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Electron Shock Waves with a Large Current Behind the Shock Front

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Electron Shock Waves with a Large Current behind the Shock Front

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Running Title: Electron Shock Waves with a Large Current behind the Shock Front

Abstract

The propagation of breakdown waves in a gas, which is primarily driven by electron gas pressure, is described by a one-dimensional, steady-state, threecomponent (electrons, ions, and neutral particles) fluid model. We consider the electron gas partial pressure to be much larger than that of the other species and the waves to have a shock front. Our set of equations consists of the equations of conservation of the flux of mass, momentum, and energy coupled with Poisson's equation. This set of equations is referred to as the electron fluid dynamical equations. In this study we are considering breakdown waves propagating in the opposite direction of the electric field force on electrons (return stroke in lightning) and moving into a neutral medium.

For Breakdown waves with a significant current behind the shock front, the set of electron fluid dynamical equations and also the boundary condition on electron temperature need to be modified. For a range of experimentally observed current values and also some larger current values which few experimentalists have been able to observe, we have been able to solve the set of electron fluid dynamical equations through the dynamical transition region of the wave. Some experimentalists have reported the existence of a relationship between return stroke lightning wave speed and current behind the shock front; however, some others are skeptical of the existence of such a relationship. Our solutions to the set of electron fluid dynamical equations within the dynamical transition region of the wave confirm the existence of such a relationship. We will present the method of solution of the set of electron fluid dynamical equations through the dynamical transition region of the wave and also the wave profile for electric field, electron velocity, electron temperature and electron number density, within the dynamical transition region of the wave.

Introduction

Electron shock waves, also known as breakdown waves, were first observed in the form of lightning and studied in laboratory discharge tubes. The phenomenon occurs when the potential difference between two points is high enough to ionize some of the neutral particles and later accelerate the resulting electrons to generate an avalanche-like shock wave. This process converts an ion-less gas into a neutral plasma and results in a high temperature electron gas that expands rapidly to produce an electron shock wave. The emitted radiation has been found to have no Doppler shift; therefore, the ions have no significant mass motion through the wave. When the net electric field force on electrons, applied plus space charge field force, acts in the same direction as the propagation of the wave, the wave is referred to as a pro-force wave. Waves for which the electric field force on electrons is in the opposite direction as the wave propagation are labeled, by definition, as antiforce waves. In the case of anti-force waves, the electron gas temperature, and therefore electron gas partial pressure, is large enough to provide the driving force for the propagation of the wave.

The breakdown wave can be broken into two distinct regions: the Debye sheath region and the quasi-neutral region. The Debye sheath region is a thin, dynamical region that follows the shock front. In the sheath region, the net electric field starts at its maximum value at the shock front and reduces to a negligible value at the trailing end of the sheath. Also electrons, starting from an initial speed behind the shock front, slow down to a speed comparable to that of heavy particles. Following the sheath region of the wave, exists a much longer region referred to as the quasi-neutral region of the wave. In the quasi-neutral region, the electron gas cools down through further ionization of the neutral particles, and ion and electron densities become approximately equal.

Model

Paxton and Fowler (1962) were first to formulate a fluid model for breakdown waves which led to a onedimensional, three component, steady state theory that described breakdown waves propagating into a non non-ionized media and in the direction of the electr field force on electrons. The set of equations included conservation of mass, momentum, and energy, and their solutions for the set of equations presented some success. Prior to 1984, Fowler and his associates (1968) added Poisson's equation to the set of fluid equations developed by Paxton (1962), and were able to solve their set of equations using an approximation method. The approximate solutions for the more developed set of equations showed better agreement developed set of equations showed better agreement
with experimental results than those presented by Paxton (1962). In the approximate solutions to the set of equations, to make solutions possible, many terms were neglected from the equation of conservation of energy. Fowler et al. (1984) added the previously neglected terms into the equation of c energy, particularly the heat conduction term, which altered the boundary condition on electron velocity and proved to be essential in an exact numerical solution of the set of electron fluid dynamical equations within the dynamical transitional region of the wave. Fowler et al. (1984) complete set of equations for breakdown waves propagating into a non-ionized medium and in the direction of the electric field force on electrons is as follows Paxton and Fowler (1962) were first to formulate a ir solutions for the set of equations present
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that described media and in the direction of the electric
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Model

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** $\frac{\lambda}{2}$ **the, e.v., n., and** T_n **and the set medium the set medium to the

distribution of the**

$$
\frac{d(nv)}{dx} = \beta n \tag{1}
$$

$$
\frac{d}{dx}\left[\,nmv(v-V)+nkT_e\right] = -enE - Kmn(v-V) \quad (2)
$$

$$
\frac{d}{dx}\left(nmv(v-V)^2 + nkT_e(5v-2V) + 2e\phi nv\right)
$$
\n(3)

$$
-\frac{5nk^2T_e}{mk}\frac{dT_e}{dx}\bigg) = -3\left(\frac{m}{M}\right)nKkT_e - \left(\frac{m}{M}\right)nmk(v-V)^2
$$

$$
\frac{dE}{dx} = \frac{en}{\epsilon_o} \left(\frac{v}{V} - 1\right) \tag{4}
$$

In the above equations, E is the electric field magnitude in the sheath region, M is the neutral particle mass, K is the elastic collision frequency, V is the wave velocity, x is the position within the sheath region, E_0 is electric field at the wave front, ϕ equations, E is the electric
the sheath region, M is the
 ζ is the elastic collision frequence
ity, x is the position within the

M. Hemmati, H.D. Moore, K. Ledbetter, and M.W. Bowman
ionization potential, and β is the ionization frequ
Also, e, v, m, n, and T_e , are electron charge, ve
first to formulate a mass, number density, and temperature Also, e, v, m, n, and T_e mass, number density, and temperature, respectively. To allow for easier solution of these equations, Fowler Fowler et al. (1984) introduced the follo dimensionless variables for proforce breakdown waves: ionization potential, and β potential, and β is the ionization frequency.
m, n, and T_e , are electron charge, velocity,
bber density, and temperature, respectively.
or easier solution of these equations, Fowler , are electron charge, velocity,

To allow for easier solution of these equations, Fowler et al. (1984) introduced the following set of dimensionless variables for proforce breakdown waves:
\n
$$
\eta = \frac{E}{E_0} \qquad \omega = \frac{2m}{M} \qquad \mu = \frac{\beta}{K}
$$
\n
$$
\xi = \frac{x e E_o}{mV^2} \qquad \psi = \frac{v}{V} \qquad \theta = \frac{T_e k}{2 e \phi}
$$
\n
$$
v = \frac{n2 e \phi}{\varepsilon_o E_o^2} \qquad \alpha = \frac{2 e \phi}{mV^2} \qquad \kappa = \frac{mV}{e E_o} K
$$
\nWhere, η , μ and ξ are dimensionless electric field ionization rate, and position within the sheath region of the wave, respectively. Also, v , ψ , and θ , are the

Where, η , μ and ξ are dimensionless electric field, ionization rate, and position within the sheath region of the wave, respectively. Also, v, ψ , and θ , are the dimensionless electron number density, velocity, and temperature. α and Substituting these dimensionless variables into equations 1 1-4 yields a set of electron fluid dynamical equations in nondimensional form for proforce waves dimensionless electron number density, velocity, and
temperature. α and K are wave parameters.
Substituting these dimensionless variables into
equations 1-4 yields a set of electron fluid dynamical
equations in nondime follows: v, m, n, and T_e , are electron charge
mber density, and temperature, represention
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(1984) introduced the followin
less variables for proforce
 $\omega = \frac{2m}{M}$
 $\frac{\dot{\phi}}{\dot{\phi}}$ $\omega = \frac{2e\phi}{$ Where, η , μ and ξ
ionization rate, and p
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Substituting these
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 $\frac{d}{d\xi} [v\psi(\psi - 1) + \alpha v\theta$
 $\frac{d}{d\xi} [v\psi(\psi - 1) + \alpha v\theta (5\psi - 1)]^2 + \alpha v\theta (5\psi - 1)^2 + \alpha v\theta (5\psi - 1)^2 + \alpha v\theta (5\psi - 1)^2$
 $+ \alpha \eta^2 - \frac{5\alpha^2 v\theta}{\kappa} \frac{d$ **Moore, K. Ledbetter, and M.W. Bowman**

ionization potential, and β is the ionization frequency.

Also, e, v, m, n, and γ_e , are electron change, velocityly.

To allow lor exaits solution of these equations. Followin easier solution of these equations, Fowler

4) introduced the following set of

s variables for proforce breakdown
 $\omega = \frac{2m}{M}$ $\mu = \frac{\beta}{K}$
 $\psi = \frac{v}{V}$ $\theta = \frac{T_e k}{2e\phi}$
 $\alpha = \frac{2e\phi}{mV^2}$ $\kappa = \frac{mV}{eE_o}K$

and ξ are wave parameters. and temperature, respectively.

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 $= \frac{2m}{M}$ $\mu = \frac{\beta}{K}$
 $= \frac{v}{V}$ $\theta = \frac{T_e k}{2e\phi}$
 $\frac{mV}{mV^2}$ $\kappa = \frac{mV}{eE_o}K$

e dimensionless following set of

oforce breakdown
 $\mu = \frac{\beta}{K}$
 $\theta = \frac{T_e k}{2e\phi}$
 $\kappa = \frac{mV}{eE_o} K$

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condless variables for proforce breakdown
 $\omega = \frac{2m}{M}$ $\mu = \frac{\mu}{K}$
 $\frac{\mu}{kE_0^2}$ mber density, and temperature, respectively.

for easier solution of these equations, Fowler

1984) introduced the following set of

less variables for proforce breakdown
 $\omega = \frac{2m}{M}$ $\mu = \frac{\beta}{K}$
 $\psi = \frac{v}{V}$ $\theta = \frac{T_e$ potential, and β is the ionization frequency.

The m, and T_e , are dectron charge, velocity,

there density, and temperature, respectively.

for casing solution of these equations, Following

1984) introduced the foll

$$
\frac{d}{d\xi}[v\psi] = \kappa \mu v \tag{5}
$$

$$
\frac{d}{d\xi} \left[\nu \psi(\psi - 1) + \alpha \nu \theta \right] = -\nu \eta - \kappa \nu (\psi - 1) \quad (6)
$$

$$
\frac{d}{d\xi} \left(v\psi(\psi - 1)^2 + \alpha v\theta(5\psi - 2) + \alpha v\psi \right)
$$
\n
$$
+ \alpha v^2 - \frac{5\alpha^2 v\theta}{v^2} \frac{d\theta}{dx} \right) = -\alpha v v [3\alpha\theta + (v - 1)^2]
$$
\n(7)

$$
\eta^2 - \frac{1}{\kappa} \frac{d\xi}{d\xi} = -\omega \kappa v [3\alpha \theta + (\psi - 1)^2]
$$

$$
\frac{a\eta}{d\xi} = \frac{v}{\alpha}(\psi - 1) \tag{8}
$$

To transform these equations into a set describing antiforce breakdown waves, some modifications are needed. Previously Sanmann and Fowler (1975) (1975) approximated solutions for antiforce waves by by considering a weak discontinuity at the wave front and used a simple sign change for K and μ . Considering waves to have a shock front, however, Hemmati (1999) yields a set of electron fluid

ondimensional form for profot

to a non-ionized media. Then
 $\frac{d}{d\xi}[v\psi] = \kappa \mu v$
 $\psi(\psi - 1) + \alpha v\theta$ = $-\nu\eta - \kappa v(\psi - 1)^2 + \alpha v\theta(5\psi - 2) + \alpha v\psi$
 $\gamma\eta^2 - \frac{5\alpha^2 v\theta d\theta}{\kappa d\xi}$ = $-\omega \kappa v[3\alpha$ equations 1-4 yields a set of electron fluid c

equations in nondimensional form for profor

propagating into a non-ionized media. The

follows:
 $\frac{d}{d\xi} [v\psi] = \kappa \mu v$
 $\frac{d}{d\xi} [v\psi(\psi - 1) + \alpha v\theta] = -v\eta - \kappa v(\psi)$
 $\frac{d}{d$

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showed Sanmann's (1975) simple change of variable showed Sanmann's (1975) simple change of variable signs were not accurate. Hemmati (1999) derived a new set of non-dimensional variables for the antiforce case and they are

Electron Shock Waves with a Large Current Behind the Shock Front
\nshowed Sammann's (1975) simple change of variable
\n
$$
\frac{d\eta}{d\xi} = \frac{K_1}{c_0 K E_0} - \frac{v}{\alpha}
$$
\n
$$
\eta = \frac{E}{E_0}
$$
\n
$$
\omega = \frac{2m}{M}, \qquad \mu = \frac{\beta}{K},
$$
\n
$$
\gamma = \frac{e}{E_0}
$$
\n
$$
\omega = \frac{2m}{M}, \qquad \mu = \frac{\beta}{K},
$$
\n
$$
\gamma = \frac{e}{\lambda_0 K E_0}
$$
\n
$$
\gamma = \frac{n e \omega}{c_0 E_0^2}, \qquad \omega = \frac{2e \phi}{m V^2}, \qquad \theta = -\frac{T_e k}{2 e \phi},
$$
\nwhere $t = \frac{1}{e \lambda_0 E_0}$ represents the value of $t_1 = 10$
\nAfter applying these new non dimensional variables.
\nAfter applying these new non dimensional variables, the elastic collision frequency
\nHermant's (1999) new set of non-dimensional, three are generally in the
\nbecome
\nd
\nHermant's (1999) new set of non-dimensional, three eigenvalues between
\nbecome
\n
$$
\frac{d}{d\xi} [w\psi(\psi - 1) + \alpha w\theta] = w\eta - \kappa v(\psi - 1)
$$
\n(10) and
\nantif $(w(\psi - 1) + \alpha w\theta) = w\eta - \kappa v(\psi - 1)$
\n
$$
= \frac{d}{\kappa} \left(\frac{w\psi(\psi - 1) + \alpha w\theta(\xi\psi - 2) + \alpha w\psi}{\frac{d\xi}{\kappa}} \right)
$$
\n(11) we have that $\frac{d}{d\xi} [w\psi(\psi - 1) + \alpha w\theta] = \frac{d}{\kappa} \left(\frac{w\psi - 1}{\psi} \right) = -\frac{w\psi}{\kappa} \frac{d\theta}{\kappa}$ \n(12)

After applying these new non dimensional variables, Hemmati's (1999) new set of non electron fluid dynamical equations for antiforce waves become After applying these new non dimensional variables,
Hemmati's (1999) new set of non-dimensional,
electron fluid dynamical equations for antiforce waves
become
 $\frac{d}{d\xi}[v\psi] = \kappa \mu v$ (9)
 $\frac{d}{d\xi}[v\psi(u-1) + \alpha v\theta] = \nu n - \kappa v(u-$

$$
\frac{d}{d\xi}[v\psi] = \kappa \mu v \tag{9}
$$

$$
\frac{a}{d\xi} \left[\nu \psi(\psi - 1) + \alpha \nu \theta \right] = \nu \eta - \kappa \nu (\psi - 1) \tag{10}
$$

$$
\frac{d}{d\xi}\bigg(v\psi(\psi-1)^2 + \alpha v\theta(5\psi-2) + \alpha v\psi\tag{11}
$$

$$
-\frac{5\alpha^2\nu\theta}{\kappa}\frac{d\theta}{d\xi}\bigg) = 2\nu\eta(\psi-1) - \omega\kappa\nu[3\alpha\theta + (\psi-1)^2]
$$

$$
\frac{d\eta}{d\xi} = -\frac{\nu}{\alpha}(\psi - 1) \tag{12}
$$

fluid dynamical equations to describe antiforce waves (return stroke in lightning) with a significant current behind the shock front. With ion number density, Ni, and ion velocity, Vi, behind the wave front, the current behind the wave front will be Hemmati et al. (2011) modified the set of electron nd the shock front. With ion number density, Ni,
ion velocity, Vi, behind the wave front, the current
nd the wave front will be
 $I_1 = eN_iV_i - env$ (13)
Ion velocity is considered to be almost equal to non dimensional variables,
set of non-dimensional,
uations for antiforce waves
= $\kappa \mu \nu$ (9)
= $\nu \eta - \kappa \nu (\psi - 1)$ (10)
+ $\alpha \nu \psi$ (11)
- 1) - $\omega \kappa \nu [3\alpha \theta + (\psi - 1)^2]$
 $-\frac{\nu}{\alpha} (\psi - 1)$ (12)
modified the set of electron
t a dimensional variables,

t of non-dimensional,

ions for antiforce waves
 $\kappa \mu \nu$ (9)
 $\nu \eta - \kappa \nu (\psi - 1)$ (10)
 $\alpha \nu \psi$ (11)
 $(\psi - 1)$ (12)
 $(\psi - 1)$ (12)

dified the set of electron

describe antiforce waves

tih a s fluid dynamical equations to describe antiforce waves
(return stroke in lightning) with a significant current
behind the shock front. With ion number density, Ni,
and ion velocity, Vi, behind the wave front, the current
b Hemmati et al. (2011) modified the set of electron

I dynamical equations to describe antiforce waves

urn stroke in lightning) with a significant current

nd the shock front. With ion number density, Ni,

ion velocity, V

$$
I_1 = eN_iV_i - env \tag{13}
$$

neutral particle speed $(V_i \cong V)$ due to lack of experimentally observed Doppler shift. No experimentally observed Doppler shift indicates that both the ions and neutral particles have insignificant speeds in the laboratory frame. Substituting V for V_i and solving for N_i from equation 13 yields experimentally observed Doppler shift indicat
both the ions and neutral particles have insign
speeds in the laboratory frame. Substituting V
and solving for N_i from equation 13 yields tron fluid dynamical equations for antifor

ome
 $\frac{d}{d\xi}[v\psi] = \kappa\mu v$
 $\frac{d}{d\xi}[v\psi(\psi - 1) + \alpha v\theta] = v\eta - \kappa v(\psi - 1)$
 $v\psi(\psi - 1)^2 + \alpha v\theta(5\psi - 2) + \alpha v\psi$
 $-\frac{5\alpha^2 v\theta d\theta}{\kappa d\xi} = 2v\eta(\psi - 1) - \omega\kappa v[3\alpha\theta + \frac{d\eta}{d\xi} = -\frac{v}{\alpha}$ Electron Shock Waves with a Large Current Behind the Shock

1975) simple change of variable

trate Hemmatic (1999) derived a

trate Hemmatic Constant Consideration is the

action of the antiformation constant and the valu it's (1999) new set of non-dimensional,

fluid dynamical equations for antiforce waves
 $\frac{d}{d\xi}[v\psi]=\kappa\mu\nu$ (9)
 $\frac{d}{d\xi}[v\psi(\psi-1)+\alpha\nu\theta]=\nu\eta-\kappa\nu(\psi-1)$ (10)
 $\nu=1)^2+\alpha\nu\theta(5\psi-2)+\alpha\nu\psi$ (11)
 $\frac{5\alpha^2\nu\theta d\theta}{\kappa}\bigg)$ experimentally observed Doppler shift. No
experimentally observed Doppler shift indicates that
both the ions and neutral particles have insignificant
speeds in the laboratory frame. Substituting V for V_i
and solving for

$$
N_i = \frac{I_1}{eV} + \frac{nv}{V} \tag{14}
$$

Substituting this into Poisson's equation, and applying the dimensionless variables for antiforce waves results in

$$
\frac{d\eta}{d\xi} = \frac{K I_1}{\varepsilon_0 K E_0} - \frac{\nu}{\alpha} \left(\Psi - 1 \right) \tag{15}
$$

reduced to

$$
\frac{d\eta}{d\xi} = \mathcal{K}t - \frac{\nu}{\alpha}(\Psi - 1) \tag{16}
$$

Where $t = \frac{t_1}{t_1}$ represents the dimensionless current. The current values behind the shock front in lightning return stroke are generally in the range of 5 to 30 kA. Using a current value of I_1 = stroke, the elastic collision frequency, K, values from stroke, from (McDaniel, 1964), and also the values of ε_0 , and E_0 one can estimate the value of ι to be of the order of 1. rn stroke are generally in the range of 5 to 30 kA.

lug a current value of I_1 = 10kA for lighting return

ke, the elastic collision frequency, K, values from

Daniel, 1964), and also the values of ε_0 , and E_0 ,
 return stroke are generally in the range of 5 to 30 kA.
Using a current value of I_1 = 10kA for lighting return
stroke, the elastic collision frequency, K, values from
(McDaniel, 1964), and also the values of ε_0 , an current values behind the shock front in lightning
in stroke are generally in the range of 5 to 30 kA.
g a current value of $I_1 = 10kA$ for lighting return
i.e, the elastic collision frequency, K, values from
Daniel, 1964 e are generally in the range of 5 to 30 l
trent value of I_1 = 10kA for lighting ret
elastic collision frequency, K, values fr $rac{a_{\eta}}{d\xi} = \frac{\ln 1}{\varepsilon_0 \text{KE}_0} - \frac{v}{\alpha} (\Psi - 1)$ (15)

n's equation is then reduced to
 $\frac{d\eta}{d\xi} = Kt - \frac{v}{\alpha} (\Psi - 1)$ (16)
 $t = \frac{I_1}{\varepsilon_0 \text{KE}_0}$ represents the dimensionless current.

rent values behind the shock

substituting it into the equation of conservation of energy for antiforce waves, equation (11), produces the final form of the equation of conservation of energy for antiforce waves with a large current behind the wave front. This completes the final form of the set of el electron fluid dynamical equations describing antiforce waves with a large current behind the shock front Waves with a Large Current Behind the Shock Front

mage of variable

ge of variable

ge of variable

ge of variable

for the antiforce

Poisson's equation is then reduced to
 $\mu = \frac{\beta}{K}$,
 $\frac{d\tau}{d\xi} = K - \frac{u}{\alpha}(\psi - 1)$ return stroke are generally in the range of 5 to 30 kA.
Using a current value of $l_1 = 10kA$ for lighting return
stroke, the elastic collision frequency, K, values from
(McDaniel, 1964), and also the values of ε_0 , an **In the Shock Front**
 $\frac{d\eta}{d\xi} = \frac{K_1}{\epsilon_0 K E_0} - \frac{v}{\alpha} (\Psi - 1)$ (15)

uation is then reduced to
 $\frac{d\eta}{d\xi} = Kt - \frac{v}{\alpha} (\Psi - 1)$ (16)
 $\frac{1}{\epsilon_0 K E_0}$ represents the dimensionless current.

values behind the shock fr **the Shock F**₁
 $\frac{d\eta}{d\xi} = \frac{K I_1}{\varepsilon_0 K E_0}$
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ge current behind the shock front
 $\frac{d}{d\xi} [v\psi] = \kappa \mu v$ (17)
 $-1 + \alpha v$ **6** quation is then reduced to
 $rac{d\eta}{d\xi} = Kt - \frac{\nu}{\alpha}(\psi - 1)$ (16)
 $\frac{1}{\epsilon_0 K E_0}$ represents the dimensionless current.

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There $t = \frac{1}{\epsilon_0 K E_0}$ represents the dimensionless current.

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964), and als Using a current value of $I_1 = 10kA$ for lighting return
stroke, the elastic collision frequency, K, values from
the CM-Daniel, 1964), and also the values of ϵ_0 , and E_0
one can estimate the value of t to be of the Where $t = \frac{t_1}{\epsilon_0 k E_0}$ represents the dimensionless current.
The current values behind the shock front in lightning
return stroke are generally in the range of 5 to 30 kA.
Using a current value of I_1 = 10kA for ligh $rac{u_f}{d\xi} = \frac{v_{\text{H}}}{v_{\text{B}}/\text{E}_0} - \frac{v}{\alpha} (\Psi - 1)$ (15)

"s equation is then reduced to
 $\frac{d\eta}{d\xi} = Kt - \frac{v}{\alpha} (\Psi - 1)$ (16)
 $= \frac{1}{\alpha_0 k v_{\text{B}}}$ represents the dimensionless current.

rent values behind the shock fro lving for $v(\Psi-1)$ from equation (16) and

iting it into the equation of conservation of

for antiforce waves, equation (11), produces the

prm of the equation of conservation of energy for

ce waves with a large current ent values behind the shock front in lion
concerner value of $I_1 = 10kA$ for the current value of $I_2 = 10kA$ for lighting
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$$
\frac{d}{d\xi}[v\psi] = \kappa \mu v \tag{17}
$$

$$
\frac{d}{d\xi}\left[\nu\psi(\psi-1)+\alpha\nu\theta\right] = \nu\eta - \kappa\nu(\psi-1) \tag{18}
$$

waves with a large current behind the shock front
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$$
\frac{d}{d\xi}[v\psi] = \kappa \mu v
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\frac{d}{d\xi}[v\psi(\psi - 1) + \alpha v\theta] = v\eta - \kappa v(\psi - 1)
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\n(18)
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$$
\frac{d}{d\xi}\left(v\psi(\psi - 1)^2 + \alpha v\theta(5\psi - 2) + \alpha v\psi\right)
$$
\n
$$
-\frac{5\alpha^2 v\theta}{\kappa}\frac{d\theta}{d\xi} + \alpha \eta^2 = 2\eta \kappa (\alpha - \omega \kappa v[3\alpha\theta + (\psi - 1)^2]
$$
\n
$$
\frac{d\eta}{d\xi} = \kappa_1 - \frac{v}{\alpha}(\psi - 1)
$$
\n(20)
\nTo solve the set of electron fluid dynamical

$$
\frac{d\eta}{d\xi} = \kappa_1 - \frac{v}{\alpha}(\psi - 1) \tag{20}
$$

equations for antiforce waves with a large current behind the shock front, Hemmati et al. (2015) had to equations for antiforce waves with a large current
behind the shock front, Hemmati et al. (2015) had to
modify the initial condition on electron temperature as well. They used the all particle (global) momentum equation to find the shock condition on electron well. They used the all particle (global) momentum
equation to find the shock condition on electron
temperature, and in dimensionless form, the electron temperature at the shock front becomes for antiforce waves, equation (11), produces the
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flhis completes the final form of the set of
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ati et al. (2015) had to
electron temperature as
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ecomes
 $\frac{1}{2}$ - $\frac{Kt}{2}$ (21)

$$
\theta_1 = \frac{\Psi_{1(1-\Psi_1)}}{\alpha} - \frac{\kappa}{\nu_1}.
$$
 (21)

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Results and Discussion

to obtain solutions for our complete set of electron fluid dynamical equations through the dynamical transition (sheath) region of the wave. For a specific wave speed, α , and dimensionless current, ι , a set of values for wave constant, K, electron number density, v_1 , and electron velocity, Ψ_1 , at the wave front were chosen, and in integration of the set of equations through the sheath region of the wave, those values were systematically changed until the integration of the set of electron fluid dynamical equations through the sheath region of the wave resulted in a successful conclusion. Meaning that, at the conclusion integration of the set of equations, our set of electron fluid dynamical equations for higher wave speed values, meaning for small α values, is not our set of electron fluid dynamical equations for higher
wave speed values, meaning for small α values, is not
very challenging; therefore, we intended to find solutions for lower ranges of wave speed values. For a certain wave speed value, α , integration of the set of electron fluid dynamical equations through the sheath region of the wave for small dimensionless current values, ι, also is relatively straight forward; however, as the dimensionless current value increases, the sheath thickness increases as well and the integration of the set of equations through the sheath region becomes region of the wave for small dimensionless current
values, *u*, also is relatively straight forward; however,
as the dimensionless current value increases, the sheath
thickness increases as well and the integration of the
 wave speed value, we intended to find the largest current value for which integration of the set of electron fluid dynamical equations through the sheath region of the wave became possible. For four wave speed values shown below and for the largest dimensionless current values for which integration of current value for which integration of the set of
electron fluid dynamical equations through the sheath
region of the wave became possible. For four wave
speed values shown below and for the largest
dimensionless current v the sheath region of the wave, for respective wave speeds became possible, the following set of initial boundary values and wave constants had to be employed. A trial and error technique of integration was used rough the sheath region of the wave, those values
ere systematically changed until the integration of the
of electron fluid dynamical equations through the
nectah region of the wave resulted in a successful
noclusion. Mea fluid dynamical equations through the dynamical
transition (sheath) region of the wave. For a specific
wave speed, α , and dimensionless current, t , a est of
values for wave constant, K , electron number
density, $v_$ electron fluid dynamical equations through the
region of the wave resulted in a successful
ion. Meaning that, at the conclusion of the
ion of the set of equations, $\Psi_2 \rightarrow 1$, and,
, at the trailing edge of the wave. Inte is increases as well and the integration c
equations through the sheath region bec
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is for lower density, v_1 , and electron velocity, Ψ_1 , at the wave front
were chosen, and in integration of the set of equations
through the sheath region of the wave, those values
were systematically changed until the integratio **EXEREM SATE ACCONOMIST CONSULTER AND CONSULTS (CONSULTS) CONSULTS** ($\psi_1 = 1$, $\psi_2 = 0$, $\psi_3 = 0$, $\psi_4 = 0.04$ ($\psi_5 = 0$) $\psi_6 = 0.04$ ($\psi_7 = 0$) $\psi_8 = 0$, $\psi_9 = 0$, trial and error technique of integration was used
tain solutions for our complete set of electron
dynamical equations through the dynamical
ion (sheath) region of the wave. For a specific
speed, α , and dimensionless cu A trial and error technique of integration was used
to obtain solutions for our complete set of electron
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wave speed, were chosen, and in integration of the set of equations
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or wave constant, K , electron
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A trial and crror technique of integration was used

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Meaning that, at the conclusion of the
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Discussion

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Discussion**

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Unitated equations at the continuing for the wore. For a specific

ties the set of equations through the sheath region becomes
more involved and time consuming. For a specific
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Results and Discussion

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for our complete set of electron** $\frac{W}{2} \approx 1$ **,** $W_2 \approx 0$ **).

region**

The following figures represent the wave profile fo antiforce waves with a significant current behind the shock front. Figures 1 through 3 show that solutions to the set of electron fluid dynamical equations within the

boundary conditions at the trailing $(\Psi_2 \rightarrow 1,$ wave all have met the required
at the trailing edge of the wave ondary conditions at the trailing edge of the \rightarrow 1, $\eta_2 \rightarrow$ 0).
Figure 1 shows dimensionless electric field, a hand of the wave all have met the required
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shows dimensionless electric field, η, as a

limensionless electron velocity, ψ, within

function of dimensionless electron velocity, ψ , within the sheath region of the wave for four values of wave speed, α , and respective dimensionless current values, ι. ndary conditions at the trailing edge of the wave \rightarrow 1, $\eta_2 \rightarrow 0$).
Figure 1 shows dimensionless electric field, η , as a tion of dimensionless electron velocity, ψ , within sheath region of the wave for four value

Figure 1. Electric field, η , as a function of electron velocity, ψ , for four dimensionless wave speed values, α dimensionless current values, *i*, of 1, 0.5; 0.25, 1.0; 0.05, 2.0 and 0.005, 5.0 within the sheath region of the wave.

function of dimensionless position, ξ, within the sheath region of the wave for four wave speed values, α , and region of the wave for four wave speed v
respective dimensionless current values, ι. Figure 2 shows dimensionless electric field, η , as a

Figure 2. Dimensionless electric field, η , as a function of dimensionless position, ξ , for four wave speed values, α , and respective dimensionless current values, ι , of 1, 0.5; 0.25, 1.0; 0.05, dimensionless position, ξ , for four wave speed values, α , and respective dimensionless current values, ι , of 1 , 0.5; 0.25, 1.0; 0.05, 2.0 and 0.005, 5.0 within the sheath region of the wave.

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as a function of dimensionless position within the sheath region of the wave for four wave speed values, sheath region of the wave for four wave speed values, α, and respective dimensionless current values, *u*. Figure 3 shows dimensionless electron velocity,

Figure 3. Dimensionless electron velocity, ψ , as a function of dimensionless position, ξ, for four wave speed values, respective dimensionless current values, 2.0 and 0.005 , 5.0 within the sheath region of the wave. wave speed
s, *u*, of 1, 0.5;
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density, *v*, as a function of dimensionless position, ξ, within the sheath region of the wave for four wave speed values, α , and respective dimensionless current values, ι. Figure 4 shows dimensionless electron number electron velocity, ψ
for four wave spe
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Figure 4. Dimensionless electron number density, *υ*, as a function of dimensionless position, ξ, for four dimensionless wave speed values, α , and respective dimensionless current values, ι , of 1, 0.5; 0.25, 1.0; 0.05, 2.0 and 0.005, 5.0 within the sheath region of the sheath wave.

ξ, within the sheath region of the wave for fou speed values, α , and respective dimensionless values, graphed with a logarithmic scale on the y axis. α =0.005 represents a relatively fast wave speed value of temperature, θ , as a function of dimensionless position,
 ξ , within the sheath region of the wave for four wave

speed values, α , and respective dimensionless values, μ ,

graphed with a logarithmic scale on th value of $3x10^6$ m/s. Short thickness, ξ , value of, 0.56 represents an actual sheath thickness of 5.6 mm Figure 5 shows temperature, θ , as a function of dimensionless position.

Figure 5. Dimensionless electron temperature, dimensionless position, ξ, for four wave speed values, respective dimensionless current values, μ , of 1, 0.5; 0.25, 1.0; 0.05, 2.0 and 0.005, 5.0 within the sheath region of the wave. 0.05 , 2.0 and 0.005 , 5.0 within the sheath region of the wave.

a relationship between wave speed values and peak peak current values in lightning return strokes. For ins For instance, Wagner (1963) has suggested that as the lightning return stroke wave speed increases, it can support larger peak current values; but others, notably Mach Mach and Rust (1989) disagree, claiming a lack of correlation between return stroke propagation spee and peak current. Our solutions indicate that a relationship does exist, as the lightning return stroke speed increases, it can support higher peak current values. Researchers have debated the possible existence of a lack of
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ugner (1963) has suggested that as the ligh
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reported minimum wave speeds for lightning stroke typically to be in the order of $10⁷$ m/s. However, as we indicated above, we have been able to integrate
our set of electron fluid dynamical equations through our set of electron fluid dynamical equations through the sheath region of the wave for lightning return reported minimum wave speeds for lightning return
stroke typically to be in the order of 10^7 m/s. However,
as we indicated above, we have been able to integrate
our set of electron fluid dynamical equations through
the predicts that antiforce waves with wave speeds below that of those experimentally measured can be detected. Investigators, for example, Rakov (2007), have typically to be in the order of 10^7 m/s. However, indicated above, we have been able to integrate

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Conclusions

Our modified set of electron fluid dynamical equations for antiforce waves with a large current behind the wave front, with modified electron temperature at the shock front, have been utilized in our integration of the set of electron fluid dynamical equations through the sheath region of the wave. Our solutions for several wave speed values, with maximum currents possible for the selected wave speeds, all meet the expected physical conditions at the trailing edge of the dynamical transition region of the wave. This indicates validity of our modified set of electron fluid dynamical equations and the extent and possible range of wave speed values and currents for lightning return strokes. Our solutions indicate that lightning return stroke speeds lower than the ranges reported by the majority of experimentalists are also possible. Our solutions also indicate, for lightning return stroke, as the wave speed increases, it can support larger currents behind the shock front. This means that in a lightning return stroke, a relationship between the wave speed values and peak currents exists.

Acknowledgements

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