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The Effects of Dynamic Graphing Utilities on Student Attitudes and Conceptual Understanding in College Algebra

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Abstract

The goal of this study is to explore and characterize the effects of using a dynamic graphing utility (DGU) on conceptual understanding and attitudes toward mathematics, measured by the responses of college algebra students to an attitude survey and concepts assessment. Two sections of college algebra taught by the primary researcher are included in the study, with one group using the dynamic graphing utility Desmos (www.desmos.com), and the other using the TI-84 (or equivalent) graphing calculator as a control. The Precalculus Concept Assessment (Carlson, Oehrtman, & Engelke, 2010) was used to measure aspects of students' conceptual understanding of course material, while the Student Attitude Survey developed at UMass-Dartmouth (Brookstein, Hegedus, Dalton, Moniz, & Tapper, 2011) was adopted to explore student attitudes. Both instruments were administered at the beginning and end of the 16-week term. Although no statistically significant overall difference was detected in the change in mean PCA scores between the two groups, analysis of results suggests that there were differences in the type of reasoning abilities whose development was supported by the use of the two devices. However, the largest PCA score improvements occurred among female students in the treatment group. The most consistent results from the SAS concerned student attitudes toward group work, indicating a negative trend in this attitude component. There were some indications that students using the DGU Desmos were more engaged with using technology as part of the learning process. Implications and limitations of the present study are discussed in detail, as well as directions and suggestions for future research.

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I am very thankful to all my family for their love and support throughout my education, and especially to my parents, Randy and Dana Thomas. They've been behind me every step of the way and I would not have been able to do any of this without their support. Most of all, I am grateful for the opportunities provided by a loving God. Not only did I come out of this thing with a (hard-earned) doctorate, but I have also made good friends and found my officemate-turned-wife, Emily. There is no way I could have done this without you, Em.

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Chapter 1 - Introduction

The Problem and Its Background

As technology advances, evolves, and becomes more widely available, educators must continue to find ways to adapt to and plan for the types of tools that will be accessible to students. In the past few decades the development and subsequent propagation of the graphing calculator has brought widespread change to mathematics curricula and instruction. Moving forward, ease of access to computers, tablets, smartphones, and other devices, as well as improvements in educational software, promise to further revolutionize both what and how we teach. Although it is impossible to know what innovations the future holds, educators should make every effort to maximize the potential benefits of educational technology for students.

The first commercially available graphing calculators appeared on the market in 1985, in the midst of a revolution in mathematics education. Plummeting test scores and fears of falling behind the rest of the world in science, technology, engineering, and mathematics (STEM) fields produced watershed documents in the form of *An Agenda for Action* (NCTM, 1980) and *A Nation at Risk* (National Commission on Excellence in Education, 1983). Calls for increased attention and accountability in education were accompanied by specific recommendations for educators to take advantage of newly available technologies, especially classroom computers and handheld calculators. When the first graphing calculators emerged, and in particular as more affordable versions hit the market, many practitioners saw the upside of the devices. The first research dissertations dealing with the use of graphing calculators as tools for teaching and learning mathematics began to appear in 1990 (Penglase & Arnold, 1996), although a number of practitioner pieces and editorials preached the potential benefits of the new technology prior to published research results. Many authors writing in the late 1980s and early 1990s (e.g., Burrill,

1992) believed that graphing calculators had the “potential to revolutionize mathematics education, both in the way it is taught and the content and emphases of curricula” (Penglase & Arnold, 1996, p. 58). Some of this promise has come to fruition; standards documents have codified the place of graphing calculators and computers in the mathematics classroom, curriculum materials have been developed that allow teachers to easily incorporate technology, and innovative teachers have found creative uses for available devices. On the other hand, basic skills (pencil-and-paper calculations, standard algorithms, etc) and solving problems that are devoid of context continue to be a significant curricular component (Dingman & Madison, 2010), doing nothing to help students see the connections between classroom mathematics and real-world applications that are relevant to their daily lives.

While educational standards moved to embrace calculator and computer use in the mathematics classroom, those opposed to reform efforts argued that students would become overly dependent on such technologies and use them as a means to avoid learning. Indeed, technology use became something of a sticking point for both sides during the “math wars” of the 1990s and early 2000s (Jackson, 1997; Schoenfeld, 2004). Ultimately, Schwartz (1996) concludes that calculators and related technology have two primary pedagogical virtues: they increase the rate at which students gain experience with mathematics, and concentrate attention on essential points of the material rather than on arithmetic and tedious calculation. However, Esty (2000a) offers an opposing interpretation of these same characteristics, noting that students may misuse technology to increase the rate at which they can “*do* traditional problems *without* gathering experience with the subject” (p. 2), and complete traditional problems without paying particular attention to any mathematically interesting point. The key bit of phrasing here is that *traditional* problems can lend themselves to such abuses of technology, and this is especially true when too powerful a tool is provided or allowed. For example, if entering “solve(x+2=5, x)” is all that is required of a student, then while that student may be learning a bit about computer science, little mathematics is occurring.

In light of this, the question then becomes, as Esty (1998) phrases it, “how can we use an exercise to teach students something worth knowing, instead of merely exercising the student’s calculator?” (p. 1). If the example in the previous paragraph was an exercise in and of itself, then its utility as a site for learning about mathematics would be questionable at best. However, if it were merely the final step in solving an open-ended problem that drew attention to key properties of linear functions, our opinion of its usefulness might be different. Graphing calculators, dynamic geometry software, and other educational technology can empower students to explore concepts on their own (Quesada & Cooper, 2010) and allow the learning process to focus less on computation and more on concepts (Esty, 1998), but only if a corresponding shift in pedagogy and curriculum seeks to use technology as a learning tool in an appropriate way. Calculators and computers were originally intended as tools to help expert mathematicians and scientists perform more calculations in less time; as they become more user-friendly, they require less knowledge of their own inner workings to operate, and thus become easier to misuse to avoid learning. Educators and policymakers should be aware of this possibility, and make conscious decisions that help to mitigate or eliminate the possibility of such misuse.

Although graphing calculators are widely accepted and used, particularly in grades 9-12 and post-secondary settings, they are not without faults. Mitchelmore and Cavanagh (2000) conducted interviews with high school students that were designed to investigate difficulties and misconceptions that the students had while using a graphing calculator in the context of linear and quadratic graphs. While some of the issues encountered were mathematical in nature, others were related directly to the design and technical limitations of the calculator itself. Anderson, Miller, and Wang (2009) identified and classified many of the common errors made by students using handheld calculators. Once again, some of these errors were caused by misconceptions or insufficient awareness of mathematical concepts (e.g., order of operations), but others, such as the somewhat specialized syntax required to perform some functions in the calculator, are a product of the inherent nature of the device. Although graphing calculators are capable of performing many useful tasks, the process required to do so can present a stumbling block for students. For example, finding the zeros of a function graphically using a calculator from the TI-84 family can be quite involved, and a student who possesses good visual intuition about where the zeros occur may yet be unable to determine their exact value due to misunderstandings about operating the device. As a result, classroom teachers may feel inclined to address technology issues directly, shifting focus from the mathematical content to the calculator. Depending on the course in question, a limited amount of direct attention to using technology may be unavoidable and even beneficial. However, this must be carefully balanced with maintaining the central focus on mathematical content, and should be an aspect of teaching practice that is subject to regular self-evaluation.

Purpose of the Study

Ease of access to the internet, whether through personal computers, smartphones, or tablets, affords the opportunity to significantly change the way that classroom technology is used. After all, the graphing calculator may be good at what it does, but how often are they used by working professionals? Rather than introduce students to a specialized tool such as the graphing calculator, using mobile technology that is both readily available and more familiar in design may provide additional benefits. In the present study, the free online graphing utility *Desmos* (www.desmos.com/calculator) was used in instruction and assessments, with the hope that students would find it more intuitive and accessible than the graphing calculator.

Desmos addresses many of the shortcomings of handheld graphing calculators. It is entirely free, uses HTML5 to run in a web browser with no additional downloads or installation required, and can be used as an app on many smartphones and tablets (“Desmos | About Us”, 2015). In addition to its availability, many features of Desmos are very intuitive; for example, zooming in or out on a graph can be accomplished by mouse scrolling on a personal computer, or by using the familiar pinch-zoom gesture on touchscreen devices. These intuitive design choices make using Desmos more relatable and familiar for students, in turn reducing the attention needed to teach students the technology and allowing for increased focus on developing skills, strategies, and mathematical concepts. Desmos also provides a fertile ground for student exploration, as the list of inputs and the graph itself are shown side-by-side and updates happen in real-time. Students can employ sliders to stand for values in equations to experiment with the effects of changing different components of an equation. Although the available literature on Desmos is limited, Beigie (2014) and Ebert (2014) found it to be a very useful tool for getting students to actively participate in mathematics. In addition, my personal experience using Desmos in the classroom has verified that many students engage with it and prefer it over the graphing calculator.

Writing in 1990, Demana and Waits observed that technology was poised to significantly impact mathematics education, writing that “the importance of, and need for, highly proficient skill in arithmetic and algebraic paper-and-pencil manipulation in the workplace has already been rendered nearly obsolete by technology” (p. 27). Twenty-five years on, this is all the more accurate. Movements within mathematics education, including the various standards movements in K-12 and the development and implementation of Quantitative Reasoning/Literacy courses in many colleges and universities, are evidence of the changing face of school mathematics. The goal of many of these efforts is to couch mathematics in real-world contexts in order to help students see the connections between what happens in the classroom and their lives (Dingman & Madison, 2010). However, in this time of transition, many students with little interest in continuing in mathematics still find themselves in traditional courses, such as college algebra.

At this critical juncture, it is vital that research into the effects of this technology on student learning be conducted to inform educational policy, curriculum and textbook design, and the practice of teachers in the field in order to utilize new tools effectively and improve the overall quality of mathematics instruction. In the present study, having taught college algebra for several years with a variety of curricular materials and pedagogical approaches, I sought a means to help students engage with the course content in a way that would be lasting and meaningful for them.

Statement of the Problem

Service-level mathematics courses like college algebra often need to serve two purposes and two groups of students: on one hand, there are those who need preparation (and often remediation) before moving on to courses in higher mathematics or other disciplines, and on the other, there are students who are in a terminal course to satisfy a general education degree requirement. At some point in the future, these terminal students may instead be required to take some type of quantitative literacy course, and will likely be better served by it, but at present many find themselves in college algebra. The question then is how to best serve both groups; in particular, engendering confidence and understanding of specific concepts and topics, while also building problem-solving skills, number sense, and a positive attitude toward mathematics. Integrating technology into the learning process has been one solution to this problem, but in many cases the technology used has failed to evolve beyond the graphing calculators introduced twenty years ago. In the present study, my goal was to use a modern graphing utility in college algebra in order to investigate any differences in student outcomes, as compared to a similar group of students using the graphing calculator in an (essentially) identical class setting.

Research Questions

The primary interest of this study is the effect of a modern, dynamic graphing utility (Desmos) on student outcomes, as compared to a control group using graphing calculators in an essentially identical learning environment. I will be examining the following research questions:

1. Does using Desmos (rather than a graphing calculator) lead to improvements in college algebra students' understanding of the concepts in the course?
2. Does the use of Desmos (rather than a graphing calculator) positively impact college algebra students' attitudes toward mathematics?

Theoretical Considerations and Definitions

One of the flaws in much of the existing research on graphing utilities is that most of the studies compare the results of classes taught using the graphing calculator to those taught using "traditional" methods on assessments over the same topics, but often without providing much detail into the context of the instructional methods used or documentation of how the calculator affects student understanding (Cavanagh & Mitchelmore, 2003; Penglase & Arnold, 1996). Introducing technology into the classroom allows for a paradigm shift; new avenues of exploration, problem solving methods, and modeling opportunities become available. However, it is also possible to adopt a new tool without making any real changes to curriculum, pedagogy, or assessment. In light of this, the classroom context in which calculators or other graphing utilities are used could significantly impact their effect on outcomes, in terms of student learning, achievement, and attitudes toward mathematics.

This study takes place in several sections of college algebra at a public, four-year university in the southern United States. The curriculum for this course was determined by state standards, and included traditional topics such as a survey of polynomial, exponential, and logarithmic functions, systems of equations, and an introduction to regression. A hybrid online model was used for the course; in particular, the class met regularly (in person) while students completed the bulk of their coursework online using the service MyLabsPlus. Class meetings used a partially “flipped” design, which required students to read pre-class lessons (PCLs) and view instructional videos, the majority of which were authored within the department and specifically tailored to the course. These materials were intended to give students familiarity with definitions, terminology, and example problems so that class time could be used more efficiently. In-class activities included small group and individual work on sample problems and worksheets, discussions in both small group and whole-class settings, and a limited amount of direct instruction. Throughout, effort was given to emphasizing the connections among course topics and the relationship between the skills required to complete exercises and the “bigger picture”. A multi-representational view of the function concept was employed for the duration of the course, and students were encouraged at various times to make use of symbolic, graphical, and numerical forms of functions in problem solving. In addition, the idea of function as a means of describing the covariation of two quantities (Thompson & Carlson, in press; Oehrtman, Carlson, & Thompson, 2008) was often discussed in class, though admittedly there were limited opportunities to assess students’ covariational reasoning.

In the interest of clarity, some terms that will be used throughout are briefly defined below. For additional details, please refer to chapter 2.

- The major curriculum reform efforts in the wake of the “Back to Basics” movement of the 1970s have all made calls for an increased focus on going beyond basic skills and rote memorization in mathematics education. This is reflected in the learning principle from the NCTM *Principles & Standards for School Mathematics* (2000), which calls for a combination of factual knowledge, procedural facility, and conceptual understanding, as well as in the CCSSM *Standards for Mathematical Practice* (2010). The concept of ***learning with understanding*** summarizes this idea nicely; Hiebert et al. (1997) define understanding by saying that “we understand something if we see how it is related or connected to other things we know” (p. 4).

- Hiebert and Lefevre (1986) argue that distinguishing between ***conceptual*** and ***procedural*** knowledge can provide a useful framework for thinking about mathematics learning. In short, procedural knowledge comprises the rules, algorithms, and procedures for completing mathematical tasks, as well as the formal language and symbolism of mathematics. On the other hand, conceptual knowledge, as defined by Hiebert and Lefevre, is that which is “rich in relationships” (p. 7) and can be thought of as a “connected web of knowledge” (p. 7). As a practical means of thinking about mathematics learning, procedural knowledge constitutes a large part of “how” to go about solving problems, while conceptual knowledge has more to do with “why” something works the way it does.
- Students in the treatment group made use of their ***personal devices*** to access Desmos during class and while completing assignments. These included laptops, tablets, and smartphones, and will collectively be referred to as students’ “personal” and/or “mobile” devices or technology. A “BYOD” (“bring your own device”) policy was in place in the classroom, and students also had access to a BYOD learning center staffed by instructors, graduate students, and learning assistants employed by the department to provide mathematical support.

- Classroom interactions involved limited direct instruction, group and individual work, open discussions intended to facilitate student inquiry, and activities meant to motivate and underscore concepts and connections. In all phases, a constructivist stance on learning drove pedagogical decisions and, as much as possible, I encouraged students to make conjectures and check for themselves, rather than "telling".
- The tool used in this experiment is the online graphing utility Desmos, but it may be the case that other similar tools exist or are developed in the future. As a result, I will use the term ***dynamic graphing utility*** (DGU) to refer to any graphing tool that shares the key characteristics of Desmos; namely, intuitive design features, instant feedback, a modern display, etc. Here *dynamic* refers to the real-time updates and single-screen design that provide students with immediate feedback when they change something. For example, graphing the slope-intercept equation of a line with sliders for the "slope" and "y-intercept" parameters (as shown in figure 1 below) allows students the opportunity to conjecture, test, and generalize the effects of changing the values of *a* and *b*.

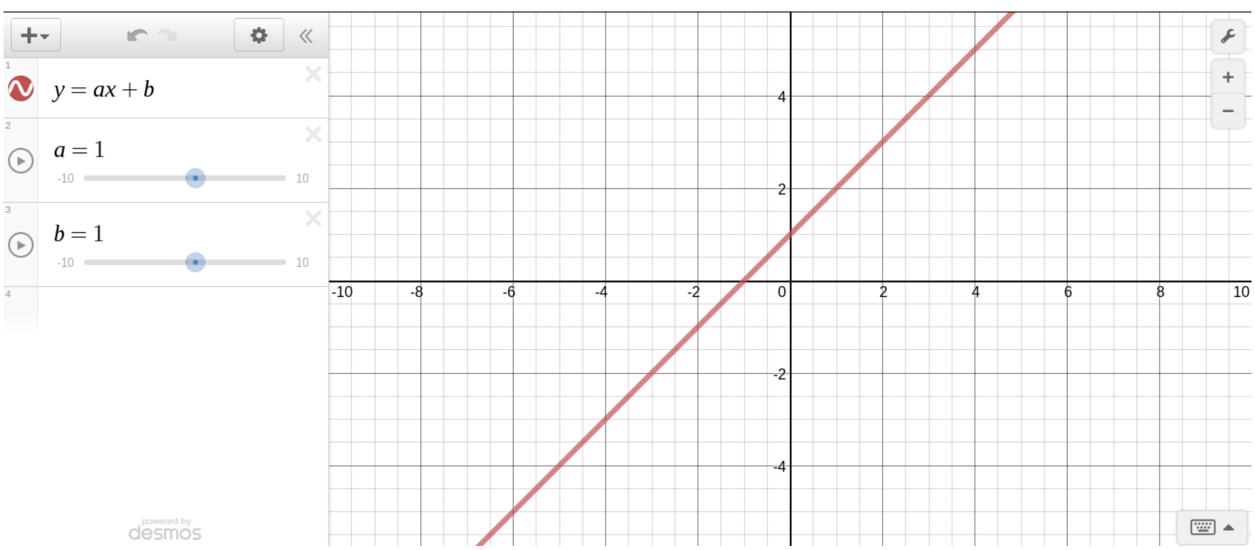

Figure 1. Example of using sliders in Desmos. This figure illustrates the dynamic nature of graphing in Desmos; dragging the sliders for the values of *a* and *b* changes the appearance of the line in real-time.

Significance of the Study

The present study is unique in the literature of teaching mathematics with technology for several reasons. First of all, it is (to the best of our knowledge) the first study to incorporate the graphing utility Desmos and examine its effects on student outcomes. Existing research using Desmos is limited to practitioner pieces (Beigie, 2014; Ebert, 2014; Locke, 2015), and although there is an existing body of research on graphing calculators and dynamic geometry software (e.g., Geometer's Sketchpad), the accessibility, versatility, and intuitive design of Desmos make it significantly different than other graphing utilities and, therefore, worth investigating in detail. Prior experience using Desmos in the classroom has indicated that it offers rich opportunities for engaging students and removing obstacles to their understanding of, and success with, mathematics. Students in previous classes (who were unable to use Desmos on formal assessments) were often observed using Desmos during in-class activities, and on more than one occasion groups of students independently adopted Desmos to perform regression on data sets and create graphs as part of an out-of-class group project.

Although not unique to this study, the design of the present research program is also uncommon among the literature, in that both treatment and control groups were taught by the same instructor, thereby controlling for any teacher effect. In addition, both groups were assessed using the same instruments and, as much as possible, were given the same learning opportunities. These design decisions were intended to address the shortcomings found by Penglase and Arnold (1996) in the existing literature; in particular, that many studies fail to distinguish "the role of the tool from that of the learning process" (p. 58) and that "claims regarding the relative effectiveness of the tool… fail decisively to account for important influences upon attitudes and conceptual understanding" (p. 82). Although it is difficult to separate the tool itself from the setting in which it is used, my hope is that by providing (as much as possible) the same learning context for both groups, the effects of the tool will become easier to identify. The sample size is admittedly small, but the results (which will be discussed in detail in chapter 4) may serve to inform decisions about the direction of related future research and classroom practice.

Summary

This study fills a gap in the existing literature concerning graphing technology in mathematics education by using a DGU (Desmos) in a treatment group and comparing student outcomes directly to a control group using TI-84-class graphing calculators. Both treatment and control groups were taught by the author. As much as possible, both sections were handled identically; they were given the same assignments, in-class activities, and learning opportunities, although class discussions and the like sometimes followed slightly different paths due to their student-centered nature. Both classes completed the same homework, quizzes, tests, and projects throughout the semester, with the exception that the treatment group used Desmos while the control used the graphing calculator. Students were given pre- and post-assessments to measure their conceptual understanding and attitudes toward mathematics. By removing teacher and pedagogy effects, the design of this experiment should isolate any effects of the tool used to a reasonable degree; however, as ever, there are many factors that could play into the results.

Chapter 2 - Review of the Existing Literature

In this chapter, an overview of the existing literature as it relates to educational technology, graphing calculators, Desmos and dynamic geometry software, conceptual understanding of mathematics, and students' attitudes toward mathematics will be given, along with discussion to illustrate its relation to the current study.

Technology

Technology use is increasingly prevalent in nearly all aspects of daily life, a trend that is reflected in the evolution of school mathematics since handheld (and later graphing) calculators and classroom computers became widely available. Evidence of the tendency toward integrating technology in mathematics education can be seen in the literature in practitioner pieces, curriculum materials, and research articles, as well as in policy statements from groups such as the NCTM and the United States Department of Education. In its 1980 call to arms, *An Agenda for Action*, the NCTM recommended that "mathematics programs take full advantage of the power of calculators and computers at all grade levels" (p. 1). This sentiment is echoed throughout the high-profile mathematics education publications of the 1980s, with similar recommendations appearing in *A Nation at Risk* (National Commission on Excellence in Education, 1983) and the NCTM *Curriculum & Evaluation Standards for School Mathematics* (1989). Subsequent standards documents, including the NCTM *Principles and Standards for School Mathematics* (2000) and the *Common Core State Standards for Mathematics* (2010) codified this call to embrace modern technology in mathematics classrooms. As a result of these developments, graphing calculators, classroom computers, and online systems for completing homework and other assignments have become commonplace in secondary and college mathematics courses. Further advances in technology are likely to make its use in the classroom even more routine, so it is important that the mathematics education community makes an effort to fully understand best practices and potential outcomes of technology integration.

Innovations in technology have heavily impacted industry as well, as noted by Demana and Waits (1990), who observed that paper-and-pencil algebraic and arithmetic manipulations were nearly obsolete in the workplace, having been replaced in many instances by emerging technologies. Although education is about more than serving the needs of industry, it is interesting to note that an observable shift in the role that mathematics plays in the professional lives of non-mathematicians could be seen twenty-five years ago, yet many mathematics classrooms still devote a great deal of effort to paper-and-pencil mechanics. Demana and Waits go on to make the case that mathematics educators should strive to help students learn how and when to make use of technology, and that students “must be trained as flexible problem solvers, capable of understanding and employing technological advances as they occur” (p. 31). This argument, particularly the need for developing flexibility in solving problems, has been one of the driving forces behind the movement to institute quantitative literacy courses as an alternative to college algebra (Steen, 2001; Dingman & Madison, 2010; Diefenderfer, 2012). However, even in a more traditional course, an approach to problem solving that involves more *thinking* and less *remembering* is beneficial. If students can learn to use technology as a tool to support their thinking, they may also come to rely on themselves for verifying mathematical validity. This is not only a useful predisposition for those continuing on to subsequent mathematics courses, but also contributes to students’ overall ability to deal with numbers in the real world.

Aziz (2010) defines educational technology as the “considered implementation of appropriate tools, techniques, or processes that facilitate the application of sense, memory, and cognition to enhance teaching processes and improve learning outcomes” (p. 1). It seems that the focus is too often solely on the latter - improving learning outcomes - without the time and reflection that constitute *considered* implementation. The official position statement of the NCTM on the role of technology in the teaching and learning of mathematics (2011) takes a similar stance, calling for strategic use of technology and arguing that simply providing access to technology is not sufficient for change to occur. In practice, the curriculum and teacher have a critical role in the use of technology (Roschelle, et al., 2010; Suh, 2010), and can heavily influence potential student outcomes. Assessing the efficacy of technology use, both in terms of the tool itself and the strategies employed in its use, should be a key part of practice. Ineffective tools or methods should be improved upon or discarded, as indiscriminate use of scarce resources is unlikely to benefit anyone. However, Aziz notes that such careless implementation is found all too often in education. In some ways, haphazard technology use mirrors the curriculum reform process; Fey and Graeber (2003) have referred to this as a cycle of “crisis-reform-reaction”, in which dramatic improvement is expected almost immediately. Without careful implementation, involving reflection and self-assessment on the part of teachers, educational technology of any sort is unlikely to produce clear, meaningful student gains.

Technology has already revolutionized many aspects of life, and education has been no exception. Computers and handheld calculators have significantly impacted school mathematics in the course of the last thirty years, but as Keengwe, Onchwari, and Wachira (2008) observe, the potential for classroom technology foreseen in the 1980s has not been fulfilled. Although there have been changes in the way classroom technology is implemented and creativity in its use is trending upward, the dramatic improvements that were speculated upon in the 1980s have not materialized. However, as emerging technologies such as tablets and online applications become more readily available, educators should explore new possibilities. In many circumstances, existing tools and methods are deeply entrenched, and the glacial pace at which things sometimes change in education stands in stark contrast to the rapid development of technology. Educators at all levels should be willing to explore new tools and strategies in the classroom, rather than becoming complacent in the status quo.

Graphing Calculators

Alongside calls for embracing graphing technology, there has been a desire among educators and researchers to determine the effect of integrating these tools into the mathematics classroom. This research has produced mixed results at times; some authors have found a positive correlation between graphing calculator use and student achievement, while others detected no significant differences in the two groups. At first glance, it appears that the existing literature fails to provide a conclusive answer to the question of whether or not graphing calculators are beneficial to students' understanding of mathematics. However, upon closer inspection, the issue is more complicated than this. Doerr and Zangor (2000), in reviewing the existing literature on graphing calculator use, found that "many, if not most, graphing calculator studies are quasi-experimental in design and seek to answer the question of whether or not graphing calculators are effective in achieving certain instructional objectives, which are often left unchanged from traditional paper-and-pencil approaches" (p. 144). Furthermore, Penglase and Arnold (1996) noted that much of the existing body of literature fails to distinguish between the use of technology and the context in which it is used, which lends more support to the argument that although the existing literature is indicative of results, it also raises many questions and should be interpreted and applied carefully.

With 25+ years of research on graphing calculators available, we have the benefit of meta-analyses that combine the results of many individual studies, providing a more comprehensive picture of the research. Ellington's (2003) meta-analysis of research on the effects of technology in pre-college mathematics integrated 54 studies and found encouraging results. In particular, Ellington found that graphing calculator use is correlated with improvements in students' conceptual and problem-solving skills, that students' operational skills benefit from calculator use, and that graphing calculators may positively influence student attitudes toward mathematics. Ellington's summary of her findings states that the greatest student gains were found when calculators took on a pedagogical role in the classroom, beyond being available for checking work or drill, which illustrates the need to modify instructional approaches to maximize the potential benefits of including a graphing utility in the classroom. It should also be noted here that the vast majority of studies included in this meta-analysis did not use curricular materials designed for instruction with calculators. However, in 36 of the 54 included studies, teachers did make pedagogical use of the calculator in instruction. This shows some initiative on the part of mathematics teachers to change classroom practice to incorporate technology, even in the absence of purpose-built curricular materials.

Also of note, Ellington indicated that in all cases, even those in which no significant improvement was detected, there was no evidence to suggest that calculator use had hindered the development of students' mathematical skills. This provides evidence counter to one of the most common arguments against the graphing calculator; namely that it would hold back students' skill development. It should be noted that the majority of the studies included in Ellington's meta-analysis were conducted in what might be called "traditional" mathematics classrooms, and that in only 6 of the 54 studies were specially-designed curricular materials used to fully integrate the graphing calculator in the learning process. However, two-thirds of the studies did feature the calculator as what Ellington describes as an active part of teaching and learning (p. 455). The results of this study also suggest that calculator use in the classroom should be part of a long-term approach to mathematics instruction, with the largest gains in operational skills and student attitudes toward mathematics occurring in those studies that lasted 9 or more weeks. Ellington recommended that future research should investigate the retention and transfer of skills, the role that the graphing calculator plays in the development of students' problem-solving abilities, and the use of curricular materials that are purposefully designed with the graphing calculator in mind.

In a 1994 study on the effects of graphing calculator use on students' performance in precalculus, Quesada and Maxwell found significant positive results in terms of student achievement on a common final exam when compared to students taught using traditional methods. Furthermore, the authors found that students in the experimental group (i.e., those using the graphing calculator) indicated that "they perceived that they had done more exploration, that they believed that the graphing calculator helped them to understand the concepts studied in the course, and that they will take following courses in sections where the graphing calculator is used" (p. 211-212). On the other hand, students in the experimental group reported spending more time and performing slightly worse than in previous mathematics classes, although this may be due at least in part to normal progression. The authors' analysis of student responses to open-ended survey questions about the students' experience with the graphing calculator found three main positive aspects; namely, that the graphing calculator facilitates understanding, allows for answer-checking, and saves time on calculations. For many of these students, "checking answers" extended beyond confirming answers graphically or numerically to a more holistic process of engaging in thinking about problems visually before attempting to solve algebraically. This supports the notion that graphing calculator use can contribute to the development of visual intuition, an important tool in the study of mathematics.

As is often the case with educational research, studies on the effects of graphing calculators must be interpreted carefully due to the difficulty of isolating a single factor. The complexity of the learning process and the huge number of elements that can affect student outcomes makes it very difficult to know exactly what causes change, and it is probable that in many cases several factors contribute to bring about improvements. Quesada and Maxwell (1994) cite several factors that may have supported students' learning and contributed to the positive effects observed when using the graphing calculator; among these, interactive presentation of topics, immediate feedback and the ability to check answers, development of visualization skills, and students' self-construction of knowledge (p. 214), stand out as qualities that go hand-in-hand with the introduction of the graphing calculator in the classroom. Esty (1998, 2000a, 2000b) emphasizes that employing graphing calculators must be accompanied by a corresponding change in pedagogy and curricular focus. In particular, the goal must become teaching students "something worth knowing, instead of merely exercising the student's calculator" (1998, p. 1). The graphing calculator (and indeed, many other technological innovations) allow educators to focus less on computational tasks and more on conceptual aspects of the curriculum; however, they can also be misused as a means to avoid involvement (2000a) and replace learning with finding answers (2000b). Thus simply adding the graphing calculator (or any other tool) to an otherwise unchanged classroom is unlikely to result in more learning or improvements in student outcomes. Instead, pedagogy, curriculum, and assessments should all be carefully adapted to accommodate the new approaches to mathematics made possible by introducing new tools.

In the context of the present study, the function concept is the linchpin of the curriculum. Many sources (e.g., Tall, McGowen, & DeMarois, 2000; Oehrtman, Carlson, & Thompson, 2008; Carlson, Oehrtman, & Engelke, 2010; Musgrave & Thompson, 2014) have noted its importance, with Carlson et al. calling it the core mathematical concept of the curriculum from algebra through calculus. In light of this, it seems appropriate to examine the documented connections between the use of graphing calculators and students' understanding of the function concept. Several sources (Leinhardt, Zaslavsky, & Stein, 1990; Wilson & Krapfl, 1994; Doerr & Zangor, 2000) have noted that the use of a graphing utility allows students to generate many examples of graphs quickly and accurately. In turn, this makes different learning opportunities feasible; for example, asking students to graph several transformations of a given function, make conjectures regarding the “rules” for graph transformations, and then verify their thinking could be a very productive and memorable activity for beginning algebra students. However, asking students to complete this same task using only pencil and paper would likely be much more time-consuming, as well as presenting additional conceptual difficulties due to the loss of precision brought on by manual graphing. This activity (and many others like it) depend on two key capabilities of the graphing calculator: being able to simultaneously view several related graphs, and the ability to change a parameter and observe the results with little additional effort required (Demana, Schoen, & Waits, 1993).

The ability to easily access multiple representations (i.e., graphic, symbolic, and tabular) of a function is another potential benefit to using graphing calculators (Hollar & Norwood, 1999), and one that can be especially useful in the context of problem solving. Depending on the context of the problem situation, the slight differences in the type of information gained by looking at each of these representations can provide insight. As students gain experience with using a graphing calculator to solve problems, they may come to realize the benefits that fluid movement among these representations offers, and in turn develop a more cohesive understanding of functions in general. Ruthven (1990) writes

> reliable access to graphic calculators is likely to encourage both students and teachers to make more use of graphic approaches in solving problems and developing new mathematical ideas, not only strengthening these specific relationships, but rehearsing more general relationships between graphic and symbolic forms (p. 447),

reinforcing the idea that multiple forms of the same function can be valuable in problem situations due to the differences in the information they present. Hollar and Norwood (1999) observed that the graphing calculator enabled students to approach problems from a different perspective, using multiple representations to explore and estimate; that is, students may use graphic forms to visualize problem situations and estimate solutions, then (if more precision is needed) explore the problem in more detail using tabular and symbolic representations.

Having access to multiple forms of the same function, coupled with the precision and accuracy of generated graphs, provides the opportunity for the graphing calculator to transform the way that mathematics students approach problem solving. Writing in 1994, Wilson and Krapfl relate the experience of Ken, a preservice mathematics teacher previously studied by Wilson (1992), as he developed strategies for using the graphing calculator. Prior to using a graphing calculator, Ken did not use graph-centric methods for solving problems due to the loss of accuracy and precision that comes with manual graphing. However, the capability of the graphing calculator to quickly create accurate graphical representations of functions makes it viable to solve (or at least obtain good estimates) using graphical methods. The authors contend that “the graphics calculator helped Ken understand functions and graphs in a fundamentally different way - as tools to help him interpret and *solve* mathematical problems” (1994, p. 255). The details of symbolic manipulation in problem solving can change from one problem to the next (e.g., solving an equation involving a quadratic expression versus one involving logarithms), which can distract students from the context when attempting to solve real-world problems and instead draw their focus to trying to decide what “type” of problem is at hand. On the other hand, using graphic and tabular representations to supplement the symbolic form of a function as tools for interpreting and solving could allow for a more connected view of mathematics. Ruthven (1990) observed that students who made consistent use of the graphing calculator in problem solving tended to show more innovation in their strategies. This follows logically; having more options available - namely, easy access to symbolic, graphic, and tabular representations of a function - allows students more opportunities to experiment, explore, and get themselves “unstuck”.

Finally, an important advantage offered by graphing calculators (and other computer tools) is that they provide students with immediate and personalized feedback in a way that is not possible using traditional pencil-and-paper methods. Ruthven (1990) observed that this feedback can reduce students’ anxiety to a degree by relieving uncertainty. Since anxiety about mathematics has been linked to poor performance and overall avoidance of the subject (Hembree, 1990), alleviating this anxiety would likely be beneficial for many students. Furthermore, research has noted that this feedback loop not only leads to reduced anxiety, but can also help build students’ confidence in their mathematical abilities (Ruthven, 1990; Dunham & Dick, 1994; Idris, 2006). Students have also reported their own perception that the graphing calculator is a useful tool that helps them to develop a more coherent understanding, or visualize certain ideas better (Smith & Shotsberger, 1997). Perception plays an important role in attitude, as perception affects self-confidence, which in turn contributes to attitude and motivation. Here again there is a role played by the duration of the exposure to technology, with sustained use resulting in higher levels of affect. Hennessy & Dunham (2001) contend that technology use can temper negative attitudes over a long period of time, but short-term use of a tool is less likely to result in lasting change.

Graphing calculators are not without fault, however, and the potential drawbacks to using the device have also been well-documented. One of the primary arguments against using calculators of any type has always been that students may become overly dependent on the devices in ways that are not mathematically productive (Leinhardt, Zaslavsky, & Stein, 1990; Wilson & Krapfl, 1994), and that this could lead the mathematical authority in the classroom to shift from teacher or textbook to the device itself, rather than to mathematical reasoning as intended. However, Doerr and Zangor (2000) found no evidence of this, arguing that this tendency can be overcome by explicit attention to the limitations of the calculator and by emphasizing that mathematical reasoning is the basis on which conjectures are proven. Through direct and repeated attention to creating mathematical meaning, the participants in this study came to view the graphing calculator as a tool, but recognized that they also needed to rely on mathematical reasoning as a check on results obtained by the calculator. Using novel problems and tasks that require contextual interpretation are also likely to mitigate an over-dependence on the calculator, since these types of activity require thinking and reasoning on the part of the learner that is independent of calculator use.

There are also aspects of working with a graphing calculator that can create problems for students, both technically and conceptually. Mitchelmore and Cavanagh (2000) traced high school students' (Australian grades 10,11) difficulties and misconceptions to four areas: scale, accuracy and approximation, linking representations, and graphic representation by pixels. Issues related to scale have also been reported by other sources (Leinhardt, Zaslavsky, and Stein, 1990; Williams, 1993), and may be due in part to students' lack of experience with graphs where the axes were not scaled equally. Curricular materials often favor examples and exercises that are easily viewable in the calculator-standard 10 x 10 viewing window, so students can encounter problems when non-standard window settings are required. This is also related to a poor understanding of using the calculator's built-in zoom operations and finding reasonable window settings manually. Typical graphing calculators have a "zoom out" feature that increases the range of both axes equally; while this is sometimes useful, in many circumstances simply zooming out results in losing important features of the graph (e.g., the local extrema of a polynomial function). The physical design of most graphing calculators can also present challenges. In particular, the most common design features a rectangular screen that is approximately 33% wider than it is tall, yet displays graphs using a square window. This results in some oddities, such as perpendicular lines appearing as though they do not intersect at a right angle (see figure 2 below). Although this may seem like a relatively minor quibble, it could conceivably cause misunderstanding or confusion on the part of some learners.

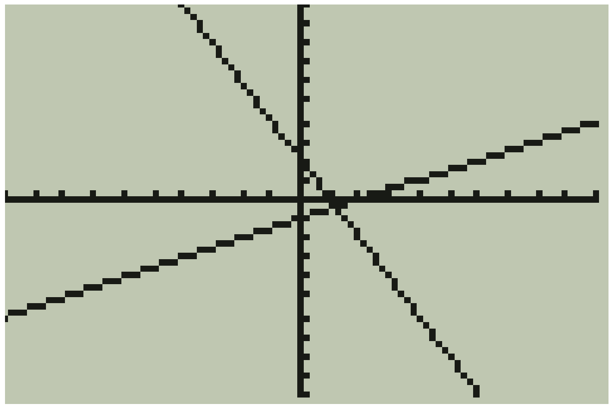

Figure 2. Perpendicular lines on a graphing calculator. Note that these lines do not appear to meet at a right angle due to the scale of the window.

Mitchelmore and Cavanagh (2000) also observed misconceptions and misunderstandings relating to accuracy and approximation among the participants in their study. For example, students tended to correlate a greater number of decimal places with increased accuracy when making approximations, but at the same time showed a “marked preference” (p. 265) for integer values. This may also be a byproduct of the curriculum, as examples and exercises tend to favor problems with integer (or other “nice”) solutions. While this presumably allows learners to focus on the point of the example instead of getting bogged down in complicated arithmetic, it may come at the cost of creating a bias against “messy” (e.g., irrational) solutions. A better understanding of how calculator and computer algorithms work, and the associated limitations, could help to alleviate some issues related to the accuracy of calculator approximations (Dick, 1992), but it would also be useful to incorporate more real-world applications and “messy” data into the curriculum.

Linking different representations of functions is another area identified as a potential source of difficulty for students. There is a tendency among students to accept the graph displayed by the calculator without necessarily thinking critically about the results; in particular, students may fail to recognize inconsistencies between the symbolic form of a given function and the graph generated by the calculator (Mitchelmore & Cavanagh, 2000). For example, the graph of a rational function can look quite different in the graphing calculator, particularly near discontinuities. In cases like this, students may not think about their knowledge of the symbolic representation of the function, leading them to misinterpret the calculator output (Ruthven, 1990; Leinhardt, Zaslavsky, & Stein, 1990). Some sources (Thompson, 1994b) have identified students' difficulties with translating among different representations of functions in general, which would certainly contribute to this phenomenon. This may also relate to issues of scale in certain circumstances, such as the perpendicular line example discussed previously. Explicit attention to finding "friendly" window settings (that is, those that provide a more visually accurate graph) could help to alleviate this problem (Vonder Embse & Engebretsen, 1996).

The final category of student difficulties in using the graphing calculator identified by Mitchelmore and Cavanagh (2000) is in the representation of graphs by a finite number of pixels. Despite dramatic improvements in LCD screen technology in the past two decades, most graphing calculators still use the same low resolution monochrome displays found in the first iterations of the devices. Mitchelmore and Cavanagh note that their students commented on the jagged appearance of graphs caused by the graphing calculator's low screen resolution. This low resolution and the irregular graphs that it causes can also lead to misunderstanding (Hector, 1992; Demana, Schoen, & Waits, 1993); for example, the asymptotic decay toward the horizontal axis of an exponential function "disappears" when the calculator can no longer display the curve as separate from the axis itself (see figure 3). This could be related to (or exacerbated by) difficulties with linking the symbolic and graphic representations as well, so it is conceivable that more accurate graphical depictions could be used by students to develop more coherent conceptions of function types.

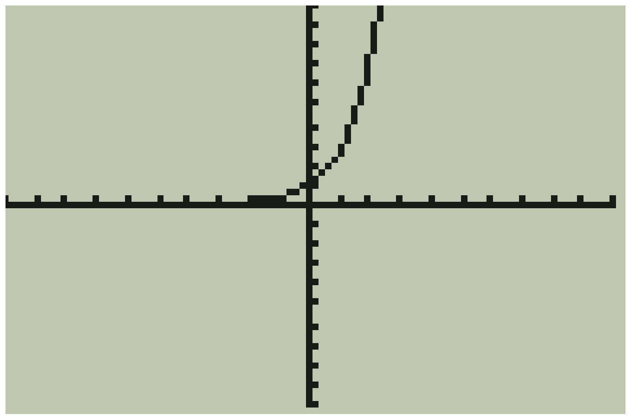

Figure 3. The graph of an exponential function. Note that the asymptotic behavior on the left-hand side of the graph is unclear as a result of the low screen resolution.

Ultimately, many of the difficulties commonly experienced by students when using a graphing calculator likely have more to do with mathematics than technology. Issues relating to scale, accuracy and approximation, and linking different function representations can likely be traced to an incomplete or inadequate understanding of the underlying mathematical concepts. However, these problems may be compounded by the design and shortcomings of the calculator, as in the case of the scale issues brought on by putting a square window in a rectangular display. The representation of graphs by a finite (and in most cases, very limited) number of pixels is due mostly to the limitations of the technology itself, but could be aggravated if the user does not have a firm understanding of the associated function properties. Furthermore, Williams (1993) found that the graphing calculator can be confusing for some students, even after instruction and experimentation, and that “teachers mistakenly assume they and their students have shared understandings when, in fact, the two groups have quite different ideas” (p. 198). This can be difficult to identify and diagnose, as graphing calculator use does not leave the same evidence as pencil-and-paper calculations. However, talking to students about their reasoning and “looking over their shoulder” as they work with the calculator (Williams, 1993) can provide important feedback about the way that a student’s understanding aligns with the teacher’s goals.

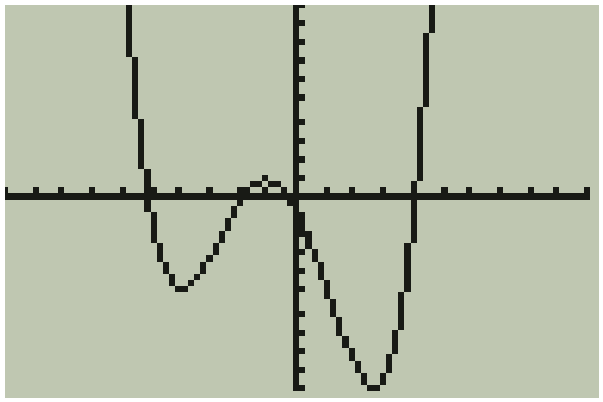

Figure 4. The graph of a polynomial function with four real roots. Finding these roots using built-in calculator functions can be tedious and presents a barrier to students’ learning.

Certain features of and calculations with the graphing calculator can be technically difficult (or at least cumbersome) as well. For example, consider the process of finding the real roots of the polynomial function whose graph is shown in figure 4 above. Identifying the real roots of this function is a perfectly reasonable exercise that could be found in a typical college algebra course, and can be accomplished using built-in functions of most graphing calculators. However, the process for finding these roots (in this case, with a calculator in the TI-84 family) involves setting an appropriate viewing window, then using the “ZERO” command once for each root, which in turn requires the user to scroll near the desired point, set left and right bounds, and finally provide a “guess” value before actually calculating the value of the root. Those who understand numerical approximation techniques for finding roots (e.g., Newton’s method, Runge-Kutta) will quickly recognize the need for each of these commands, but for students they can be intimidating and present a potential barrier to learning. Kissane, Bradley, and Kemp (1994) argue that “it is unlikely that the necessary technical and discrimination skills associated with graphics calculators will be picked up by students without us paying much explicit attention to them" (p. 32). Classroom experience supports this claim, as students sometimes need repeated explanations of certain techniques, such as finding real zeros. In these cases, the focus shifts to be too much about the technology itself, rather than the mathematics at hand.

Graphing calculators are a powerful tool that, when used appropriately and accompanied by a shift in classroom culture, have the potential to change the way that students learn mathematics. Research indicates that students who use the graphing calculator in the learning process tend to outperform their peers, and emerge with more innovative approaches to problem solving. There are potential drawbacks and pitfalls to using graphing calculators, however. Aspects of the devices can intensify students' misconceptions if not treated carefully, and some built-in calculator operations are cumbersome and can become barriers to understanding. Developing classroom practices, curricular materials, and assessments that maximize the potential benefits of the graphing calculator, while also minimizing drawbacks by keeping the focus on mathematics, is a difficult but important process. It should also be noted that at present, the primary concern is calculator operations related to graphing and representation of functions. Any use of the graphing calculator as a computer algebra system (CAS) will be beyond our current scope, but previous research has explored this usage as well (e.g., Cannon & Madison, 2003).

Desmos & DGS

Aside from graphing calculators, some in the mathematics education community have advocated the use of classroom or personal computers along with dynamic geometry software (DGS) such as *Geometer's Sketchpad* or *GeoGebra*. One of the primary strengths of such programs is that they allow students to interact with geometric figures in a dynamic setting; Hull and Brovey (2004) argue that the dynamic environment provided by this software allows students to investigate a large number of possibilities relatively quickly, enabling them to explore and conjecture. In short, DGS can reduce tedium in some of the tasks involved in learning Euclidean geometry (e.g., ruler-and-compass constructions) in the same way that graphing calculators can diminish the need for monotonous calculations, allowing students' focus to shift to more mathematically-rich aspects of the curriculum (Quesada & Cooper, 2010). Since DGS use is typically situated in the context of geometry courses, making direct inferences about how it relates to the learning of algebraic concepts may be difficult. However, the effects that dynamic interactions with technology have on students' understanding and attitudes may shed light on the present study.

Hull and Brovey (2004) conducted a study at a public high school in the southern U.S. aimed at investigating the effects of using DGS (in this case, *Geometer's Sketchpad*) in a ninth grade geometry class. The students used the software during a three-week long unit on circles, which included teacher demonstrations and hands-on experience with the software in a computer lab. The researchers measured student learning by comparing the scores of students in the treatment group with those of a similar group of students from the previous year on a traditional geometry assessment. Post-intervention mean test scores showed a slight improvement for the treatment group, though the authors do not indicate whether this difference was statistically significant. An attitude survey was also administered, but differences in the pre- and post-intervention survey results were slight. The authors concede that these results are inconclusive, and that the impact of the use of DGS on these students cannot be determined based solely on this study. However, the very short duration (three weeks) of this intervention may have prevented the students from becoming comfortable using the software, and the authors suggest that a longer study could produce stronger results. In spite of the lack of conclusive findings, Hull and Brovey did observe that students were "more convinced of the veracity of a theorem when it is explored using a software program" (p. 7), perhaps due to the larger volume of examples that can be quickly examined using DGS than with traditional methods. In addition, the participants in this study reported having enjoyed DGS lessons (conducted in a computer lab) more than their usual classroom lessons. This is evidence of the ability of DGS and other computer-aided tools to foster a student-centric learning environment, and suggests that its sustained use could positively impact attitudes toward mathematics.

More recently, and of primary interest in the present study, the free online graphing utility *Desmos* offers a powerful alternative to the graphing calculator. Desmos was selected for use in this study for a variety of reasons, including its availability and ease of use. Since it runs in a web browser and is compatible with a wide range of devices, students can quickly and easily access Desmos from anywhere. Desmos also offers a mobile app, available for smartphones and tablets, that allows students to use their preferred personal device during class and while completing assignments. Allowing students a degree of ownership in this matter may affect attitudes toward the technology, as well as reducing the learning curve. Friendly design features, including intuitive scrolling and the familiar "pinch" zooming on touchscreen devices, make using Desmos very similar to web browsing and other everyday interactions with technology. On a strictly practical level, Desmos is also completely free to use, whereas a new graphing calculator requires a significant expense. Furthermore, graphing calculator technology is outdated, and lack of competition among manufacturers has created a near-monopoly (Smith, 2015) in the high school market. However, iPads and other mobile technologies are becoming more common in K-12 classrooms. In the spring of 2015, the Eanes (TX) school district conducted a pilot study using a restricted version of Desmos on iPads for their 8th grade state standardized assessment (Yenca, 2015; Locke, 2015; Smith, 2015). As this relatively new alternative to the traditional graphing calculator gains popularity and support, it is important to carefully examine its use and effects on students.

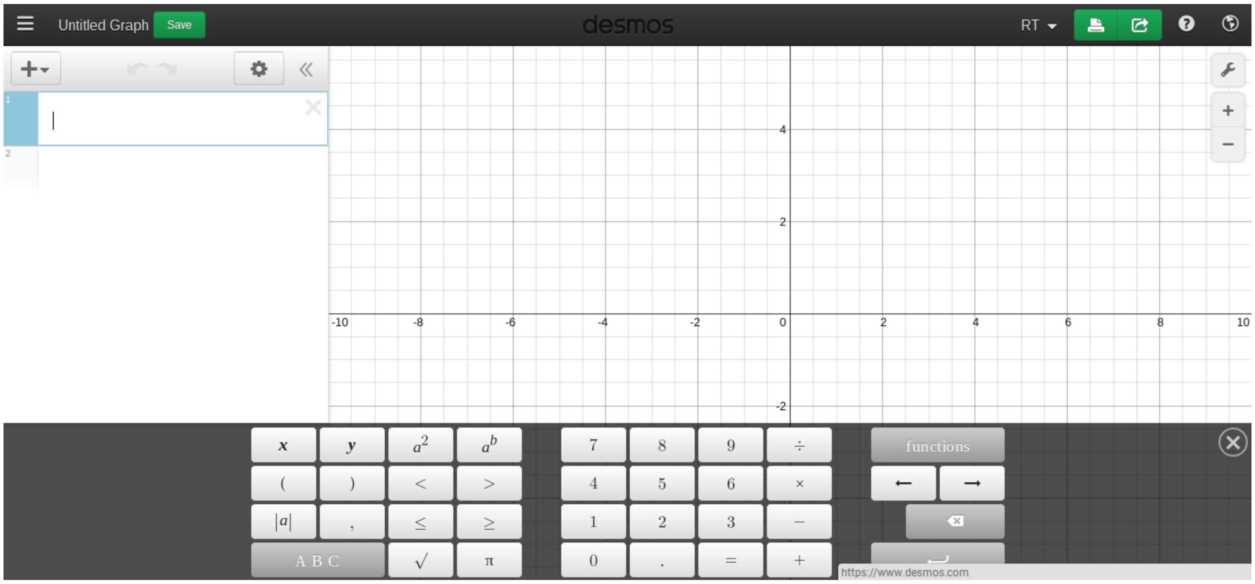

Figure 5. The Desmos user interface. Commands can be entered via the pop-up virtual keyboard shown here, or by typing the appropriate symbols.

Desmos is a powerful graphing utility that performs nearly all of the same functions as a TI-84 graphing calculator, but is much more user-friendly and intuitive (Ebert, 2014; Smith, 2015; Liang, 2016). The user interface shows the input and graphing window side-by-side, allowing students to see updates to their graphs in real-time. Beigie (2014) contends that these instantaneous changes in the graphing window provide immediate feedback, and this is very conducive to informal experimentation and conjecturing. Compare this to the graphing calculator, which requires students to switch back and forth between input and graphing windows and will often bog down as it redraws graphs, and the potential benefits are easy to see. In addition, students can add sliders in place of parameters (such as coefficients) that allow them to vary these values and observe the changes, all in real-time and without having to change windows (see figure 1). This could be a powerful way for students to gain experience with many core concepts in a class such as college algebra. For example, instead of presenting “rules” to be memorized for graph transformations, students could conjecture and experiment to construct their own understanding of how the transformations work. Sliders and the dynamic nature of graphing in Desmos allow students to see the effects of changing parameters immediately, providing individualized feedback and a way for students to verify their hypotheses.

Desmos also provides a powerful tool for problem solving. Multiple graphs can be displayed in the same window in up to six different colors, whereas the majority of graphing calculators use a monochrome display. Graphing calculators with color displays have appeared on the market recently, but retain their predecessors' low screen resolution, pixelated appearance, and limited processing power. On the other hand, Desmos displays graphs in sufficiently high resolution as to appear smooth and very rarely shows any sign of slowdown or lag in normal usage (see figure 6). Perhaps more importantly for applications, Desmos makes finding points of interest on a graph a straightforward process. Relative extrema, *x*- and *y*-intercepts, and points of intersection can be found by simply clicking on the curve near the point. While the complicated processes involved in finding such points with a graphing calculator tend to obscure the mathematics at hand, the effortless nature of using Desmos to accomplish the same task allows students to focus on the big picture. Finally, users have the option to save graphs for later use and can easily share their work with others, providing a greater capacity for application as a collaborative tool than the graphing calculator.

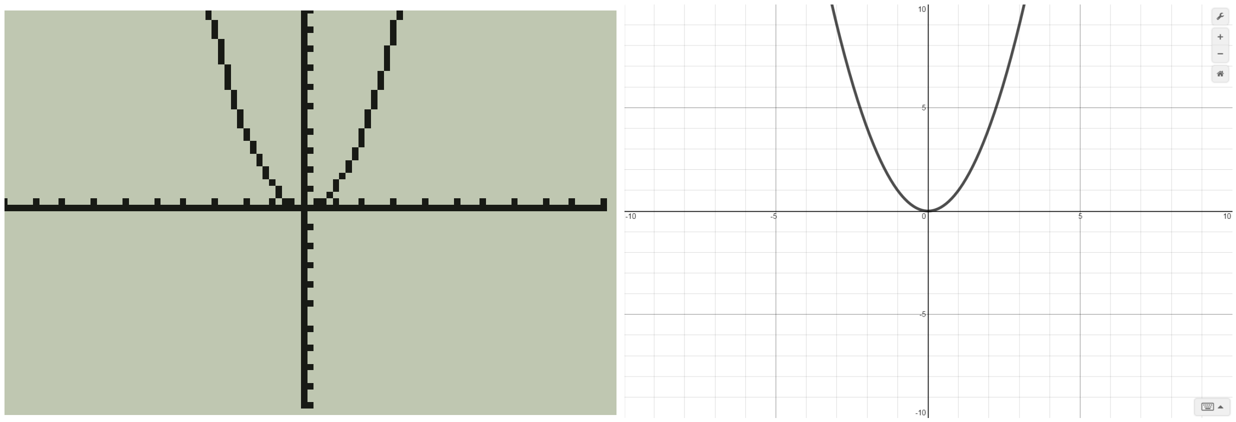

Figure 6. Side-by-side comparison of the same graph as displayed by the TI-84 graphing calculator and Desmos. Note the pixelated appearance of the image on the left.

Existing research using Desmos is very limited, and consists primarily of practitioner pieces. Beigie (2014) and Ebert (2014) used Desmos as the medium for open-ended graphing projects that tasked students (in high school Algebra 1 and Algebra 2, respectively) with creating a piece of art using graphs of mathematical relations. This project requires students to apply their knowledge of different types of relations, as well as reinforcing their understanding of graph transformations and restrictions on domain and range. Although neither author presents any real data, anecdotal evidence from these articles suggests that students are able to engage with Desmos in a deep and meaningful way. The project described by Beigie required students to use a minimum of twelve graphs to create the drawing, but the work eventually handed in by students included an average of sixty-three graphs per drawing. Ebert experienced a similar phenomenon, writing that “many students go far above and beyond the requirements of this project and are extremely proud of their work” (p. 390). The degree to which these students engaged with Desmos in the context of this project is evidence of the potential it holds for changing students’ attitudes about mathematics. Yenca (2015), in describing the Texas pilot program to utilize a restricted version of Desmos in 8th grade standardized testing, reports that an informal survey was conducted on a small (*n*=49) sample of students. Of these, 76% indicated that they preferred using the TI-84; however, detailed survey data shows that the reason for this preference was primarily due to built-in calculator functions, such as descriptive statistics computations and conversion from decimal to fraction form. On the other hand, close analysis of students' survey responses indicates that 88% of these same students favored using Desmos for any tasks that involved graphing. It is reasonable to assume that students' preference is strongly correlated with the content of the curriculum; if these students needed to do more graphing and less statistics, the results could look quite different.

Although the exact effects of using Desmos on students' understanding, achievement, and attitudes toward mathematics and technology are as yet unknown, the existing literature and experience with the tool seem to indicate that it has the potential to significantly improve student outcomes. Determining the extent to which its use impacts students will require extensive research, but it is our hope that the present study will provide encouraging preliminary results and perhaps provide direction for future investigation.

Conceptual Understanding of Mathematics

Improving mathematics education has been an ongoing and contentious issue for over a century, with research mathematicians, educators, classroom teachers, administrators, parents, and policymakers all contributing to the various reform movements that have brought about the current state. At the core of many of these debates have been the questions of how people learn mathematics, and what kinds of mathematical knowledge are worth developing. Gray and Tall (1994) summarize the dichotomy surrounding mathematics by saying that "mathematics has been notorious over the centuries for the fact that so many of the population fail to understand what a small minority regard as being almost trivially simple" (p. 116). The conclusion that the authors reach is that the experience of doing mathematics is qualitatively different for these two groups;

while some see mathematics as a series of tedious computations, standard algorithms, and formulas to be memorized and repeated, others are able to focus on the bigger picture and can encapsulate these things into mathematical objects that are more easily manipulated. Fields medalist William Thurston, commenting on the state of mathematics education in the United States in 1990, observed the following:

> by the time students are in college, they are inhibited from thinking for themselves and from admitting out loud what they are thinking. Instead, they try to figure out what routines they are supposed to learn. When there is any departure in class from the syllabus or the text, someone invariably asks whether it's going to be on the test (p. 4).

The Common Core State Standards for Mathematics (2010) sought to remedy this by calling for a decrease in emphasis on standard algorithms and more opportunities for students to develop their own thinking, starting in the earliest grades. However, proper implementation is vital for this change to take hold (Gaddy, Harmon, Barlow, Milligan, & Huang, 2014) and there is some concern that schools may misinterpret the standards, simply presenting alternative algorithms in addition to the standard ones rather than allowing students to think for themselves (Sun & Strauss, 2016).

If the goal of mathematics educators is to improve the quality of students' education in mathematics, it is critical to carefully consider the matter of what it means to learn something. Contributions from psychology, including the work of Piaget (1970), Vygotsky (1978), and von Glasersfeld (1989), have formed the basis for the current prevailing notion that mathematical knowledge is developed through an internal construction process. This may be influenced by a great many things, but ultimately it occurs within the individual learner. The phrase "learning with understanding" has been used to describe the desired student outcome, which the NCTM *Principles & Standards* (2000) define as a combination of factual knowledge, procedural facility, and conceptual understanding. As defined by Hiebert and Lefevre (1986), the *conceptual* aspect of mathematical knowledge is rich in relationships, while the *procedural* consists of the formal language and symbolism, together with the algorithms and rules that comprise step-by-step instructions for completing mathematical tasks. Furthermore, Hiebert and Carpenter (1992) and Hiebert et al. (1997) relate *understanding* to a network or web of interconnected ideas, claiming that we really understand something when we begin to see how it is related to other things we know, and that as these connections grow in strength and number, so does the degree of understanding. The argument made by Thurston (1990) and others (White & Mitchelmore, 1996; Carlson, Oehrtman, & Engelke, 2010; Thompson, 2013) is that one of the chief failings of the mathematics education program has been the focus on procedures and skills without sufficient attention to meaning, coherence, and creating connections between procedural and conceptual aspects of the content in the curriculum. To some degree, this is likely a product of the movement toward evaluation by standardized testing, which tends to privilege students who are able to quickly recall and apply procedures to solve familiar problem types. However, when confronted with novel problem situations, these same students may struggle and find themselves unable to cope.

Explicit attention to meaning has often been overlooked or ignored in the mathematics classroom in the United States, instead being replaced by a focus on “learning” to take prescribed steps in response to certain stimuli (White & Mitchelmore, 1996; Thompson, 2013). International studies such as TIMSS indicate that this paradigm has not been effective, either in developing mathematical understanding and critical thinking skills in American students or in terms of the relative strengths of the average American mathematics student as compared to their counterparts abroad (McKnight & Schmidt, 1998). On the other hand, the question of how to go about attending to meaning in the classroom is difficult to answer, especially when one considers that many classroom mathematics teachers will have been trained in the same procedurally-focused environment. Harel, Fuller, and Rabin (2008) observed instances of classroom teachers using mathematical terms and symbols in ways that were sometimes conflicting or differed from typical usage, and without attention to their meaning. For example, in discussing percent change, the teacher used the equations “$b = 1 + \%$” and “$b = 1 - \%$” as a sort of mnemonic intended to aid students in carrying out prescribed procedures, rather than as part of a meaningful discussion or statement about related quantities. The students in this episode would have been better served by using language and symbols in a precise and consistent manner, and may have also come to better appreciate the reasoning behind studying the concept had it been treated in a more meaningful way.

Mathematics is a subject that can be continually reorganized and compressed within the mind of the individual. As we gain experience with an idea, we begin to move beyond the minutiae and can see things from a coherent, contextualized perspective. This and related phenomena are given various names in the literature, including encapsulation (Dubinsky, 1997), mental compression (Thurston, 1990), and assimilation (Piaget, 1970). Gray and Tall (1994) explain this occurrence as a cognitive process, by which a dynamic process is transformed into a static “conceptual entity”, or a “cognitive object that can be manipulated as the input to a mental procedure” (p. 118). As an example, Thurston (1990) recalls his childhood realization of the connection between fractions and division; in particular, rather than interpreting “134 divided by 29” as a tedious computation to be carried out, likely following an algorithm learned by rote, Thurston was able to instead conceptualize the fraction 134/29 as a mathematical object in its own right. This mental reorganization allows one to use a process or idea as part of a larger mathematical task; for example, the algebraic manipulations (factoring, etc) that take place in the context of finding a partial fraction decomposition. While these symbolic operations may have been of primary interest at some point in the history of an individual learner's mathematical training, by the time partial fraction decomposition is relevant, they are (or should be) a background tool on the same level as basic arithmetic.

A framework for the construction of mathematical knowledge developed by Dubinsky (1997) breaks this development into three phases: *action*, *process*, and *object*. Learners will initially conceptualize a new idea as an action, which operates on and transforms objects, but is perceived as being external to some degree and occurs in response to some outward cue. At this stage of development, the learner likely needs to rely on external resources that give precise instructions to be followed; for example, a student who has an action conception of function may be able to evaluate a given function at specified points, but could be expected to struggle with finding its inverse. Repeated experience and individual reflection on actions can lead to the learner internalizing the action into a process. Although the outcome of this process is the same as the action that preceded it, a process is more internal and does not depend on external stimuli in the same way as an action. Furthermore, a process conception provides greater capacity for reflection and description, as well as enabling a learner to mentally reverse steps without necessarily needing to explicitly perform them (Dubinsky, 1997). In the case of function, developing a process conception allows a learner to comprehend more advanced ideas, including composition and inversion of functions. If an individual is further able to perceive the process as a whole, it is said to be "encapsulated" as an object, to which subsequent higher-order actions and processes can be applied (e.g., recognizing that the derivative of a function is a function itself). This is difficult to accomplish, and pedagogical strategies for leading students toward developing this type of conception are limited (Dubinsky, 1997; Voskoglou, 2013). Voskoglou argues that "because encapsulation entails a radical shift in the nature of one's conceptualization" (p. 32), it requires time, experience, and reflection to accomplish. For Dubinsky, the highest cognitive level is the creation of *schema* that organize actions, processes, and objects. Although examining students' understanding in great detail using the APOS framework described here will not be attempted in the present study, it does provide a useful way of thinking about students' reasoning that has helped to influence instructional decisions.

David Tall and his collaborators have conducted significant research into determining the bases of mathematical learning (Tall & Vinner, 1981; Tall, 1992; Gray & Tall, 1994; Tall, McGowen, & DeMarois, 2000), introducing the terms *concept image* and *concept definition* into the literature. A learner's concept image for a mathematical concept is the total cognitive structure that is associated with that concept, including mental pictures and related properties and processes. Tall and Vinner write that the concept image is "built up over the years through experiences of all kinds, changing as the individual meets new stimuli and matures" (p. 2). On the other hand, a learner's concept definition is a group of words used to specify a concept. This may be a *formal* (i.e., mathematically accepted) concept definition or a *personal* concept definition that is constructed by the individual, and may or may not be developed in a meaningful way. Rote memorization of a formal concept definition is unlikely to serve the learner in novel situations, but if a meaningful personal concept definition is developed through experience, then the learner may develop strong connections among related concept images in a way that promotes understanding. At a more fundamental level, Tall (1992) presents the idea of *cognitive roots* as the basis for developing mathematics concepts. According to Tall, a cognitive root should be simultaneously familiar to students and a good foundation for later mathematical development. Supposing that a sound cognitive root can be identified for a topic, it could then be used to influence decisions in curriculum development and pedagogy. For example, limit is typically the first concept that students experience in calculus, but it is also an idea that may be difficult for students to learn in a meaningful way in a short period of time. Tall suggests investigating "local straightness" as an alternative; this is an idea that is relatively easy to investigate and discover using technology (e.g., repeatedly zooming in on a curve) and may provide a better support for topics appearing later in the curriculum, including the derivative. For more on this, Bingolbali and Monaghan (2008) investigate the development of first-year undergraduate students' concept images of the derivative.

Returning to the function concept, some of the difficulties that students often encounter in mathematics can be characterized through the lens of these theories of learning. Vinner and Dreyfus (1989) extended the work of Tall and Vinner (1981) in investigating concept image and definition by studying constructions for the function concept in Israeli first-year college students and junior high mathematics teachers who had not majored in mathematics (n=271 and n=36, respectively). In the course of this study, the authors found that many of the participants held inconsistent images and definitions for this concept. The term *compartmentalization* describes the occurrence of potentially incompatible images and definitions in an individual's cognitive structure, which may conflict when they are evoked. In a detailed analysis of their results, Vinner and Dreyfus determined that those students whose major field of study required more mathematics tended to show symptoms of compartmentalization less often, which the authors attribute to these students being "more aware of their thought processes and… more likely to reflect on them" (p. 365). For those students in non-STEM disciplines, the inconsistent concept images and definitions evidenced here may never reconcile themselves, which could negatively impact these students' attitudes and self-confidence about mathematics moving forward.

Several sources have noted that among the factors that contribute to students' overall weak understanding of the function concept is a tendency toward a static view of function. Tall (1992) writes that the central idea of function as a *process* is often overlooked. Here Tall uses the term "process" in a different sense than Dubinsky (1997), and this is clarified by Tall, McGowen, and DeMarois (2000), who propose that the “function machine” is a viable cognitive root for the function concept. This “function machine” is a pedagogical tool designed to help students view function as a process which takes a given input and transforms it into a particular output, with the goal of developing a sense of function as a way of relating two quantities. Thompson (1994b) also cautions against placing too much emphasis on multiple representations of functions at the cost of creating meaning; in short, that curricula should initially employ graphs, expressions, and tables as representations of a specific situation or context, rather than starting with a formal definition of function and then developing the various representations. Oehrtman, Carlson, and Thompson (2008) observe that “strong emphasis on procedures without accompanying activities to develop deep understanding of the concept has not been effective for building foundational function conceptions” (p. 151), which perhaps indicates that whatever model is used to initially convey the function concept, instruction and curricular materials should make a concerted effort to create meaning with and for function. As learners develop understanding of the function concept and begin to internalize it as a process (in the sense of Dubinsky), they will also exhibit mental actions associated with covariational reasoning (Carlson, Oehrtman, & Engelke, 2010).

The cognitive structures that constitute an individual’s mathematical knowledge are dynamic, shifting and reorganizing as new connections are made in response to experience. Exactly how this happens, whether it be Piaget’s assimilation to a scheme (1970), Thurston’s mental compression (1990), Dubinsky’s APOS framework (1997), or something else entirely, is not fully known. However, its effects on mathematics education can be seen by considering some of the common difficulties encountered in practice. Thurston writes that “after mastering mathematical concepts, even after great effort, it becomes very hard to put oneself back in the frame of mind of someone to whom they are mysterious” (p. 5). Mathematics is unique in that each individual can essentially re-discover everything for themselves - apart from organizational tools like notation, there are very few things that *must* be memorized. This distinguishes the subject from other school curricula (e.g., language arts or history) where *remembering* is an essential skill for success. As a result, teaching mathematics involves carefully bridging the gap between one's own mathematical knowledge, carefully constructed and curated over a lifetime of experience and training, and that of the students, each with a unique set of prior experiences and preconceptions. It is challenging but crucial to allow students the room to develop on their own.

Student Attitudes Toward Mathematics

Attitudes toward mathematics, and in particular math anxiety, have been shown to relate to mathematics achievement, and there is evidence to indicate a reciprocal relationship between the two (Aiken, 1970); that is, attitudes affect achievement, and achievement affects attitude. Therefore, any potential effects on attitude as a result of technology use may be valuable to educators and beneficial for learners. Those students who also experience math anxiety are further impeded in learning. Hembree (1990) finds that math anxiety is linked to poor performance on achievement tests, and in particular that higher anxiety levels consistently correlated with lower performance. Furthermore, Hembree's results indicate an inverse relationship between math anxiety levels and both positive attitudes toward the subject and self-confidence in mathematics. Students with high levels of math anxiety are also more likely to exhibit behaviors that indicate avoidance of the subject, including taking fewer mathematics classes. Hembree indicates that "mathematics anxiety seems to be a learned condition more behavioral than cognitive in nature" (p. 45). Although these behaviors likely start at an early age, if they are indeed the result of a learned condition, then there is some hope of overcoming this anxiety.

Another source of possible difficulty for students who experience math anxiety involves the method of instruction. Based on prior findings indicating that teacher characteristics may be tied to attitudes toward mathematics (Aiken, 1970), Clute (1984) conducted a study to determine if instructional style and math anxiety interact in affecting student achievement. Two similar student groups were taught using different instructional methods - one group was given a series of organized lectures designed to guide students toward “mastering” material, while the other used a discovery-based approach involving questioning sequences to develop the subject matter. Both groups were given an achievement test after the intervention to quantify the results. In this experiment, the discovery group attained a higher overall mean score than the expository group (though not statistically significant), and scores among students with low and medium levels of math anxiety in the discovery group were both higher (in the mean) and more homogeneous than their counterparts in the expository group. However, among those students with high levels of math anxiety, the students in the discovery group had lower mean test scores with a higher level of variance. Math anxiety level had a significant impact on achievement level, with high anxiety correlating with lower achievement. Furthermore, there was a statistically significant interaction effect between instructional method and anxiety level, with low- and medium-level anxiety groups performing better (on average) with discovery-based instruction, while high-level anxiety students performed better with expository instruction. This is evidence that supports using a variety of instructional strategies, as different groups of students will respond differently to various pedagogical approaches.

Although empirical data concerning the relationship between dynamic graphing utility use and student attitudes toward mathematics is severely limited, there are some indications that there may be a positive correlation between the two. Hollar and Norwood (1999) found slight positive effects related to graphing calculator use among college students enrolled in intermediate algebra, though their results were not statistically significant. Other sources (Ruthven, 1990; Dunham & Dick, 1994; Smith & Shotsberger, 1997; Hennessy & Dunham, 2001; Idris, 2006) also provide anecdotal evidence that lends credence to the possibility for graphing calculator use to improve students' engagement with and enjoyment of the subject. Chazan (1999), in discussing his experience teaching a functions-based approach to algebra supported by technology, observed that the latter provided a support system for students' involvement. Hull and Brovey (2004), in their study using the DGS Geometer's Sketchpad, found that their students enjoyed using the computer software more than traditional paper-and-pencil methods. Beigie (2014) and Ebert (2014), in practitioner pieces about using Desmos, displayed evidence that students were able to engage with Desmos and went far beyond minimum requirements in completing assignments. Though far from conclusive, these results are indicative of potential positive effects on student attitude toward mathematics as a consequence of prolonged experience with using Desmos.

Summary

The existing body of research on graphing calculators is extensive; however, the literature contains few references to DGUs apart from a few practitioner pieces. While it seems reasonable to think that a DGU would offer the same benefits to students as a graphing calculator, the added dynamic features and overall more intuitive user experience may provide powerful advantages as well. Investigating any differences in student outcomes when using these tools should reveal if this is in fact the case, and provide direction for future studies.

Chapter 3 - Methodology

In this chapter, the methods and design of the study will be presented. More specifically, the instruments used in data collection, the experience gained in conducting a pilot study, and the implementation in the primary experiment will be discussed.

Data Collection Instruments

There were two primary instruments used for data collection in this experiment, each intended to address one of the principal research questions. Since the primary goal of the study was to examine the effects of using a DGU (Desmos) as a support for students' developing understanding of algebraic concepts, it was necessary to find an appropriate tool for measuring this knowledge. In reviewing the relevant literature, the Precalculus Concept Assessment (PCA) developed by Carlson, Oehrtman, and Engelke (2010) was identified as a viable option and adopted after careful review. The PCA is a 25-item multiple-choice exam designed to probe for the "reasoning abilities and understandings central to precalculus and foundational for beginning calculus" (Carlson et al., 2010, p. 113). Although designed for use in the context of precalculus, the majority of the items in the PCA are also appropriate for college algebra students, particularly those focused on aspects of the function concept, rate of change, and interpreting models in sample applications. A sample of assessment items is included in the appendix, and full details of the results are provided along with the data analysis. For an example item from the assessment, see Figure 7.

Central to the reasoning behind developing an instrument like the PCA is the evidence that emphasizing the importance of procedures, without a simultaneous focus on overall understanding and coherence, is not an effective means for developing "conceptions that support meaningful interpretation and use of concepts when solving novel problems" (Carlson et al., 2010, p. 115;

Hiebert & Carpenter, 1992; Thompson, 1994a; White & Mitchelmore, 1996). The authors developed the PCA and related taxonomy with a primary motive to “provide useful knowledge about the central ideas of precalculus that are needed for learning calculus” (Carlson et al., 2010, p. 119). Through a series of qualitative studies (Carlson, 1997, 1998; Carlson et al., 2002; Oehrtman, 2008; Oehrtman, Carlson, & Thompson, 2008) and four phases of development, these reasoning abilities and understandings became part of the PCA taxonomy, the full reproduction of which can be found in Table 1. This taxonomy also indicates the breakdown of PCA items, which will be used in data analysis.

Table 1

The PCA Taxonomy

Reasoning Abilities	
R1	*Process view of function* (items 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 20, 22, 23) View a function as a generalized process that accepts input and produces output. Appropriate coordination of multiple function processes
R2	*Covariational reasoning* (items 15, 18, 19, 24, 25) Coordinate two varying quantities that change in tandem while attending to how the quantities change in relation to each other
R3	*Computational abilities* (items 1, 3, 4, 10, 11, 14, 16, 17, 21) Identify and apply appropriate algebraic manipulations and procedures to support creating and reasoning about function models
Understandings	
Understand meaning of function concepts	
ME	Function evaluation (items 1, 5, 6, 11, 12, 16, 20)
MR	Rate of change (items 8, 10, 11, 15, 19, 22)
MC	Function composition (items 4, 5, 12, 16, 17, 20, 23)
MI	Function inverse (items 2, 4, 9, 10, 13, 14, 23)
Understand growth rate of function types	
GL	Linear (items 3, 10, 22)
GE	Exponential (item 7)
GR	Rational (items 18, 25)
GN	General non-linear (items 15, 19, 24)
Understand function representations (interpret, use, construct, connect)	
RG	Graphical (items 2, 5, 6, 8, 9, 10, 15, 19, 24)
RA	Algebraic (items 1, 4, 7, 10, 11, 14, 16, 17, 18, 21, 22, 23, 25)
RN	Numerical (items 3, 12, 13)
RC	Contextual (items 3, 4, 7, 8, 10, 11, 15, 17, 18, 20, 22)

Table 1. The PCA Taxonomy consists of the reasoning abilities and understandings identified as being fundamental to success in calculus (Carlson, Oehrtman, & Engelke, 2010).

The key reasoning abilities identified in the development of the PCA include a view of function as a *process* rather than an *action*, the ability to coordinate the covariation of two quantities, and certain related computational skills. The taxonomy also includes conceptual understanding of the meaning of function concepts, representations, and growth rates. Many of the actual assessment items require students to interpret functions defined verbally or in context and apply elements of the taxonomy. For example, one item (see Figure 7) asks students to identify the formula that defines the area of a square in terms of its perimeter, requiring the ability to recognize the need to invert a familiar function (e.g., $p = 4l$ to find the perimeter of a square) prior to composing two functions.

Which one of the following formulas defines the area, A, of a square in terms of its perimeter, p?

a) $A = \frac{p^2}{16}$
b) $A = s^2$
c) $A = \frac{p^2}{4}$
d) $A = 16s^2$
e) $p = 4\sqrt{A}$

Figure 7. An item from the PCA. The correct response (a) requires students to recognize the need to invert a familiar function prior to composing two functions. (Carlson, Oehrtman, & Engelke, 2010).

Development of the PCA was motivated by internal research at Arizona State University (Thompson et al., 2007), which found that less than 30% of students earning a grade of C or better in university-level precalculus went on to success (A-B-C) in Calculus I. While many of those who persisted were successful, nearly two-thirds of the precalculus students did not continue on to an initial calculus course. This is symptomatic of the issues facing STEM education in general, as many of these students likely left STEM fields. The PCA itself is a valid tool for assessing students' learning of the ideas and abilities identified in the taxonomy, with Cronbach's α=.73 (2010, p. 137) and support from qualitative studies conducted during the development process indicating that there is overall coherence in the assessment. Due to the complex interactions among the reasoning abilities and conceptual understandings in question, each PCA item is associated with multiple taxonomy categories and no two items assess the same combination. This allows the instrument to remain at a reasonable length (25 items), but also makes it difficult to establish "meaningful correlations among items" (p. 137) and necessitates some reliance on the qualitative data conducted during the development process. Nevertheless, follow-up data using the PCA as a placement test into first-semester calculus (n=248) found that "77% of the students scoring 13 or higher… passed the first-semester calculus course with a grade of C or better while 60% of the students scoring 12 or lower failed with a D or F or withdrew" (p. 140). This lends further evidence to the validity and reliability of the PCA as a predictor of success in calculus, and hence as an assessment of the foundational reasoning abilities and conceptual understandings identified in the PCA taxonomy.

In order to investigate any changes to students' attitudes toward mathematics, an appropriate attitude survey was also needed. The Student Attitude Survey (SAS) developed at the University of Massachusetts-Dartmouth by Brookstein, Hegedus, Dalton, Moniz, and Tapper (2011) was reviewed and subsequently adopted. The SAS is a 27 item, 5-point Likert-scale survey that explores "students' deeply held beliefs about mathematics and learning of mathematics, as well as their propensity for sharing private thinking" (Brookstein et al., 2011, p. 1), and was originally developed as part of an NSF-funded project examining the use of technology (SimCalc MathWorlds software and curriculum) in 9th grade Algebra 1 classrooms. Slight modifications were made to the survey to accommodate the present study; since the survey was designed to be administered to high school students, two items refer to middle school classes and were changed to instead reference “previous math classes”. One additional item was added to gather more information about students’ attitudes about the use of technology in learning and understanding mathematics. The post-intervention survey also included an optional space for students’ written comments, which will be discussed along with the survey results in the data analysis. The survey itself is included in the appendix in its entirety, and a few sample items are reproduced in Table 2.

Table 2

SAS Sample Items

I think mathematics is important in life.	SD	D	N/U	A	SA
I like math.	SD	D	N/U	A	SA
Using technology helps me understand mathematics.	SD	D	N/U	A	SA

Table 2. Some sample items from the SAS. Here the response choices range from “strongly disagree” (SD) to “strongly agree” (SA). (Brookstein et al., 2011)

Validity testing of the SAS was conducted as part of the development process at UM-Dartmouth. To establish concurrent validity, the SAS was compared to the Fennema-Sherman Mathematics Attitude Scale. This instrument has been extensively vetted over nearly forty years, and “is regarded as valid and reliable” (Brookstein et al., 2011, p. 2), but measures different aspects of students’ attitudes toward mathematics and was originally intended to determine gender-related differences among students. However, the Fennema-Sherman scale and the SAS share similar items on the dimensions of disposition and confidence related to mathematics, so it was chosen as a comparison to demonstrate concurrent validity. A sample of students (n = 139) were given both the Fennema-Sherman survey and the SAS, and one-sample t-tests were conducted on mean scores of similar test items, with non-significant differences being interpreted as demonstrations of concurrence between the two instruments. Of six SAS items compared to items on the Fennema-Sherman survey, three had non-significant differences, and those that were significant were “generally small - less than one category (of the five in the scale)” (p. 3). Furthermore, the authors found that “items related to confidence received the highest level of concurrence” (p. 2).

The original project for which the SAS was designed hypothesized that “attitudes and/or motivations of students would change over time… if the SAS could be demonstrated to detect pre/post changes in student attitudes, it could be said to have predictive validity” (p. 3-4). The ability to detect such changes is also crucial to the present study, since attitude-related effects of using a DGU are one of the primary questions under investigation. In order to verify the validity of the SAS, the developers considered the instrument as composed of four components or categories of items; namely, positivity toward learning mathematics and school, working collaboratively and related effect, working privately, and use of technology. For these four components, Cronbach’s α ranged from .637 to .744 in an initial dataset (n = 408), indicating some measure of internal consistency and reliability. Furthermore, testing with this initial dataset showed that the SAS was able to detect changes at the 99% confidence level for two components (positivity toward learning mathematics and school, use of technology) and at the 90% confidence level for a third (working collaboratively and related effect). This suggests that the SAS does indeed possess predictive validity in aggregate, and can be used as a reliable measure of changes in student attitudes. Brookstein et al. (2011) found a relatively small number of significant changes to mean scores on SAS items. The authors cite one possible explanation as the short time span (6-12 weeks) between pre/post tests, arguing that “this is a relatively short amount of time to expect changes in specific attitudes - results for individual variables may change over longer periods of time” (p. 4-5). The primary data collection in the present study follows a longer schedule (approximately 15 weeks), but attitudes and dispositions can be deep-seated and slow to change.

This will need to be kept under consideration during data analysis, but we can be reasonably confident that the SAS will detect any changes in attitude that do occur.

The Pilot Study

A pilot study was conducted to test course materials and data collection instruments, the results of which were then used to inform the design of the primary research program. The participants in the pilot study were 14 students enrolled in a 5-week summer college algebra course taught by the primary researcher. These participants came from a wide variety of academic classifications and major fields of study, with the majority taking the course as a general education requirement. Students were made aware of their participation in the study and informed consent was obtained on the first day of class. In addition, students were asked to complete an attitude survey (SAS) on the first day of class as a pre-intervention measure of attitudes toward mathematics and technology. Participants responded to this same survey again during the last week of class to determine any changes in attitudes during the course.

The course itself followed a typical precalculus algebra curriculum, with an emphasis on function as a unifying concept and multiple function representations (symbolic, graphic, tabular) as a tool for interpretation and problem solving. A hybrid instructional design was used, with students expected to prepare prior to class by reading pre-class lesson notes and watching instructional videos, allowing for in-class time to be spent pursuing active learning opportunities. The 90-minute class sessions typically included whole-class and small group discussion about mathematical concepts, small group and individual activities (in the form of worksheets or individual exercises), and limited direct instruction. Due to the rapid pace of a summer course, the balance among these components differed slightly from day to day, but always with the goal of guiding students toward developing an understanding of the conceptual “big picture”.

Students in the pilot study completed homework and quizzes using the online service MyLabsPlus, allowing them to receive immediate feedback (in the form of a correct/incorrect prompt) and limited guidance if desired. They were allowed full access to the Desmos graphing utility on their personal devices (smartphones, tablets, and/or laptops) in class and for all assignments, with many students often using Desmos to explore or check aspects of their thinking during classroom activities. During exams, students worked in a proctored computer lab and were allowed access to Desmos on the provided desktop computers. Although the core features are present regardless of the method of access, there are slight differences as a result of the input methods available on touchscreen devices and personal computers, such as scrolling rather than “pinching” to zoom or using a physical rather than a digital keyboard. While the author considers these differences to be negligible for most since they are also present in a typical web browsing experience, students were encouraged to use Desmos on both touchscreen and non-touchscreen devices while working on homework assignments in order to mitigate any difficulties in transitioning to a desktop computer for testing.

Due to the time constraints of a summer course, the PCA was not administered at the beginning of the pilot study, and so no pre-/post-intervention comparison is possible for the pilot data. However, students did perform well overall on course assessments. Several questions found on the final examination were common to the final from the same course in the previous summer, also taught by the author. Students in the pilot study performed better than their counterparts on over 60% of these questions, and did no worse than the previous group on nearly 75%. This is highly anecdotal, of course; the sample size is small (approximately 14 students in each class) and there is insufficient background information to draw any substantial conclusions. However, it is at least an intriguing bit of information and suggests that gains may indeed be possible.

Students were asked to complete the SAS on the first day of class, after the study parameters and informed consent were discussed. Those opting out of the study were still asked to complete the SAS for pedagogical purposes, but their responses have been omitted from analysis. Students were given the survey again during the last week of class. A paired *t*-test was used to test the hypothesis that post-intervention scores on the SAS differed significantly from the pretest scores on each of the 28 survey items. For the pilot data, significant differences ($\alpha = 0.05$) were detected on 4 survey items. In particular, item 8, "technology can make mathematics easier to understand" ($t(27) = 0.034$), item 26, "when using technology for learning mathematics, I feel like I am in my own private world" ($t(27) = 0.041$), and item 28, "using technology helps me understand mathematics" ($t(27) = 0.018$) showed statistically significant improvement in the post-intervention responses. On the other hand, item 17, "I receive good grades on math tests and quizzes" ($t(27) = 0.041$) showed a significantly lower score in the post-intervention responses. Although this change ostensibly represents a loss of confidence in mathematics, this may (at least in part) be due to the natural progression of course difficulty, as well as the breakneck pace of a 5 week summer course.

Although no statistically significant differences were detected in the other 24 SAS items, differences in pre- and post-intervention mean scores indicate that many students experienced gains in their interest in mathematics, confidence in problem solving and talking out loud in front of classmates, and overall valuation of the importance of mathematics in life. Again, these results were not statistically significant, but do at least suggest movement in the desired direction. In addition, students were given an opportunity to provide written comments on the post-survey. Few exercised this option, but one student wrote that they "went from being apprehensive about ever doing well at math to considering taking calculus for fun", indicating gains in this particular student's attitude and confidence toward mathematics in general.

The small sample size (n = 14) for the pilot study makes it difficult to draw conclusions from these data with much confidence. In addition, the short duration (approximately 5 weeks) of the study may have mitigated some effects that would be more apparent in a longer setting. However, the overall results are encouraging and suggest avenues of exploration for the primary data collection. First and foremost, an entry score on the PCA for students participating in the pilot would have provided an additional dimension of investigation into the effects of using a DGU. As a result, participants in the primary study were asked to complete the PCA during the first week of class as well as the SAS. In addition, the pilot study provided an opportunity to develop in-class activities, including group worksheets and class demonstrations in Desmos. In many cases, these materials were refined as a result of lessons learned during the pilot study.

Primary Data Collection

For the primary study, two sections of college algebra (approximately 30 students each) were taught by the principal researcher, with one section serving as the treatment group and the other as a control. The treatment group made frequent use of Desmos for instruction and class activities and were allowed to use it for testing, while the control group instead used TI-84-type graphing calculators. Apart from this difference, students in both sections completed all of the same assignments and were given the same learning opportunities in class. The two classes were similar in size and comprised almost entirely of first-semester college students from a variety of majors. The course served as a sort of middle ground between remedial mathematics and "standard" college algebra; in particular, students placed into this course, which included some elements of review and remediation, typically entered the university with slightly lower placement scores in mathematics. However, students were asked to complete the same exams and other assignments as their counterparts in the “standard” college algebra course, with the primary difference being that classes met four times per week rather than two.

Students in both groups were made aware of the research study on the first day of class, and their informed consent was obtained at this time. An overview of the study was given to students in both sections, both verbally and in the consent form. These descriptions did not mention the use of Desmos or the graphing calculator specifically; the stated purpose of the study was to “develop theories on students’ conceptual understanding of and attitudes towards college algebra”. This was done in an attempt to mitigate any biasing in favor of the treatment group. However, it is highly probable that students in the treatment group became aware of their status at some point during the course of the semester. This section of approximately 30 students was alone among ~2000 college algebra students at the university that had sanctioned use of Desmos for assignments and tests, so it seems safe to assume that these students were aware of the difference. It is conceivable that the control group did not know the role they played in the study. On the other hand, all students were made aware of their participation in a research program, so the Hawthorne effect (French, 1953) must be considered in analyzing the results.

Having obtained the students’ informed consent, participants were asked to complete both the SAS and PCA during the first week of class. Students were given a full class period (50 minutes) to complete the PCA, and had access to Desmos or a graphing calculator (as appropriate) if they so desired. Prior to beginning the PCA, students were told that it would provide a sense of what everyone knew coming into the class and would not be counted for a grade. Obtaining these course entry data points (for both the SAS and PCA) did require a substantial time investment during an already hectic first week of class, but the benefits to the final analysis outweigh the cost. Having pre- and post-course data points for both instruments, as well as control and treatment groups that received the same instruction and completed the same course work, will allow for a much more direct examination of any treatment effects.

Summary

The primary data collection instruments, the PCA and SAS, will allow investigation of the principle research interests; in particular, any differences in the development of students' understanding of the concepts central to the college algebra curriculum and any shifts in attitudes toward mathematics. Obtaining pre- and post-intervention responses on both instruments will allow for exploration of any changes that occur over the course of the term. Furthermore, the PCA taxonomy and detailed analysis of the results could indicate any qualitative differences in the type of skills or reasoning abilities that are privileged by the use of graphing calculator or DGU technology. In the following chapter, analysis of the study results will be presented alongside discussion of the consequences.

Chapter 4 - Results

In this chapter, the collected data and results of the study will be examined. Data analysis will include discussion of the results of both the Precalculus Concepts Assessment (PCA) and Student Attitude Survey (SAS), as well as observations about student engagement. As a reminder to the reader, the primary focus of this study is the effects of a dynamic graphing utility (in this case, Desmos) on student outcomes. In particular, a treatment group of students using Desmos as part of instruction and assessment was compared to a control group in a similar learning environment, with the key difference that these students were permitted only the use of a TI-84 (or equivalent) graphing calculator. The research questions of primary interest are as follows:

1. Does using Desmos (rather than a graphing calculator) lead to improvements in college algebra students' understanding of the concepts in the course?
2. Does the use of Desmos (rather than a graphing calculator) positively impact college algebra students' attitudes toward mathematics?

By analyzing the PCA and SAS results, we hope to gain some indications as to the effects of DGU use on these aspects of student outcomes.

PCA Results

The PCA was administered to participants in both the treatment and control groups during both the first and last weeks of class, separated by an interval of about 15 weeks. Students were given a full class period (50 minutes) to complete the assessment, and were informed that the results would not affect their grades. During the assessment, students in the control section were permitted to use a graphing calculator (TI-84 or equivalent), while students in the treatment section were allowed access to Desmos on their personal devices (smartphones, tablets, or laptops) with wireless connectivity functions disabled. The overwhelming majority of items on the PCA are calculator neutral (Cannon & Madison, 2003); that is, apart from basic arithmetic, calculator capabilities are unlikely to be of much benefit in completing assessment items. For example, questions about the graph of a function may be accompanied by the graph, but not a symbolic representation that could be entered in a graphing utility. As a result, the assessment tests the reasoning abilities that students have developed related to (for example) graphical representations without requiring accurate use of the tool. Students' examinations were scored by hand, and each students' responses to individual items were recorded in a spreadsheet. Students were assigned randomly generated 4-digit identification numbers in order to pair their results, but this information was stored separately and used only by the primary researcher. Once the results of the post-test had been recorded, numerical scores were calculated for each student, as well as a change score found by subtracting each student's pre- score from their post- score.

Statistical analysis began by computing descriptive statistics and verifying the normality of each data set. A total of 37 students (17 control, 20 treatment) completed both the pre- and post-administration of the PCA, so these results formed the primary data. While testing the results for normality, one outlier was identified that significantly affected the analysis. This score, which came on the post-test in the treatment group, was nearly 3.5 standard deviations above the mean and was excluded from analysis. With this result removed, histograms and Q-Q plots were used to verify that the data were sufficiently normal. Results of the initial administration of the PCA showed that the treatment group scored higher in the mean than the control group, so an independent two-sample *t*-test was conducted to compare PCA scores on the initial assessment between the treatment and control groups. There was a significant ($\alpha=0.05$) difference in the scores for the DGU ($M=9.37$, $SD=2.67$) and graphing calculator ($M=6.41$, $SD=3.86$) groups; $t(34)=2.697$, $p=0.011$. This suggests that there were significant differences in the two groups initially, and more specifically, that the students in the DGU (treatment) group scored higher on the pre-intervention PCA. As a result of these initial differences in the two classes, analysis of the end-of-course examination requires care to ensure that any disparity in the groups is not simply the result of this initial gap. Note that a summary of statistical results is included in Table 3.

Table 3

Summary of PCA Results

		DGU (*n*=20)	GC (*n*=17)	*t*-test results
Pre-				
	M	9.37	6.41	$t(34)=2.697$
	SD	2.67	3.86	$p=0.011$*
Post-				
	M	9.68	7.71	$t(34)=2.02$
	SD	2.47	3.39	$p=0.052$*
Difference:				
	M	0.32	1.29	$t(34)=0.96$
	SD	3.20	2.89	$p=0.344$

Table 3. Results of independent two-sample *t*-tests conducted on the PCA pre- and post-test results for both treatment (DGU) and control (GC) groups. Mean (M) and standard deviation (SD) are reported in each case, along with the *t*-test results.
* denotes significance at the $\alpha=0.05$ level.

To get an initial sense of the post-intervention results, an independent two-sample *t*-test was conducted to compare PCA scores on the final assessment between the two groups. This test revealed marginally significant results at the $\alpha=0.05$ level; the DGU group (M=9.68, SD=2.47) showed overall gains and continued to score higher than the graphing calculator group (M=7.71, SD=3.39), but the difference lies at the edge of statistical significance ($t(34)=2.02$, $p=0.052$).

While the mean score of both groups increased over the course of the semester, greater gains were seen in the control group mean (+1.29) than in the treatment group mean (+0.32), leading to a weaker overall result. In an attempt to mitigate any effects from the initial between-groups difference, a subsequent two-sample *t*-test was conducted on the change scores, calculated by finding the difference in each student's pre- and post- scores. The results of this test indicate that there was no significant difference in the change score between the treatment (M=0.32, SD=3.20) and control (M=1.29, SD=2.89) groups; ($t(34)=0.96$, $p=0.344$). This lends further evidence to suggest that the graphing utility used by the students in the two groups -- the DGU Desmos or the graphing calculator -- did not significantly impact the development of the knowledge and reasoning skills needed for the participants to score well overall on the PCA. Analysis of covariance (ANCOVA), which is robust against preexisting differences in groups and reduces within-group error variance, was administered as an additional check on these findings. In particular, a one-way ANCOVA was conducted to determine if there was a statistically significant difference between the DGU and graphing calculator groups on the end-of-course PCA scores, controlling for beginning-of-course PCA scores. There was no significant effect of the type of graphing tool used on end-of-course PCA score after controlling for beginning-of-course PCA score; $F(1, 33)=0.407$, $p=0.528$. This further supports the conclusions drawn above; although the mean scores of both groups improved over the course of the semester, there was no significant difference in the amount of improvement between the two groups.

In the process of organizing and analyzing the data, a gender disparity emerged among the participants in the treatment group. In the control group, the mean pre-post change score was very similar for both female and male students (+1.5 and +1 points, respectively). However, among the treatment group, the mean change score for female participants, excluding the aforementioned outlier, was over 3 points higher than that of the male participants (+2.1 and -1 points, respectively). This may indicate that some aspect of working with Desmos holds greater benefits for female students, but additional research will be needed to explore this possibility. Once the already-small sample has been subdivided by gender, there are simply too few data points for any meaningful conclusions. Nevertheless, these observed gender differences could serve as a guide for future research and provide an interesting direction for investigation.

To better understand any differences in the two groups, the results of individual assessment items were compiled and compared. Since each item on the PCA is multiple choice, counting the number of students choosing each answer option and comparing pre-/post- results between the two groups may uncover any distinction in the way that participants' reasoning skills developed in the course of the study. In order to obtain this data, the number of responses for each answer option on both the pre- and post-test were used to calculate a delta score representing the change in number of participants choosing each answer option from the pre-test to the post-test. Of particular interest is the change in the number of correct responses; for brevity, this will be denoted as "ΔCR". For example, on Item 1 there were three more correct responses on the post-test than on the pre-test among the control group participants, so ΔCR for this item in the control group is +3. As a means of comparing the two groups, the difference in ΔCR scores between the treatment and control groups is of primary interest. While there were relatively few PCA items for which these ΔCR scores differed substantially, there were five items that were found to have a difference of 4 (>10% of total respondents) or more points. These are summarized in Table 4.

Table 4

Comparison of ΔCR Scores for Treatment and Control Groups

Group	Item 2	Item 10	Item 17	Item 19	Item 25
Control	-6	-4	+2	-4	+3
Treatment	+2	0	-2	+1	-3
Difference	+8	+4	-4	+5	-6

Table 4. There were five PCA items whose ΔCR score showed a difference (T-C) of 4 or more points, indicating that one group *improved* or *maintained* better than the other on these items, but not necessarily that the actual number of correct responses differed.

There are three PCA items for which the treatment ΔCR was at least 4 greater than that of the control group: Items 2, 10, and 19. On the other hand, Items 17 and 25 showed a difference of at least 4 in favor of the control group. Referring to the PCA Taxonomy (see Table 1, chapter 3) allows for a close examination of the implications in terms of the reasoning abilities and understandings demonstrated by these responses. The reasoning abilities -- process view of function, covariational reasoning, and computational abilities -- assessed by these items do not demonstrate a marked difference between the two groups; the variance in ΔCR scores consists of instances where each group outperformed the other on each of these three reasoning abilities. However, the types of understanding measured by the PCA -- those related to the meaning of function concepts, growth rates of function types, and function representations -- do indicate that there may be some difference in the development of the two groups. The items on which the DGU group attained a higher ΔCR score indicate that their understanding of rate of change, function inverse, and graphical representations may have been improved or maintained better than that of their counterparts in the graphing calculator group. On the contrary, the graphing calculator group demonstrated more improvement on items related to function composition and algebraic representations than did the DGU group. Once again, the small overall sample size and relatively small differences in ΔCR scores prohibit any strong inference, but may provide direction for future research.

The PCA results do not indicate a significant difference in the impact of the type of graphing utility used for the course between the two groups. Although there was a statistically significant difference on the pre-test and a marginally significant difference on the post-test, controlling for the initial disparity in the two groups suggests that the gains made by the participants over the course of the study were essentially the same for both groups. The results of other assessments, including common computer-based and written comprehensive examinations taken at the end of the term, lend more evidence to this conclusion; there were no significant differences between the scores of the two groups on either of these assignments. As detailed above, analysis of the PCA results suggest that students in the DGU group may have developed stronger graphical reasoning skills than those using only the graphing calculator. On the other hand, students in the graphing calculator group may have exited the course with a better understanding of algebraic representations. This difference in student outcomes may be due in part to the nature of working with these tools; while Desmos accepts input in nearly any form, the graphing calculators used by the control group require students to enter expressions as "y =". As a result, some initial symbol manipulation is often required of students, and in certain situations students may choose to apply algebraic (rather than graphical) methods instead. This could lead to these students becoming more familiar with algebraic representations. On the contrary, the relative ease of working with Desmos, as well as the larger and more visually accurate graphical depictions that it produces, likely contributed to students in the DGU group developing stronger graphical reasoning skills. Again, the evidence here is far from conclusive; however, it does point the way for future research and suggest some possible differences in the types of reasoning abilities supported by the graphing calculator or DGU.

An unexpected result that emerged in the process of analyzing the data was that although mean PCA scores increased for both groups, this increase was not significant at the 0.05 level for either group. Paired two-sample *t*-tests were conducted on pre- and post-test PCA scores for both the DGU and graphing calculator groups in order to determine the effects of the course as a whole on the types of understanding assessed by the PCA. Neither group improved their PCA scores by a significant ($\alpha=0.05$) amount over the course of the term. The overall results of the course in terms of student performance were not unusual; grades were similar to the population of college algebra students in the same cohort, and there is no reason to think that the performance of study participants on assessments and in final course grades was anything but typical. This suggests that the typical college algebra curriculum, regardless of technological support available, does not foster the development of the types of skills and reasoning abilities assessed by the PCA.

SAS Results

The version of the SAS used in this experiment is a 28 item, 5 point Likert-scale questionnaire that explores aspects of students' attitudes toward mathematics and technology. Participants were asked to complete the SAS once during the first week of class, and again during the last week, with an interval of about 15 weeks separating the two. Both pre- and post- surveys also included a prompt and space for students' written comments, though few students took advantage of this opportunity. Each time the SAS was administered, participants were given about 15 minutes to complete the survey. As a result, it was much more feasible to obtain SAS results from students who were absent when it was given; they could easily complete the survey in a few minutes before or after the next class. Making up a missed PCA would require a significant time commitment on the part of the student, and furthermore would need to be done within a day or two of the scheduled administration in order to be consistent with the rest of the data. Consequently, no examinations were completed in this way. Thus the final data set, which consists only of those students who completed all four instruments, was driven largely by the PCA response rate. In addition, there were three individuals who completed the four instruments but did not consent to have their results included in the research study at the beginning of the experiment. Once again, student responses were paired using randomly generated identification numbers. To conduct analysis of the results, survey responses were coded using a 0-4 scale, with 0 representing “strongly disagree” and 4 representing “strongly agree”.

The initial phase of data analysis involved examining the results of the SAS administered during the first week of class. Participants’ individual responses to each survey item were recorded numerically, using the 0-4 scale previously described. In order to analyze the data in aggregate, the mean response was calculated for each survey item among each of the two study groups. This created two sets of 28 data points, with one datum for each item on the SAS (28 total) and one set for each of the treatment and control sections. To analyze the initial data, a two-sample paired *t*-test was conducted to determine if there were initial differences in SAS responses between the treatment and control groups. The results of this test indicate that there was no significant ($\alpha=0.05$) difference between the two groups on the initial SAS iteration; $t(27)=0.28$, $p=0.78$. This suggests that there were no statistically significant differences in the initial SAS responses between the two groups. It should be noted that a paired *t*-test was used here due to the nature of the data sets; in particular, a given entry in each set corresponds to the same SAS item, so pairing the data restricts the test to differences in the mean responses in an item-by-item fashion. These test results support the hypothesis that the participants did not differ significantly in the aspects of attitude toward mathematics measured by the SAS at the beginning of the course.

A similar strategy was applied to analyze the data obtained from the second administration of the survey, completed by the participants at the end of the semester. Again, mean responses were calculated for each survey item within each of the two study groups. These data sets were used to conduct a two-sample paired *t*-test to determine if there were between-group differences in SAS responses at the end of the course. This test revealed that there was a significant ($\alpha=0.05$) difference in the two groups on the post-test; $t(27)=2.38$, $p=0.02$. Since the initial SAS results failed to indicate any distinction between the DGU and graphing calculator groups in the aspects of attitude toward mathematics that are measured by the survey, it is reasonable to suppose that the contrast in the two groups that developed has some relation to the difference in participants' experiences over the course of the semester. This is a complicated issue; each individual will have a unique set of experiences, perceptions of those experiences, and subsequent developments related to mathematics (see, e.g., Piaget, 1972). The current experiment was designed with the intent to control as many factors related to the course itself as possible; participants in both treatment and control groups completed the same assignments, assessments, and classroom activities. Since the primary difference (at the group level) in the two groups is the type of graphing utility in use, it is likely that this was a contributing factor to any observed differences in the groups on the end-of-course SAS.

The mean responses of several survey items did not differ much from pre- to post-test, so analysis was conducted to determine which individual items changed over the course of the semester. For each survey item, a paired two-sample *t*-test was conducted to determine if there were significant differences in participants' responses on the initial and final administration of the SAS within each of the two study groups. Missing responses were replaced with the mean for that item. The results of these tests are summarized in Table 5, which includes summary statistics and *t*-test results for those items with a statistically significant difference. The column labeled "ΔM" shows the change in mean score from pre- to post-test, and was calculated by subtracting the former from the latter.

There were a total of nine items that showed a statistically significant (α=0.05) difference in the initial and final SAS results for one or both groups. Of those, four were significantly different in both groups, while the other five differed for only one group. The four items common to both groups (items 3, 5, 20, and 22) all relate to social components of learning: preference for working alone or in groups, perception of how well the respondent learns on their own, and attitude towards listening to the ideas of peers. The results of all of these items suggest that the participants grew to view group work in mathematics less favorably in the course of completing this class. As mentioned previously, group activities were a regular feature of class sessions, and in addition, students were required to complete one out-of-class group project during the second half of the term. For the project and majority of in-class activities, students chose their own partners (typically groups of 3-4); however, the out-of-class project in particular can be taxing for students and it has not been uncommon historically for friction to arise within groups. This may have influenced responses to these items to some degree, as there was a relatively short interval between the project due date and the second SAS administration. In addition, item 12, which asks about group activities outside of school, differed significantly from pre- to post- for the treatment group only. These participants' responses to the statement "I do not participate in many group activities outside of school" at the end of the course were more neutral than at the beginning of the course, when the mean response fell near the "disagree" option.

Table 5

Significant Results of Paired t-tests by Group

Item	Group	M (pre-)	SD (pre-)	M (post-)	SD(post-)	ΔM	*p*
2	T	2.52	1.12	3.00	1.13	+0.48	0.02**
	C	2.36	0.90	2.73	0.98	+0.36	0.10
3	T	1.43	0.84	2.48	1.16	+1.04	0.00*
	C	1.18	0.73	2.05	1.17	+0.86	0.01*
5	T	1.48	1.04	2.74	1.14	+1.26	0.00*
	C	1.41	1.37	1.95	1.09	+0.54	0.01*
8	T	2.61	0.84	2.65	1.40	+0.04	0.88
	C	2.55	0.74	3.00	0.93	+0.45	0.02**
12	T	1.17	0.98	1.78	1.09	+0.61	0.02**
	C	1.27	0.83	1.50	0.91	+0.23	0.26
15	T	2.35	1.19	2.48	1.20	+0.13	0.48
	C	1.68	0.99	2.18	1.10	+0.50	0.00**
16	T	1.83	1.27	2.27	1.05	+0.45	0.05**
	C	2.59	1.14	2.68	1.21	+0.09	0.69
20	T	2.61	1.03	1.91	1.08	-0.70	0.01*
	C	2.59	1.05	2.09	1.11	-0.50	0.02*
22	T	2.91	0.60	2.43	0.95	-0.48	0.02*
	C	2.82	0.80	2.05	1.09	-0.77	0.00*

Table 5. There were a total of 9 SAS items with a statistically significant difference in scores from the pre- to the post-test in one or both groups. Mean (M) and standard deviation (SD), as well as change in mean (ΔM) are reported.
* denotes statistical significance at the α=0.05 level.
** denotes statistical significance at the α=0.05 level in one group only

Two other items differed significantly from the beginning to the end of the term in the treatment group only. Both of these (items 2 and 16) deal with students' past experiences in mathematics classes and may indicate a shift in perception. There were also two items (8 and 15) for which the post-test responses differed significantly from the pre-test in only the control group. Item 15, which concerns students' confidence in their ability to solve mathematics problems, exhibits growth in mean response in both groups; however, this difference was statistically significant for only the control group. Similarly, the post-test results for item 8 showed growth among both groups, suggesting that both groups experienced gains in their perception that technology can make mathematics easier to understand, although in the treatment group these fell short of statistical significance.

The survey items found in the SAS can be grouped into four factor categories that describe the components of student attitude measured by those items: positivity towards learning mathematics and school (deep affect), working collaboratively and related effect, working privately, and use of technology (Brookstein et al., 2011). To analyze differences in the two groups across these factors, the change in mean response scores from the pre- to post-survey was calculated for each survey item among all participants in both groups. These change scores were then used to find the difference in change in the two groups, calculated by subtracting the change in the mean response among the control group from the change in mean response among the treatment group. For example, on item 1 ("I think mathematics is important in life"), the mean response in the control group decreased by 0.045 from the pre- to post-test, while the treatment group's mean response increased by 0.083, giving a total difference of 0.13. Once these differences had been calculated for each item in the SAS, items were grouped according to the four factors for analysis. The items related to each factor can be found in Table 6, and a copy of the instrument used in the study can be found in the appendix.

Table 6

SAS Items by Attitude Component

Factor	Survey Items
Positivity towards learning mathematics and school (deep affect)	1,2,6,7,13,14,22,23
Working collaboratively and related effect	4,11,12,15,16,17,18,19,21,24
Working privately	3,5,10,20
Use of technology	8,9,25,26,27,28

Table 6. The 28 items on the SAS correlate to four components of student attitude.

Responses to the items corresponding to positivity towards learning mathematics and school (deep affect) do not indicate major shifts in student attitude. Of these items, one (item 22) was significantly different from pre- to post- in both groups, while one other (item 2) was significantly different in only the treatment group. Considering the change in mean response from pre- to post- among the items related to this attitude component suggests that respondents in the control group tended to fare better in overall attitude towards school (item 13) and mathematics (item 14) in general. On the other hand, mean responses for those in the treatment group were more favorable regarding the importance of mathematics in life (item 1) and personal interest in the subject (item 23). This presents a somewhat convoluted picture; participants in the treatment group indicated enjoying mathematics less but being more interested in the subject, while the responses of their counterparts in the control group suggest the converse. Both groups appear to have grown more anxious about school (item 6) in the duration of the course, although it may be worthwhile to remember that nearly all of the participants were first-semester college students, so some anxiety about one's first round of final examinations may be expected.

As previously discussed, many of the largest changes in students' responses were on those items related to working collaboratively or privately when learning mathematics. In particular, mean responses to items 3, 5, 12, 15, 16, and 20 all differed significantly from pre- to post-test, and each of these items relates to the working collaboratively or working privately attitude components. This evidence consistently suggests a negative trend in participants' inclination towards group work and an increase in preference for working individually. Even among those items whose pre-/post- differences were only statistically significant in one group or the other, the direction of change is consistent across both groups and indicative of this same trend. Item 15, which concerns student confidence, shows that while mean confidence levels increased for both groups, the increase was significantly more pronounced among the control group. While the general increase in confidence across both groups is encouraging, future studies involving the use of a DGU should attempt to discern how or why it relates to changes in student confidence, and develop strategies for addressing these differences.

Items related to the use of technology were relatively consistent from pre- to post-test; among these, only one (item 8) had significantly different responses at the end of the course, and this in the control group only. This item concerns student perceptions about the ability of technology to make mathematics easier to understand. There was a slight increase in the mean response for this item in both groups, but it was more pronounced in the graphing calculator group ($\Delta M=+0.45$) than the DGU group ($\Delta M=+0.04$). Among the other items related to this component of attitude, there are again some conflicting indications. Mean scores for items 26 and 28 increased more in the treatment group, suggesting that these students developed a stronger sense that technology use helped them to understand mathematics, while also experiencing feelings of being in their "own private world" when using technology. Meanwhile, the responses of students in the control group increased on item 27, indicating that these students became more comfortable using technology as part of class. It is possible that some of the conflict observed in these responses is due to some unintended ambiguity in the wording of survey items. Although disparities between the use of a DGU and graphing calculator are the primary focus of the study, the students also spent a significant amount of time working with an online learning management system (MyLabsPlus) to complete homework, quizzes, and tests. It is entirely possible that some inconsistencies exist in how individual respondents interpreted survey items about technology; participants could be thinking primarily of the graphing utility, the learning management system, or some conflation of the two in response to the term “technology”. In future studies, care should be taken to refine these findings by using more specific language.

There are several considerations that must be taken into account in interpreting these results. Although the present study was conducted over the course of a 16-week semester, this is a brief period of time relative to the participants’ years of prior experience with school in general, and mathematics in particular. Students’ previous experiences with mathematics will invariably have an influence on attitudes towards the subject, and the beliefs, perceptions, and feelings that are the result of these prior experiences can be deep-seated and slow to change. In light of this fact, it seems unlikely that attitudes will change significantly in the span of a single-semester course. It is possible that some of the participants also had previous experience with graphing calculators. No data was collected to determine the frequency with which this occurred, but graphing calculator use is common in U.S. secondary classrooms and has been for some time. Hennessy and Dunham (2001) argue that technology use can ease negative attitudes, but is more likely to produce lasting change when employed over a long period of time. Therefore it is conceivable that for a student with years of experience using a graphing calculator, a few weeks’ worth of contact with a DGU may result in only minor relative changes.

Student Engagement and Perception

Although not a primary focus of the present study, student engagement is also an important consideration in regards to technology use in the classroom. There is some evidence to suggest that the DGU group was more engaged with the technology than those in the graphing calculator group. As part of an out-of-class project, students in both classes were required to complete an art project in which graph transformations, domain & range restrictions, and familiar function types were used to “draw” a picture. The minimum requirements given to students for this project were that at least three different types of functions (or other relations) must be used, and that projects should include no fewer than fifteen individual pieces. As one might expect, many students stayed close to these necessary conditions; however, students using Desmos went beyond the minimum requirements more frequently than their counterparts in the graphing calculator group. On average, students in the DGU group used over three more graphs than those in the graphing calculator group (mean 24 and 20.96, respectively) and the median number of graphs used in the DGU group was higher than in the graphing calculator group by four (median 20 and 16, respectively). While this is far from conclusive, it may indicate that working with a DGU presents a more engaging experience for students. Additional research is needed in this area before anything can be inferred with confidence, but it does present an additional dimension to consider.

Students’ perceptions of the tool may also contribute to its overall effects. To gather information about the way students perceived using a DGU or graphing calculator and the course in general, the SAS included a prompt for any written comments. Unfortunately, few students exercised this option; however, one student did write that “the inclusion of technology in this course is the only reason that I was successful”. This appears to indicate that this particular student did feel that technology use throughout the course was beneficial, but may also suggest that the student had come to depend on the technology. The design of course materials included some measures to safeguard against over-reliance on technology (e.g., requiring students to find exact solutions in terms of radical or logarithmic expressions, or interpreting a solution in context), but it still presents a concern. Another student commented that Desmos "really helped me visualize the problem, and therefore the answer". This supports the findings of previous research related to the development of visualization skills when using technology (e.g, Quesada & Maxwell, 1994; Smith & Shotsberger, 1997), and lends evidence to the idea that the graphical advantages that Desmos holds over the graphing calculator may contribute to this development.

Summary

Overall results of the PCA do not indicate significant differences between students using the DGU and those using the graphing calculator. Although the treatment group achieved higher mean scores on both the pre- and post-tests, there was not a statistically significant difference in the two groups. However, detailed item analysis suggests that there were differences in the types of skills and reasoning abilities that students developed while working with one tool or the other. The results of the SAS indicate a marked negative trend in students' attitudes toward working in groups, but findings related to other dimensions are less clear.

Chapter 5 - Summary and Conclusions

In order to explore any differences in student outcomes as a result of using a DGU rather than graphing calculator, two groups of college algebra students were allowed to use one type of graphing utility in different sections of the same course, taught by the same instructor, and using the same course materials and assessments. One group (treatment) was allowed to use the DGU Desmos for tests and other assessments, and encouraged or required to use it as part of instruction and other in-class activities, while the other (control) group was allowed the use of a graphing calculator (TI-84 or equivalent). These students were asked to complete research instruments at the beginning and end of the 16-week term to collect data pertaining to the primary research questions:

1. Does using Desmos (rather than a graphing calculator) lead to improvements in college algebra students' understanding of the concepts in the course?
2. Does the use of Desmos (rather than a graphing calculator) positively impact college algebra students' attitudes toward mathematics?

In this chapter, the results of the experiment (as detailed in chapter 4) are summarized briefly. In addition, the implications of these findings, limitations of the present study, and directions for future research will be discussed.

Summary of Results

The Precalculus Concept Assessment (PCA) developed by Carlson, Oehrtman, and Engelke (2010) was used as a means of measuring the development of participants' understanding of algebraic concepts. This instrument was administered twice during the term - once during the first week of class, and again a few days prior to the final exam - with an interval of about 15 weeks separating the two. Comparing the pre- and post-test PCA results showed that while mean scores of both groups improved, there was not sufficient evidence to conclude that this change was statistically significant within either group. Furthermore, after controlling for the initial disparity at the beginning of the term in the two groups, there was no significant between-groups difference detected in the amount of improvement; that is, mean scores in both groups improved by the same amount, speaking statistically. Although the small sample size discourages over-reliance on inferential statistics, the lack of a pronounced difference between the groups suggests that the type of tool used did not significantly impact overall performance on the PCA. It is interesting to note that the largest gains in PCA score from pre- to post-test occurred among female students in the treatment group. Once subdivided by gender, the samples are much too small to draw any conclusions, but this would appear to support the hypothesis of prior research (e.g., Ruthven, 1990; Dunham, 1990) that the feedback and exposure to graphic representations provided by a graphing utility may be likely to improve the performance of female students relative to their male counterparts. It is possible that some dimension of working with a DGU amplifies this effect, but additional research is required to better understand these phenomena.

Although the overall effects of DGU use on PCA scores were not markedly different from the graphing calculator group, distinctions emerged in the type of skills and reasoning abilities that students in each group developed. These may be indicative of relative strengths and weaknesses of using each tool as a support for students' developing understanding of the core concepts in the college algebra curriculum. While those students in the graphing calculator group improved more on items related to symbolic representations and function composition, the DGU group showed greater gains on items involving rate of change, function inverse, and graphical representations (see Table 4, chapter 4). The observed differences in symbolic and graphical representations may stem from the nature of working with these graphing utilities; in order to graph with a graphing calculator, some initial symbolic manipulation is often required whereas Desmos is capable of accepting many forms of input, including implicitly defined functions. On the other hand, Desmos provides graphical representations that are more visually appealing as a result of higher display resolution, color graphics, and intuitive interaction (e.g., pinch-to-zoom). Differences in the concepts that appear to have been privileged will require additional research to fully explain. However, the rate of change items could be related to dynamic features of Desmos such as sliders, since these students had the opportunity to see function values and secant lines in motion, while those using the graphing calculator did not.

To measure changes in students' attitudes toward mathematics, the Student Attitude Survey (SAS) developed by Brookstein, Hegedus, Dalton, Moniz, and Tapper (2011) was administered to participants at the beginning and end of the course, approximately 15 weeks apart. In addition to the 28 Likert-scale items on the SAS, a prompt for written comments was included, though regrettably few exercised this option. Student responses were recorded using a 0-4 scale ("strongly disagree" to "strongly agree", respectively) and results of the pre- and post-intervention surveys were paired for each individual student. Statistical analysis in the form of a two-sample paired *t*-test indicated that there was no significant difference in the initial mean responses between the two groups, suggesting that it is reasonable to assume that the students in each group entered the class holding comparable attitudes on the dimensions measured by the SAS. However, a similar test revealed statistically significant between-group differences on the post-test, indicating that some factor or factors experienced by the students in the course of the term caused attitudes among the two groups to shift in different ways. Subsequent paired *t*-tests were used within the groups to determine which survey items changed over the course of the semester, resulting in a total of nine items for which the pre- to post-test change was significant.

Four of these were significantly different within both groups, while the other five showed a significant change in only one group.

The most consistent findings obtained from the SAS results were on those items related to working collaboratively or privately in mathematics. These results were indicative of a marked negative trend in participants' attitudes toward working in groups and an increase in preference for working individually. This was somewhat surprising, as students regularly worked in groups as part of in-class activities and there were no obvious indications in the classroom about any ill feelings toward group work. However, a group project, which required research, writing, and collaboration outside of the classroom and took place in the latter part of the term, may have contributed to these results. Other findings showed evidence that the control group experienced a larger gain in confidence related to their ability to solve mathematics problems and perceived that technology can make mathematics easier to understand, while participants in the treatment group may have experienced a shift in perception about mathematics classes. Items related to technology were largely inconsistent; while one item may show larger gains for one group than the other, the next item may indicate the opposite. This could be the result of non-specific language in the item prompts as a consequence of the presence of multiple technologies within the course; students not only used the graphing calculator or DGU, but also interacted with an online learning management system to complete course assignments.

Implications of Findings

The overall results of the PCA do not give a clear sense of one tool being inarguably better than the other. However, this should not come as a surprise; the history of mathematics education presents a compelling argument against the existence of any cure-all. Instead, educators should attempt to better understand the tools and methods available so that they may be effectively employed. A closer examination of the results shows that the type of tool used by the students -- the graphing calculator or a DGU like Desmos -- may have an impact on the type of reasoning abilities and skills that students develop. In the current study, participants using Desmos appear to have experienced more growth related to working with graphical representations, rate of change, and function inverse. On the other hand, those students limited to using only the graphing calculator showed more improvement on items related to symbolic representations and function composition. With these differences in mind, it may be possible to adjust pedagogy and curriculum in future courses to both embrace the strengths and curtail the weaknesses of either tool.

The results of the SAS also fail to reveal a clear distinction between the graphing calculator and Desmos. The most consistent indications from the SAS results across both groups suggest that the students' attitudes toward group work and collaboration suffered during the course of the term, but this is more closely related to course design and pedagogy than the type of technology in use. The control group did show significant improvement on item 15 ("I feel confident in my abilities to solve mathematics problems"), but the mean score for the treatment group also improved on this item and due to initial differences between the groups, the post- score for the treatment group was still slightly higher (2.48) than that of the control group (2.18). Survey items related directly to technology present some inconsistent information about this component of student attitudes; while some items (e.g., item 8) suggest that the graphing calculator may be more advantageous, others (items 25, 28) appear to indicate that using Desmos has a greater positive impact. The relatively short duration (16 weeks) of the study may have been a contributing factor to the slight overall changes in SAS results. Attitudes tend to be deeply entrenched and slow to change, so a single semester may be too short a time for many significant differences to appear. However, the results of this study do provide some information about the effects of technology on student attitudes, and also the impact that emphasis on group work had on these students. These findings can be a valuable asset to inform the design of follow-up studies, particularly as it relates to measuring aspects of student attitudes.

There is also reason to believe that working with Desmos may provide a more engaging experience for students than approaches that rely on the graphing calculator. The results of the art project described previously, as well as those of similar projects (Beigie, 2014; Ebert, 2014) and anecdotal evidence from this and previous classes, suggest that some students prefer Desmos to the graphing calculator. Additional research is needed to further explore this hypothesis. For many students in a course like college algebra, engagement and interest in the material can be an ongoing source of struggle. However, if using a DGU can be shown to facilitate improvements in student engagement, there would be strong motivation to adopt such utilities on a wider basis.

Limitations of the Study

There are some limitations with the present study that must be considered when interpreting these results, and that may provide direction for future research. First of all, the small sample size discourages us from relying too heavily on the results of inferential statistics. While the findings do provide some information about possible differences in student outcomes when using a DGU as compared to a graphing calculator, further research is needed to explore these initial indications in greater detail. Additional data could provide a more complete picture of the effects that intentional and prolonged DGU integration has on students. Subsequent research should strive to expand the data set not only in terms of the number of participants, but also by collecting additional types of information. The PCA is a multiple-choice instrument and the vast majority of student work in the course was done using an online learning management system, so there are very few artifacts that could be used to examine students' thought process in problem solving. Looking closely at student work could provide some useful insight, but the research instrument requires some modification to collect this type of information. Student interviews or think aloud sessions would also contribute valuable data about student thinking to future study. Items related to the use of technology on the SAS lack specificity that may have caused confusion for respondents, as they regularly interacted with both graphing technology (in the form of the graphing calculator or Desmos) and the learning management system. There were also very few written comments or responses to working with the graphing utilities, which could have provided valuable information about students' perceptions. Follow-up interviews with students might yield additional data that would clarify survey responses.

Other limitations concern the course itself, rather than the research instruments. Although this study was conducted for a full 16 week term, the timeframe is arguably still short relative to students' prior experiences with mathematics. Graphing calculator use is commonplace in school mathematics in the United States, so it is not unreasonable to assume that many college algebra students will have several years of experience using a graphing calculator prior to entering the course. It is also entirely possible that some students may have had previous experience using Desmos or other graphing tools. In future studies, survey data about students' prior use of graphing utilities can be collected to address this uncertainty. Attitudes can be slow to change, and it may be the case that these previous experiences had some effect on the observed student outcomes. Presenting another source of restraint is the design of the experiment itself. In an attempt to make the comparison of the treatment and control groups as straightforward as possible, Desmos was used almost exclusively as a graphing calculator replacement. While there are important differences from the graphing calculator in this regard (e.g., sliders, higher screen resolution), it also restricts the use of some other features that may be beneficial for students. The Desmos activity builder, which allows one to create self-paced, guided activities for students and view the results of graphing exercises and writing prompts, presents an opportunity for building student engagement. However, because this does not translate well to paper for a more traditional classroom environment, it was employed only once during the course. Conceivably, using Desmos to its full potential by introducing more nontraditional items into the curriculum could further differentiate it from the graphing calculator.

Future Research

The results of this study, as well as the experiences gained in conducting the experiment, have provided a great deal of information to guide the direction of future research. The PCA provided insight into learning outcomes and revealed possible differences in the type of reasoning abilities developed by students in the two study groups. However, as a consequence of its multiple-choice format, it does not encourage students to show work or produce artifacts of their thinking apart from the answer choice. In the future, augmenting the PCA results with open-response items could provide fertile ground for studying students' problem solving strategies, symbolic skills, and organizational methods. Modifying the curriculum to include more nontraditional assessments and decrease the emphasis on computer-based homework problems (often devoid of context and evaluated on an all-or-nothing basis) would also give a more detailed picture of how these technologies are impacting student learning. The literature related to graphing calculator and computer use in the classroom repeatedly cites the potential benefits of students using the technology to conjecture, check, and refine their thinking; the dynamic features of Desmos (e.g., sliders) make this process even more straightforward, so introducing more opportunities for students to "play" could lead to changes in student outcomes.

Attitude outcomes could be explored in greater detail by collecting not only additional SAS data, but also by refining the survey items and gathering a wider variety of evidence. Survey items related to technology do not specify the type of technology to which they refer, which may have resulted in students responding to these items in an inconsistent fashion. Carefully phrasing these items to clearly reflect the research questions could alleviate this problem. Subsequent research should also supplement SAS results by collecting more qualitative data, such as written responses and records of student interviews. Although the version of the SAS used in this study included a prompt for students' written comments, it was used very infrequently. More deliberate attempts to gather information about the way students perceived using a DGU or graphing calculator, in the students' own words, could provide valuable insight into how and why students use graphing utilities, and shed light on any differences experienced by those using each tool.

Anecdotal evidence, both from this study and previous experience using Desmos in the classroom, indicates that it may provide a viable tool for engaging students in doing mathematics. Gathering data to explore this should be a higher priority in any future research, which will require developing methods to study this aspect of students' interactions with the technology. Some of these questions may be answered by gathering qualitative data such as written or interview responses from students, but additional information may be needed as well. Classroom observations by a third party are a potential source of evidence, as are nontraditional assessments in the same vein as the art project used in the present study. These measures may also provide more information about the indications that participants experienced a negative trend in those attitude components related to working collaboratively, and allow subsequent studies to determine what factors may have caused this apparent shift in favor of working individually.

Although the current sample is too small to make conclusive inferences, the observed gender differences could be indicative of larger forces at play. Gender disparity in STEM fields is well documented (e.g., Landivar, 2013), so research that may contribute to improving the experience of female students in mathematics is worthwhile. Future studies could be conducted with the goal of further exploring the gender differences observed in the current project.

Considering possible gender disparity as part of the experimental design, and intentionally seeking information about the differences in the experience of female and male students while learning mathematics using a DGU, could yield valuable findings to help narrow the gender gap. More research is needed, but these preliminary results suggest that this may be a viable avenue of future study and is deserving of further attention.

The biggest question in any small-scale study is how well it can be replicated or generalized to other situations. Although the size of the current sample discourages any strong conclusions, it has provided useful direction for developing further research. In future studies, a research program should be implemented across multiple sections of a course taught by instructors not directly involved in the research design. This would allow for the principal investigator(s) to observe classroom interactions and gather field data, providing an opportunity for a more detailed analysis of students' day-to-day classroom usage of the technology. Such a program would not necessarily need to be limited to college algebra; since the function concept is central to the curriculum through the calculus sequence, the use of a DGU could be applicable to students in precalculus or calculus courses, as well as translating well to algebra in the 9-12 setting. By expanding the scope of the research beyond the classroom of a single instructor and course, a more comprehensive body of data could be collected that would allow for a stronger indication as to the efficacy of using a DGU such as Desmos.

Summary

The goal of this study was to explore any potential differences or advantages to using a dynamic graphing utility as a replacement for the graphing calculator in college algebra. The study was conducted in two sections of college algebra at a four-year public university in the southern United States over the course of a 16-week semester. These two classes were randomly assigned as the treatment and control groups, with the treatment group using the DGU Desmos (including for tests) and the control group using the TI-84 (or equivalent; non-CAS) graphing calculator. Both groups of students completed the same homework, tests, and other assessments throughout the course, and were taught by the same instructor. Students were informed of the study parameters on the first day of class, and consent was obtained from the research participants (please see appendix for a copy of the consent form). During the first week of class, students were asked to complete an attitude survey (SAS) and an instrument to measure their understanding of concepts identified as fundamental to success in future mathematics courses (PCA), which they would again be asked to complete approximately 15 weeks later, near the end of the term. The results of these instruments were used to explore the primary research questions:

1. Does using Desmos (rather than a graphing calculator) lead to improvements in college algebra students' understanding of the concepts in the course?
2. Does the use of Desmos (rather than a graphing calculator) positively impact college algebra students' attitudes toward mathematics?

In addition to these primary questions, the matter of student engagement with technology emerged as a potential source of difference in the two graphing utilities. Although the present study offers only limited evidence on this subject, it does provide direction for future study.

The results of the Precalculus Concept Assessment (PCA) indicated that there were significant differences in mean scores between the two groups on the pre-test, with the treatment group scoring about 3 points (12%) higher than the control. Post-test results showed that both groups improved, but that this was slightly more pronounced in the control group. While the post-test between-group differences were still marginally significant, the difference in mean scores closed somewhat so that the treatment group outscored the control by only about 2 points on the final assessment. Within the two groups, paired *t*-tests of students' pre- and post-test scores showed that although the mean scores of both groups improved over the course of the term, the pre-/post- differences were not statistically significant in either group, which may speak to the efficacy of the curriculum as preparation for the types of knowledge and skills assessed by the PCA. Close analysis of the PCA results suggests that the type of graphing utility used by students may have influenced the type of reasoning abilities that they developed during the course. While students in the DGU group appear to have experienced some benefits relating to rate of change, function inverse, and graphical representations, their counterparts in the graphing calculator group improved more on items related to symbolic representations and function composition. This may relate in part to the mechanics of working with the graphing utilities, since the graphing calculator often requires symbolic manipulation prior to graphing but Desmos will accept implicitly defined functions as input. Differences in items related to rate of change may owe to dynamic features of Desmos, such as sliders.

The Student Attitude Survey (SAS) results provide some indications as to how students' attitudes and perceptions about mathematics and working with technology changed during the term. The most consistent results across both groups were for the items involving attitudes about working collaboratively or privately. These give the overall impression that students in both groups experienced a negative shift in attitudes toward group work, along with a corresponding positive shift in attitudes toward working individually. Working in groups during class meetings was common, and an out-of-class group project near the end of the semester may have influenced these results somewhat. Other findings suggest that the control group may have experienced growth in their self-confidence about solving mathematics problems, and that perceptions about mathematics classes may have shifted for participants in the treatment group. Results of many of the items related to technology were inconsistent, with conflicting indications about students' attitudes toward working with technology in learning mathematics. This suggests some directions for future research, as collecting additional data in the form of written responses, student interviews, or think-aloud work sessions could help to clarify these issues. The version of the survey used in the study also failed to distinguish between the graphing utility and other instances of technology (such as the online learning management system), so more specific language in the survey itself may also provide results that are more transparent.

The current study does not provide clear evidence of either Desmos or the graphing calculator offering students a clearly defined advantage in terms of conceptual understanding of college algebra course topics (as measured by the PCA) or attitudes toward mathematics. However, the lack of clear distinctions in student outcomes suggests that there are also no significant drawbacks to the use of either device, just that there may be some differences in the type of reasoning abilities that students develop as they work with one or the other. There is some evidence to suggest that using a DGU (such as Desmos) may provide students with a more engaging experience than the graphing calculator. This warrants further study in future research, but if subsequent evidence continues to support this theory, it would lend support for more widespread adoption of dynamic graphing utilities as an alternative to the graphing calculator. Additional research, collecting a wide variety of data, could provide a more cohesive picture of the effects of students' use of a DGU. Records of think-aloud sessions and written work would shed some light on the problem-solving strategies that students use when interacting with a DGU, while student interviews or written responses to prompts could be used to further explore dimensions of student attitudes. Classroom observations by a third party would provide a more complete and more reliable record of interactions in the classroom that could supply additional evidence about student engagement with the content and technology.

Although the findings of the present study are not sufficient to make a conclusive judgment in favor of or against the use of a DGU as a graphing calculator replacement to support college algebra students' learning, the data collected here do provide direction and motivation for future research. Furthermore, factors beyond the dimensions considered here, including availability and equity considerations, make an interesting case for the use of a modern tool like Desmos. Current results suggest that it does indeed present the same benefits to students as the typical graphing calculator, with the added advantages of encouraging student engagement and being free-to-use from nearly any web-capable device. As 1:1 mobile device environments become increasingly prevalent in schools at all levels, exploring options that leverage these technologies to support student learning becomes imperative. If subsequent research can expound on the attitude and engagement dimensions of DGU integration, then the case in favor of using such tools would become even stronger.

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Appendix

1. Research Compliance Protocol Letter

UNIVERSITY OF ARKANSAS

Office of Research Compliance
Institutional Review Board

May 14, 2015

MEMORANDUM

TO:	Ryan Thomas Shannon Dingman
FROM:	Ro Windwalker IRB Coordinator
RE:	New Protocol Approval
IRB Protocol #:	15-04-703
Protocol Title:	*The Effects of Using Desmos on Student Attitudes and Conceptual Understanding in College Algebra*
Review Type:	☒ EXEMPT ☐ EXPEDITED ☐ FULL IRB
Approved Project Period:	Start Date: 05/13/2015 Expiration Date: 05/12/2016

Your protocol has been approved by the IRB. Protocols are approved for a maximum period of one year. If you wish to continue the project past the approved project period (see above), you must submit a request, using the form *Continuing Review for IRB Approved Projects*, prior to the expiration date. This form is available from the IRB Coordinator or on the Research Compliance website (https://vpred.uark.edu/units/rscp/index.php). As a courtesy, you will be sent a reminder two months in advance of that date. However, failure to receive a reminder does not negate your obligation to make the request in sufficient time for review and approval. Federal regulations prohibit retroactive approval of continuation. Failure to receive approval to continue the project prior to the expiration date will result in Termination of the protocol approval. The IRB Coordinator can give you guidance on submission times.

This protocol has been approved for 75 participants. If you wish to make *any* modifications in the approved protocol, including enrolling more than this number, you must seek approval *prior to* implementing those changes. All modifications should be requested in writing (email is acceptable) and must provide sufficient detail to assess the impact of the change.

If you have questions or need any assistance from the IRB, please contact me at 109 MLKG Building, 5-2208, or irb@uark.edu.

109 MLKG • 1 University of Arkansas • Fayetteville, AR 72701-1201 • (479) 575-2208 • Fax (479) 575-6527 • Email irb@uark.edu
The University of Arkansas is an equal opportunity/affirmative action institution.

2. Student Consent Form

Student Name (Print):

In signing this form, you are agreeing to participate in a research study designed to develop theories on students' conceptual understanding of and attitudes towards college algebra. I, Ryan Thomas, a graduate student in the mathematics department at the University of Arkansas, will be the primary researcher, and I will be working under the supervision of Dr. Shannon Dingman.

You will be asked to complete a short survey (5-10 minutes) at the beginning and end of the semester that is designed to gauge your attitude toward mathematics. Your responses to this survey will be used for research purposes only with your consent. By signing this consent form, you are also giving your permission for field notes containing descriptions of classroom interactions and activities, as well as results of regular classwork including exams, to be used for research purposes. Your participation in the study will begin on 24 August 2015, and end no later than 18 December 2015.

Participation in this study is voluntary and may be ended at any time. Your decision whether or not to participate will not affect the services normally provided by the University of Arkansas, and will not lead to the loss of any benefits to which you are otherwise entitled. Whether you choose to participate or not will have no effect on your grade or your relationship with your teacher. You are not waiving any legal claims, rights, or remedies because of your participation in this research study. By signing this form, you are asserting that you are at least 18 years of age.

Any information that is obtained in connection with this study and that can be identified with you will remain confidential and will be disclosed only with your permission or as required by law. Confidentiality will be maintained. Should you have any questions or desire further information, please email me at rthomas@uark.edu, or contact my faculty advisor, Dr. Shannon Dingman, at sdingman@uark.edu. You may also contact the University of Arkansas Research Compliance office if you have any questions about your rights as a participant, or to discuss any concerns about, or problems with the research: Ro Windwalker, CIP, Institutional Review Board Coordinator, Research Compliance, University of Arkansas, 109 MLKG, Fayetteville, AR 72701-1201, Phone 479-575-2208, Email irb@uark.edu.

Ryan Thomas

Signature of student: **Date:**

3. Precalculus Concept Assessment (PCA) Taxonomy

Reasoning Abilities

R1 *Process view of function* (items 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 17, 20, 22, 23)
View a function as a generalized process that accepts input and produces output. Appropriate coordination of multiple function processes

R2 *Covariational reasoning* (items 15, 18, 19, 24, 25)
Coordinate two varying quantities that change in tandem while attending to how the quantities change in relation to each other

R3 *Computational abilities* (items 1, 3, 4, 10, 11, 14, 16, 17, 21)
Identify and apply appropriate algebraic manipulations and procedures to support creating and reasoning about function models

Understandings

Understand meaning of function concepts

ME Function evaluation (items 1, 5, 6, 11, 12, 16, 20)
MR Rate of change (items 8, 10, 11, 15, 19, 22)
MC Function composition (items 4, 5, 12, 16, 17, 20, 23)
MI Function inverse (items 2, 4, 9, 10, 13, 14, 23)

Understand growth rate of function types

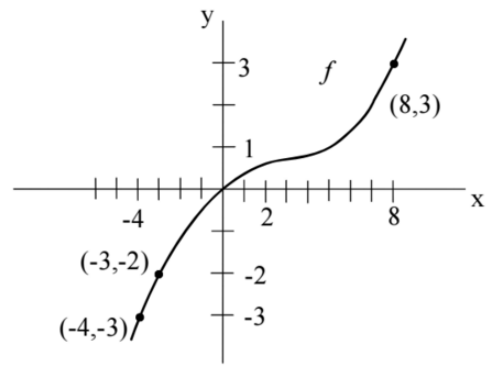
GL Linear (items 3, 10, 22)
GE Exponential (item 7)
GR Rational (items 18, 25)
GN General non-linear (items 15, 19, 24)

Understand function representations (interpret, use, construct, connect)

RG Graphical (items 2, 5, 6, 8, 9, 10, 15, 19, 24)
RA Algebraic (items 1, 4, 7, 10, 11, 14, 16, 17, 18, 21, 22, 23, 25)
RN Numerical (items 3, 12, 13)
RC Contextual (items 3, 4, 7, 8, 10, 11, 15, 17, 18, 20, 22)

4. Sample PCA Items

2) Use the graph of f to solve $f(x) = -3$ for x.

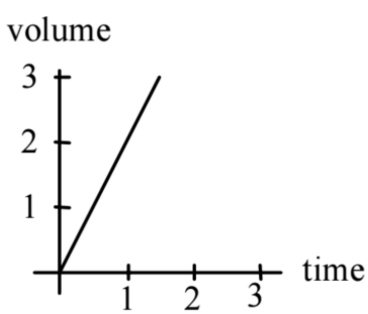


a) (-3,-2)
b) -4
c) (-4,-3)
d) -2
e) -3

4) Which one of the following formulas defines the area, A, of a square in terms of its perimeter, p?

a) $A = \frac{p^2}{16}$
b) $A = s^2$
c) $A = \frac{p^2}{4}$
d) $A = 16s^2$
e) $p = 4\sqrt{A}$

10) A hose is used to fill an empty wading pool. The graph shows volume (in gallons) in the pool as a function of time (in minutes). Which of the following defines a formula for computing the time, t, as a function of the volume, v?

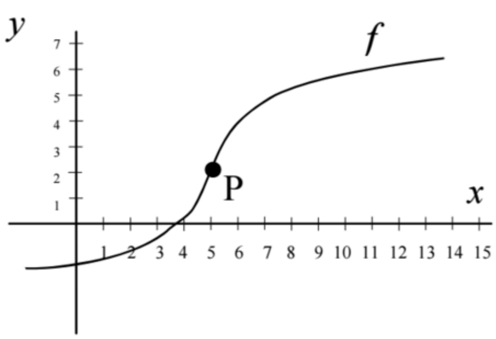


a) $v(t) = \frac{t}{2}$
b) $t(v) = 2v$
c) $t(v) = \frac{v}{2}$
d) $v(t) = 2t$
e) None of the above

17) A ball is thrown into a lake, creating a circular ripple that travels outward at a speed of 5 cm per second. Express the area, A, of the circle in terms of the time, t, (measured in seconds) since the ball hit the lake.

a) $A = 25\pi t$
b) $A = \pi t^2$
c) $A = 25\pi t^2$
d) $A = 5\pi t^2$
e) None of the above

19) Using the graph below, explain the behavior of function f on the interval from $x = 5$ to $x = 12$.



a) Increasing at an increasing rate.
b) Increasing at a decreasing rate.
c) Increasing at a constant rate.
d) Decreasing at a decreasing rate.
e) Decreasing at an increasing rate.

25) Which of the following best describes the behavior of the function f defined by,

$$f(x) = \frac{x^2}{x-2}.$$

I. As the value of x gets very large, the value of f approaches 2.
II. As the value of x gets very large, the value of f increases.
III. As the value of x approaches 2, the value of f approaches 0.

a) I only
b) II only
c) III only
d) I and III
e) II and III

5. Student Attitude Survey

Student ID number: ______________________________

		SD	D	N/U	A	SA
1	I think mathematics is important in life.	0	1	2	3	4
2	In previous math classes, my teachers listened carefully to what I had to say.	0	1	2	3	4
3	I learn more about mathematics working on my own.	0	1	2	3	4
4	I do not like to speak in public.	0	1	2	3	4
5	I prefer working alone rather than in groups when doing mathematics.	0	1	2	3	4
6	I get anxious in school.	0	1	2	3	4
7	In previous math classes, I learned more from talking to my friends than from listening to my teacher	0	1	2	3	4
8	Technology can make mathematics easier to understand.	0	1	2	3	4
9	Cell phones are an important technology in my life.	0	1	2	3	4
10	I like my own space outside school the majority of the time.	0	1	2	3	4
11	I enjoy being part of large groups outside school.	0	1	2	3	4
12	I do not participate in many group activities outside school.	0	1	2	3	4
13	I do not like school.	0	1	2	3	4
14	I like math.	0	1	2	3	4
15	I feel confident in my abilities to solve mathematics problems.	0	1	2	3	4
16	In the past, I have not enjoyed math class.	0	1	2	3	4
17	I receive good grades on math tests and quizzes.	0	1	2	3	4
18	When I see a math problem, I am nervous.	0	1	2	3	4
19	I am not eager to participate in discussions that involve mathematics.	0	1	2	3	4
20	I enjoy working in groups better than alone in math class.	0	1	2	3	4
21	I like to go to the board or share my answers with peers in math class.	0	1	2	3	4
22	I enjoy hearing the thoughts and ideas of my peers in math class.	0	1	2	3	4
23	Mathematics interests me.	0	1	2	3	4
24	I sometimes feel nervous talking out loud in front of my classmates.	0	1	2	3	4
25	I enjoy using a computer when learning mathematics.	0	1	2	3	4
26	When using technology for learning mathematics, I feel like I am in my own private world.	0	1	2	3	4
27	I am not comfortable using technology in math class.	0	1	2	3	4
28	Using technology helps me understand mathematics.	0	1	2	3	4

Please use this space for any written comments:

SD: strongly disagree
D: disagree
N/U: neutral/undecided
A: agree
SA: strongly agree