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Logical Transition from Magnetic Poles to Current Loops

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ABSTRACT

Consideration of the relationship between the magnetic field produced by a magnetic shell and that produced by a current loop requires the evaluation of the line integral \( \oint \mathbf{H} \cdot d\mathbf{l} \), where a portion of the path must cross the dipole layer. The correct line integral to be evaluated must be \( \oint \mathbf{B} \cdot d\mathbf{l} \). The evaluation of this integral leads immediately to the correct Maxwell equation in \( \nabla \times \mathbf{H} \). Gaussian units are used.

The assumption of the existence of magnetic poles leads to Maxwell's fourth electromagnetic equation by the evaluation of the line integral

\[ \oint \mathbf{H} \cdot d\mathbf{l}, \]

where a portion of the path must traverse a magnetic dipole layer. In the Gaussian system of units, the magnetic field \( \mathbf{H} \) is defined as the force per unit of magnetic charge. Using techniques familiar in electrostatics, one defines a magnetic potential \( \Phi \) such that \( \mathbf{H} = \nabla \Phi \), and for the dipole layer the potential is given by

\[ \Phi = -4\pi \phi, \]

\( \phi \) being the solid angle subtended by the double layer of magnetic charge at the point of the potential and \( \phi \) being the strength of the dipole layer. The work necessary to carry a unit of magnetic charge from one side of the dipole layer to the other is found to be

\[ 4\pi \phi. \]

The transition from magnetic effects attributed to magnetic poles to those attributed to Amperian currents usually is passed over lightly or ignored completely. Page (1952) makes the assumption that for a dipole layer the path integral for \( \mathbf{H} \) is given by

\[ \oint \mathbf{H} \cdot d\mathbf{l} = 4\pi \phi. \]

This assumption leads to difficulties when Amperian currents are included in the theory. The inclusion of these currents, forces a redefinition of the field \( \mathbf{H} \) if Maxwell's equation for curl \( \mathbf{H} \) is to be derived correctly. The redefinition of \( \mathbf{H} \), if carried back through the equations, leads to inconsistencies.

The fault lies with the assumption by Page (1952) that the closed path integral is equal to \( 4\pi \phi \). Examination shows that the integral should not be taken over a closed path, but must be written as

\[ \oint \mathbf{H} \cdot d\mathbf{l} = 4\pi \phi, \]

where \( \phi \) is an arbitrary open external path leading from the side of lower to the side of higher potential for the dipole layer. If the path to be closed, allowance must be made for that portion of the path passing through the dipole layer. No matter how thin the layer becomes, even in the limit of zero thickness, one must allow for the finite discontinuity in the potential as discussed by Stratton (1941).

It is easily shown that the \( \mathbf{H} \) field within the dipole layer is given by

\[ \mathbf{H} = -4\pi \mathbf{M}. \]

Thus for the case of the closed path integral for \( \mathbf{H} \), one finds

\[ \oint \mathbf{H} \cdot d\mathbf{l} = \oint \mathbf{B} \cdot d\mathbf{l} - 4\pi \mathbf{M} \cdot d\mathbf{l}. \]

The closed path integral may be used on the right hand side as \( \mathbf{M} = \mathbf{0} \) in the region external to the dipole layer. It is now evident that for the case of a magnetic dipole layer the path integral must be written as

\[ \oint \mathbf{B} \cdot d\mathbf{l} = 4\pi \phi, \]

and not \( \oint \mathbf{H} \cdot d\mathbf{l} \) as Page (1952) assumes. The magnetic potential is that which results from all currents, both conduction and bound if such exist.

The Amperian point of view for magnetism requires that bound current be given by the expression

\[ \mathbf{i}_b = c \oint \mathbf{M} \cdot d\mathbf{l} \]

as derived by Page (1952). Thus using \( \phi = i/c \), the path integral for \( \mathbf{B} \) may be written as

\[ \oint \mathbf{B} \cdot d\mathbf{l} = 4\pi (i + i_b)/c \]

and substituting the value for \( i_b \) into the equation yields

\[ \oint \mathbf{H} \cdot d\mathbf{l} = 4\pi i/c \]

which is the correct integral form for Maxwell's equation in curl \( \mathbf{H} \) for the static field case.

Consideration of the path integral as it penetrates the magnetic dipole layer gives a smooth transition from a theory based on the existence of magnetic poles to one requiring only currents produced by the motion of electric charge. In achieving the transition, the magnetic field \( \mathbf{H} \) is defined precisely as the force per unit of magnetic pole and requires no redefinition when Amperian currents are included and magnetic pole density is set equal to zero.

LITERATURE CITED
