossover specified by the user) is selected for cross over. A cross over location is randomly determined. The two randomly selected chromosomes exchange their genes from this location to the rest of the chromosome. The two new chromosomes (offspring) are then used to replace the original two parents.

If the two parent chromosomes, each with 15 genes, are:

1 1 1 0 0 1 0 1 0 1 0 0 1 1 0,

and 1 0 0 1 0 1 1 0 0 1 0 1 0 0 0,

and the crossover location is right after the 6th gene, the two offsprings, which replace the two parents become:

1 1 1 0 0 1 1 0 0 1 0 1 0 0 0,

and 1 0 0 1 0 1 0 1 0 1 0 0 1 1 0.

Mutation.--Mutation is the process in which some genes change their genetic codes. In this implementation, mutation causes a gene to change its value from 0 to 1, or vice versa. After several generations, it is possible that a solution which is superior to the others but not really acceptable could take control of the entire population by spreading its genetic codes to others. A better solution would then become impossible. Mutation injects diversity to the population and often helps to move the evolution out from a local optimum situation.

Results and Discussion

As an example, a beam is to be designed for a bending moment (M_u) of 2,000,000 lb-in (226 kN-m) using the concrete strength (f'_c) of 4,000 psi (27.6 MPa) and the steel yield strength (f_y) of 60,000 psi (414 Mpa), as shown in Fig. 3.

Rectangular Beam Designer – (c) 1994 by S. Malasri

Concrete Strength - f'c (psi) : ? 4000
Steel Yield Strength - fy (psi) : ? 60000
Required Moment - Mu (in-lb) : ? 2000000

Press any key to continue

Fig. 3. Input screen.

Other input parameters including the population size, the crossover probability, the mutation probability, and the number of generations are shown in Fig. 4. After 20 generations, a 11.96" by 30.29" section is obtained with the moment capacity (M_d) of 2,013,032 in-lb (which is very close to the required M_u). The steel ratio (Rho) of 0.0035 is also within the minimum steel ratio of 0.0033 and the maximum steel ratio of 0.0214. The width/depth ratio is 0.39 which is not too far from the desired 0.5. This, in fact, is a good design.

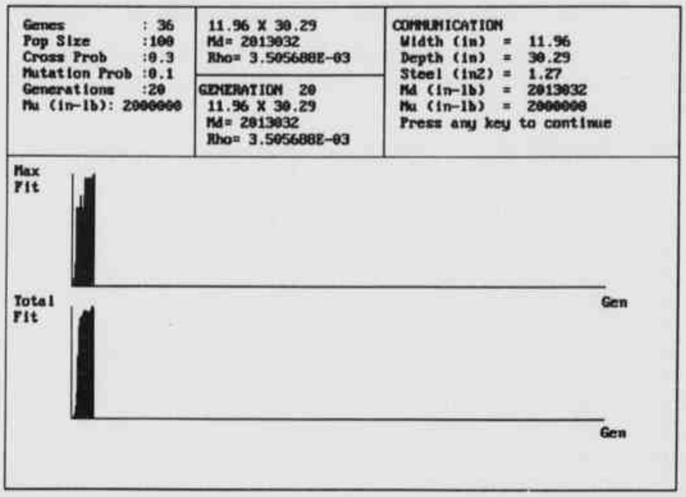

Fig. 4. Screen Showing the Evolution Process and Results.

Ten consecutive runs were made using different values of population size, crossover and mutation probabilities, and number of generations. They are summarized in Table 1. Most of the runs give good designs, except for the following:

1) Run number 3 has the steel ratio of 0.0012 which is lower than the minimum of 0.0033 allowed by the American Concrete Institute Code. The design engineer would reject this design.

2) Run number 6 has the steel ratio of 0.0248 which is greater than the maximum of 0.0214. This is not too bad, since theoretically, the maximum steel ratio in this case can go up to 0.0285. However, a conservative designer would reject this design.

3) Run numbers 7 and 9 are unnecessarily large, since they give the bending capacity of over 3,000,000 in-lb as compared to the required moment of 2,000,000 in-lb. This solution is safe but uneconomical.

Out of these 10 runs, six give acceptable solutions, two give safe but uneconomical solutions, one gives a working solution with less safety margin, and one gives an undesirable solution. Three of the four runs that have problems (run numbers 3, 7, and 9) use the same population size of 50. This population size probably does not provide enough diversity. By increasing the mutation probability from 0.1 to 0.2 as in run number 10, an acceptable solution is obtained. Table 2 shows the evolution process that took place in run number 10. The solution starts from a very large section in the first generation that gives almost six times the desired bending capacity to an acceptable solution after 9 generations. After the only minor changes occur until the 51st generation. No better solution was found from the 51st generation to the 100th generation.

Table 1. Various Runs for the Same Design Problems.

Input*:

No.	Population Size	Crossover Probability	Mutation Probability	Number of Generations
1	150	0.3	0.1	20
2	100	0.3	0.1	20
3	50	0.3	0.1	20
4	100	0.3	0.3	20
5	100	0.3	0.1	20
6	100	0.3	0.1	20
7	50	0.3	0.1	20
8	150	0.3	0.1	20
9	50	0.3	0.1	40
10	50	0.3	0.2	100

* Other input data is shown in Fig. 3.

Output:

No.	Section b x d	b/d	M_d in-lb	Steel Ratio**	A_s in^2
1	8.63" x 18.02"	0.48	2,066,671	0.0159	2.47
2	10.83" x 19.90"	0.54	2,071,534	0.0098	2.11
3	18.98" x 40.94"	0.46	2,034,294	0.0012	0.93
4	11.28" x 13.93"	0.81	2,059,934	0.0214	3.38
5***	11.96" x 30.29"	0.39	2,013,032	0.0035	1.27
6	8.31" x 15.35"	0.54	2,051,439	0.0248	3.17
7	13.16" x 26.18"	0.50	3,479,880	0.0077	2.64
8	12.06" x 23.05"	0.52	2,046,162	0.0062	1.74
9	13.61" x 27.65"	0.49	3,789,192	0.0072	2.71
10	8.24" x 17.93"	0.46	2,099,687	0.0173	2.56

** Minimum Steel Ratio = 0.0033, Maximum Steel Ratio = 0.0214
*** Also shown in Fig. 4.

Table 2. Evolving from an Initial Random Solution to an Acceptable Solution.

Generation	Section	Md (in-lb)	Steel Ratio
1	21.75" x 38.75"	11,473,060	0.0069
2	11.99" x 16.35"	3,372,077	0.0250
3	8.22" x 14.29"	674,143	0.0080
4	21.75" x 38.77"	1,948,608	0.0011
6	14.35" x 18.09"	2,307,335	0.0100
8	8.22" x 14.29"	1,609,766	0.0220
9	7.95" x 18.09"	2,114,676	0.0179
11	8.23" x 18.09"	2,128,144	0.0173
12	8.27" x 18.09"	2,123,180	0.0171
24	7.95" x 17.93"	2,092,471	0.0180
25	7.99" x 17.93"	2,087,830	0.0179
51	8.24" x 17.93"	2,099,687	0.0173

Conclusion

This paper demonstrates that it is possible to automate the design process using the evolution process as seen in the reinforced concrete beam design example. The cumulative selection (as opposed to pure random selection) is a very powerful mechanism in evolution. As shown in the example, acceptable solutions are obtained quickly (within 20 generations). In this problem, the goal is to optimize the bending capacity with the three constraints: M_d is greater or equal to M_u, section proportion is around 0.5, and steel ratio should lie within the acceptable range. For a more complex problem with more constraints, more generations may be needed.

To a structural engineer, the design of a reinforced concrete beam is a simple problem and many design aids are available. But for other more complex problems where design aids are not available and a resonable trial section is hard to guess, this evolution approach becomes very useful. The current work includes the design of structural steel columns. This problem has more complex constraints. For example, steel sections come in standard sizes, a data base of the available standard section must be checked. This puts severe restrictions to the corss over and mutation mechanisms.

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S. Melati, D.A. Halijan and N.L. Keough

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