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The Physical Meaning of the Field \( H \)

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ABSTRACT

Through the introduction of the concept of magnetic charge the physical meaning of the magnetic field \( H \) is established within the rationalized MKS system of units.

In developing the mathematical structure of electromagnetic theory certain postulates must be made. These postulates may take several different forms. The particular form chosen is dependent upon the system of units used and the goals one wishes to achieve in the development of the electromagnetic equations.

In the case of Gaussian units one would choose as the starting point Coulomb's law for the force between two electric charges and an identical form for magnetic charges, whereas in the case of the rationalized MKS system one chooses Coulomb's law for electrical charge and the relationship between a current element and the magnetic field produced by that element at some point in the space surrounding the conductor.

In the Gaussian system the meaning of the magnetic field \( H \) is made clear through its definition as the force per unit of magnetic charge, but within the rationalized MKS system the concept of magnetic charge is avoided and \( H \) usually is defined through the Biot-Savart law given by

\[
H = \frac{1}{4 \pi} \frac{1}{d} \frac{m}{r^2}, \tag{1}
\]

where \( H \) is specified by the physical configuration of the circuit and the current it carries. Through the Biot-Savart law, \( H \) is introduced as an abstract vector field whose physical meaning is obscure. The nearest approach to magnetic charge is the concept of magnetic moment which is defined entirely in terms of a torque exerted on a current loop. The use of (1) as the basic definition of the \( H \) field in the rationalized MKS system fails to give insight as to the physical meaning of the field. The concept of magnetic charge is useful in establishing a physical meaning for the magnetic field \( H \) when working in the MKS system of units.

The first step in the introduction of magnetic charge into the rationalized MKS system is to assume Coulomb's law for the force between two point magnetic poles, given by

\[
F = \frac{k}{4 \pi} \frac{m^* m^*}{r^2}. \tag{2}
\]

Through (2) a field \( H^* \) is defined as

\[
H^* = \frac{m^*}{r^2}. \tag{3}
\]

The defined field \( H^* \) represents the force per unit of magnetic charge \( m^* \). In the case of rationalized MKS units the value of \( k \) is determined through the definition of the ampere.

The use of \( H^* \) as the definition of magnetic field leads to difficulties when an attempt is made to incorporate the effects of magnetic materials into the field equations. A redefinition of the magnetic field and magnetic charge as

\[
H = \frac{H^*}{4 \pi} \frac{k}{m} \tag{4}
\]

and

\[
m = 4 \pi k \frac{m^*}{m} \tag{5}
\]

removes the difficulties encountered with the field \( H^* \). With the new definitions, Coulomb's law is given by

\[
F = \frac{m}{4 \pi} \frac{k}{r^2}. \tag{6}
\]

It has been said \( ^* \) that this form for Coulomb's law leads to difficulties with units. This is not necessarily true because the dimensions of magnetic charge are free to be chosen subject to their being consistent with (6). The newly defined field \( H \) is

\[
H = m / (4 \pi)^3 r^2. \tag{7}
\]

Here \( H \) represents the force per unit of magnetic charge \( m \). Inclusion of the effects of a magnetic material now leads to the familiar expression for the magnetic flux density

\[
B = 4 \pi k (H + M) \tag{8}
\]

where \( M \) is related to the bound charge density given by

\[
\rho = \nabla \cdot M \tag{9}
\]

rather than the \( \rho \) associated with the charge \( m \) in (6). Through the use of the concept of a magnetic shell \( ^* \) of strength \( \Phi = I \), one may show that the \( H \) defined through (7) and used to define the magnetic flux density \( B \) is established from the current through the Biot-Savart law (1). Therefore \( H \) as normally defined in the rationalized MKS system represents the force per unit of magnetic charge as defined through (6) with \( k \) set equal to \( \frac{\mu_0}{4 \pi} \).

The units of \( H \) are established from (1) as the amperes/meter. Through the definition of the ampere, \( \mu_0 \) is given the units of newton/(amperes)\(^2 \). The units of magnetic charge \( m \) are then defined by (6) to be the newton-meter/ampere which is the weber. The unit for the magnetic charge \( m^* \) as used in (2) is the ampere-meter. It is normal to use the ampere-meter \( ^* \) as the unit for magnetic dipole moment as opposed to the weber-meter associated with the charge \( m \).

The expression for (8), is an example where quantities involving units of magnetic charge are mixed. The case is that \( H \) is the force per unit of magnetic charge \( m \), but \( M \) is the magnetic moment per unit of volume associated with the dipole moment given by \( m^* d \) rather than \( md \) as one might expect.

Although magnetic charge has never been found to exist, it easily fits into the mathematical framework of electromagnetic theory and through it a useful physical meaning may be established for the magnetic field \( H \). This meaning is found through (7) where \( H \) is defined to be the force per unit of magnetic charge \( m \) satisfying Coulomb's law in the form (6).

LITERATURE CITED


