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Detailed Inventory Record Inaccuracy Analysis

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ABSTRACT

This dissertation performs a methodical analysis to understand the behavior of inventory record inaccuracy (IRI) when it is influenced by demand, supply and lead time uncertainty in both online and offline retail environment separately. Additionally, this study identifies the susceptibility of the inventory systems towards IRI due to conventional perfect data visibility assumptions. Two different alternatives for such methods are presented and analyzed; the IRI resistance and the error control methods. The discussed methods effectively countered various aspects of IRI; the IRI resistance method performs better on stock-out and lost sales, whereas error control method keeps lower inventory. Furthermore, this research also investigates the value of using a secondary source of information (automated data capturing) along with traditional inventory record keeping methods to control the effects of IRI. To understand the combined behavior of the pooled data sources an infinite horizon discounted Markov decision process (MDP) is generated and optimized. Moreover, the traditional cost based reward structure is abandoned to put more emphasis on the effects of IRI. Instead a new measure is developed as *inventory performance* by combining four key performance metrics; lost sales, amount of correction, fill rate and amount of inventory counted. These key metrics are united under a unitless platform using fuzzy logic and combined through additive methods. The inventory model is then analyzed to understand the optimal policy structure, which is proven to be of a control limit type.

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CHAPTER 1 INTRODUCTION

1.1 Background and Motivation

Supply chain and inventory management has always been a major concern in the business world as well as in the academic domain. It can be referred to as the planned course of action against random consumption of the items, products, goods, etc. The scope entails physical holding, lead times, holding costs, replenishment, defective goods, quality control, transportation, storage, and inventory visibility. Hence, inventory models can be regarded as one of the most widely studied topics in industrial engineering and operations management. Due to the uncertain nature of the world, these models are known to have a complex structure.

There is countless number of research studies related to inventory management in the literature. The main goal for most of these studies is to reach efficient solutions that would provide cost effective realizations in practice. Keeping specific levels of inventory is a must to attain optimal values for cost or profit (Rinehart, 1960). Relph et al. (2003) categorize the basic reasons for inventory in three sets: lead-time, uncertainty, consumer satisfaction. Lead time is the time lags present in the supply chain - from suppliers to end costumers – that requires a certain amount of inventory to be used. However, in practice, inventory is to be maintained for consumption during variations in lead time, forcing decision makers to hold extra items to account for the time lag. Decisions are made under different levels of uncertainty which forces extra items to be maintained as buffers to meet uncertainties in demand, supply and movements of goods. Ideal condition of "one unit at a time at a place where a consumer needs it, when he/she needs it" principle incur lots of costs in terms of logistics. So bulk buying, movement and storing brings in economies of scale, thus extra inventory. Hence, items must be ordered periodically, stored and managed efficiently or else, the business will lose money. To avoid such undesirable situations companies pay a lot of attention to inventory and its management.

In practice, dealing with all the uncertain factors, satisfying the high service levels and reaching optimal solutions at the same time is challenging. Starting from late 70s, theoretical studies began addressing the difficulties faced in inventory management (Boxx, 1979; Covin, 1981; French, 1980). In industries where the competition is fierce and profit margins are thin, companies have automated the inventory management processes to better meet customer demand and reduce operational costs. Such schemes significantly decreased the response time of the decision makers, making it dramatically easy to keep track of the records and avoid human intervention as much as possible. However, the automation of management processes transferred the entire critical decision making - such as what products are where and in what quantity - from humans to computers.

The effectiveness of automated systems depends on data gathering and passing it through the chain with the aim of effectively coordinating the movement of the goods. This, according to Boritz (2003), raises the issue of data accuracy. Most companies make substantial investments in innovating systems and thus enabling them to improve the level of automation of their supply chain processes (Boritz, 2003). However, majority of the inventory models operate under the assumption of perfect data accuracy. In other words, the quantities of the various goods in stock at any time are known accurately. Such models have limited liability, especially if the number of inventory is large with high turnover rates. In such a setting, inventory records are likely to be incorrect, and ignoring this fact often results in failed re-procurement cycles and quantities.

The lack of theoretical studies in this conjecture has left the practices vulnerable to unrequired replenishment, unnecessary procurement and occasional delays in supplying customers. Iglehart and Morey (1972) and Raman et al. (2001) quantify the effect of the data assumptions on data accuracy; Iglehart and Morey (1972) also report that out of 20,000 total items 25% revealed discrepancies, which corresponded to roughly 4% of monthly inventory. Similarly, Raman et al. (2001) reports that 65% of 370,000 units of inventory, did not match the physical stocks.

The objectives of this dissertation are to understand the concept of inventory record inaccuracy (IRI), explore the effects induced by uncertainty on IRI, and apply methods to control the impact of IRI. In this context, IRI is defined as the error when the stock record is not in agreement with the physical stock. Such discrepancies are generally introduced to the system during three operations: inbound transactions, shelving operations and outbound transactions. These errors force the system to operate with inaccurate information and make wrong decisions, often followed by a stock-out. The susceptibility in this setting arises from two factors, the capabilities of technological systems and the shortcomings of the theoretical inventory models used in the system. Hence, it is clear that policies that are more resistant to IRI and technologies that can capture data more effectively are needed.

1.2 Literature review

Inventory problems, in general, have been studied extensively in the literature. Since 1950s artifacts, barcode readers and universal tags have been used in order to decrease the complexity of decision making. One of the major benchmarks in the gradual progress of supply chain and inventory management, particularly in inventory management, was in the early 1980s. The development of technology reached to a point in which easy and cheap utilization of stronger computers with faster processing power became possible. Companies started to take advantage of these computers and began to automate their inventory management processes using specialized software for inventory management. According to Lee and Ozer (2007), the specialized software that emerged is referred to as automatic replenishment systems (ARS). The ARS gather the point-of-sales statistics under one platform by tracking the changes in inventory records. In addition, replenishment orders are placed automatically based on the gathered data and the implemented control policy. With the support of various inquiries, these systems significantly reduced the complexity of decision making by providing superior utilization of statistics. Automatic replenishment systems operate by keeping track of every stock keeping unit (SKU) in the inventory through recording the fluctuations due to demand, supply, and any other possible cause at the same time. With this SKU information in hand, such systems can react to predetermined circumstances (such as low on-hand inventory or a sharp increase in holding costs) without the need of frequent cycle counting.

1.2.1 Inventory Record Inaccuracy

An essential shortcoming of the ARS is the regular implicit assumption that the quantities of the various goods in stock at any time are accurately known. In other words, the actual on-hand inventory and the recorded inventory is equal or very close. However, empirical observations have found this implicit assumption to be incorrect, DeHoratius and Raman (2008) and Iglehart and Morey (1972), show that make such assumptions have limited viability. Surveys and empirical studies have also shown that the difference between inventory records and actual inventory has a critical effect on the resulting operating costs and revenue (Agrawal, 2001; Kang & Gershwin, 2004). If the information provided to an automated replenishment system is incorrect and if the control mechanisms do not account for inventory discrepancy, then the system fails to order when it should or it carries more inventory than necessary. The outcome is either lost sales or an inventory surplus.

Early studies conducted by Rinehart (1960) observe that the larger the supply operations, the more susceptible it will be to discrepancies between inventory records and physical stocks. In his research, a case study conducted on a government agency reveals that there is 33% discrepancy out of 6,000 randomly picked items during a specific period of time. Furthermore, the study concludes that small discrepancies with little impact on inventory control operations and re-ordering procedures could lead to huge inconsistencies over a period of time. Thus, in terms of identification purposes all discrepancies are significant regardless of their size.

Iglehart and Morey (1972) discuss the same issue by looking at a report conducted at a naval supply depot. This report shows that 25% of the 20,000 total SKUs have discrepancies. These discrepancies correspond to an error rate approximately 4% of the monthly inventory turnover. Furthermore, an alternative case is also addressed in their investigations. A retailer with 400 units of monthly demand with a fixed standard error deviation is considered. They analyze how rapidly the errors grow between cycle counts. Their study shows that the cumulative error after 26 months reaches to approximately 20% of the monthly demand.

Several studies (Iglehart & Morey, 1972; Rinehart, 1960), realize the importance of the accuracy of inventory records and introduce the concept of IRI. Starting in the late 1970s, IRI has been extensively researched, especially under material requirements planning (MRP) (Boxx, 1979; Covin, 1981; French, 1980). With the development of manufacturing simulation systems in the 1980s, the interest in IRI jumped to various fields. Ritzman et al. (1984) focus on the standardization of the product and the corresponding IRI rate. Krajewski et al. (1987) show that the probability of incorrect inventory transaction is 0.02, if a fixed order quantity is used for lot sizes. Viewing IRI as a reoccurring problem, Bragg (1984) addresses the long term impact on inventory delivery and supply chain performance. The conventional ways of reducing the inaccuracy is first discussed by Plossl (1977). According to Plossl, management can control the accuracy by formalized training of personnel, cycle counting, barcoding, limiting access to the stockroom, and higher wages for personnel who track inventory data. These procedures imply incurring additional costs on employee downtime. Baudin (1996) and Millet (1994) utilized a similar approach and argued that improving employee traits such as incentives, motivation, training and tools can achieve higher accuracy levels.

The majority of the literature on IRI used the same functional definition for inventory accuracy, which is based on the discrete counts of inventory components called SKU. Inventory accuracy is then defined to be the ratio of the number of SKUs counted and found to be correct with a small tolerance for error. In this setting, the magnitude of the error is defined to be the size of the discrepancy between the physical stock and recorded inventory; see studies by Buker (1979) to Robison (1994). However, in the 1990s with the advent of computer aided systems for record keeping, the main focus of research began to shift towards identifying the underlying reasons for IRI and analyzing their long term effects. In the earlier studies the most commonly encountered problems in IRI are categorized as misplacement errors, theft, perished products, supplier frauds, and transaction errors. Mosconi et al. (2004) capture the interaction between IRI and the variability due to scanning and receiving processes. In the study, a mathematical model is proposed defining the amount of inventory on-hand and the level of demand. By focusing on the impacts of factors that lead to IRI, Atali et al. (2006) analyze inventory shrinkage problem under three categories: permanent inventory shrinkage (such as theft and damage), temporary inventory shrinkage (such as misplacement) and the final group of factors (such as scanning error) that affects only the inventory record without changing the physical inventory level.

Studies tend to agree on grouping IRI in two categories: shrinkage and transaction errors. One of the earliest analyses on transaction errors is by Iglehart and Morey (1972), which entails a single-item periodic-review inventory system with a predefined stationary stocking policy. Another paper by Kok and Shang (2007) explores an inventory replenishment problem together with an inventory audit policy to correct transaction errors. They consider transaction errors as a source for discrepancy and assume that these error terms are identically and independently distributed with mean zero. They consider a periodic-review stationary inventory system in which transaction errors accumulate until an inventory count is triggered. Hence, the manager incurs a linear ordering, holding and penalty cost and a fixed cost per count. When inventory is not counted periodically, the total error gradually increases thus contributing to the amount of uncertainty. The question is whether to deal with a larger uncertainty, or to count and incur an additional fixed cost and subsequently deal with lesser uncertainty. For a finite horizon problem Kok and Shang (2007) shows that the adjusted base-stock policy is close to optimal through a numerical analysis. The policy claims that if the inventory record is below a threshold, an inventory counting is requested to correct the errors. They model cost analysis framework to compare two classic periodic-review inventory control problems for which the base-stock policy is the optimal. They compare the cost of a periodic review systems facing demand uncertainty at each period. Comparing the two, the authors observe that the costs can be reduced by around 11% if the manager can eliminate all transaction errors.

Shrinkage is the second source of discrepancies influencing IRI. Shrinkage can be categorized as the general unavailability of products due to various reasons such as theft, spoilage or damage. Kang and Gershwin (2004) investigate inventory movement when the errors are caused only by shrinkage. They illustrate how shrinkage increases lost-sales and results in an indirect cost of losing customers (due to unexpected out of stock), in addition to the direct cost of losing inventory. Their objective is to illustrate the effect of shrinkage on lost-sales through simulation. They do not consider transaction errors and misplacement, nor do they consider optimal inventory counting decision. However, they provide some plausible methods to compensate for inventory inaccuracy.

The reoccurring encounter of IRI forced industry to pursue different methods and to invest in computer aided systems that provide automatic identification (Auto-ID), such as barcoding. According to Steidtmann (1999), US retailers spend 1% of total annual sales on automated inventory management tools to track sales, forecast demand, plan product assortment, determine the replenishment quantities, and control inventory. In his paper, Agrawal (2001) points out that the barcode system became the most commonly used data capture technology in practice. Approximately five billion codes are scanned every day in 140 countries. Utilization of a barcode system reduced the effects IRI caused by transaction errors; however, it did not account for other types of errors. A more recent work on IRI, (Raman et al., 2001), report that out of 370,000 SKUs investigated in 37 of two leading apparel retail stores, more than 65% of the inventory records did not match the physical stock. Ton and

Raman (2004) conduct similar empirical analysis to show that the discrepancy problem still exists today. Gentry (2005) also report an IRI around $142,000 in a well-known apparel retailer, The Limited. Comparison of these case studies reveals two important observations. First, retail environments that have a high inventory turnovers and more contact with customers accumulate much more discrepancy than distribution centers that have lower inventory turnovers and less contact with customers. Second, the recent developments in information technology have not yet addressed or eliminated the inventory discrepancy problem. Presumably, with a real-time tracking technology the manager can have complete visibility of inventory movement within the company at any point in time.

The focus of the studies of IRI in the literature is generally on monetary effects. Our study on the other hand focuses on modelling the behavior of IRI, analyzing various methods to control the behavior and limit the impact of IRI. Furthermore, most of the studies only use random demand and do not include the lead time or supply uncertainty in the inventory framework. This dissertation however, scrutinizes the effects of supply and lead time uncertainties as well as their influence on IRI. Combining all of the analysis, a general formulation for IRI is presented including the uncertainties faced. Finally, two different alternatives for compensating IRI are developed: limiting the impact of IRI on inventory model and controlling the behavior of IRI.

1.2.2 Multi-Objective Inventory Models

Multi-objective inventory models are also commonly studied in the literature, i.e. (Lewis, 1970; Naddor, 1966; Silver & Peterson, 1985), under various constraints. Modelling an inventory problem involves inventory costs, purchasing and selling prices in the objectives and constraints, which are seldom known in real life. So due to the specific requirements and local conditions, uncertainties are associated with these variables and the mentioned objectives are vague and imprecise. This motivated researchers to use fuzzy logic in formulating inventory models, especially in the multi-objective setting (Roy & Maiti, 1998; Tsou, 2008; Wee et al., 2009; Xu & Liu, 2008). ARTICTE (1995) group multi-objective optimization problems into two categories, complementary and conflicting. In complementary objective multi-objective decisions can often be solved through a hierarchical extension of the multi-criteria evaluation process i.e. (Carver, 1991). With conflicting objective multi-objective decisions are often prioritized to give rank order. The most common way of solving such problems involves optimization of a choice function (Feiring, 1986) or goal programming (Ignizio, 1985). Please refer to Marler and Arora (2003) for a comprehensive review of methods on multi-objective optimization.

The first publication in fuzzy set theory, (Zadeh, 1965), presents methods to accommodate uncertainty in a non-stochastic sense rather than the presence of random variables. After that, fuzzy set theory is applied to many fields including inventory management. One of the first applications of fuzzy dynamic programming to inventory problem is by Kacprzyk and Staniewski (1982). Instead of minimizing the average inventory cost, they reduced it to a multi-stage fuzzy decision making problem. Another paper, Park (1987), focus on the EOQ formula in the fuzzy set theoretic perspective, associating the fuzziness with the cost data. The Eoq model is then transformed to a fuzzy optimization problem. Petrovic and Sweeney (1994) fuzzify the demand, lead time and inventory level into triangular fuzzy numbers in an inventory control model. Vujosevic et al. (1996) extend EOQ model by introducing the fuzziness of ordering and holding cost. Roy and Maiti (1997) also develop an EOQ model where unit prices vary with demand, cost and production. They evaluate the optimal order quantity by both fuzzy nonlinear programming and fuzzy geometric programming. Chang et al. (2006) investigate mixture inventory model involving variable lead time with backorder and lost sales. They obtain the total cost by fuzzifing the lead time demand with a triangular membership function.

Fuzzy multi-objective inventory models are a developing field. Roy and Maiti (1998) investigate a multi-item inventory model of deteriorating items with stock-dependent demand in a fuzzy environment. Their objective is maximizing the profit and minimizing the wastage cost which are fuzzy. They express the impreciseness in the fuzzy objective and constraint goals by fuzzy linear membership functions and that in inventory costs and prices by triangular fuzzy numbers. Chen and Tsai (2001) reformulate the fuzzy additive goal programming by incorporating different important and preemptive priorities of the fuzzy goals. An interactive fuzzy method for multi-objective non-convex programming problems using genetic algorithms is proposed by Sakawa and Yauchi (2001). Mandal et al. (2005) formulate a multi-item multi-objective fuzzy inventory model with storage space, number of orders and production cost restrictions. The multi-objective fuzzy inventory model was solved by geometric programming method. Xu and Liu (2008) concentrate on developing fuzzy random multi-objective model for multi-item inventory problems in which all inventory costs are assumed to be fuzzy. They use trapezoidal fuzzy numbers to represent the impreciseness of objectives and constraints. They provide a fuzzy random multi-objective model and a hybrid intelligent algorithm to provide solutions to inventory models. Wee et al. (2009) study a fuzzy multi-objective joint replenishment inventory problem of deteriorating items. Their model maximizes the profit and return on inventory investment under fuzzy demand and shortage cost constraint. The fuzzy multi-objective models are formulated using fuzzy additive goal programming method and also a novel method inverse weight fuzzy non-linear programming is proposed.

The general focus of multi-objective inventory models is usually on various conflicting return on investment type of objectives. In this dissertation we define a new measure called the inventory performance. This measure is a fuzzy combination of four key parameters that are directly influenced by IRI; lost sales, expected correction, stock-out amount and service level. These parameters are then used to develop a multi-objective setting for a fuzzy additive goal programming

1.3 Organization of the Dissertation

The organization of this dissertation is shown in Table 1-1. The model column in the table shows the utilized setup for each chapter. The data source column presents the sources of information used in the model. In this context, inventory records refer to the traditional stock keeping methods where the number of inventory on-hand is calculated based on order and sales. In Chapter 2, Appendix A and Chapter 4, the only source of information on the inventory level is the inventory records; however, in Chapter 5 another source of information is introduced as the visibility information, obtained through automated data capturing systems (e.g. RFID). The IRI policy column shows the decision maker’s perception of IRI. Ignorant means that the decision maker assumes that IRI is non-existent; whereas, in the informed policies the decision maker is aware of IRI and the inventory system is constructed accordingly. The focus column shows the focus of the analyses done. The final column denotes the environment the system belongs to. Offline retail is the traditional brick and mortar type of retailing and online retail is the channel where customers make their purchases from internet.

Table 1-1: Dissertation organization

Chapter	Model	Data Source	IRI Policy	Focus	Environment
Chapter 2	Simulation	Inventory Records	Ignorant and Informed	IRI	Offline Retail
Chapter 3		Inventory Records	Informed	IRI	Online Retail
Chapter 4		Inventory Records	Informed	Cost	Online & Offline
Chapter 5	Optimization	Inv Records & AutoID	Informed	Inventory Performance	Offline Retail

In the dissertation, Chapter 2 and Appendix A conduct simulation studies to replicate and understand IRI behavior under demand, supply and lead time uncertainty for online and retail setting. Furthermore, various methods to control the behavior and limit the impact of IRI are analyzed. In these models the only source of information is inventory records. Combining all of the analysis, a general formulation is presented including the errors and the uncertainties faced. This general formulation is then separated into two separate cases (the best and the worst case) representing different order of events. Furthermore, for each case two different alternatives for compensating IRI are categorized: limiting the impact of IRI on inventory model and controlling the behavior of IRI.

Chapter 4 continues the analysis on IRI by introducing a cost framework. The general cost structure is divided into three categories; IRI related costs, penalty costs and operating costs. This cost structure is then used to create a common platform for important performance metrics. This model is designed for online and offline retail setting separately.

In Chapter 5, two new concepts are introduced to the system. First, a new source of information on inventory level is defined as Auto-ID. With two separate sources of information about the inventory level, the decision maker hopes to optimize the Inventory problem by reducing or controlling IRI. Second, the cost structure is improved further by using fuzzy logic. Multiple fuzzy cost parameters are defined and then combined in a multi-objective setting. With both sources of information the inventory problem is modeled as an infinite horizon discounted MDP with fuzzified multi-objective. This model is extensively analyzed to understand the optimal policy structure.

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CHAPTER 2 IRI ANALYSIS IN OFFLINE RETAILING WITH LEAD TIME AND SUPPLY UNCERTAINTY

Inventory models are one of the widely studied topics in supply chain management. Due to the uncertain nature of the system parameters such as demand, supply, lead times and errors, these models are known to have a complex structure. In practice, dealing with these uncertain factors, satisfying the high service levels, and reaching optimal solutions at the same time are very difficult.

Majority of the research on inventory models operate under the assumption perfect record accuracy. According to Bensoussan et al. (2005), a limited number of studies investigate IRI mainly due to retailers do not publicize their lack of full inventory information, and because the information must be inferred from surrogate measures. Moreover, the assumption of perfect accuracy assumption greatly reduces the theoretical complexity of the inventory problems. However, in real settings, inventory records are likely to be incorrect. The lack of theoretical studies in this conjecture has left practices susceptible to unrequired replenishment, unnecessary procurement and occasional delays in supplying customers. Empirical studies try to quantify and reveal the impact of data accuracy assumption. In Iglehart and Morey (1972), out of 20,000 total items 25% revealed discrepancies which corresponded roughly 4% of monthly inventory. In Raman et al. (2001) 65% of 370,000 units of inventory, did not match the physical stocks.

The objective of this dissertation is to understand the IRI concept in the offline environment, explore the effects induced by uncertainty, and apply methods to control the impact of IRI. In the offline retail setting, IRI can briefly be defined as the error when the stock record is not in agreement with the physical stock. Such discrepancies are generally introduced to the system during three operations, inbound transactions, shelving operations and outbound transactions. These errors force the system to operate with inaccurate information and make wrong decisions, often followed by a stock-out. The susceptibility in this setting arises from two factors, the capabilities of technological systems and the shortcomings of the theoretical inventory models used in the system. Hence, it is clear that inventory policies that are more resistant to IRI and technologies that can capture data more effectively are needed.

Lead time and supply uncertainty are extensively researched topics in inventory management problems. However, the literature on IRI commonly operates under the assumption that the lead time and the supply are deterministic. This dissertation also investigates the influence of the additional uncertainty caused by the random supply and the random lead time. Supply uncertainty. The introduction of general random lead time mechanism often causes disruptions in the supply chain coordination due to loss of tractability (Bashyam & Fu, 1998). Furthermore, it enhances the stock-out risk faced during the lead time. The supply uncertainty, on the other hand, is often caused by two factors, random capacity and random yield of the suppliers (Henig & Gerchak, 1990). In our study, we use simulation analyses to model these extra uncertainties and estimating performance.

The outline of this chapter is as follows, Section 2.1 presents the general characterization for errors under demand, supply and lead time uncertainty. In Section 2.2 for a continuous review inventory problem, methods for compensating the impact of IRI are examined. Also the analysis is tested on a numerical example. Finally, in Section 2.3 a discussion about this study is presented.

2.1 Model

As previously mentioned, the IRI is affected by many uncertain factors such as random demand, random supply, and lead time, in addition to the inventory management related errors. The complication is that the relationship is not one sided; IRI also affects all of these factors. Various enumerations of IRI can be found in the literature. Underlying error factors are typically categorized based on dependent variables. In this study however, we will categorize these error factors based on their impacts on inventory as follows: (1) Inbound errors: Errors that occur during ordering and receiving process; (2) Shelving errors: Errors that are due to damaged, stolen, or expire SKUs, which cause the physical stock to change without informing inventory records; (3) Outbound errors: Errors that occur during check-out processes (e.g. scanning errors). When left uncorrected, these errors will significantly lower retailer performance by increasing the stock-out rate. According to Gruen et al. (2002) the stock-out rates on average fall in range 5-10% which roughly corresponds to 4% of sales.

Figure 2-1 shows a typical continuous inventory behavior subject to errors. In the considered model, the start of each period is identified as points at which a replenishment order is given. In order to create a generalized model, let $\tilde{x}_{t_k}$ denote the amount of actual inventory and x_{t_k} denote the inventory record at time t_k, at the replenishment order time in period k.

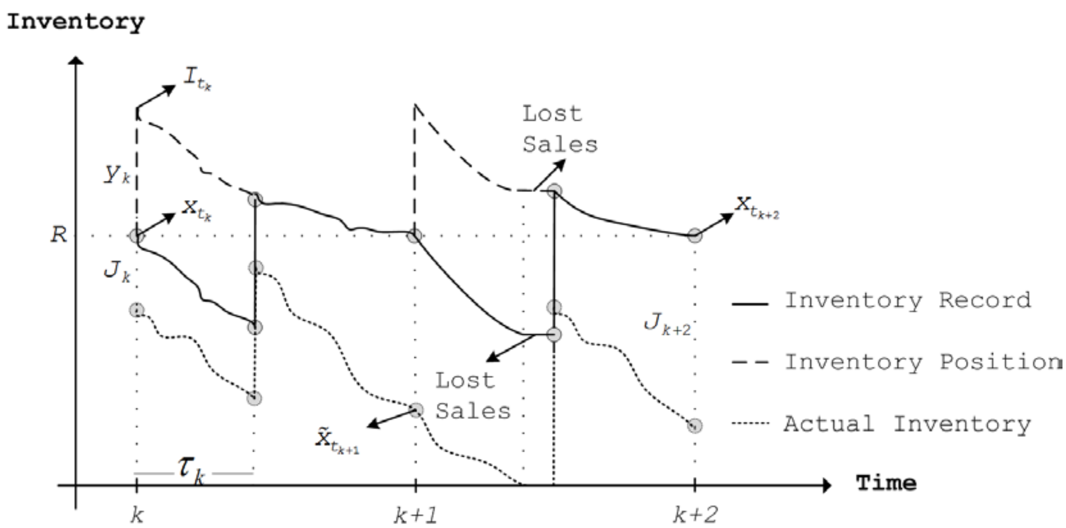

Figure 2-1: Behavior of the physical stock, inventory stock, and inventory position

Also let y_k be the order quantity given by the decision maker and D_k be the total demand in period k. Hence, at the start of period, the inventory records are checked and updated by reordering y_k units of inventory. After τ_k units of time, the order is received. The standard procedure continues until period $k+1$, when records reach to the reorder level R. Up until this point T_k time periods passed and ε_k amounts of error occurred which made total error equal to J_{k+1}. As seen in the figure, after the second replenishment decision, the inventory keeps decreasing rapidly until none left. Eventually this rapid decrease results in a stock-out. The demand occurring during this interval is lost until inventory is replenished.

Suppose that demand has a known distribution function $F_D(z) = P\{D \le z\}$ with a density $f_D(z)$. Furthermore, let S_k be the amount of sales and ε_k be the discrepancy between the actual and the recorded inventory during period k. In a perfect world where there is no IRI, no lead time, and no random supply, $\tilde{x}_{t_k} = x_{t_k}$. Hence, inventory progression can be formulated as

$$\tilde{x}_{t_k} = x_{t_k} = x_{t_{k-1}} + y_k - S_k \,, \tag{2.1}$$

where

$$S_k = \min\{D_k, x_{t_k}\}. \tag{2.2}$$

Since there is no lead time and no randomness in yield, the orders will arrive at the beginning of the next cycle. The total physical inventory will be updated upon the arrival of the order. The demand will be satisfied afterwards. In order account for IRI as it occurs in real life we modify equation (2.1) as,

$$\tilde{x}_{t_k} + \sum_{k \in K} \varepsilon_k = x_{t_k} = x_{t_{k-1}} + y_k - S_k \,. \tag{2.3}$$

Furthermore, the randomness in supply is implemented using two most commonly encountered methods: yield and capacity (Erdem & Ozekici, 2002; Henig & Gerchak, 1990).

In random capacity models, typically the supplier has a replenishment power which is a random variable, represented by K with a known distribution function $F_K(v)$ that has a density $f_K(v)$. When an order is placed for y_k units, the suppliers will ship y_k if the total amount of on-hand inventory they have, K, is greater than y_k. Or else, they will send their entire inventory, which is K.

In random yield models, it is assumed that the amount ordered could be different from the amount received so that only a fraction enters the stockpile. The randomness in this case is represented by a random variable U with a known distribution function $F_U(u)$ that has a density $f_U(u)$. When an order is placed for y_k units, the amount received will be Uy_k.

When the supply uncertainty is caused by both sources, equation (2.3) becomes,

$$E\left[x_{t_k}\right]=E\left[x_{t_{k-1}}\right]+E\left[U_k \min\{K_k, y_k\}\right]-E\left[\min\{D_k, x_{t_k}\}\right]. \tag{2.4}$$

For practicality let Y_k denote the random order received after ordering y_k units of inventory. In other words

$$Y_k = U_k \min\{K_k, y_k\}.$$

2.1.1 Error Modeling

We consider the errors as previously classified. In this classification inbound and outbound errors occur during receiving and selling transactions. Errors during receiving and selling are modeled as $\varepsilon_k^r = \gamma Y_k$ and $\varepsilon_k^s = \delta S_k$ respectively, where $\gamma \in [-1,\infty]$ and $\delta \in [-1,\infty]$. More information on transaction errors can be found in Morey (1985) and Rosetti et al. (2010). Due to the nature of the transaction procedures, selling and receiving errors can be positive or negative. In this context, a negative selling error corresponds to the multiple scanning of the same product; whereas, a positive selling error is the mistake of not scanning an item during check-out. Similarly, negative receiving error is getting more items than the ordered quantity due to the supply or the loading errors; whereas, a positive receiving error corresponds to getting less than the ordered quantity. The parameters for the transaction errors γ and δ are both bounded by -1 because the highest possible negative error that can be done cannot exceed the total order quantity and the total sales, respectively. In other words, the maximum amount of negative selling errors that can be done is equal to the amount of total sales.

Shelving errors are caused primarily due to stock-loss. Stock-loss has three main components: Theft $\varepsilon_k^t = \min\{\alpha D, x_{t_k}\}$, misplacement $\varepsilon_k^m = \theta x_{t_k}$ and expiration $\varepsilon_k^e = \beta \max\{x_{t_k} - S_k, 0\}$ where $\alpha \geq 0$ is the rate of theft, $\theta \in [0,1]$ is the percentage of items misplaced and $\beta \in [0,1]$ is the rate of expiration/spoilage. More information can be found in Rekik et al. (2009), Yan et al. (2011), and Rekik et al. (2008). The parameters for the shrinkage errors are all non-negative numbers because it is assumed that a non-existing product cannot become salable (e.g. an expired item being unexpired). Finally, the shrinkage errors ε_k^t, ε_k^e and ε_k^m are all bounded by the total inventory available since it is not possible to lose an item that the system does not currently have.

Equation (2.4) can be rewritten using the relation in (2.3) as

$$\begin{aligned} E\left[\tilde{x}_{t_k}\right] &= E\left[x_{t_{k-1}}\right] + E\left[U_k \min\{K_k, y_k\}\right] - E\left[\min\{D_k, x_{t_k}\}\right] \\ &+ \sum_{j=0}^{k} E\left[\gamma U_j \min\{K_j, y_j\}\right] + \sum_{j=0}^{k} E\left[\delta \min\{D_j, x_{t_j}\}\right] \\ &- \theta \sum_{j=0}^{k} E\left[x_{t_j}\right] - \sum_{j=0}^{k} E\left[\min\{\alpha D_j, x_{t_j}\}\right] - \beta \sum_{j=0}^{k} E\left[\max\{x_{t_j} - S_j, 0\}\right]. \end{aligned} \tag{2.5}$$

Furthermore, by subtracting equation (2.4) from equation (2.5), the inventory record error can be formulated as

$$\begin{aligned} E\left[\tilde{x}_{t_k} - x_{t_k}\right] &= \sum_{j=0}^{k} E\left[\gamma U_j \min\{K_j, y_j\}\right] + \sum_{j=0}^{k} E\left[\delta \min\{D_j, x_{t_j}\}\right] \\ &+ \beta \sum_{j=0}^{k} \left(E\left[\min\{D_j, x_{t_j}\}\right]\right) - \sum_{j=0}^{k} \left(E\left[\min\{\alpha D_j, x_{t_j}\}\right]\right) \\ &- (\theta + \beta) \sum_{j=0}^{k} E\left[x_{t_j}\right]. \end{aligned} \tag{2.6}$$

We introduce the lead time uncertainty to equation (2.6) by separating the demand into two parts: lead time demand D', and the demand for the rest of the period, D''. Then, equation (2.6) becomes

$$
\begin{aligned}
E[J_k] = {} & \sum_{j=0}^{k} E\left[\gamma U_j \min\{K_j, y_j\}\right] + \sum_{j=0}^{k} E\left[\delta \min\{D_j, x_{t_j}\}\right] \\
& + \beta \sum_{j=0}^{k} \left(E\left[\min\{D_j', x_{t_j}\}\right] + E\left[\min\{D_j'', Y_j + w_j\}\right] \right) \\
& - \sum_{j=0}^{k} \left(E\left[\min\{\alpha D_j', x_{t_j}\}\right] + E\left[\min\{\alpha D_j'', Y_j + w_j\}\right] \right) \\
& - (\theta + \beta) \sum_{j=0}^{k} E\left[x_{t_j} + Y_j + w_j\right]
\end{aligned}
\tag{2.7}
$$

where J_{t_k} is the total error made until period k and w_k is the safety stock for period k.

2.1.2 General Inventory Formulation

The underlying problem for equation (2.7) is not easy to solve due to recursive relationship between parameters. In order to reduce this complexity, we implemented two models in which the best and the worst possible situations are analyzed. The difference between them is the order of events. Figure 2-2 shows the order of events for each model. Each period k is divided into two phases: the first phase contains lead time demand and the second phase contains the demand for the rest of the period. Replenishment time determines the end of the first phase.

In the best case scenario the demand is assumed to be fulfilled first and then errors occur. Since the sold items are outside of the feasible space for errors, this scenario maximizes the demand fill rate and minimizes the IRI. In the worst case scenario, the errors occur first and then the demand is fulfilled; thus, the fill rate is minimized. In reality, the inventory behaves somewhere between best and worst case situations; hence the two characterizations provide a lower and an upper bound. In this model ε_k^1 denotes outbound plus shelving errors during lead time, ε_k^2 denotes inbound errors, and ε_k^3 denotes outbound plus shelving errors during the remainder of period k.

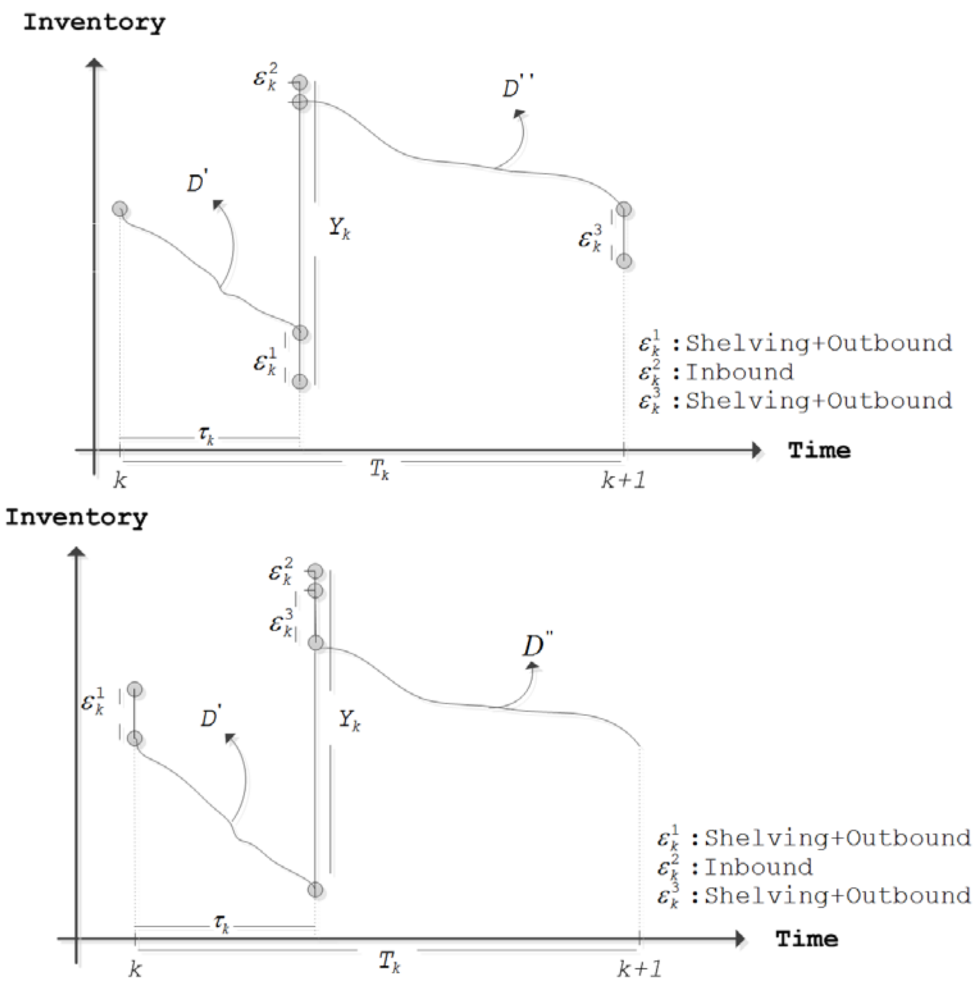

Figure 2-2: The best (left) and the worst (right) case inventory behavior

Full formulations of the error functions for the best and the worst case scenarios can be found in Appendix A.I and A.II. Error function structures for both scenarios are dependent on input parameters α, β, θ, γ and δ. Based on their configuration, the error function can be increasing or decreasing with lead time demand. However, the error function value increases as R, W, and Y increase (i.e., more inventory elevates error). Lead time demand has direct and indirect effect on error in both scenarios. As demand increases, the system observes more outbound errors but leads to fewer inventory and less shelving errors.

2.1.3 Numerical Study

Characterization and behavior of the developed error function are analyzed using a case study provided by an appliance and furniture company (The data provided ranges from 1990 - 2003). In the case study, a continuous (Q,R) policy is utilized with $(600,80)$. Weekly demand D and lead time τ are normally distributed with $(50,12^2)$ and $(1.14,0.33^2)$ respectively.

The parameters for the errors are selected from various examples in the literature. The transaction errors are assumed to be uniformly distributed, $\delta \sim unif(-1\%,1\%)$ and $\gamma \sim unif(-2\%,2\%)$ (Morey, 1985; Rosetti et al., 2010). The shelving parameters are defined as $\alpha = 1\%$ and $\beta = 0.5\%$ for theft and misplacement, respectively (Rekik et al., 2008, 2009; Yan et al., 2011).

Over 2000 random numbers for D and τ with 5 replications are generated to obtain the expected errors in a single period. There are seven different sources of errors in each period. We categorize these errors into 3 types: three of these errors are first phase errors, three of them are second phase errors and the last one is the inbound error in between phases.

We conducted a discrete event simulation for 52 periods with 60 replications for (Q,R) policy. It is assumed that the model starts with 0 IRI when the inventory records and actual physical stock equal to the reorder level. The model is depicted in Figure 2-3.

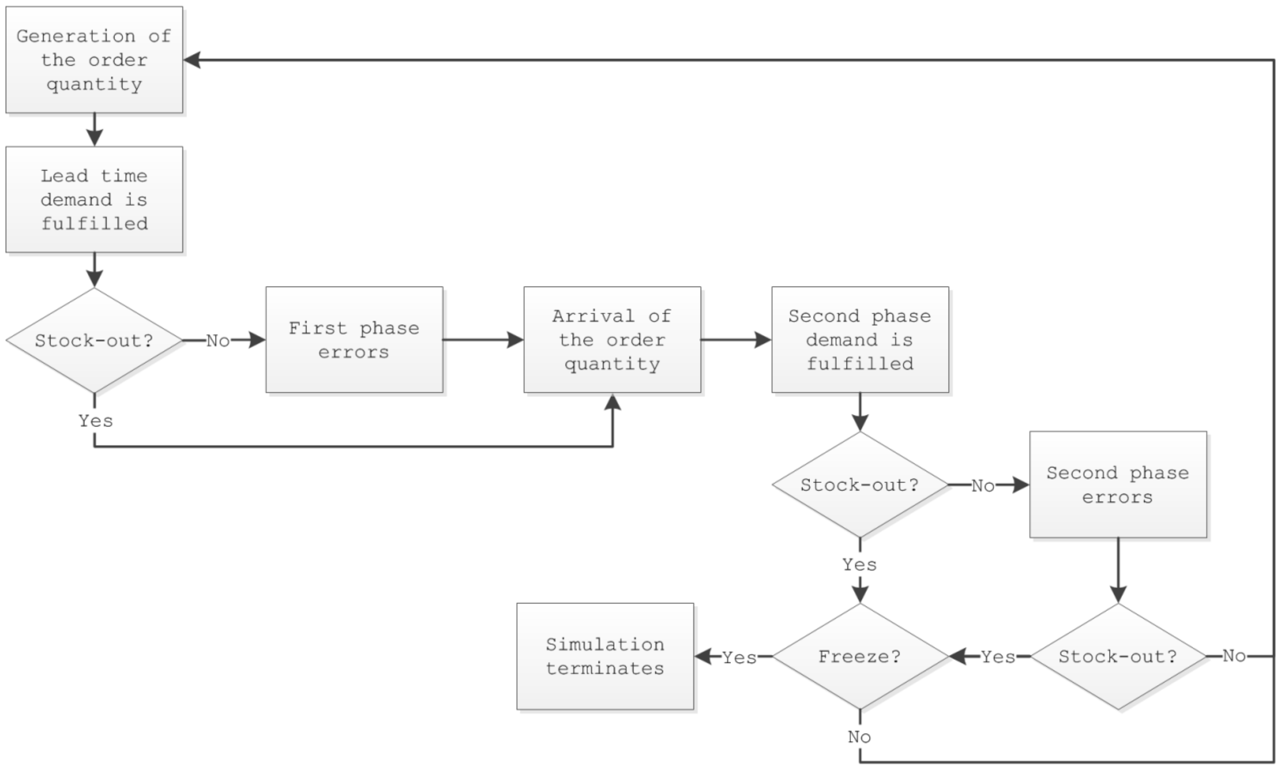

Figure 2-3: Flowchart of the simulation model

In this setting, the simulation is terminated by one of the two possible outcomes. Either the gradual error build up becomes too big and causes the inventory to freeze, or the system reaches period 52 and terminates itself normally. In this context, we use the term *freeze* to describe the situations in which the IRI becomes exceeds the reorder level; hence, no more replenishment orders can be given. Details about the characterization and calculation of freezing are presented in the next section.

Remark: Freezing is a frequently encountered problem in practice. Upon encountering a freeze situation the sales immediately stop until the errors are corrected. A common practice to overcome this situation is using a zero sales check mechanism (Raman et al., 2001). Using this method, the sales are tracked for a specified duration. If they remain constant over this interval than a cycle count is performed to correct the errors. In the simulation study, we are not implementing a correction model, hence once freezing is observed the simulation terminates.

Table 2-1: Correlation matrix of 7 different types of errors in the best case

Corr	D'	D''	**1st** ε_k^s	**1st** ε_k^t	**1st** ε_k^e	**1st** ε_k^r	**2nd** ε_k^s	**2nd** ε_k^t	**2nd** ε_k^e	**1st Phase**	**2nd Phase**
D'	1										
D''	0.014	1									
1st ε_k^s	-0.16	-0.00	1								
1st ε_k^t	0.006	-0.00	0.14	1							
1st ε_k^e	-0.97	-0.01	0.11	-0.16	1						
1st ε_k^r	0.004	0.01	-0.01	-0.01	-0.01	1					
2nd ε_k^s	-0.03	-0.21	0.007	-0.01	0.035	-0.01	1				
2nd ε_k^t	-0.12	-0.32	0.008	-0.02	0.128	-0.02	0.193	1			
2nd ε_k^e	-0.12	-0.94	0.023	-0.01	0.125	-0.04	0.135	0.143	1		
1st Phase	-0.27	-0.01	0.943	0.434	0.173	-0.00	0.009	0.015	0.033	1	
2nd Phase	-0.08	-0.38	0.011	-0.01	0.085	-0.02	0.935	0.514	0.264	0.016	1

Tables 2-1 to 2-6 summarize the results of simulation studies. In the tables D' denotes lead time demand, D'' is the remaining demand, ε_k^s denotes outbound (selling) errors, ε_k^t denotes errors due to theft, ε_k^e denotes errors due to expiration, and ε_k^r denotes inbound (receiving) errors. Recall that theft and expiration forms shelving errors. Correlation and covariance matrices of the demand, 7 types errors and total errors in the first and the second phases are given in Table 2-1 and Table 2-2. Based on the correlation matrix, errors have no strong dependence between each other.

Table 2-2: Covariance matrix of 7 different types of errors in the best case

Cov	D'	D''	**1st** ε_k^s	**1st** ε_k^t	**1st** ε_k^e	**1st** ε_k^r	**2nd** ε_k^s	**2nd** ε_k^t	**2nd** ε_k^e	**1st Phase**	**2nd Phase**
D'	469.76										
D''	49.405	2357									
1st ε_k^s	-2.353	-0.9	0.40								
1st ε_k^t	0.03	-0.10	0.02	0.058							
1st ε_k^e	-2.01	-0.20	0.01	-0.01	0.009						

Cov	D'	D''	1st ε_k^s	1st ε_k^t	1st ε_k^e	1st ε_k^r	2nd ε_k^s	2nd ε_k^t	2nd ε_k^e	1st Phase	2nd Phase
1st ε_k^r	0.348	9.61	-0.01	-0.01	-0.01	11.889					
2nd ε_k^s	-4.295	-175	0.02	-0.01	0.018	-0.261	29.3				
2nd ε_k^t	-5.67	-105	0.01	-0.01	0.025	-0.184	2.21	4.457			
2nd ε_k^e	-1.67	-91.2	0.01	-0.00	0.007	-0.09	0.46	0.191	0.396		
1st Phase	-4.335	-1.2	0.43	0.075	0.011	-0.022	0.03	0.023	0.015	0.525	
2nd Phase	-11.64	-372	0.04	-0.02	0.051	-0.536	32.0	6.860	1.051	0.076	39.96

The variance of the errors per period is 51.417, or standard deviation 7.17. The mean of the errors per period is 3.83; 0.49 from phase 1 and 3.34 from phase 2. In order to validate the mean and variance results, we performed a goodness of fit test for the error values based on the generated data. The results indicate that they are normally distributed with (4.07, 7.12) (The mean squared error for the fit is below 0.0005 and p-value for the Chi-Squared test is below 0.5). Looking from this perspective the errors can be treated as another source of demand with mean 3.83 and standard deviation 7.17. Table 3 shows the summary of the results obtained from this study.

Table 2-3: Summary of statistics of the best case

Stock-out	Error	Length	Time	Sold	n(R)	Shelving	Inbound	Outbound
9	85	12.301	251.421	9,595	36	89	(3)	(1)

Table 2-3 shows the observed results for 9 inventory performance measures obtained as a result of the numerical study for the best case. The stock-out refers to the amount of time where the actual physical stock dropped to zero. The error column denotes the average number of IRI accumulated in the system. The length column displays the average time length for a period and time column displays the average overall length of the simulation study. Sold column shows the average number of units sold. The remaining three columns represent the average errors.

Based on this table, we observe low values for stock-out, sales and errors because of early terminations. The gradual increase of IRI forces the inventory system to freeze which in turn forces the simulation to stop early. The average length of a period is recorded as 12.301 weeks and the overall time of the simulation study is at 251.421 weeks. The average lifecycle of the simulations is at 18 periods instead of 52. Inbound and outbound errors are close to zero. This is because of the assumption on the distribution functions; it is equally likely for inbound and outbound errors to be positive or negative.

Table 2-4: Correlation matrix of 7 different types of errors in the worst case

Corr	D'	D''	**1st** ε_k^s	**1st** ε_k^t	**1st** ε_k^e	**1st** ε_k^r	**2nd** ε_k^s	**2nd** ε_k^t	**2nd** ε_k^e	**1st Phase**	**2nd Phase**
D'	1										
D''	-0.00	1									
1st ε_k^s	-0.00	-0.01	1								
1st ε_k^t	1	-0.00	-0.001	1							
1st ε_k^e	-0.30	0.01	-0.951	-0.306	1						
1st ε_k^r	-0.01	0.01	-0.005	-0.010	0.01	1					
2nd ε_k^s	-0.00	-0.01	-0.001	-0.004	0.002	0.012	1				
2nd ε_k^t	-0.00	1	-0.013	-0.002	0.013	0.008	-0.005	1			
2nd ε_k^e	-0.11	-0.94	0.012	-0.118	0.025	-0.02	-0.026	-0.94	1		
1st Phase	0.306	-0.01	0.95	0.306	-1	-0.01	-0.003	-0.01	-0.02	1	
2nd Phase	-0.01	0.15	-0.003	-0.017	0.01	0.01	0.986	0.152	-0.16	-0.00	1

The summary of the results for the worst case scenario is presented in Table 2-4 and Table 2-5. The results from Table 2-4 indicate that contrary to the best case, errors show dependence between each other for the worst case.

Table 2-5: Covariance matrix of 7 different types of errors in the worst case

Cov	D'	D''	**1st** ε_k^s	**1st** ε_k^t	**1st** ε_k^e	**1st** ε_k^r	**2nd** ε_k^s	**2nd** ε_k^t	**2nd** ε_k^e	**1st Phase**	**2nd Phase**
D'	468,4										
D''	-6.63	2268									

Cov	D'	D''	**1st** ε_k^s	**1st** ε_k^t	**1st** ε_k^e	**1st** ε_k^r	**2nd** ε_k^s	**2nd** ε_k^t	**2nd** ε_k^e	**1st Phase**	**2nd Phase**
1st ε_k^s	-0.01	-1.34	0.451								
1st ε_k^t	4.68	-0.06	-0.00	0.046							
1st ε_k^e	-0.02	0.007	-0.00	-0.00	0.00						
1st ε_k^r	-0.76	4.548	-0.01	-0.01	0.00	11.89					
2nd ε_k^s	-0.55	-4.18	-0.01	-0.00	0.00	0.258	33.78				
2nd ε_k^t	-0.06	226.8	-0.01	-0.00	0.00	0.045	-0.04	2.268			
2nd ε_k^e	-1.57	-87.4	0.004	-0.01	0.00	-0.05	-0.09	-0.87	0.379		
1st Phase	4.649	-1.40	0.448	0.046	-0.00	-0.01	-0.01	-0.01	-0.01	0.49	
2nd Phase	-2.19	135.1	-0.01	-0.02	0.00	0.253	33.65	1.351	-0.59	-0.03	34.41

Using the covariance matrix, the variance of the errors per period is 47.192, or the standard deviation 6.86. The mean of the errors per period is 6.78; 0.97 from the first phase and 5.81 from the second phase. Again normal distribution is tested on the generated data, it turns out the data fits into a normal distribution with (6.69, 7) (The mean squared error is under 0.001 and the Chi-Squared p-value is below 0.005). As expected, the average errors in the worst case are larger than the best case. Still, the errors can be treated as another source of demand with mean 6.78 and standard deviation 6.86.

Table 2-6: Summary statistics of the worst case

Stock-out	Error	Length	Time	Sold	n(R)	Shelving	Inbound	Outbound
7	88	11.8211	173.632	5,622	29	89	(1)	0

The early terminations are observed in worst case as well. On average the system maintains its lifecycle for 10 periods. The ratio between the average number of items sold and lost sales is 193.86 which is lower than the best case. The inbound and the outbound errors are again close to zero.

2.2 Evaluation of the Impact of IRI

In the previous section we presented the underlying reasons for IRI and their influence on different inventory parameters. Additionally, a general framework for modeling the behavior of errors in terms of inventory is presented. In this section we explore two approaches to manage errors. When the existence of the IRI is acknowledged, two alternatives are considered: Increasing the resistance of the current inventory policy, or controlling the factors that cause IRI. These two alternatives are systematically analyzed in this section to reveal the impact of IRI and the influence of other important key measures, such as average number of units sold in a period, the amount of stock-outs, the expected amount of lost sales, etc.

2.2.1 Freezing Potential

In previous section it is demonstrated that when there are no correction procedures present both cases end up in freeze. Hence, it is possible to create a characterization for the probability of freezing for both scenarios.

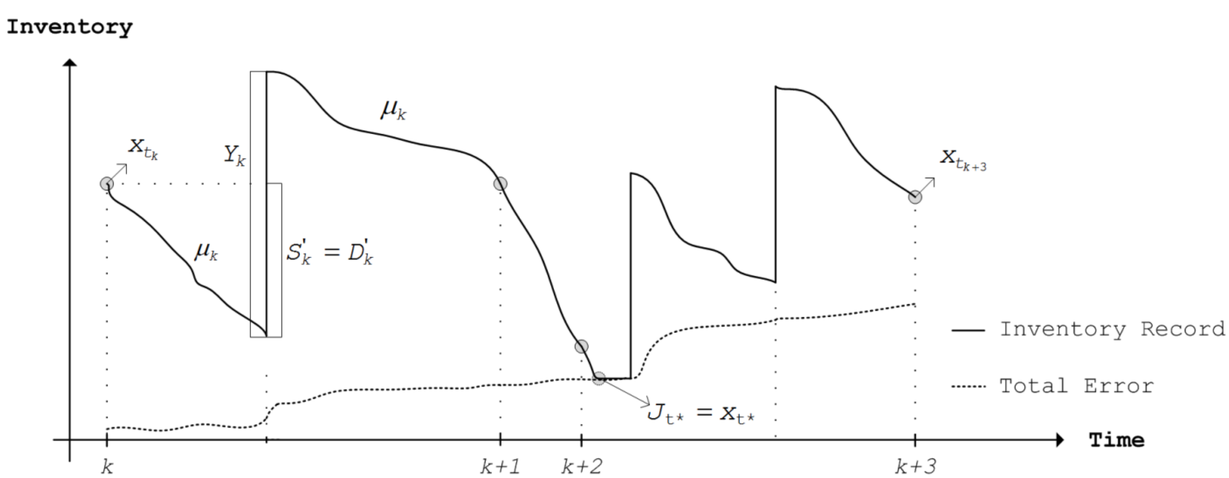

Figure 2-4: The relation between records and errors

Figure 2-3 illustrates a freeze situation by looking at the total errors. At time t_k the total amount of error is J_{t_k}. At the end of period k, ε_k is added to J_{t_k}. The same setup continues throughout the timeline.

An interesting observation at this point is the time at which the inventory records are equal to total errors, $x_{t^*} = J_{t^*}$. Briefly, when total errors catch up to the point at which they are equal to inventory records, an unobserved stock-out occurs. In other words the demand is still there and the records show positive inventory, but the sales stays at zero since the actual physical stock is zero. This situation is different than a normal stock-out in the sense that unless there is a replenishment order on the way, the inventory system is frozen. Such an occurrence has drastic effects both on short and long term.

i. Best Case

Since in both phases the demand is fulfilled first, in a freezing situation the inventory has to be frozen at a value above reorder level before the complete demand for the second phase is fulfilled. Based on the period layout it is not possible for the second phase demand to be fulfilled before inventory records reach the reorder level, R. In other words,

$$\begin{aligned} P\{\text{Freezing at } k\} &= P\{x_k > R \cap \tilde{x}_k = 0\} = P\{x_k - \tilde{x}_k > R \cap \tilde{x}_k = 0\} \\ &= P\{J_k + \varepsilon_k^1 + \varepsilon_k^2 > R \cap \tilde{x}_k = 0\}. \end{aligned} \tag{2.8}$$

When $k = 0$ or $J_k = 0$, the remaining part $\varepsilon^1 + \varepsilon^2$ constitutes the total errors done in the first phase (see Figure 2-2).

Note that if $D' > R$ than all the actual physical stock on-hand is sold during the lead time; which implies that the first phase errors are 0. If $D' \le R$, ε^1 for the best case can be formulated as

$$\varepsilon^1 = \begin{cases} E\left[\min\{\delta D', R-D'\}\right] & \text{if } D' > \dfrac{R}{1+\delta} \\ E\left[\min\{\delta D', R-D'\}\right] + E\left[\min\left\{\begin{matrix}\alpha D', R-D' \\ -\min\{\delta D', R-D'\}\end{matrix}\right\}\right] & \text{if } \dfrac{R}{1+\delta} < D' \le \dfrac{R}{1+\alpha+\delta} \\ E\left[\min\{\delta D', R-D'\}\right] + E\left[\min\left\{\begin{matrix}\alpha D', R-D' \\ -\min\{\delta D', R-D'\}\end{matrix}\right\}\right] & \\ +\beta E\left[\begin{matrix} R-D' - \min\{\delta D', R-D'\} \\ -\min\left\{\begin{matrix}\alpha D', R-D' \\ -\min\{\delta D', R-D'\}\end{matrix}\right\}\end{matrix}\right] & \text{if } D' \le \dfrac{R}{1+\alpha+\delta} \end{cases} . \qquad (2.9)$$

Remark: Equation (2.9) shows the calculation of the first phase errors, when $D' \le R$, based on the lead time demand D' and the input parameters $\delta \in [-1,\infty]$, $\beta \in [0,1]$ and $\alpha \ge 0$. In this formulation when $(1+\delta)D' > R$, the actual physical stock drops to zero after the outbound errors in the first phase; in the second line the actual physical stock depletes after theft errors; and when $(1+\alpha+\delta)D' \le R$ there is enough inventory left in the system for all the first phase errors. For practicality we make a notational change for the rest of the dissertation. Hence, equation (2.9) is rewritten as

$$\begin{aligned}\varepsilon^1 &= E\left[\min\{\delta D', R-D'\}\right]_{\{D' \le R\}} + E\left[\min\{\alpha D', R-D' - \min\{\delta D', R-D'\}\}\right]_{\{(1+\delta)D' \le R \cap D' \le R\}} \\ &\quad + \beta E\left[\begin{matrix} R-D' - \min\{\delta D', R-D'\} \\ -\min\{\alpha D', R-D' - \min\{\delta D', R-D'\}\}\end{matrix}\right]_{\{D'(1+\alpha+\delta) \le R \cap D' \le R\}} .\end{aligned} \qquad (2.10)$$

Similarly, ε^2 is $\gamma E[Y]$. Thus,

$$\begin{aligned}\varepsilon^1 + \varepsilon^2 &= E\left[\min\{\delta D', R-D'\}\right]_{\{D' \le R\}} + E\left[\min\left\{\begin{matrix}\alpha D', R-D' \\ -\min\{\delta D', R-D'\}\end{matrix}\right\}\right]_{\{(1+\delta)D' \le R \cap D' \le R\}} \\ &\quad + \beta E\left[\begin{matrix} R-D' - \min\{\delta D', R-D'\} \\ -\min\{\alpha D', R-D' - \min\{\delta D', R-D'\}\}\end{matrix}\right]_{\{D'(1+\alpha+\delta) \le R \cap D' \le R\}} + \gamma E[Y].\end{aligned} \qquad (2.11)$$

The highest value β can get is 1, which intuitively means everything left at the end of each phase will be lost. Hence, the whole equation becomes $R-D'$, which implies that first phase errors, ε^1 cannot be greater than R, This immediately provides an easy upper bound for freeze probability based on the distribution of ε^2. Inbound errors can be positive or negative by definition.

If in fact they assume a negative value freezing cannot occur since errors up to inbound cannot be greater than R. In other words, $0 \le P\{\text{Freezing}\} \le P\{\varepsilon^2 \ge 0\}$

A tighter bound can be computed using $\varepsilon^1 + \varepsilon^2$ as

$$P\{\text{Freezing}\} \le P\left\{\begin{array}{l} R < E\left[\min\{\delta D', R - D'\}\right]_{\{D' \le R\}} \\ \quad + E\left[\min\{\alpha D', R - D' - \min\{\delta D', R - D'\}\}\right]_{\{(1+\delta)D' \le R \cap D' \le R\}} \\ \quad + \beta E\left[\begin{array}{l} R - D' - \min\{\delta D', R - D'\} \\ - \min\{\alpha D', R - D' - \min\{\delta D', R - D'\}\} \end{array}\right]_{\{D'(1+\alpha+\delta) \le R \cap D' \le R\}} \\ \quad + \gamma E[Y]. \end{array}\right\} \qquad (2.12)$$

since $P\{\text{Freezing}\} = P\{\varepsilon_k^1 + \varepsilon_k^2 > R \cap \tilde{x}_k = 0\}$. Following a similar logic, by conditioning on the lead time demand the inequality in (2.12) can be transformed into,

$$P\{R < \varepsilon^1 + \gamma E[Y] \| D' \le R\} P\{D' \le R\} + 0 P\{D' \ge R\}. \qquad (2.13)$$

This can be written as,

$$\begin{array}{l} P\{R < R - D' + \gamma E[Y] \| (1+\delta)D' \ge R \cap D' \le R\} P\{(1+\delta)D' \ge R \cap D' \le R\} \\ + P\{R < \varepsilon^1 + \gamma E[Y] \| (1+\delta)D' \le R \cap D' \le R\} P\{(1+\delta)D' \le R \cap D' \le R\}. \end{array} \qquad (2.14)$$

Equation (2.14) can be further expanded by conditioning on the second term. In other words,

$$\begin{array}{l} P\{R < R - D' + \gamma E[Y] \| (1+\delta)D' \ge R \cap D' \le R\} P\{(1+\delta)D' \ge R \cap D' \le R\} \\ + P\{R < R - D' + \gamma E[Y] \| D'(1+\alpha+\delta) \ge R \cap D' \le R\} P\{D'(1+\alpha+\delta) \ge R \cap D' \le R\} \\ + P\left\{\begin{array}{l} R < \beta E[R - D'(1+\alpha+\delta)] \\ \quad + E[(\alpha+\delta)D' + \gamma Y] \end{array} \middle\| D'(1+\alpha+\delta) \le R \cap D' \le R\right\} P\{D'(1+\alpha+\delta) \le R \cap D' \le R\}. \end{array} \qquad (2.15)$$

Based on the distributions of δ, α, β, γ and D this function can be calculated as an upper bound for the best case framework freeze probability.

ii. Worst Case

The formulation for freezing in the worst case is similar,

$$\begin{array}{l} P\{\text{Freezing at } k\} = P\{x_k > R \cap \tilde{x}_k = 0\} = P\{x_k - \tilde{x}_k > R \cap \tilde{x}_k = 0\} \\ \qquad = P\{J_k + \varepsilon_k^1 + \varepsilon_k^2 + \varepsilon_k^3 > R \cap \tilde{x}_k = 0\}. \end{array} \qquad (2.16)$$

The function can be rewritten as

$$P\left\{J_k + \varepsilon_k^1 + \varepsilon_k^2 + \varepsilon_k^3 > R\right\} = P\left\{\begin{array}{l} R < \delta E\left[\min\left\{D^{'}, R\right\}\right] + E\left[\min\left\{\alpha D^{'}, R - \delta D^{'}\right\}\right]_{\left\{\delta D^{'} \leq R\right\}} \\ \quad + \beta E\left[R - \delta \min\left\{D^{'}, R\right\} - \min\left\{\alpha D^{'}, R - \delta D^{'}\right\}\right]_{\left\{D^{'}(\alpha+\delta) \leq R\right\}} \\ \quad + \gamma E[Y] + \varepsilon_k^3 \end{array}\right\}. \qquad (2.17)$$

When $k=0$ or $J_k=0$ and $\beta=1$ the function becomes $P\left\{0 < Y - E\left[D^{'}\right]\right\}$. Based on the distribution of D this function can be calculated as an upper bound.

2.2.2 IRI Resistance Method

We have shown that the amount of errors is not the only important factor, the IRI susceptibility of the inventory policy is equally important as well. In this context, the term IRI resistance refers to the adopted inventory policy’s susceptibility to errors. We utilize a scheme that aims to account for the error and incorporate replenishment decisions accordingly.

Our approach is similar to the myopic model designed by Dehoratius et al. (2008). The researchers model the multi-period problem as an infinite horizon with no fixed ordering cost and zero lead time. When lead times are non-zero, the myopic solution becomes a heuristic for an infinite horizon problem. In their heuristic myopic algorithm, they adjust the inventory records based on the stock-out probability. However, in our model we develop a formulation for errors and utilize this formulation to generate the expected error, which is then used to adjust the safety stock level.

Figure 2-4 illustrates the behavior of inventory under a (Q,R) policy with gradually increasing safety stock. At each decision epoch the replenishment order is given based on the adjusted (Q,R) policy. The order quantity remains unchanged but the reorder level increases. As the figure illustrates, this method aims to compensate for the IRI by increasing the inventory records so that the actual physical stock remains relatively stable. The situation continues until a cycle count is triggered, and the model reverts back to period zero.

Remark: This method does not require any additional investments from the practices for implementation. By implementing a predictive approach, it generates the order decisions earlier than they are scheduled to counter IRI.

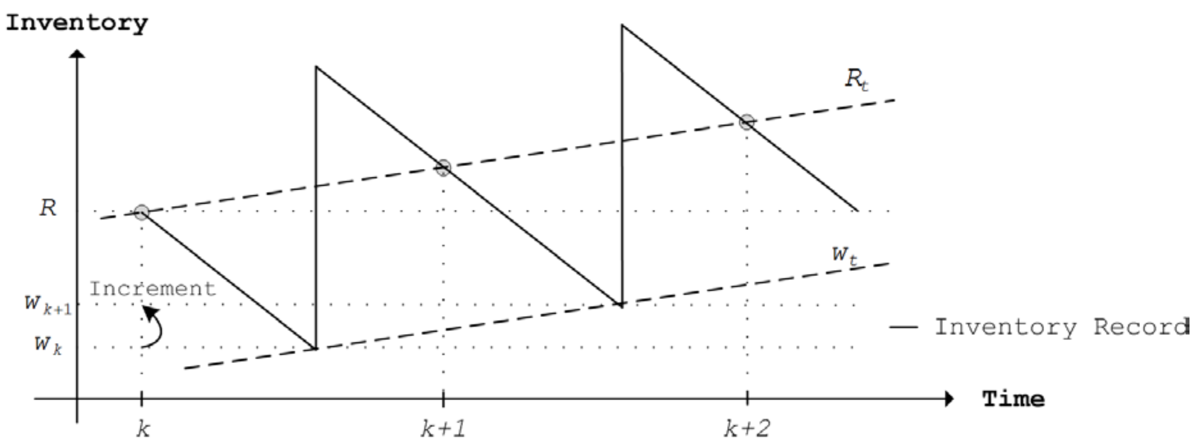

Figure 2-5: Increasing the safety stock

For the remainder of this dissertation we use the term *increment* to indicate the amount of increase for the safety stock at each period. As shown in Figure 2-4, the increment amount is critical for this method. We utilize the formulations derived for the best and the worst cases to obtain a mean and a variance for periodical errors. Then, these values are used to obtain an estimate for the increment amount. We used the same numerical study in Section 2.1.2 to demonstrate our results.

Using the variance and the mean, an estimate for the increment can be obtained for this method. To demonstrate this, we used on the same case study simulation and applied the IRI resistance method.

For the best and the worst case scenarios, we generated separate simulation models. Both models continue for 52 periods and replicated 60 times for integer increments 0 to 30. When the increment is 0, the algorithm is not being utilized therefore the model reverts back to original (Q,R) policy. For values greater than 0 the algorithm is active.

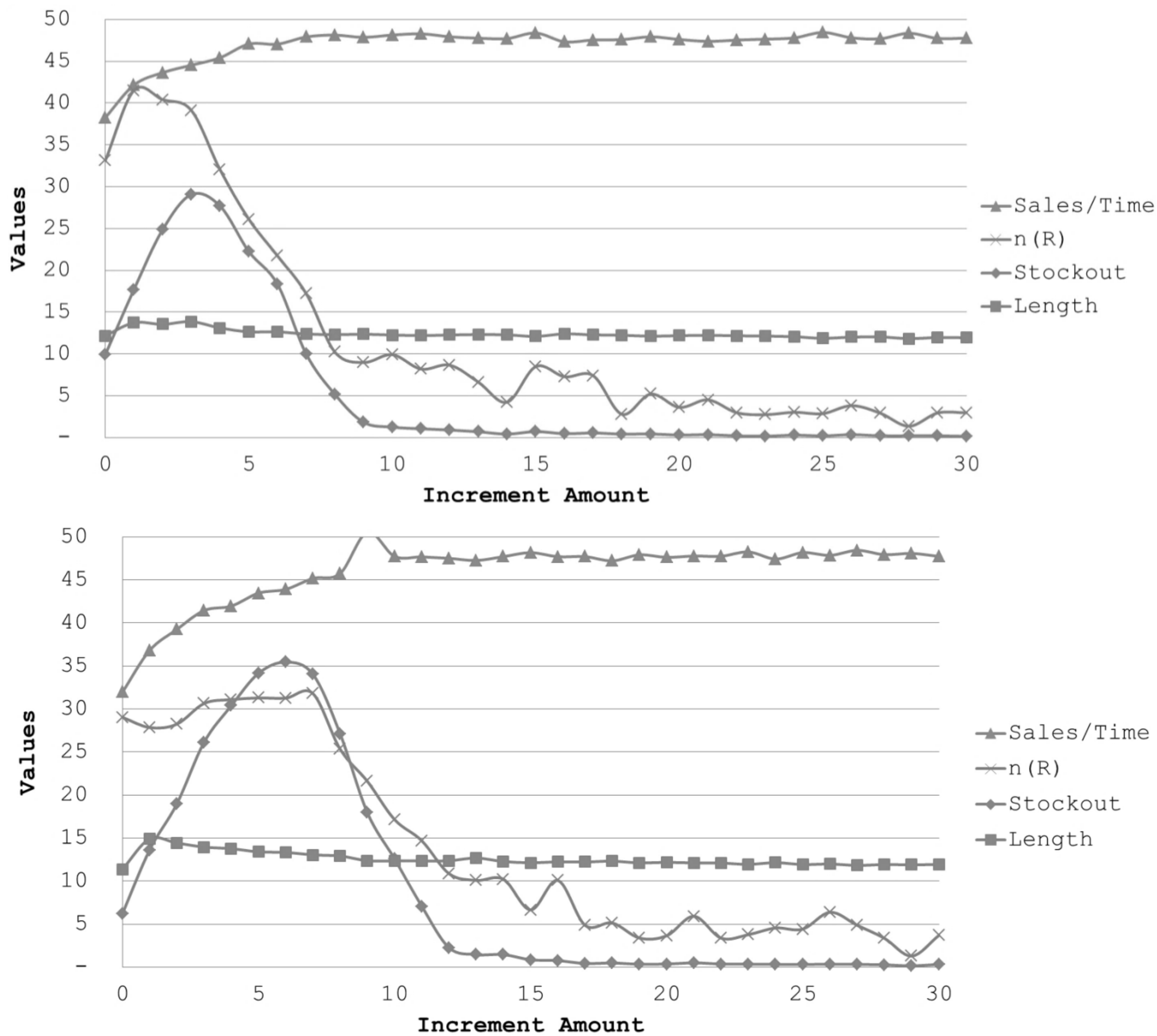

Figure 2-6: IRI Resistance method for the best and the worst cases

Figure 2-5 shows the combined results of the simulation study for each case. The horizontal axis denotes the increment amounts and the vertical axis represent the total numbers of the following parameters respectively: Sales per unit time, lost sales, stock-out and period length. As the increment increases, the sales per unit time also increases (unit time is a week) for both cases. This is because IRI resistance method compensates for some portion of the demand lost due to IRI. Moreover as the increment amount increases, the lost sales exhibits an increasing behavior first (for small increments) but then drops as increment keeps increasing. The reason behind this behavior for lost sales is freezing. For smaller increment levels, the inventory system is unable to reach period 52 due to the IRI. As a result the total lost sales remains low. But for bigger increment levels the inventory system is more resistant to IRI, and therefore can complete the simulation duration without freezing. Similar situation is observed for the total number of stock-outs as well. Finally, the average length of the period behaves independently from the increment.

Inbound and Outbound errors (not present in the figure) act erratically around 0. That is an expected result since they do not depend on the inventory on-hand.

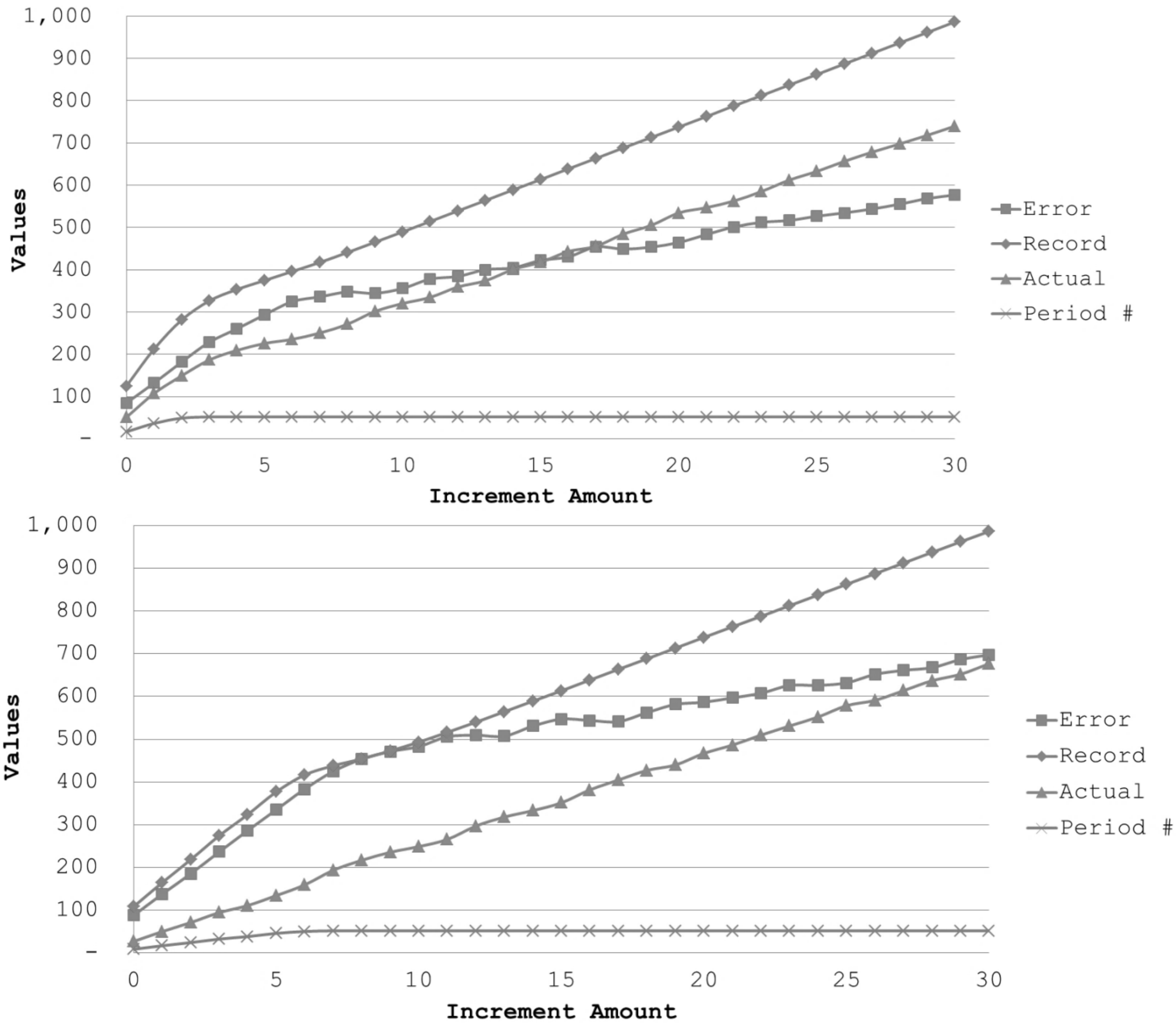

Figure 2-7: IRI Resistance method for the best and the worst cases cont.

Figure 2-6 shows the results of the remaining parameters from the simulation. As expected both the total error and total stock-loss increases with the increment. Similarly recorded and actual inventory is also increasing with increment. The reason behind these is, IRI resistance method increases the average inventory on-hand gradually to compensate for the IRI. Thus, as a result the system operates with higher amounts of inventory than normal which in turn increases IRI. The final line is the period number that the system managed to reach before the simulation time ended.

According to Figure 2-6, there is a break point in the graphs where the slopes changes. Also based on the simulation results higher increment levels hurt the system more than it benefits. Therefore, there is a range where the increment works best. Depending on preference (due to line of business), certain parameters can be chosen and the increments that maximize or minimize those values can be selected.

2.2.3 Error Control and Correction Method

One of the fundamental methods of error control is counting full physical inventory. This is a process where the entire inventory is reckoned physically (Iglehart & Morey, 1972; Opolon, 2010; Young & Nie, 1992). However, this procedure is long and costly, especially if there is a large number of inventory.

The literature on cycle counting and inventory auditing is vast (Iglehart & Morey, 1972; Kok & Shang, 2007; Kumar & Arora, 1991; Meyer, 1990; Rosetti et al., 2010; Young & Nie, 1992). While considering cycle counting practices there are several issues to be discussed. Kok and Shang (2007) present detailed analysis about cycle counting and consider many key aspects. The first aspect is the trade-off between inspection frequency and IRI related costs. The more frequent the cycle counts are, the lower the IRI is. Hence, choosing an appropriate mechanism to determine the frequency is important. On the other hand, inspection policies directly affect the amount of inventory stored which in turn affects the replenishment policy. It is beneficial to choose a replenishment policy while considering the inspection frequency. Additionally, the effectiveness of the cycle count itself is crucial since this process is prone to errors as well.

Remark: This method does not require any additional investments from the practices for implementation. However, it includes a counting cost based on the setup of the business and the magnitude of on-hand inventory.

As repeatedly mentioned in literature, the predominant factors for determining the frequency of cycle counting are costs and disruptions associated with it. Apart from these, there are other key measures that can determine the effectiveness of the counting mechanism: lead time, amount of expected error correction, triggering condition, amount of expected lost sales and IRI. Our model focuses on these key metrics rather than a cost based structure. It is assumed that the correction procedures are done error free.

Determining the best possible triggering condition is not an easy task. In our study we utilize the relation between the lead time sales and the expected demand during lead time to configure a *trigger mechanism*. In this relation, the expected lead time demand is a known value and the lead time sales is an observed value. The logic behind this trigger mechanism is: If the lead time sales is considerably lower than the expected demand during lead time, then it can be concluded that the system contains high amounts of IRI. However, determining the sensitivity of the trigger mechanism still remains as a daunting issue. To overcome this burden we inserted a modifier, called the *trigger value*. The main purpose of the trigger value is to adjust the expected demand during lead time. Via this method we can effectively change the sensitivity of the trigger mechanism.

Remark: In this context, sensitivity of the trigger mechanism determines how often cycle counts are triggered. If the mechanism has high (or low) sensitivity to IRI than higher (or lower) number of cycle counts will be observed.

The trigger value itself is a positive number; when it approaches 0, the trigger mechanism becomes extremely sensitive and as it increases the mechanism loses its sensitivity to IRI.

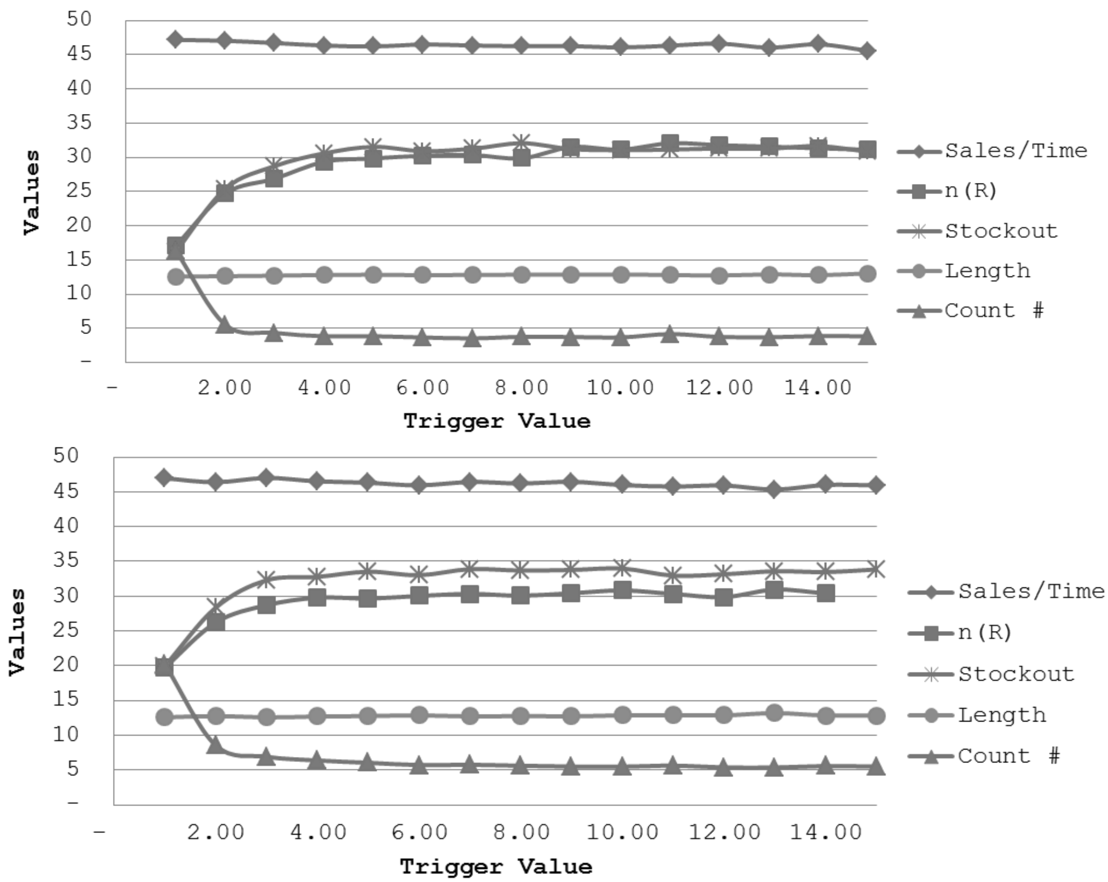

Figure 2-8: The error control method for the best (left) and the worst (right) case

For the best and the worst case, we generated separate simulation models. Both models use 60 replications for 52 periods for trigger values 1 to 15. Figure 2-7 and Figure 2-8 show the combined results of these simulation studies. The horizontal axis denotes the trigger values and the vertical axis represent the average total values after 60 replications.

Based on Figure 2-7, the sales per unit time and average period length are not affected by the changes trigger value. Lost sales and number of stock-outs in both cases increases and number of cycle counts done decreases with the trigger value. These are expected results, since increasing trigger value decreases the sensitivity of the trigger mechanism. Furthermore, the effect of the trigger value diminishes for values greater than 5.

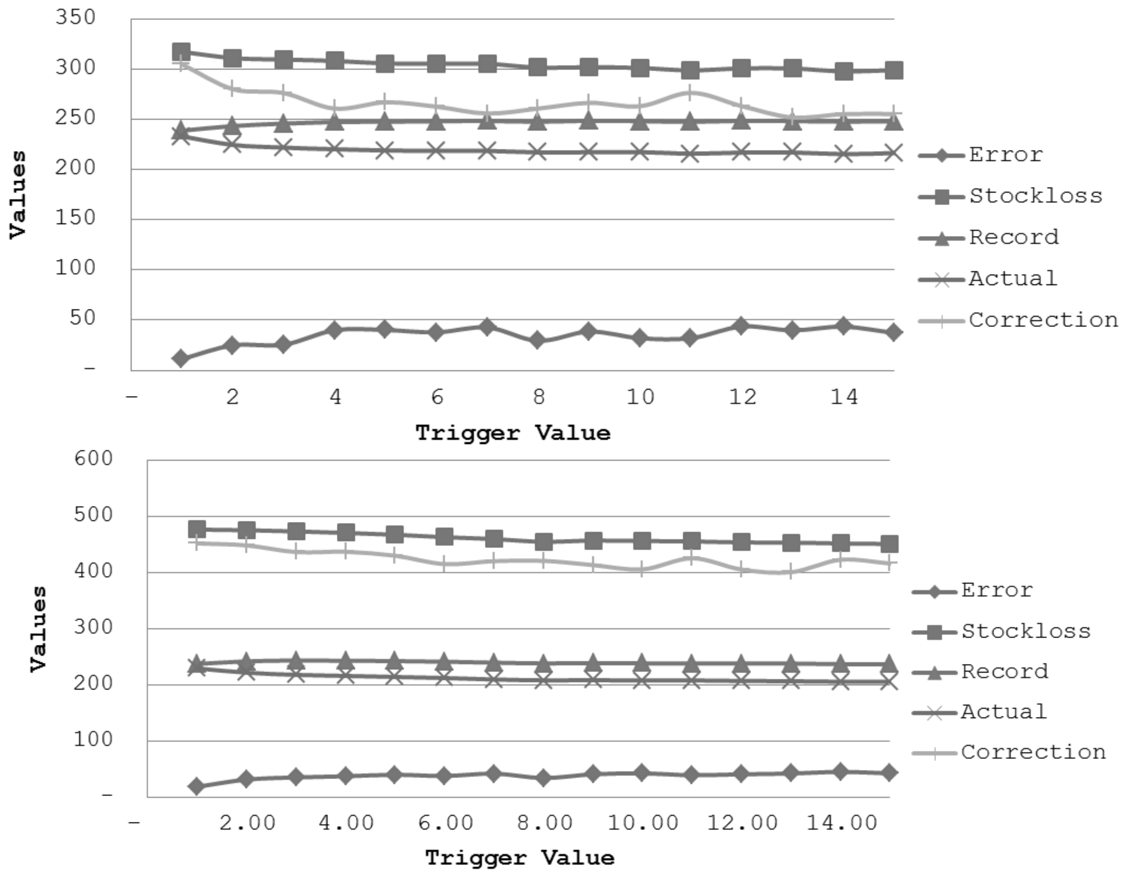

Figure 2-9: The error control method for the best (left) and the worst (right) case cont.

Figure 2-8 shows the results of the remaining parameters. Based on the figure, the number of errors and stock-loss slowly decreases with trigger value. The reason behind this is, the bigger trigger value means fewer counts, which implies that the system operates with more errors. Another important observation is the gap between inventory records and actual physical stock. As the trigger value increases this gap gets larger. This is because when the actual physical stock is low, the potential for making errors is also low

2.2.4 IRI Resistance and Error Control

Figure 2-9 depicts the combined framework in which both compensating methods are utilized simultaneously. With each period, the IRI resistance method increases the safety stock. At the same time, depending on the trigger mechanism, the system performs cycle counts. When a cycle count is triggered both the inventory records and the safety stock levels are reset.

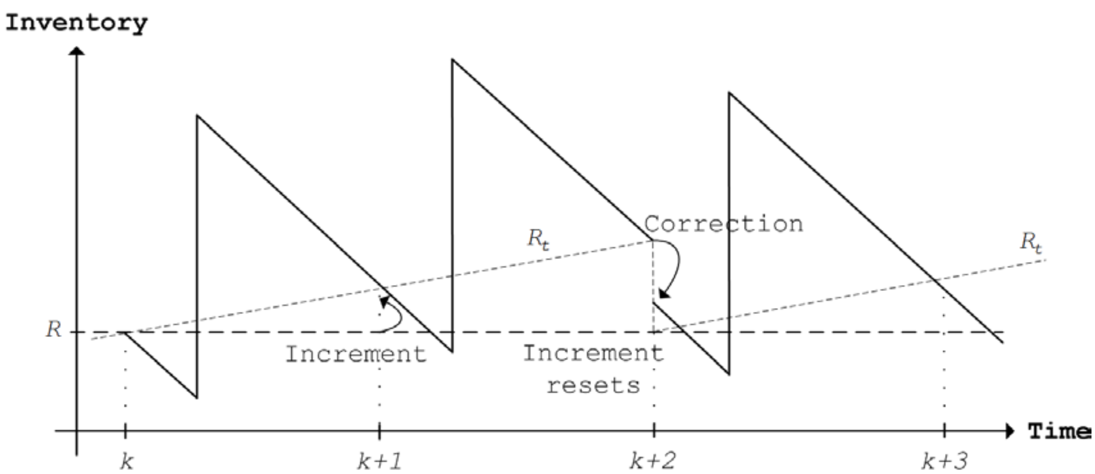

Figure 2-10: Combined compensation framework

The algorithm is applied to the numerical study in Section 2.1.2. Figure 2-10 depicts the actual and the recorded inventory behavior for the best case when two previously mentioned methods are combined. Both the increment and the trigger value increase the inventory records. The actual inventory increases with the increment but decreases with the trigger value. This is because when the trigger value is high there are fewer number of cycle counts which results in fewer number of corrections and higher number of IRI in the system.

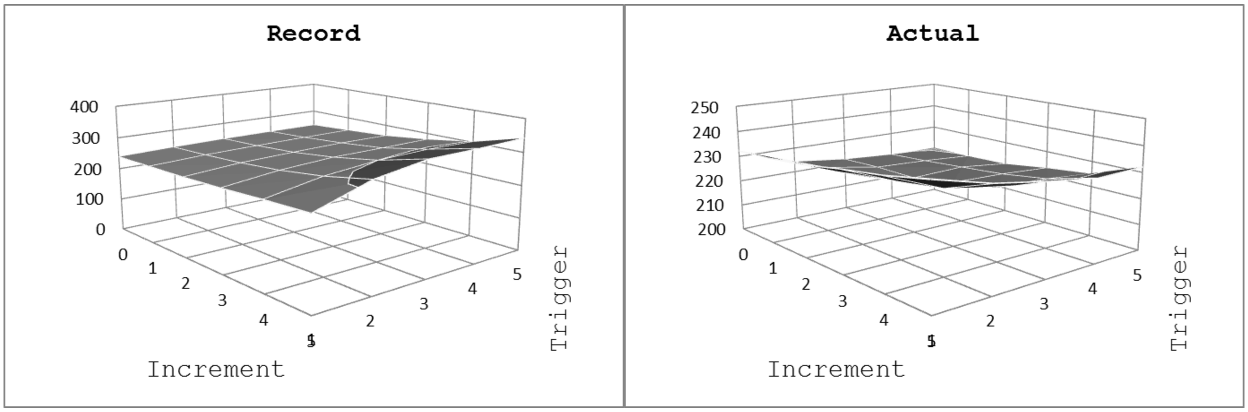

Figure 2-11: Combined method results for recorded and actual inventory for the best case

Figure 2-11 compares the lost sales to the stock-out amounts. With the increment they both decrease; but the stock-out decreases faster. With the trigger value they both increase; again the stock-out increases faster.

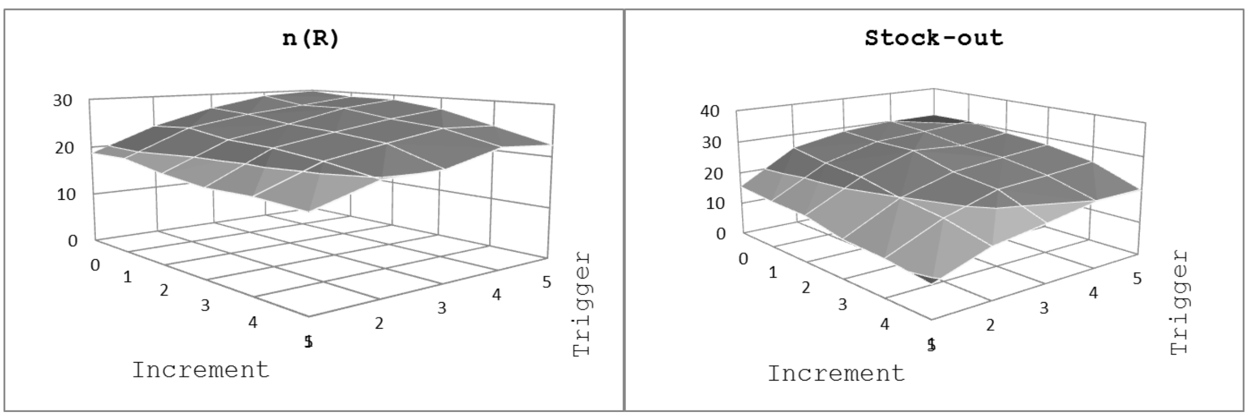

Figure 2-12: Combined method results for lost sales and stock-outs for the best case

Figure 2-12 compares the effectiveness of error correction with the number of counts done. Normally the IRI resistance method does not offer any correction procedures. Thus, we expected a linear line on the x-axis; however, as increment increases both the count number and the error correction decreases. Intuitively, when the IRI resistance method is in effect the system does not need to activate trigger mechanism often due to the reduced number of stock-outs and lost sales. This outcome shows that the effectiveness of the trigger mechanism is reduced as the increment gets higher.

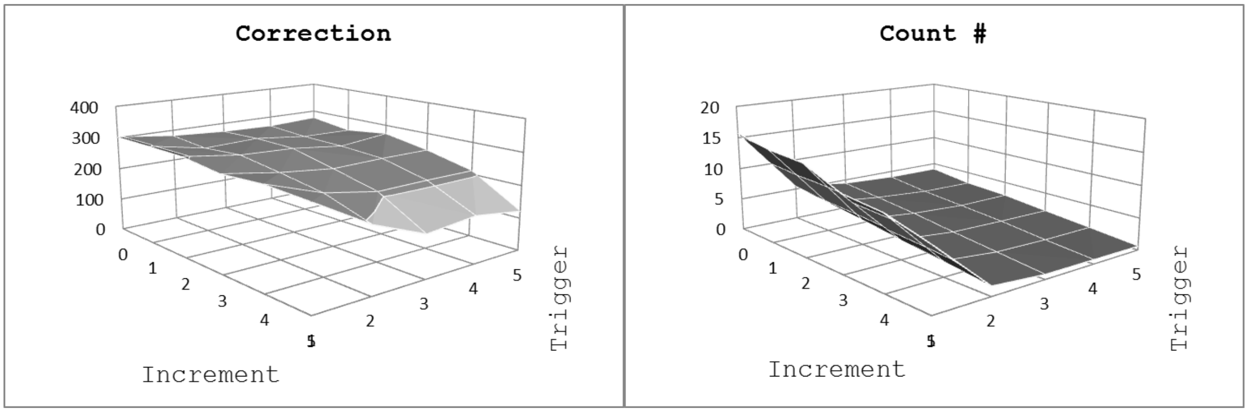

Figure 2-13: Combined method results for error correction and count number for the best case

Figure 2-13 presents sales per unit time and the behavior of the errors. Sales rise with the increment and remain relatively constant the trigger value. Errors, on the other hand, remain constant with the increment and rise with the trigger value. Both graphs present some chaotic behavior; this is largely due to the natural uncertainty faced due to randomness in demand, supply and lead time.

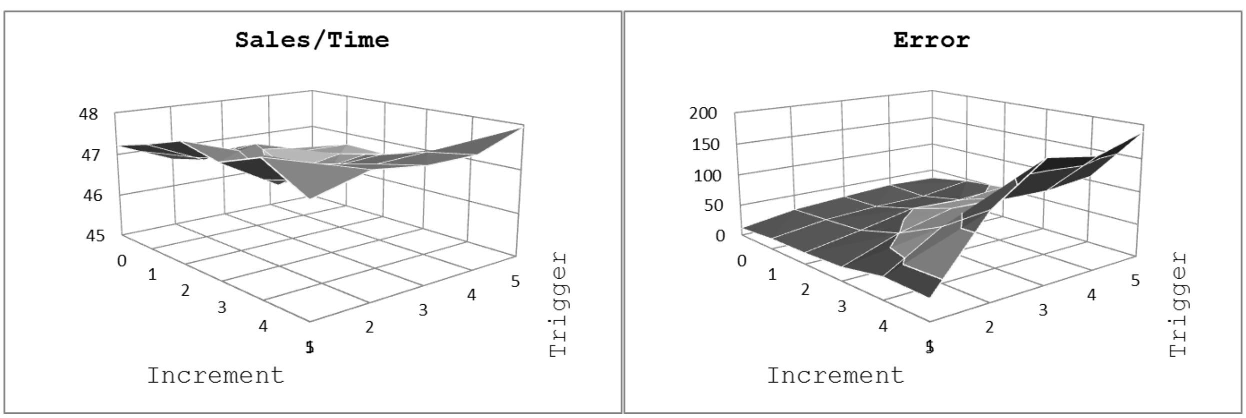

Figure 2-14: Combined method results for sales and error for the best case

Remark: The same numerical study is conducted for the worst case as well. The details and the results are in Appendix A.III

Table 2-7: Final result statistics table with optimal range selection

Case	Sales per unit time	(Scaled*) Stock-out	Correction per Count	(Scaled*) Error	Average Record	Average Actual	(Scaled*) Lost Sales
Original (Best)	38.25	31.14*	-	250*	255	216	90*
Original (Worst)	31.96	36.37*	-	400*	251	210	110*
Increment (Best) 8	40-50	5-10	-	350-400	400-450	250-300	15-10
Increment (Worst) 12	30-50	5-10	-	500-550	500-550	260-300	20-10
Trigger (Best) 4	45-50	30	50-75	25-40	225-250	220-230	30
Trigger (Worst) 6	45-50	35	70-90	35-40	200-250	200-250	30-35
Combined (Best) 5,2	47-50	10-15	130-140	30-35	300-320	230-250	19-20
Combined (Worst) 5,3	47-50	20-25	120-130	30-35	280-300	220-250	25-26

Table 2-7 presents the comparison between all the numerical studies done in this chapter. As discussed in the previous section the IRI resistance method performs better on the stock-out and the lost sales, whereas error control method operates with lower inventory levels. But when the two correction methods are combined, both inventory and stock-out parameters decrease considerably.

2.3 Conclusion and Future Work

In the first part of this chapter inventory inaccuracy is analyzed extensively to understand its behavior when influenced by demand, supply and lead time uncertainty. Different factors that constitute IRI are defined and formulated. Moreover, the impact of the stochastic nature is incorporated as random demand, lead time and supply. The effects of these uncertainties are demonstrated. Combining all of the analyses, a general formulation is presented as the best and the worst case framework. Then, a numerical study using simulation is conducted to show the sensitivity of the inventory replenishment policy to IRI. The highlights can be summarized as;

- In terms of lead time demand, there is no conclusive result on the behavior of the error function. Depending on input parameters α, β, θ, γ and δ it can decrease or increase with the lead time demand.

- In terms of R, W and Y the error function is increasing.

- Errors have no strong dependence between each other in the best case. This dependence is much higher in the worst case.

- In both cases the biggest effect is done by the outbound errors. Hence, parameter β has the highest impact.

In the second part methods for reducing the effect of IRI are developed. Two different alternatives for such methods are presented; The IRI resistance and the error correction. Then, a numerical analysis is performed to observe the behavior of IRI and to quantify the effects of the applied solution alternatives. The primary findings can be concluded as:

- The IRI resistance method positively influences sales.

- The IRI resistance method positively influences errors because as the increment gets higher there will be more average inventory in the system.

- The determining factor in choosing the trigger value is the count number. High count number means fewer stock-outs and fewer lost sales but more frequent counts. Therefore, a range based on the certain parameters can be chosen and the values that maximize or minimize those parameters can be selected.

- The IRI resistance method performs better on stock-outs and lost sales, whereas the error control method can keep low inventory levels. When combined, achieving lower levels of inventory becomes possible while keeping stock-out and lost sales low.

Finally, using the best and worst case frameworks we presented lower and upper bounds for the behavior of errors. The derivations and the numerical analyses provided insights about the relations between IRI and key parameters. The sensitivity of these relations is shown to be similar in direction but different in magnitude for each case. Thus, the values for the trigger and increment in the best and the worst cases can be used as bounds real practices.

The compensation methods described in this chapter are static. In other words the same level of increment or trigger value is implemented throughout the duration of the inventory system. Hence for future work utilizing dynamic correction methods that change for each period will be a considerable contribution.

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APPENDIX A

A.I Best Case Error Calculation

The errors made in the first phase can be written as,

$$\begin{aligned}&\delta E\left[Outbound_{phase\,1}\left(D',Inv_1\right)\right]+E\left[Theft_{phase\,1}\left(D',Inv_2\right)\right]\\&+\beta E\left[Expiration_{phase\,1}\left(D',Inv_3\right)\right].\end{aligned} \tag{2.18}$$

Sales cause the outbound errors to occur, which is a function of the demand and the remaining inventory. The term "Inv" refers to the remaining actual inventory after each step. Following the outbound, theft takes place; again it is a function of demand and remaining inventory. Lastly, expiration and spoilage occurs with a similar structure. First phase is concluded upon receiving the replenishment orders and with them the inbound errors. Which is, $\gamma Inbound_{phase\,1}(Y)$. Then the error structure for the first phase becomes,

$$\begin{aligned}&E\left[\min\left\{\delta D',R-D'\right\}\right]_{\{D'\le R\}}+E\left[\min\left\{\alpha D',R-D'-\min\left\{\delta D',R-D'\right\}\right\}\right]_{\{(1+\delta)D'\le R\cap D'\le R\}}\\&+\beta E\left[R-D'-\min\left\{\delta D',R-D'\right\}-\min\left\{\alpha D',R-D'-\min\left\{\delta D',R-D'\right\}\right\}\right]_{\{D'(1+\alpha+\delta)\le R\cap D'\le R\}}\\&+\gamma E[Y].\end{aligned} \tag{2.19}$$

The calculation of the second phase errors is more complicated since it dependents on the inventory left after the first phase. But the structure is the same as equation (2.18). The outbound errors on the second phase can be formulated as,

$$E\left[\min\left\{\begin{array}{l}\delta D'',Y(1-\gamma)+R-D'-\min\left\{\delta D',R-D'\right\}\\-\min\left\{\alpha D',R-D'-\min\left\{\delta D',R-D'\right\}\right\}\\-\beta\left(\begin{array}{l}R-D'-\min\left\{\delta D',R-D'\right\}\\-\min\left\{\alpha D',R-D'-\min\left\{\delta D',R-D'\right\}\right\}\end{array}\right)\\-D''\end{array}\right\}\right]_{\{Region\}} \tag{2.20}$$

where

$$
\begin{aligned}
&Y(1-\gamma)+R-D' - \min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\
&-\beta\left(R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\right)
\end{aligned}
$$

is the actual inventory left before second phase starts. Note that this value is always non-negative since,

$$
0\leq\left(\begin{aligned}&R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\&-\beta\left(R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\right)\end{aligned}\right)_{\{D'(1+\alpha+\delta)\leq R\cap D'\leq R\}}
$$

And $0\leq Y(1-\gamma)$. Then,

$$
\left(\begin{aligned}&Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\&-\beta\left(R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\right)-D''\end{aligned}\right)_{\{Region\}}
$$

The equation above can assume non-negative values only if there are some physical stocks left on the shelves. This condition is forced by defining an area, referred as *region*. The area defined by region corresponds to the actual inventory left just before the second phase begins. Hence, the total visible plus non-visible (IRI) demand for that time frame cannot exceed the available actual physical stock. That is why the condition forces the equation to stay non-negative. The formulation for region is

$$
\begin{aligned}
D''\leq{}&Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\
&-\beta\left(R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\right)
\end{aligned}
$$

Similarly, theft for phase 2 can be written as,

$$
E\left[\min\left\{\alpha D'',\left(\begin{aligned}&Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}\\&-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\&-\beta\begin{pmatrix}R-D'-\min\{\delta D',R-D'\}\\-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\end{pmatrix}-D''\\&-\min\left\{\begin{aligned}&\delta D'',Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}\\&-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\&-\beta\begin{pmatrix}R-D'-\min\{\delta D',R-D'\}\\-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\end{pmatrix}\\&-D''\end{aligned}\right\}\end{aligned}\right)\right\}\right]_{\{Region\ 2\}} \tag{2.21}
$$

The equation (2.21) can only assume non-negative values if there are some physical stocks left on the shelves. This time condition is forced by defining another area which forces the result to be non-negative; this area is referred as *region2*. The characterization for region2 is

$$
\begin{aligned}
D'' \le\ & Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\} \\
& -\beta\left(R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\right) \\
& -\min\left\{\begin{array}{l} D'',Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\} \\ -\beta\left(R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\right)-D'' \end{array}\right\}
\end{aligned}
$$

Similarly, region2 defines a smaller area that corresponds to the inventory left on-hand after the demand and the outbound errors takes place in the second phase. The final type of errors can be formulated similarly,

$$
\beta E\left[\begin{array}{l}
Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\} \\
-\beta\left(R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\right)-D'' \\
-\min\left\{\begin{array}{l} \delta D'',Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\} \\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\} \\ -\beta\left(\begin{array}{l} R-D'-\min\{\delta D',R-D'\} \\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\} \end{array}\right) \\ -D'' \end{array}\right\} \\
-\min\left\{\alpha D'',\left(\begin{array}{l} Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\} \\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\} \\ -\beta\left(\begin{array}{l} R-D'-\min\{\delta D',R-D'\} \\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\} \end{array}\right)-D'' \\ -\min\left\{\begin{array}{l} \delta D'',Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\} \\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\} \\ -\beta\left(\begin{array}{l} R-D'-\min\{\delta D',R-D'\} \\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\} \end{array}\right) \\ -D'' \end{array}\right\} \end{array}\right)\right\}
\end{array}\right]_{\{Region\ 3\}} \tag{2.22}
$$

In the above equation *region3* can be characterized as,

$$
\begin{aligned}
D^{''} \le\; & Y(1-\gamma)+R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\}-\min\left\{\alpha D^{'},R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\}\right\} \\
& -\beta\left(R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\}-\min\left\{\alpha D^{'},R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\}\right\}\right) \\
& -\min\left\{\begin{array}{l}
\delta D^{''},Y(1-\gamma)+R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\} \\
-\min\left\{\alpha D^{'},R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\}\right\} \\
-\beta\left(\begin{array}{l} R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\} \\ -\min\left\{\alpha D^{'},R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\}\right\}\end{array}\right) \\
-D^{''}
\end{array}\right\} \\
& -\min\left\{\alpha D^{''},\left(\begin{array}{l}
Y(1-\gamma)+R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\} \\
-\min\left\{\alpha D^{'},R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\}\right\} \\
-\beta\left(\begin{array}{l} R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\} \\ -\min\left\{\alpha D^{'},R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\}\right\}\end{array}\right)-D^{''} \\
-\min\left\{\begin{array}{l}
\delta D^{''},Y(1-\gamma)+R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\} \\
-\min\left\{\alpha D^{'},R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\}\right\} \\
-\beta\left(\begin{array}{l} R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\} \\ -\min\left\{\alpha D^{'},R-D^{'}-\min\left\{\delta D^{'},R-D^{'}\right\}\right\}\end{array}\right) \\
-D^{''}
\end{array}\right\}
\end{array}\right)\right\}
\end{aligned}
$$

The area defined by region3 is even smaller compared to the others. This equation ensures that the last part of errors cannot exceed available inventory left on-hand after all the visible and invisible demand is satisfied. The final characterization of the expected error for a period is obtained by combining equations (2.19), (2.20), (2.21) and (2.22) as

$$
\begin{aligned}
&E\left[\min\{\delta D',R-D'\}\right]_{\{D'\le R\}}+E\left[\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\right]_{\{(1+\delta)D'\le R\cap D'\le R\}}+\gamma E[Y]\\
&+\beta E\left[R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\right]_{\{D'(1+\alpha+\delta)\le R\cap D'\le R\}}\\
&+E\left[\min\left\{\begin{array}{l}\delta D'',Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}\\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\ -\beta\left(\begin{array}{l}R-D'-\min\{\delta D',R-D'\}\\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\end{array}\right)\\ -D'\end{array}\right\}\right]_{\{Region\}}\\
&+E\left[\min\left\{\alpha D'',\left(\begin{array}{l}Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}\\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\ -\beta\left(\begin{array}{l}R-D'-\min\{\delta D',R-D'\}\\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\end{array}\right)-D''\\ -\min\left\{\begin{array}{l}\delta D'',Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}\\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\ -\beta\left(\begin{array}{l}R-D'-\min\{\delta D',R-D'\}\\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\end{array}\right)\\ -D''\end{array}\right\}\end{array}\right)\right\}\right]_{\{Region\ 2\}}\\
&+\beta E\left[\begin{array}{l}Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\ -\beta\left(R-D'-\min\{\delta D',R-D'\}-\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\right)-D''\\ -\min\left\{\begin{array}{l}\delta D'',Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}\\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\ -\beta\left(\begin{array}{l}R-D'-\min\{\delta D',R-D'\}\\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\end{array}\right)\\ -D''\end{array}\right\}\\ -\min\left\{\alpha D'',\left(\begin{array}{l}Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}\\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\ -\beta\left(\begin{array}{l}R-D'-\min\{\delta D',R-D'\}\\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\end{array}\right)-D''\\ -\min\left\{\begin{array}{l}\delta D'',Y(1-\gamma)+R-D'-\min\{\delta D',R-D'\}\\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\\ -\beta\left(\begin{array}{l}R-D'-\min\{\delta D',R-D'\}\\ -\min\{\alpha D',R-D'-\min\{\delta D',R-D'\}\}\end{array}\right)\\ -D''\end{array}\right\}\end{array}\right)\right\}\end{array}\right]_{\{Region\ 3\}}
\end{aligned}
$$

The error characterization is increasing in R since equations (2.18), (2.19), (2.20), (2.21) and (2.22) are all increasing in R. This is an intuitive result; more inventory means more mistakes. Demand however, has direct and indirect effects.

A.II Worst Case Error Calculation

Similar to equation (2.18),

$$\begin{aligned}&\delta E\left[\min\{D',R\}\right]+E\left[\min\{\alpha D',R-\delta D'\}\right]_{\{\delta D'\le R\}}\\&+\beta E\left[R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}\right]_{\{D'(\alpha+\delta)\le R\}}+\gamma E[Y].\end{aligned}\tag{2.23}$$

The calculation of the second phase errors start with the outbound,

$$\delta E\left[\min\left\{\begin{array}{l}D'',Y(1-\gamma)+R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}_{\{D'\le R\}}\\-\beta\left(R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}\right)_{\{D'(\alpha+\delta)\le R\}}-D'\end{array}\right\}_{\{Region\}}\right]\tag{2.24}$$

where

$$\begin{aligned}&Y(1-\gamma)+R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}_{\{D'\le R\}}\\&-\beta\left(R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}\right)_{\{D'(\alpha+\delta)\le R\}}-D'\end{aligned}$$

is the actual inventory left before the second phase starts. Equation (2.24) is forced to be non-negative by region which is,

$$\begin{aligned}0\le\ &Y(1-\gamma)+R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}_{\{D'\le R\}}\\&-\beta\left(R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}\right)_{\{D'(\alpha+\delta)\le R\}}-D'\end{aligned}$$

Similarly, theft for the second phase can be written as,

$$E\left[\min\left\{\begin{array}{l}\alpha D'',Y(1-\gamma)+R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}_{\{D'\le R\}}\\-\beta\left(R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}\right)_{\{D'(\alpha+\delta)\le R\}}-D'\\-\delta\min\left\{\begin{array}{l}D'',Y(1-\gamma)+R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}_{\{D'\le R\}}\\-\beta\left(R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}\right)_{\{D'(\alpha+\delta)\le R\}}-D'\end{array}\right\}\end{array}\right\}_{\{Region\ 2\}}\right].\tag{2.25}$$

The equation (2.25) can only assume non-zero values if there are some physical stocks left on shelves. The region2 in which this happens can be characterized as,

$$\begin{aligned}0\le\ &Y(1-\gamma)+R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}_{\{D'\le R\}}\\&-\beta\left(R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}\right)_{\{D'(\alpha+\delta)\le R\}}-D'\\&-\delta\min\left\{\begin{array}{l}D'',Y(1-\gamma)+R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}_{\{D'\le R\}}\\-\beta\left(R-\delta\min\{D',R\}-\min\{\alpha D',R-\delta D'\}\right)_{\{D'(\alpha+\delta)\le R\}}-D'\end{array}\right\}\end{aligned}$$

The final phase of errors can be formulated similarly,

$$
\beta E\left[\left(\begin{array}{l}
Y(1-\gamma)+R-\delta\,\mathrm{min}\{D^{'},R\}-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}_{\{D^{'}\leq R\}}\\
-\beta\left(R-\delta\,\mathrm{min}\{D^{'},R\}-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}\right)_{\{D^{'}(\alpha+\delta)\leq R\}}-D^{'}\\
-\delta\,\mathrm{min}\left\{\begin{array}{l}
D^{''},Y(1-\gamma)+R-\delta\,\mathrm{min}\{D^{'},R\}-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}_{\{D^{'}\leq R\}}\\
-\beta\left(R-\delta\,\mathrm{min}\{D^{'},R\}-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}\right)_{\{D^{'}(\alpha+\delta)\leq R\}}-D^{'}
\end{array}\right\}\\
-\mathrm{min}\left\{\begin{array}{l}
\alpha D^{''},Y(1-\gamma)-D^{'}\\
+(1-\beta)\left(\begin{array}{l}R-\delta\,\mathrm{min}\{D^{'},R\}\\-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}\end{array}\right)_{\{D^{'}(\alpha+\delta)\leq R\}}\\
-\delta\,\mathrm{min}\left\{\begin{array}{l}
D^{''},Y(1-\gamma)-D^{'}\\
+(1-\beta)\left(\begin{array}{l}R-\delta\,\mathrm{min}\{D^{'},R\}\\-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}\end{array}\right)_{\{D^{'}(\alpha+\delta)\leq R\}}
\end{array}\right\}
\end{array}\right\}
\end{array}\right)_{\{Region\ 3\}}\right]. \qquad (2.26)
$$

In the above equation region is the same as before, however region3 is even a smaller zone which can be defined as,

$$
\begin{array}{l}
0\leq Y(1-\gamma)+R-\delta\,\mathrm{min}\{D^{'},R\}-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}_{\{D^{'}\leq R\}}\\
\quad-\beta\left(R-\delta\,\mathrm{min}\{D^{'},R\}-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}\right)_{\{D^{'}(\alpha+\delta)\leq R\}}-D^{'}\\
\quad-\delta\,\mathrm{min}\left\{\begin{array}{l}
D^{''},Y(1-\gamma)+R-\delta\,\mathrm{min}\{D^{'},R\}-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}_{\{D^{'}\leq R\}}\\
-\beta\left(R-\delta\,\mathrm{min}\{D^{'},R\}-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}\right)_{\{D^{'}(\alpha+\delta)\leq R\}}-D^{'}
\end{array}\right\}\\
\quad-\mathrm{min}\left\{\begin{array}{l}
\alpha D^{''},Y(1-\gamma)+R-\delta\,\mathrm{min}\{D^{'},R\}-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}_{\{D^{'}\leq R\}}\\
-\beta\left(R-\delta\,\mathrm{min}\{D^{'},R\}-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}\right)_{\{D^{'}(\alpha+\delta)\leq R\}}-D^{'}\\
-\delta\,\mathrm{min}\left\{\begin{array}{l}
D^{''},Y(1-\gamma)+R-\delta\,\mathrm{min}\{D^{'},R\}-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}_{\{D^{'}\leq R\}}\\
-\beta\left(R-\delta\,\mathrm{min}\{D^{'},R\}-\mathrm{min}\{\alpha D^{'},R-\delta D^{'}\}\right)_{\{D^{'}(\alpha+\delta)\leq R\}}-D^{'}
\end{array}\right\}
\end{array}\right\}
\end{array}
$$

The expected error formulation for a period is obtained similarly by combining equations (2.23), (2.24), (2.25) and (2.26).

A.III Worst Case Combined

Same set of simulation studies with the combined method is generated to for the worst case.

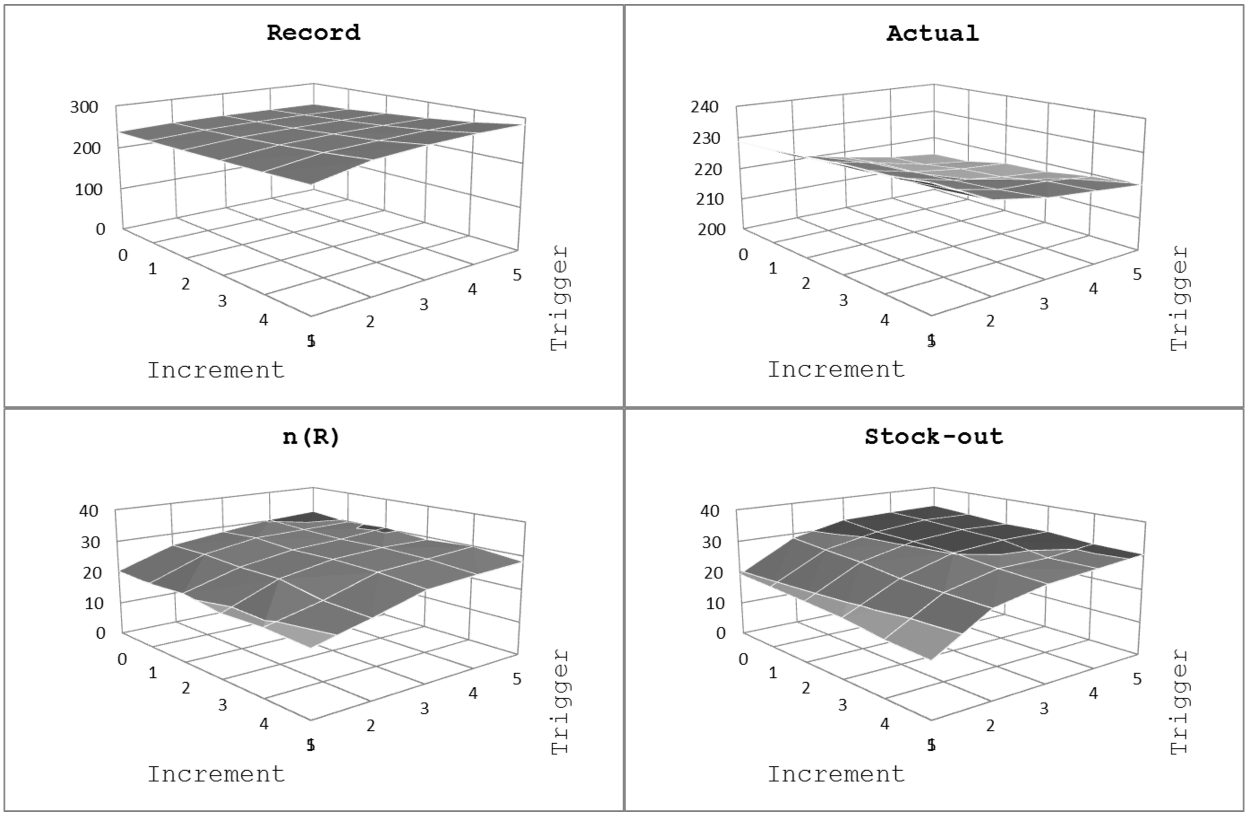


Figure A-1: Combined method results for the worst case

In the simulation, same parameters are used as done in previous cases. The results are very similar to the best case. Figure A-1 and Figure A-2 depict the simulation results of the worst case study. Lost sales and stock out parameters decrease with higher increment and lower trigger values. Inventory levels increase with increment and decrease with trigger.

Similar results are obtained through Figure A-2. The correction and count number decreases as the increment increase. The trigger value is not effective for the correction and count number. Sales and error graphs also exhibit a similar behavior.

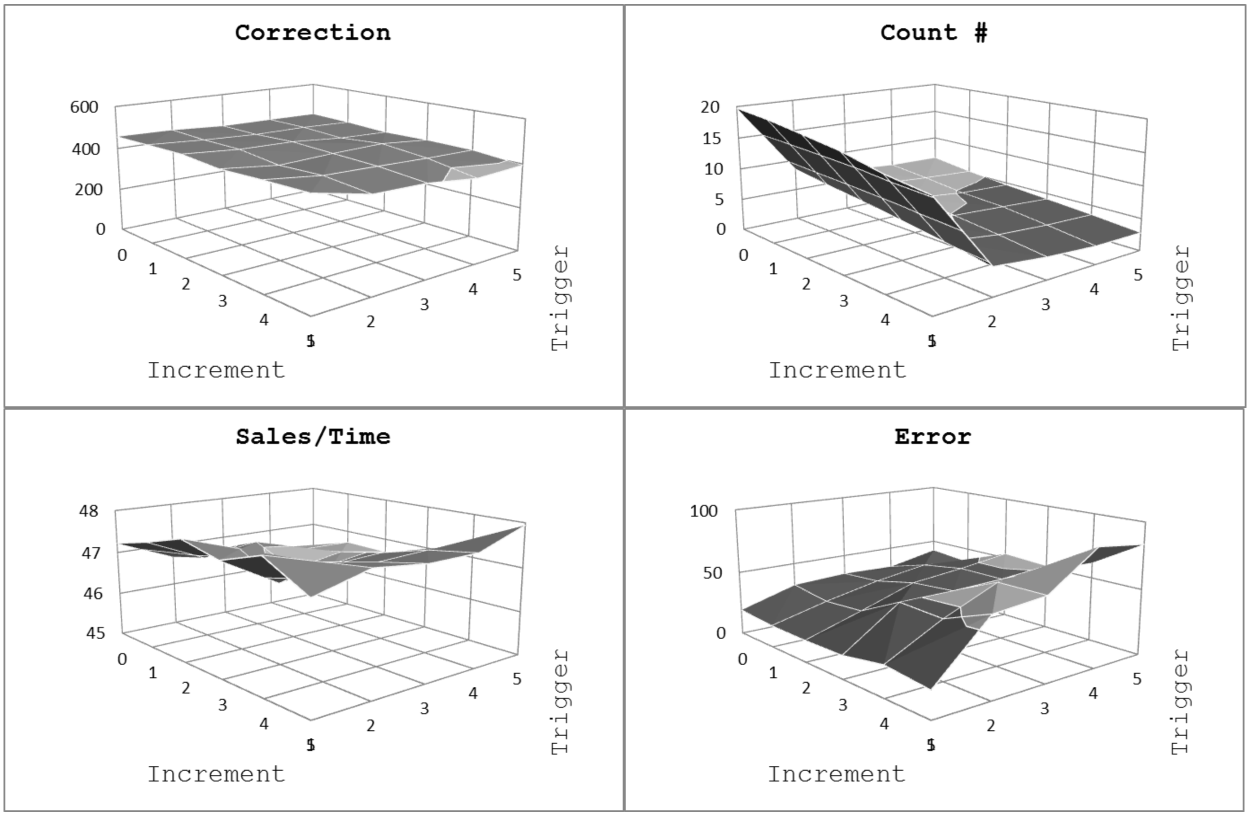


Figure A-2: Combined method results for the worst case (cont.)

The combined method managed to utilize the positive sides of the both compensation methods. With the extra buffer the incrementing method provides the trigger mechanism is able to function effectively for low trigger values which considerably decreases the total count number; hence, reducing the costs associated with it. On the other side, trigger mechanism keeps the records in check and reduces the extra error caused by excessive incrementing.

CHAPTER 3 IRI ANALYSIS IN ONLINE RETAILING WITH LEAD TIME AND SUPPLY UNCERTAINTY

With the advancement of the Internet, online retailing becomes an important channel for retails. According to Mangalindan (2005), 5.5% of all retail sales (excluding travel) are done online in 2004. This potential is recognized by many organizations as demonstrated in Tsay and Agrawal (2004). The major difference of the online retail environment is that customers do not have access to the goods during purchase. This fact greatly enhances the importance of inventory records since the amount of sales is highly dependent on the accuracy of the records.

IRI is a well-known problem for both the online and the offline retail environment. In this context IRI is regarded as the mismatch between the inventory records and the actual physical stock. In large scale retailing, the inventory records are likely to be incorrect, and ignoring this fact often leads to failed re-procurement cycles and quantities. Inventory models with IRI are commonly studied in the offline retail environment, but there are limited studies for the online retails.

The online and the offline retail models are similar on many aspects. But one crucial difference is: In the online retails, customers make their purchasing decisions based on inventory records where as in the offline retails customer are allowed to pick the product from shelves directly. This difference has various implications on the IRI behavior. Our aim in this chapter is to develop a model to identify the effects of customers' interaction with physical stock and understand the behavior IRI in the online retailing environment.

This chapter also classifies IRI in three groups: inbound, shelving and outbound errors. While the customer interaction has little effect on inbound and outbound types of IRI, it is quite influential on shelving errors. In fact it is commonly known that customer interaction in the supply chain is a major source of shelving errors (Rekik et al., 2009). The online model lacks this interaction; however, a substantial amount of shelving errors can still take place even without the presence of customers. According to Center for Retail Research (2005) the average rate for shelving errors in UK is 1.4% percent of sales, which is one of the highest in Europe. The study also identifies that 14.4% of shelving errors can be attributed to internal errors, such as processing errors, accounting mistakes and pricing discrepancies.

In this chapter, we investigate the online retail models where the customers are not allowed to directly interact with products; instead they make their purchasing decision based solely on the inventory records (Grewal et al., 2004). Furthermore, we characterize the structure of IRI when it is influenced by random supply, random demand and random lead time in the online setting where the inventory records are reviewed continuously.

3.1 Model

IRI is influenced by many factors such as demand, supply and lead time apart from the direct causes (shrinkage, transaction and misplacement errors). The difficulty in modeling IRI generally lies in the fact that the relationship is not one sided, IRI affects all of these factors back. In addition, the online retail environment has three main distinctive features. First, the customers do not have access to the inventory; therefore, the demand is satisfied based on the inventory records and not by the actual amount. Second, the actual physical stock that is above the inventory records is unsalable. And lastly, customers can continue purchasing even if the actual physical stock is zero. This case may be perceived as backordering in the offline retail settings; however, this situation only occurs if the actual inventory reaches zero while the inventory records are positive. We refer to this case as *penalty sales*. Figure 3-1 provides a graphical explanation.

In our model, we define a three-way categorization for the errors in order to understand the behavior: (1) inbound errors: Errors that occur during ordering and receiving processes; (2) storing errors: Received SKUs get damaged or expire, which causes the physical stock to change without updating the inventory records; and (3) outbound errors: The errors that occur during selling and shipment of SKUs. When left uncorrected, these errors can lower retailer performance by increasing the stock-out rates.

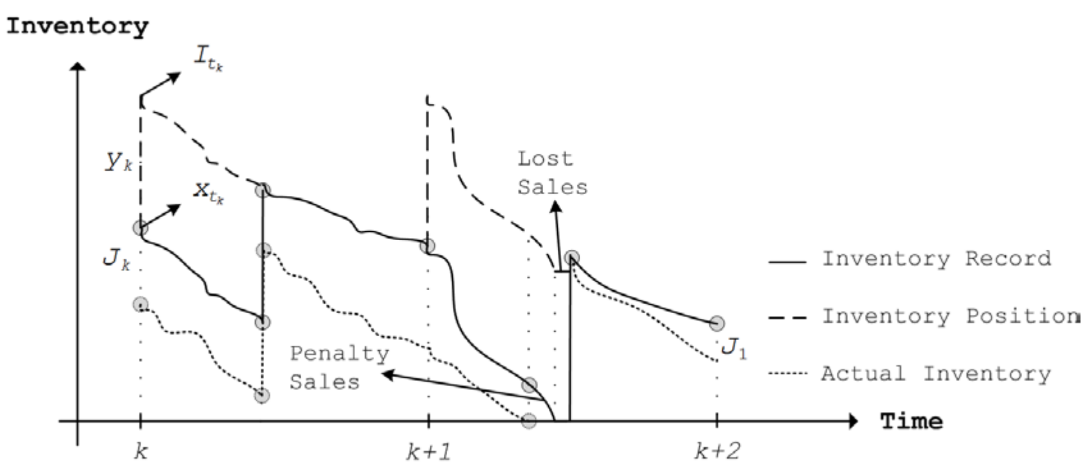


Figure 3-1: Behavior of the physical stock, the inventory record, and the inventory position

In this model, the classification for the errors and the inventory behavior is similar to Chapter 2. Figure 3-1 shows a typical continuous inventory behavior subject to errors. At period k the inventory records, x_{t_k}, are checked and updated by ordering y_k units of inventory. After τ_k units of time, the order arrives. The standard procedure continues until period $k+1$ when the records reach to the reorder level. Up to this point, T_k units of time passed and ε_k amounts of error occurred which made the total error equal to J_{k+1}.

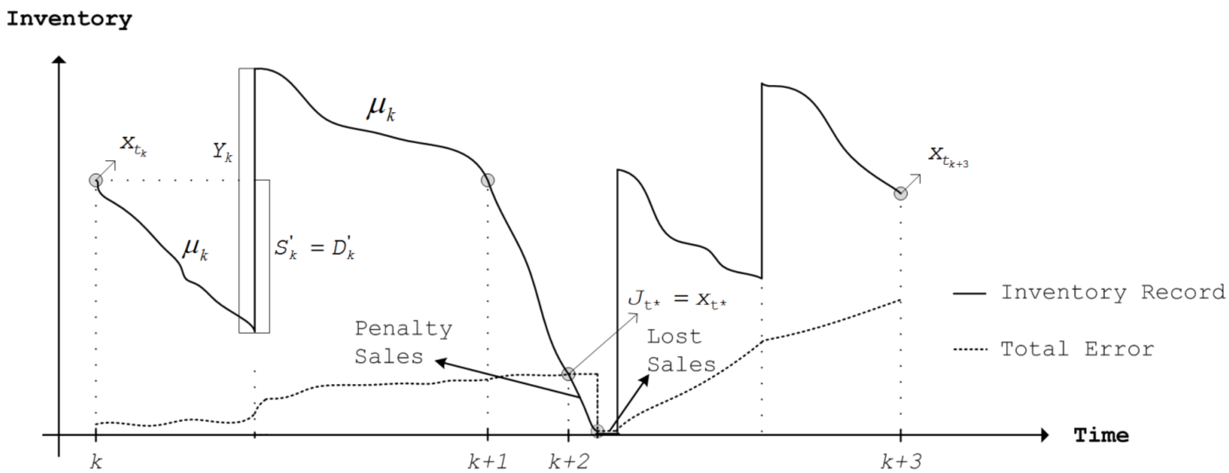


Figure 3-2: The relation between the records and the errors

The natural randomness of the model may result in situations where there is no actual inventory on-hand. However, in the online environment the customers are allowed to continue purchasing even when the actual physical stock drops to zero. We use the term penalty sales to describe this situation. Penalty sales can continue to occur as long as the inventory records are positive. If the demand continues to drain the inventory records all the way down to zero, a stock-out for the inventory records happens. At this point customers cannot purchase any more items and the remaining demand is lost. Another important factor is, the stock-out for inventory records is fully observable and when it happens, the present IRI is automatically corrected at $\tilde{x}_t = x_t = 0$. Figure 3-2 depicts such a stock-out situation by comparing the behavior of the total error versus the inventory records.

Let $\tilde{x}_{t_k}$ denote the amount of actual inventory and x_{t_k} denote the inventory record at time t_k in period k. Also let, y_k be the order quantity and D_k be the total demand in period k. Suppose that demand has a known distribution function $F_D(z) = P\{D \le z\}$ with a density $f_D(z)$. Furthermore, let S_k be the amount of sales and ε_k be the discrepancy between the actual and the recorded inventory during period k. When there is, no lead time and no random supply, inventory progression can be formulated as

$$\tilde{x}_{t_k} + \sum_{k \in K} \varepsilon_k = x_{t_k} = x_{t_{k-1}} + y_k - S_k \,, \tag{3.1}$$

where $S_k = \min\{D_k, x_{t_k}\}$ is the amount of sales for $0 \le S_k \le x_k$ and ε_k is the mismatch at period k. Furthermore, we introduce the supply uncertainty using the same setup presented in Section 2.1 as $E[U \min\{K, y_k\}]$, where K represents the random capacity and U denotes the random yield of the supplier (Erdem & Ozekici, 2002; Henig & Gerchak, 1990). Then, equation (3.1) can be rewritten as

$$E\left[\tilde{x}_{t_k}\right] = x_{t_{k-1}} + E[U_k \min\{K_k, y_k\}] - E[S_k] - E\left[\sum_{i=0}^{k} \varepsilon_i\right]. \tag{3.2}$$

3.1.1 Error Modeling

The total errors incurred until period k is denoted by

$$E\left[\sum_{i=0}^{k} \varepsilon_i\right].$$

Note that we consider errors as previously classified. In this classification the shelving errors are only caused by expiration or spoilage (ε_k^e) since customers are not allowed to physically interact with the products. Yan et al. (2011) provide a general characterization for expired and damaged items in inventory models. We modify their model to fit the online retail environment as, $\varepsilon_k^e = \beta \max\{x_{t_k} - S_k, 0\}$ where $\beta \in [0,1]$ is the rate of expiration/spoilage. Expiration errors occur from the unsold inventory at the end of each period; which can be simplified as, $\varepsilon_k^e = \beta(x_{t_k} - D_k)$. Moreover, ε_k^r denotes the inbound errors which are related to the order quantity; whereas, the outbound errors, ε_k^s, are dependent on the number of units sold. Rosetti et al. (2010) provide detailed insights about the structure of the transaction errors. In this setting $\varepsilon_k^r = \gamma y_k$ and $\varepsilon_k^s = \delta S_k$ where

$$\gamma \in \left[1, \frac{R - S'}{D'}\right]$$

and

$$\delta \in \left[1, -\frac{\beta + \beta(1-\beta)}{(1-\beta)^2}\right].$$

The general formulation derived in Section 2.1.2 is modified according to the online retail framework as

$$\begin{aligned} E\left[\tilde{x}_{t_k}\right] = x_{t_{k-1}} + E\left[U_k \min\{K_k, y_k\}\right] - E\left[D_k\right]_{\{D_k \le x_k\}} - \beta \sum_{i=0}^{k} E\left[(x_{t_i} - D_i)\right]_{\{D_i \le x_i\}} \\ + \sum_{i=0}^{k} E\left[\gamma U_i \min\{K_i, y_i\}\right] + \delta \sum_{i=0}^{k} E\left[D_i\right]_{\{D_i \le x_i\}}. \end{aligned} \tag{3.3}$$

Then, the inventory formulation can be modified accordingly to obtain the error function, $E\left[J_{t_k}\right] = E\left[x_{t_k} - \tilde{x}_{t_k}\right]$. Using the inventory relation in equation (3.1), equation (3.3) can be rewritten as

$$E\left[\tilde{x}_{t_k} - x_{t_k}\right] = -\beta \sum_{i=0}^{k} E\left[x_{t_i} - D_i\right]_{\{D_i \le x_i\}} + \delta \sum_{i=0}^{k} E\left[D_i\right]_{\{D_i \le x_i\}} + \gamma \sum_{i=0}^{k} E\left[U_i \min\{K_i, y_i\}\right]. \tag{3.4}$$

Therefore, the total expected error at period k when the current period is i can be modeled as,

$$E[J_k|i] = -\beta\sum_{j=i}^{k} E[x_{t_j} - D_j]_{\{Dj \le xj\}} + \delta\sum_{j=i}^{k} E[D_j]_{\{Dj \le xj\}} + \gamma\sum_{j=i}^{k} E[U_j \min\{K_j, y_j\}] + E[J_i]. \tag{3.5}$$

Additionally, for $i = k-1$ and $J_{k-1} = 0$ the formulation turns in to one-step error calculation for the single-period inventory problem.

3.1.2 General Inventory Formulation

As done in Chapter 2 the underlying problem in equation (3.5) is reformulated to analyze the best and the worst possible cases. The difference in these models is the order of events.

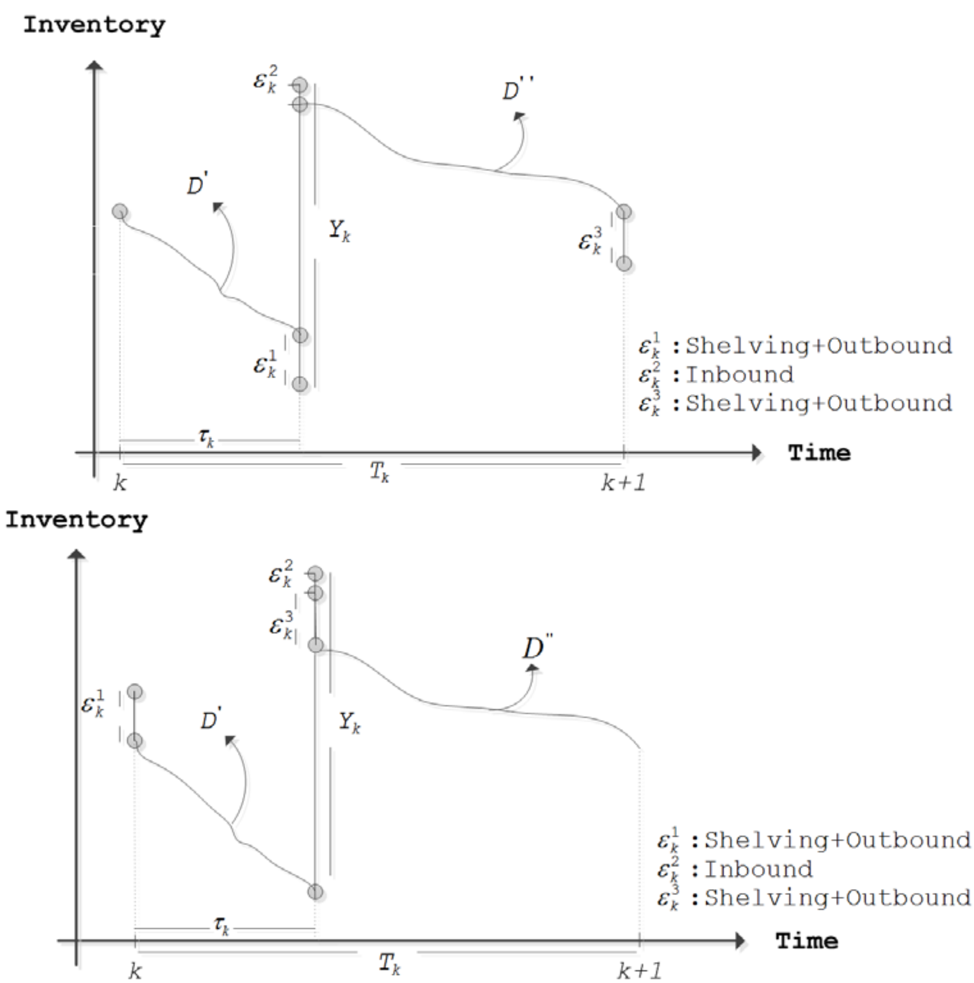


Figure 3-3: The best (left) and the worst (right) case inventory behavior

Figure 3-3 shows the order of events for each model. Each period is divided into two phases; the first phase contains the lead time demand and the second phase contains the demand for the rest of the period. Replenishment time determines the end of the first phase.

In the best case framework the demand is fulfilled first and then errors occur. Since sold items are outside of the feasible space for errors, this case maximizes the demand fill rate and minimizes the IRI. In the worst case, the errors occur first and then the demand is fulfilled; thus, minimizing the fill rate. In reality, the inventory behaves somewhere between the best and the worst case situations; hence, the two characterizations provide a lower and an upper bound. In this case ε_k^1 denotes the outbound and storing errors during lead time, ε_k^2 denotes the inbound errors, and ε_k^3 denotes the outbound and the storing errors during the remainder of period k.

i. Best Case

The errors done in the first phase for the best case are,

$$E\left[\min\{\delta D', R-S'\}\right]+\beta E\left[R-S'-\min\{\delta D', R-S'\}\right]+\gamma E[Y]. \tag{3.6}$$

The calculation of the second phase errors depends on the inventory left which is

$$E[Y](1-\gamma)+R-E[S']-E\left[\min\{\delta D', R-S'\}\right]-\beta E\left[R-S'-\min\{\delta D', R-S'\}\right] \tag{3.7}$$

Note that equation (3.7) is always non-negative and can assume positive values only if there are some physical stocks left on shelves. This can easily be proven by choosing the highest $\beta=1$, which corresponds to the extreme situation where all the inventory is lost at the end of the phase. In that case, equation (3.7) becomes $E[Y](1-\gamma)$ which is always non-negative.

The second phase in the best case starts off with the demand, which can be written as

$$E[Y](1-\gamma)+R-E[S^{'}]-E\left[\min\{\delta D^{'},R-S^{'}\}\right]-\beta E\left[R-S^{'}-\min\{\delta D^{'},R-S^{'}\}\right]-E[D^{''}]. \tag{3.8}$$

Equation (3.8) represents the available inventory before the next source of error occurs. Note that, equation (3.8) is always positive as long as $Y(1-\gamma)\leq D^{'}$. This provides a lower bound for γ. The outbound errors on the second phase can be formulated as

$$E\left[\min\{\delta D^{''},Y(1-\gamma)+R-S^{'}-D^{''}-\min\{\delta D^{'},R-S^{'}\}-\beta\left(R-S^{'}-\min\{\delta D^{'},R-S^{'}\}\right)\}\right]. \tag{3.9}$$

Finally the storing errors can be formulated similarly,

$$\beta E\begin{bmatrix}Y(1-\gamma)+R-S^{'}-D^{''}-\min\{\delta D^{'},R-S^{'}\}-\beta\left(R-S^{'}-\min\{\delta D^{'},R-S^{'}\}\right)\\ -\min\{\delta D^{''},Y(1-\gamma)+R-S^{'}-D^{''}-\min\{\delta D^{'},R-S^{'}\}-\beta\left(R-S^{'}-\min\{\delta D^{'},R-S^{'}\}\right)\}\end{bmatrix} \tag{3.10}$$

The final characterization of the expected error for a period is obtained by combining equations (3.6), (3.9) and (3.10) as

$$\begin{aligned}E[J]=&E\left[\min\{\delta D^{'},R-S^{'}\}\right]+\beta E\left[R-S^{'}-\min\{\delta D^{'},R-S^{'}\}\right]+\gamma E[Y]\\ &+E\left[\min\begin{Bmatrix}\delta D^{''},Y(1-\gamma)+R-S^{'}-D^{''}-\min\{\delta D^{'},R-S^{'}\}\\ -\beta\left(R-S^{'}-\min\{\delta D^{'},R-S^{'}\}\right)\end{Bmatrix}\right]\\ &+\beta E\begin{bmatrix}Y(1-\gamma)-D^{''}+(1-\beta)\left(R-S^{'}-\min\{\delta D^{'},R-S^{'}\}\right)\\ -\min\{\delta D^{''},Y(1-\gamma)+(1-\beta)\left(R-S^{'}-\min\{\delta D^{'},R-S^{'}\}\right)\}\end{bmatrix}.\end{aligned} \tag{3.11}$$

Reorder level dependency

$E[J]$ is increasing in R since the partial derivative with respect to R is positive. This can be shown by separating equation (3.11) into three terms. For practicality, we let $E[J^{i}]$ denote the line $i=1,2,3$ for equations (3.6), (3.9) and (3.10) respectively, so that $E[J]=E[J^{1}]+E[J^{2}]+E[J^{3}]$. Then, the derivation of $E[J^{1}]$ is,

$$\frac{\partial E[J^{1}]}{\partial R}=\left(1-1_{\{D^{'}\geq R\}}\right)_{\{R-S^{'}\leq\delta D^{'}\}}+\beta\left(1-1_{\{D^{'}\geq R\}}-\left(1-1_{\{D^{'}\geq R\}}\right)_{\{R-S^{'}\leq\delta D^{'}\}}\right).$$

The derivative is non-negative since $\left(1-1_{\{D^{'}\geq R\}}\right)_{\{R-S^{'}\leq \delta D^{'}\}}\geq 0$ and $\beta\in[0,1]$. The derivative of $E[J^2]$ is,

$$\begin{aligned}\frac{\partial E[J^2]}{\partial R}&=\frac{\partial}{\partial R}E\left[\min\left\{\delta D^{''},Y(1-\gamma)-D^{''}+(1-\beta)\left(R-S^{'}-\min\left\{\delta D^{'},R-S^{'}\right\}\right)\right\}\right]\\&=(1-\beta)\left(1-1_{\{D^{'}\geq R\}}-\left(1-1_{\{D^{'}\geq R\}}\right)_{\{R-S^{'}\leq\delta D^{'}\}}\right)_{\left\{\delta D^{''}\geq Y(1-\gamma)-D^{''}+(1-\beta)\left(R-S^{'}-\min\left\{\delta D^{'},R-S^{'}\right\}\right)\right\}}.\end{aligned}$$

Since $\beta\in[0,1]$ and $1-1_{\{D^{'}\geq R\}}\geq 0$, the above equation is again always non-negative. The final part can be written as,

$$\begin{aligned}\frac{\partial E[J^3]}{\partial R}&=\frac{\partial}{\partial R}\beta E\begin{bmatrix}Y(1-\gamma)-D^{''}+(1-\beta)\left(R-S^{'}-\min\left\{\delta D^{'},R-S^{'}\right\}\right)\\-\min\left\{\delta D^{''},Y(1-\gamma)+(1-\beta)\left(R-S^{'}-\min\left\{\delta D^{'},R-S^{'}\right\}\right)\right\}\end{bmatrix}\\&=\beta\begin{pmatrix}(1-\beta)\left(1-1_{\{D^{'}\geq R\}}-\left(1-1_{\{D^{'}\geq R\}}\right)_{\{R-S^{'}\leq\delta D^{'}\}}\right)\\-(1-\beta)\left(1-1_{\{D^{'}\geq R\}}-\left(1-1_{\{D^{'}\geq R\}}\right)_{\{R-S^{'}\leq\delta D^{'}\}}\right)_{\left\{\delta D^{''}\geq Y(1-\gamma)+(1-\beta)\left(R-S^{'}-\min\left\{\delta D^{'},R-S^{'}\right\}\right)\right\}}\end{pmatrix}\end{aligned}$$

Hence, the summation of $E[J^1]+E[J^2]+E[J^3]$ is increasing in R when $R\geq D^{'}$ or it is zero.

Lead time demand dependency

The function behavior with the first phase demand can be found by taking the derivative with respect to $D^{'}$. Let $E[J]=E[J^1]+E[J^2]+E[J^3]$ then,

$$\begin{aligned}\frac{\partial E[J^1]}{\partial D^{'}}&=\delta_{\{R-S^{'}\geq\delta D^{'}\}}-1_{\{R-D^{'}<\delta D^{'}\}}-\beta-\beta\left(\delta_{\{R-S^{'}\geq\delta D^{'}\}}-1_{\{R-D^{'}<\delta D^{'}\}}\right)\\&=\left(\delta_{\{R-S^{'}\geq\delta D^{'}\}}-1_{\{R-D^{'}<\delta D^{'}\}}\right)(1-\beta)-\beta\\&=\begin{cases}\delta-\delta\beta-\beta & R-S^{'}\geq\delta D^{'}\\-1 & R-S^{'}<\delta D^{'}\end{cases}.\end{aligned}$$

$E[J^1]$ behaves differently based on δ and β. When δ is big enough $E[J^1]$ is decreasing with $D^{'}$, but when δ is small it depends on both β and δ. The derivative of $E[J^2]$ with respect to $D^{'}$ is,

$$\frac{\partial E[J^2]}{\partial D'} = E\left[\min\left\{\delta D'', Y + R - S' - D'' - E[J^1]\right\}\right]$$
$$= \left(-1_{\{D' \le R\}} - \frac{\partial E[J^1]}{\partial D'}\right)_{\{\delta D'' \ge Y + R - D - E[J^1]\}}.$$

$E[J^2]$ is also decreasing for big enough δ, otherwise it also depends on both δ and β through $E[J^1]$. Finally, the derivative of $E[J^3]$ is,

$$\frac{\partial E[J^3]}{\partial D'} = \beta \frac{\partial}{\partial D'}(Y + R - S' - D'' - E[J^1] - E[J^2])$$
$$= \beta\left(-1_{\{D' \le R\}} - \frac{\partial E[J^1]}{\partial D'} - \frac{\partial E[J^2]}{\partial D'}\right).$$

Once again the behavior is dependent on δ and β.

ii. Worst Case

In the worst case, it is assumed that IRI takes place prior to demand fulfillment. Similar to the best case, the inbound errors during the first phase can be obtained as,

$$\delta E[S'] + \beta E[R - \delta S'] + \gamma E[Y]. \tag{3.12}$$

The calculation of the second phase errors start with the outbound errors,

$$\delta E\left[\min\left\{D'', Y(1-\gamma) + R - S' - \delta S' - \beta(R - \delta S')\right\}\right] \tag{3.13}$$

where $Y(1-\gamma) + R - S' - \delta S' - \beta(R - \delta S')$ is the actual remaining inventory on the shelves. When $\beta = 1$, the on-hand inventory becomes $Y(1-\gamma) - S'$. Recall that, due to the nature of the online retail sales, customers can continue purchasing even when the actual physicals stock is zero. Hence, $Y(1-\gamma) - S'$ can be negative, but $Y(1-\gamma)$ is always positive. The final phase of errors can be formulated similarly,

$$\beta E[(Y(1-\gamma) + R - S - \delta S' - \beta(R - \delta S') - \delta \min\{D'', Y(1-\gamma) + R - S' - \delta S' - \beta(R - \delta S')\})]. \tag{3.14}$$

Also note that, equation (3.14) is always positive when $Y(1-\gamma)\geq S^{'}$. The expected error formulation for a period is obtained similarly by combining equations (3.12), (3.13) and (3.14) as,

$$\begin{aligned}&\delta E\left[S^{'}\right]+\beta E\left[R-\delta S^{'}\right]+\gamma E[Y]+\delta E\left[\min\left\{D^{''},Y(1-\gamma)+R-S^{'}-\delta S^{'}-\beta\left(R-\delta S^{'}\right)\right\}\right]\\&+\beta E\left[\left(Y(1-\gamma)+R-S^{'}-\delta S^{'}-\beta\left(R-\delta S^{'}\right)-\delta\min\left\{D^{''},Y(1-\gamma)+R-S^{'}-\delta S^{'}-\beta\left(R-\delta S^{'}\right)\right\}\right)\right].\end{aligned}\tag{3.15}$$

Reorder level dependency

$E[J]$ in the worst case setup is again increasing in R since the partial derivative with respect to R is positive. This can be shown as

$$\begin{aligned}\frac{\partial E[J]}{\partial R}&=\beta+\delta(1-\beta)_{\left\{D^{''}\leq Y(1-\gamma)+R-\delta D^{'}-\beta\left(R-\delta D^{'}\right)-D^{'}\right\}}+\beta\left(1-\beta-\delta(1-\beta)_{\left\{D^{''}\leq Y(1-\gamma)+R-\delta D^{'}-\beta\left(R-\delta D^{'}\right)-D^{'}\right\}}\right)\\&=\beta+\beta(1-\beta)+\delta(1-\beta)(1-\beta)_{\left\{D^{''}\leq Y(1-\gamma)+R-\delta D^{'}-\beta\left(R-\delta D^{'}\right)-D^{'}\right\}}.\end{aligned}$$

This equation is non-negative, as long as,

$$\delta\geq-\frac{\beta+\beta(1-\beta)}{(1-\beta)^{2}}.\tag{3.16}$$

Equation (3.16) is decreasing in terms of β since the derivative with respect to β is negative. Thus, the lower bound for δ decreases as β increases.

Lead time demand dependency

The function behavior with the first phase demand can be found by taking the derivative with respect to $D^{'}$. In other words,

$$\begin{aligned}\frac{\partial E[J]}{\partial D^{'}}&=\delta-\beta\delta+\delta(\beta\delta-\delta-1)_{\left\{D^{''}\geq Y(1-\gamma)+R-\delta D^{'}-\beta\left(R-\delta D^{'}\right)-D^{'}\right\}}\\&\quad+\beta\left(-\delta+\beta\delta-1-\delta(\beta\delta-\delta-1)\right)_{\left\{D^{''}\geq Y(1-\gamma)+R-\delta D^{'}-\beta\left(R-\delta D^{'}\right)-D^{'}\right\}}\\&=(\beta\delta-\delta-1)_{\left\{D^{''}\geq Y(1-\gamma)+R-\delta D^{'}-\beta\left(R-\delta D^{'}\right)-D^{'}\right\}}(\delta+\beta-\beta\delta)+\delta(1-\beta).\end{aligned}$$

Just like the best case this function's behavior depends on the relation between δ and β.

3.1.3 Numerical Study

The model is applied to the numerical study presented in Section 2.1.2. In the case study, a continuous (Q,R) policy is utilized with $(600,80)$. Weekly demand D and lead time τ are normally distributed with $(50,12^2)$ and $(1.14,0.33^2)$ respectively. Parameters for shelving errors are $\alpha = 1\%$ and $\beta = 0.5\%$ whereas parameters for transaction errors are uniformly distributed with $\delta \in [-1\%,1\%]$ and $\gamma \in [-2\%,2\%]$. Over 2000 of random numbers for D and τ with 5 replications are generated to obtain the expected errors in a single period.

The duration of the simulation study is 52 periods. Moreover, the model starts with zero IRI. In this setting, the simulation terminates by one of the two possible outcomes: (1) gradual error build up becomes too big and causes the inventory to freeze or (2) the system reaches period 52 and terminates normally.

Table 3-1: Correlation matrix of 5 different types of errors in the best case

Corr	D'	D''	**1st** ε_k^s	**1st** ε_k^e	**1st** ε_k^r	**2nd** ε_k^s	**2nd** ε_k^e	**1st Phase**	**2nd Phase**
D'	1								
D''	-0.016	1							
1st ε_k^s	-0.145	0.00	1						
1st ε_k^e	-0.976	0.0151	0.084	1					
1st ε_k^r	-0.01	-0.01	-0.00	0.0216	1				
2nd ε_k^s	-0.02	-0.20	-0.003	0.0225	0.002	1			
2nd ε_k^e	-0.1	-0.94	0.017	0.1083	-0.01	0.125	1		
1st Phase	-0.28	0.002	0.98	0.2295	0.0027	0.000	0.033	1	
2nd Phase	-0.03	-0.26	-0.00	0.0404	0.5318	0.842	0.192	0.004	1

Table 3-1 and Table 3-2 summarize the results of simulation studies. In the tables D' denotes the lead time demand, D'' denotes the remaining demand, ε_k^s denotes the outbound (selling), ε_k^e denotes the storing (expiration and spoilage), ε_k^r and denotes the inbound (receiving) errors. Table 3-1 presents the results of correlation study done on the different types of errors; the study includes the total errors done in both the first and the second phase. Based on this table, errors have no strong dependence between each other. The demand in each phase has a negative correlation with errors; however, this relation is dependent on the δ and β as shown in the previous section.

Table 3-2 demonstrates the covariance matrix between the same parameters. This table is used to calculate the variation in multiple dimensions.

Table 3-2: Covariance Matrix of 5 different types of errors in the best case

Cov	D'	D''	**1st** ε_k^s	**1st** ε_k^e	**1st** ε_k^r	**2nd** ε_k^s	**2nd** ε_k^e	**1st Phase**	**2nd Phase**
D'	463.09								
D''	-53.07	22626							
1st ε_k^s	-1.980	0.002	0.400						
1st ε_k^e	-2.007	0.215	0.005	0.009					
1st ε_k^r	-1.478	-4.531	-0.001	0.007	12.1				
2nd ε_k^s	-2.590	-163.8	-0.01	0.011	0.052	29.5			
2nd ε_k^e	-1.389	-88.59	0.006	0.006	-0.02	0.42	0.386		
1st Phase	-3.986	0.206	0.405	0.014	0.006	0.00	0.013	0.4194	
2nd Phase	-5.467	-256.9	-0.00	0.025	12.17	29.97	0.786	0.0197	42.88

In the equations derived in the previous section we failed to conclude a strong linear relation between the demand and the total errors. Same conclusion is also observed from Table 3-1 and Table 3-2.

Table 3-3 summarizes the statistics obtained through the simulation study. The results in this table are used to create a baseline to assess the impact of IRI through comparison.

Table 3-3: Summary of statistics of the best case

Stock-out	**Error**	**Length**	**Time**	**Sold**	**n(R)**	**Stock-loss**	**Inbound**	**Outbound**
22	38	12.8	662	30,522	40	37	(2)	3

The total variance of errors is 43.365, or standard deviation 6.58. The mean of the errors is 0.1; 0.05 from phase 1 and 0.05 from phase 2. The correlation matrix for the worst case is shown in Table 3-4

Table 3-4: Correlation matrix of 5 different types of errors in the worst case

Corr	D'	D''	**1st** ε_k^s	**1st** ε_k^e	**1st** ε_k^r	**2nd** ε_k^s	**2nd** ε_k^e	**1st Phase**	**2nd Phase**
D'	1								
D''	0.007	1							
1st ε_k^s	0.001	0.002	1						
1st ε_k^e	-0.001	-0.002	-1	1					
1st ε_k^r	-0.005	0.001	-0.005	0.005	1				
2nd ε_k^s	-0.006	-0.004	0.015	-0.015	-0.002	1			
2nd ε_k^e	-0.12	-0.94	-0.004	0.004	-0.02	-0.03	1		
1st Phase	0.002	0.002	0.999	-0.99	-0.004	0.015	-0.004	1	
2nd Phase	-0.02	-0.09	0.010	-0.01	0.509	0.853	0.054	0.01	1

Contrary to the best case, errors have strong dependence between each other. The outbound and the storage errors in the first phase have strong negative correlation. The demand again has no strong correlation with errors except the storing errors in the second phase. Recall that this correlation is dependent on the relation between δ and β.

Table 3-5: Covariance matrix of 5 different types of errors in the worst case

Cov	D'	D''	**1st** ε_k^s	**1st** ε_k^e	**1st** ε_k^r	**2nd** ε_k^s	**2nd** ε_k^e	**1st Phase**	**2nd Phase**
D'	478.78								
D''	21.28	22855.4							
1st ε_k^s	0.011	0.169	0.446						
1st ε_k^e	0.00	-0.0008	-0.002	0.00					
1st ε_k^r	0.008	0.555	0.008	0.00	12.1				
2nd ε_k^s	-1.355	-0.72	0.026	-0.00	-0.041	33.7			
2nd ε_k^e	-1.723	-89.6	-0.002	0.00	-0.044	-0.12	0.391		
1st Phase	0.017	0.181	0.444	-0.002	0.008	0.027	-0.002	0.441	

2nd Phase	-3.089	-89.8	0.033	-0.00	12.02	33.5	0.225	0.033	0.225

Table 3-5 depicts the covariance matrix for the worst Case. Based on the table, the total variance of errors is 46.29, or standard deviation 6.8. And the mean of the errors is 1.44; 0.04 from the first phase and 0.09 from the second phase.

Table 3-6: Summary of statistics of the worst case

Stock-out	Error	Length	Time	Sold	n(R)	Stock-loss	Inbound	Outbound
31	180	12.8	670	30,425	39.5	180	(6)	5

Table 3-6 summarizes the final result statistics for the worst case simulation study. By comparing Table 3-3 and Table 3-6, we observe that the average errors in the worst case are considerably larger. This is intuitive since the order of events are arranged to maximize the errors in the worst case and minimize them in the best case. Unexpectedly, the average lost sales in both cases are similar. The reason behind this result is the effect of customer’s inability to access the actual physical stock. Lost sales in the online retail model can only occur after the records reach to zero. At that point both cases behave similarly.

3.2 Evaluation of the Impact of IRI

In the previous section we modeled IRI in the online retail environment. In the model the decision maker is assumed to be blind to IRI. When this assumption on IRI is removed, inventory manager has to adjust the inventory policies accordingly. In this section we analyze the two alternative methods for compensating IRI; increasing IRI resistance of the current inventory policy and controlling the factors that cause IRI.

3.2.1 IRI Resistance Method

It is known that even small amount of IRI can cause significant losses (DeHoratius & Raman, 2008). The main focus here is minimizing the impact of IRI via controlling the level of inventory on-hand. As done in Section 2.2.2, we characterize a framework for error formulation and utilize this formulation to generate expected error, which is then used to adjust the inventory records.

In a traditional (Q,R) framework, the inventory manager observes the inventory records continuously and makes a replenishment decision of Q when the records fall below R. In this setup, R is defined based on three parameters; lead time, supply/demand uncertainty and service level. Using these parameters the reorder level is defined as the safety stock plus the lead time demand. The exact calculation of Q and R levels is out of the scope of this study. We are interested in modifying the current safety stock each period to account for the IRI.

The formulation of IRI in the previous section provides expected one-step error given certain parameters. Using this expectation we gradually increment the safety stock levels at each decision epoch. This behavior is depicted graphically in Figure 3-4. As the figure illustrates, this method forces the inventory records to increase gradually while keeping the actual physical stock relatively constant.

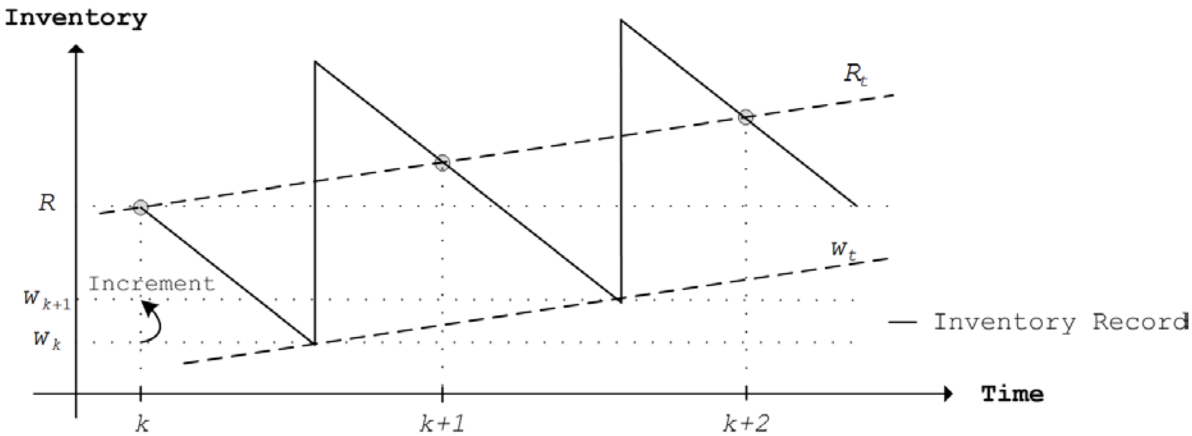


Figure 3-4: Increasing safety stock

Determination of the increment value is critical for IRI resistance. One method of obtaining a suitable increment value is using the defined error characterization to obtain a mean and a variance for errors. Then these values can be utilized to devise an estimate for the increment value. However, in the case study we explore various increment values to account for the high standard deviation. For the best and the worst case scenarios, we generated separate simulation models. Both models have 52 period duration and replicated 60 times for increments 0 to 30.

Figure 3-5 shows the results of the simulation studies for each case. The horizontal axis denotes the increment value and the vertical axis represent the total values of the parameters. According to the figure, sales per unit time (unit time is a day) is not effected with the increment because customers make their purchasing decision based on records; however, lost sales drops as the increment increases. The total number of stock-outs also decreases with the increment. Penalty sales is considerably different for the best and worst case; in the worst Case, penalty sales are larger. But in both cases they both decrease as the increment increases. After increment 12, all the parameters remain relatively constant.

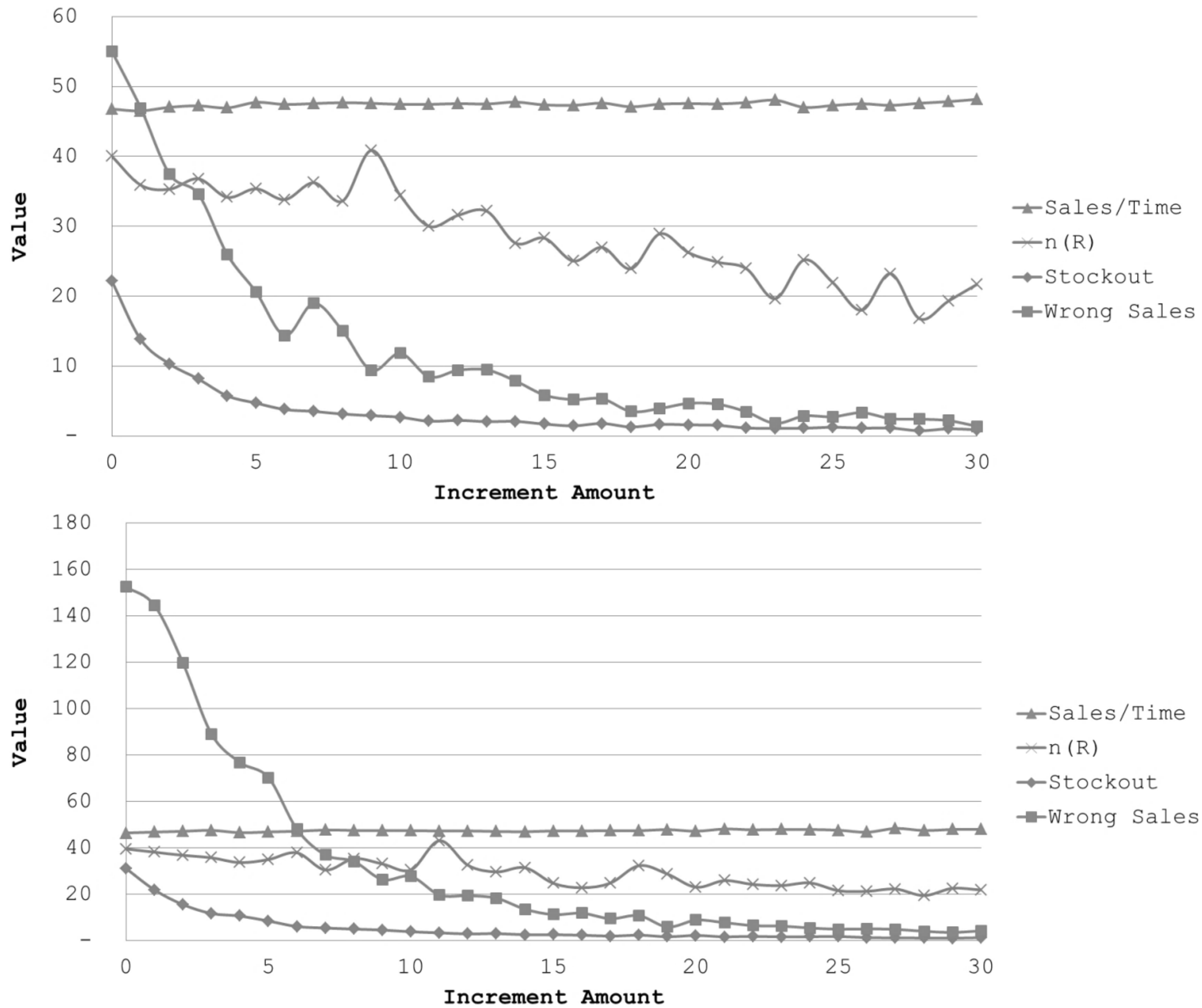
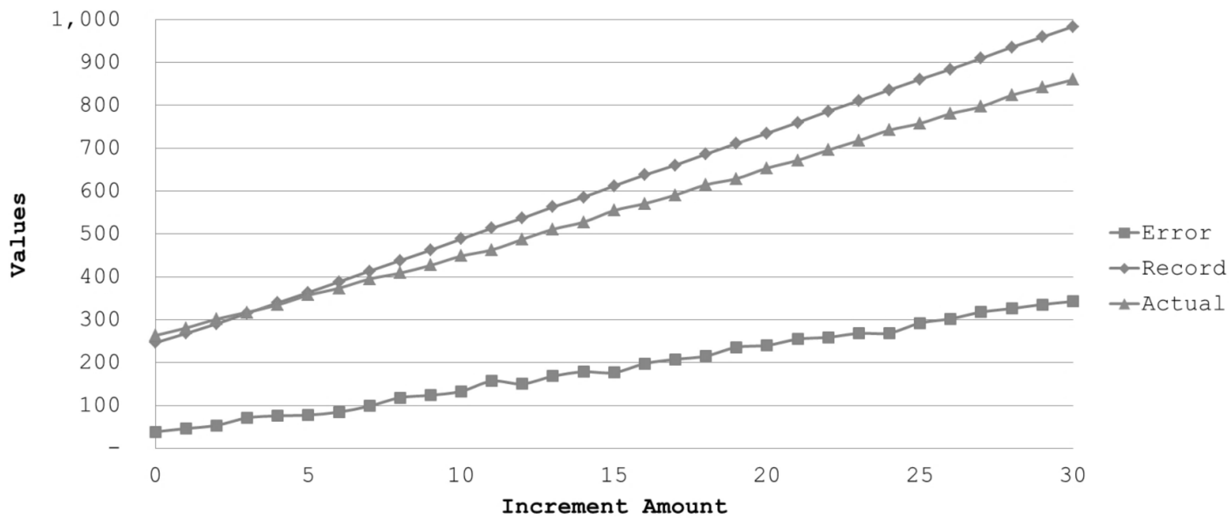


Figure 3-5: The IRI resistance method for the best (left) and the worst (right) case

The average length of the period, the inbound and the outbound errors behave independently from the increment. That is an expected result since they do not depend on the inventory on-hand.



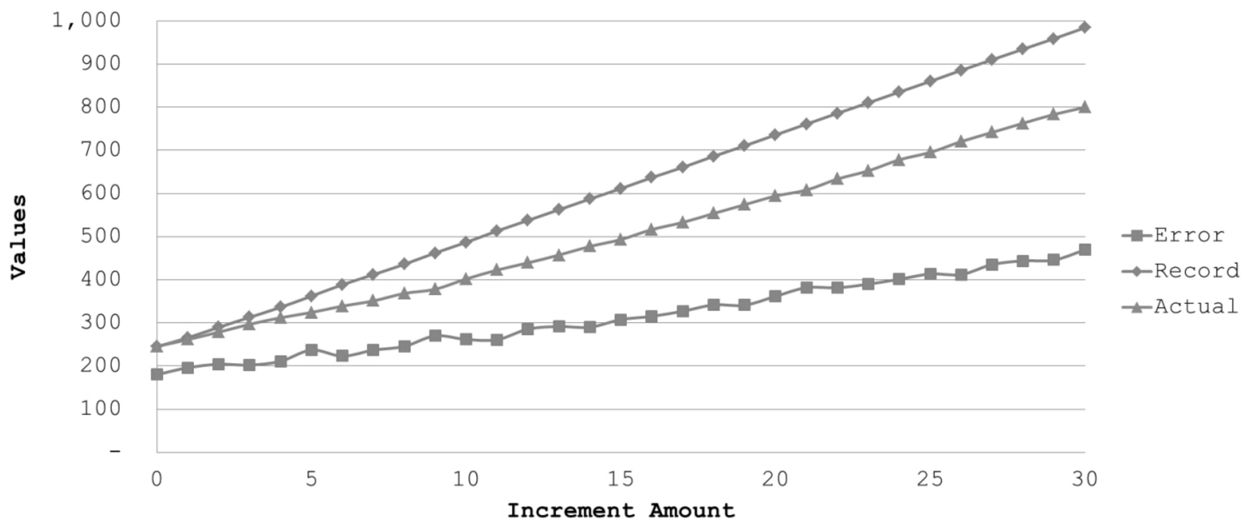


Figure 3-6: The IRI resistance method for the best (left) and the worst (right) case cont.

Figure 3-6 shows the results of the remaining parameters after the simulation. As expected the total error increases with the increment. Recorded and actual inventory are also increasing with increment. The reason is the increment raises the average inventory levels. Hence, choosing a very high increment is going to hurt the system more than it benefits it. Depending on preference (due to line of business), certain parameters can be chosen and the increments that maximize or minimize those values can be selected.

3.2.2 Error Control and Correction Method

Cycle counting is one of the fundamental methods of error controlling methods. This is a process where the entire inventory is reckoned physically; see (Iglehart & Morey, 1972; Opolon, 2010; Young & Nie, 1992). The literature on cycle counting and inventory auditing is vast, see (Iglehart & Morey, 1972; Kok & Shang, 2007; Kumar & Arora, 1991; Meyer, 1990; Rosetti et al., 2010; Young & Nie, 1992).

As mention in Section 2.2.3, determining the best possible triggering condition is not an easy task. Once again we perform simulation studies with various trigger configurations. In these studies we utilize the relation between the lead time sales and the expected demand during lead time to configure a *trigger mechanism*. The expected lead time demand is a known value and the lead time sales is an observed value. The logic behind this trigger mechanism is: If the lead time sales is considerably lower than the expected demand during lead time, then it can be concluded that the system contains high amounts of IRI. However, determining the sensitivity of the trigger mechanism still remains as a daunting issue. We overcome this burden by inserting a modifier called the trigger value. The main purpose of the trigger value is to adjust the expected demand during lead time. Via this method we can effectively change the sensitivity of the trigger mechanism.

For the best and the worst case, we generated separate simulation models. Both models have 52 period duration and use 60 replications for trigger values 1 to 15.

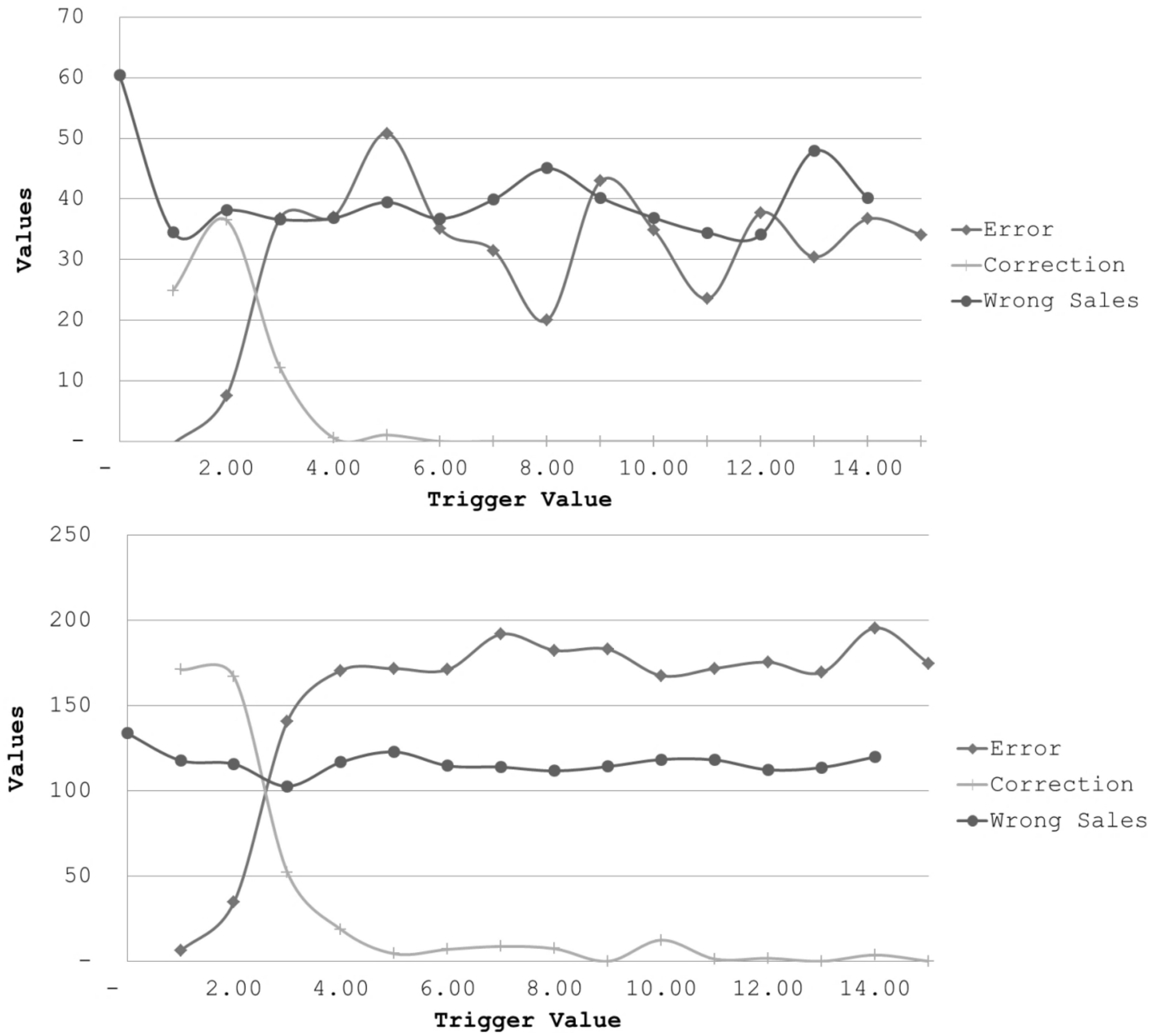


Figure 3-7: The error control method for the best (left) and the worst (right) case

Figure 3-7 shows the results of the simulation study. The horizontal axis denotes the trigger values and the vertical axis represent the average value after 60 replications. In both cases, as the trigger value increases expected correction decreases, penalty sales increases and errors slightly decrease. Comparatively, the changes are bigger in the worst case.

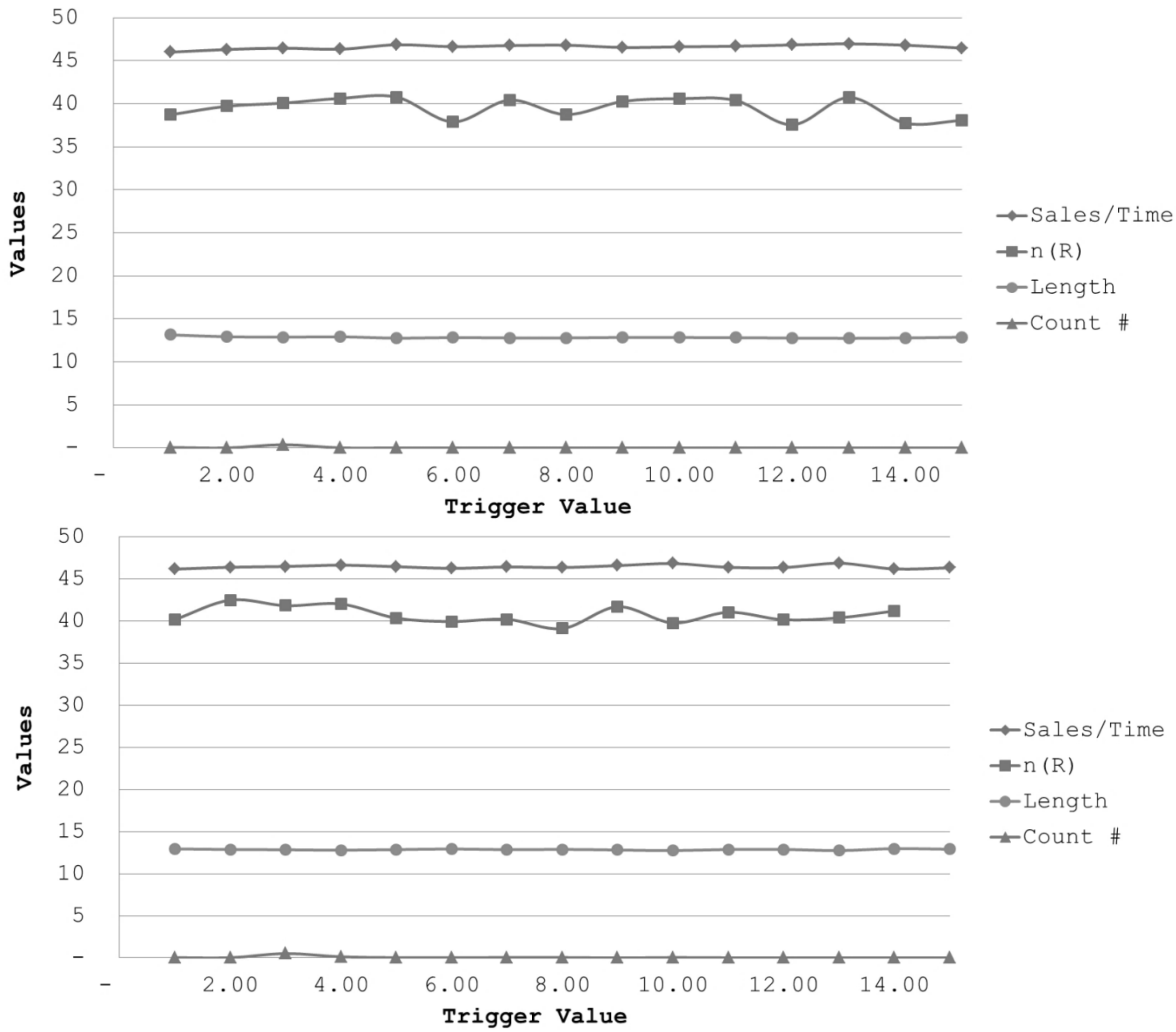


Figure 3-8: The error control method for the best (left) and the worst (right) case cont.

Figure 3-8 presents the results of the remaining parameters. According to the figure, the sales per unit time, the lost sales and the average period length does not change over time. The behavior of these parameters for the best and worst case is very similar to each other. Once again the main reason behind this outcome is the customer’s lack of access to the actual physical stock.

3.2.3 IRI Resistance and Error Control

In this section we apply IRI resistance and error control method simultaneously. As depicted in Figure 3-9, with each period, the IRI resistance method increases the reorder level. At the same time, depending on the trigger mechanism, cycle counts are triggered. After cycle count is triggered all the IRI is corrected and the reorder level resets to its original value.

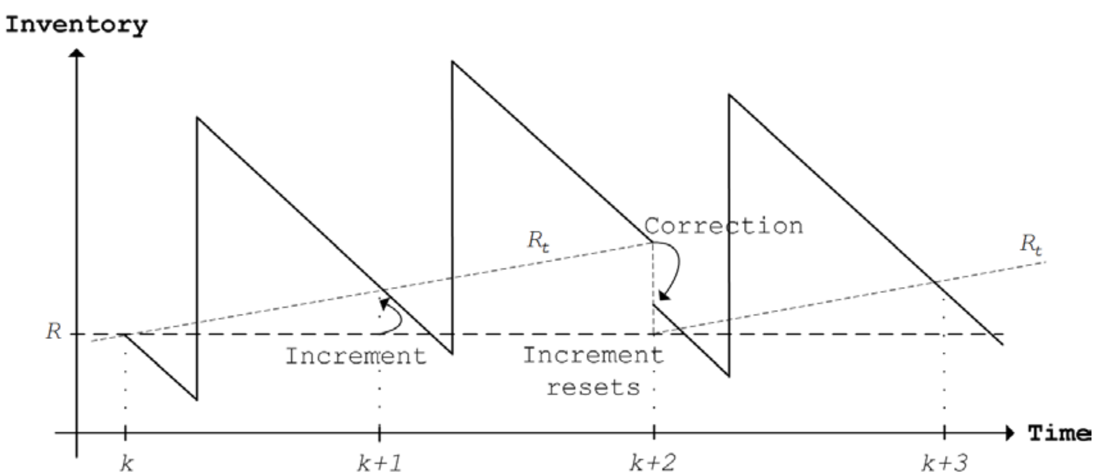


Figure 3-9: Combined compensation framework

The combined method is applied to the numerical study presented in Section 2.1.2; Figure 3-10 and Figure 3-11 show the result of the study.

Figure 3-10 depicts the penalty sales and the lost sales for the best case. The lost sales remain relatively constant with the increment level but a small decrease is observable as the trigger value increases. Penalty sales, on the other hand, decrease with both methods. For increment or trigger values greater than 2, the system observes no penalty sales.

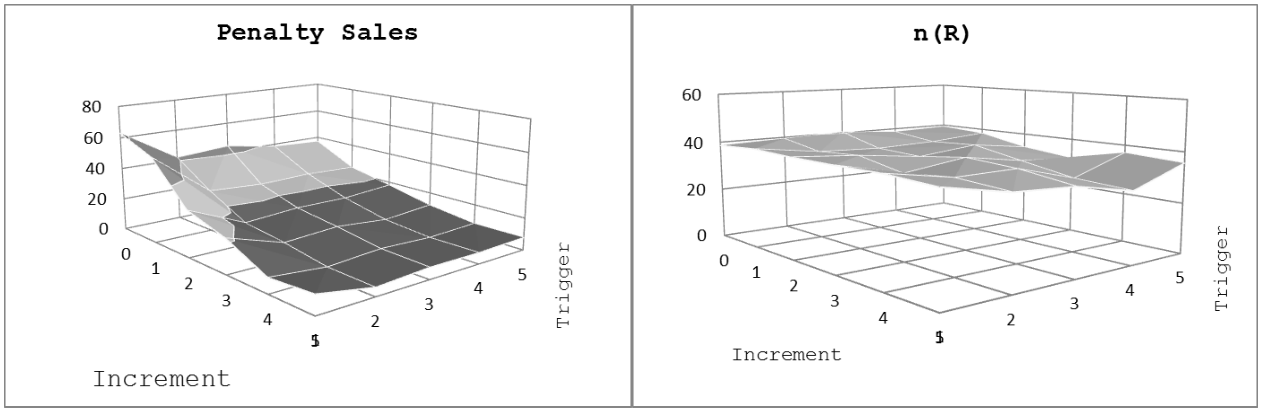


Figure 3-10: Combined method results for penalty sales and lost sales for the best case

Figure 3-11 compares error correction with the number of triggered cycle counts and the levels of increments. As can be seen from the graphs, the increment value increases the count number. This is because the IRI resistance method increases the total errors for higher increment levels and higher errors cause more cycle counts. For values greater than 2, the trigger mechanism starts to become ineffective. This is because the mechanism is not sensitive enough to generate any counts for values above 2.

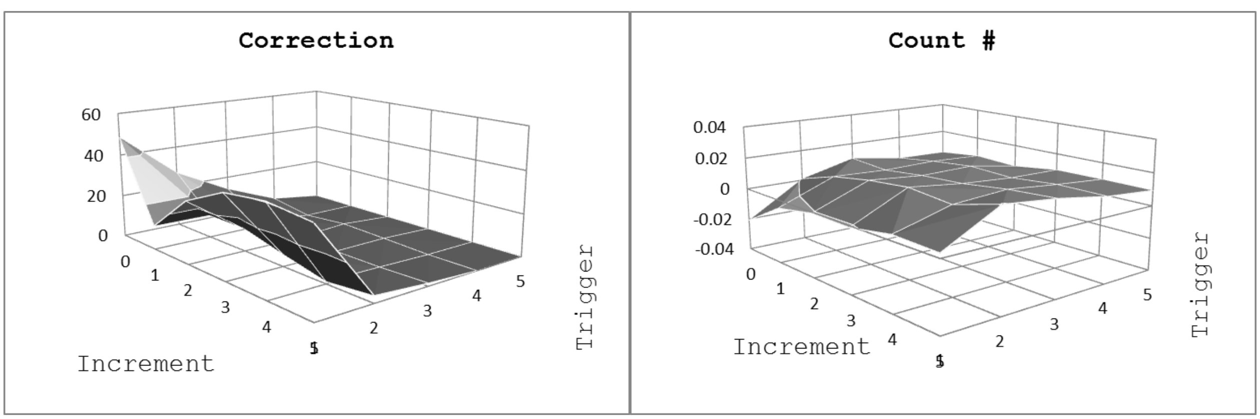


Figure 3-11: Combined method results for correction and count number for the best case

The results obtained by the comparison between the original case and compensation methods are presented in Table 3-7. In this table, original case refers to the classical (Q,R) model without implementing any IRI correction method.

Table 3-7: Final result statistics table with optimal range selection

Compensation Methods	Sales per unit time	Stock-out	Correction per Count	Error	Average Record	Average Actual	Lost Sales
Original (Best)	45-50	20-25	-	40-50	250-260	150	40
Original (Worst)	45-50	30-25	-	150-200	240-250	250	40
IRI Res. (Best) 8	45-50	0-5	-	100-150	400-450	400-450	30-35
IRI Res. (Worst) 9	45-50	0-5	-	200-250	400-450	350-400	30-40
Error Control (Best) 1	45-50	15-20	25-30	0-5	240-250	240-250	30-40
Error Control (Worst) 1	45-50	20-25	170-190	0-5	240-250	240-250	40-45
Combined (Best) 2,1	47-50	10-15	45-50	5-10	260-270	260-270	35-40
Combined (Worst) 2,1	47-50	15-20	170-200	10-15	250-260	250-260	35-40

According to the table the sales per unit time does not change between compensation methods. This is an intuitive result because the sales are mainly influenced by demand; which is same in all the models. The original case has the highest total stock-out; all the other methods decrease this statistic. The best result for the stock-out is observed by the IRI resistance method. Error correction is done only in the error control and the combined method; and it is higher in the combined method. The reason is, the combined method utilizes the IRI resistance method as well, and as mentioned, the IRI positively influences the errors. Same outcome is observed for the actual inventory as well. The lost sales decreases with each method however, the best result is observed by the combined method.

3.3 Conclusion and Future Work

IRI behavior in the online retail under the influence of demand, supply and lead time uncertainty is analyzed. Factors contributing to IRI are defined and formulated. Then, a framework for calculating the errors is derived in two separate cases: The best and worst case. Then, a numerical study using simulation is conducted to show the sensitivity of the inventory replenishment policy to IRI. The highlights can be summarized as;

- In terms of the lead time demand, there is no conclusive result on the behavior of the error function. Depending on the input parameters β and δ it can decrease or increase with the lead time demand.
- In terms of R, W and Y the error function is increasing.
- In both phases the biggest effect is done by the outbound errors. Hence, the parameter β has the highest impact.

Two alternatives for compensating IRI are presented; the IRI resistance and the error control method. Then, a numerical analysis is performed to observe the behavior of IRI and to quantify the effects of the applied solution alternatives. Based on these studies, the IRI resistance method positively influences IRI because it increases the average inventory on-hand. The IRI resistance method performs better on stock-out and lost sales, whereas the error control method can keep low inventory levels. For the trigger mechanism high count number means, fewer stock-outs and fewer lost sales but more frequent counts. Therefore, a range can be chosen and the values that maximize or minimize desired parameters can be selected.

Similar to Chapter 2, the compensation methods described in this chapter are static. Hence again a good opportunity for future work is utilizing dynamic correction methods that change for each period.

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CHAPTER 4 COMPARATIVE ANALYSIS

In the previous chapters we derived characterizations for error behavior in online and offline retail environment with random demand, supply and lead time. Furthermore, we provided compensation methods that limit and control the impact of IRI and conducted numerical analyses using simulation. In this chapter we provide a comparative analysis using a cost framework for each setting. Then, we apply the cost framework to the previously developed compensation methods. Finally, we demonstrate the results with the same case study.

We develop a cost model by dividing the general cost structure into three categories: IRI related costs, penalty costs and operating costs. IRI related costs are the ones caused by errors. As explained in the previous chapters they can further be categorized as shelving, inbound and outbound errors. The inbound and outbound errors are similar in the online and the offline retail settings. Typically, the inbound errors occur during ordering and receiving processes whereas the outbound errors occur during check-out processes (e.g. scanning errors). Shelving errors on the other hand are different for the online and the offline retail environment. Due to customer interaction in the offline retail environment, items are subject to theft and spoilage whereas in the online environment only spoilage occurs.

Penalty costs consist of cycle counting costs and lost sales for the offline retail setting. In addition to those, the online settings also have penalty sales. Recall that, penalty sales denote the number of units sold when the actual inventory is zero but the inventory records are positive (see Appendix A). Operating costs are typically incurred in routine inventory management processes: holding, purchasing and selling price (as a negative cost). These are known by the decision maker at all times. Moreover, they are used as the basis for estimating the remaining cost parameters.

4.1 Model

Let purchasing cost, holding cost for one period, and selling price of one unit of inventory be c, h, and p, respectively, where $p > c > h > 0$. The total cost in the (Q,R) framework as defined in Zipkin (2000) is,

$$TC(Q,R) = h\left(\frac{Q}{2} + R - E[D^{'}]\right) + k\frac{E[D]}{Q} + \rho\frac{E[D]}{Q}n(R)$$

where

$$n(R) = E[\max\{D - R, 0\}] = \int_{R}^{\infty}(z - R)f_D(z)dz\,.$$

In the above equations h, k, ρ, $D^{'}$ and $n(R)$ are respectively holding, ordering, shortage cost, lead time, the expected unit time average demand and the expected lost sales. Additionally, the term $E[D]/Q$ denotes the expected unit time average demand per order (Nahmias, 2008). In our model we modify this cost function by introducing IRI and penalty cost. The costs are estimated as follows:

IRI related costs: IRI has two consequences. First of all, the items that are affected by IRI become unsalable; thus, the opportunity of selling those items is vanished. The cost of losing the opportunity to sell an item is p. Secondly, the unsalable items also become unobservable; hence the inventory replenishment policy receives incorrect information. Finding a cost for incorrect information is challenging. For practicality, we use λ to represent the cost of one unit of mismatch between actual inventory and records. Furthermore, it is possible to experience a positive or negative IRI. In this context positive IRI corresponds to losing inventory due to shrinkage.

Negative IRI is observed when unexpected items are obtained due to transaction errors. Then, the total value for IRI related unit cost becomes $p+\lambda$ for positive and $-p+\lambda$ for negative IRI.

Penalty costs: These costs are incurred when certain conditions are met. In our model we have three such costs, penalty sales, cycle count costs, and lost sales. Counting cost is incurred based on the number of items counted and it is the same for both the offline and the online setting. Let υ denote the cost of counting a unit. Penalty sales is only active for online setting; it only occurs when actual inventory is less than or equal to zero and inventory records are positive. Under a penalty sale situation, customers pay for the full price of the inventory that the system does not currently have. Let ν denote the cost of one unit of penalty sales. Finally, lost sales occurs when actual physical stock drops to zero in the offline setting; whereas in the online setting it occurs when inventory records drops to zero. Note that lost sales is not simply equal to the selling price (p) because lost sales also has long term effects such as loss of goodwill and inaccurate demand estimation. For practicality, let ρ denote the lost sales cost per unit in both the online and the offline settings. These cost parameters are summarized in Table 4-1,

Table 4-1: Cost structure

Cost	**IRI Related Cost**				**Penalty Cost**			**Operating Cost**	
	Theft	**Inbound**	**Outbound**	**Spoilage**	**Pnlty sales**	**Count**	**Lost Sales**	**Holding**	**Purchasing**
Offline	$\pm p+\lambda$	$\pm p+\lambda$	$\pm p+\lambda$	$\pm p+\lambda$	-	υ	ρ	h	c
Online	-	$\pm p+\lambda$	$\pm p+\lambda$	$\pm p+\lambda$	ν	υ	ρ	h	c

One of the biggest problems of this setup is the fact that it does not account for errors. Additionally, in our model we use purchasing cost c instead of using a fixed ordering cost k . Hence, the total cost function can be rewritten as

$$TC(Q,R) = h\left(\frac{Q}{2} + R - E[D']\right) + cQ\frac{E[D]}{Q} + \rho\frac{E[D]}{Q}n(R) \\ + (\pm p + \lambda)\frac{E[D]}{Q}E[IRI] \tag{4.1}$$

where $E[IRI]$ is the average amount of errors in a period. When cycle counting is implemented the equation becomes even more complex. We use the same function to model cycle counting mechanism as done Section 2.2.3. The relation between the lead time sales and the expected demand during lead time is utilized to configure the trigger mechanism. The expected lead time demand is a known value and the lead time sales is an observed value. The logic behind this trigger mechanism is: If the lead time sales are considerably lower than the expected demand during lead time, then it can be concluded that the system contains high amounts of IRI. A modifier called the trigger value is inserted to adjust the expected demand during lead time. Via this the sensitivity of the trigger mechanism can be controlled effectively.

Note that the maximum number of times a cycle count can happen in a period is 1. This is because the decision to trigger a cycle count is only available to decision maker once per period, when inventory record is equal to the reorder level. Hence, equation (4.1) can be modified as,

$$TC(Q,R) = h\left(\frac{Q}{2} + R - E[D']\right) + cQ\frac{E[D]}{Q} + \rho\frac{E[D]}{Q}n(R) \\ + (\pm p + \lambda)\frac{E[D]}{Q}(E[IRI] - E[Correction]_{\{count=1\}}) \\ + \upsilon\frac{E[D]}{Q}E[Count]_{\{count=1\}} \tag{4.2}$$

where $E[Correction]$ denotes the expected number of correction when a cycle count is triggered. Additionally, $E[Count]$ denotes the expected number of inventory counted when a cycle count is triggered. We use equation (4.2) in our simulation studies to calculate the inventory related costs.

In order to compare similarities and differences of the online and offline setting we conducted series of simulation studies. We first examine two groups of studies for the offline setting. The first group is done using the IRI resistance method (Section 2.2.2); and the second group is done using the error control method (Section 2.2.3). In both groups several separate simulation studies are performed to understand the sensitivity of the total cost with respect to IRI and penalty costs. Furthermore, the best and the worst case framework are also implemented during these analyses. The same methodical analyses are then performed for the online setting.

The main goal of these studies is to use the costs as a generic measure for all the key performance metrics. Then, the total cost function in equation (4.2) can be used to determine the effectiveness of the compensation methods described in Chapter 2 and Appendix A.

The simulation study uses the same numerical study in Section 2.1.2. In the case study, a continuous (Q,R) policy is utilized with $(600,80)$. Weekly demand and lead time are normally distributed with $(50,12^2)$ and $(1.14,0.33^2)$ respectively. Parameters for shelving errors are $\alpha = 1\%$ and $\beta = 0.5\%$ whereas parameters for transaction errors are uniformly distributed with $\delta \in [-1\%,1\%]$ and $\gamma \in [-2\%,2\%]$.

4.2 Numerical Study: The Offline Retail Setting

We first discuss the findings for the offline retail environment. Using the error characterization derived in Chapter 2 and the cost structure presented equation (4.2) we conducted two groups of simulation studies. The first group is done to assess the best levels of increment for the IRI resistance method and the second one is done for finding the most suitable trigger mechanism for the error control method.

4.2.1 IRI Resistance Method

Table 4-2 and Table 4-3 summarize the key performance measures while the IRI resistance method is being implemented for the best and the worst case, respectively. In both tables, columns represent the key measures that form the total cost function presented in equation (4.2). In these tables, the sales per unit time represents the average amount of sales done per week; the total sales denotes the total amount of sales done throughout 52 periods; The stock-out column shows the total number of time actual physical stock drops to zero; $n(R)$ represents the total amount of lost sales; IRI column represents the total number of mismatch; the actual and the record columns denote the levels of inventory; and total actual column shows the total actual physical stock purchased in 52 periods. Furthermore, each row in these tables denote a set of simulation studies performed by using a specified level of increment for the IRI resistance method. For comparison, 7 different values for increment level are selected and presented, which are 0, 5, 10, 15, 20, 25 and 30. Note that the first row (with 0 increment level) is the base setup where the IRI resistance method is not applied.

Table 4-2: Summary of statistics for the best case with the IRI resistance

Increment	Sales/Time	Total Sales	Stock-out	n(R)	IRI	Avg Actual	Avg Record	Total Actual
0	38.25	9,915	9.90	327.76	92.78	50.79	124.78	10,000
5	47.05	30,046	22.25	581.19	304.63	225.60	374.51	30,340
10	48.12	30,093	1.25	12.39	367.73	320.10	489.53	30,449
15	48.37	29,840	0.72	6.08	416.33	417.77	613.34	30,262
20	47.60	29,587	0.30	1.08	474.57	534.50	737.64	30,051
25	48.44	29,333	0.20	0.57	523.21	632.70	862.00	29,860
30	47.76	29,080	0.15	0.44	576.33	739.91	986.49	29,657

Based on the table, the stock-out and the lost sales values decrease with the increment amount; conversely IRI, inventory records and the actual physical stock levels increase as the increment increases. The main reason behind this is, the increment level increases the average on-hand inventory to reduce the effects of IRI (i.e. stock-outs, lost sales); but the system experiences more errors as a result of having excessive amounts of inventory. This suggests that having higher increment levels (15 to 30) are undesirable. Moreover, smaller increment levels (0 to 4) results in higher amounts of stock-outs and lost sales due to early freezing. As discussed in the previous chapters, when the increment level is too low, the system freezes before it reaches period 52. This can be observed by comparing the total sales. Finally, for moderate increment levels (5 to 14) the stock-out and the lost sales decrease considerably while the changes on the other measures are relatively small. Hence, using moderate levels of increment levels are more preferable.

Table 4-3: Summary of statistics for the worst case with the IRI resistance method

Increment	Sales/Time	Total Sales	Stock-out	n(R)	IRI	Avg Actual	Avg Record	Total Actual
0	31.96	5,252	6.23	181.01	82.69	27.09	27.09	108.36
5	43.47	20,743	34.15	1068.5	327.15	134.29	134.29	376.73
10	47.74	30,043	12.58	215.8	484.71	248.92	248.92	493.20
15	48.18	29,840	0.87	5.77	532.45	351.42	351.42	613.21
20	47.63	29,587	0.37	1.34	588.46	467.04	467.04	738.25
25	48.18	29,333	0.32	1.4	642.56	578.49	578.49	862.11
30	47.74	29,080	0.33	1.23	689.93	676.84	676.84	986.10

The results observed from Table 4-3 are very similar; again, moderate levels of increment works better. The biggest difference between the two cases is: the amount of IRI experienced is larger in the worst case;

therefore, the most suitable level of increment is expected to be higher for the worst case.

Table 4-4 presents the baseline and the remaining scenarios. The values for holding (h), purchasing (c) and selling (p) is selected. In the literature the most frequently used relation between h and c is $h = 0.2c$; however there is no general relation for the selling price. For our purposes the accuracy of p, h and c is not relevant as long as they satisfy $h > c > p$ and justifies a profitable opportunity for the decision maker to be in the business. Hence, we utilized the following setup: $h = 0.2c$ and $p = 2c$. In other words, if the purchasing cost is 1 unit then the holding cost is 0.2 and the selling price is 2 units. The cost values for the lost sales and IRI are subjective; so different configurations of values are considered for each scenario.

Table 4-4: Cost scenarios in the offline setting with the IRI Resistance

Cost Setup	Holding	Purchasing	Price	n(R)	IRI
Base	0.2	1	2	1	2
Scenario 1	0.2	1	2	1	6
Scenario 2	0.2	1	2	1	12
Scenario 3	0.2	1	2	3	2

Table 4-4 demonstrates 6 different costs in the columns and four different scenarios in the rows. The base scenario uses small penalty costs for lost sales and IRI. In scenarios 1 and 2, the unit cost of IRI is increased. Finally in scenario 3 the cost of lost sales is increased.

We calculated the costs using the scenarios presented in Table 4-4 and the total cost function derived in equation (4.2). Then, we calculated the revenues by using the selling price and the total sales. Finally, the profit function is obtained as shown in Figure 4-1. With the increment, the profit obtained increases sharply up to a certain point than it decreases slowly.

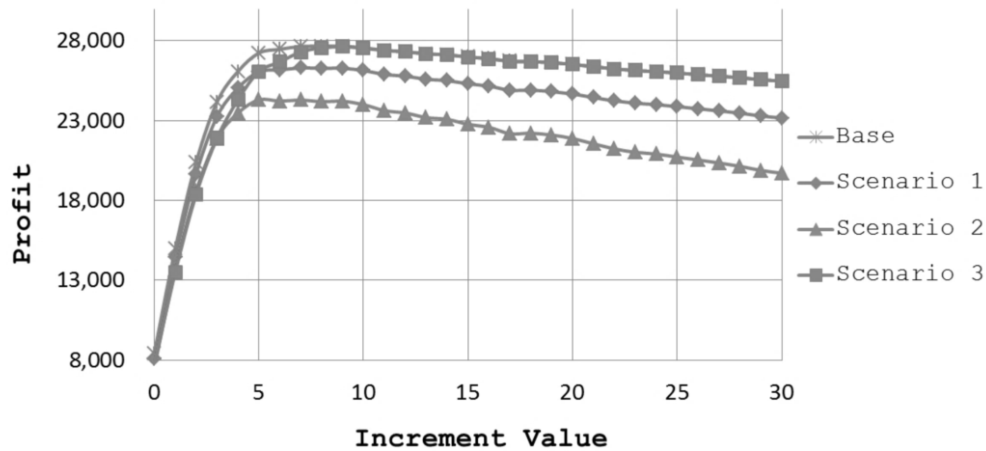

Figure 4-1: Profit function for each scenario for the best case with the IRI resistance method

As the unit cost of IRI and the unit cost of lost sales change, the profit functions behaves differently. In the base cost structure the optimal increment value is 8. When the penalty for IRI increases, the optimal increment value decreases; the optimal increment for scenario 1 is 7 and scenario 2 is 5. As presented in Section 2.2.2, the average inventory increases with the increment value, this in turn causes more shelving errors. In scenario 3 we increase the unit cost of lost sales, and the best increment value jumps to 9. Finally, the base case profit function is always greater than the other 3 as expected since the scenario has higher penalty costs.

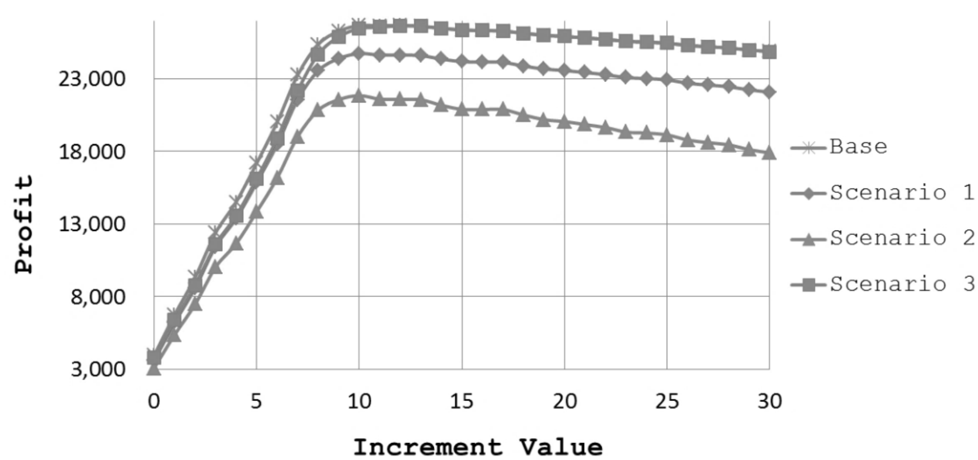

Figure 4-2: Profit function for each scenario for the worst case with the IRI resistance method

The profit function behavior in the worst case with each scenario is demonstrated in Figure 4-2. In the base case, the optimal increment value is 12. The optimal increment is at 10 for scenario 1 and 2 but for scenario 3 it is back at 12. The overall behavior of the profit function is the same.

4.2.2 Error Control and Correction Method

We implement the same procedures for the error control method. Table 4-5 and Table 4-6 summarize the key performance measures while the error control method is being implemented for the best and the worst case, respectively. In both tables, columns represent the key measures that form the total cost function presented in equation (4.2). Each row denotes a set of simulation studies performed by using a specified trigger level.

For comparison, 7 different trigger values are selected and presented, which are 0, 1, 2, 3, 4, 5 and 10. Note that the first row (with 0 trigger value) is the base setup where the error control method is not applied.

Table 4-5: Summary of statistics for the best case with the error control method

Trigger	Sales per Time	Total Sales	Stock-out	n(R)	IRI	Count #	Avg Actual	Avg Record	Total Actual
0	38.25	9,915	9.90	327.76	92.78	-	50.79	124.78	10,000
1	47.17	30,295	16.22	277.18	317.21	16.38	233.05	238.68	30,611
2	47.04	30,319	25.40	627.82	310.98	5.58	224.51	243.27	30,624
3	46.71	30,323	28.73	772.90	309.39	4.38	221.79	245.69	30,625
4	46.35	30,329	30.60	898.53	308.12	3.88	220.15	247.30	30,629
5	46.22	30,293	31.55	941.97	305.59	3.88	218.87	247.54	30,600
10	46.07	30,136	31.05	966.89	301.13	3.72	217.33	248.14	30,432

Recall that as the trigger value increases, the sensitivity of the trigger mechanism decreases, and as a result fewer cycle counts are observed. Based on Table 4-5, for higher trigger values the system experiences more stock-outs, higher lost sales, higher IRI and lower counts while the remaining parameters do not fluctuate. On the other hand, when the trigger value is zero the system does not experience any cycle counts. As a result IRI buildups and causes the system to freeze early. This can be observed by comparing the total sales. Finally, for the moderate trigger values, system keeps the values for lost sales, IRI and stock-out low. Hence, choosing a moderate values for trigger (1 or 2) is more preferable.

Table 4-6: Summary of statistics for the worst case with the error control method

Trigger	Sales per Time	Total Sales	Stock-out	n(R)	IRI	Count #	Avg Actual	Avg Record	Total Actual
0	38.25	9,915	9.90	327.76	92.78	-	50.79	124.78	10,000
1	46.97	30,149	19.95	393.95	476.59	20.22	229.11	237.20	30,619
2	46.40	30,133	28.35	743.34	475.40	8.57	221.57	241.87	30,612
3	47.01	29,954	32.30	927.38	473.18	6.92	217.76	243.19	30,426
4	46.53	29,754	32.78	976.50	470.57	6.38	215.85	242.80	30,228
5	46.33	29,530	33.53	995.25	467.32	6.05	213.89	242.25	30,000
10	46.00	28,715	33.98	1,034.31	456.11	5.48	207.63	238.25	29,162

The results observed from Table 4-6 are very similar; moderate values of trigger works better.

Table 4-7: Cost scenarios in the offline setting with the error control method

Cost Setup	Holding	Purchasing	Price	Count	n(R)	IRI
Base	0.2	1	2	0.2	1	2
Scenario 1	0.2	1	2	0.2	1	6
Scenario 2	0.2	1	2	0.2	1	12
Scenario 3	0.2	1	2	0.2	3	2
Scenario 4	0.2	1	2	0.4	1	2

We examine the costs associated with the error control method using Table 4-7. In scenarios 1 and 2, the unit cost of IRI is increased. In scenario 3 the cost of lost sales is boosted. Finally, the cycle counting unit cost is increased in scenario 4.

Figure 4-3 depicts the profit function for each of the cost scenarios for the best case framework. The behavior of the error control method for compensating IRI is considerably different from the IRI resistance method. In the IRI resistance method, with the increment the profit obtained increases sharply up to a certain point than it decreases slowly. But in this setup, the change in the profit function is small. But it is still possible to observe an increasing motion followed by a slow decrease. The recorded optimal trigger value for both best and worst cost structure is 2. Scenario 1, 2 and 4 does not change the optimal value for both cases but in Scenario 3 the optimal trigger jumps to 3 for both cases.

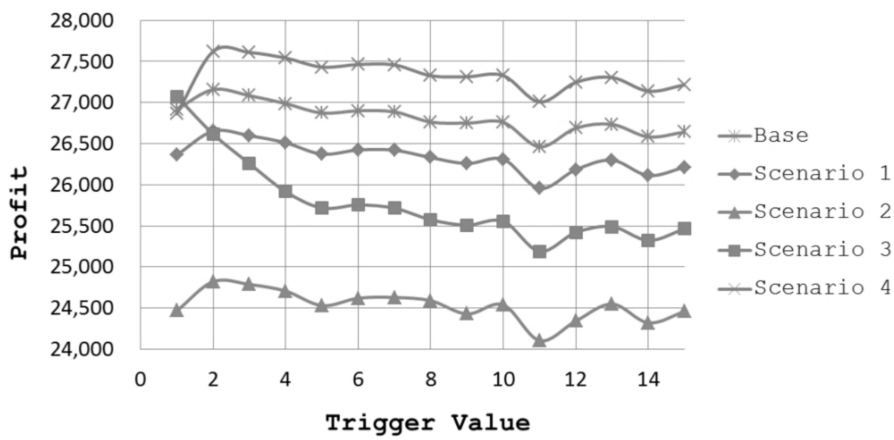

Figure 4-3: Profit function for each scenario for the best case with the error control method

As demonstrated in previous chapters, when applied, the error control method brings the best and the worst case setup closer to each other. This result is clearer in Figure 4-3 and Figure 4-4.

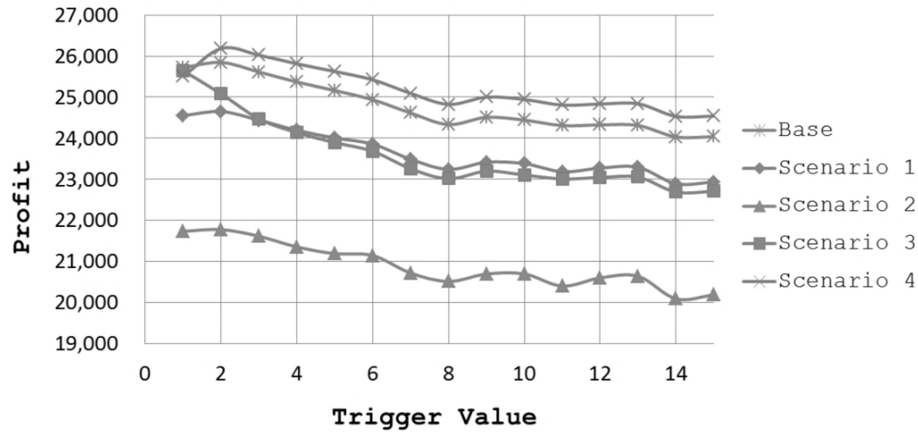

Figure 4-4: Profit function for each scenario for the worst case with the error control method

In all of the scenarios errors are penalized; therefore, the system wants to keep low errors in general. Since errors are strongly dependent on the actual inventory, the system also wants to keep low levels of physical stock. Specifically in Scenario 3 the IRI is penalized severely, that is why the behavior in that scenario is a little different than the rest.

4.3 Numerical Study: The Online Setting

We conduct the same set of simulation studies for the online setting by using the error characterizations derived in Appendix A and the cost structure presented in equation (4.2).

4.3.1 IRI Resistance Method

Table 4-8 and Table 4-9 summarize the key performance measures while IRI resistance method is being implemented for the best and the worst case, respectively. In both tables, columns represent the key measures that form the total cost function presented in equation (4.2). And each row denotes a set of simulation studies performed by using a specified level of increment for the IRI resistance method.

Table 4-8: Summary of statistics for the best case with the IRI resistance method

Increment	Sales/Time	Total Sales	Stock-out	n(R)	IRI	Avg Actual	Avg Record	Total Actual
0	47.50	28,505	12.17	289.93	26.38	227.18	229.14	28,523
5	47.84	30,256	1.43	8.78	75.33	334.72	365.02	30,329
10	48.04	30,052	0.67	3.27	127.82	435.96	489.33	30,186
15	47.46	29,807	0.53	2.11	180.15	546.24	613.70	30,010
20	48.34	29,586	0.42	1.29	237.65	647.43	737.36	29,828
25	47.42	29,333	0.33	1.04	294.41	756.74	861.54	29,621
30	47.48	29,070	0.32	0.61	348.57	861.22	986.39	29,417

According to Table 4-8 as the increment level gets larger, the stock-out and the lost sales decrease but IRI, the actual and the recorded inventory increase. Higher increment levels (20 to 30) have small improvements on the stock-outs, the lost sales and IRI. Thus, it can be concluded that the smaller increment levels (0 to 5) perform better.

Table 4-9: Summary of statistics for the worst case with the IRI resistance method

Increment	Sales/Time	Total Sales	Stock-out	n(R)	IRI	Avg Actual	Avg Record	Total Actual
0	47.09	29,506	29.75	618.56	165.44	217.11	232.36	29,671
5	47.34	30,341	6.45	21.28	195.77	273.36	364.73	30,536
10	48.01	30,053	0.97	5.98	244.56	369.95	489.44	30,306
15	47.93	29,830	0.72	2.72	303.77	486.21	613.22	30,132
20	48.15	29,586	0.57	2.08	354.63	586.42	737.67	29,949
25	47.62	29,333	0.50	1.16	408.55	692.71	861.85	29,743
30	47.84	29,080	0.28	0.8	461.37	796.89	986.45	29,549

The results observed from Table 4-9 are similar; again, lower values for the increment are preferable.

Table 4-10 shows the cost structure for 4 different scenarios. In these scenarios, the higher or the lower limits for the underlying costs are utilized in order to fully understand the influence of each factor.

Table 4-10: Cost scenarios in the online setting with IRI resistance method

Cost Setup	Holding	Purchasing	Price	n(R)	IRI
Base	0.2	1	2	1	2
Scenario 1	0.2	1	2	1	6
Scenario 2	0.2	1	2	1	12
Scenario 3	0.2	1	2	3	2

The base scenario uses small values for the lost sales and IRI costs. In scenarios 1 and 2 the unit cost of IRI is increased. And in scenario 3 the cost of lost sales is increased.

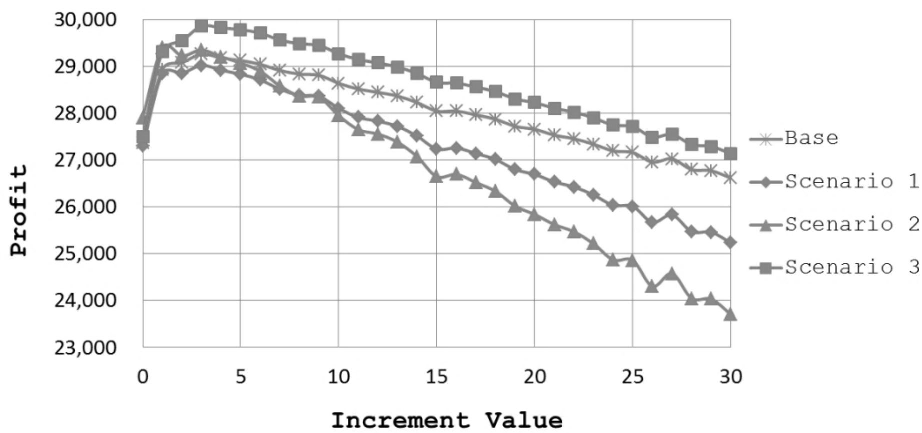

Figure 4-5: Profit function for each scenario for the best case with the IRI resistance method

Figure 4-5 depicts the profit function for each of the cost scenarios for the best case framework. With the increment the profit obtained increases sharply up to a certain point than it decreases slowly.

The behavior of the profit function in the online setting is similar to the offline setting. The online setting has higher profit in general for the same cost parameters. This is a direct result of the lack customer interaction with products. In the offline setting higher customer interaction causes more errors, which in turn reduces profit.

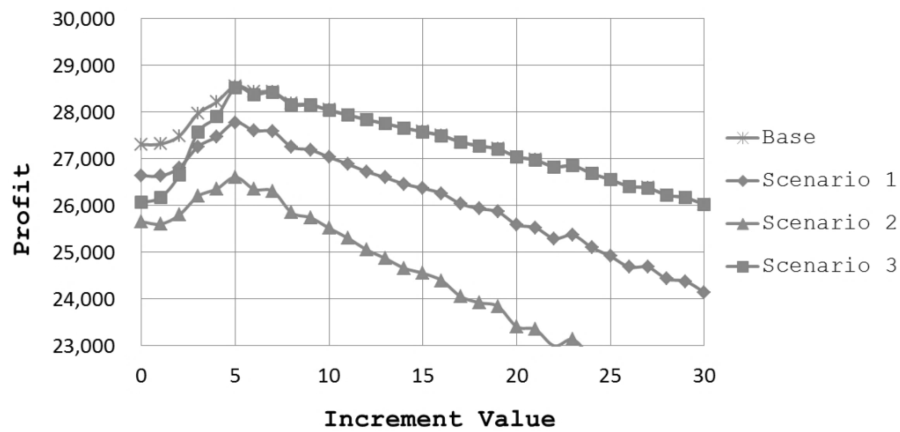

Figure 4-6: Profit function for each scenario for the worst case with the IRI resistance method

The profit function behavior in the worst case with each scenario is demonstrated in Figure 4-6. In the base case, the optimal increment value is 3; however the optimal increment is 5 for the worst case.

4.3.2 Error Control and Correction Method

We implement the same procedures for the error control method. Table 4-11 and Table 4-12 summarize the key performance measures while the error control method is being implemented for the best and the worst case, respectively. In both tables, columns represent the key measures that form the total cost function presented in equation (4.2). Each row denotes a set of simulation studies performed by using a specified level of trigger for the error control method.

Table 4-11: Summary of statistics for the best case with the error control method

Trigger	Sales per Time	Total Sales	Stock-out	n(R)	IRI	Count #	Avg Actual	Avg Record	Total Actual
0	47.50	28,505	12.17	289.93	26.38	-	227.18	229.14	28,523
1	47.57	28,446	12.70	295.38	26.44	7.90	227.96	228.86	28,466
2	47.42	28,844	12.58	330.41	26.45	0.73	226.14	231.34	28,876
3	47.78	28,838	14.58	367.28	25.42	0.25	225.96	231.37	28,866
4	47.47	28,603	12.58	349.92	26.06	0.08	227.53	229.00	28,622

5	47.07	28,623	13.00	320.41	26.04	0.07	227.96	229.66	28,644
10	47.28	28,604	12.92	348.46	25.82	-	242.05	229.51	28,618

According to Table 4-11 the influence of the trigger value on the stock-out, the lost sales and IRI is considerably small compared to previous cases. The other parameters remain relatively constant with trigger except count number. The results observed from Table 4-12 are similar in behavior.

Table 4-12: Statistics summary for the worst case with the error control method

Trigger	Sales per Time	Total Sales	Stock-out	n(R)	IRI	Count #	Avg Actual	Avg Record	Total Actual
0	47.09	29,506	29.75	618.56	165.44	-	217.11	232.36	29,671
1	47.69	29,331	29.17	595.18	166.09	9.60	219.18	230.62	29,485
2	47.43	29,426	31.17	586.04	165.31	1.60	217.24	231.44	29,591
3	47.54	29,368	31.88	613.07	164.28	1.60	215.16	231.16	29,520
4	47.07	29,449	30.73	628.19	165.19	1.03	216.56	231.55	29,625
5	48.40	29,223	32.27	677.88	164.07	1.13	215.03	229.39	29,394
10	47.60	29,071	32.67	666.32	163.39	-	214.80	228.91	29,222

The same cost structure is used again in Table 4-13. 4 scenarios are created by systematically adjusting the error related costs. In these scenarios, the higher or the lower limits for the underlying costs are utilized in order to fully understand the influence of each factor.

Table 4-13: Cost scenarios in the online setting with the IRI Resistance

Cost Setup	Holding	Purchasing	Price	Count	n(R)	IRI
Base	0.2	1	2	0.2	1	2
Scenario 1	0.2	1	2	0.2	1	6
Scenario 2	0.2	1	2	0.2	1	12
Scenario 3	0.2	1	2	0.2	3	2
Scenario 4	0.2	1	2	0.4	1	2

The base scenario uses small lost sales and IRI costs. In scenarios 1 and 2, the unit cost of IRI is increased. In scenario 3 the cost of lost sales is increased. Finally, the cycle counting unit cost is increased in scenario 4.

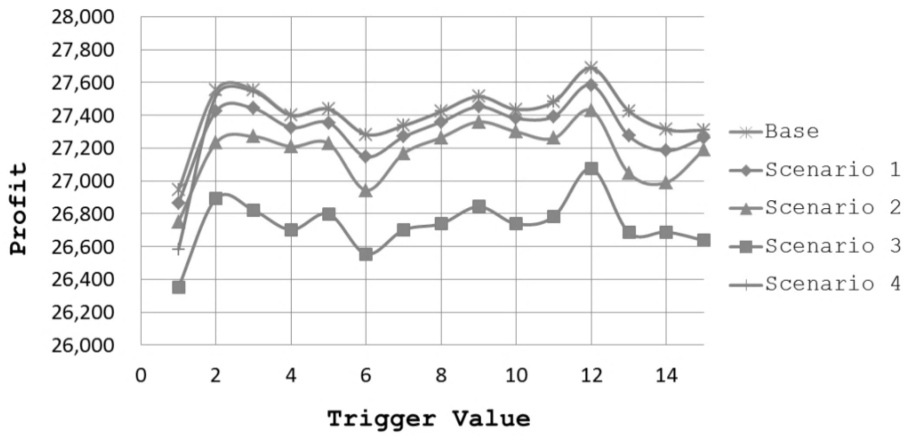

Figure 4-7: Profit function for each scenario for the best case with the error control method

Figure 4-7 depicts the profit function for each of the cost scenarios for the best case framework. The behavior of the error control method for compensating IRI is again considerably different than the previous method. In this setup, the change in the profit function is small; but, it is still possible to observe an increasing motion followed by a slow decrease.

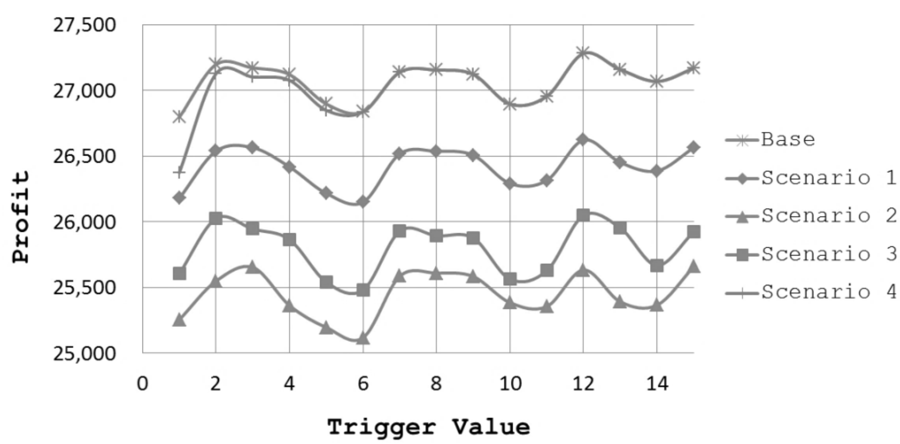

Figure 4-8: Profit function for each scenario for the worst case with the error control method

Figure 4-8 shows the profit function for the worst case setup. The behavior is similar.

The profit function in both figures increases when the trigger is positive than it remains relatively stable. The trigger mechanism becomes redundant when the trigger value is above 5 for both cases.

4.4 Comparison of Retail Environments

Chapter 2 contains detailed analyses on the structure of IRI for the offline retail environments. Furthermore, two compensation methods are developed to account for the impact of IRI for the offline retail setting. The results are then demonstrated on a numerical study. Similar analyses are performed for the online retail environment in Appendix A. Using the results obtained in Chapter 2 and Appendix A, this chapter provides comparative analyses for the online and offline retail settings.

In this chapter the classical cost function, (Zipkin, 2000), is modified by introducing error related factors impacting the decision making process, such as IRI penalty cost, cycle counting cost etc. The effects of these factors are then compared as a function of cost and sales. Finally the results are demonstrated on the same numerical analysis in Section 2.1.2.

Table 4-14 summarizes the results obtained from Chapter 2, Appendix A and Chapter 4. In this table, the preferable levels for each compensation method are tabulated based on the retail environment they are implemented upon.

Table 4-14: Preferable **l**evels of compensation methods based on the retail environment

Chapter #	**Chapter 2**		**Chapter 3**		**Chapter 4**			
Environment	Offline Retail		Online Retail		Offline Retail		Online Retail	
Method	Incr.	Trigger	Incr.	Trigger	Incr.	Trigger	Incr.	Trigger
Best Case	5-10	1-2	0-5	0-1	8	2	3	2
Worst Case	10-15	1-2	5-10	1-2	12	2	5	2

Remark: The results in Chapter 2 and Appendix A do not utilize the cost structure introduced in this chapter.

Investigations in Chapter 2 and Appendix A show that the IRI resistance method performs better on keeping stock-out and lost sales low, whereas the error control method can operate with lower inventory. However, when the cost structure is implemented then same behavior cannot be observed. Based on the results obtained in this chapter, IRI resistance method reaches higher profit values in all of the cases, especially in the online setting.

The overall results of the analyses done in this chapter are summarized into three groups: Retail environments, compensation methods, and IRI sensitivity

4.5 Retail environments

Major difference between the online and the offline retail environments is the lack of customer access to the goods during purchase. This difference has three major outcomes that greatly affect the inventory model. First, as a direct result, customers make their purchasing decisions based on the recorded inventory. This setup increases the importance of record accuracy and also creates a new type of IRI measure called the penalty sales. Second, the lack of customer access results in the absence of certain error factors that are present in the offline setting, such as theft. Because of this, the preferable levels for each compensation method is lower in the online setting as can be seen from Table 4-14. Finally, one of the biggest challenges faced in the offline setting is freezing as described in Section 2.2.1. The online setting on the other hand, is completely resistant to freezing. The main reason is, freezing can only occur during a stock-out and in the online setting these stock-outs are observable.

Compensation methods

The profit obtained with the IRI resistance method, in all situations, increases with the increment level first and then decreases. This suggests that there is a range of values preferable for the increment value as shown in Table 4-14.

Also the error control method becomes ineffective in the online setting. Intuitively, in the error control method the main goal is managing the difference between the actual and recorded inventory. But in the online setting, customers use records instead of actual physical stock. As a result the error control method loses its effectiveness.

IRI sensitivity

The behavior of the profit function under different scenarios is depicted in Figure 4-1 to Figure 4-8. According to these graphs, the IRI resistance method is not sensitive to changes in lost sales and unit IRI costs. Different cost values just change the magnitude of the profit function not the behavior. In both settings the influence of IRI is much greater than lost sales, mainly because the IRI resistance method operates with low lost sales.

The profit function movement when the error control method is implemented shows greater sensitivity to changes in lost sales and IRI unit costs. The counting cost has small effect on the magnitude of the profit function. Whereas, lost sales and IRI costs are considerably influential. Moreover, slight changes in the behavior of the profit function are observed upon increasing the lost sales cost. This outcome is intuitively because error control method operated with high lost sales.

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CHAPTER 5 FUZZY MULTI-OBJECTIVE MDP MODEL FOR INVENTORY RECORD INACCURACY

Supply chains suffer greatly from inventory inaccuracy, which is a well-studied problem in the literature. Due to this inaccuracy, the complete information about the current state of the inventory does not always exist. In the literature this issue is often referred as inventory visibility. In this study we concentrate on investigating the value of visibility through methodically analyzing the benefits of using a secondary source of information (i.e., automated data capturing) along with traditional inventory record keeping methods to control the effects of inventory record inaccuracy (IRI).

In order to fully understand the value of visibility we define a secondary source of information which is referred to as visibility information for the rest of this study. We assume that the visibility information is obtained through an automated data capturing system which may or may not work with 100% accuracy. Hence the inventory manager has access to the conventional records and the new visibility data while making decisions.

The inventory management and supply chain related problems are known to have a complex structure for optimization purposes. The conventional approach to solve these problems generally involves a cost estimation to bring the key metrics such as stock-outs, lost sales, or holding etc. to one generic platform, dollar value. Then an objective function is defined by assigning scalar weights to these metrics. In the multi-objective setting, the decision maker tries to optimize two or more objectives simultaneously under various restrictions. For a multi-objective optimization problem, a complete optimal solution seldom exists, and a Pareto-optimal solution is usually used. A number of methods, such as weighting method, assigning priorities to the objectives and setting aspiration levels for the objectives are used to derive a compromise solution (Rosenthal, 1985).

In general, inventory models involve uncertainty since certain values like shortage, or penalty cost are not known exactly. Furthermore, the decision maker often has vague goals such as keeping the shortage costs to a minimal, or keeping the customers satisfied. For such cases, fuzzy set theory and fuzzy mathematical programing methods are suitable (Bellman & Zadeh, 1970; Zimmermann, 1978).

In this study, in order to put more emphasis on the effects of IRI, a new measure is developed as inventory performance by combining four key performance metrics: lost sales, amount of correction, fill rate and amount of inventory counted. These key metrics are combined under a unitless platform using fuzzy logic and combined through additive methods. In a single item infinite horizon setting we develop a fuzzy multi-objective inventory model influenced by IRI with cycle counting under random supply, demand and lead time with no backordering. The multi-objective setup for fuzzy goals, on the other hand, is formulated using a fuzzy goal programming (FGP) approach involving different importance levels. Fuzzified goals are then assigned weights and combined using an additive model to maximize the sum of all the fuzzy goals. Extra information on additive methods in inventory problems can be found in Xu and Liu (2008) and Wee et al. (2009).

Our goals are (1) to define an inventory system to assess the value of visibility, (2) to apply cycle counting methods to the new inventory system in order to gauge the IRI susceptibility of the created system, (3) to create a new performance measure (inventory performance) which combines the key inventory metrics in a platform where they can be compared, and (4) to find the optimal policy that maximizes the inventory performance

The inventory problem is modeled as an infinite horizon discrete-time discounted Markov decision process with fuzzified multi-objective subjected to random demand, supply and lead time. This model is extensively analyzed to understand the optimal policy structure. Finally, a numerical example is solved using policy iteration algorithm to provide insights.

5.1 Model

Earlier studies the inventory problem is commonly perceived as a sequential decision making problem. In such a problem, at a specified time the decision maker observes the state of a system. Based on this state decision maker chooses an action and receives an immediate reward. The action choice produces results and the system evolves to a new state at a different point in time according to a probability distribution determined by the action choice. Therefore, the main goal is to find a policy that provides a prescription for choosing actions in any possible future states. In this study we focus on a particular sequential decision model referred to as Markov decision process (MDP). In MPDs the set of available actions, the rewards and the transition probabilities depend only on the current state and action. Please refer to Puterman (2009).

In inventory problems various types of uncertainties and imprecisions are inherent; such as demand, supply and lead time randomness. These are often modeled using approaches from the probability theory. Yet, it is not always possible to treat all types of uncertainties by probabilistic models (i.e. shortage cost, stock-out cost etc.). For such imprecise parameters we use fuzzy numbers defined on bounded intervals on the axis of real numbers. The fuzziness in inventory models can be present on multiple levels such as decision variables (S. P. Chen, 2011), costs (Vujosevic et al., 1996), goals

(L. H. Chen & Tsai, 2001), parameters (Ouyang & Chang, 2002) or objectives Wee et al. (2009).

Fuzzy set theory, introduced by Zadeh (1965), is an extension of traditional set theory. In fuzzy theory the elements of the set are no longer required to belong to the set; instead, these elements have a degree of membership that quantifies how well they belong to the set. Fuzzy sets use a membership function, μ_A, for a set A, that extends the range of $f_A : U \rightarrow \{0,1\}$ to $\mu_A : U \rightarrow [0,1]$ (Kosko, 1992). Triangular and trapezoidal membership function are very commonly used because they fit most of the cases and provide fast computation time (Xexéo). Other curves like Gaussian and sigmoid may provide smooth results but require higher computation time.

The goal of finding the optimal Q and R values have been studied extensively and therefore not in our scope. Instead we analyze alternatives that will maximize the potential of the selected (Q,R) policy by managing IRI. We are formulating a single-item multi-objective continuous-time stochastic inventory problem over an infinite horizon where the decision maker is following a (Q,R) policy with random lead time, random supply, lost sales and unobservable IRI. The inventory problem is modeled as an infinite horizon MDP with multiple objectives with the components discussed below. The multi-objective setup is defined as the overall inventory performance which is a combination of four fuzzy parameters: Lost sales, expected error correction, service level and expected amount counted. These parameters are represented with triangular membership functions and combined together using fuzzy additive goal programming. A similar approach is present in Wee et al. (2009).

5.1.1 State Space:

Consider the inventory problem where X denotes the inventory level obtained from the records, $\bar{X}$ denotes the inventory level obtained from automated data capturing system (i.e. Auto-ID, RFID etc.) and $\tilde{X}$ denotes the actual physical stock available for sales. The first two sources of information X and $\bar{X}$ are fully observable at any given time. However, $\tilde{X}$ becomes observable only when inventory count is triggered. Using these variables we define state ξ as follows,

$$\xi = \left(X - \tilde{X}\right) + \left(\tilde{X} - \bar{X}\right) \tag{5.1}$$

Recall in previous chapters, $\left(X - \tilde{X}\right)$ denotes the total error ε. The second term $\left(\tilde{X} - \bar{X}\right)$ shows the discrepancy of the automated data capturing system, $\bar{\varepsilon}$. Therefore, the state space equation can be rewritten as, $\xi = \varepsilon + \bar{\varepsilon}$.

By defining state space this way we implicitly made the assumption that $X \geq \tilde{X} \geq \bar{X}$. In other words, the actual inventory that is available for sales is bounded by inventory records from above and automated data from below. In reality this may not be true. As we demonstrated in the previous chapters the actual inventory can be larger than inventory records due to negative transaction errors. However, in general this situation rarely occurs or lasts for very short durations. The same justification can also be claimed for the visibility as well. In reality automated data capturing systems are known to overestimate on occasions due to multiple readings. These outcomes are observed when the items (or the packaging) have the ability to reflect (such as metals) the radio waves. In order to decrease the complexity, in our models we assume that $X \geq \tilde{X} \geq \bar{X}$ holds; however, this assumption can be relaxed by sacrificing computational efficiency (Note that when $X \geq \tilde{X} \geq \bar{X}$ is true, this inequality implicitly assumes $\bar{\varepsilon} \geq 0$ and $\varepsilon \geq 0$ are also true). Figure 5-1 shows the graphical representation of the state space and its evolution as time moves forward.

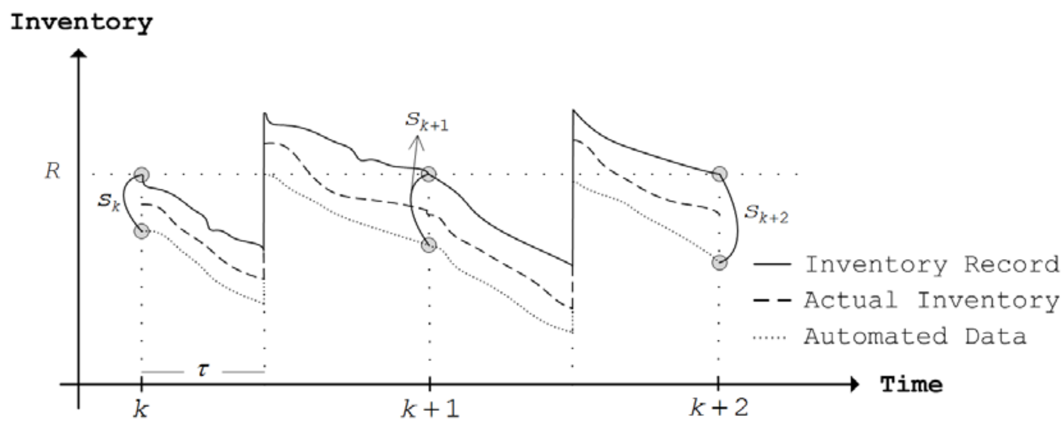

Figure 5-1: State Space

With the assumption of $X \geq \tilde{X} \geq \bar{X}$ the state space becomes finite, $s \in \{0,1,...,R-1,R,F\}$ where R is the reorder level and F denotes the freeze state (Absorbing state). The reason for the state space to be bounded by reorder level comes from the analysis done in the previous chapter. As shown, the maximum value that ε can reach is R. That is because whenever $\varepsilon = R$ the system stops and making more errors becomes impossible. The visibility discrepancy on the other hand is always positive, $\bar{\varepsilon} \geq 0$. So whenever $s = i$ for $i \in \{0,1,...,R\}$, then $\varepsilon \in \{0,1,...,R\}$ and $\bar{\varepsilon} \in \{0,1,...,R\}$ as long as $s = \varepsilon + \bar{\varepsilon}$ is satisfied. The freezing scenario is observed only when $s = F$; meaning $\varepsilon = R$ and $\bar{\varepsilon} = 0$. Moreover, freezing state F is designed to represent the worst possible scenario. When the system is in this state the problem terminates with a big penalty m. This penalty is the main reason for the system to avoid freezing and triggering early cycle counts. Technically, the state space can be seen as

$$S = \{0,1,...,R-1,R,R+1\}.$$

The state space in this setup shows the sum of the total error and the total visibility discrepancy at each decision epoch. This setup greatly reduces the state space; instead of tracking the entire inventory, the system only tracks R states. However, the effect of having a high vs. low amounts of overall inventory is not reflected in this setup. For example, having $s = 10$ when the overall inventory is 100 is different than having $s = 10$ with thousands of overall inventory. In order to overcome this problem we first assume that if a practice has high inventory levels it also has a high reorder level. There are situations where this assumption does not hold such as just in time delivery systems. Moreover, the error formulation is a function of the reorder level and in Section 2.1.2 it is shown that errors increase with the reorder level. Thus, our states are sufficient to characterize the system changes in the overall inventory levels.

5.1.2 Action Space

As constructed in the previous chapters, the inventory problem evolves similarly. Figure 5-2 depicts this behavior and the decision epochs. According to the graph inventory records are replenished based on a (Q,R) system. In this setting, the beginning and the ending of each stage are determined by inventory record level. In the figure τ denotes the lead time and a_k denotes the action at period k.

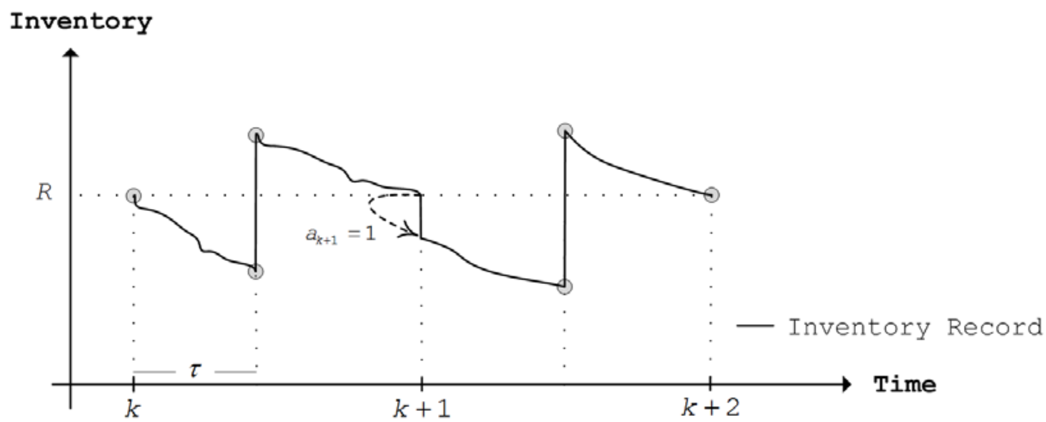

Figure 5-2: Action space

During a decision epoch, whenever the record reaches the reorder level, the decision maker has to take an action yielding a reward. The action space $A_s^k = \{0,1\}$ for $\forall k \in \{1,2,...,N\}$ and $\forall s \in \{0,1,2,...,R+1\}$ describes the cycle counting decision. For example in Figure 5-2 a cycle count is triggered at $a_{k+1} = 1$ and a cycle count is not triggered at stage $a_k = 0$. Furthermore, for each $s \in S$, $d_k(s) \in A_s^k$ is the Markovian decision rule. In our study we are looking at an infinite horizon problem; hence, for the remainder of the research we use a to denote the action instead of a_k. Also since $A_s^k = \{0,1\}$ for $\forall k \in \{1,2,...,N\}$ and $\forall s \in \{0,1,2,...,R+1\}$ we replace A_s^k with A and $d_k(s)$ with $d(s)$.

5.1.3 Transition Probabilities

The transition matrix **P** is formulated using two principle values, the probability of making an error and the probability of having one unit of visibility discrepancy. Note that, *error* is obtained by looking at the mismatch between the actual physical stock and the inventory records; whereas, the *visibility discrepancy* is the difference between the information obtained from the actual physical stock and the automated data capturing systems. In this study we are interested in maximizing the inventory performance policy when the system is subjected to IRI.

The error from automated identification systems (i.e. visibility discrepancy), on the other hand, is dependent on many factors such as item types, warehouse shape, location, reader distance, packaging and many more. But we are interested only in the dependency to actual physical stock available. Although the remaining factors can be important contributions in a future study, they fall out of this study's scope. In literature there are various papers dealing with the capabilities of automated data capturing systems (Agrawal, 2001; Raman et al., 2001; Ton & Raman, 2004). In our study we use visibility accuracy as an input parameter that shows the accuracy of the automated data capturing systems and investigate the various scenarios involving different levels of visibility accuracy. These levels can be high, medium and low.

Let ε', $\bar{\varepsilon}'$ and s' denote the error, visibility discrepancy and the state space for the next period. Then, $P(\varepsilon' - \varepsilon = i | a = 0) \equiv g^i$ for $i = 0, \ldots, R$ denotes the probability of having i errors at the current period. In this study we assume that errors are not corrected unless a cycle count is triggered, which means $\varepsilon' - \varepsilon \geq 0$ is always true. Let $P(\bar{\varepsilon} = t | s, a) \equiv r_s^t$ be the probability of having t units of visibility discrepancy when the system is in state s where $t \in \{0, 1, \ldots, R\}$ and the action a is taken. Unlike errors, visibility discrepancy can be automatically corrected. The following condition holds for every s and t,

$$P(\bar{\varepsilon} = t | s, a) = \begin{cases} 0 & t > s \\ r_s^t & t \leq s \end{cases}. \tag{5.2}$$

To calculate $P(s' | s, a)$, we condition on visibility discrepancy

$$P(s'|s,a)=\sum_t P(s'|s,a,\bar{\varepsilon}=t)P(\bar{\varepsilon}=t|s,a) \tag{5.3}$$

Then, $P(s'|s,a,\bar{\varepsilon}=t)$ can be calculated by conditioning on the discrepancy in the next observation

$$\begin{aligned}
P(s'|s,a,\bar{\varepsilon}=t) &= P(\bar{\varepsilon}'+\varepsilon'=s'|s,a,\varepsilon=s-t) \\
&= \sum_{l=0}^{R} P(\bar{\varepsilon}'=l,\varepsilon'=s'-l|s,a,\varepsilon=s-t) \\
&= \sum_{l=0}^{s'-s+t} P(\bar{\varepsilon}'=l,\varepsilon'=s'-l|s,a,\varepsilon=s-t) \quad \text{(Joint probability rule)} \\
&= \sum_{l=0}^{s'-s+t} P(\bar{\varepsilon}'=l|s,a,\varepsilon'=s'-l)P(\varepsilon'=s'-l|s,a,\varepsilon=s-t) \\
&= \sum_{l=0}^{s'-s+t} P(\bar{\varepsilon}=l|s=s',a)P(\varepsilon'=s'-l|s,a,\varepsilon=s-t) \quad \text{(Markov property)} \\
&= \sum_{l=0}^{s'-s+t} r_{s'}^{l} g^{s'-s+t-l} .
\end{aligned} \tag{5.4}$$

In equation (5.4) the first line shows the general equality of the conditional probability. The second equality extends the equation using the joint probabilities. Both ε' and $\bar{\varepsilon}'$ are non-negative so the summation starts from $l=0$ to R. But since for the given observations certain values for s' are not attainable, the summation bounds are reduced in the third equality. In the fourth line, the joint probability is converted to the conditional. Then, the probability of visibility discrepancy is reduced to $P(\bar{\varepsilon}=l|s=s',a)$ using the Markov property and time independence of visibility observations.

Then, transition probability can be calculated as,

$$P(s'|s,a=0)=\begin{cases} \sum_{t=0}^{s} r_s^t \left(\sum_{l=0}^{s'-s+t} g^{s'-s+t-l} r_{s'}^{l} \right) & s \le s' \\ \sum_{t=s-s'}^{s} r_s^t \left(\sum_{l=0}^{s'-s+t} g^{s'-s+t-l} r_{s'}^{l} \right) & s > s' \end{cases} \tag{5.5}$$

and

$$P(s'|s,a=1)=r_0^0 \left(\sum_{l=0}^{s'} g^{s'-l} r_{s'}^{l} \right) \quad s \le s' . \tag{5.6}$$

As state s gets larger, the probability of reaching to smaller states decreases. In other words, the more the IRI is in the current period, the higher the IRI will be in the next period. This is mainly because errors cannot be corrected until a cycle count is triggered.

Note that the transition matrix becomes time independent since both the error and the discrepancy probabilities are time independent. In other words, the transition matrix **P** is fixed over time.

For example, let $s \in \{0,1,2,3\}$ and action be $a \in \{1,0\}$. Then, the transition matrix **P** can be calculated as follows: If the current state is $s = 1$, then the probability that the system will be in state $s' = 2$ for the next period is $P(2|1,a)$. This probability is calculated by conditioning on the visibility discrepancy two times.

$$
\begin{aligned}
P(2|1,a) &= P(2|1,a,\overline{\varepsilon}=0)P(\overline{\varepsilon}=0|1,a) + P(2|1,a,\overline{\varepsilon}=1)P(\overline{\varepsilon}=1|1,a) \\
&\quad + P(2|1,a,\overline{\varepsilon}=2)P(\overline{\varepsilon}=2|1,a) + P(2|1,a,\overline{\varepsilon}=3)P(\overline{\varepsilon}=3|1,a) \\
&= r_1^0 \begin{bmatrix} P(\varepsilon'=0,\overline{\varepsilon}'=2|1,a,\overline{\varepsilon}=0) + P(\varepsilon'=1,\overline{\varepsilon}'=1|1,a,\overline{\varepsilon}=0) \\ +P(\varepsilon'=2,\overline{\varepsilon}'=0|1,a,\overline{\varepsilon}=0) \end{bmatrix} \\
&\quad + r_1^1 \begin{bmatrix} P(\varepsilon'=0,\overline{\varepsilon}'=2'|1,a,\overline{\varepsilon}=1) + P(\varepsilon'=1,\overline{\varepsilon}'=1|1,a,\overline{\varepsilon}=1) \\ +P(\varepsilon'=2,\overline{\varepsilon}'=0|1,a,\overline{\varepsilon}=1) \end{bmatrix} \\
&= r_1^0 \begin{bmatrix} P(\overline{\varepsilon}'=1|1,a,\overline{\varepsilon}=0,\varepsilon'=1)P(\varepsilon'=1|1,a,\overline{\varepsilon}=0) \\ +P(\overline{\varepsilon}'=0|1,a,\overline{\varepsilon}=0,\varepsilon'=2)P(\varepsilon'=2|1,a,\overline{\varepsilon}=0) \end{bmatrix} \\
&\quad + r_1^1 \begin{bmatrix} P(\overline{\varepsilon}'=2|1,a,\overline{\varepsilon}=1,\varepsilon'=0)P(\varepsilon'=0|1,a,\overline{\varepsilon}=1) \\ +P(\overline{\varepsilon}'=1|1,a,\overline{\varepsilon}=1,\varepsilon'=1)P(\varepsilon'=1|1,a,\overline{\varepsilon}=1) \\ +P(\overline{\varepsilon}'=0|1,a,\overline{\varepsilon}=1,\varepsilon'=2)P(\varepsilon'=2|1,a,\overline{\varepsilon}=1) \end{bmatrix} \\
&= r_1^0\left(g^1 r_1^0 + g^0 r_1^1\right) + r_1^1\left(g^2 r_1^0 + g^1 r_1^1 + g^0 r_1^2\right).
\end{aligned}
$$

5.1.4 Reward: Inventory Performance

One of the biggest challenges the researchers have been facing is creating a standard platform for the key metrics in the inventory problem. This platform is almost always the estimated dollar value of the mentioned metrics such as shortage or penalty cost. This setup enables the use of objective functions that aim to maximize profits or minimize. In this paper, our focus is on IRI, we create a different platform. In the inventory framework the error (the mismatch between the records and the actual) has substantial impacts on various performance metrics such as service level, probability of stock-out, average inventory, lost sales, sales, inventory freezing, cycle counting, error correction, etc. Unfortunately the traditional models fail to address the importance of all of these metrics at the same time. Because of this, some of the metrics are constantly overlooked. By limiting our focus on IRI we limit our research to four key elements that are directly influenced by errors.

In order to understand the value of errors we develop a new measure called “inventory performance”. This measure is a combination of the following metrics, (1) expected lost sales, (2) expected amount of correction, (3) service level and (4) expected amount of inventory to be counted. Unfortunately, as mentioned these metrics do not share a uniform unit. To overcome this challenge we fuzzify each of these metrics into a unitless platform, combine them into one measure and maximize it.

The correlation between the chosen four performance metrics is complicated. Based on the simulation analysis presented in the previous chapters, the following results are concluded: as the frequency of cycle counting increases (1) the number of correction per count decreases, (2) the number of stock-outs decreases, (3) lost sales decreases and (4) the total number of errors made decreases. Additionally, the graphs presented in pervious chapters show the dependency between these parameters

Theoretically, lost sales is a function of the reorder level, which is a function of safety stock. And safety stock is a function of service level; therefore, there is a direct dependency between lost sales and service level. However, the general characterization for lost sales and service level assumes that there are no errors. This poses a major problem since the effects of error on lost sales and service level are very different. Hence, the correlations between the discussed performance metrics are dependent on the level of errors present in the inventory system. The relation between correction and the count amount is also similar. That is essentially the reason why these four metrics are chosen: lost sales, service level, error correction and count amount.

i. Reward 1: Lost Sales

The first part of the system performance is the lost sales. In the earlier studies, many inventory models have considered the uncertainty of shortage costs. However, papers dealing with the fuzzification of shortage costs are few (Chang et al., 2006; Lin, 2008). In this section, we aim to fuzzify the value of lost sales. Applications of fuzzy set concepts on EOQ inventory models have been proposed earlier (Dutta et al., 2007; Vujosevic et al., 1996; Yao & Chiang, 2003). However, these studies almost always concentrate on the simple EOQ models, in which restrictive assumptions are implied.

In a general (Q,R) setting the expected lost sales is formulated as the expected number of difference between demand and available inventory for a specified interval. In other words,

$$n(R) = E\left[\max\{D_\tau - R, 0\}\right] = \int_R^\infty (z-R) f_D(z)\,dz$$

where f_D is the demand distribution. Note that the term $\max\{D_\tau - R, 0\}$ uses reorder level instead of inventory level. This characterization is possible due to the decision structure. Recall that decision epochs are identified as the points at which inventory records reaches reorder level. Since the decision maker is only allowed to make decisions at decision epochs, the inventory level can be substituted with reorder level.

Lost sales value is generally inserted in the objective function with a scalar weight p denoting the shortage cost per unit. We utilize a similar setup for lost sales, where the expected lost sales is calculated with

$$n(R|s,a) = E\left[\max\{D_\tau - R, 0\}|s,a\right]. \tag{5.7}$$

Equation (5.7) can be rewritten as,

$$n(R|s,a) = \begin{cases} E\left[\max\{D_\tau - R, 0\}\right] & a = 1 \\ E\left[\max\{D_\tau - R + E[\varepsilon|s,a=1], 0\}\right] & a = 0 \end{cases}. \tag{5.8}$$

Note that in equation (5.8) the second line uses $E[\varepsilon|a=1]$ although the realization is only possible when $a=0$. This would seem unintuitive at a quick glance; however, at any decision epoch for a given s when a cycle count is not triggered than the actual inventory on-hand is less than or equal to R (due to IRI). The difference between R and the errors can be estimated by $E[\varepsilon|s,a=1]$, since it gives the expected amount of error correction if a cycle count is triggered. Equation (5.10) provides more details on this calculation.

Then, to describe the fuzzy objective we define an acceptable interval $[n(R)_l, n(R)_u]$.

This interval is subjective and it represents the tolerable range for the lost sales values. The boundary limits $n(R)_l$ and $n(R)_u$, as shown in the Figure 5-3 part A, the optimistic and pessimistic situations respectively. Naturally, the lower the expected lost sales value is the better. The membership function $\mu(n(R|s,a))$, which may be linear or non-linear, is then used to describe the tolerance rating of the lost sales; the lower the expected lost sales is, the higher the membership value will be.

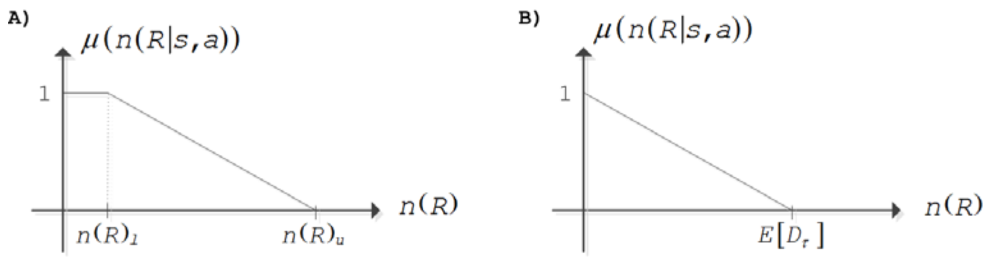

Figure 5-3: (A) Membership function for Lost sales and (B) membership function with bounds

However, it is also possible to make more objective characterizations for lower and upper bounds, as shown in Figure 5-3 part B. The lower bound for the value can easily be obtained by looking at the demand distribution. The demand is assumed to be non-negative by nature. Thus, the smallest value for $n(R|s,a)$ is zero. An easy upper bound for the value can be obtained by looking at the expected demand. The expected demand during lead time is always larger than expected lost sales by definition. In other words, $E[D_\tau] > n(R|s,a)$; which can be written as

$$E[D_\tau] \geq n(R|s,a) \geq 0.$$

Then, the triangular membership function $\mu(n(R)|s,a)$ is characterized as

$$\mu(n(R|s,a)) = \begin{cases} 1 & n(R|s,a) \leq n(R)_l \\ \dfrac{n(R)_u - n(R|s,a)}{n(R)_u - n(R)_l} & n(R)_u > n(R|s,a) > n(R)_l \\ 0 & n(R|s,a) \geq n(R)_u \end{cases}. \tag{5.9}$$

Equation (5.9) is used as the first part of the reward structure.

ii. Reward 2: Expected Error Correction

In this research we concentrate our attention on determining the right size of mismatch (state space) for counting. It is assumed that cycle counting is done with perfect accuracy

From this perspective, the expected amount of correction can easily be characterized as

$$E[\varepsilon|s,a]=\begin{cases}E[\varepsilon|s] & a=1,\forall s=0,1,...,R\\ 0 & a=0\end{cases} \tag{5.10}$$

where $E[\varepsilon|s]$ is the expected error when the system is in state s upon triggering the cycle count. Since for a given s and a, $s=E[\varepsilon|s,a]+E[\overline{\varepsilon}|s,a]$. Then expected error can be found by conditioning on probability of visibility discrepancy, or

$$\begin{aligned}E[\varepsilon|s]&=\sum_{i=0}^{s} iP(\varepsilon=i|s)\\ &=\sum_{i=0}^{s} iP(s-\overline{\varepsilon}=i|s)\\ &=\sum_{i=0}^{s} iP(\overline{\varepsilon}=s-i|s).\end{aligned} \tag{5.11}$$

Let $P(\overline{\varepsilon}=i|s)=r_s^i$, then the above equation can rewritten as,

$$E[\varepsilon|s]=\sum_{i=0}^{s} ir_s^{s-i} . \tag{5.12}$$

Hence, to describe the fuzzy objective, as shown in Figure 5-4 part A, we define an acceptable interval $[E[\varepsilon]_l ,E[\varepsilon]_u]$. Naturally, it is better to have higher values for expected correction. Hence, the triangular membership function is greater for higher correction values.

The lower bound for the value is zero since the smallest number correction can get is zero. The upper bound for the value is one since the maximum correction equals to the reorder level. In other words, as shown in Figure 5-4 part B the bounds can written as $0\leq E[\varepsilon|s,a]\leq R$. Figure 5-4 shows the membership function of fuzzified error correction parameter. For simplicity we use a triangular membership set as depicted in figure.

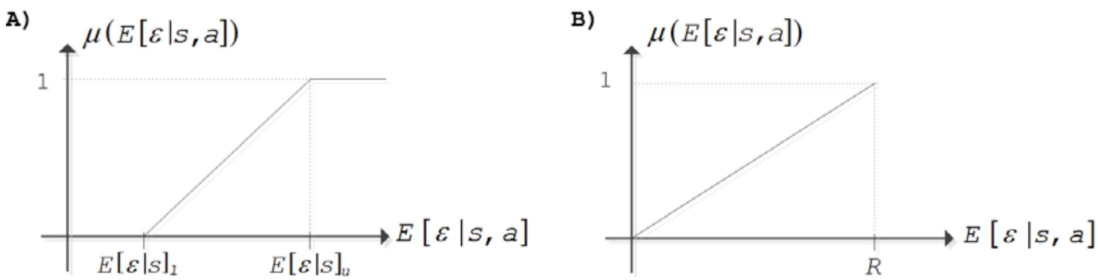

Figure 5-4: (A) Membership function for correction and (B) membership function with bounds

The triangular membership function $\mu(E[\varepsilon|s,a])$ is characterized as,

$$\mu(E[\varepsilon|s,a]) = \begin{cases} 1 & E[\varepsilon|s,a] \geq E[\varepsilon|s]_u \\ \dfrac{E[\varepsilon|s,a]-E[\varepsilon|s]_l}{E[\varepsilon|s]_u - E[\varepsilon|s]_l} & E[\varepsilon|s]_l \leq E[\varepsilon|s,a] \leq E[\varepsilon|s]_u \\ 0 & E[\varepsilon|s,a] < E[\varepsilon|s]_l \end{cases} \tag{5.13}$$

Equation (5.13) is used as the second part of the reward structure.

iii. Reward 3: Service level

Service level is a commonly used metric in inventory replenishment problems. Many definitions of service levels are used in the literature as well as in practice. These may differ not only with respect to their scope and to the number of products considered, but also with respect to the time interval they are related to. In our research the cycle service rate is the probability that there is no stock-out while waiting for an order to come in. In the long run, this corresponds to the percentage of order periods where there is no stock-out.

In supply chain and inventory management literature, service level metrics are commonly used as a constraint (Bashyam & Fu, 1998; Ouyang & Wu, 1997; Tarim & Kingsman, 2004). The reward associated with the service level is characterized as the probability of no stock-out. In a stochastic inventory problem, a stock-out can only happen during the lead time after an order is given, assuming that the inventory is not frozen due to errors (Thiel et al., 2010). Hence, the probability of having no stock-out can be calculated as,

$$P\{No\ \ Stockout\} = P\{D_\tau < R\}.$$

In this formulation, D_τ denotes the lead time demand. There is a very definite relationship between service level and the amount of inventory being stocked. Generally, the more the inventory level is the higher the service level will be but at a decreasing rate. The traditional setup of safety stock (w) in (Q,R) is obtained as follows. Let $R = w + \mu_\tau$ be the reorder level where w is the safety stock and μ_τ is the lead time demand. Then, w can be formulated as

$$w = z^* \sqrt{\tau\sigma_\mu^2 + \mu^2\sigma_\tau^2}\ .$$

where τ is the mean lead time, σ_μ^2 is demand variance and σ_τ^2 is the lead time variance (Zipkin, 2000). In this formulation z^* is the value obtained from the z-table for the desired service level (complementary type 1 error: all customer orders arriving within a given time interval will be completely delivered from stock on-hand). When the lead time is deterministic $\mu^2\sigma_\tau^2$ becomes zero making $w = z^* \sqrt{\tau\sigma_\mu^2}$ and when there is no lead time w and R becomes zero. These results are very intuitive because if there is no lead time than there is no reason to have extra inventory. Obviously, such generalizations have limited viability in practice. Due to the difficulty in characterization of safety stock, in general the service level commonly becomes an input parameter inserted as a constraint.

In our research, the way the decision scheme is set up, inventory records are forced to be at R during each decision epoch. However, the current inventory record observation involves errors. In reality the actual physical stock can be obtained by subtracting the error from R during each decision epoch. Hence, at each decision epoch depending on the observed state the probability of observing a stock-out will be different, and is given by

$$P\{D_\tau \geq R - E[\varepsilon|s,a]\} \quad \forall s \in \{0,1,...,R\}, \forall a \in \{0,1\}.$$

Similarly no stock-out probability can be obtained as

$$P\{D_\tau < R - E[\varepsilon|s,a]\} \quad \forall s \in \{0,1,...,R\}, \forall a \in \{0,1\}.$$

For simplicity of notation let $P\{No\ \ Stockout\} = P\{NS\}$. Note that the no stock-out probability characterized in the equation above is dependent on the action;

$$P\{NS|s,a\} = \begin{cases} P\{D_\tau < R\} & a=1 \\ P\{D_\tau < R - E[\varepsilon|s]\} & a=0 \end{cases} \tag{5.14}$$

To describe the fuzzy objective, as shown in Figure 5-5 part A, we define an acceptable interval for probability of no stock-out, $[P\{NS\}_l, P\{NS\}_u]$.

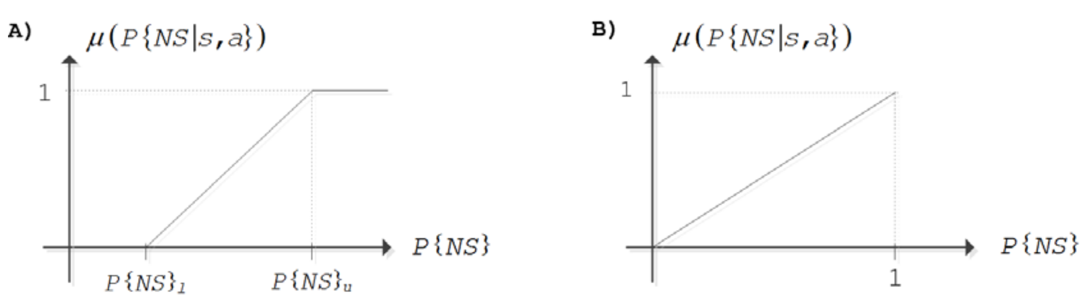

Figure 5-5: (A) Membership function for service level and (B) membership function with bounds

Similar to correction, higher service levels are more desirable; so, higher service levels will yield higher membership values. Since service level is a probability, it is naturally bounded by 0 and 1, see Figure 5-5 part B. However, the shape of the function's bounds is still subjective. Once again we utilize a triangular function.

The triangular membership function $\mu(P\{NS\})$ is characterized as,

$$\mu(P\{NS\}|a)=\begin{cases}1 & P\{NS|s,a\}\geq P\{NS\}_u\\ \dfrac{P\{NS|s,a\}-P\{NS\}_l}{P\{NS\}_u-P\{NS\}_l} & P\{NS\}_l\leq P\{NS|s,a\}\leq P\{NS\}_u\\ 0 & P\{NS|s,a\}<P\{NS\}_l\end{cases}\tag{5.15}$$

Equation (5.15) is used as the third part of the reward structure.

iv. Reward 4: Expected Amount Counted

When a cycle count is triggered, a lot of time and effort is put into the counting procedure, especially if there is a lot of inventory. In literature, the tediousness of cycle counting and opportunity costs associated with it is often overlooked. Yet there are some considered the effects of the number of units counted (Gumrukcu et al., 2008). Moreover, the accuracy of the cycle counting is another issue. In practice the counting procedure itself is prone to errors as well. And the accuracy drops as the number of inventory to be counted increases (Stevenson & Hojati, 2007). To address these issues we implement fourth reward as the expected amount counted.

Theoretically, when the actual physical stock level is high (or low), initiating a count yields a lower (or higher) reward (Kiefer & Novack, 1999). In our model we utilize the same reward structure for counting.

Let c denote the amount counted after a cycle count is triggered. Then, $E[c|s,a]$ denotes the expected amount counted given s and a; which can be formulated as,

$$E[c|s,a] = \begin{cases} R - E[\varepsilon|s] & a = 1 \\ 0 & a = 0 \end{cases} \tag{5.16}$$

To describe the fuzzy reward, as shown in Figure 5-6 part A, we define an acceptable interval for $E[c|s,a]$ as,

$$[E[c]_l, E[c]_u].$$

This number can easily be bounded with 0 from below, since that is the lowest possible countable number. From above it can be bounded by maximum allowable error, which is R. This is because when the errors in the system reaches R, a freeze is observed and the problem terminates. Figure 5-6 part B shows this relation.

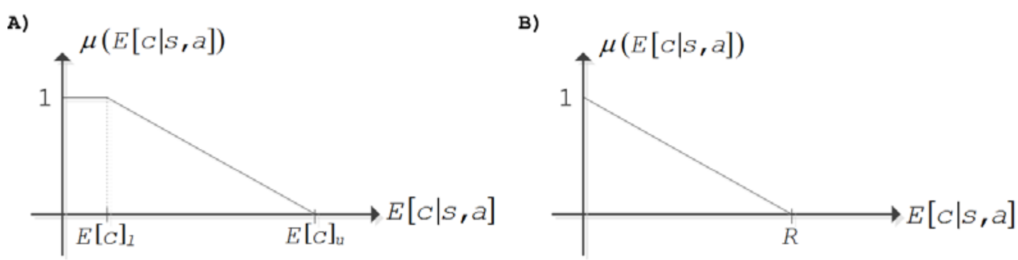

Figure 5-6: (A) Membership function for counting amount and (B) membership function with bounds

We again we utilize a triangular function with the following membership function,

$$\mu(E[c|s,a]) = \begin{cases} 1 & E[c|s,a] \le E[c]_l \\ \dfrac{E[c]_u - E[c|s,a]}{E[c]_u - E[c]_l} & E[c]_u > E[c|s,a] > E[c]_l \\ 0 & E[c|s,a] \ge E[c]_u \end{cases} \tag{5.17}$$

Equation (5.17) is used as the first part of the reward structure.

v. The Aggregate Reward

We coined the term chained reward to symbolize the combined fuzzy rewards defined in equations (5.9), (5.13), (5.15) and (5.17). The chained reward in state s for the action a, $r(s,a)$, can be obtained through fuzzy additive goal programming (Roy & Maiti, 1998; Xu & Liu, 2008).

In our study the chained reward can be characterized as,

$$r(s,a) = W_1\mu_1 + W_2\mu_2 + W_3\mu_3 + W_4\mu_4 . \tag{5.18}$$

In the equation, μ_i ($i=1,2,3,4$) is the fuzzy reward obtained from each of the reward function i. The term W_i ($i=1,2,3,4$) denotes the associated weight of each reward. In this setup, $i=1$ to 4 represents lost sales, error correction, service level and the counting amount respectively. Moreover, the assigned weights are assumed to be constant over stages.

The formulation below in equation (5.19) shows one-step maximization problem. The left side shows the general characterization and the right side shows the bounded version. This characterization is rewritten as an infinite horizon model in the following sections.

$$\begin{array}{llll}
Max & r(s,a) & Max & r(s,a) \\
s.t. & \dfrac{n(R)_u - n(R|s,a)}{n(R)_u - n(R)_l} = \mu_1 & s.t. & \dfrac{E[D_\tau] - n(R|s,a)}{E[D_\tau]} = \mu_1 \\
 & \dfrac{E[\varepsilon|s,a] - E[\varepsilon|s]_l}{E[\varepsilon|s]_u - E[\varepsilon|s]_l} = \mu_2 & & \dfrac{E[\varepsilon|s,a]}{R} = \mu_2 \\
 & \dfrac{P\{NS|s,a\} - P\{NS\}_l}{P\{NS\}_u - P\{NS\}_l} = \mu_3 & & P\{NS|s,a\} = \mu_3 \\
 & \dfrac{E[c]_u - E[c|s,a]}{E[c]_u - E[c]_l} = \mu_4 & & \dfrac{R - E[c|s,a]}{R} = \mu_4 \\
 & a \in \{0,1\} \quad \forall i = \{1,2,...,N-1\} & & a \in \{0,1\} \\
 & \mu_i \in (0,1) \quad \forall i = \{1,2,3\} & & \mu_i \in (0,1) \quad \forall i = \{1,2,3\}
\end{array} \tag{5.19}$$

The weights W_1, W_2, W_3 and W_4 reflect the relative importance of each goal in the decision model. FAGP method is commonly applied to solve multiple criteria decision problems (Yaghoobi et al., 2008). The basic concept is to use a single utility function to express the overall preference of the decision maker to draw out the relative importance of each criterion (Lai & Hwang, 1994). In this case, we obtain a linear weighted utility function by multiplying each membership function of fuzzy goals with their corresponding weights and then adding the results together. The weighted additive model, proposed by Tiwari et al. (1987), belongs to the convex fuzzy models outlined by Bellman and Zadeh (1970). One important shortcoming of additive models is that the weights are assumed to be known. In reality this is not always correct; hence, we conducted sensitivity analysis on various weights to study the influence of the weights and the effect of error.

5.1.5 Infinite-Horizon Discounted MDP

Optimizing sequential decision making problems requires the computation of objective function for each combination of values. This becomes a significant obstacle when the dimension of the state variable is large. As pointed out by Rust (1997) this can considerably reduce the ability to solve continuous MDPs accurately. In literature such problems are often referred as the curse of dimensionality (Bellman, 1957; Bellman & Dreyfus, 1962).

In this study we model an infinite-horizon discrete-time discounted MDP. In our problem the state space is the difference of two data sources (records and visibility), which are both fully observable. At each decision epoch decision maker has two alternatives; cycle counting or not. The immediate rewards obtained after the actions are bounded and stationary. The transition probabilities do not change over time and the state space is finite.

By using the expected total discounted reward as the objective function we solve our problem using policy iteration algorithm. The policy iteration algorithm, as described in Puterman (2009), first selects an arbitrary policy and then calculates the corresponding value function. Then, one by one different policies are generated and corresponding value functions are compared iteratively. At the end of each iteration, the generated policies are either updated or discarded based on the value function comparison. This cycle continues until no improvement is possible.

Let $v_{\lambda}^{\pi}(s)$ represent the expected total discounted reward of policy π, where $0 \le \lambda \le 1$. Then infinite-horizon optimality equations can be formed as

$$v_{\lambda}^{\pi}(s) = \sup_{a \in A} \left\{ r(s,a) + \sum_{s' \in S} \lambda P(s'|s,a) v_{\lambda}^{\pi}(s') \right\}$$

see (Puterman, 2009).

5.2 Structural Properties

In this section, we examine the sufficient conditions that ensure the existence of an optimal control limit policy. The control limit type policy in MDP framework can be briefly explained as follows: The optimal decision rule is to trigger a cycle count when the current state is above a threshold state s^* and do nothing if it is below s^*. In other words, the cycle count is done if and only if the observed state is among $s^*, s^*+1, ..., R+1$ (Barlow & Proschan, 1996). In this paper we refer this type as threshold type policy. Also, it is widely known that when the optimal policy is of threshold type the problem typically can be solved more efficiently, as demonstrated in Puterman (2009).

Below, we discuss some observations that are used to specify special structures about the optimal policy. Interested readers should refer to (Barlow & Proschan, 1996; Puterman, 2009).

5.2.1 Transition Matrix

Property 1: $q(k|s,a)$ is non-decreasing in s for all a, where

$$q(k|s,a) = \sum_{s'=k}^{\infty} P(s'|s,a).$$

Proof. From equation (5.6), $q(k|s,a)$ is independent of s for $a=1$ so it is non-decreasing. For $a=0$, let $q(k|s+1,0) - q(k|s,0) = q_s^k$ so that

$$q_s^k = \sum_{s'=k}^{\infty} P(s'|s+1,a) - \sum_{s'=k}^{\infty} P(s'|s,a). \tag{5.20}$$

Then, q_s^k is non-decreasing due to the transition probability structure as mentioned in Section 5.1.3. □

5.2.2 Rewards

The following discussions are important observations about the structure of the aggregate reward.

Property 2: For $i \in \{1,2,3\}$, $\mu_i(a=1) \geq \mu_i(a=0)$ for all s.

Proof. This can be shown by looking at the formulation of each μ_i in equation (5.19).

For $i=1$,

$$\mu_1 = \frac{E[D_\tau] - n(R|s,a)}{E[D_\tau]} = 1 - \frac{n(R|s,a)}{E[D_\tau]}. \tag{5.21}$$

In equation (5.21), as $n(R|s,a)$ gets larger μ_1 gets smaller. Also the expected lost sales is always smaller when a cycle count is triggered,

$$n(R|s,a=0) \geq n(R|s,a=1).$$

This can be proven by examining equation (5.8); since $E[\varepsilon|s,a=1]$ is always non-negative, $n(R|s,a=0) \geq n(R|s,a=1)$ is always true. Then, for all s, $\mu_1(a=1) \geq \mu_1(a=0)$ holds. Similarly, same statement can be proven for

μ_2 and μ_3 using equations (5.10) and (5.14). They both increase when a count is triggered ($a=1$). Hence, $\mu_i(a=1) \geq \mu_i(a=0)$ holds for $i \in \{1,2,3\}$.
□

Property 3: $\mu_4(a=0) \geq \mu_4(a=1)$ for any s.

Proof. This can be proven by examining equation (5.16). Since $E[\varepsilon|s]$ is non-negative, $E[c|s,a=1] \leq E[c|s,a=0]$ holds. Hence $\mu_4(a=0) \geq \mu_4(a=1)$ also holds for any s. □

Property 4: $\mu_1(a=0)$ and $\mu_3(a=0)$ are monotonically decreasing with s.

Proof. First of all, as s increases the expected number of inventory error increases

$$E[\varepsilon|s,a] \leq E[\varepsilon|s+1,a]. \tag{5.22}$$

Equation (5.22) can be shown as follows:

$$0 \leq (s+1)r_{s+1}^{0} \tag{5.23}$$

is always true since both r and s are non-negative. Equation (5.23) can be expanded by adding

$$\sum_{i=0}^{s} i r_s^{s-i}$$

to both sides of the inequality, in other words

$$\begin{aligned}\sum_{i=0}^{s} i r_s^{s-i} &\leq \sum_{i=0}^{s} i r_s^{s-i} + (s+1)r_{s+1}^{0} \\ &\leq \sum_{i=0}^{s+1} i r_{s+1}^{s+1-i}\end{aligned} \tag{5.24}$$

According to equations (5.10) and (5.12),

$$E[\varepsilon|s,a] = \begin{cases} \sum_{i=0}^{s} i r_s^{s-i} & a=1 \\ 0 & a=0 \end{cases}.$$

Hence, $E[\varepsilon|s,a] \le E[\varepsilon|s+1,a]$ is correct. Consequently, this implies that the following also holds

$$n(R|s+1,a=0) \ge n(R|s,a=0), \tag{5.25}$$

since

$$n(R|s,a=0) = E\left[\max\{D_\tau - R + E[\varepsilon|s,a=1],0\}\right].$$

Finally, equation (5.25) implies that $\mu_1(a=0)$ is monotonically decreasing in s. The proof for $\mu_3(a=0)$ is very similar: when the system is in higher states the probability of having a stock out is also higher which decreases the fill rate. In other words, for any s

$$P(D_\tau \ge R - E[\varepsilon]|s+1,a) \ge P(D_\tau \ge R - E[\varepsilon]|s,a). \tag{5.26}$$

Equation (5.26) implies

$$P(NS|s+1,a) \le P(NS|s,a),$$

since

$$P\{NS|s,a\} = \begin{cases} P\{D_\tau < R\} & a=1 \\ P\{D_\tau < R - E[\varepsilon]|s\} & a=0 \end{cases}.$$

Thus, $\mu_1(a=0)$ and $\mu_3(a=0)$ are monotonically decreasing with s. □

Property 5: $\mu_1(a=1)$ and $\mu_3(a=1)$ are constant with respect to s.

Proof. Since $n(R|s,a)$ and $P\{NS|s,a\}$ are independent of s when $a=1$. □

Property 6: $\mu_2(a=1)$ and $\mu_4(a=1)$, are both monotonically increasing with s.

Proof. Since equation (5.22) is true for any a and s,

$$\mu_2 = \frac{E[\varepsilon|s,a]}{R}$$

also becomes monotonically increasing with s. Likewise

$$\mu_4 = \frac{R - E[c|s,a]}{R}$$

and

$$E[c|s,a] = \begin{cases} R - E[\varepsilon|s] & a = 1 \\ 0 & a = 0 \end{cases} \tag{5.27}$$

decreases with s by equation (5.22). □

Property 7: $\mu_2(a=0)$ and $\mu_4(a=0)$ are constant with respect to s

Proof. Since they are both 0 as shown in equations (5.10) and (5.16). □

Property 8: $r(s,a)$ is non-decreasing with s for $a=1$ and non-increasing with s for $a=0$.

Proof. The aggregate reward function can be expended as

$$r(s,a) = W_1\mu_1 + W_2\mu_2 + W_3\mu_3 + W_4\mu_4 .$$

In this equality, W_i is independent of s and $W_i \geq 0$, $i = 1,2,3,4$. From propositions 4 through 7, it can be shown that $r(s,a=0)$ is non-increasing and $r(s,a=1)$ is non-decreasing. □

Theorem 1: Let $f(s) = r(s,a=1) - r(s,a=0)$, then $f(s)$ is a monotonically increasing function with s.

Proof. This can be proven by showing $r(s,a=1)$ is monotonically increasing and $r(s,a=0)$ is monotonically decreasing with s. In other words $r(s+1,a=1) \geq r(s,a=1)$ and $r(s,a=0) \geq r(s+1,a=0)$ for every $s \in [0,R)$.

For $s \in S$ and $i \in \{1,2,3,4\}$, $r(s,a=1)$ is a convex combination of $\mu_i(a=1)$ and $r(s,a=0)$ is a convex combination of $\mu_i(a=0)$ where $W_i \in [0,1]$ and

$$\sum_{i=1}^{4} W_i = 1,$$

as shown in (5.18). Since $\mu_1(a=0)$, $\mu_3(a=0)$ are monotonically decreasing and $\mu_2(a=0)$, $\mu_4(a=0)$ are constant with s. Any convex combination of them is also monotonically decreasing with s. Similarly, $\mu_1(a=1)$, $\mu_3(a=1)$ are constant and $\mu_2(a=1)$, $\mu_4(a=1)$ are monotonically increasing with s. Any convex combination of them is also monotonically increasing with s. Hence, $f(s)$ is monotonically increasing with s. □

5.2.3 Value Function and Policy

According to Puterman (2009) under following conditions:

1. Stationary rewards and transition probabilities: $r(s,a)$ and $P(s'|s,a)$ do not change from decision epoch to decision epoch.

2. Bounded rewards: $|r(s,a)| \leq M < \infty$ for all a and s.

3. Discounting: future rewards are discounted according to discount factor λ, with $0 \leq \lambda < 1$.

4. Discrete state space; S is finite and countable.

Theorem 2: For any stationary policy π, there is a unique solution v_λ^π.

Theorem 3: A policy π^* optimal if and only if $v_\lambda^{\pi^*}$ is a solution of the optimality equation.

Theorem 4: For a discrete S if A_s is compact, $\tilde{r}(s,a)$ is continuous for each s and $P(s'|s,a)$ is continuous in a for each s and s'. Then there exists an optimal deterministic stationary policy.

Proof. See Puterman (2009) for theorems 2 through 4.

Theorem 5: The structure of the optimal policy is a control limit type as long as assumptions 1 through 4 are satisfied and $W_i \in [0,1]$ for every i,

$$\sum_{i=1}^{4} W_i = 1 .$$

Proof. Assume that there exists an s^* such that $d(s)=0$ for all $s<s^*$ and $d(s)=1$ for all $s \geq s^*$. This assumption implies that for every $s<s^*$

$$r(s,1)+\lambda\sum_{s'=0}^{R+1} P(s'|s,1)v(s') \leq r(s,0)+\lambda\sum_{s'=0}^{R+1} P(s'|s,0)v(s')$$

and for every $s \geq s^*$

$$r(s,1)+\lambda\sum_{s'=0}^{R+1} P(s'|s,1)v(s') \geq r(s,0)+\lambda\sum_{s'=0}^{R+1} P(s'|s,0)v(s') .$$

Let's assume the converse of the assumption is true. In other words, there exists an $s<s^*$ such that $d(s)=0$ and $d(s-1)=1$. This can be formulated as

$$r(s,1)+\lambda\sum_{s'=0}^{R+1} P(s'|s,1)v(s') \leq r(s,0)+\lambda\sum_{s'=0}^{R+1} P(s'|s,0)v(s') \tag{5.28}$$

and

$$r(s-1,1)+\lambda\sum_{s'=0}^{R+1} P(s'|s-1,1)v(s') > r(s-1,0)+\lambda\sum_{s'=0}^{R+1} P(s'|s-1,0)v(s') . \tag{5.29}$$

Equations (5.28) and (5.29) can be respectively rewritten as

$$r(s,1)-r(s,0)\leq\lambda\left(\sum_{s'=0}^{R+1}v(s')(P(s'|s,0)-P(s'|s,1))\right) \tag{5.30}$$

and

$$r(s-1,1)-r(s-1,0)>\lambda\left(\sum_{s'=0}^{R+1}v(s')(P(s'|s-1,0)-P(s'|s-1,1))\right) \tag{5.31}$$

In property 9 it is already proven that $f(s)=r(s,1)-r(s,0)$ is a monotonically increasing function in terms of s. This implies that $f(s-1)-f(s)\leq 0$. Then, by subtracting equation (5.30) from equation (5.31) we get

$$\begin{aligned} f(s-1)-f(s) &> \lambda\left(\begin{array}{l}\sum_{s'=0}^{R+1}v(s')(P(s'|s-1,0)-P(s'|s-1,1))\\ -\sum_{s'=0}^{R+1}v(s')(P(s'|s,0)-P(s'|s,1))\end{array}\right)\\ 0 &> \lambda\left(\begin{array}{l}\sum_{s'=0}^{R+1}v(s')(P(s'|s-1,0)-P(s'|s-1,1))\\ -\sum_{s'=0}^{R+1}v(s')(P(s'|s,0)-P(s'|s,1))\end{array}\right)\end{aligned} \tag{5.32}$$

In equation (5.32), λ is a non-negative number between 0 and 1; removing it does not violate the inequality. Also, by definition $P(s'|s,1)=P(s'|0,0)$ for any s, so we can rewrite the equation as

$$0>\sum_{s'=0}^{R+1}[v(s')(P(s'|s-1,0)-P(s'|0,0)-P(s'|s,0)+P(s'|0,0))]. \tag{5.33}$$

Subsequently, it can be simplified as

$$\sum_{s'=0}^{R+1}v(s')P(s'|s,0)>\sum_{s'=0}^{R+1}v(s')(P(s'|s-1,0)) \tag{5.34}$$

where both $v(s')$ and $P(s'|s,0)$ are non-negative for any s. Hence, equation (5.34) is equivalent to saying $P(s'|s,0)>(P(s'|s-1,0))$ for all $s<s^*$. This is simply not true for every possible realization. Which means there exists a contradiction, meaning $d^*(s)=0$ implies $d^*(s-1)=0$ for $s<s^*$.

Similarly, $d(s)=1$ for $s \geq s^*$ can easily be proven by employing the same logic. Hence, $d^*(s)=0$ implies $d^*(s-1)=0$ for $s<s^*$ and $d^*(s)=1$ implies $d^*(s+1)=1$ for $s \geq s^*$. Meaning the optimal policy is a threshold type. □

5.3 Numerical Analysis

To demonstrate the performance of our algorithm we solved the same case study provided in previous chapters. Recall, in the case study a (Q,R) policy is utilized with $(600,80)$. The weekly demand is normally distributed with $(50,12^2)$ and lead time is also normally distributed with $(1.14,0.33^2)$. Furthermore, by utilizing the error characterization provided in chapter 3 we used normal distribution with $(3.83,7.17^2)$ for errors. We compare three level visibility performance with truncated normal distribution.

As mentioned the mean is set to zero to ensure that the system is not intentionally making errors. The standard deviation however, depends on the current state and it is the factor that defines the performance. Intuitively, in an ideal setup the unbiased visibility yields $P(\bar{\varepsilon}=0|s)=1$ if t is zero and $P(\bar{\varepsilon}=t|s)=0$ if $t>0$; this is a matrix with ones in column zero and zeroes everywhere else. Similarly the worst unbiased visibility would assume a uniform shape across t. In other words, in such a scenario $P(\bar{\varepsilon}=t|s)=1/s$ for every t and s. In reality the visibility performance will behave somewhere in between these two extremes.

5.3.1 Transition Matrix

In our example, $P(\bar{\varepsilon}=t|s)$ follows a truncated normal distribution with mean zero and variance $\sigma_{\bar{\varepsilon}}$. If $\sigma_{\bar{\varepsilon}}$ is zero, then the visibility will be perfect and as $\sigma_{\bar{\varepsilon}}$ increases the visibility performance decreases. We utilize three different settings for our problem. In the high visibility case $\sigma_{\bar{\varepsilon}}^{H}$ is assumed to be $0.1s$. Similarly, $\sigma_{\bar{\varepsilon}}^{M} = 0.2s$ and $\sigma_{\bar{\varepsilon}}^{L} = 0.3s$ are for medium and low visibility performance respectively.

The structure of visibility discrepancy probability respect to states for high performance case is demonstrated in Figure 5-7Figure 5-8. Lines in the graph correspond to states; 10, 25, 50 and 80. The x-axis shows the amount of discrepancy and the y-axis shows the corresponding probability of having that many discrepancies for the given state.

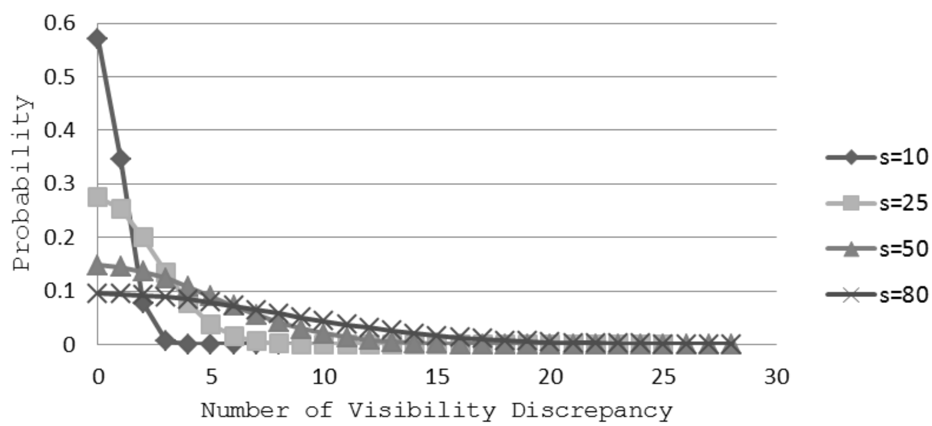

Figure 5-7: Probability of visibility discrepancy for 4 states for high performance

For example in Figure 5-7, when the current state is 10 (the first line in the figure) the probability of having no visibility discrepancy (0 on the x-axis) is close to 0.6; the probability of having small visibility discrepancy (1 to 5 on the x-axis) sharply decreases, and for higher visibility discrepancies this probability becomes almost 0. Whereas, if the current state is 80 (the last line in the figure) no visibility discrepancy probability is 0.1; small discrepancy probability is still around 0.1, and for higher values of discrepancy the probability slowly decreases.

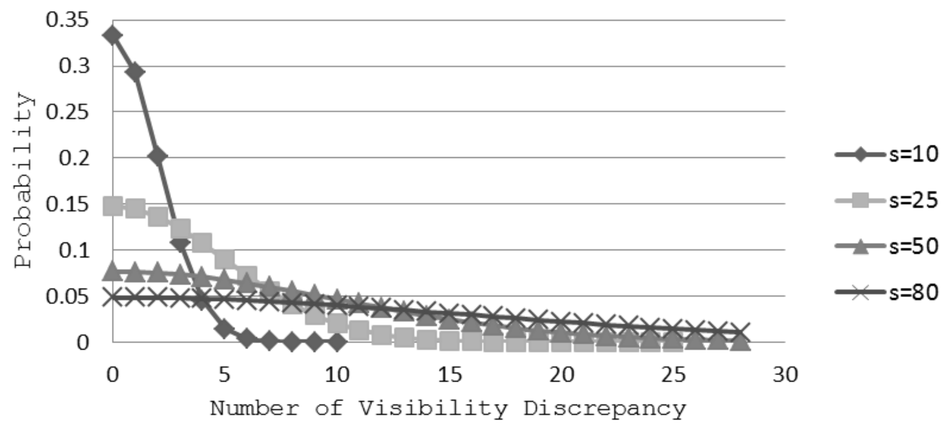

Figure 5-8: Probability of visibility discrepancy for 4 states for medium performance.

Similarly, the visibility discrepancy probabilities for medium and low performance are demonstrated in Figure 5-8 and Figure 5-9. As can be seen from the figures, in each performance level the visibility discrepancy probability decreases as state increases. The reason behind this behavior is because the visibility discrepancy is bounded and the distribution is truncated with state. If the current state is $s = 10$ then $P(\bar{\varepsilon} = t|s)$ can only take positive values for $t \in [0,10]$.

As shown in the previous sections, the transition probability matrix is generated using the visibility probabilities. Figure 5-10 summarizes the transition probability matrix for high performance by depicting the transition behavior given the current state. For simplicity, once again 4 states are chosen for representation; 10, 25, 50 and 80. Moreover, the x-axis represents the observations for the next state (s') and the y-axis shows the probability of reaching to s' given the current state (or line).

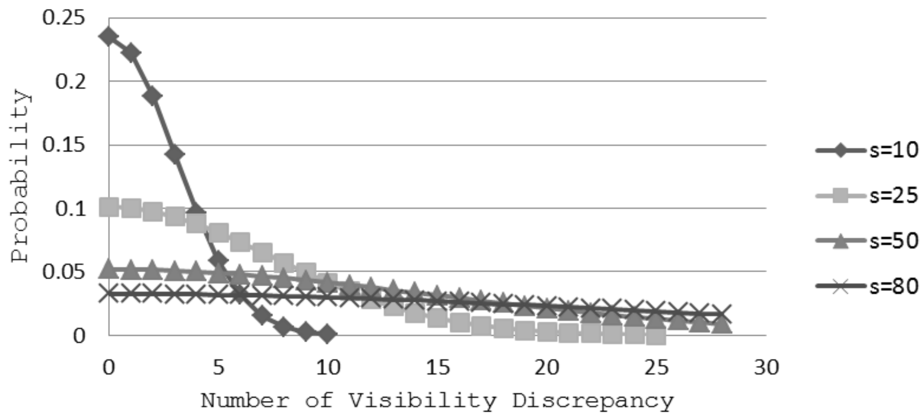

Figure 5-9: Probability of visibility discrepancy for 4 states for low performance

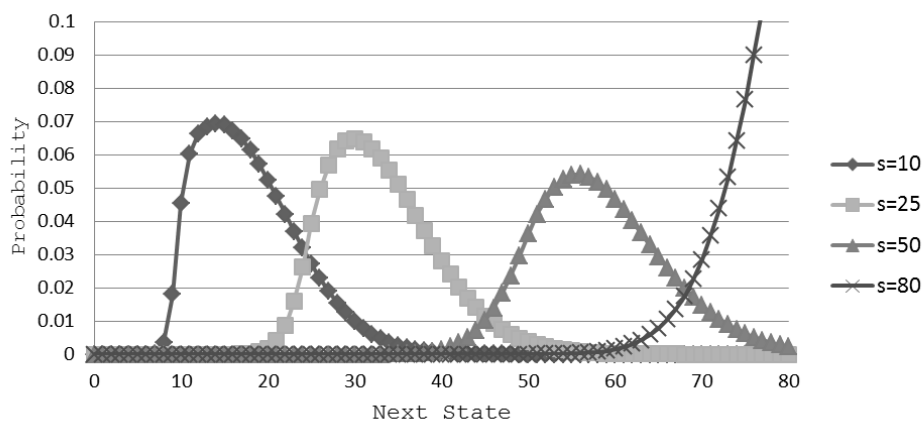

Figure 5-10: Transition probability matrix for 4 states for high performance

For example in Figure 5-10, when the current state is in 10 (the first line in the figure) the probability that the next state will be in between 10 and 30 is very high; and the probability of being in the remaining state is very low. Also, the transition matrix displays increasing failure rate type of behavior (IFR); the system is likely to move towards higher states and stay there.

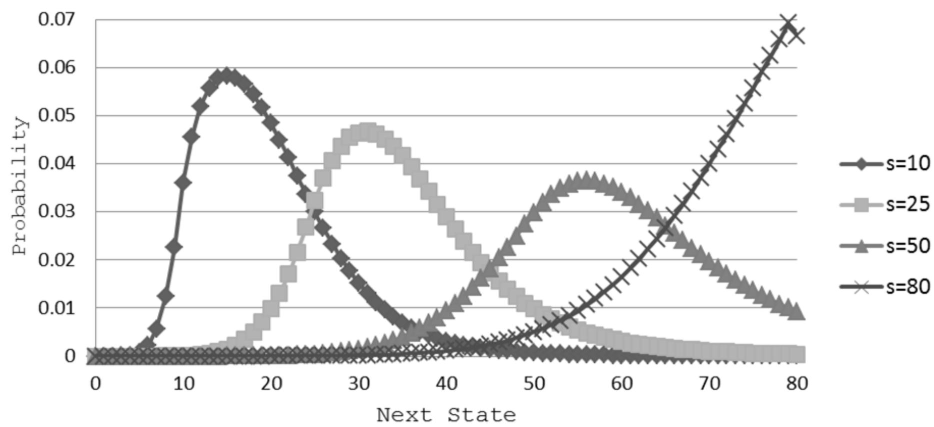

Figure 5-11: Transition probability matrix for 4 states for medium performance

Same observations can be seen in Figure 5-11 and Figure 5-12 as well. By looking at the figures, the transition matrix exhibits a similar behavior at each level. When the state is small the probability of transitioning to immediate vicinity is higher than jumping to a distant state. But as the state gets larger, the jump range increases; especially if the visibility level is low.

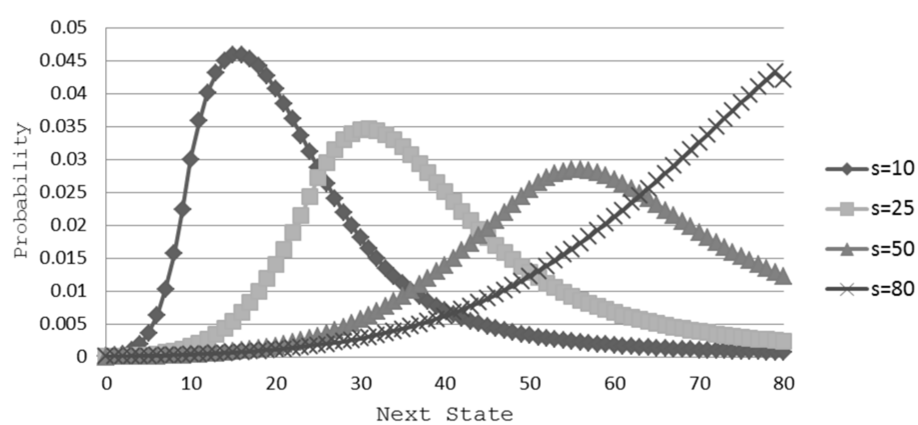

Figure 5-12: Transition probability matrix for 4 states for low performance

Moreover, in each scenario for each state the system is more likely to move up in state in each transition. This means if left unattended state space will reach to R and eventually results freezing; however, when the system is close to R the structure of the transition probabilities change considerably. This is because the system cannot go beyond R due to freezing.

5.3.2 Rewards

The reward setup, using weights w_i is presented, in equations (5.18) and (5.19). The weights play an important role in determining the optimal policy. Finding standardized weights for the objective function that apply to all inventory problems is difficult. To overcome this problem we utilize various weight distributions, which are shown in Table 5-1. But before that we first take a look at the rewards.

Table 5-1: Rewards table with three performance levels

$\tilde{r}(s,a)$		High				Low			
a	s	μ_1	μ_2	μ_3	μ_4	μ_1	μ_2	μ_3	μ_4
0	0	0.9693	0.9693	0	0	0.9693	0.9693	0	0
	10	0.9334	0.9693	0	0.1184	0.9408	0.9693	0	0.0988
	25	0.8365	0.9693	0	0.2912	0.8703	0.9693	0	0.2417
	50	0.5514	0.9693	0	0.5789	0.6646	0.9693	0	0.4797
	80	0.1006	0.9693	0	0.9241	0.3143	0.9693	0	0.7652
1	0	0.9693	0	0	0.8395	0.9693	0	0	0.8395
	10	0.9693	0	0.1184	0.7097	0.9693	0	0.0988	0.7341
	25	0.9693	0	0.2912	0.4644	0.9693	0	0.2417	0.5377
	50	0.9693	0	0.5789	0.123	0.9693	0	0.4797	0.2149
	80	0.9693	0	0.9241	0.0073	0.9693	0	0.7652	0.0321

Table 5-1 shows the structure of immediate gains obtained from each component of the reward function. The table summarizes these rewards with respect to action, various states (0, 10, 25, 50 and 80) and different visibility performance levels. Recall that, μ_i for $i = 1,2,3,4$ denotes the objective function coefficient for lost sales, error correction, fill rate and count amount respectively.

5.3.3 Value Function and Policy

The numerical example is solved, by combining fuzzified key performance metrics into rewards. To see the sensitivity of the system with respect to weights we performed various combinations. Table 5-2 summarizes the weight selections. According to the table, seven different combinations of W_i's are used; recall that, W_i for $i = 1,2,3,4$ denotes the weights for lost sales, error correction, fill rate and count amount respectively. In the first combination, equal values are assigned to each as 0.25. In the remaining combinations different values are methodologically assigned to each W_i to assess the relative effect on the objective function.

Table 5-2: Different weight selections

#	W_1	W_2	W_3	W_4
1	1/4	1/4	1/4	1/4
2	3/8	1/8	3/8	1/8
3	1/8	3/8	1/8	3/8
4	5/8	1/8	1/8	1/8
5	1/8	5/8	1/8	1/8
6	1/8	1/8	5/8	1/8
7	1/8	1/8	1/8	5/8

The results obtained by using the information presented in Table 5-2 are summarized in Table 5-3. The table is designed to show changes in the control limit state by different weight selections and for different visibility levels. Additionally the table presents the optimal values for the selected weight distribution and visibility level.

According to the results, the optimal policy is greatly influenced by the visibility level and weight distribution. As the visibility decreases the threshold state increases. This is an unexpected result and also very hard to prove mathematically, but intuitively when the visibility is lowered, the expected error given at any observed state increases.

Table 5-3: Optimal value vs. control limit state table for each weight selection

	High		**Medium**		**Low**	
#	Control Limit	Value	Control Limit	Value	Control Limit	Value
1	22	12.782	26	12.847	29	12.965
2	12	14.972	14	14.995	16	15.060
3	39	11.272	43	11.382	44	11.442
4	16	15.828	18	15.852	20	15.914
5	21	7.332	25	7.465	27	7.7041
6	10	14.181	12	14.198	13	14.266
7	51	15.478	54	15.512	57	15.467

Furthermore, as the state increases the value function also increases; however, this is only true when the state is above the threshold state.

5.4 Conclusion and Future Work

In this study, a multi-objective single-item continuous-review stochastic inventory problem over an infinite horizon where the decision maker is following a (Q,R) policy with random lead time, lost sales and IRI where the objective is fuzzy is formulated. To show the value of inventory visibility a secondary source of information is used along with traditional inventory record keeping methods to control the effects of IRI. Using both measures the decision maker chooses the best time to generate a cycle count. Furthermore, in the multi-objective setting, the traditional cost based reward structure is abandoned to put more emphasis on the effects of IRI. Instead a new measure is developed as inventory performance by combining four key performance metrics; lost sales, amount of correction, fill rate and amount of inventory counted. These key metrics are combined under a unitless platform using fuzzy logic and combined through additive methods.

The inventory problem is modeled as an infinite horizon discounted MDP with fuzzified multi-objective. The optimal policy is shown to be a control limit policy. Finally, the results are shown in a brief numerical example solved by policy iteration algorithm.

The dynamic programming model in this chapter is designed to find the optimal inventory performance using the error correction and control method. This model can be extended by implementing the IRI resistance method. In that case, the decision maker has two decision available at each decision epoch, whether to do a cycle count or not and to decide on the best increment level.

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CHAPTER 6 CONCLUSION

Supply chain and inventory management has always been a major concern in the business world as well as in the academic domain. The scope entails physical holding, lead times, holding costs, replenishment, defective goods, quality control, transportation, storage, and inventory visibility. Hence, inventory models can be regarded as one of the most widely studied topics in industrial engineering and operations management. The main goal for most of these studies is to reach efficient solutions that would provide cost effective realizations in practice. Due to the uncertain nature of the world, these models are known to have a complex structure. In practice, dealing with all the uncertain factors, satisfying the high service levels and reaching optimal solutions at the same time is challenging. Starting from late 70s, theoretical studies began addressing the difficulties faced in inventory management. In industries where the competition is fierce and profit margins are thin, companies have automated the inventory management processes to better meet customer demand and reduce operational costs. Such schemes significantly decreased the response time of the decision makers, making it dramatically easy to keep track of the records and avoid human intervention as much as possible. However, the automation of management processes transferred the entire critical decision making - such as what products are where and in what quantity - from humans to computers. As a result understanding the value of data accuracy and controlling the impact of data inaccuracy became a crucial part of inventory management problems. The aims to answer the following two questions: What is the impact of IRI? And how can we control IRI?

In this dissertation a methodical analysis is performed to understand the behavior of inventory record inaccuracy (IRI) when it is influenced by demand, supply and lead time uncertainty in both the online and the offline retail environment separately. Additionally, this study identifies the susceptibility of the inventory systems towards IRI due to conventional perfect data visibility assumptions. In terms of lead time demand, there is no conclusive result on the behavior of the error function. Depending on input parameters for errors, the function can decrease or increase with the lead time demand. In terms of the reorder level, the safety stock and the order quantity the error function is increasing. It is also shown that, errors have no strong dependence between each other. And, in both best and worst cases the biggest effect on IRI is done by the outbound errors. To compensate for IRI two different alternatives are presented and analyzed; the IRI resistance and the error control methods. The discussed methods effectively countered various aspects of IRI. The IRI resistance method performs better on stock-out and lost sales but influences errors; whereas, the error control method keeps lower inventory but has more stock-out, higher stock-out and additional counting cost.

By shifting the focus from IRI to cost, this dissertation, also provides a detailed comparison between the retail environments, the compensation methods and the IRI sensitivity. In terms of the retail environments, it is shown that in the online retail, the importance of record accuracy is elevated, a new type of IRI measure called the penalty sales is revealed and the freezing problem is vanished. The studies on the compensation methods reveal that the IRI resistance method generates higher levels of profit in all situations and the error control becomes ineffective in the online setting. Finally, in terms of the IRI sensitivity, the IRI resistance method is not as sensitive as the error control method to changes in IRI unit costs.

Furthermore, this research also investigates the value of using a secondary source of information (automated data capturing) along with traditional inventory record keeping methods to control the effects of IRI. To understand the combined behavior of the pooled data sources and a multi-objective infinite-horizon single-item continuous-review problem with (Q,R) policy, random lead time, random lost sales, IRI and fuzzy objective is formulated. Moreover, the traditional cost based reward structure is abandoned to put more emphasis on the effects of IRI. Instead a new measure is developed as *inventory performance* by combining four key performance metrics; lost sales, amount of correction, fill rate and amount of inventory counted. These key metrics are united under a unitless platform using fuzzy logic and combined through additive methods. The inventory model is then analyzed to understand the optimal policy structure, which is proven to be of a control limit type.

The work done in this dissertation can be extended by including combined retail environments where the customers can use the offline or the online platform simultaneously. The store pickup and home delivery models could be added into the model. Furthermore, price changes and multi-inventory setting could be introduced to the system to make it more realistic. The discussed compensation methods are designed as static decision; they could be remodeled as dynamic decision so that at each period the decision maker can adjust the values based on the system performance. Finally the dynamic programming model in the final is designed to find the optimal inventory performance using the error correction and control method. This model can be extended by implementing the IRI resistance method. In that case, the decision maker has two decision available at each decision epoch, whether to do a cycle count or not and to decide on the best increment level.