7-2015

An Adaptive Large Neighborhood Search Heuristic for the Inventory Routing Problem with Time Windows

Mina Hadianniasar
University of Arkansas, Fayetteville

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Inventory Routing Problem with Time Windows

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science in Industrial Engineering

By

Mina Hadianniasar
Amirkabir University of Technology
Bachelor of Science in Industrial Engineering, 2012

July 2015
University of Arkansas

This thesis is approved for recommendation to the Graduate Council

____________________________________-

Dr. Ashlea Bennett Milburn
Thesis Director

____________________________________-

Dr. John A. White
Committee Member

____________________________________-

Dr. Justin R. Chimka
Committee Member

Abstract

This research addresses an integrated distribution and inventory control problem which is faced by a large retail chain in the United States. In their current distribution network, a direct shipping policy is used to keep stores stocked with products. The shipping policy specifies that a dedicated trailer should be sent from the warehouse to a store when the trailer is full or after five business days, whichever comes first. Stores can only receive deliveries during a window of time (6 am to 6 pm). The retail chain is seeking more efficient alternatives to this policy, as measured by total transportation, inventory holding and lost sales costs. More specifically, the goal of this research is to determine the optimal timing and magnitudes of deliveries to stores across a planning horizon. While dedicated shipments to stores will be allowed under the optimal policy, options that combine deliveries for multiple stores into a single route should also be considered. This problem is modeled as an Inventory Routing Problem with time window constraints. Due to the complexity and size of this NP-hard combinatorial optimization problem, an adaptive large neighborhood search heuristic is developed to obtain solutions. Results are provided for a realistic set of test instances.

Acknowledgments

I would like to express my deepest gratitude to my advisor, Dr. Ashlea Bennett Milburn, for her excellent guidance, caring, and patience. She provided me with an excellent atmosphere for doing my research. I have been amazingly fortunate to have an advisor who gave me the freedom to explore on my own, and at the same time the guidance to recover when my steps faltered. I would also like to thank my committee members, Dr. John A. White and Dr. Justin R. Chimka for their time, interest, helpful comments, and insightful questions.

Contents

List of Figures

List of Tables

1 Introduction and Motivation

As companies have grown, so have their needs for a more comprehensive and accurate system for monitoring, controlling and analyzing the storage and flow of finished products between warehouses and customers. This is in order to ensure that the right product is available to the right customer at the right time with the right quality and quantity (Simchi-Levi et al., 1999). Logistics management is known to be the heart of many economic decisions in industry, creating value for customers through product availability and customer service (Campbell et al., 1998).

According to the Council of Supply Chain Management Professionals (CSCMP), the U.S. expenditures on logistics were 1.39 trillion USD in 2013, representing 8.2% of gross domestic product (GDP) (https://cscmp.org/media-center/facts-global-supply-chain). Moreover, the U.S. Census Bureau News 2013 reported revenue for transportation and warehousing in the second quarter of 2013 of approximately $204.3 billion, representing a more than 7% increase over the first quarter of 2013 (http://www.census.gov/newsroom/press-releases/2014/cb14-205.html).

Many companies today are struggling with logistics management, trying to understand the relationship between the stock levels at their customers and the frequency with which customers should receive orders (http://www.ftpress.com/articles/article.aspx?p=2263501). As depicted in Figure 1, transportation, warehousing, and inventory carrying comprise the majority of logistical expenses. It has been recognized that the transportation and inventory management components of logistics systems are not completely independent. In fact, decisions in one area can affect the performance of the other. Thus, collaboration between the distribution and inventory control functions is considered a key issue in logistics management (Campbell et al., 1998).

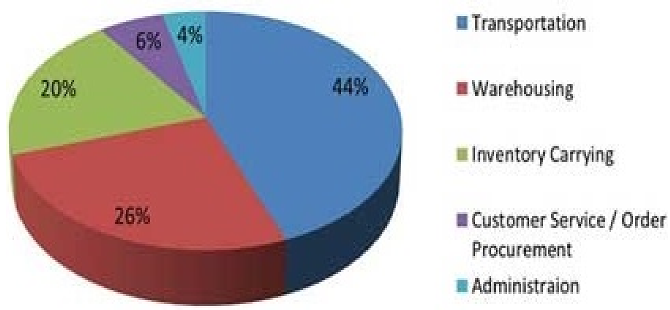


Figure 1: Logistical Expense Breakdown
Source: adopted from http://www.westpak.com/page/supply-chain

Vendor Managed Inventory (VMI) is an example of a supply chain management strategy that reduces logistics costs and adds value to the system (Coelho et al., 2012a). It is a replenishment policy in which suppliers monitor and analyze the inventory level at their customers and determine the timing and quantities of deliveries to customers. In the traditional retailer managed inventory (RMI), customers are responsible for monitoring and replenishing their own inventory levels. Upon receiving orders from customers, the supplier prepares and makes deliveries using a fleet of trailers. In contrast, using VMI, a distributor is responsible for making integrated distribution and inventory decisions using customer demand information. Specifically, the distributor keeps track of inventory levels at the customers and specifies which customers must be replenished at which times and with how much inventory (see Figure 2). VMI has been referred to as a win-win situation, because it creates value for both suppliers and customers. It enables suppliers to reduce their distribution costs by better coordinating deliveries to customers. Also, customers do not have to devote any resources to inventory management (Campbell et al., 1998). This replenishment policy can be modeled using a combinatorial inventory control and routing model referred to as the Inventory Routing Problem (IRP). The complexity of solving this NP-hard problem, especially for large scale networks, is known as a drawback of this replenishment policy (Coelho et al., 2012b).

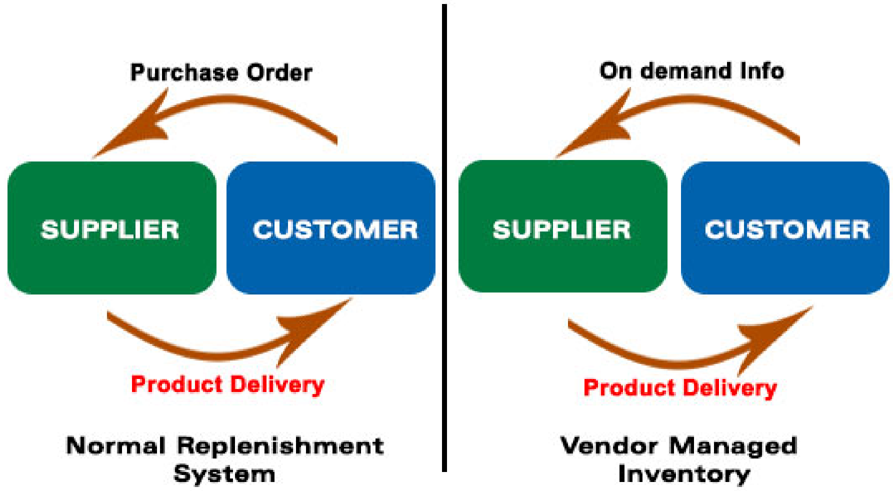


Figure 2: Vendor Managed Inventory
Source: adopted from http://www.tanktel.com/vmi.html

One of the reasons for the complexity of IRPs is the interaction between distribution and inventory management costs. As shipment frequency increases, transportation costs increase. However, inventory levels are lowered, reducing inventory holding costs. On the other hand, transportation costs are reduced by decreasing the frequency of shipments. But, inventory holding costs at stores are increased through delivering more inventory to stores at each visit. Therefore, a trade-off between transportation costs and inventory holding costs exists (Figure 3). In addition, lost sales costs can be reduced by delivering larger quantities at each visit or increasing the frequency of shipments, but doing so could increase transportation and inventory holding costs (Figure 4). Such trade-offs add to the complexity of determining the frequency of visits and quantities of deliveries to stores.

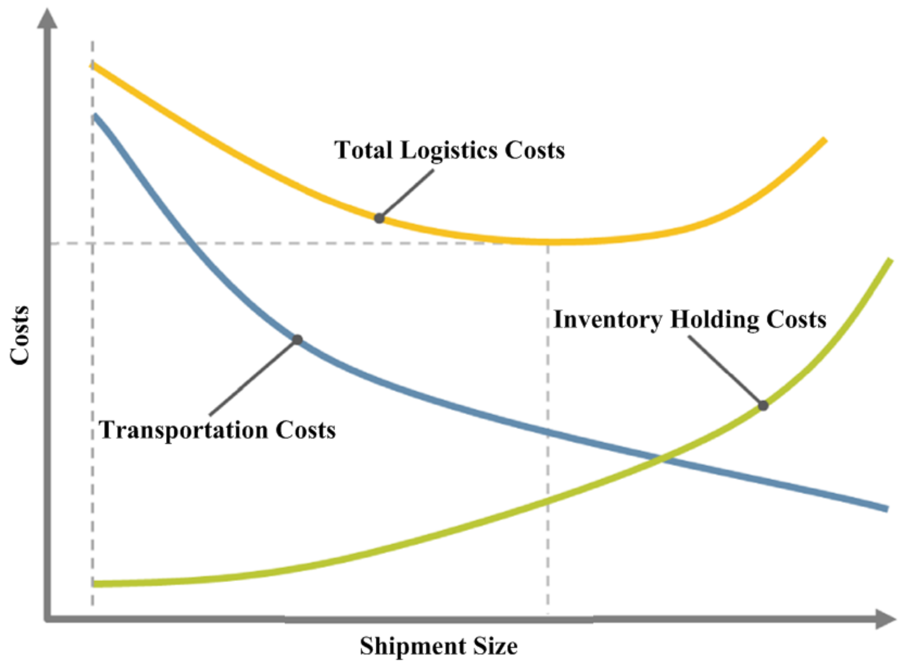


Figure 3: Total Logistics Costs Trade-off
Source: McKinnon, A. ”The Effects of Transport Investment on Logistical Efficiency”, Logistics Research Centre, Heriot-Watt University, Edinburgh, UK.

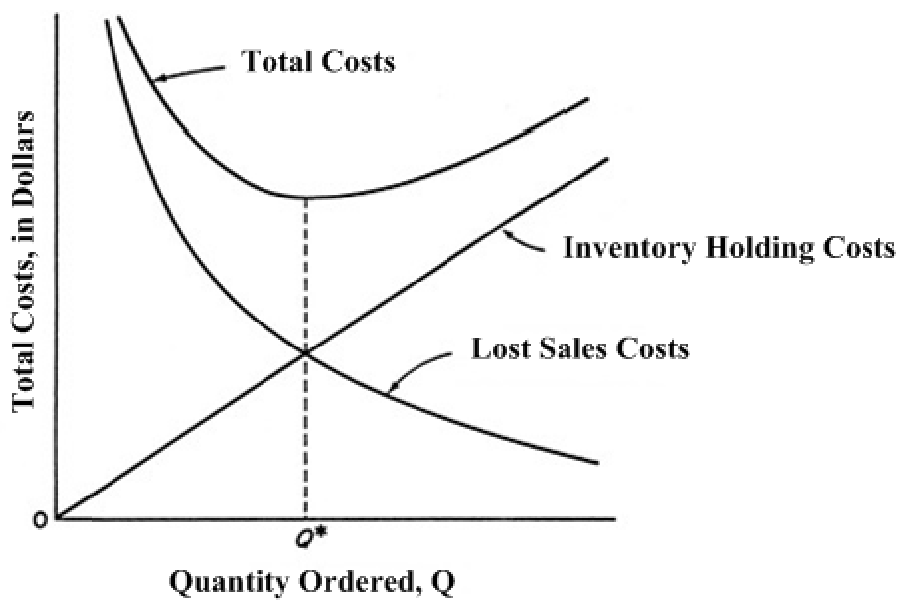


Figure 4: Basic Inventory Cost Trade-offs
Source: United States Department of Transportation - Federal Highway Administration

One of the companies that is facing a challenge in its logistics operations is a large department store chain in the United States operating approximately 300 stores in 29 states. With approximately 30,000 employees and annual sales of more than $6.5 billion, it ranks among the largest chains in the nation offering a variety of merchandise from cosmetics and high fashion apparel to home goods.

This retail chain is facing a problem for determining how often each store should be visited (frequency of visit) and how much inventory (quantity of delivery) should be delivered to each store during a visit. Currently, trailers (shipments) are sent to stores only when they are full or after five business days (whichever comes first). This replenishment policy focuses on maximizing trailer capacity utilization without explicitly considering the impact on inventory carrying costs and lost sales. Specifically, each trailer is dedicated to a single store. Stores are served independently and in separate routes starting and ending at the depot. Thus, routes containing more than one store are not allowed. This policy is referred to as Direct Shipping (DS) policy (Bertazzi and Speranza, 2012). The retail chain is seeking more efficient alternatives to this policy. Specifically, balancing freight costs, inventory holding costs, and lost sales costs is the primary goal of this research.

The following three customers example explains a situation where a direct shipping policy is not an optimal strategy for store replenishment. Distances between locations and daily demands of stores across a three-day planning horizon are given in Tables 1 and 2, respectively. An unlimited number of trailers each with a capacity of 45 units, fixed cost of $100 and variable cost of $1 per mile are available to make deliveries. The inventory holding rate is $0.007 per unit of inventory held per day for each customer. Finally, lost sales are not allowed, for simplicity of exposition.

Two solutions are developed for this instance of IRP. One is developed using a direct shipping

Table 1: Distance

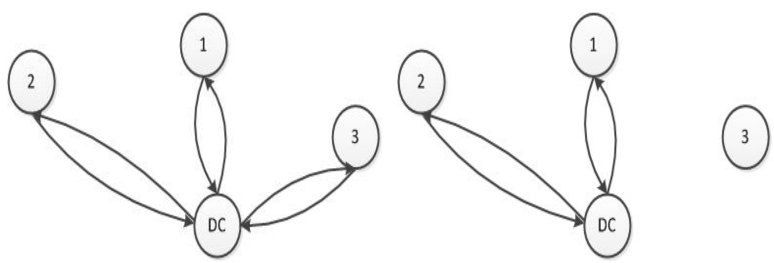
Location	DC	1	2	3
DC	0	20	35	15
1	20	0	10	20
2	35	10	0	25
3	15	20	25	0

Table 2: Demands

Stores \days	1	2	3
1	15	12	20
2	25	20	28
3	10	18	14

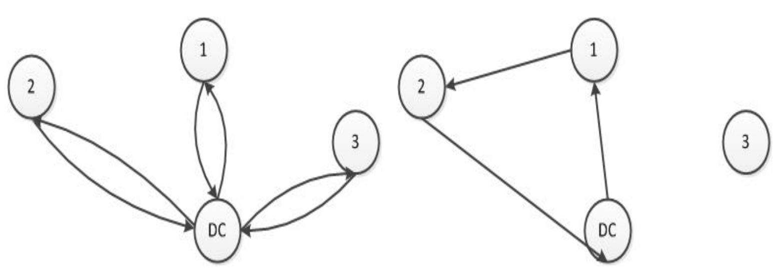
policy, and the other is developed following a policy that allows for combining stores on routes. The results are depicted in Figures 5 and 6 and summarized in Table 3. Under both policies, all three stores receive direct shipments from the DC on day one. No stores receive shipments on day two under either policy. However, differences in the solutions can be observed on day three. Under the direct shipping strategy, stores one and two both receive direct shipments. Under the alternative policy, these stores are combined in a route. By using the shared trailer, total costs are reduced by 19.33%.

In summary, this research project seeks alternative replenishment policies for the retail chain



(a) Day 1 (b) Day 3

Figure 5: Results of DS Policy (Cost = $750.77)



(a) Day 1 (b) Day 3

Figure 6: Results of Optimal Solution (Cost = $605.59)

Table 3: Results for Three Customers Example

Day	Direct Shipping		Optimal	
	Transportation	Inventory Holding	Transportation	Inventory
1	440	0.595	440	0.469
2	0	0.175	0	0.119
3	310	0	165	0
Total	750	0.77	605	0.588

in question. While dedicated shipments to stores are permitted under the new policy, options that combine deliveries for multiple stores into a single route are also considered. The objective of this research is to determine the best delivery schedule for the retail chain taking into account the need to balance freight costs, the costs of holding inventory in the store, and the lost sales costs that occur when there are stock-outs (stock-outs occur when the demand for products exceeds the inventory in the store available for sale). This problem is modeled as an Inventory Routing Problem (IRP) variant, where decisions regarding the timing and quantities of deliveries to stores must be made, as well as vehicle routing (customer delivery sequencing) decisions. This thesis develops models and solution approaches capable of answering the following questions for a vendor-managed logistics network:

- How often should each store/customer receive deliveries?
- How much inventory should be delivered to each store/customer in each visit?
- What delivery routes should be used?

This thesis is structured as follows. First, a formal problem statement and mathematical formulation of the problem are provided in Section 2. Then, a review of the IRP literature along with discussion of the proposed methods and research outcomes for solving IRP in the literature are presented in Section 3. Section 4 includes a description of the solution methodology introduced in this thesis. The results of the developed methodology validation are discussed in Section 5. The results of a computational study are given in Section 6. Finally, Section 7 presents ideas for future research.

2 Problem Statement and Formulation

The notation that is used in the mathematical formulation of the integrated distribution and inventory problem is defined in the following.

There are a set of $\mathcal{N}$ nodes representing the locations of retail stores and a set of $\mathcal{T}$ days in a planning horizon. For each retail store $i \in \mathcal{N}$, the daily demand magnitude u_{it} is known, measured in units of sales per day $t \in \mathcal{T}$. Parameter D_i, the total demand of store i over the planning horizon, is defined as $\sum_{t \in \mathcal{T}} u_{it}, \forall i \in \mathcal{N}$. Also, associated with each retail store i is a lost sales cost per unit of stock-out inventory per day p_i. A depot located at node 0 represents the distribution center (DC) where goods are held prior to delivery to the retail store. There is an inventory holding cost, h, measured in dollars per unit of inventory held per day for all retail stores $i \in \mathcal{N}$. A homogeneous fleet of trailers stationed at the DC, each having capacity Q measured in units of inventory and fixed cost r measured in dollars. An unlimited number of trailers are assumed to be available. Deliveries to stores must be made during an allowable delivery window $[a, b]$, where a is the earliest time a trailer can begin unloading at a retail store and b is the latest time the trailer can finish unloading. The proposed inventory routing problem is defined on a complete network denoted by the graph $G = (N_0, \mathcal{A})$, where N_0 represents all retail stores in $\mathcal{N}$ and the DC. The set $\mathcal{A}$ includes arcs (i, j) connecting all nodes in N_0 with nonnegative travel distances and travel times, c_{ij} and f_{ij}, respectively. Finally, m represents the cost per mile of traveling arc $(i, j) \in \mathcal{A}$.

The problem is to determine a delivery schedule that specifies on which days each store should receive deliveries and the quantities that should be delivered each time. Additionally, a set of trailer routes capable of satisfying the established delivery schedule must be designed. In order for a route to be feasible, it should begin and end at the distribution center and the total demand loaded onto the trailer should not exceed Q. The objective of this problem is to minimize the total transportation costs, inventory holding costs and lost sales costs associated with the selected delivery schedule and set of trailer routes.

The mixed integer programming (MIP) representation of this IRP variant is defined next.

2.1 MIP Model

Let X_{ijt} be a binary variable that is equal to 1 if arc (i,j) is traversed in period t and zero otherwise. Also, let q_{ijt} denote the delivery quantity on arc (i,j) in period t. Decision variable d_{it} represents the delivery quantity to retailer i in period t. The continuous variable s_{it} represents the start time of service at retailer i in period t. Finally, I^+_{it} and I^-_{it} denote the inventory level of retailer i at the end of period t, if positive and negative, respectively.

$$min \sum_{i\in N_0}\sum_{j\in N_0}\sum_{t\in\mathcal{T}} m\times c_{ij}\times x_{ijt} + \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} r\times x_{0it} + \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} h\times I^+_{it} + \sum_{i\in\mathcal{N}}\sum_{t\in\mathcal{T}} p_i\times I^-_{it} \tag{1}$$

Subject to

$$\sum_{t\in\mathcal{T}} d_{it} \leq D_i \quad \forall i\in\mathcal{N} \tag{2}$$

$$q_{0jt} \leq Q \quad \forall j\in\mathcal{N} \quad ,\forall t\in\mathcal{T} \tag{3}$$

$$\sum_{j\in FS(i)} q_{ijt} - \sum_{j\in RS(i)} q_{jit} = -d_{it} \quad \forall i\in\mathcal{N} \quad \forall t\in\mathcal{T} \tag{4}$$

$$q_{ijt} \leq M\times x_{ijt} \quad \forall i,j\in N_0 \quad ,\forall t\in\mathcal{T} \tag{5}$$

$$\sum_{i\in N_0} x_{ijt} = \sum_{i\in N_0} x_{jit} \quad j\in\mathcal{N} \quad ,\forall t\in\mathcal{T} \tag{6}$$

$$\sum_{j\in N_0} x_{ijt} \leq 1 \quad i\in\mathcal{N} \quad ,\forall t\in\mathcal{T} \tag{7}$$

$$I^+_{it} - I^-_{it} = I^+_{i,t-1} + d_{it} - u_{it} \quad \forall i\in\mathcal{N} \quad ,\forall t\in\mathcal{T} \tag{8}$$

$$s_{0t} = 0 \quad \forall t\in\mathcal{T} \tag{9}$$

$$s_{it} + x_{ijt}\times f_{ij} - s_{jt} \leq (1-x_{ijt})\times M \quad \forall i\in N_0 \quad ,\forall j\in\mathcal{N} \quad ,\forall t\in\mathcal{T} \tag{10}$$

$$s_{it} + x_{ijt}\times f_{ij} - s_{jt} \geq (x_{ijt}-1)\times M \quad \forall i\in N_0 \quad ,\forall j\in\mathcal{N} \quad ,\forall t\in\mathcal{T} \tag{11}$$

$$e \leq s_{it} \leq l \quad \forall i\in\mathcal{N} \quad ,\forall t\in\mathcal{T} \tag{12}$$

$$x_{ijt} \in \{0,1\} \quad \forall i,j\in\mathcal{N} \quad ,\forall t\in\mathcal{T} \tag{13}$$

$$d_{it} \geq 0 \quad \forall i\in\mathcal{N} \quad ,t\in\mathcal{T} \tag{14}$$

$$q_{it} \geq 0 \quad \forall i \in \mathcal{N} \quad, t \in \mathcal{T} \tag{15}$$

$$I_{it}^{+} \geq 0 \quad \forall i \in \mathcal{N} \quad, t \in \mathcal{T} \tag{16}$$

$$I_{it}^{-} \geq 0 \quad \forall i \in \mathcal{N} \quad, t \in \mathcal{T} \tag{17}$$

$$s_{it} \geq 0 \quad \forall i \in \mathcal{N} \quad, t \in \mathcal{T} \tag{18}$$

The objective function in Equation (1) minimizes the total transportation, inventory holding and lost sales costs. Constraint set (2) ensures combined delivery quantities to each retail store do not exceed the total demand of that store over the entire planning horizon. Constraint set (3) ensures trailer capacity is never violated for any trailer.

Constraint set (4) describes the mass balance constraints for the retail stores. They ensure that in each period t, the total delivery quantity emanating from the retailer i minus the total delivery quantity entering i is equal to the magnitude of delivery to i at t. Constraint set (5) relates the route-oriented variables, x_{ijt}, to the delivery quantity variables q_{ijt}. More specifically, it ensures that an inventory flow is carried on arc (i, j) in period t only if the arc (i, j) is traversed by a trailer in that period. If arc (i, j) is not being traversed by any trailer in period t, its inventory flow in the same period must be zero.

Constraint set (6) imposes that if an arc enters retailer i in period t, it should leave i in the same time period. Constraint set (7) ensures each retail store should be visited at most once during each time period. Constraint set (8) is an inventory balance equation. It determines the inventory level of retailer i at the end of period t by taking its inventory level at the end of period $t-1$, adding the sum of the quantities delivered to that retailer in period t, and subtracting its demand during period t. Constraint sets (9) - (12) calculate arrival times for each retail store visit and ensure those times are within the allowable windows. They also serve to eliminate subtours. Finally, constraint sets (13) - (18) define restrictions on the decision variables.

3 Literature Review

The Inventory Routing Problem (IRP) has been extensively studied within the context of supply chain and logistics. Applications of IRP are found in many industries, for example, the grocery and gasoline distribution industries (Gaur and Fisher (2004), Li et al. (2014)). Specific problem features encountered in each application influence the particular variant of IRP that is used to model the problem. For example, the objective may be to minimize the overall cost of the system or to minimize the route travel time (Li et al., 2014). The IRP is classified into different categories in the literature based on various factors including the main characteristics of the problem or the purpose of the study. For example, Coelho et al. (2013) classifies IRP based on the structural variants of the problem and the availability of demand information. In this study, the IRP literature is categorized along two dimensions: whether the time horizon is finite or infinite, and whether product consumption rate (demand) is deterministic or stochastic.

The combination of inventory management and distribution yields a long-term, possibly infinite horizon, problem in which decisions regarding the timing and magnitude of replenishments in different periods are not independent. Also, the actual objective is to minimize the long term costs of the system. Given this long-term nature, it is difficult, if not impossible, to model and solve the inventory routing problem. Therefore, many approaches in the literature reduce the long-term planning horizon to a short-term one and solve a simplified version of the problem by incorporating a finite time horizon (e.g., one week) in their models. With a finite time horizon, the specific situation of each problem determines the planning horizon (e.g., Gaur and Fisher (2004)); while with an infinite time horizon, the objective is to determine a long-term distribution plan and minimize the long-term costs (e.g., Bertazzi and Speranza (2012)).

In addition to time horizon, the usage rate of products (i.e., demand) can be taken into account as one of the most important components of the inventory routing problem. Deterministic demands occur when demand rates are stable and known with certainty (Ramtin et al. (2010), Ramtin and Pazour (2014), Ramtin and Pazour (2015)). However in most real-world cases, the consumption rate of products or demand at customers is not known with certainty. In this case, referred to as the stochastic inventory routing problem, random demand is described by a probability distribution representing uncertain factors.

3.1 Finite Time Horizon with Deterministic Demand

IRP variants with finite time horizons and deterministic demand are the focus of a number of papers in the literature (e.g., Miller (1987), Chien et al. (1989), Carter et al. (1995), Kim et al. (2000), Bertazzi et al. (2002), Campbell and Savelsbergh (2004), Al-Khayyal and Hwang (2007), Archetti et al. (2007), Savelsbergh and Song (2007), Savelsbergh and Song (2008), Yu et al. (2008), and Hemmelmayr et al. (2009)). For instance, Miller (1987) offers a network flow model for the inventory scheduling problem for a chemical corporation and is among the first to consider an IRP with a finite time horizon and deterministic demand. The problem presented is to determine the timing and quantities of deliveries that minimize a total cost function comprised of components such as voyage-related ship costs, in-transit inventory costs and terminal inventory costs. A constructive heuristic approach is used to obtain a feasible initial delivery schedule. Then, an interactive decision support system having both manual and automatic solution improvement procedures is applied.

Chien et al. (1989) introduces an integrated inventory allocation and vehicle routing problem with a short time horizon, known demand rates and a capacitated distribution center. The combinatorial optimization problem is modeled as a multi-commodity flow-based mixed integer program. The objective is to maximize total profit, defined as the revenue obtained from delivering inventory minus stock-out costs and fixed and variable transportation costs. Inventory holding cost is not considered. The multi-period problem is decomposed into a series of single period problems for simplicity. The periods are linked by modifying revenues of each period based on the penalty costs of the previous period. Finally, a Lagrangian relaxation for decomposing the problem into two subproblems and a Lagrangian based heuristic for constructing feasible solutions are proposed. Additional details regarding the decomposition are available in Geoffrion (1974) and Fisher (1981). Computational results indicate that good quality solutions are obtained.

Abdelmaguid and Dessouky (2006) formulates a deterministic IRP as a mixed integer program with a nonlinear function for transportation costs. Customers are assumed to have limited storage capacity and back orders on unsatisfied demands may occur. The complete version of the model is accessible in Abdelmaguid (2004). A genetic algorithm focusing on delivery schedule decision variables is developed. The routing subcomponent is handled via the Clarke-Wright Savings algorithm Clarke and Wright (1964). Computational results on test instances with 5, 10 and 15 customers over 5 and 7 day planning horizons indicate the proposed heuristic is capable of generating solutions that are within 20% of the optimal solution.

Herer and Levy (1997) considers the addition of a new concept to the IRP, resulting in a new problem termed the Metered IRP (MIRP). The new element in MIRP is that rather than paying for a delivery when it is made, customers pay for the inventory as they use it. Therefore, the supplier (not the customer) pays for the inventory that is held at the customer. Moreover, a probability of stocking out in customers with stochastic demands is considered in this study. A number of simplifying assumptions are made, for example, customers are assumed to have unlimited capacity and there are no time window limitations regarding the timing of deliveries. A solution procedure based on a modified Clarke-Wright algorithm is developed.

Savelsbergh and Song (2007) introduces a new version of the IRP named the Inventory Routing Problem with Continuous Moves (IRP-CM). This new problem is introduced in order to investigate conditions where availability of products are limited, customers can not be visited using out-and-back tours and delivery routes span several days. A minimum delivery quantity at customers is enforced and a greedy randomized adaptive search procedure (GRASP) is proposed (Feo and Resende, 1995). GRASP determines a delivery schedule for the entire planning horizon that specifies the timing and magnitudes of deliveries to customers as well as vehicle routes. Finally, a delivery volume optimization model is proposed to maximize the total volume of products delivered to customers over the planning horizon. In this optimization model, the timing and magnitudes of deliveries to customers are recalculated after removing the minimum delivery requirement. However, the vehicle routes are not changed, thus the total transportation costs remain the same. The proposed strategy is able to generate optimal or near optimal solutions for test instances including up to approximately 200 customers in a reasonable amount of time. Computational times for larger instances are higher.

Archetti et al. (2007) are among the first to implement an exact branch-and-cut algorithm to obtain an optimal solution for a class of distribution problems that follow a vendor-managed inventory policy. In this problem, a supplier must determine the delivery quantity for each customer using a policy named deterministic order-up-to level discussed in Bertazzi et al. (2002). In this policy, the quantity delivered to each customer has to be determined in such a way that the given maximum inventory level for each customer is not violated.

IRP-CM is further investigated in Savelsbergh and Song (2008) with more focus on developing optimization algorithms. A multi-period multi-commodity flow formulation is suggested along with a solution strategy for reducing the size of the problem, developing a set of valid inequalities, and building a branch and cut algorithm. More specifically, the quality of solutions obtained from the randomized greedy heuristic, GRASP, is improved by developing an optimization algorithm.

Hemmelmayr et al. (2009) studies the problem of managing blood product delivery for the American Red Cross. In the first presented solution approach, which is an Integer Programming approach, fixed delivery routes visiting all customers on each day are obtained using a vehicle routing algorithm. Then, these routes are passed to an integer program to decide which customers can be skipped each day without stock-outs occurring. Finally, a variable neighborhood search is suggested as an alternative and compared with the first method. Computational results reveal that the two methods have similar performance, but the IP approach requires more CPU time.

Moin et al. (2011) studies a many-to-one distribution network consisting of a warehouse, an assembly plant, and N suppliers over a finite time horizon with multi-periods and deterministic demands. There is a fleet of capacitated vehicles transporting products from the suppliers to satisfy the demand of the assembly plant for each period. A linear mathematical representation of this problem is suggested and is used to obtain lower and upper bounds. Finally, a hybrid genetic algorithm is used to solve the problem.

Qin et al. (2014) is one of the most recent studies about the periodic inventory routing problem over a finite planning horizon with a set of retailers having deterministic demands. The problem is decomposed into two sections: the inventory problem and the vehicle routing problem for which a local search method and a tabu search algorithm are proposed, respectively. The performance of the proposed solution approach is tested on ten data sets given in Zachariadis et al. (2009). Results demonstrate the proposed algorithm provides solutions that reduce the total costs of the system by 6.79%.

3.2 Finite Time Horizon with Stochastic Demand

Due to the complexity associated with considering uncertainty and obtaining probability distributions for product usage rates, stochastic demand has attracted less attention in the literature. This type of demand in the context of a finite time horizon is first introduced in Bell et al. (1983) where improving the distribution of air products to a set of customers is studied. Before formulating this problem as a mixed-integer linear program, a set of feasible routes are generated. Then, the model is solved using Real-Time Optimizer for Vehicle Routing (ROVER) to obtain a schedule determining the timing and quantities of deliveries at customers for a subset of generated routes. A feasible solution to the route selection model is obtained by developing a Lagrangian relaxation algorithm with the branch and bound framework. The Lagrangian algorithm determines routes to be included in the delivery schedule as an input for the MIP. Then the MIP is solved to determine the timing and magnitudes of deliveries to customers within the selected routes.

Federgruen and Zipkin (1984) studies the problem of simultaneously allocating and distributing a scarce resource among a set of customers. Demands are uncertain and are modeled using a cumulative distribution function. The objective function is comprised of stock-out costs for unsatisfied demands, inventory holding costs and transportation costs. A single period planning horizon is used. A nonlinear mixed integer programming model and an exact algorithm based on Bender's decomposition are proposed to solve the problem. This problem is extended in Federgruen et al. (1986) to allow for multiple types of perishable products. A new component, out of date costs, is introduced in the objective function of the extended problem. This component models the cost that must be paid for discarding unused products that have expired.

Dror and Ball (1987) develops a procedure for reducing the long term stochastic IRP to a short term problem. The set of customers to be replenished each day is selected based on fixed delivery, distribution and stock out costs. If a customer is not included in the replenishment plan and consequently experiences a stock-out on a given day, it will be served immediately by an emergency service with a high cost. The computational results for this problem are given in a companion paper Dror et al. (1985).

Campbell et al. (1998) studies two IRP variants, one with deterministic and one with stochastic demands. The possibility of customer stock-outs are only considered in the latter. The problem is analyzed for just one and two customers in order to develop insights for this version of the IRP. An integer programming approach based on the concept of clusters and a dynamic programming approach based on a discrete time Markov decision process (MDP) are proposed as solution methods for the deterministic and stochastic IRP variants, respectively. The need for a large amount of parameter tuning and significant computational time are listed as implementation challenges of the MDP.

Gaur and Fisher (2004) develops a system to solve a periodic inventory routing problem with a finite time horizon and stochastic demands in a supermarket. The problem is to determine the vehicle route and visiting time for each store with the objective of minimizing the transportation costs. This problem is similar to the IRP in the case study of this thesis, since it is a real problem (IRP at a supermarket chain) and time horizon is finite. The fixed partition policy is applied to solve this complex problem. Under this policy, the set of customers is divided into different clusters and for each, independent routes are created. Each cluster is visited and replenished separately from other clusters. It is concluded that annual distribution costs are decreased by 4% by implementing the proposed solution methodology in the supermarket chain.

In a complex distribution optimization problem which is faced by a Norwegian production company discussed in Dauzère-Pérès et al. (2007), customers' demands are time varying. This inventory routing problem is formulated as a mixed-integer program with a nonlinear objective function to determine both shipment quantities and schedules. Efficiency and planning process are improved by applying a memetic algorithm combining a local search heuristic and genetic algorithm.

Liu and Lee (2011) proposes a two phase heuristic for solving IRP with soft time windows, where retailers demands are stochastic and lost sales costs are known. An initial solution based on a construction approach is generated in the first phase. Then, the initial solution is improved by a combination of variable neighborhood search and tabu search. The developed heuristic is compared with three algorithms designed for the vehicle routing problem with time windows.

3.3 Infinite Time Horizon with Deterministic Demand

Infinite time horizon with deterministic demand has been considered in various studies including Blumenfeld et al. (1985), Anily and Federgruen (1993), Bramel and Simchi-Levi (1995), Bertazzi et al. (1997), Chan and Simchi-Levi (1998), Jung and Mathur (2007), Raa and Aghezzaf (2008), and Raa and Aghezzaf (2009). For example, Blumenfeld et al. (1985) is one of the earliest works studying the optimal shipment policy by evaluating a trade-off between transportation, inventory, and production set-up costs in an infinite time planning horizon. Various topologies such as consolidation terminals with direct shipping are considered and analyzed by using Economic Order Quantity (EOQ) models.

According to Andersson et al. (2010), the common objective is not to minimize the total cost of the system in inventory routing problems with an infinite time horizon. In this case, because the optimal replenishment strategy is repeated over the planning horizon, minimizing the total cost of one repetition divided by the length of the repetition is the most appropriate goal. This strategy is used by Anily and Federgruen (1990) to determine feasible long-term replenishment policies in a system with a single depot and many customers with deterministic demands.

Also, Aghezzaf et al. (2006) studies an economic order quantity policy for replenishing a set of customers and allowing multi-tours for a set of vehicles to serve customers. For this purpose, a new IRP model is introduced where demands of customers are stable and the objective function is to minimize the total transportation and inventory holding costs. A fixed vehicle cost component is not considered in this study, thus the suggested transportation cost is proportional to travel times between customers. In addition to a mixed integer formulation for the IRP, a solution method is suggested based on a column generation approach. The proposed model is distinguished from other IRP models in the literature due to the consideration of a vehicle multi-tour in the model, implying that a vehicle's travel plan may contain more than one tour. Computational results suggest an 11% savings in total cost in comparison to a situation where multiple-routes are not allowed for vehicles.

3.4 Infinite Time Horizon with Stochastic Demand

More complicated IRP models are developed in studies where demands are considered to be random over the long-term planning horizon. The majority of studies in this area, including Minkoff (1993), Barnes-Schuster and Bassok (1997), Reiman et al. (1999), Kleywegt et al. (2002), Kleywegt et al. (2004), and Hvattum et al. (2009) use Markov decision processes and simulation methods to find an optimal solution to this problem.

Larson (1988), for instance, studies a strategic IRP, where there is a significant lead time between purchasing or leasing a fleet of vehicles and the actual delivery times to customers. The primary focus of this version of IRP is to estimate the minimum number of vehicles required to service a set of customers from a depot. Customers are divided into a set of clusters and within each, all customers are visited on a single route. Such a method is not always efficient, especially if the frequencies of required deliveries for all customers in a single cluster differ a lot. This inefficiency is addressed and improved in Webb and Larson (1995) through the inclusion of period and phase of replenishment as additional decision variables. It is concluded that the developed period/phase strategy reduces costs by maximizing vehicle utilization leading to minimizing number of required vehicles.

Barnes-Schuster and Bassok (1997) studies a single depot/multi-trailer system with stochastic demands under a specific policy of direct shipments. Inventory level at the depot and lead time between warehouse and customers are assumed to be zero. The study results in obtaining a lower bound on the long run average cost per period whose effectiveness is evaluated using simulation.

Reiman et al. (1999) investigates three queuing control problems with a single warehouse and a single capacitated vehicle. The study focuses on two types of inventory routing problems: fixed and dynamic. Two variants for the fixed routing IRP are suggested: Traveling Salesman Problem (TSP) and Direct Shipping (DS). In the IRP with TSP routing, an optimized tour has to be visited by the vehicle for a subset of retail stores. However, all the contents of the vehicle have to be delivered to a single store in the IRP with DS. The decision to whether use a DS or TSP tour has to be made in the dynamic IRP. The study concludes that by making larger and less frequent visits to customers, DS can lead to smaller transportation costs, while inventory costs can be decreased by visiting customers more often and delivering fewer inventories in TSP routing.

The IRP with multiple items and uncertain demands is addressed by Huang and Lin (2010). A modified ant colony optimization (ACO) algorithm is proposed for this problem and compared with the conventional ACO. The computational results show the superiority of the proposed algorithm over the classic ACO with respect to the total costs.

Table 4 provides the classification of the IRP literature based on type of demand, planning horizon and whether or not lost sales costs are considered. Also, Table 5 shows the classification of previous studies with similar features to the problem considered in this thesis, deterministic demand over a finite planning horizon, highlighting their solution strategies. These papers have been introduced in the previous sections.

3.5 Contribution to Existing Literature

As discussed, one of the main distinguishing factors of IRP variants is the objective function used. The literature has focused mainly on IRPs with the objective of minimizing transportation costs, inventory carrying costs, or both. Rather than allowing lost sales and balancing these costs against other costs of the system, the majority of the literature has focused on preventing stock-outs

Table 4: Classification of IRP Literature Based on Basic Features

Article	Time Horizon		Demand		Lost sales
	Finite	Infinite	Deterministic	Stochastic	
Bell et al. (1983)	✓			✓	No
Federgruen and Zipkin (1984)	✓			✓	Yes
Federgruen et al. (1986)	✓			✓	Yes
Chien et al. (1989)	✓		✓		Yes
Barnes-Schuster and Bassok (1997)		✓		✓	Yes
Reiman et al. (1999)		✓		✓	Yes
Anily and Federgruen (1990)	✓		✓		No
Minkoff (1993)		✓		✓	Yes
Campbell et al. (1998)	✓		✓		No
Kleywegt et al. (2002)		✓		✓	Yes
Kleywegt et al. (2004)		✓		✓	Yes
Campbell and Savelsbergh (2004)	✓			✓	No
Abdelmaguid and Dessouky (2006)	✓		✓		Yes
Dauzère-Pérès et al. (2007)	✓			✓	No
Raa and Aghezzaf (2008)		✓	✓		No
Yu et al. (2008)	✓		✓		No
Hvattum et al. (2009)		✓		✓	Yes
Raa and Aghezzaf (2009)		✓	✓		No
Moin et al. (2011)	✓		✓		No
Li et al. (2014)	✓		✓		No

at customers. One of the shortcomings of such a method is that it is difficult, if not impossible, to completely eliminate lost sales in reality. Therefore, the absence of lost sales costs in the IRP formulations decreases the flexibility of such models in real world applications. In addition, using a constraint to eliminate lost sales does not allow trade-offs in lost sales, inventory holding costs and transportation costs to be considered. Including lost sales in the objective function allows for (and requires) valuing them appropriately. Therefore, to overcome this limitation, balancing freight costs, inventory holding costs, and lost sales costs is the objective function of the deterministic IRP studied in this research.

Moreover, the majority of IRP papers with lost sales incorporated in the objective function considered demands of customers to be stochastic, as discussed in Sections 3.2 and 3.4. Only a few papers incorporate stock-out costs in IRP problem variants with deterministic demand (e.g.,

Table 5: Classification of Related Literature to Retail Chain's Problem

Article	Solution Methodology
Miller (1987)	Constructive heuristic/decision support system
Chien et al. (1989)	MIP/lagrangian based heuristic
Abdelmaguid and Dessouky (2006)	Genetic algorithm
Savelsbergh and Song (2007)	Enhanced greedy heuristic /GRASP
Archetti et al. (2007)	Branch & cut algorithm
Savelsbergh and Song (2008)	Branch & cut algorithm
Hemmelmayr et al. (2009)	Integer programming/variable neighborhood search
Moin et al. (2011)	Hybrid genetic algorithm
Qin et al. (2014)	Inventory/routing decomposition local search, tabu search

Chien et al. (1989) and Abdelmaguid and Dessouky (2006)). However, they are different from this research with respect to both suggested constraints and solution methods. For example, both of the mentioned studies impose order-up-to policies at customers. This makes the delivery quantity decisions easier, as evidenced in Campbell et al. (2001), which states that allowing changes in delivery volumes adds to the complexity of the IRP. Also, neither Chien et al. (1989) nor Abdelmaguid and Dessouky (2006) consider time window constraints for customer deliveries. In summary, there is a gap in the IRP literature with respect to deterministic problem variants that incorporate inventory holding costs, lost sales costs and time windows limitations. This research seeks to fill this gap.

4 Methodology

The problem described in Section 2 is first attempted to be solved optimally. The MIP is implemented in AMPL with CPLEX used as the solver. However, it was not possible to obtain solutions to instances of the size required in this research (58 stores over a one week horizon). CPLEX failed due to lack of memory. This result is expected, because according to the literature, large instances of IRP (e.g., 60 customers over a one week planning horizon) cannot be solved optimally. As an example, Archetti et al. (2007) claimed their Branch-and-Cut algorithm was able to obtain optimal solutions for instances with up to 50 customers and a time horizon of 3 days. However, as the time horizon increased to six days, the largest instances that could be solved optimally were limited to 30 customers. Another study in the IRP literature, Li et al. (2014), reported CPLEX could only solve instances with up to nine customers and two vehicles optimally.

Because the case study in this thesis includes instances with at least 50 retail stores and a one week planning horizon, a heuristic solution method is developed.

4.1 Adaptive Large Neighborhood Search Algorithm

In this study, an Adaptive Large Neighborhood Search (ALNS) strategy that is an extension of the Large Neighborhood Search introduced in Shaw (1998) is developed. This specific approach is chosen because impressive results for the performance of similar methods are reported in the literature. It is shown in Kilby et al. (2000) that large neighborhood search is well suited for the vehicle routing problem with some side constraints. Given that distribution decisions comprise a major part of IRPs, ALNS shows promise for this class of combinatorial optimization problem. Indeed, large neighborhood search algorithms have demonstrated good results for two IRP variants in the literature (Coelho et al. (2012a), Zhao et al. (2008)).

The ALNS that is developed in this thesis consists of two phases, described below.

4.1.1 Initial Solution Construction Phase

Algorithm 1 contains details of the initial solution generation method developed in this study. The algorithm begins by setting a mandatory delivery quantity for each customer $i \in N$ at the beginning of each period $t \in T$ equal to d_{it}, the demand of i at time t. Then, subproblems for each $t \in T$ are solved. In a given period, all customers are initially placed in an unvisited customer list, u, and a route is opened. A customer i is randomly removed from the list u for insertion. Open routes are examined to find feasible insertion locations for customer i. An insertion location in a route r is feasible for customer i if inserting i in that location will not violate the time windows of the route, and if adding delivery quantity d_{it} will not violate the capacity of the trailer. If no feasible insertion locations exist in open routes, a new route is opened. If multiple feasible insertion locations are available, the one with cheapest insertion cost is selected. This means customer i is inserted in a location that yields the minimum increase in transportation cost. This is repeated until u is empty (there are no customers left to insert), and then the overall process is repeated for each $t \in T$. Finally, to improve the sequencing decisions within each route, a sweep ordering of the customers assigned to each route is created, and the customers are then visited in the order specified by the sweep sequence (Gillett and Miller, 1974). In sweep ordering, the polar coordinates of each customer with respect to the depot are calculated and customers are sorted in increasing order of polar angle.

The aim of Algorithm 1 is to generate an initial solution with high vehicle capacity utilization. This is because transportation costs, especially fixed trailer costs, comprise a significant portion of the total costs, according to the parameters of the case study in this thesis. The routing decisions that result from Algorithm 1 are also expected to be good, as the customer sequences are developed using the sweep methodology.

In addition, the developed scheme for initial solution generation works well for instances where transportation costs is not the dominant cost. One example for this situation is when lost sales multipliers have high values with respect to the transportation costs. Under this condition, it would be more cost efficient to prevent lost sales occurrence and visit the majority of the stores. This is taken care of in the developed algorithm, because for each day, mandatory delivery quantities are set to the store demand. Thus, lost sales do not occur. Also in the proposed method, the vehicle's capacities are used as much as possible, because customers are inserted into routes unless doing so is not feasible.

4.1.2 Solution Improvement Phase

ALNS explores the search space by moving from the current solution to its neighbors using pre-defined operators including removing, reinserting, swapping and exchanging.

Because lost sales are allowed in the IRP that is studied here, the same concept regarding partial fulfillment of customers that is explained in Coelho et al. (2012a) is applied. The main idea is that rather than using a pair of removal and reinsertion operators (that is the case in the majority of local

Algorithm 1 Initial Solution

```
t = 1
while t ≠ T do
    set delivery quantity to customer i at period t be equal to demand of i at t
    list of unvisited customers = u ← {1, 2, ..., n}
    set of open routes = o ← ∅
    while u ≠ ∅ do
        randomly select i ∈ u and remove it from the list u
        assign customer i to first route having feasible insertion location
        if such a route does not exist then open a new route for i
            add the new opened route to the set o
        end if
    end while
    apply sweep on all obtained routes
    t = t + 1
end while
return initial solution
```

search algorithms), it is allowed here to only remove a customer from a solution and not reinsert it. More specifically, in traditional neighborhood search algorithms, each removal operator is followed by an insertion operator. But in this thesis, the option of using the removal and insertion operators independently is allowed. This adds more flexibility to the neighborhood search mechanism. The ALNS variant developed in this thesis is detailed below. First, the proposed operators are described and then, the flow of the algorithm is discussed in detail.

4.1.3 Move Operators

Recall that route r refers to a sequence of customers assigned to a vehicle for replenishment, and also denotes the magnitude of delivery to each customer in the route. Also, o refers to the set of open routes which are updated throughout the algorithm. Whenever it is mentioned that a customer is inserted in a route, the insertion is carried out only if doing so will not violate the feasibility of the route with respect to trailer capacity and time windows. A description of operators used in this implementation of ALNS is given below.

- Remove all days: A customer i is randomly selected and all of its assigned visits across all days of the planning horizon are removed.

- Remove worst distance per delivery: A trailer route $r$is randomly selected. The customer in the route having the smallest ratio of delivery quantity to transportation cost is removed.
- Remove random: A route r is randomly selected and a customer is randomly selected from r and removed.
- Remove k: The remove random operator is repeated k times, where k is a user-defined parameter.
- Remove smallest delivery: A route r is randomly selected and the customer in the route having smallest delivery quantity is removed.
- Remove worst: A route r is randomly selected and the customer whose removal maximizes travel distance savings is removed.
- Adjust delivery quantity: A customer i is randomly selected. All delivery quantities to i across all days of the planning horizon are adjusted to ensure that the total quantity delivered to i is not more than what is needed (its total demand over the planning horizon).
- Improve capacity utilization: An open route r with available capacity ($r \in o$) is chosen randomly. Next, customer $i \in r$ with having minimum delivery quantity is identified. The delivery quantity to i is augmented in an amount equal to the remaining capacity of the trailer. Note that the *adjust delivery quantity* operator that is described earlier accounts for the situation where customer i does not need this large delivery.
- Insert biggest demand: A day t is selected randomly. Among all customers that are not visited in t, the one with the biggest demand is selected. A route r from all available routes on day t is selected randomly. Then, the customer is inserted into the cheapest feasible insertion location in r. This operator was first introduced in Rosenkrantz et al. (1977). Note that if there are no open routes on day t or there is not a feasible insertion location for customer i in r, a new route is opened for inserting i.
- Insert random: An open route r is selected randomly. A customer i that is not visited within r is selected randomly. Customer i is inserted into the cheapest feasible insertion location in r. The delivery quantity to i is set equal to the remaining capacity of the trailer. Note that if

there is no feasible insertion location for customer i in r, another route is selected randomly and the remaining steps of the process are repeated. Note that the *adjust delivery quantity* operator that is described earlier accounts for the situation where customer i does not need this large delivery.

- Change a route: A route r is randomly selected. The period t in which this route will be executed is changed. Specifically, the period number is divided by two and rounded up to the next integer number, if necessary. For example, if the selected route is originally scheduled for day six of the planning horizon, it will be moved to day 3. Or if the selected route is in day 3 of the planning horizon, it will be moved to day 2.
- Break a route: A route r is randomly selected and split into two routes, such that the two resulting routes will contain the same or almost the same number of customers. For example, a route 0-5-3-7-2-0 will be split into the two routes 0-5-3-0 and 0-7-2-0.

Algorithm 2 summarizes the flow of the ALNS scheme. The improvement phase is initialized by assigning equal weights for all operators so that they are all equally likely to be selected in each iteration. For the first n iterations, where n is a user-defined parameter, all operators are applied to the current solution s to generate a set v of neighbors for s. At the end of each iteration, the admissible neighbor in v with minimum total cost is selected as the candidate neighbor $s\prime$ of s. In order for a neighbor to be admissible, it must not appear on a list that contains the solutions from the last w iterations of the algorithm (this is, effectively, the tabu list as in tabu search heuristics). Moreover, if candidate neighbor $s\prime$ has total cost lower than the best solution found so far, s_{best}, then s_{best} is updated to $s\prime$. Also, solutions that are in the tabu list, but have a total cost less than the best solution found so far are accepted. Finally, $s\prime$ is the starting point of the next iteration.

Note that before moving to the next iteration, operator weights are adjusted based on their performance. Specifically, associated with each operator is a score that is initially set to zero. Then, one point is added to the score of an operator, each time it is the best admissible neighbor. Also, two and three points are added to the scores of operators yielding solutions with total costs lower than s and s_{best}, respectively. Finally, selection probabilities of operators (weights) are calculated

Algorithm 2 ALNS

Initialize weights of all operators
$s \leftarrow$ initial solution
$s_{best} \leftarrow s$
set of neighbor solutions $v \leftarrow \emptyset$
while stopping criteria not met **do**
    select operators based on their past performances
    apply operators on s to get set of neighbor solutions v
    choose the best admissible neighbor from v to get a neighbor solution $s\prime$
    if $f(s\prime) < f(s_{best})$ **then** $s_{best} \leftarrow s\prime$
    end if
    if A neighbor that is in the tabu list has a total cost less than s_{best} **then** accept neighbor
    end if
    update weight of operators
end while
apply sweep on all routes to improve routing aspect of the solution
return final solution

using the points associated with each operator divided by the total points across all operators.

After n iterations, these operator weights are used to select a single operator per iteration. Operators with higher weights are more likely to be selected. This procedure is continued until the stopping criteria is met. The outline of the algorithm is described in Figure 7.

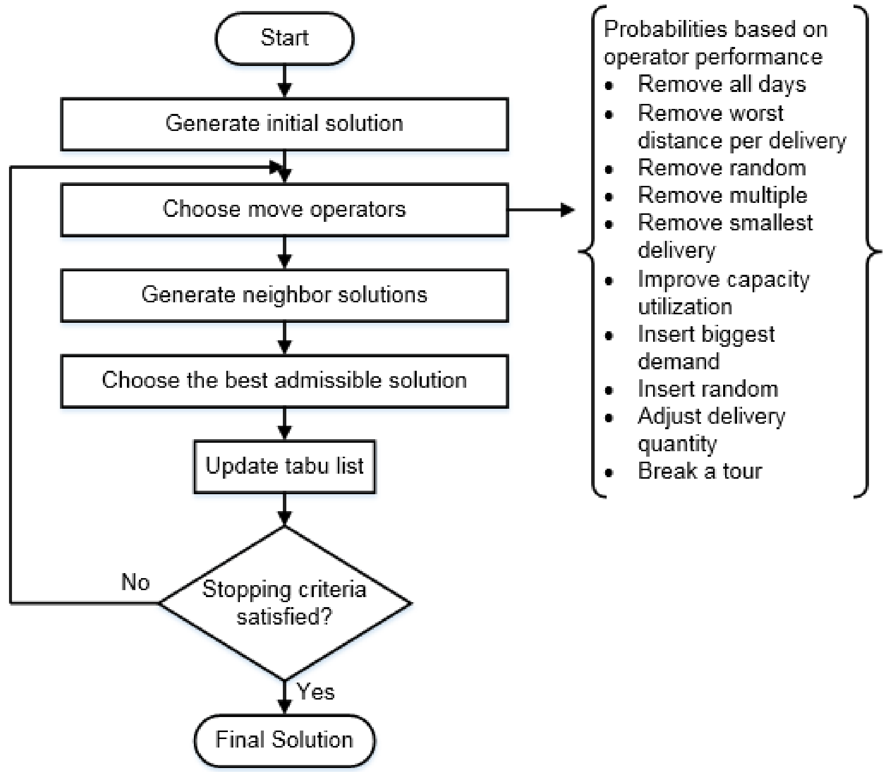


Figure 7: Heuristic Outline

5 Heuristic Validation

The performance of the solution methodology presented in Section 4 is investigated across a number of test instances by comparing results obtained from the heuristic to results obtained from commercial optimization software. Section 5.1 describes test instance development and Section 5.2 presents the comparison of the two solution methods.

5.1 Test Instance Development

In addition to test instances that resemble actual data from the retail chain, a broader set of test instances resembling real-world situations are developed. Test instances have several constant parameters including fixed and variable transportation costs and allowable time windows for deliveries at stores. These parameters along with associated values are described in Table 6.

Table 6: Instance Parameters

Parameter	Description	Value
r	fixed trailer cost	\$106.9
m	transportation cost per mile	\$1.883
a	the earliest time a trailer can begin unloading	6 a.m.
b	the latest time the trailer can finish unloading	6 p.m.

There are five parameters that vary across test instances. These are referred to as factors in the experimental design, and each factor has a number of levels. Additionally, five test instance replications are generated randomly for each factor combination, yielding a total of 240 test instances. The factors and levels are summarized in Table 7 and described below.

Table 7: Experiment Development

Factors	Level	Level Description
Demand	3	1: Low week - N (400, 150) 2: Medium week - N (800,150) 3: High week - N (1200,200)
Location	2	1: Uniformly distributed customers 2: Clustered customers
Lost Sales Cost	2	1: N (0.3025, 0.0247) 2: N (3.025, 0.0247)
Holding Cost	2	1: 13% 2: 18%
Trailer Capacity	2	1: Small truck 1100 units 2: Large truck 2900 units

- Demand: Historical demand of the retail chain can be classified into three levels: low, medium and high. Data series from low, medium and high volume weeks were used separately to fit three corresponding normal distributions.
- Location: Store locations are generated using a uniform and clustered location distribution. In the first, store locations are distributed uniformly throughout the rectangular area with length 720 miles and width 570 miles. These limits are calculated using the distances between maximum and minimum latitudes and longitudes of the locations provided by the retail chain. In the second, c points are randomly generated as cluster seeds. Then, locations

are randomly generated within a radius r of those seed locations. For the purposes of this study, r is 48 miles. This results in c clusters of locations. Figure 8 and Figure 9 depict examples of uniform and clustered location distributions, respectively.

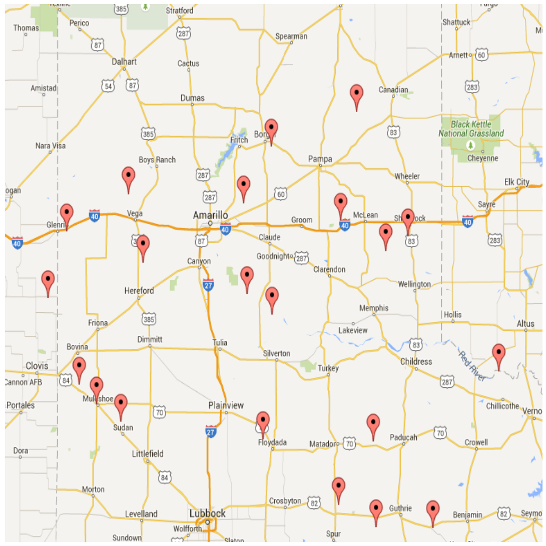


Figure 8: Example of Uniform Locations

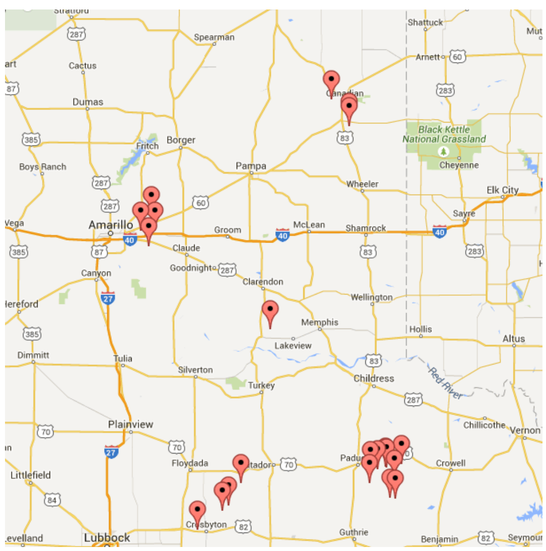


Figure 9: Example of Clustered Locations

- Lost Sales Cost: A goodness of fit test performed on the retail chain lost sales costs indicates the data follows a normal distribution with mean 0.3025 and standard deviation 0.0247. The first level of this factor uses this distribution. Initial experiments revealed that this magnitude of lost sales costs leads to frequent stock-outs in resulting solutions. Therefore, a second level of this factor that infrequently allows lost sales was determined through experimentation to be ten times larger, with a mean of 3.025 and the same standard deviation. Therefore, these two levels are believed to represent "extreme" cases to be considered: solutions with many stock-outs versus few stock-outs. Additional sensitivity analysis on this parameter was performed for a subset of the experimental design and is described in Section 6 of this thesis.

- Holding Cost: A common range in industry for inventory carrying costs is 5% - 20% (http://www.lokad.com/definition-inventory-costs). In this research, 13% and 18% are used.
- Trailer Capacity: Small and large trailers with capacities of 1100 and 2900 units are considered. These two values permit scenarios where there are many opportunities to combine deliveries to multiple stores in the same route (the larger trailer) and scenarios where there are fewer such opportunities (the smaller trailer).

This experimental design yields 48 instances types, described in Table 8.

Table 8: Instance Types

Instance	Demand	Location	Lost Sales Costs	Holding Costs	Trailer Capacity
1	Low	Cluster	0.3	0.13	Small
2	Low	Uniform	0.3	0.13	Small
3	Low	Cluster	3	0.13	Small
4	Low	Uniform	3	0.13	Small
5	Medium	Cluster	0.3	0.13	Small
6	Medium	Uniform	0.3	0.13	Small
7	Medium	Cluster	3	0.13	Small
8	Medium	Uniform	3	0.13	Small
9	High	Cluster	0.3	0.13	Small
10	High	Uniform	0.3	0.13	Small
11	High	Cluster	3	0.13	Small
12	High	Uniform	3	0.13	Small

Instance	Demand	Location	Lost Sales Costs	Holding Costs	Trailer Capacity
13	Low	Cluster	0.3	0.18	Large
14	Low	Uniform	0.3	0.18	Large
15	Low	Cluster	3	0.18	Large
16	Low	Uniform	3	0.18	Large
17	Medium	Cluster	0.3	0.18	Large
18	Medium	Uniform	0.3	0.18	Large
19	Medium	Cluster	3	0.18	Large
20	Medium	Uniform	3	0.18	Large
21	High	Cluster	0.3	0.18	Large
22	High	Uniform	0.3	0.18	Large
23	High	Cluster	3	0.18	Large
24	High	Uniform	3	0.18	Large
25	Low	Cluster	0.3	0.13	Large
26	Low	Uniform	0.3	0.13	Large
27	Low	Cluster	3	0.13	Large
28	Low	Uniform	3	0.13	Large
29	Medium	Cluster	0.3	0.13	Large
30	Medium	Uniform	0.3	0.13	Large
31	Medium	Cluster	3	0.13	Large
32	Medium	Uniform	3	0.13	Large
33	High	Cluster	0.3	0.13	Large
34	High	Uniform	0.3	0.13	Large
35	High	Cluster	3	0.13	Large
36	High	Uniform	3	0.13	Large
37	Low	Cluster	0.3	0.18	Small
38	Low	Uniform	0.3	0.18	Small
39	Low	Cluster	3	0.18	Small
40	Low	Uniform	3	0.18	Small
41	Medium	Cluster	0.3	0.18	Small
42	Medium	Uniform	0.3	0.18	Small

Instance	Demand	Location	Lost Sales Costs	Holding Costs	Trailer Capacity
43	Medium	Cluster	3	0.18	Small
44	Medium	Uniform	3	0.18	Small
45	High	Cluster	0.3	0.18	Small
46	High	Uniform	0.3	0.18	Small
47	High	Cluster	3	0.18	Small
48	High	Uniform	3	0.18	Small

5.2 Comparison of Solution Approaches

Both the heuristic and a commercial optimization solver were used to obtain solutions to all test instances described in Section 5.1. The commercial optimization solver that is used is CPLEX. Specifically, all computations are executed on a CPLEX Version 12.6 on an Intel(R) Core(TM) i7 CPU @ 2.93GHz 8.00 GB RAM PC. Table 9 provides the results of this comparison for all instances. The row number gives the instance type. The heuristic was given a stopping criterion of 1000 iterations. The table provides the total cost of the heuristic solution, the total cost of the best integer feasible solution obtained by CPLEX in 60 minutes, and the percentage gap between the heuristic and CPLEX solutions. It should be noted that CPLEX did not find verifiably optimal solutions for any of the test instances with this time limit. Based on the results, the average gap between the heuristic and CPLEX solutions for this set of test instances is 3.24%. That is, the solutions obtained via CPLEX are 3.24% lower, on average. However, one hour was required to obtain the CPLEX solutions while on average five minutes were required to obtain the heuristic solutions. Further details of these comparisons with respect to the components of transportation, inventory holding and lost sales costs are presented in Appendix B.

Table 9: Average Results Across All Instances

Instance Type	Average Cost - CPLEX	Average Cost - Heuristic	Gap(%)
1	48536.38	50107.13	3.14
2	41125.81	41464.43	0.82
3	152123.37	158109.17	3.77
4	141220.4	146908.05	3.84
5	82713.97	82967.12	0.31
6	83706.38	84009.83	0.36
7	330022.28	352473.25	6.3
8	340431.82	352858.78	3.6
9	126166.68	126470.69	0.24
10	106463.47	107273.69	1.02
11	204809.46	206484.96	0.74
12	465428.33	471012.86	1.19
13	33089.27	35140.8	5.81
14	35756.37	37229.27	4
15	52292.76	53968.66	3.22
16	63300.52	67571.39	6.33
17	88746.9	92191.38	3.78
18	75261.08	78606.96	4.25
19	117858.93	126821.19	7.09
20	114132.59	122895.49	7.13
21	130084.36	132017.48	1.45
22	106934.35	111255.89	3.91
23	131274.89	133071.54	1.36
24	158665.79	166532.88	4.74
25	42957.79	44198.28	2.75
26	31993.66	33723.96	5.31
27	65349.9	70015.08	6.64
28	61221.2	62727.38	2.37

Instance Type	Average Cost - CPLEX	Average Cost - Heuristic	Gap
29	68976.06	71025.31	2.91
30	72685.89	75537.54	3.77
31	108952.93	114797.32	4.87
32	117490.92	124924.53	5.99
33	157402.51	159847.03	1.52
34	112791.13	114961.65	1.86
35	125588.44	129624.17	3.07
36	161228.23	174432.29	7.57
37	67959.19	72052.59	5.62
38	45620.57	46062.39	0.9
39	135347.62	139295.21	2.83
40	193959.19	203523.87	4.68
41	86044.7	86625.73	0.68
42	85474.69	86009.5	0.61
43	284431.45	299718.55	5.07
44	329291.94	341158.91	3.48
45	124555.26	125085.25	0.42
46	123383.27	124771.6	1.11
47	470950.51	477094.05	1.3
48	476161.26	484799.33	1.78

Another method for comparing the performance of the heuristic with that of CPLEX is to investigate the solutions discovered by each within the same amount of runtime. Because the average runtime for the developed heuristic is around five minutes, a five minute runtime limitation is set for CPLEX. Table 10 reports the results of this comparison for a representative subset of instances. The negative percentage gaps indicate the heuristic is obtaining better solutions than CPLEX within this runtime limitation.

Finally, the objective function value obtained by the heuristic as a function of iteration number is depicted in Figure 10. It demonstrates the heuristic is capable of finding good solutions in the

Table 10: Comparison of Heuristic and CPLEX After 5 Minutes

Instance Type	Run Time (S)	Total Cost-Heuristic	Total Cost-CPLEX	Gap
17	175	119532.04	123814.89	-3.58%
18	166	127940.68	132080.12	-3.24%
19	233	101932.11	106153.02	-4.14%
20	284	104008.4	108489.04	-4.31%
29	176	115373.6	118580.2	-2.78%
30	187	114176.7	117634.42	-3.03%
31	220	109598.57	112954.46	-3.06%
32	249	105371.95	108718.61	-3.18%

first 100 iterations.

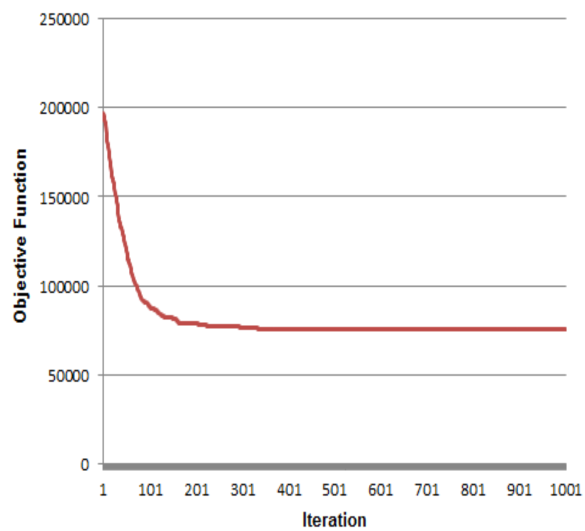


Figure 10: Heuristic Convergence as a Function of Iteration Number

6 Result

Computational results for the full experimental design are discussed in this section. Additionally, a comparison is made between solutions developed using the current policy of the retail chain and solutions developed using the IRP policy presented in this thesis. For this comparison, the IRP results are obtained via the ALNS heuristic.

6.1 Results From Experimental Design

Table 11 provides the average cost of solutions developed using the ALNS heuristic for all instance types. For example, the first row provides results from all 16 instances in the experimental design that have low demand. As can be seen in the Table 11, the instances with the higher value for the lost sales multiplier have the highest total cost among all instances.

As expected, total costs are also increased by moving from low to high demand. In these

Table 11: Average Results for All Instance Levels

Instance level	number of instances	Average Cost - Heuristic
Low Demand	16	79006.10
Medium Demand	16	155788.83
High Demand	16	202796.03
Uniform Locations	24	152510.56
Clustered Locations	24	139216.74
Lost Sales 1	24	84109.86
Lost Sales 2	24	207617.45
Holding Cost 1	24	139914.82
Holding Cost 2	24	151812.49
Small Truck	24	194430.70
Large Truck	24	95926.22

situations, there is more product to be delivered and stored, and more opportunities for stock-outs as well. Problems with uniformly distributed locations have higher total cost than those with clustered locations. One potential explanation for this is there are more opportunities to realize savings in transportation costs for the clustered location distributions because there are subsets of stores in close proximity to each other that can be served using shared trailers.

Large differences in total costs can be seen when moving between the levels of the lost sales and trailer size factors. For example, changing the value for the lost sales multiplier from \$0.3 to \$3 leads to an approximate \$120,000 increase in total costs. Also, decreasing the trailer size from large to small results in doubling the total costs. This demonstrates that transportation costs are a significant portion of the total costs. This is as expected, given the relatively high fixed and variable costs associated with each trailer that is used.

Another interesting fact that can be ascertained from Table 11 is that by increasing the inventory holding rate from 13% to 18%, the behavior of the system remains relatively unchanged. The reason is that inventory holding rates are small when compared with the multipliers for the other two cost components (transportation and lost sales). Therefore, increasing the inventory holding rate by 5% is not significant enough for system avoidance of this cost. That is, the cost is not high enough for holding less inventory to be preferred over making fewer shipments.

6.2 Comparison of Current and Proposed Retail Chain Replenishment Policies

The focus of this section is on comparing the solutions obtained using the retail chain's current replenishment policy to solutions obtained when direct shipments are no longer required for all stores. The results from the ALNS are used to make this comparison.

6.2.1 Current Policy

This section provides details of the current policy of the retail chain. The main components of the case study data are first introduced and discussed. Then, the cost of the current policy for these case study instances are presented.

6.2.2 Description of the Case Study Data

This study has been focused on a real data set from the retail chain with the following elements:

- Locations of 58 retail stores and the depot
- Distances between retail stores and the depot
- Daily demands of stores, measured in units of sales per day, based on historical data from one low, one medium and one high volume week for all stores
- Fixed transportation costs associated with the trailer drop fee of $106.9 per trailer
- Variable transportation costs based on a mileage rate of $1.883 per mile traveled
- Inventory holding rate of 13%, multiplied by the quantity of inventory at the end of each day for each store to calculate inventory holding cost

- Lost sales rate of $0.3, multiplied by the number of stock-outs at the end of each day for each store to calculate lost sales cost.

The total costs of the current replenishment policy of the retail chain for five sample stores are provided in Table 12. Figure 11 depicts the weekly cost for all stores included in the retail chain network. In both the table and figure, a breakdown of the total cost by its three components is provided. Transportation costs comprise the major part of total costs which is an expected result. The reason is that as mentioned earlier, a dedicated trailer is sent to a store after five days or whenever it becomes full (whichever comes first). Thus, lost sales costs rarely occur.

Table 12: Total Weekly Costs of Retail Chain's Stores

Store	Transportation Cost	Inventory Holding Cost	Lost Sales Cost	Total Cost
1	5884.534	53.697	230.416	6168.647
2	5045.298	63.608	0	5108.905
3	1094.441	125.088	0	1219.529
4	3922.749	47.143	45.752	4015.644
5	4788.327	74.087	0	4862.413

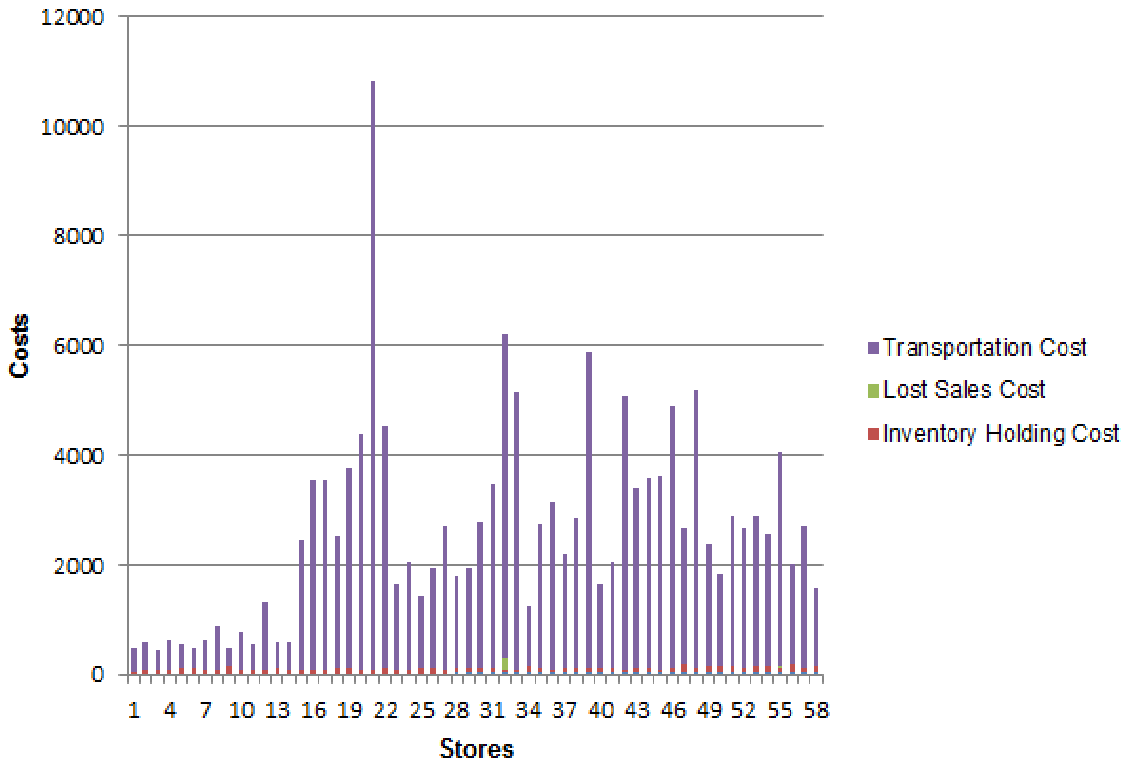


Figure 11: Results of Current Method

The number of stores receiving two, three, four and five visits per week under the current policy is provided in Figure 12. Note that approximately half of all stores receive two visits per week. An additional one-quarter of stores receive three visits per week.

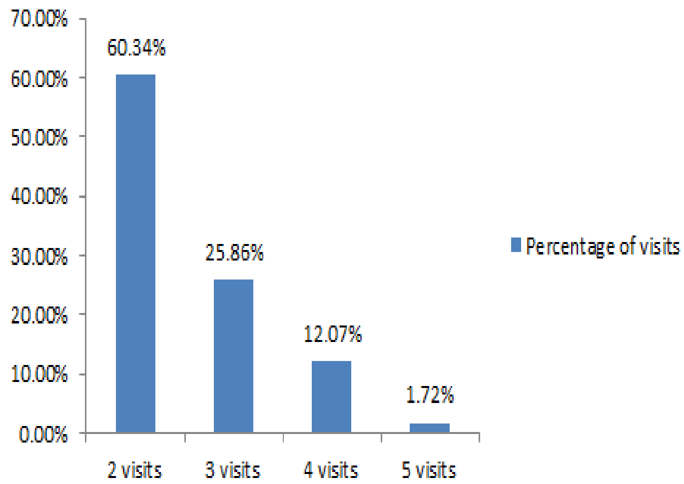


Figure 12: Weekly Visits to Stores Under Current Policy

As can be seen in Figure 12, the majority of stores have the same frequency of visits per week. Also based on Figure 13, which depicts the locations of stores in the retail chain network, there are clusters of stores that are far from the DC. Stores in some of these clusters have the same frequency of delivery which makes them good candidates for replenishment via a shared route. To illustrate this point, see the stores identified by blue and red circles in Figure 13. These require two and three visits per week, respectively. Because these stores are within a cluster and share the same visit frequency, they can be served together. This implies that the option of allowing combined deliveries may reduce the total replenishment costs.

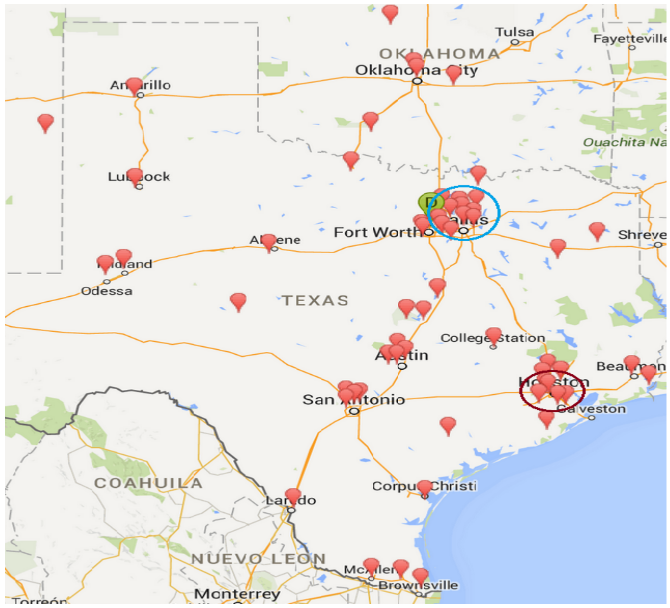


Figure 13: Clusters of Retail Chain's stores

In addition, according to Gallego and Simchi-Levi (1990), a direct shipping strategy is effective when the Economic Order Quantity (EOQ) of customers is at least 71% of the trailer capacity. The EOQ which is first introduced in Harris (1990) determines the order quantity by minimizing total inventory costs including holding, ordering, and lost sales costs. The quantity representing 71% of trailer capacity is 1781 units of inventory. However, the average EOQ for stores during low, medium and high weeks is 681, 805, and 1422 units of inventories, respectively. This provides further evidence that a direct shipping policy may not be the most effective strategy for the retail chain (Gallego and Simchi-Levi, 1990).

6.2.3 Proposed Policy

Results obtained by the ALNS heuristic described in Section 4 for the case study instances are discussed here. Heuristic results with respect to transportation, inventory holding, and lost sales costs for all stores included in the retail chain's network are shown in Figure 14.

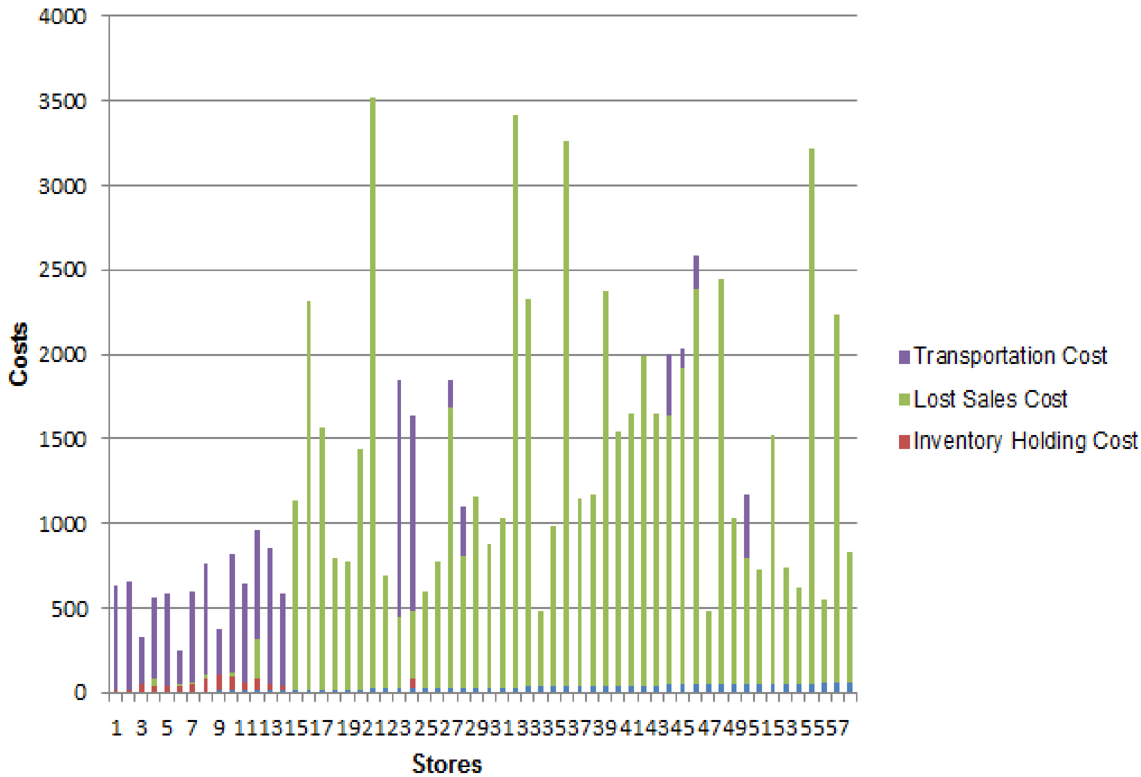


Figure 14: Results of Heuristic for All Stores

Solutions produced by the heuristic decrease total costs by 51.39% when compared with the current policy of the retail chain. As can be seen in Figure 14, the total cost reduction is mainly achieved by decreasing the frequency of visits to stores. Therefore, transportation costs decrease, but the frequency of lost sales occurrences greatly increases. The total number of weekly visits across all stores is shown in Figure 15.

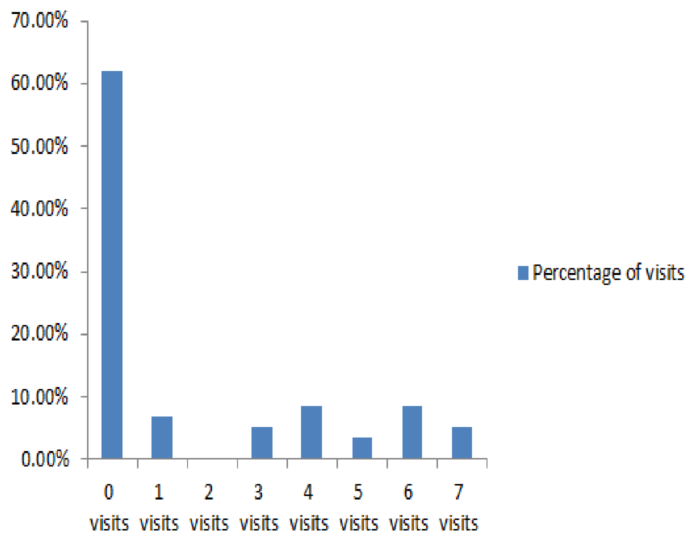


Figure 15: Weekly Visits to Stores Under New Policy

Because many stores do not receive visits under the new policy, it suggests that some parameters, for example the lost sales multiplier or the transportation costs (fixed and/or variable), may need modification. The magnitude of the lost sales multiplier currently being provided by the retail chain is very low ($0.3) with respect to variable transportation cost ($1.883 per mile). Therefore, optimal solutions tend to allow lost sales to occur for the majority of stores to avoid higher transportation costs for visiting them. This concept is depicted in Figure 16.

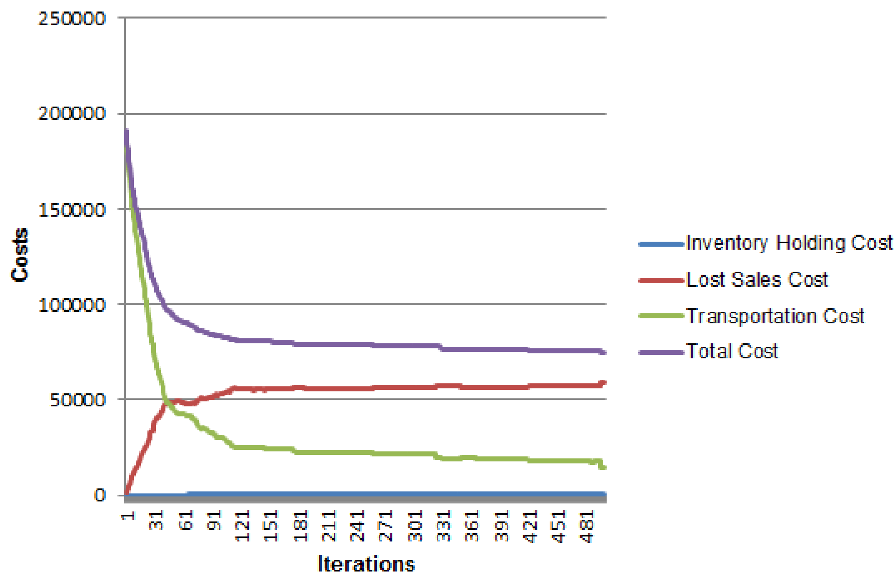


Figure 16: Lost Sales Multiplier

While these parameters may need tuning, it is important to note that even if lost sales are not allowed, the new policy (by combining stores in share routes) is still capable of generating savings when compared with a strict direct shipping policy for all stores. Evidence of such is provided in Table 13, where the cost of the retail chain's current replenishment policy is compared with the results of the proposed policy with a no-stock-outs modification.

Table 13: No Stock-outs Results

Strategy	Transportation Cost	Holding Cost	Lost Sales Cost	Total Cost
Retail Chain's Current Policy	144534.30	4746.91	276.17	149557.38
Developed Heuristic	127208.84	3275.80	0	130536.06

7 Future Research

It is always desirable to develop models and solution approaches that as much as possible capture elements of the real-world problems. In order for IRP models to be implementable in industry, they should include challenges that companies are facing in their supply chain and logistics systems. Elements whose addition to the IRP model would benefit companies are discussed in the following.

- Soft Time Windows: The allowable time duration that is considered in this thesis for making deliveries to stores is called a hard time windows. It means that violating these constraints would yield infeasible solutions. However, allowing deliveries to arrive at customers outside a specified time windows at penalty, which is called soft time windows, will increase the flexibility of the model.

- Multiple Products: In many real-world situations, several types of products are distributed among stores using the same fleet of vehicles. In this thesis, products are aggregated at the store level meaning that only one type of product is available. Thus, the model that is proposed here does not distinguish between different types of product which makes the problem more tractable. However in the case of multi-products, in addition to determining which customers need to be visited at each period and by which vehicle route, decisions regarding the magnitude of each product that must be delivered to each store during each visit should be made as well. The significant increase in complexity of resulting models would likely require heuristic solution approaches.

- Reconsidering Assignment of Customers to Depot: The retail chain that is considered in this thesis has more than one distribution center (DC) in its network for replenishing stores. Because DCs are added to their network gradually during years, the current assignment of stores to DCs may not be efficient. Based on the location of DCs and their assigned stores, the possibility of reducing total costs by changing the current assignment of stores to DCs exists. Future studies could focus on determining the optimal assignment of stores to the available DCs.

A Sensitivity Analysis for Lost Sales Multiplier

As described, the majority of stores do not receive any visits over the planning horizon when the lost sales multiplier is \$0.3. Also with the value of the lost sales multiplier being equal to \$3, lost sales costs rarely occur. A sensitivity analysis around this parameter is performed to better illustrate the effect of the lost sales multiplier in the behavior of the system. The two mentioned situations are extreme, since lost sales are experienced for the majority of stores in the first one and lost sales rarely occur in the second. Thus, values less than \$0.3 and higher than \$3 are not tested. Instead, some intermediate points in the range \$0.3 3 including \$1, \$1.7, and \$2.4 are considered in the sensitivity analysis. The ALNS results obtained for a subset of test instances with these lost sales multipliers are shown in Tables 15 - 19. Also, features of the selected instances with respect to the factors in the experimental design are shown in Table 14. Note that all instances involve 20 customers over a five days planning horizon.

According to the results, as the magnitude of the lost sales multiplier is increased, the possibility of lost sales occurrence and/or its value is decreased. Also, instances for which the value of the lost sales multiplier is not significantly different from the transportation cost multiplier are harder to solve in comparison with those instances having a huge difference between these two parameters (based on the obtained lower bound by CPLEX in a 60 minute time limitation).

Table 14: Features of Instances

Demand	Location	Holding Cost	Trailer Capacity
Medium week	Clustered customers	13%	Small truck

Table 15: Lost Sales Multiplier = 0.3

Instance	Transportation Costs	Holding Costs	Lost Sales Costs	Total Costs
1	11778.46	172.45	1136.72	13087.64
2	13030.34	162.73	86.09	13279.16
3	8835.29	151.47	0	8986.76
4	11822.69	78.05	1052.17	12952.9
5	12770.51	133.07	404.87	13308.44
6	9494.31	168.71	45.79	9708.81
7	9354.07	163.52	1479.13	10996.71
8	12789.77	123.77	235.15	13148.7
9	13476	76.88	34.68	13587.56
10	16129.35	171.46	163.13	16463.95

Table 16: Lost Sales Multiplier = 1

Instance	Transportation Costs	Holding Costs	Lost Sales Costs	Total Costs
1	14788.63	179.08	212.66	15180.38
2	14646.68	81.99	0	14728.67
3	8762.36	161.23	0	8923.58
4	16216.07	76.17	0	16292.24
5	15304.74	92.12	0	15396.86
6	10328.43	107.28	0	10435.71
7	12329.08	108.83	295.64	12733.55
8	15302.34	81.33	0	15383.67
9	14785.65	88.73	0	14874.38
10	17167.52	150	0	17317.52

Table 17: Lost Sales Multiplier = 1.7

Instance	Transportation Costs	Holding Costs	Lost Sales Costs	Total Costs
1	14025.06	142.11	474.22	14641.39
2	14919.91	67.84	0	14987.75
3	9023.05	75.61	0	9098.67
4	15742.97	94.22	0	15837.1
5	15331.75	98.15	0	15429.9
6	10615.71	117.97	0	10733.68
7	12601.07	118.63	0	12719.7
8	15571.47	101.03	0	15672.49
9	15027.13	3.72	0	15030.86
10	17714.69	114.63	0	17829.32

Table 18: Lost Sales Multiplier = 2.4

Instance	Transportation Costs	Holding Costs	Lost Sales Costs	Total Costs
1	14760.17	108.8	0	14868.97
2	14957.05	26.3	0	14983.35
3	9092.84	124.31	0	9217.16
4	16738.53	60.95	0	16799.48
5	15425.17	64.71	0	15489.88
6	10709.42	120.38	0	10829.8
7	12372.73	120.98	25.42	12519.13
8	15441.99	77.5	0	15519.49
9	15027.13	3.24	0	15030.37
10	17375.82	171.27	0	17547.09

Table 19: Lost Sales Multiplier = 3

Instance	Transportation Costs	Holding Costs	Lost Sales Costs	Total Costs
1	14969.32	141.99	0	15111.31
2	14572.94	104.49	0	14677.43
3	9051.16	151	0	9202.16
4	16179.58	95.01	0	16274.59
5	15432.58	73.26	0	15505.84
6	10399.6	134.83	0	10534.43
7	12950.51	68.73	0	13019.24
8	14745.28	114.72	0	14860.01
9	14934.35	25.4	0	14959.75
10	17114.68	184.04	21.39	17320.11

B Results

The computational results that are obtained from both the heuristic and CPLEX for all test instances described in Section 5.1 are shown here. Specifically, Tables 20 - 24 provide the components of the total cost (transportation, inventory holding and lost sales costs) of the heuristic solution, the components of the total cost of the best integer feasible solution obtained by CPLEX in 60 minutes, and the percentage gap between the heuristic and CPLEX solutions.

Table 20: Results for Run 1

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
1	5836.47	15.9	44741.9	50594.27	11695.03	20.74	39884.41	51600.17	1.95
2	1221.38	37.84	39492.4	40751.62	567.38	2.38	40834.03	41403.79	1.58
3	129077	765.92	9426.62	139269.54	140806.55	61.47	3604.84	144472.85	3.6
4	138494	964.77	3293.53	142752.3	146654.32	142.31	2907.41	149704.05	4.64
5	3508.17	26.12	77520.8	81055.09	2341.14	14.89	79242.34	81598.36	0.67
6	4564.47	61.53	77824.6	82450.6	679.29	4.34	82641.52	83325.14	1.05
7	302140	837.21	2750.74	305727.95	323426.67	42.29	743.53	324212.49	5.7
8	323482	836.57	13503.6	337822.17	353483.46	40.89	413.52	353937.87	4.55
9	4850.13	10	119541	124401.13	4402.47	6.16	120176.77	124585.4	0.15
10	1938.28	1.75	126221	128161.03	1107.59	0	127251.29	128358.88	0.15
11	296452	107.46	109373	405932.46	306004.76	134.98	101941.67	408081.41	0.13
12	333825	127.95	125555	459507.95	346999.57	154.7	118534.27	465688.54	1.33
13	10416.2	1097.73	27244.2	38758.13	28032.61	622.92	12261.96	40917.49	5.28
14	3403.48	490.72	35685.8	39580	451.92	90.57	40041.21	40583.71	2.47
15	46895	3790.91	2675.46	53361.37	45774.76	2497.73	5357.1	53629.59	0.5
16	66997.4	2697.35	3725.76	73420.51	63553.92	2754.18	11898.57	78206.67	6.12
17	36402.7	622.4	53944.7	90969.8	50733.23	682.36	42893.53	94309.12	3.54

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
18	17732.2	985.31	63121.5	81839.01	3825.22	291.47	81355.1	85471.8	4.25
19	112011	3761.01	3716.6	119488.61	128880.45	128	500.79	129509.23	7.74
20	106954	3714.14	6966.49	117634.63	126951.88	224.81	0.02	127176.72	7.5
21	35747.9	531.61	98783.9	135063.41	105692.73	171.47	30022.46	135886.65	0.61
22	33475.7	1212.98	78414.8	113103.48	24074.3	194	92894.03	117162.33	3.46
23	126912	3450.26	767.32	131129.58	132272.13	281.63	0	132553.76	1.07
24	152473	3788.23	658.9	156920.13	164515.08	292.88	0	164807.96	4.79
25	14749.87	159.01	36509.4	51418.28	35603.42	353.65	17197.22	53154.29	3.27
26	8797.58	795.42	27901.8	37494.8	20890.56	394.11	16879.85	38164.52	1.75
27	64111.2	2688.82	8442.29	75242.31	80319.56	358.85	893.41	81571.82	7.76
28	63598.1	2576.62	2520.45	68695.17	61494.54	428.77	9385.97	71309.28	3.67
29	18155.1	371.67	58801	77327.77	37790.46	166.13	40414.46	78371.06	1.33
30	19757.8	852.97	56886.9	77497.67	4774.83	293.1	75928.12	80996.05	4.32
31	101531	2407.35	2214.45	106152.8	66343.69	399.7	45835.7	112579.1	5.71
32	117305	76.78	8955.26	126337.04	125923.42	2669.15	2439.05	131031.62	3.58
33	146065.3	312.48	19902	166279.78	140538.99	62.51	26554.5	167156	0.52
34	30100.3	839.91	85910.1	116850.31	13838.04	337.63	105684.8	119860.47	2.51
35	129482	2317.57	5258.91	137058.48	137672.78	700.06	1500.98	139873.81	2.01
36	159866	2775.85	2527.98	165169.83	178360.87	78.31	0	178439.17	7.44
37	31724.95	0.36	41829.5	73554.81	45806.9	12.07	31599.73	77418.69	4.99
38	0	0	44528.3	44528.3	0	0	44527.21	44527.21	0
39	128444	1144.29	11608.1	141196.39	140990.73	112.83	5925.55	147029.11	3.97
40	193694	1611.76	2178.72	197484.48	185751.75	174.23	23414.43	209340.4	5.66
41	2317.18	51.73	82693.2	85062.11	2206.61	118.19	82958.28	85283.08	0.26
42	0	0	83039.7	83039.7	0	0	83039.7	83039.7	0
43	279511	1199.67	6441.46	287152.13	299837.37	57.92	342.2	300237.48	4.36

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
44	329694	1394.12	2174.55	333262.67	341457.35	86.27	913.98	342457.6	2.68
45	0	0	127244	127244	0	0	127244.07	127244.07	0
46	1437.16	4.64	122906	124347.8	1437.16	8	122905.57	124350.74	0
47	330951	195.94	117499	448645.94	352694.27	279.91	104975.45	457949.63	2.03
48	354806	194.04	114706	469706.04	368132.51	117.12	107278.12	475527.76	1.22

Table 21: Results for Run 2

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
1	8294.37	5.89	42790.1	51090.36	17898.45	9.8	34624.48	52532.73	2.75
2	1498.83	43.92	39268.9	40811.65	450.28	9.04	40700.72	41160.04	0.85
3	135284	828.8	7362.8	143475.6	147514.61	74.04	3154.53	150743.18	4.82
4	144413	1001.54	3531.71	148946.25	152332.76	133.61	1932.11	154398.48	3.53
5	0	0	83574.8	83574.8	0	0	83574.8	83574.8	0
6	967.65	16.15	83836.2	84820	483.82	2.34	84468.9	84955.06	0.16
7	298372	702.03	10943.5	310017.53	329521.67	54.2	757.04	330332.91	6.15
8	316579	848.54	7333.1	324760.64	334318.43	70.25	550.24	334938.91	3.04
9	0	0	128936	128936	0	0	128935.71	128935.71	0
10	3330.81	4.19	122358	125693	2876.03	5.21	122978.26	125859.5	0.13
11	373932	97.53	122182	496211.53	383121.08	162	118116.07	501399.14	1.03
12	377312	135.12	112062	489509.12	392918.53	168.32	100919.92	494006.77	0.91
13	8423.07	141.24	31221.9	39786.21	28223.94	354	13905.28	42483.22	6.35
14	7469.62	977.78	31009	39456.4	2819.75	437.83	37554.24	40811.82	3.32
15	54344.4	3510.75	6200.89	64056.04	64631.98	466.78	177.81	65276.58	1.87

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
16	53814.9	2989.65	3999.9	60804.45	60774.49	630.43	5221	66625.92	8.74
17	23623	288.47	76701.3	100612.77	76603.58	409.28	27009.33	104022.2	3.28
18	21481.4	1376.91	57736.1	80594.41	8644.34	508.04	74087.44	83239.81	3.18
19	117183	3668.35	6228.64	127079.99	133898.54	242.15	154.51	134295.2	5.37
20	112409	3618.11	2552.64	118579.75	127312.78	260.88	0	127573.66	7.05
21	34019.1	644.51	103152.4	137816.01	113919.37	229.78	27072.03	141221.18	2.41
22	21231.5	882.08	90767.3	112880.88	6582.56	470.35	109044.26	116097.16	2.77
23	134712	3328.47	1168	139208.47	139594.02	62.76	0	139656.78	0.32
24	158899	3423.61	6511.86	168834.47	174818.95	151.28	0	174970.23	3.51
25	14972	661.87	29833.4	45467.27	29592.04	284.26	15845.41	45721.7	0.56
26	10646.1	772.67	27523.4	38942.17	5172.18	216.65	35964.85	41353.68	5.83
27	55815.5	2688.48	7207.52	65711.5	69047.23	269.52	1003.16	70319.91	6.55
28	56800.4	2511.31	2971.82	62283.53	54566.17	496.21	9299.6	64361.98	3.23
29	24892.5	624.01	46132.2	71648.71	37714.59	482.92	36816.61	75014.12	4.49
30	13816.3	703.16	62649	77168.46	16343.69	399.7	62435.7	79179.1	2.54
31	114403	2312.33	7099.08	123814.41	131744.83	174.87	1212.4	133132.11	7
32	111697	232.79	7488.47	119418.26	124373	2118.96	0	126491.96	5.59
33	126242.3	444.22	19656.6	146343.12	126412.44	185.39	21859.2	148457.03	1.42
34	23403.6	512.1	96481.8	120397.5	9492.03	158.76	113478.14	123128.94	2.22
35	119920	2476.52	2457.21	124853.73	125328.36	164.02	1900.51	127392.89	1.99
36	159983	2696.47	2700.82	165380.29	177372.42	121.21	346.98	177840.62	7.01
37	31694.38	15.67	42176.2	73886.25	52396.5	39.63	27122.21	79558.34	7.13
38	0	0	42474.78	42474.78	0	0	42474.78	42474.78	0
39	132354	1101.14	7588.82	141043.96	142200.51	128.95	1039.96	143369.43	1.62
40	194116	1429.71	4526.33	200072.04	186399.38	276.85	23861.02	210537.25	4.97
41	0	0	87038.1	87038.1	0	0	87038.12	87038.12	0

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
42	0	0	84114.2	84114.2	0	0	84114.23	84114.23	0
43	268599	1150.6	4892.83	274642.43	282391.24	53.76	0	282445	2.76
44	329280	1486.77	3961.52	334728.29	345942.54	135.3	960.11	347037.96	3.55
45	11244.3	17.8	112268	123530.1	7656.86	1.54	115997.09	123655.49	0.1
46	3907.19	13.69	120909	124829.88	2623.11	1.07	122341.36	124965.54	0.11
47	376346	224.92	110274	486844.92	391805.25	193.12	97281.05	489279.43	0.5
48	353532	146.46	119064	472742.46	365103.47	248.17	115798.33	481149.97	1.75

Table 22: Results for Run 3

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
1	6928.54	3.85	43101.5	50033.89	25764.82	7.16	26053.39	51825.37	3.46
2	424.54	13.28	40221.5	40659.31	0	0	40870.18	40870.18	0.52
3	147078	775.4	8958.57	156811.97	154206.65	140.4	3190.83	157537.89	0.46
4	141170	1008.4	8870.2	151048.6	149687.91	135.65	8656.04	158479.61	4.69
5	0	0	83273.4	83273.4	0	0	83273.39	83273.39	0
6	2004.16	7.34	82273.6	84285.1	0	0	84314.11	84314.11	0.03
7	296244	798.72	9761.17	306803.89	322584.54	74.64	584.03	323243.21	5.09
8	318406.42	797.87	9358.94	328563.23	348304.98	21.51	4359.39	352685.88	6.84
9	225.45	28.7	115396.34	115650.49	776.01	56.24	115403.86	116236.11	0.5
10	3137.3	36.95	122076.34	125250.6	4070.9	62.4	122478.38	126611.67	1.08
11	1574.16	68.79	38789.4	40432.35	181.4	9.87	40722.77	40914.04	1.18
12	369808.84	82.45	110842.58	480733.88	385029.46	64.16	99816.86	484910.48	0.86
13	919.91	88.58	30002.48	31010.97	20334.88	249.84	12802.21	33386.92	7.12

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
14	33.54	925.12	29789.58	30748.24	5069.31	333.67	26451.17	31854.16	3.47
15	46841.24	3458.08	4981.47	55280.79	56742.92	362.62	925.25	58030.79	4.74
16	46311.74	2936.98	4780.48	54029.2	52885.43	526.27	4117.93	57529.63	6.08
17	16119.84	235.8	75481.88	91837.53	68714.52	305.12	25906.27	94925.91	3.25
18	13978.24	1324.24	56516.68	71819.16	755.27	403.87	72984.37	74143.52	3.13
19	109679.84	3615.68	5009.22	118304.74	126009.48	137.98	948.55	127096.02	6.92
20	104905.84	3565.44	2333.22	110804.5	119423.72	156.72	1103.07	120683.51	8.19
21	26515.94	591.84	101932.98	129040.76	106030.31	125.62	25968.96	132124.89	2.33
22	13728.34	829.41	89547.88	104105.63	1306.51	366.19	107941.19	109613.88	5.03
23	127208.84	3275.8	51.42	130536.06	131704.96	41.4	1103.07	132849.42	1.74
24	151395.84	3370.94	5292.44	160059.22	166929.89	47.11	1103.07	168080.07	4.77
25	7468.84	609.21	28613.98	36692.03	21702.98	180.09	14842.34	36725.41	0.09
26	3142.94	720	26303.98	30166.92	2716.89	112.49	28861.78	31691.16	4.81
27	48312.34	2635.81	5988.1	56936.25	61158.17	165.36	99.91	61423.44	7.31
28	49297.24	2458.64	1752.4	53508.28	46677.11	392.05	8196.53	55265.68	3.18
29	17389.34	571.34	44912.78	62873.46	29825.53	378.76	35713.54	65917.83	4.62
30	6313.14	650.49	61429.58	68393.22	8454.63	295.54	61332.63	70082.8	2.41
31	106899.84	2259.66	5879.66	115039.16	123855.77	70.71	109.34	124035.82	7.25
32	104193.84	180.12	6269.05	110643.01	116483.94	2014.8	1103.07	119601.8	7.49
33	152520	19.76	8797.66	161337.42	157314.24	67.08	6419.5	163800.82	1.5
34	25024.72	801.21	81765.44	107591.37	8659.56	275.23	100911.89	109846.68	2.05
35	124406.42	2278.87	1114.25	127799.54	132494.3	637.66	3271.93	136403.89	6.31
36	154790.42	2737.15	1616.68	159144.25	169182.38	15.91	4772.91	173971.21	8.52
37	26649.37	38.34	37684.84	64372.55	40628.42	50.33	26826.81	67505.56	4.64
38	5075.58	38.4	39963.64	45077.62	5278.48	62.4	39754.3	45095.18	0.04
39	123368.42	1105.59	7463.44	131937.45	135812.25	50.43	1152.64	137015.32	3.71

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
40	188618.42	1573.06	1965.94	192157.42	180573.26	111.83	18641.52	199326.61	3.6
41	2758.4	13.03	78548.54	81319.97	2971.87	55.79	79185.37	82213.04	1.09
42	5075.58	38.7	78895.04	84009.32	5178.48	62.4	79266.79	84507.66	0.59
43	292482	236.57	13503.6	306222.17	323483.46	40.89	413.52	323937.87	5.47
44	321776.84	1434.1	2742.1	325953.04	338053.48	31.14	142.95	338227.57	3.63
45	3741.14	34.87	111048.58	114824.59	232.2	102.63	114894.02	115228.85	0.35
46	3595.97	38.97	119689.58	123324.53	5265.95	103.09	121238.29	126607.34	2.59
47	353532	146.46	119064	472742.46	365103.47	248.17	115798.33	481149.97	1.75
48	345297.84	202.67	117725.83	463226.34	358399.12	207.95	111818.08	470425.15	1.53

Table 23: Results for Run 4

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
1	6795.84	4.92	31891.7	38692.46	10846.09	8.53	29191.73	40046.35	3.38
2	0	0.02	42974.1	42974.12	0	0.02	42974.08	42974.1	0
3	159520	819.76	9797.66	170137.42	173314.24	67.08	4419.5	177800.82	4.31
4	127089	1015.33	2718.41	130822.74	129687.91	135.65	4656.04	134479.61	2.72
5	5719.87	66.56	75631.2	81417.63	4462.18	45.51	77374.86	81882.55	0.57
6	1176.7	18.01	82565.8	83760.51	470.68	2.5	83518	83991.18	0.27
7	386014	779.93	4354.56	391148.49	418055.07	87.52	2497.17	420639.77	7.01
8	309075.84	795.88	6113.68	315985.4	326429.36	33.91	552.83	327016.1	3.37
9	7503.16	52.67	127716.58	135272.41	7889.06	104.16	127832.64	135825.87	0.41
10	4172.35	48.48	121138.58	125359.41	5013.04	98.95	121875.19	126987.17	1.28
11	424.54	13.28	40221.5	40659.31	0	0	40870.18	40870.18	0.52

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
12	328749.42	89.25	121410.34	450249.01	341821.09	92.3	113761.36	455674.75	1.19
13	5340.62	1059.03	23099.54	29499.19	22854.13	560.52	7489.05	30903.7	4.54
14	1672.1	452.02	31541.14	33665.26	3726.56	28.18	32268.3	36023.04	6.55
15	41819.42	3752.21	469.2	46040.83	40596.28	2435.34	4584.19	47615.8	3.31
16	61921.82	2658.65	418.9	64999.37	58375.44	2691.79	7125.65	68192.88	4.68
17	31327.12	583.7	49800.04	81710.86	45554.75	619.96	38120.62	84295.33	3.07
18	12656.62	946.61	58976.84	72580.07	1353.26	229.08	76582.19	78164.53	7.14
19	106935.42	3722.31	428.06	111085.79	118701.97	65.6	272.12	119039.69	6.68
20	101878.42	3675.44	2821.83	108375.69	113773.4	162.42	4272.89	118208.71	8.32
21	30672.32	492.91	94639.24	125804.47	100514.25	109.07	25249.54	125872.86	0.05
22	28400.12	1174.28	74270.14	103844.54	18895.82	131.61	88121.11	107148.54	3.08
23	121836.42	3411.56	3377.34	128625.32	127093.65	219.23	4772.91	132085.8	2.62
24	147397.42	3749.53	3485.76	154632.71	159336.6	230.48	4772.91	164339.99	5.91
25	9674.29	120.31	32364.74	42159.35	30424.94	291.25	12424.31	43140.5	2.27
26	3722	756.72	23757.14	28235.87	17712.08	331.71	12106.94	30150.73	6.35
27	59035.62	2650.12	4297.63	65983.37	65141.08	296.45	3879.5	69317.03	4.81
28	58522.52	2537.92	624.21	61684.65	56316.06	366.38	5613.06	62295.49	0.98
29	13079.52	332.97	54656.34	68068.84	32611.98	103.74	35641.55	68357.27	0.42
30	14682.22	814.27	52742.24	68238.73	403.65	230.71	71155.21	71789.56	4.95
31	96455.42	2368.65	1930.21	100754.28	61165.21	337.3	41062.79	102565.31	1.77
32	112229.42	38.08	4810.6	117078.1	120744.94	2606.75	2333.86	125685.56	6.85
33	139394.08	467.4	18365.05	158226.53	141627.49	62.51	22756.33	164446.33	3.78
34	15900.44	459.43	95262.38	111622.25	1602.97	48.6	112375.08	114026.64	2.11
35	112416.84	2423.85	1237.79	116078.48	117439.3	59.86	797.44	118296.6	1.88
36	152479.84	2643.8	1481.4	156605.04	169483.36	17.05	756.08	170256.49	8.02
37	24191.22	37	40956.78	65185	44507.44	64.53	26019.14	70591.11	7.66

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
38	7503.16	52.09	41219.78	48775.04	7889.06	104.16	41371.71	49364.94	1.19
39	124850.84	1048.47	6369.4	132268.71	134311.45	24.79	63.1	134399.34	1.59
40	186612.84	1377.04	3306.91	191296.79	178510.32	172.69	22757.96	201440.96	5.04
41	7503.16	52.67	85818.68	93374.51	7889.06	104.16	85935.05	93928.28	0.59
42	7503.16	52.67	82894.78	90450.61	7889.06	104.16	83011.17	91004.39	0.61
43	274435.42	1160.97	2296.8	277893.19	294658.88	4.48	4430.71	299094.07	7.09
44	324618.42	1355.42	1970.11	327943.95	336278.87	23.87	3858.94	340161.68	3.59
45	5075.58	38.7	122099.34	127213.62	5178.48	62.4	122471.16	127712.03	0.39
46	3438.42	34.06	116761.34	120233.82	3941.32	54.39	118132.66	122128.38	1.55
47	354806	194.04	114706	469706.04	368132.51	117.12	107278.12	475527.76	1.22
48	379935.29	190.39	116891.55	497017.23	394233.99	150	112257.48	506641.47	1.9

Table 24: Results for Run 5

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
1	2687.94	2.69	49580.3	52270.93	23060.97	5.39	31464.67	54531.03	4.14
2	1574.16	68.79	38789.4	40432.35	181.4	9.87	40722.77	40914.04	1.18
3	141179	815.48	8927.87	150922.35	156430.19	104.09	3456.84	159991.12	5.67
4	128038	1090.3	3403.83	132532.13	133481.04	249.06	3748.38	137478.48	3.6
5	4619.55	38.39	79591	84248.94	921.66	2.24	83582.6	84506.51	0.3
6	1232.01	19.09	81964.6	83215.7	492.8	2.11	82968.75	83463.66	0.3
7	328049	806.54	7558.01	336413.55	363483.46	40.89	413.52	363937.87	7.56
8	288902.71	33.43	106091.51	395027.65	298542.17	54.49	97118.49	395715.15	0.17
9	3435.81	9.55	123128	126573.36	4082.76	8.61	122678.99	126770.36	0.16

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
10	2866.91	1023.7	23962.71	27853.32	20570.02	542.43	7438.78	28551.23	2.44
11	1498.83	43.92	39268.9	40811.65	450.28	9.04	40700.72	41160.04	0.85
12	326362.41	47.46	120731.82	447141.69	339450.29	80.67	115252.78	454783.74	1.68
13	2953.61	1017.24	22421.02	26391.87	18483.33	548.89	8980.47	28012.69	5.79
14	4059.11	410.23	30862.62	35331.96	7097.36	16.55	29759.72	36873.63	4.18
15	38225.47	2423.7	2075.61	42724.78	39432.41	3710.42	2147.72	45290.55	5.67
16	59534.81	2616.86	1097.42	63249.09	56004.63	2680.16	8617.08	67301.86	6.02
17	28940.11	541.91	49121.52	78603.54	43183.94	608.33	39612.04	83404.32	5.76
18	10269.61	904.82	58298.32	69472.74	3724.07	217.45	68073.61	72015.13	3.53
19	104548.41	3680.52	5106.58	113335.51	121331.16	53.97	2780.7	124165.83	8.72
20	109491.41	3633.65	2143.31	115268.37	119402.6	150.78	1281.47	120834.85	4.61
21	28285.31	451.12	93960.72	122697.15	98143.44	97.44	26740.97	124981.84	1.83
22	26013.11	1132.49	73591.62	100737.22	16525.02	119.98	89612.54	106257.53	5.2
23	119449.41	3369.77	4055.86	126875.04	124722.85	207.6	3281.49	128211.94	1.04
24	145010.41	3707.74	4164.28	152882.43	156965.79	218.85	3281.49	160466.13	4.73
25	7287.28	78.52	31686.22	39052.02	28054.14	279.62	13915.73	42249.48	7.57
26	1334.99	714.93	23078.62	25128.54	13341.27	320.08	13598.36	27259.71	7.82
27	56648.61	2608.33	3619.11	62876.05	64770.27	284.82	2388.08	67443.17	6.77
28	56135.51	2496.13	1302.73	59934.37	53945.25	354.74	6104.48	60404.48	0.78
29	10692.51	291.18	53977.82	64961.51	30241.17	92.11	37132.97	67466.25	3.71
30	19295.21	772.48	52063.72	72131.4	2774.46	219.07	72646.63	75640.16	4.64
31	94068.41	2326.86	2608.73	99004	58794.41	325.67	42554.21	101674.29	2.63
32	109842.41	3.71	4132.08	113978.2	118374.14	2595.12	842.44	121811.7	6.43
33	133076.39	17.98	21731.32	154825.69	138516.01	238.46	16620.51	155374.98	0.35
34	6375.44	257.14	100861.62	107494.21	22551.01	765.88	84628.61	107945.5	0.42
35	118210.18	619.57	3322.2	122151.95	121932.71	2243.54	1977.42	126153.67	3.17

Ins	CPLEX				Heuristic				Gap (%)
	Trans	Hold	Lost	Total	Trans	Hold	Lost	Total	
36	152316.71	2701.82	4823.18	159841.71	170898.27	2.18	753.51	171653.96	6.88
37	24175.66	73.67	38548.01	62797.34	38344.3	68.42	26776.55	65189.27	3.67
38	7462.59	80.49	39704.03	47247.12	7549.29	73.73	41226.81	48849.83	3.28
39	120894.71	1070.26	8326.61	130291.58	133528.14	32.34	1102.37	134662.85	3.25
40	186144.71	1537.73	1102.77	188785.21	178289.15	93.74	18591.25	196974.14	4.16
41	5255.99	37.7	78135.1	83428.79	5232.11	22.3	79411.71	84666.12	1.46
42	7462.59	80.49	78216.52	85759.6	7549.29	74.03	79758.21	87381.53	1.86
43	271961.71	1125.64	3159.97	276247.32	292374.77	22.57	480.98	292878.32	5.68
44	322144.71	1320.09	1106.94	324571.74	333994.76	5.78	3909.2	337909.74	3.95
45	7462.59	80.49	122420.89	129963.97	7549.29	74.03	123962.51	131585.83	1.23
46	6025.43	72.49	118082.39	124180.31	6112.13	69.39	119624.51	125806.03	1.29
47	373246	129.18	103438	476813.18	383157.5	543.57	97862.4	481563.47	0.99
48	360569.61	196.4	117348.24	478114.25	380189.25	183.76	109879.3	490252.32	2.48

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