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Essays on Optimization and Modeling Methods for Reliability and Reliability Growth

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Essays on Optimization and Modeling Methods for Reliability and Reliability Growth

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by

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Abstract

This research proposes novel solution techniques in the realm of reliability and reliability growth. We first consider a redundancy allocation problem to design a system that maximizes the reliability of a complex series-parallel system comprised of components with deterministic reliability. We propose a new meta-heuristic, inspired by the behavior of bats hunting prey, to find component allocation and redundancy levels that provide optimal or near-optimal system reliability levels. Each component alternative has an associated cost and weight and the system is constrained by cost and weight factors. We allow for component mixing within a subsystem, with a pre-defined maximum level of component redundancy per subsystem, which adds to problem complexity and prevents an optimal solution from being derived analytically.

The second problem of interest involves how we model a system's reliability growth as it undergoes testing and how we minimize deviation from planned growth. We propose a Grey Model, GM(1,1) for modeling reliability growth on complex systems when failure data is sparse. The GM(1,1) model's performance is benchmarked with the Army Materiel Systems Analysis Activity (AMSAA) model, the standard within the reliability growth modeling community. For continuous and discrete (one-shot) testing, the GM(1,1) model shows itself to be superior to the AMSAA model when modeling reliability growth with small failure data sets.

Finally, to ensure the reliability growth planning curve is followed as closely as possible, we determine the best level of corrective action to employ on a discovered failure mode, with corrective action levels allowed to vary based upon the amount of resources allocated for failure mode improvement. We propose a Markov Decision Process (MDP) approach to handle the stochasticity of failure data and its corresponding system reliability estimate. By minimizing a weighted deviation from the planning curve, systems will ideally meet the reliability milestones specified by the planning curve, while simultaneously avoiding system over-development and unnecessary resource expenditure for over-correction of failure modes.

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Finally, I'd be remiss if I didnt acknowledge the immeasurable sacrifices made by my wife, Valerie, for her unfailing love and support and for shouldering far more than her fair share of the family burdens while I pursued this final degree.

Dedication

To my loving wife, Valerie, and my daughters, Madelyn and Abigail, without whose patience and support this would not have been possible.

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Talafuse, T.P. and Pohl, E. A., "A Bat Algorithm (BA) For the Redundancy Allocation Problem (RAP)," Engineering Optimization, 2016, Vol. 48, No. 5, pp. 900-910.

Chapter 3:

Talafuse, T.P. and Pohl, E.A., "Small Sample Continuous Reliability Growth Modeling Using a Grey Systems Model," Reliability Engineering and System Safety (in review) (2016).

1. Introduction

This research considers problems dealing with the design and test of complex systems undergoing developmental testing for purposes of reliability growth. In general, optimizing a system's reliability, planning its growth, and modeling that growth throughout testing has been widely studied over the past several decades. In this research, we contribute to this field by considering a new method for determining optimal design for a system comprised of components with deterministic reliabilities, as well as introduce a new method for modeling reliability growth and allocating resources to corrective actions during developmental testing.

During development of a new complex system, prototypes produced will generally contain design, manufacturing and/or engineering deficiencies. Chapter 2 is dedicated to a meta-heuristic approach for developing and designing such a system. Because of these deficiencies, the initial reliability of the prototypes may be below the system's reliability goal or requirement. In order to identify and correct these deficiencies, the prototypes are often subjected to a rigorous testing program to expose them to stresses that are likely to be encountered in the operational environment. During testing, problem areas, or failure modes, are identified and appropriate corrective actions or redesign are taken to mitigate the occurrence of the failure mode and improve the system's reliability. This improvement is referred to as reliability growth, and is formally defined as the positive improvement in a reliability parameter over a period of time due to changes in product design or manufacturing processes (Department of Defense, 2011).

In the field of reliability growth, there are three major areas including: planning, tracking, and projection. Reliability growth planning focuses on the construction of a reliability growth planning curve, which identifies the planned reliability achievement as a function of test duration, in addition to other program resources. Reliability growth tracking focuses on the analysis of a systems current demonstrated reliability. Reliability growth projection focuses on estimating system reliability following implementation of corrective actions to known failure modes. Each of these areas of reliability growth apply to complex systems whose test durations are continuous, as well as to complex systems whose test durations are discrete. Chapter 3 focuses on reliability growth tracking and how a system's reliability demonstrated reliability is modeled for continuous testing, while Chapter 4 will focus on reliability growth tracking for discrete (one-shot) testing. Chapter 5 is dedicated to determining how resources are allocated to corrective actions to most closely follow the reliability growth planning curve initially developed during reliability growth planning.

In Chapter 2, we introduce the Bat Algorithm (BA), a fairly new meta-heuristic that can be used for optimizing initial system design. For this specific research effort, we use this meta-heuristic to provide high quality solutions to the reliability redundancy allocation problem (RAP). The BA was previously introduced as a powerful meta-heuristic for continuous functions but was not originally formulated for handling combinatorial problems. Modeled after the behavior of bats hunting prey, the virtual bats search the feasible and near-feasible region for component allocation and redundancy levels that provide optimal or near-optimal system reliability levels. Each component alternative has a known, deterministic reliability with an associated cost and weight. The system is constrained by cost and weight factors. We allow for component mixing within a subsystem, with a pre-defined maximum level of component redundancy per subsystem. We ensure the feasible boundary of our search space is explored by allowing the meta-heuristic to consider infeasible solutions at a penalized value. Designed experimentation was conducted to determine parameter settings for the virtual bat behavior. The impact of parameter sensitivity on solution quality was evaluated, along with how the algorithm performed as the system's cost constraint was tightened. The work on generating the BA can be generalized as a suitable meta-heuristic for any type of combinatorial problem whose optimal solution cannot be analytically derived.

Chapters 3 and 4 study the way in which reliability growth is modeled and focuses on systems whose test duration is continuous, and discrete, respectively. Since the 1950's, a multitude of models have been developed for reliability growth tracking and is the most well-developed area of reliability growth (Hall, 2008). However, when failure data are sparse, as is often the case when performing system-level testing, the estimated reliability parameters from these models have a great level of uncertainty. We therefore introduce a model based upon Grey Systems Theory, the GM(1,1) model, for prediction of system reliability growth parameters. The GM(1,1) model has shown to be effective for handling small data sets, making it a plausible candidate for modeling reliability growth. Chapter 3 introduces a GM(1,1) model tailored for modeling reliability growth for systems undergoing continuous testing, with effectiveness compared to the AMSAA model. Results from Monte Carlo simulation on a system whose failures follow a poly-Weibull distribution demonstrate the GM(1,1) model's superiority across the response surface when handling small samples of failure observations. Likewise, chapter 4 focuses on using the GM(1,1) model for reliability growth modeling of discrete (one-shot) systems when failure data are not ample. Simulation demonstrates its capability for providing more accurate growth modeling parameters more reflective of true reliability growth.

Chapter 5 introduces a Markov Decision Process (MDP) methodology for modifying the level of corrective action taken to improve discovered failure modes. Given an initial reliability growth planning curve, it is likely that developmental testing will produce system reliability estimates that deviate from the desired milestone reliability levels. Should demonstrated reliability fall below the planned level for a given milestone, there is a greater risk of the system not meeting reliability requirements at the end of developmental testing, resulting in additional time and resources needed to ensure the final product meets its stated reliability requirements. On the other hand, should demonstrated reliability exceed the planned level for a given milestone, employing corrective actions on observed failure modes may be unnecessary to meet the stated requirements, and could potentially be a waste of resources that could be better served in some other area. We propose a dynamic approach to determine the optimal level of corrective action to employ to ensure the system's reliability growth follows the initial reliability growth planning curve as closely as possible. The costs of employing a corrective action rise as the level of corrective action increases to reflect the greater level of effort needed to provide a higher quality improvement. This research indicates that this dynamic approach can produce a system whose reliability growth follows the desired plan more accurately than assuming a fixed level of corrective action, and do so at a lesser cost. When determining optimal corrective action policy, penalties are considered for exceeding the stated reliability milestones, however, we consider application of a more heavily weighted penalty for deviations falling below the desired reliability level to encourage systems to exceed reliability milestones in lieu of falling short of the desired levels. We also consider weighting deviations from the desired reliability more heavily in each progressive phase of testing to more accurately reflect the realistic desire for reliability to be as close to the planned level as developmental testing ends.

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2. A Bat Algorithm (BA) For the Redundancy Allocation Problem (RAP)

Thomas P. Talafuse   Edward A. Pohl

Abstract: This paper uses a recently developed Bat Algorithm (BA) meta-heuristic optimization method to solve the reliability redundancy allocation problem (RAP). The RAP is a well-known NP-hard problem which has been the subject of much prior work, generally of a restricted form where each component must consist of identical components in parallel to make computations tractable. Meta-heuristic methods overcome this limitation and allow for larger instances to be solved for a more general case where different components can be placed in parallel. The BA has not yet been used in reliability design, as it was a method initially designed for continuous problems. A BA is devised and tested on a well-known suite of problems from the literature. It is shown that the BA is competitive with the best known heuristics for redundancy allocation.

2.1 Introduction

The reliable performance of a system for a predefined time under various conditions is very important in many industrial applications. To maximize the reliability of a system, which is typically comprised of a number of components, either the component reliability can be enhanced, or redundant components can be added in parallel (Kuo and Prasad, 2000). In many real-world problems, the reliability of components utilized to construct a system are fixed, meaning the only way to improve system reliability is to increase the redundancy of utilized components. However, increasing the redundancy of components requires more resources. Thus, it is imperative to optimally allocate redundancy to components under some resource constraints, referred to as the redundancy allocation problem (RAP) (Fyffe et al., 1968). The RAP is one of the most important reliability optimization problems with regards to improving the reliability of real-world systems in the design phase. It has attracted many researchers in the past several decades due to reliabilitys critical importance in various kinds of systems, such as electrical systems, mechanical system, and software systems (Kuo and Prasad (2000); Kuo and Wang (2007)).

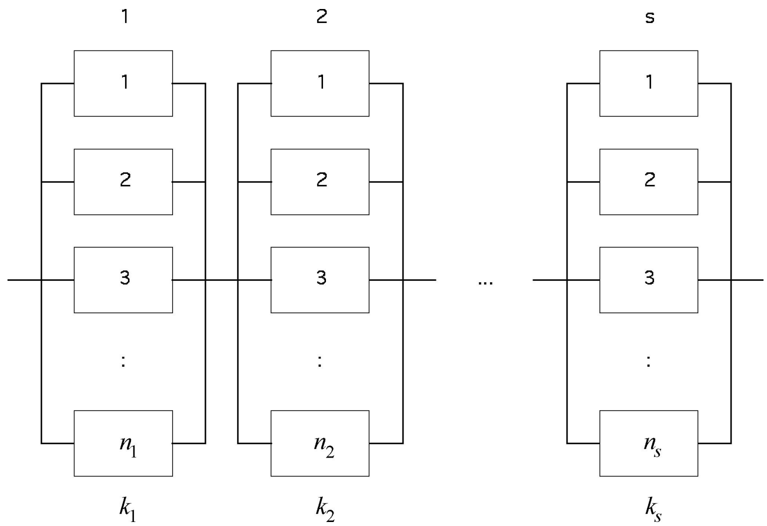

Figure 2.1: Series-parallel system configuration

A well-studied design configuration for the RAP is a parallel-serial system with s independent k-out-of-n:G subsystems, as illustrated in Figure 2.1. Components in each subsystem are parallel and a subsystem i is functioning properly if at least k_i of its n_i components are operational, and a series-parallel system is where $k_i = 1$ for all subsystems. For parallel-serial systems, the RAP is always formulated as a nonlinear integer programming problem with the objective to select the optimal combination of components and redundancy to maximize the system reliability under some constraints, such as cost, volume, and weight. Initially, RAPs were relatively simple as all the components of a subsystem were considered to be of the same type. Later, extensions were made to consider different types of components in one subsystem, significantly increasing the difficulty of RAPs. The RAP is known to be NP-hard (Chern, 1992), and has been thoroughly studied in many forms (Tillman et al., 1977b; Kuo and Prasad, 2000).

The RAP can be formulated to maximize system reliability, R, given restrictions on system cost C and system weight W. It is assumed that system weight and system cost are linear combinations of component weight and cost, resulting in:

$$\max R = \prod_{i=1}^{s} R_i(\mathbf{y}_i | k_i) \tag{2.1}$$

subject to the constraints

$$\sum_{i=1}^{s} C_i(\mathbf{y}_i) \leq C, \tag{2.2}$$

$$\sum_{i=1}^{s} W_i(\mathbf{y}_i) \leq W. \tag{2.3}$$

where $\mathbf{y}_i$ is a vector of the quantity of each component type used in subsystem i, $R_i(\mathbf{y}_i|k_i)$ is the reliability of subsystem i given k_i: the minimum number of components in parallel required for subsystem i to function, and $C_i(\mathbf{y}_i)$ and $W_i(\mathbf{y}_i)$ are the total cost and weight, respectively, of subsystem i.

We assume the following for the problem:

- The state of the components and the system are either good or failed;
- Failed components are not repaired and have no impact on other component performance;
- Component attributes (reliability, cost, weight) are known and deterministic;
- Component supply is unlimited.

Traditional exact optimization approaches to the RAP, including dynamic programming (e.g. Bellman and Dreyfus, 1958; Fyffe et al., 1968; Nakagawa and Miyazaki, 1981) integer programming (e.g. Bulfin and Liu, 1985; Gen et al., 1993; Ghare and Taylor, 1969; Misra and Sharma, 1991), and mixed-integer nonlinear programming (e.g. Tillman et al., 1977a) have been investigated. However, the exponential increase in the search space with problem size makes heuristic approaches a viable alternative for the RAP.

Coit and Smith (1996a) first proposed a GA which searches over feasible and infeasible regions to identify a final, feasible optimal, or near optimal solution to a relaxed version of the RAP. Coit and Smith (1996b) also constructed a hybrid algorithm using a combination of GA and neural network approaches. Hsieh (2003) developed a linear programming approach to approximate the integer nonlinear RAP. Ramirez-Marquez et al. (2004) reformulated the objective of this problem, maximizing the minimum subsystem reliability, and then solved using integer programming. Liang and Smith (2004) used ant colony optimization (ACO) to effectively explore the feasible region and the infeasible region near the border of the feasible area.

A number of other methods have been investigated for solving the RAP, including tabu search (Kulturel-Konak et al., 2003), simulated annealing (Kim et al., 2004), immune algorithm (Chen and You, 2005), heuristic method (You and Chen, 2005), variable neighborhood descent algorithm (Liang and Wu, 2005), variable neighborhood search (Liang and Chen, 2007), hybrid algorithm (Nahas et al., 2007), memetic algorithm (Safari and Tavakkoli-Moghaddam, 2010), and an exact method based on the improved surrogate constraint method (Onishi et al., 2007). Because of the large search space size of the RAP and the lack of a dominant solution technique, it is a good candidate for other meta-heuristic approaches including the focus of this paper, BA optimization.

2.2 The BA Approach

2.2.1 Mainframe of BA

Meta-heuristic algorithms are powerful methods for solving many tough optimization problems. There are many emerging meta-heuristic algorithms derived from the behavior of biological and/or physical systems found in nature, including simulated annealing, genetic algorithms, particle swarm optimization (PSO), harmony search, and the firefly algorithm. Each of these algorithms possesses certain advantages and disadvantages. For example, simulated annealing can almost guarantee to find the optimal solution if the cooling process is slow enough and the simulation is running long enough (Granville et al., 1994). However, the fine adjustment in parameters affects the convergence rate of the optimization process. The BA is a meta-heuristic optimization method, inspired by the behavior of real bats, which seeks to combine the major advantages of other algorithms to develop a potentially better algorithm.

There is a vast array of bat species, with each varying in its use of a type of sonar, called echolocation. Micro-bats use echolocation extensively, allowing them to detect prey, avoid obstacles, and discriminate different types of insects, even in complete darkness. These bats emit a very loud sound pulse, generally at a constant frequency, and listen for the echo that bounces back from the surrounding objects. As bats identify and converge toward their prey, the rate of pulse emission is increased and the loudness of the pulse decreased to effectively hone in on their prey.

First introduced by Yang (2010), the BA demonstrated effective results on a benchmark set of unconstrained problems with continuous and real search spaces. The BA idealizes some of the echolocation characteristics of bats, which are following a set of idealized rules:

- All bats use echolocation to sense distance and they know the difference between food/prey and background barriers.
- Bats fly randomly with velocity v_i at position x_i with a fixed frequency f_{min}, varying wavelength λ and loudness A_0 to search for prey. They can automatically adjust the wavelength (or frequency) of their emitted pulses and adjust the rate of pulse emission $r \in [0,1]$ depending on the proximity of their target.
- Loudness varies from a large (positive) A_0 to a minimum constant value A_{min}.

Each bat is defined by its position x_i^t, velocity v_i^t, frequency f_i, loudness A_i^t, and emission pulse rate r_i^t in a d-dimensional search space. New solutions x_i^t and velocities v_i^t at time step t are given by:

$$f_i = f_{min} + (f_{max} - f_{min})\beta \tag{2.4}$$

$$v_{i,j}^t = v_{i,j}^{t-1} + (x_{i,j}^{t-1} - x_*(j))f_i \quad \forall j \in d \tag{2.5}$$

$$x_i^t = x_i^{t-1} + v_i^t \tag{2.6}$$

where $\beta \in [0,1]$ is a random vector drawn from a uniform distribution and $x_*(j)$ is the j^{th} element of the best solution at that iteration. Once a solution is selected among the current best solutions, a new solution for each bat is generated locally using a random walk

$$x_{new} = x_{old} + \varepsilon A^t \tag{2.7}$$

where $\varepsilon \in [-1, 1]$ is a scaling factor which is a random number, while $A_t = < A_i^t >$ is the average loudness of all the bats at time step t.

Velocity and position updates are similar to the procedure used in PSO, as f_i essentially controls the pace and range of the movement of the bats, just as it does the movement of the swarming particles. Beji et al. (2010) provides additional information on how a PSO can be applied to solve the RAP. Loudness A_i and pulse rate emission r_i update accordingly as the iterations proceed using

$$A_i^{t+1} = \alpha A_i^t, \quad r_i^{t+1} = r_i^0 [1 - \exp(-\gamma t)], \tag{2.8}$$

where α and γ are constants, with α being similar to the cooling factor in simulated annealing. Kim et al. (2004) provides additional details on the application of simulated annealing to the RAP.

The BA uses a frequency-tuning technique to increase the diversity of solutions while simultaneously using automatic zooming to try to balance exploration and exploitation during the search process by mimicking the variations of pulse emission rates and loudness of bats when searching for prey. One of the main advantages of the BA is that it can provide very quick convergence at a very initial stage by switching from exploration to exploitation, making it an efficient algorithm when a quick solution is needed. If switching to an exploitation stage too quickly leads to solution stagnation after some initial stage, loudness and pulse rates can be varied at a slower rate to encourage greater exploration. This robustness allows the BA to efficiently find high quality solutions across a vast array of applications.

2.2.2 Recent Adaptations and Applications of BA

The efficient nature of the BA has led to a wide range of variants. These include the Fuzzy Logic BA (FLBA), the Multi-Objective BA (MOBA), K-Means BA (KMBA), Chaotic BA (CBA), Binary BA (BBA), Differential Operator and Levy Flights BA (DLBA), and the Improved BA (IBA) (Yang and He, 2013). With numerous variants, the applications of the BA are quite diverse; it has been applied in areas of optimization, classifications, image processing, feature selection, scheduling, data mining, and more. The BA has shown to be a very effective and efficient search algorithm and is competitive with other highly efficient algorithms, such as Cuckoo Search (Natarajan et al., 2013).

Of particular mention, Gandomi et al. (2013) expanded Yang's original work to solve complex constrained nonlinear optimization problems. Yang (2012) extended the BA to solve multi-objective optimization problems, such as welded beam design, via the MOBA. Mallick et al. (2015) applied the MOBA to obtain the optimal design point for trailing edge flap configuration and flap location to simultaneously achieve minimum hub vibration levels and flap actuation power. Nakamura et al. (2012) and Mirjalili et al. (2014) developed the BBA which restricted bat position and movement to only binary values and applied it to solve classification and feature selection problems. While there has been significant expansion of the literature on the BA in the past five years, there has been no variant developed for application to large-scale discrete problems, such as the RAP, which is the motivation for this paper.

2.2.3 BA Modification for RAP

In order to solve the RAP directly, one of the key issues is to transform the bat position to only allow discrete values. Using an approach similar to that used in the BBA (Mirjalili et al., 2014; Nakamura et al., 2012), frequency and velocity are calculated using (2.4) and (2.5). If $\rho < |\frac{2}{\pi}\tan^{-1}(\frac{\pi}{2}) * v_{i,j}|$, where $\rho \in [0,1]$ is a random draw from a uniform distribution, then position is updated using

$$x_{i,j}^t = x_*(j) + k, \tag{2.9}$$

where $k \in [-1, 0, 1]$ is a discrete random variable. Furthermore, if $\rho > r_i$, then $x_{i,j}$ is set to $x_*(j)$. Bat fitness is calculated and penalizes infeasible solutions using the method described in Liang and Smith (2004), via

$$R_{x_i p} = R_{x_i} \cdot \left(\frac{W}{W_{x_i}} \right)^{\eta} \cdot \left(\frac{C}{C_{x_i}} \right)^{\eta} \tag{2.10}$$

where $R_{x_i p}$ is the penalized objective function value, R_{x_i} is the unpenalized objective function value, W_{x_i} and C_{x_i} are the total system weight and cost of solution x_i, and η is a preset amplification parameter. This encourages the algorithm to explore the feasible region and the infeasible region that is near the border of the feasible area and discourages, but permits, search further into the infeasible region.

The new bat position is accepted if there is improvement in the objective function and $\rho < A_i$. Loudness and pulse rate emission are updated via (2.8), but are reset to their initial values if an improved feasible solution is found. The BA algorithm for the RAP is expressed as follows:

Begin

Initialization: Set generation counter $t = 1$. Initialize population of N bats randomly, with each bat corresponding to a potential solution of the given problem; define loudness A_0 and pulse rate r_0; set penalty parameter η

While termination criteria not satisfied or $t <$ max generation

Calculate fitness and identify best bat x_*

For each bat:

Assign frequency and update velocity using 2.4 and 2.5

Update position using 2.9

Set $x_{i,j} = x_*(j)$ if $\rho > r_i$

Calculate fitness

If fitness improves and $\rho < A_i$ **then**

Accept new position

End if

If new position is feasible **then**

Update loudness and pulse rate, setting $A_i = A_0$ and $r_i = r_0$

Reset $t = 1$

Else update using 2.8

End if

Set $t = t + 1$

End

End while

End

2.3 Experimentation and Results

To evaluate the performance of the proposed BA, a typical example taken from Fyffe et al. (1968) is solved. A series-parallel system is connected by 14 subsystems, with each subsystem having three or four components of choice. Component mixing is allowed in each subsystem so in order to reduce the size of the search space, a maximum of 8 of each component type is allowed per subsystem, still resulting in a search space size larger than 7.6×10^{33}. For each subsystem, component reliability (R_i), cost (C_i), and weight (W_i) are given for each component alternative i, in Table 2.1, with the objective of maximizing system reliability given the constraints for system cost and weight. 33 test instances as devised by Nakagawa and Miyazaki (1981) were generated, where system cost $C = 130$, and system weight W is decreased incrementally from 191 to 159.

Table 2.1: Input Data for RAP (Fyffe et al., 1968)

	Component Alternatives											
	1			2			3			4		
Subsystem	R_1	C_1	W_1	R_2	C_2	W_2	R_3	C_3	W_3	R_4	C_4	W_4
1	0.9	1	3	0.93	1	4	0.91	2	2	0.95	2	5
2	0.95	2	8	0.94	1	10	0.93	1	9	*	*	*
3	0.85	2	7	0.9	3	5	0.87	1	6	0.92	4	4
4	0.83	3	5	0.87	4	6	0.85	5	4	*	*	*
5	0.94	2	4	0.93	2	3	0.95	3	5	*	*	*
6	0.99	3	5	0.98	3	4	0.97	2	5	0.96	2	4
7	0.91	4	7	0.92	4	8	0.94	5	9	*	*	*
8	0.81	3	4	0.9	5	7	0.91	6	6	*	*	*
9	0.97	2	8	0.99	3	9	0.96	4	7	0.91	3	8
10	0.83	4	6	0.85	4	5	0.9	5	6	*	*	*
11	0.94	3	5	0.95	4	6	0.96	5	6	*	*	*
12	0.79	2	4	0.82	3	5	0.85	4	6	0.9	5	7
13	0.98	2	5	0.99	3	5	0.97	2	6	*	*	*
14	0.9	4	6	0.92	4	7	0.95	5	6	0.99	6	9

The bat population was initialized to ten bats. Preliminary investigation on population size discovered that use of a smaller population did not adequately cover the search space, resulting in degradation of solution quality, while an increase in the number of bats provided no improvement of solution quality to warrant the corresponding increase in computational effort. The maximum number of iterations without an improvement to system reliability was set to 30,000 to ensure ample examination of the search space. Each instance was replicated ten times using different random number seeds to enhance exploration of the search space. The BA was coded in MATLAB and run using an Intel i7 2.80GHz PC with 4.0GB of RAM, with final system reliability rounded to four places after the decimal. Results from the BA are compared with those of Liang and Smith (2004), as they are the heuristic benchmark for the RAP when component mixing is allowed.

Due to the unknown behavior of the BA on combinatorial problems, designed experimentation was performed to tune maximum frequency f_{max}, initial loudness A_0 and its update parameter α, and initial pulse emission rate r_0 and its update parameter γ, for each test instance. This led to setting parameters $f_{max} = 4$, $A_0 = 5$, and $r_0 = 0.05$. The loudness update parameter α took on values ranging from 0.7-0.99, taking on larger values as total system weight W decreased. Likewise, γ took on values ranging from 0.3-0.35, increasing as W increased. The penalty parameter η was initialized between 0.1-0.5, taking on larger values for less constrained instances, and was incrementally decreased to a minimum of 0.05 as the number of iterations increased and the bats converged to a feasible solution.

Two special cases of the BA were also considered: for the first case, frequency is replaced by a random parameter, $A_i = 0$, and $r_i = 1$, making this equivalent to a PSO. For the second case, velocities were not used, and loudness and pulse rate were fixed to constants, reducing the BA to a Harmony Search. As expected, these more restrictive cases of the BA limited the flexibility and robustness of the BA, resulting in inferior performance without a significant decrease in computational efforts. For brevity, these results have been omitted.

Results from the BA are summarized in Table 2.2, where the comparisons between the results from the ACO and BA are divided into three categories: maximum, mean, and minimum system reliability. In 13 of the 33 instances the ACO outperformed the BA, but with never more than a 0.075% gap. The BA was equivalent or superior to the ACO in 20 (60%) of the cases. In general, the ACO performed better on the lesser constrained problems and performance was similar as the instances became more constrained. By allowing infeasible solutions to be considered at a penalty, high quality solutions can be discovered quickly, with a solution generated for these instances in roughly 60 seconds. This rapid exploration, coupled with the ability to simultaneously exploit good solutions, allows the bats to converge to locally optimal solutions in a short time. However, across the ten replications, there was significantly more variation in BA than the

ACO, as can be seen in Figure 2.2. Due to variations in hardware, software, and coding, coupled with no published computational times for the ACO, it is impossible to compare if the increase in variability for the BA is worth the trade-off in computational speed. However, due to the short computational time required for the BA, multiple replications of the BA can be run and a maximum reliability on par with benchmark solutions can be found.

2.4 Conclusions

In this paper, we have proposed an adaptation of the Bat Algorithm (BA) to solve a mathematical model of a redundancy allocation problem for a series-parallel system with component mixing. This problem is not easy to solve, especially for large sizes, motivating the use of meta-heuristic methods. The BA has demonstrated its effectiveness on continuous problems and our adaptation for discrete problems provides a robust method for obtaining quality solutions. From our computational results, we have demonstrated that our proposed BA provides solutions on par with that of benchmark meta-heuristics, and in a few cases, outperforms them. Because the BA has attributes of flexibility, robustness, and implementation ease, it seems a very promising general method for other NP-hard reliability design problems. Future research will seek to investigate further tailoring of model parameters and bat population size, as well as investigate applicability of the BA to other problem types.

Table 2.2: Comparison of ACO and BA Results

			L&S ACO-RAP - 10 Runs			BA - 10 Runs			
No	C	W	Max R	Mean R	Min R	Max R	Mean R	Min R	Gap %
1	130	191	0.9868	0.9862	0.9860	0.9866	0.9844	0.9824	0.020%
2	130	190	0.9859	0.9858	0.9857	0.9856	0.9839	0.9821	0.029%
3	130	189	0.9858	0.9853	0.9852	0.9854	0.9838	0.9809	0.037%
4	130	188	0.9853	0.9849	0.9848	0.9850	0.9834	0.9803	0.030%
5	130	187	0.9847	0.9841	0.9837	0.9844	0.9833	0.9794	0.034%
6	130	186	0.9838	0.9836	0.9835	0.9842	0.9833	0.9824	–
7	130	185	0.9835	0.9830	0.9828	0.9834	0.9798	0.9782	0.007%
8	130	184	0.9830	0.9824	0.9820	0.9826	0.9797	0.9763	0.035%
9	130	183	0.9822	0.9818	0.9817	0.9815	0.9795	0.9777	0.075%
10	130	182	0.9815	0.9812	0.9806	0.9812	0.9795	0.9777	0.024%
11	130	181	0.9807	0.9806	0.9804	0.9807	0.9790	0.9770	–
12	130	180	0.9803	0.9798	0.9796	0.9803	0.9785	0.9770	–
13	130	179	0.9795	0.9795	0.9795	0.9795	0.9780	0.9762	–
14	130	178	0.9784	0.9784	0.9783	0.9784	0.9771	0.9755	–
15	130	177	0.9776	0.9776	0.9776	0.9776	0.9761	0.9747	–
16	130	176	0.9765	0.9765	0.9765	0.9767	0.9758	0.9747	–
17	130	175	0.9757	0.9754	0.9753	0.9757	0.9751	0.9741	–
18	130	174	0.9749	0.9747	0.9741	0.9749	0.9742	0.9741	–
19	130	173	0.9738	0.9735	0.9731	0.9738	0.9728	0.9727	–
20	130	172	0.9730	0.9726	0.9714	0.9730	0.9713	0.9708	–
21	130	171	0.9719	0.9717	0.9710	0.9719	0.9701	0.9674	–
22	130	170	0.9708	0.9708	0.9708	0.9708	0.9691	0.9674	–
23	130	169	0.9693	0.9693	0.9693	0.9693	0.9670	0.9651	–
24	130	168	0.9681	0.9681	0.9681	0.9681	0.9644	0.9627	–
25	130	167	0.9663	0.9663	0.9663	0.9663	0.9642	0.9627	–
26	130	166	0.9650	0.9650	0.9650	0.9650	0.9638	0.9627	–
27	130	165	0.9637	0.9637	0.9637	0.9636	0.9596	0.9571	0.014%
28	130	164	0.9624	0.9624	0.9624	0.9618	0.9595	0.9566	0.065%
29	130	163	0.9606	0.9606	0.9606	0.9602	0.9566	0.9566	0.044%
30	130	162	0.9592	0.9592	0.9592	0.9589	0.9579	0.9565	0.026%
31	130	161	0.9580	0.9580	0.9580	0.9580	0.9561	0.9544	–
32	130	160	0.9557	0.9557	0.9557	0.9557	0.9553	0.9544	–
33	130	159	0.9546	0.9546	0.9546	0.9546	0.9542	0.9534	–

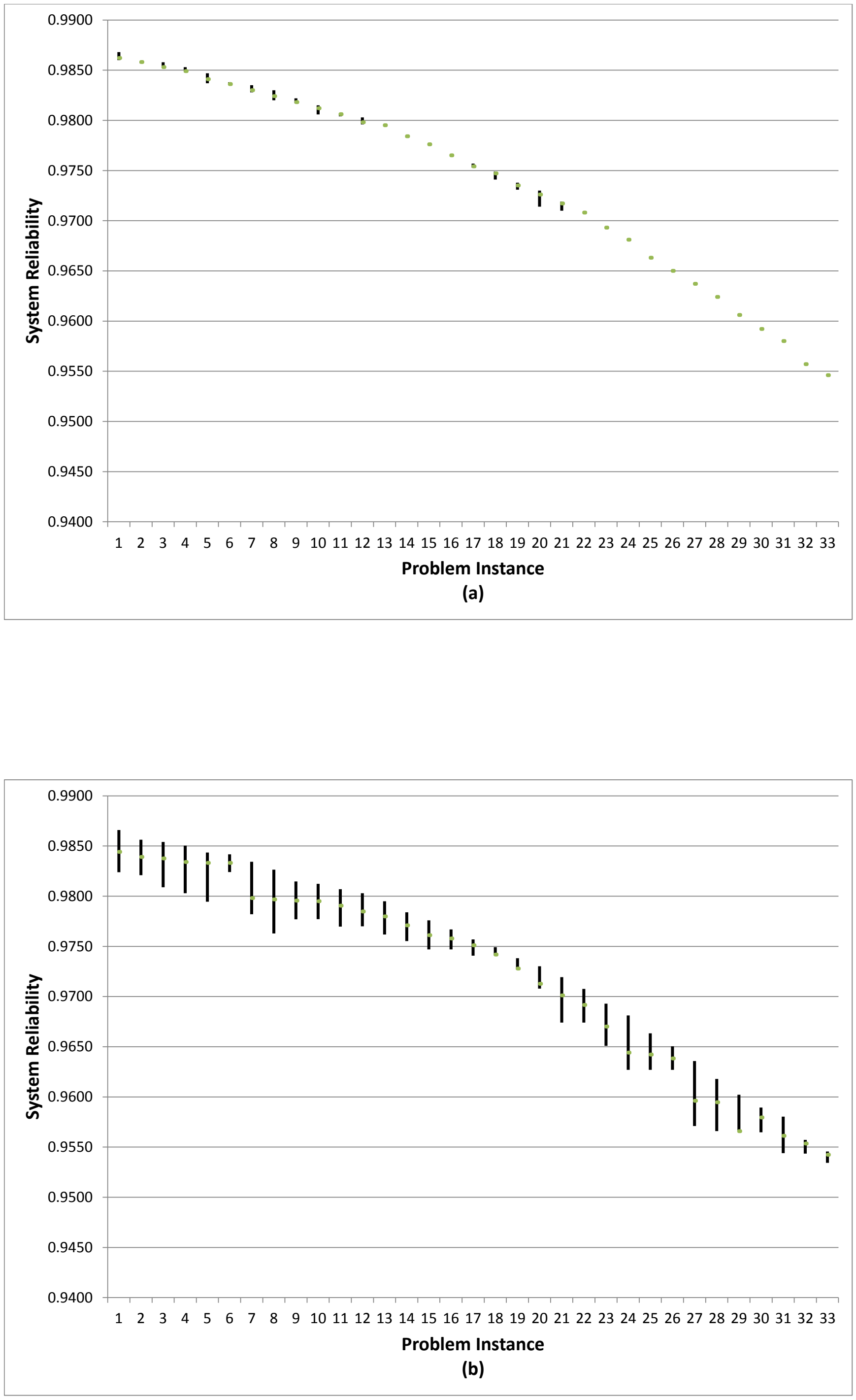

Figure 2.2: Range of Performance Over 10 Replications. (a) ACO; (b) BA

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Appendix

2.A Certification of Student Work

UNIVERSITY OF ARKANSAS

College of Engineering
Department of Industrial Engineering

MEMORANDUM

TO: Graduate School, University of Arkansas

FROM: Edward A. Pohl, Professor and Department Head

DATE: July 12, 2016

SUBJECT: Certification of Student Effort

I certify that greater than 51% of the work conducted for this chapter entitled "A Bat Algorithm (BA) For the Redundancy Allocation Problem (RAP)" was conducted by Thomas P. Talafuse.

Sincerely,

Edward A. Pohl
ephol@uark.edu
479-575-6029
Professor and Department Head
Department of Industrial Engineering
University of Arkansas

3. **Small Sample Continuous Reliability Growth Modeling Using a Grey Systems Model**

Thomas P. Talafuse Edward A. Pohl

Abstract: When performing system-level developmental testing, time and expenses generally warrant a small sample size for failure data. Upon failure discovery, redesigns and/or corrective actions can be implemented to improve system reliability. Current methods for estimating reliability growth, namely the Crow (AMSAA) growth model, stipulate that parameter estimates have a great level of uncertainty when dealing with small sample sizes. For purposes of handling limited failure data, we propose the use of a modified GM(1,1) model to predict system reliability growth parameters and investigate how parameter estimates are affected by systems whose failures follow a poly-Weibull distribution. Monte-Carlo simulation is used to map the response surface of system reliability, and results are used to compare the accuracy of the modified GM(1,1) model to that of the AMSAA growth model. It is shown that with small sample sizes and multiple failure modes, the modified GM(1,1) model is more accurate than the AMSAA model for prediction of growth model parameters.

3.1 Introduction

Reliability growth is the progressive improvement of reliability performance measures over time through the discovery of failure modes via testing and implementation of solutions to mitigate these failure modes (IEC 61014, 2003). During developmental testing of a complex system, there is considerable interest in assessing how system reliability grows to ensure the finished product meets user reliability requirements. Developmental testing is typically limited by cost, schedule, resource, and other constraints, often resulting in small data samples and making it imperative to identify and correct reliability deficiencies in a new design.

A number of models are available for systems undergoing testing in both the continuous and discrete (one-shot) cases. These models include both parametric and non-parametric methods for modeling reliability growth. Continuous models are prolific and include (but are not limited to): AMSAA (Crow, 1975), Cox & Lewis (Cox and Lewis, 1996), Duane (Duane, 1964), and the nonparametric-Bayes (Robinson and Dietrich, 1987). With small samples, however, it proves difficult for these models to confidently obtain accurate parametric estimators and reliability growth prediction results. For system-level testing, it is reasonable to assume that a system contains an unknown number of independent competing failure modes whose respective failure times follow Weibull failure rates, resulting in failure data following a poly-Weibull distribution (Freels, 2013). Furthermore, the AMSAA model, one of the most popular models, assumes failures occur according to a non-homogeneous Poisson process (NHPP) with a Weibull intensity function, expressed as $\lambda(T) = \lambda\beta T^{\beta-1}$. To date, little investigation has been conducted on how this assumption impacts reliability growth modeling on system-level testing for systems following a poly-Weibull failure distribution.

In this paper we present a new reliability growth model for continuous systems based on a modified GM(1,1) model. The model may not be suitable for application to all continuous development programs but it is useful in cases where budgetary and/or time constraints result in a small set of failure data. This model can obtain better prediction results, especially for data of small sample sizes. We focus on developmental testing at the system level, as schedule and cost constraints often preclude sufficient testing to generate a meaningful reliability estimate from the data obtained in these tests.

The remainder of this paper is organized as follows: an introduction to Grey systems theory, the original GM(1,1) model and the modified GM(1,1) model will be presented in section 3.2. Section 3.3 tailors the modified GM(1,1) model for reliability growth modeling, with section 3.4 applying the GM(1,1) model on systems undergoing failure-terminated reliability growth testing with a varying number of failure modes, different assumed fix effectiveness factor (FEF) levels, and the number of failures observed for test termination. Conclusions and future work are provided in section 3.5.

3.2 Grey Model for Reliability Growth

3.2.1 Grey Systems Background

In systems theory, a system can be defined with a color that represents the amount of clear information about that system. For instance, a system can be called a black box system if its internal characteristics or mathematical equations that describe its dynamics are completely unknown. On the other hand if the description of the system is, completely known, it can be named as a white system. Similarly, a system that has both known and unknown information is defined as a grey system (Liu and Lin, 2006). In real life, every system can be considered as a grey system because there are always some uncertainties. Information that can be obtained from a system is always uncertain and limited in scope due to noise from both inside and outside of the system of concern (Liu and Lin, 2006). Systems achieving reliability growth in developmental testing are no exception, making investigation of modeling reliability growth using grey models warranted.

First proposed by Deng (1982), Grey systems theory has become increasingly popular with its ability to deal with systems that have partially unknown parameters. Unlike conventional statistical models, grey models require only a limited amount of data to estimate the behavior of an unknown system (Deng, 1989), and has been widely applied to a broad spectrum of fields, including social, economic, agricultural, industrial, ecological, and biological arenas (Wang, 2002; Chang et al., 2007; Hsu, 2003; Hsu and Chen, 2003; Jović et al., 2005; Mao and Chirwa, 2006; Huang and Jane, 2009). These previous studies have shown that grey system theory-based approaches can achieve good performance characteristics when applied to real-time systems, since grey predictors adapt their parameters to new conditions as new outputs become available. Because of this reason, grey predictors are more robust with respect to noise and lack of modeling information when compared to conventional methods.

The main task of grey system theory is to extract realistic governing laws of the system using available data. This process is known as the generation of the grey sequence. It is argued that even though the available data of the system, which are generally white numbers, is too complex or chaotic, they always contain some governing laws. If the randomness of the data obtained from a grey system is somehow smoothed, it is easier to derive any special characteristics of that system. (Liu and Lin, 2006). Grey models predict the future values of a time series based only on a set of the most recent data depending on the window size of the predictor. It is assumed that all data values to be used in grey models are positive, and the sampling frequency of the time series is fixed (Kayacan et al., 2010).

In grey systems theory, GM(*n,m*) denotes a grey model, where *n* is the order of the difference equation and *m* is the number of variables. Although various types of grey models can be studied, most research has focused attention on GM(1,1) models because of its computational efficiency. This is mainly due to most applications valuing computational efforts second only to model performance (Kayacan et al., 2010).

3.2.2 The GM(1,1) Model

The Grey Model First Order, One Variable, or GM(1,1), has been the most widely model discussed in literature. This model is a time series forecasting model in which the differential equations of the GM(1,1) model have time-varying coefficients, meaning that the model is renewed as new data become available to the prediction model. The GM(1,1) model can only be used in positive data sequences (Deng, 1989). In this paper, system failure times are used as the raw data points and are positive, allowing grey models to be used to forecast the future values of the raw data points. In order to smooth the randomness, the raw data obtained from the system to form the GM(1,1) is subjected to an operator, named the Accumulating Generation Operator (AGO) (Deng, 1989). The differential equation (i.e. GM(1,1)) is solved to obtain the *n*-step ahead predicted value of the system. Finally, using the predicted value, the Inverse Accumulating Generation Operator (IAGO) is applied to find the predicted values of the original data sequence. Given that

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)) \tag{3.1}$$

is a non-negative sequence of raw data, then

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)) \tag{3.2}$$

is a sequence generated from applying the first-order AGO to $X(0)$, where,

$$X^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i),\ k = 1, 2, ..., n. \tag{3.3}$$

The least square estimate sequence of the grey differential equation of GM(1,1) is defined as follows (Deng, 1989):

$$X^{(0)}(k) + az^{(1)}(k) = b, \tag{3.4}$$

where the generated mean sequence of $X^{(1)}$ is defined as:

$$Z^{(1)} = (z^{(1)}(1), z^{(1)}(2), ..., z^{(1)}(n)) \tag{3.5}$$

and $z^{(1)}(k)$ is the generated mean value of adjacent data, calculated as:

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1),\ k = 2, 3, ..., n. \tag{3.6}$$

The GM(1,1) whitening equation is then given by:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b. \tag{3.7}$$

Least squares estimators can be derived using:

$$\hat{u} = [a, b]^T = (B^T B)^{-1} B^T Y, \tag{3.8}$$

where

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \\ -z^{(1)}(n) & 1 \end{bmatrix}, \tag{3.9}$$

and parametric estimators can be obtained as follows (Liu and Lin, 2006):

$$\begin{cases} a = \dfrac{\sum_{k=2}^{n} x^{(0)}(k) \cdot \sum_{k=2}^{n} z^{(1)}(k) - (n-1) \cdot \sum_{k=2}^{n} x^{(0)}(k) \cdot z^{(1)}(k)}{(n-1) \sum_{k=2}^{n} \left[z^{(1)}(k)\right]^2 - \left[\sum_{k=2}^{n} z^{(1)}(k)\right]^2} \\ b = \frac{1}{(n-1)} \cdot \left[\sum_{k=2}^{n} x^{(0)}(k) + a \cdot \sum_{k=2}^{n} z^{(1)}(k)\right]^2 \end{cases}. \tag{3.10}$$

By equation 3.7, the time response solution can be expressed as:

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a}. \tag{3.11}$$

Predicted values of the original raw data sequence can then be obtained by applying the IAGO, resulting in:

$$\begin{aligned} \hat{x}^{(0)}(k+1) &= \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \\ &= (1 - e^{a}) \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak}. \end{aligned} \tag{3.12}$$

3.2.3 Modification of GM(1,1) Model

Wang et al. (2010) introduced a modification to the original GM(1,1) model to take better advantage of new pieces of information in the raw data sequence. The time response solution of the whitened equation is expressed as:

$$x^{(1)}(k) = ce^{-ak} + \frac{b}{a}\, t = 1, 2, \ldots n \tag{3.13}$$

where c is a constant, and parameters a and b are derived according to equation 3.8. For equation 3.13, if $k = 1$, then

$$x^{(1)}(1) = ce^{-a} + \frac{b}{a} \tag{3.14}$$

and for $k = n$

$$x^{(1)}(n) = ce^{-an} + \frac{b}{a}. \tag{3.15}$$

In order to fully use new information in the raw data sequence while also maintaining the initial conditions of the original GM(1,1) model, a new initial condition is set to

$$0.5\left(x^{(1)}(1) + x^{(1)}(n)\right) \tag{3.16}$$

and c is derived to be:

$$c = 2(e^{-a} + e^{-an})^{-1}\left(\frac{x^{(1)}(1) + x^{(1)}(n)}{2} - \frac{b}{a}\right). \tag{3.17}$$

This results in a new time response solution of

$$x^{(1)}(k) = \frac{2}{1 + e^{-a(n-1)}}\left(\frac{x^{(1)}(1) + x^{(1)}(n)}{2} - \frac{b}{a}\right)e^{-a(k-1)} + \frac{b}{a} \tag{3.18}$$

and predicted raw data values as:

$$\hat{x}^{(0)}(k) = 2(1 - e^{a})(1 + e^{-a(n-1)})^{-1} \times (\frac{x^{(1)}(1) + x^{(1)}(n)}{2} - \frac{b}{a})e^{-a(k-1)}. \tag{3.19}$$

The new initial condition derived in equation 3.19 from the first and last observations in the raw data sequence preserve the format of the initial condition for the original GM(1,1) model and make full use of new observations and can be utilized to more accurately predict raw observation values.

3.3 Application to Reliability Growth

It was desired to see how the modified GM(1,1) model performed when applied to continuous reliability growth modeling. Some assumptions are made regarding testing and evaluation:

- The system has a fixed number of independent competing failure modes whose respective failure times follow Weibull failure rates.
- Multiple system prototypes are undergoing concurrent testing.
- Failure of any one failure mode results in system failure.
- Upon failure, discovery is immediate and the corresponding failure mode is known with certainty.
- All failure modes are classified as type BD. That is, corrective action was implemented during test.
- Upon any given failure, testing on all system prototypes is halted and corrective action taken to improve the identified failure mode for all systems. Upon completion, testing on all systems is resumed.
- Corrective action does not remove a failure mode. Rather, it improves the characteristic life, η_i, by an assumed constant FEF.
- Testing is terminated once a predetermined number of failures are observed.

For purposes of assessment, system MTBF ($MTBF_i$) is calculated upon termination of testing and is compared to the instantaneous MTBF estimates derived from the AMSAA model and the GM(1,1) model. Since it is common practice to assume that failures will continue at a constant rate once improvements stop being made to the system, instantaneous MTBF is an appropriate metric to use for assessment. It is assumed that upon test termination and after final corrective actions are taken that the system is as good as new. Then the $MTBF_i$ for the true system, with failures following a poly-Weibull distribution with J failure modes, can be expressed as:

$$MTBF_i = \int_0^\infty t \times f(t|\eta,\beta)dt = \int_0^\infty t \times \left\{ \exp\left[-\sum_{j=1}^{J} \left(\frac{t}{\eta_j} \right)^{\beta_j} \right] \right\} \times \sum_{j=1}^{J} \frac{\beta_j t^{\beta_{j-1}}}{\eta_j^{\beta_j}} dt, \tag{3.20}$$

and derived using numerical integration.

For a sequence of n observed failures with cumulative failure times $(T_1, T_2, ..., T_n)$, we derive parameter values for the AMSAA model using unbiased estimators, as the number of observations is small. The parameters for the AMSAA model are estimated using maximum likelihood estimation (MLE) with likelihood function:

$$L = \lambda^n \beta^n e^{-\lambda T^{*\beta}} \prod_{i=1}^{n} T_i^{\beta-1} \tag{3.21}$$

where $T^* = T_n$ is test termination time for failure-terminated testing. The resulting parameter estimates are then:

$$\hat{\beta} = \frac{N-2}{N} \frac{n}{n \ln T^* - \sum_{i=1}^{n} \ln T_i} \tag{3.22}$$

$$\hat{\lambda} = \frac{n}{T^{*\hat{\beta}}} \tag{3.23}$$

With the parameter estimates obtained in equations 3.22 and 3.23, the Weibull intensity function is used to calculate the instantaneous MTBF at time T^*, resulting in:

$$MTBF_{AMSAA} = \frac{1}{\lambda_i(t)} = \frac{1}{\hat{\lambda}\hat{\beta}T^{*(\hat{\beta}-1)}}. \tag{3.24}$$

For the modified GM(1,1) model, the raw data sequence is the cumulative time between failures, $X^{(0)} = (x^{(0)}(1) = T_1, x^{(0)}(2) = T_2 - T_1, ..., x^{(0)}(n) = T_n - T_{n-1})$. The modified GM(1,1) model is applied to obtain $\hat{X}^{(0)}$, or the predicted cumulative time between observed failures. Assuming that the number of failures observed follows a power law process based upon the cumulative time between failures, we have:

$$E\left[N(t)\right] = \lambda t^{\beta}. \tag{3.25}$$

Defining

$$F(\lambda, \beta, \hat{x}^{(0)}(i)) = \left(\frac{\hat{x}^{(0)}(i)}{\lambda}\right)^{\left(\frac{1}{\beta}\right)} \quad i = 1, \ldots, n, \tag{3.26}$$

we derive estimates parameters $\hat{\lambda}$ and $\hat{\beta}$ via nonlinear least squares using the following:

$$\underset{\lambda,\beta}{\text{minimize}} \quad \sum_{i=1}^{n} \left(F\left(\lambda, \beta, \hat{x}^{(0)}(i)\right) - N\left(\hat{x}^{(0)}(i)\right)\right)^2 \tag{3.27}$$

Instantaneous MTBF can then be estimated as:

$$MTBF_{GM} = \left(\frac{T^*}{\hat{\lambda}}\right)^{\left(\frac{1}{\hat{\beta}}\right)}. \tag{3.28}$$

3.4 Numerical Experimentation

3.4.1 Initial Investigation

The quality of modeling reliability growth via the GM(1,1) model was initially explored to determine if it provided more accurate estimates than the AMSAA model. Via Monte-Carlo simulation, failure data were generated for a hypothetical system undergoing continuous developmental testing with a fixed number of independent competing failure modes whose respective failure times follow Weibull failure rates. The number of failure modes in the system ranged from one, equivalent to testing on an individual component, to as many as ten failure modes. Failure mode parameters were randomly generated with each β_i drawn from a uniform distribution in the range (1, 3.5) and each η_i from a uniform distribution in the range (1,000, 10,000). These values were chosen to reflect parameters that may be seen in real-world failure modes undergoing developmental testing. Table 3.1 lists the parameter values for the failure modes initially investigated.

The impact of assuming a constant level of corrective action was also investigated by assuming FEF values of 50%, 60%, 70%, and 80%. Test termination conditions varied from as few as three failures to as many as ten failures.

Table 3.1: Failure Mode (FM) Parameter Data

FM(i)	β_i	η_i
1	2.3585	3505.3245
2	2.0613	8602.9852
3	1.0118	2094.1221
4	2.6769	8432.6748
5	1.3418	6175.8399
6	3.2283	2882.8191
7	1.4633	1975.3920
8	1.5492	9807.6141
9	3.0292	2547.4691
10	3.0406	3466.6637

Test instances were developed for all possible combinations of failure modes, corrective action levels, and termination conditions. To account for the stochastic nature of failure times and its impact on MTBF estimates, each test instance was replicated $n = 1000$ times. Both models were evaluated using the absolute relative error between $MTBF_i$ and their respective estimate for instantaneous MTBF, with no preference being shown for either conservative or optimistic estimates, and are expressed as:

$$\delta_{AMSAA_k} = \frac{|MTBF_{i_k} - MTBF_{AMSAA_k}|}{MTBF_{i_k}}, \quad k = 1, \ldots, n \tag{3.29}$$

$$\delta_{GM_k} = \frac{|MTBF_{i_k} - MTBF_{GM_k}|}{MTBF_{i_k}}, \quad k = 1, \ldots, n \tag{3.30}$$

From these replications, we derive sample means and standard deviations:

$$\bar{\delta}_{AMSAA} = \frac{1}{n} \sum_{k=1}^{n} \delta_{AMSAA_k} \tag{3.31}$$

$$S_{AMSAA} = \sqrt{\frac{1}{n-1} \sum_{k=1}^{n} \left(\delta_{AMSAA_k} - \bar{\delta}_{AMSAA_k}\right)} \tag{3.32}$$

$$\bar{\delta}_{GM} = \frac{1}{n}\sum_{k=1}^{n}\delta_{GM_k} \tag{3.33}$$

$$S_{GM} = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}\left(\delta_{GM_k} - \bar{\delta}_{GM_k}\right)} \tag{3.34}$$

Using the values calculated in equations 3.31 through 3.34, confidence intervals were constructed to assess if any statistical difference existed between the AMSAA and modified GM(1,1) when estimating the true system MTBF. Because of the large number of replications, the central limit theorem permits use of the z-statistic for computing interval half-widths. The confidence interval is then calculated as:

$$\left(\left(\bar{\delta}_{AMSAA} - \bar{\delta}_{GM}\right) \pm z_{1-\alpha/2} \times \sqrt{\left(\frac{S_{AMSAA}}{\sqrt{n}}\right)^2 + \left(\frac{S_{GM}}{\sqrt{n}}\right)^2}\right), \tag{3.35}$$

with intervals strictly above zero indicating superiority of the GM(1,1) model, and intervals strictly below zero indicating superiority of the AMSAA model. To convey the instances where the GM(1,1) model outperforms the AMSAA model, figures 3.1 through 3.4 show the lower confidence bounds for the various combinations of assumed FEF, number of failure modes, and terminating condition, with results above zero indicating the statistically superior performance of the GM(1,1) model. The complete list of the lower and upper confidence bounds can be found in the tables of Appendix 3.A.

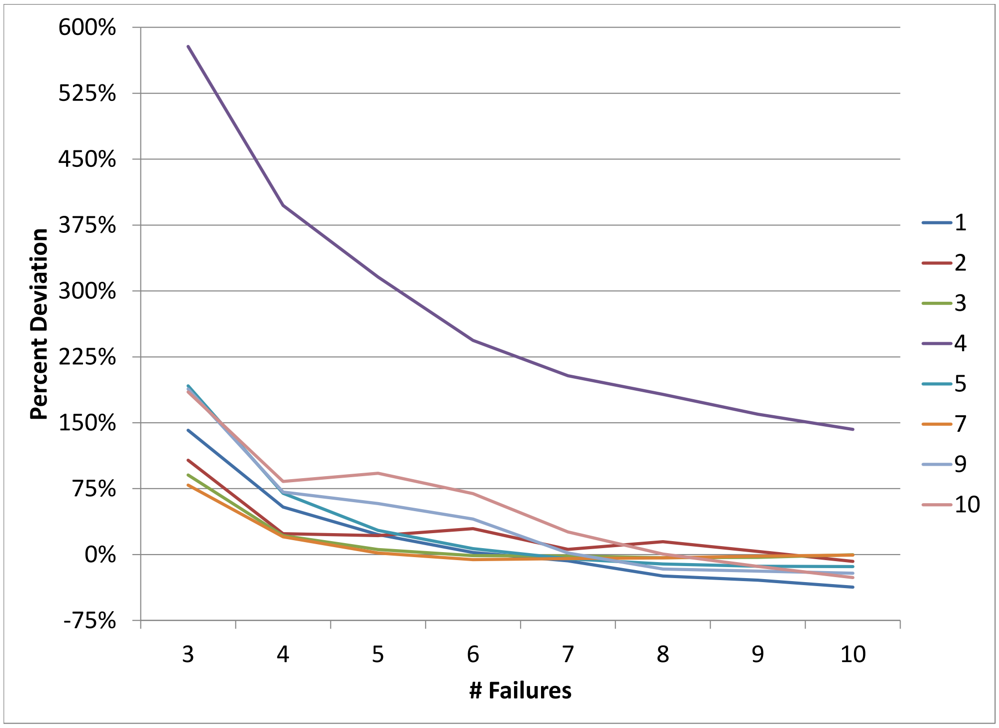

Figure 3.1: Lower Bounds for Difference Between GM(1,1) and AMSAA - 50% FEF

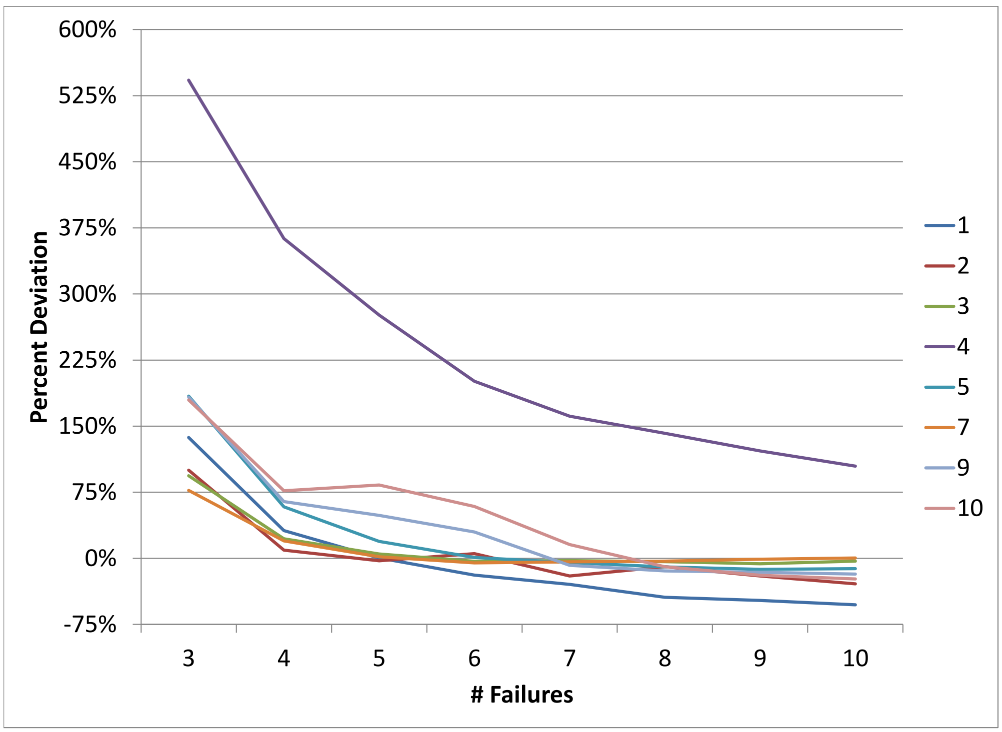

Figure 3.2: Lower Bounds for Difference Between GM(1,1) and AMSAA - 60% FEF

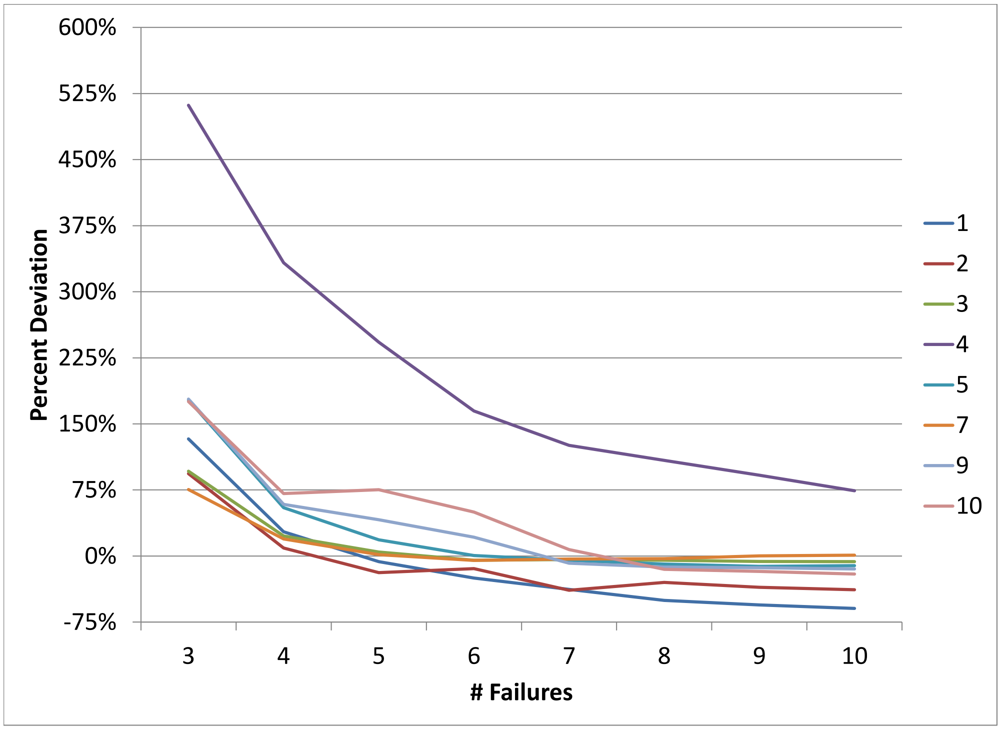

Figure 3.3: Lower Bounds for Difference Between GM(1,1) and AMSAA - 70% FEF

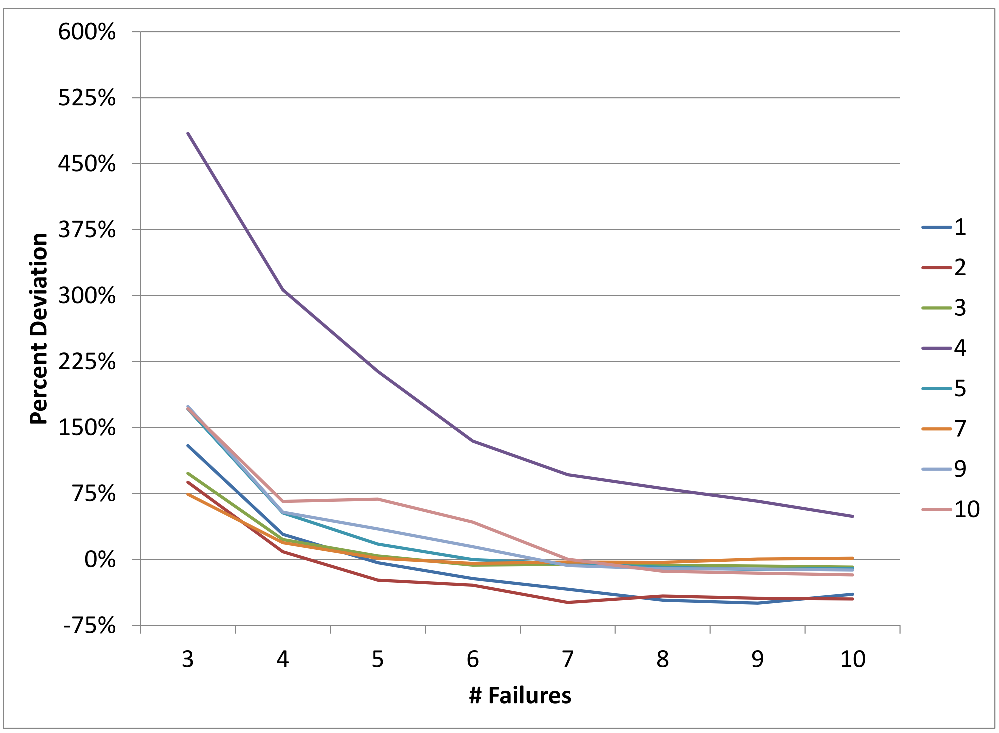

Figure 3.4: Lower Bounds for Difference Between GM(1,1) and AMSAA - 80% FEF

Initial analysis of the randomly generated set of failure mode parameters indicates the assumed FEF level has insignificant impact on the relative performance of the GM(1,1) model, as results are fairly consistent across all assumed FEF levels. With terminating conditions of five or less observed failures, performance of the GM(1,1) model was on par or superior to the AMSAA model for all FEF levels and number of failure modes. As the number of observed failures increased beyond five, the AMSAA model tended to provide better estimates, especially for systems with few failure modes. It is of significance to note the drastic superiority of the GM(1,1) model for systems with four failure modes. When compared to the parameters of other failure modes in the system, the relatively larger beta and eta values result in highly sparse observed failures stemming from this failure mode. As a result, it has impact on calculating the system's true MTBF, but virtually no impact on parameter estimates determined via the AMSAA and GM(1,1) models. While an outlier, this instance is particularly useful in showing how system reliability estimates may be drastically biased by the presence of one failure mode whose relative reliability prevents it from being observed in testing, and demonstrates the superiority of the GM(1,1) model for deriving these estimates under these conditions.

3.4.2 Response Surface Mapping

The promising results from the single set of randomly generated failure mode parameters warranted investigation of a larger area of the response surface. Due to the highly variable nature of the response surface, a designed grid search with four values of β and three values for η was established, with ten combinations considered for simulation to determine if any general conclusions could be drawn on the performance of the GM(1,1) model. Table 3.2 lists the failure mode parameter values and the order in which each of these values occurred.

Table 3.2: Failure Mode Parameters for Designed Test Points

β	η	Set 1	Set 2	Set 3	Set 4	Set 5	Set 6	Set 7	Set 8	Set 9	Set 10
1.5	2000	1	10		2		2	5	10	9	
1.5	5000	8	6	6	10	6	7	1		8	8
1.5	8000	10		5		7	6	7	3	10	2
2	2000	7	2	7	5	1	9			4	5
2	5000	2	1	2	4	8		2	6	5	
2	8000	4	3	10	9	4	4	6	1		3
2.5	2000	6	5	8	6	5	3		5	3	6
2.5	5000	9			1	9	8	10	4	7	9
2.5	8000	5	9	1	7	2	10	9	2	6	10
3	2000	3	7	4		3	1	3	7		7
3	5000		8	9	3	10		4	9	2	1
3	8000		4	3	8		5	8	8	1	4

In order to reduce the computational time needed to map the response surface, only FEF values of 60% and 70% were investigated, and the number of replications for each combination of failure mode, terminating condition, and assumed FEF level was reduced to $n = 500$. Parameter estimates for the AMSAA model and GM(1,1) model were formulated for each replicate via the methodology in Section 3.3, with equations 3.29 through 3.34 providing a sample mean and standard deviation for each of the ten failure mode parameter sets.

Results from simulation show that across the response surface, on average, the GM(1,1) model is superior to the AMSAA model when estimating reliability growth parameters for all test instances, except for the case of one failure mode with terminating conditions of seven or more failures. Table 3.3 contains the average difference between the GM(1,1) model and the AMSAA model across the ten sets, with shaded cells indicating where AMSAA performance was superior. Furthermore, confidence interval construction on the average difference via the t-distribution with $\alpha = 0.05$, demonstrate the GM(1,1) model is on par or superior to the AMSAA model for all instances except the aforementioned cases where the average difference favored the AMSAA model. The lower and upper bounds on the confidence intervals can be seen in Tables 3.4 and 3.5, respectively. Highlighted cells indicate those instances where the GM(1,1) model fails to statistically dominate the AMSAA model. In general, the relative performance of the GM(1,1) model follows a decreasing trend as either the number of failure modes or the terminating number of failures increases. Standard deviation of the differences is minimally impacted by the assumed FEF level and terminating conditions, but is greatly impacted by the number of failure modes in the system. The standard deviations see a noticeable increase when testing a system with two or three failure modes, but decrease and appear to stabilize to a constant level as more failure modes are introduced. Figures 3.5 and 3.6 illustrate this trend.

Table 3.3: Average Difference of GM(1,1) Model and AMSAA Model Across Parameter Sets

60% FEF								
	# Failure Modes							
# Fails	1	2	3	4	5	7	9	10
3	175.94%	246.96%	234.72%	200.90%	135.64%	124.28%	106.54%	107.20%
4	72.69%	118.60%	111.62%	85.74%	46.95%	38.81%	29.66%	38.56%
5	27.15%	78.63%	92.87%	62.90%	61.23%	23.60%	13.73%	18.68%
6	16.58%	63.98%	62.11%	75.18%	51.27%	18.52%	13.02%	16.22%
7	0.97%	31.21%	55.26%	56.74%	41.15%	19.82%	11.48%	10.18%
8	-18.45%	48.23%	31.23%	44.27%	36.35%	15.44%	9.94%	10.53%
9	-10.95%	25.85%	30.07%	31.60%	33.36%	18.70%	9.94%	10.68%
10	-39.01%	20.88%	14.29%	34.47%	25.79%	18.61%	8.46%	8.27%
70% FEF								
# Fails	1	2	3	4	5	7	9	10
3	162.79%	223.40%	218.16%	189.58%	127.92%	119.53%	103.29%	104.18%
4	64.02%	106.82%	99.61%	83.93%	52.86%	37.69%	31.44%	40.08%
5	20.00%	61.74%	79.21%	53.15%	54.74%	20.78%	11.92%	16.85%
6	9.79%	47.51%	49.42%	62.29%	44.60%	15.53%	10.78%	14.30%
7	-5.22%	19.04%	40.28%	44.88%	34.92%	16.53%	8.51%	7.48%
8	-24.02%	33.43%	18.75%	34.55%	30.25%	13.02%	7.07%	7.99%
9	-15.86%	11.63%	20.31%	22.51%	26.50%	15.69%	6.67%	7.91%
10	-34.50%	9.92%	7.13%	26.50%	20.00%	15.64%	5.29%	5.40%

Table 3.4: Lower Confidence Bounds on Average Difference of GM(1,1) and AMSAA Models Across Parameter Sets

60% FEF								
	# Failure Modes							
# Fails	1	2	3	4	5	7	9	10
3	133.38%	82.86%	115.24%	147.26%	78.95%	101.28%	96.21%	84.61%
4	48.16%	10.34%	40.62%	43.60%	20.26%	21.04%	19.88%	26.11%
5	12.43%	-4.71%	42.52%	32.59%	17.71%	-0.41%	5.91%	11.41%
6	7.42%	3.70%	23.61%	31.40%	-6.50%	-3.94%	1.83%	8.75%
7	-4.09%	-13.98%	17.12%	17.69%	-11.02%	-6.04%	-0.57%	0.49%
8	-19.77%	-4.96%	4.60%	10.68%	-7.20%	-1.70%	1.34%	3.78%
9	-12.72%	-10.71%	4.31%	6.23%	-10.24%	-2.82%	2.73%	1.28%
10	-47.93%	-8.60%	-7.50%	6.85%	-8.67%	0.32%	1.86%	-2.66%
70% FEF								
# Fails	1	2	3	4	5	7	9	10
3	123.90%	77.92%	106.90%	140.08%	74.76%	97.98%	93.16%	82.20%
4	44.80%	14.73%	39.77%	45.54%	17.29%	22.81%	22.19%	29.41%
5	9.16%	-6.53%	36.36%	28.04%	16.10%	-0.66%	4.56%	10.33%
6	4.37%	0.90%	17.28%	25.77%	-6.96%	-4.11%	0.69%	7.65%
7	-6.78%	-15.69%	9.41%	12.89%	-9.66%	-5.85%	-2.49%	-1.49%
8	-28.08%	-5.86%	0.37%	7.82%	-4.65%	-1.02%	-0.10%	2.36%
9	-19.57%	-12.07%	1.30%	3.31%	-7.60%	-1.53%	0.41%	-0.22%
10	-46.12%	-9.22%	-9.88%	5.65%	-6.89%	1.28%	-0.53%	-4.05%

Table 3.5: Upper Confidence Bounds on Average Difference of GM(1,1) and AMSAA Models Across Parameter Sets

60% FEF								
	# Failure Modes							
# Fails	1	2	3	4	5	7	9	10
3	218.50%	411.07%	354.19%	254.53%	192.33%	147.29%	116.87%	129.78%
4	97.21%	226.85%	182.61%	127.88%	73.63%	56.58%	39.45%	51.00%
5	41.87%	161.97%	143.22%	93.21%	104.75%	47.61%	21.53%	25.95%
6	25.73%	124.26%	100.60%	118.96%	109.03%	40.98%	24.22%	23.69%
7	6.03%	76.41%	93.41%	95.79%	93.31%	45.69%	23.54%	19.86%
8	-17.13%	101.44%	57.85%	77.85%	79.91%	32.59%	18.54%	17.28%
9	-9.17%	62.41%	55.82%	56.97%	76.96%	40.23%	17.14%	20.07%
10	-30.08%	50.36%	36.10%	62.09%	60.25%	36.89%	15.06%	19.21%
70% FEF								
# Fails	1	2	3	4	5	7	9	10
3	201.68%	368.87%	329.41%	239.06%	181.08%	141.07%	113.42%	126.15%
4	83.23%	198.90%	159.43%	122.32%	88.43%	52.57%	40.69%	50.75%
5	30.84%	130.01%	122.05%	78.24%	93.37%	42.21%	19.27%	23.36%
6	15.19%	94.11%	81.56%	98.80%	96.16%	35.18%	20.86%	20.95%
7	-3.66%	53.79%	71.13%	76.86%	79.51%	38.91%	19.51%	16.46%
8	-19.96%	72.73%	37.13%	61.28%	65.15%	27.07%	14.25%	13.62%
9	-12.15%	35.33%	39.32%	41.70%	60.61%	32.92%	12.91%	16.04%
10	-22.87%	29.06%	24.16%	47.34%	46.90%	29.98%	11.12%	14.86%

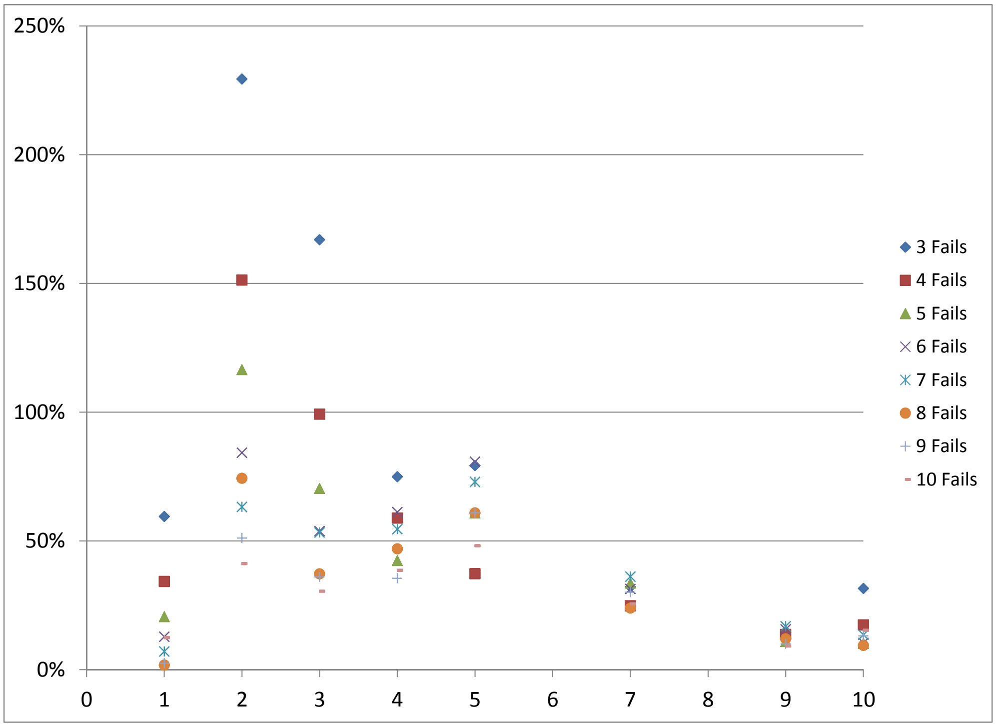

Figure 3.5: Standard Deviation in Differences Across Parameter Sets - 60% FEF

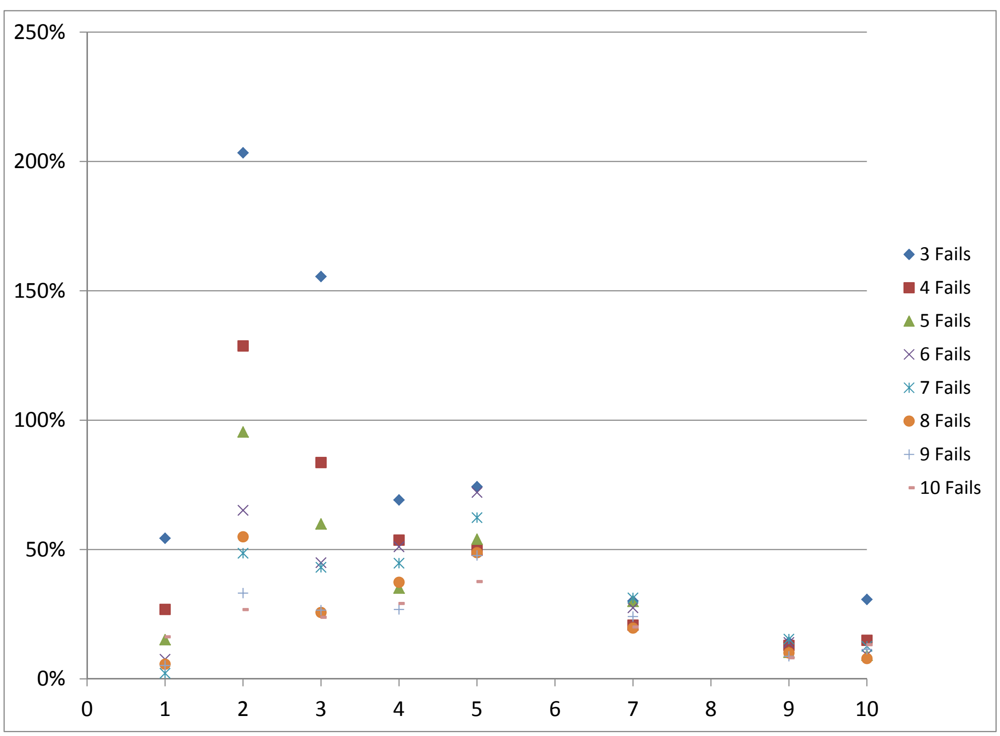

Figure 3.6: Standard Deviation in Differences Across Parameter Sets - 70% FEF

3.5 Conclusions and Future Work

In this paper, we have proposed the modified GM(1,1) model for continuous reliability growth modeling when dealing with limited failure data. To compare its effectiveness, Monte-Carlo simulation was conducted to compare prediction accuracy with the AMSAA model when handling a system whose failures follow a poly-Weibull distribution. Results of simulation across the response surface indicates that, on average, the GM(1,1) model performs better than the AMSAA model for modeling reliability growth. These results, however, validate continued usage of the AMSAA model for component level testing when ample failures are observed, but when failures are sparse and constraints, be they budgetary or time, limit the quantity of observed failures, the GM(1,1) model provides growth parameter estimates that are statistically better than the AMSAA model and are more in line with true system reliability growth. Since the GM(1,1) model is capable of handling as few as three observed failures and is easily implemented, it shows itself to be a viable alternative to the AMSAA model for modeling the reliability growth of complex systems. Future work will seek to establish confidence bounds on the GM(1,1) estimates and expand the GM(1,1) model to small sample reliability growth modeling in the discrete case.

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Appendix

3.A Confidence Bounds for Difference Between GM(1,1) and AMSAA Monte Carlo Simulation

Table 3.A.1: Terminating Condition: 3 Failures

	Number of Failure Modes							
	1		2		3		4	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	141.43%	168.50%	107.45%	148.24%	90.48%	120.86%	578.21%	669.73%
60%	137.03%	162.73%	100.08%	139.64%	93.67%	124.21%	542.63%	628.92%
70%	133.04%	157.46%	93.54%	132.35%	96.38%	127.61%	511.70%	593.51%
80%	129.30%	152.57%	87.70%	125.33%	97.92%	128.82%	484.52%	562.43%

	5		7		9		10	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	191.97%	237.10%	79.03%	104.27%	188.46%	239.72%	184.99%	238.16%
60%	183.98%	227.41%	77.09%	101.81%	182.89%	232.56%	179.81%	231.37%
70%	177.22%	219.21%	75.55%	99.95%	178.08%	226.40%	175.34%	225.53%
80%	170.97%	211.31%	74.15%	98.27%	173.91%	221.06%	171.46%	220.45%

Table 3.A.2: Terminating Condition: 4 Failures

	Number of Failure Modes							
	1		2		3		4	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	54.17%	73.30%	23.82%	52.60%	21.66%	36.38%	397.34%	447.84%
60%	31.41%	49.45%	9.29%	37.53%	22.13%	36.55%	362.85%	410.16%
70%	27.56%	44.59%	9.07%	35.72%	22.75%	36.60%	332.88%	377.43%
80%	28.46%	44.54%	8.63%	34.15%	22.56%	36.42%	306.59%	348.76%

	5		7		9		10	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	69.73%	95.07%	20.13%	37.44%	71.12%	101.22%	83.31%	116.20%
60%	58.47%	82.58%	19.75%	36.69%	64.35%	93.32%	76.64%	108.39%
70%	54.92%	77.83%	19.41%	36.11%	58.60%	86.60%	70.94%	101.70%
80%	52.95%	75.10%	19.08%	35.60%	53.65%	80.83%	65.99%	95.91%

Table 3.A.3: Terminating Condition: 5 Failures

	Number of Failure Modes							
	1		2		3		4	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	22.74%	40.34%	21.55%	47.58%	5.79%	16.37%	315.85%	350.71%
60%	0.45%	17.06%	-2.70%	22.66%	5.02%	15.39%	276.08%	308.58%
70%	-6.34%	9.32%	-18.82%	5.53%	4.52%	14.97%	242.69%	273.19%
80%	-3.80%	10.96%	-23.66%	0.59%	3.97%	14.02%	213.86%	242.78%

	5		7		9		10	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	27.58%	45.81%	1.73%	14.95%	57.95%	77.33%	92.55%	115.90%
60%	19.25%	36.46%	1.71%	14.57%	48.72%	67.30%	83.25%	105.69%
70%	18.38%	34.70%	1.55%	14.23%	41.11%	58.99%	75.36%	97.01%
80%	17.42%	33.16%	1.41%	13.87%	34.60%	51.89%	68.59%	89.57%

Table 3.A.4: Terminating Condition: 6 Failures

	Number of Failure Modes							
	1		2		3		4	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	2.38%	17.75%	29.45%	55.37%	-1.30%	6.94%	243.77%	270.38%
60%	-19.09%	-4.50%	5.36%	29.76%	-3.32%	5.32%	200.88%	226.18%
70%	-24.99%	-11.19%	-14.26%	9.78%	-4.77%	4.54%	164.62%	188.75%
80%	-21.72%	-8.69%	-29.44%	-5.77%	-6.62%	3.60%	134.61%	157.63%

	5		7		9		10	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	6.66%	22.65%	-5.86%	3.61%	40.33%	52.68%	69.32%	84.28%
60%	1.04%	16.05%	-5.22%	3.85%	29.95%	41.85%	58.81%	72.99%
70%	0.55%	14.89%	-5.00%	3.88%	21.46%	32.94%	49.92%	63.44%
80%	-0.13%	13.84%	-4.53%	4.04%	14.35%	25.49%	42.33%	55.28%

Table 3.A.5: Terminating Condition: 7 Failures

	Number of Failure Modes							
	1		2		3		4	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	-7.08%	7.45%	5.92%	30.89%	-1.65%	5.53%	203.42%	228.26%
60%	-29.62%	-15.80%	-20.02%	4.56%	-2.56%	4.56%	161.21%	185.26%
70%	-37.82%	-24.26%	-38.84%	-14.11%	-4.00%	3.28%	125.53%	149.05%
80%	-34.00%	-21.53%	-48.99%	-23.81%	-5.48%	2.06%	96.20%	119.30%

	5		7		9		10	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	-4.78%	6.84%	-4.67%	1.44%	1.67%	10.04%	25.68%	36.92%
60%	-4.82%	6.21%	-4.05%	1.87%	-7.87%	-0.09%	15.67%	26.17%
70%	-4.84%	5.81%	-3.31%	2.39%	-8.07%	-0.79%	7.32%	17.22%
80%	-5.03%	5.43%	-2.99%	2.55%	-6.97%	-0.14%	0.24%	9.63%

Table 3.A.6: Terminating Condition: 8 Failures

	Number of Failure Modes							
	1		2		3		4	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	24.40%	-10.12%	14.71%	32.29%	-3.54%	2.44%	182.25%	202.34%
60%	-44.25%	-30.54%	-9.54%	8.23%	-3.75%	2.03%	141.88%	161.62%
70%	-50.29%	-37.09%	-29.97%	-9.92%	-4.73%	1.53%	108.64%	128.13%
80%	-46.36%	-33.59%	-41.65%	-21.79%	-6.64%	0.63%	80.62%	100.09%

	5		7		9		10	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	-10.80%	-0.50%	-3.72%	0.23%	-16.39%	-10.10%	0.52%	8.57%
60%	-9.79%	0.06%	-3.23%	0.78%	-14.38%	-8.55%	-9.45%	-1.98%
70%	-9.05%	0.61%	-2.93%	1.23%	-12.55%	-7.09%	-15.00%	-8.07%
80%	-8.98%	0.71%	-3.29%	1.25%	-10.76%	-5.64%	-13.52%	-7.06%

Table 3.A.7: Terminating Condition: 9 Failures

	Number of Failure Modes							
	1		2		3		4	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	-29.19%	-15.72%	3.56%	21.18%	-3.21%	1.94%	159.80%	177.70%
60%	-47.80%	-34.77%	-20.12%	-1.51%	-6.07%	0.10%	121.77%	139.43%
70%	-55.51%	-42.83%	-35.48%	-15.90%	-6.11%	-0.18%	91.82%	108.91%
80%	-49.77%	-37.45%	-44.29%	-24.38%	-7.62%	0.63%	66.12%	83.04%

	5		7		9		10	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	-13.37%	-5.63%	-1.84%	1.57%	-18.91%	-13.78%	-13.43%	-7.44%
60%	-12.53%	-4.89%	-0.95%	2.32%	-15.95%	-11.22%	-19.29%	-13.80%
70%	-11.64%	-3.93%	0.24%	3.41%	-13.34%	-8.98%	-17.37%	-12.31%
80%	-11.43%	-3.78%	0.40%	3.59%	-10.88%	-6.83%	-15.60%	-10.92%

Table 3.A.8: Terminating Condition: 10 Failures

	Number of Failure Modes							
	1		2		3		4	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	-37.10%	-23.66%	-7.81%	12.21%	-0.16%	4.54%	142.54%	157.95%
60%	-52.70%	-39.55%	-28.88%	-8.19%	-3.16%	2.49%	104.62%	120.04%
70%	-59.43%	-46.74%	-38.29%	-19.49%	-6.45%	0.35%	74.05%	89.33%
80%	-39.66%	-27.88%	-45.05%	-25.86%	-8.67%	-1.29%	48.89%	64.13%

	5		7		9		10	
FEF	Lower	Upper	Lower	Upper	Lower	Upper	Lower	Upper
50%	-13.72%	-7.52%	-0.57%	2.26%	-21.26%	-16.71%	-26.09%	-20.71%
60%	-11.61%	-5.73%	0.39%	3.34%	-17.91%	-13.74%	-23.33%	-18.47%
70%	-10.87%	-4.63%	0.97%	3.95%	-14.76%	-10.86%	-20.49%	-16.04%
80%	-10.16%	-3.76%	1.32%	4.35%	-12.11%	-8.40%	-17.69%	-13.55%

3.B Certification of Student Work

UNIVERSITY OF ARKANSAS

College of Engineering
Department of Industrial Engineering

MEMORANDUM

TO: Graduate School, University of Arkansas

FROM: Edward A. Pohl, Professor and Department Head

DATE: July 12, 2016

SUBJECT: Certification of Student Effort

I certify that greater than 51% of the work conducted for this chapter entitled "Small Sample Continuous Reliability Growth Modeling Using a Grey Systems Model" was conducted by Thomas P. Talafuse.

Sincerely,

Edward A. Pohl
ephol@uark.edu
479-575-6029
Professor and Department Head
Department of Industrial Engineering
University of Arkansas

4. **Small Sample Discrete Reliability Growth Modeling Using a Grey Systems Model**

Thomas P. Talafuse Edward A. Pohl

Abstract: When performing system-level developmental testing, time and expenses generally warrant a small sample size for failure data. Upon failure discovery, redesigns and/or corrective actions can be implemented to improve system reliability. Current methods for estimating discrete (one-shot) reliability growth, namely the Crow (AMSAA) growth model, stipulate that parameter estimates have a great level of uncertainty when dealing with small sample sizes. For purposes of handling limited failure data, we propose the use of a modified GM(1,1) model to predict system reliability growth parameters and investigate how parameter estimates are affected by systems whose failures follow a poly-Weibull distribution. It is shown that with small sample sizes and multiple failure modes, the modified GM(1,1) model is more accurate than the AMSAA model for prediction of growth model parameters.

4.1 Introduction

Reliability growth is the progressive improvement of reliability performance measures over time through the discovery of failure modes via testing and implementation of solutions to mitigate these failure modes (IEC 61014, 2003). During developmental testing of a complex system, there is considerable interest in assessing how system reliability grows to ensure the finished product meets user reliability requirements. Developmental testing is typically limited by cost, schedule, resource, and other constraints, often resulting in small data samples and making it imperative to identify failure modes and correct reliability deficiencies via corrective action and/or redesign.

A number of models are available for systems undergoing testing in both the continuous and discrete (one-shot) cases. These models include both parametric and non-parametric methods for modeling reliability growth. Discrete models include (but are not limited to): AMSAA (Crow, 1983), Lloyd Lipow (Lloyd and Lipow, 1984), Gompertz (Virene, 1968), and Duane (Duane, 1964). With small samples, however, it proves difficult for these models to confidently obtain accurate parametric estimators and reliability growth prediction results. For system-level testing, it is reasonable to assume that a system contains an unknown number of independent competing failure modes whose respective failure times follow Weibull failure rates, resulting in failure data following a poly-Weibull distribution (Freels, 2013). Furthermore, the AMSAA model, one of the most popular models, assumes failures occur according to a non-homogeneous Poisson process (NHPP) with a Weibull intensity function, expressed as $\lambda(T) = \lambda\beta T^{\beta-1}$. To date, little investigation has been conducted on how this assumption impacts reliability growth modeling on system-level testing for systems following a poly-Weibull failure distribution.

In this paper we present a new reliability growth model for discrete systems based on a modified GM(1,1) model. The model may not be suitable for application to all discrete development programs but it is useful in cases where budgetary and/or time constraints result in a small set of failure data. This model can obtain better prediction results, especially for data of small sample sizes. We focus on developmental testing at the system level, as schedule and cost constraints often preclude sufficient testing to generate a meaningful reliability estimate from the data obtained in these tests.

The remainder of this paper is organized as follows: an introduction to Grey systems theory, the original GM(1,1) model and the modified GM(1,1) model will be presented in section 4.2. Section 4.3 tailors the modified GM(1,1) model for reliability growth modeling, with section 4.4 applying the GM(1,1) model to one-shot reliability growth testing with a varying number of failure modes, different assumed fix effectiveness factor (FEF) levels, and the number of failures observed for test termination. Conclusions and future work are provided in section 4.5.

4.2 Grey Model for Reliability Growth

4.2.1 Grey Systems Background

In systems theory, a system can be defined with a color that represents the amount of clear information about that system. For instance, a system can be called a black box system if its internal characteristics or mathematical equations that describe its dynamics are completely unknown. On the other hand if the description of the system is, completely known, it can be named as a white system. Similarly, a system that has both known and unknown information is defined as a grey system (Liu and Lin, 2006). In real life, every system can be considered as a grey system because there are always some uncertainties. Information that can be obtained from a system is always uncertain and limited in scope due to noise from both inside and outside of the system of concern (Liu and Lin, 2006). Systems achieving reliability growth in developmental testing are no exception, making investigation of modeling reliability growth using grey models warranted.

First proposed by Deng (1982), Grey systems theory has become increasingly popular with its ability to deal with systems that have partially unknown parameters. Unlike conventional statistical models, grey models require only a limited amount of data to estimate the behavior of an unknown system (Deng, 1989), and has been widely applied to a broad spectrum of fields, including social, economic, agricultural, industrial, ecological, and biological arenas (Wang, 2002; Chang et al., 2007; Hsu, 2003; Hsu and Chen, 2003; Jović et al., 2005; Mao and Chirwa, 2006; Huang and Jane, 2009). These previous studies have shown that grey system theory-based approaches can achieve good performance characteristics when applied to real-time systems, since grey predictors adapt their parameters to new conditions as new outputs become available. Because of this reason, grey predictors are more robust with respect to noise and lack of modeling information when compared to conventional methods.

The main task of grey system theory is to extract realistic governing laws of the system using available data. This process is known as the generation of the grey sequence. It is argued that even though the available data of the system, which are generally white numbers, is too complex or chaotic, they always contain some governing laws. If the randomness of the data obtained from a grey system is somehow smoothed, it is easier to derive any special characteristics of that system. (Liu and Lin, 2006). Grey models predict the future values of a time series based only on a set of the most recent data depending on the window size of the predictor. It is assumed that all data values to be used in grey models are positive, and the sampling frequency of the time series is fixed (Kayacan et al., 2010).

In grey systems theory, GM(*n,m*) denotes a grey model, where *n* is the order of the difference equation and *m* is the number of variables. Although various types of grey models can be studied, most research has focused attention on GM(1,1) models because of its computational efficiency. This is mainly due to most applications valuing computational efforts second only to model performance (Kayacan et al., 2010).

4.2.2 The GM(1,1) Model

The Grey Model First Order, One Variable, or GM(1,1), has been the most widely model discussed in literature. This model is a time series forecasting model in which the differential equations of the GM(1,1) model have time-varying coefficients, meaning that the model is renewed as new data become available to the prediction model. The GM(1,1) model can only be used in positive data sequences (Deng, 1989). In this paper, system failure times are used as the raw data points and are positive, allowing grey models to be used to forecast the future values of the raw data points. In order to smooth the randomness, the raw data obtained from the system to form the GM(1,1) is subjected to an operator, named the Accumulating Generation Operator (AGO) (Deng, 1989). The differential equation (i.e. GM(1,1)) is solved to obtain the *n*-step ahead predicted value of the system. Finally, using the predicted value, the Inverse Accumulating Generation Operator (IAGO) is applied to find the predicted values of the original data sequence. Given that

$$X^{(0)} = (x^{(0)}(1), x^{(0)}(2), ..., x^{(0)}(n)) \tag{4.1}$$

is a non-negative sequence of raw data, then

$$X^{(1)} = (x^{(1)}(1), x^{(1)}(2), ..., x^{(1)}(n)) \tag{4.2}$$

is a sequence generated from applying the first-order AGO to $X(0)$, where,

$$X^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i),\ k = 1,2,...,n. \tag{4.3}$$

The least square estimate sequence of the grey differential equation of GM(1,1) is defined as follows (Deng, 1989):

$$X^{(0)}(k) + az^{(1)}(k) = b, \tag{4.4}$$

where the generated mean sequence of $X^{(1)}$ is defined as:

$$Z^{(1)} = (z^{(1)}(1), z^{(1)}(2), ..., z^{(1)}(n)) \tag{4.5}$$

and $z^{(1)}(k)$ is the generated mean value of adjacent data, calculated as:

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1),\ k = 2,3,...,n. \tag{4.6}$$

The GM(1,1) whitening equation is then given by:

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b. \tag{4.7}$$

Least squares estimators can be derived using:

$$\hat{u} = [a,b]^T = (B^T B)^{-1} B^T Y, \tag{4.8}$$

where

$$Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \\ -z^{(1)}(n) & 1 \end{bmatrix}, \tag{4.9}$$

and parametric estimators can be obtained as follows (Liu and Lin, 2006):

$$\begin{cases} a = \dfrac{\sum\limits_{k=2}^{n} x^{(0)}(k) \cdot \sum\limits_{k=2}^{n} z^{(1)}(k) - (n-1) \cdot \sum\limits_{k=2}^{n} x^{(0)}(k) \cdot z^{(1)}(k)}{(n-1) \sum\limits_{k=2}^{n} \left[z^{(1)}(k)\right]^2 - \left[\sum\limits_{k=2}^{n} z^{(1)}(k)\right]^2} \\ b = \frac{1}{(n-1)} \cdot \left[\sum\limits_{k=2}^{n} x^{(0)}(k) + a \cdot \sum\limits_{k=2}^{n} z^{(1)}(k)\right]^2 \end{cases} . \tag{4.10}$$

By equation 4.7, the time response solution can be expressed as:

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak} + \frac{b}{a}. \tag{4.11}$$

Predicted values of the original raw data sequence can then be obtained by applying the IAGO, resulting in:

$$\begin{aligned} \hat{x}^{(0)}(k+1) &= \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) \\ &= (1-e^{a}) \left(x^{(0)}(1) - \frac{b}{a}\right) e^{-ak}. \end{aligned} \tag{4.12}$$

4.2.3 Modification of GM(1,1) Model

Wang et al. (2010) introduced a modification to the original GM(1,1) model to take better advantage of new pieces of information in the raw data sequence. The time response solution of the whitened equation is expressed as:

$$x^{(1)}(k) = ce^{-ak} + \frac{b}{a}\ t = 1, 2, ...n \tag{4.13}$$

where c is a constant, and parameters a and b are derived according to equation 4.8. For equation 4.13, if $k = 1$, then

$$x^{(1)}(1) = ce^{-a} + \frac{b}{a} \tag{4.14}$$

and for $k = n$

$$x^{(1)}(n) = ce^{-an} + \frac{b}{a}. \tag{4.15}$$

In order to fully use new information in the raw data sequence while also maintaining the initial conditions of the original GM(1,1) model, a new initial condition is set to

$$0.5\left(x^{(1)}(1)+x^{(1)}(n)\right) \tag{4.16}$$

and c is derived to be:

$$c=2(e^{-a}+e^{-an})^{-1}\left(\frac{x^{(1)}(1)+x^{(1)}(n)}{2}-\frac{b}{a}\right). \tag{4.17}$$

This results in a new time response solution of

$$x^{(1)}(k)=\frac{2}{1+e^{-a(n-1)}}\left(\frac{x^{(1)}(1)+x^{(1)}(n)}{2}-\frac{b}{a}\right)e^{-a(k-1)}+\frac{b}{a} \tag{4.18}$$

and predicted raw data values as:

$$\hat{x}^{(0)}(k)=2(1-e^{a})(1+e^{-a(n-1)})^{-1}\times(\frac{x^{(1)}(1)+x^{(1)}(n)}{2}-\frac{b}{a})e^{-a(k-1)}. \tag{4.19}$$

The new initial condition derived in equation 4.19 from the first and last observations in the raw data sequence preserve the format of the initial condition for the original GM(1,1) model and make full use of new observations and can be utilized to more accurately predict raw observation values.

4.3 Application to Reliability Growth

It was desired to see how the modified GM(1,1) model performed when applied to discrete reliability growth modeling. Some assumptions are made regarding testing and evaluation:

- Each system undergoing test is identical and has a fixed number of independent competing failure modes whose respective failure times follow Weibull failure rates.
- Failure of any one failure mode results in system failure.

- There are n system configurations being tested.
- Testing of configuration i is comprised of N_i trials to determine success/failure for a mission length L.
- For systems that failed during a stage of testing, discovery of the causal failure mode is known with certainty.
- Corrective actions on all identified causal failure modes are implemented after completion of a stage of testing and improves the characteristic life, η_i, by an assumed constant FEF.

For purposes of assessment, system reliability for the configuration undergoing testing, $R_i(M)$, is calculated upon completion of a stage of testing and implementation of corrective actions. It is assumed that after implementation of corrective actions that the system is as good as new. The CDF for a poly-Weibull distribtion with J failure modes that each follow a unique Weibull distribution, is expressed as:

$$F(t,\eta,\beta) = 1 - \left\{ \exp\left[-\sum_{j=1}^{J} \left(\frac{t}{\eta_j} \right)^{\beta_j} \right] \right\},\ \eta_j,\beta_j > 0,\ t \geq 0. \tag{4.20}$$

Then the true system reliability for mission length L is calculated as:

$$R_i(L) = \exp\left[-\sum_{j=1}^{J} \left(\frac{L}{\eta_j} \right)^{\beta_j} \right], \tag{4.21}$$

and can be used as the basis of comparison to measure the accuracy of the reliability estimates derived from the AMSAA model and the GM(1,1) model.

To derive AMSAA parameter values, we let N_i be the number of trials during configuration i, with M_i failures in each configuration. The cumulative number of trials, T_i, and failures, K_i, are expressed as:

$$T_i = \sum N_i \tag{4.22}$$

$$K_i = \sum M_i \tag{4.23}$$

The expected number of failures for configuration i can be expressed as $E[K_i]$. Applying the learning curve property from the AMSAA model implies:

$$E[K_i] = \lambda T_i^{\beta} \tag{4.24}$$

The probability of failure for configuration 1, denoted as f_1, can be expressed in terms of T and N as:

$$E[K_1] = \lambda T_1^{\beta} = f_1 N_1 \tag{4.25}$$

and the failure rate therefore, is:

$$f_1 = \frac{\lambda T_1^{\beta}}{N_1}. \tag{4.26}$$

The expected number of failures by the end of configuration 2 is the sum of the expected number of failures in configuration 1 and the expected number of failures in configuration 2 is:

$$E[K_2] = \lambda T_2^{\beta} \tag{4.27}$$

$$= f_1 N_1 + f_2 N_2, \tag{4.28}$$

resulting in a probability of failure of:

$$f_2 = \frac{\lambda T_2^{\beta} - \lambda T_1^{\beta}}{N_2}. \tag{4.29}$$

By inductive reasoning, we can express the probability of failure for configuration i as:

$$f_i = \frac{\lambda T_i^{\beta} - \lambda T_{i-1}^{\beta}}{N_i}. \tag{4.30}$$

The reliability for configuration i is then expressed as:

$$R_{AMSAA_i} = 1 - f_i. \tag{4.31}$$

Using this formulation, AMSAA parameter values can then be derived via maximization of the likelihood function:

$$\prod_{i=1}^{n} \binom{N_i}{M_i} \left(\frac{\lambda T_i^{\beta} - \lambda T_{i-1}^{\beta}}{N_i} \right)^{M_i} \left(\frac{N_i - \lambda T_i^{\beta} + \lambda T_{i-1}^{\beta}}{N_i} \right)^{N_i - M_i}. \tag{4.32}$$

Exact maximum likelihood estimators for λ and β are then values that satisfy the following two equations:

$$\sum_{i=1}^{n} H_i \times S_i = 0 \tag{4.33}$$

$$\sum_{i=1}^{n} U_i \times S_i = 0, \tag{4.34}$$

where

$$H_i = T_i^{\beta} \ln T_i - T_{i-1}^{\beta} \ln T_{i-1} \tag{4.35}$$

$$S_i = \frac{M_i}{\lambda T_i^{\beta} - \lambda T_{i-1}^{\beta}} - \frac{N_i - M_i}{N_i - \lambda T_i^{\beta} + \lambda T_{i-1}^{\beta}} \tag{4.36}$$

$$U_i = T_i^{\beta} - T_{i-1}^{\beta}. \tag{4.37}$$

Following paremeter derivation, AMSAA reliability estimates for each configuration can be derived via equations 4.30 and 4.31 and compared to the true system reliability calculated in equation 4.21.

For the modified GM(1,1) model, the raw data sequence, $X^{(0)}$, is the point estimate of the reliability for configuration i:

$$X^{(0)} = \left(x^{(0)}(1) = 1 - \frac{M_1}{N_1}, \ldots, x^{(0)}(n) = 1 - \frac{M_n}{N_n} \right). \tag{4.38}$$

The modified GM(1,1) model discussed in sections 4.2.2 and 4.2.3, is applied to obtain $\hat{X}^{(0)} = (R_{GM_i}, \ldots, R_{GM_n})$, or the predicted reliability for each configuration. Reliability estimates can then be compared to the true system reliability, allowing for model performance, relative to the AMSAA model, to be assessed.

4.4 Numerical Experimentation

The quality of modeling reliability growth via the GM(1,1) model was explored to determine if it provided more accurate estimates than the AMSAA model. Via Monte-Carlo simulation, failure data were generated for a hypothetical system undergoing discrete developmental testing with a fixed number of independent competing failure modes whose respective failure times follow Weibull failure rates. The number of failure modes in the system ranged from five to as many as 20 failure modes.

Failure mode parameters were randomly generated with each β_i drawn from a uniform distribution in the range (1, 3.5) and each η_i from a uniform distribution in the range (1,000, 10,000). These values were chosen to reflect parameters that may be seen in real-world failure modes undergoing developmental testing. Table 4.1 lists the parameter values for the failure modes initially investigated. The impact of assuming a constant level of corrective action was also investigated by assuming FEF values of 50%, 60%, 70%, and 80%.

A total of $n = 5$ configurations were considered, with each progressive configuration incorporating the corrective actions introduced from failure mode discovery in the previous configuration. Each test configuration consisted of $N_i = 10,\ \ i = 1, \ldots, 5$, systems undergoing testing to determine success/failure for mission length $L = 1000$.

Table 4.1: Failure Mode (FM) Parameter Data

FM(i)	β_i	η_i
1	2.3585	3505.3245
2	2.0613	8602.9852
3	1.0118	2094.1221
4	2.6769	8432.6748
5	1.3418	6175.8400
6	3.2283	2882.8191
7	1.4633	1975.3920
8	1.5492	9807.6141
9	3.0292	2547.4691
10	3.0406	3466.6637
11	2.0793	9460.2684
12	3.0441	4025.0076
13	1.4385	4355.4884
14	1.0142	3271.8372
15	2.9892	1137.2947
16	2.4971	6434.2409
17	1.2629	4437.4910
18	1.0912	9013.7041
19	3.4523	1539.4779
20	3.2264	6192.1135

Test instances were developed for all possible combinations of failure modes and corrective action levels. To account for the stochastic nature of failure times and its impact on reliability estimates, each test instance was replicated $r = 500$ times. Both the AMSAA model and the GM(1,1) model were evaluated using the absolute error between the true reliability and their respective estimate for reliability, with no preference being shown for either conservative or optimistic estimates, and are expressed as:

$$\delta_{AMSAA_i} = |R_i - R_{AMSAA_i}|, \quad i = 1, ..., r \tag{4.39}$$

$$\delta_{GM_k} = |R_i - R_{GM_i}|, \quad i = 1, ..., r \tag{4.40}$$

From these replications, we derive sample means and standard deviations:

$$\bar{\delta}_{AMSAA} = \frac{1}{r}\sum_{i=1}^{r}\delta_{AMSAA_i} \tag{4.41}$$

$$S_{AMSAA} = \sqrt{\frac{1}{r-1}\sum_{i=1}^{r}\left(\delta_{AMSAA_i} - \bar{\delta}_{AMSAA_i}\right)} \tag{4.42}$$

$$\bar{\delta}_{GM} = \frac{1}{r}\sum_{i=1}^{r}\delta_{GM_i} \tag{4.43}$$

$$S_{GM} = \sqrt{\frac{1}{r-1}\sum_{i=1}^{r}\left(\delta_{GM_i} - \bar{\delta}_{GM_i}\right)} \tag{4.44}$$

Using the values calculated in equations 4.41 through 4.44, confidence intervals were constructed to assess if any statistical difference existed between the AMSAA and modified GM(1,1) when estimating the true system reliability. Because of the large number of replications, the central limit theorem permits use of the z-statistic for computing interval half-widths. The confidence interval is then calculated as:

$$\left(\left(\bar{\delta}_{AMSAA} - \bar{\delta}_{GM}\right) \pm z_{1-\alpha/2} \times \sqrt{\left(\frac{S_{AMSAA}}{\sqrt{r}}\right)^2 + \left(\frac{S_{GM}}{\sqrt{r}}\right)^2}\right), \tag{4.45}$$

with intervals strictly above zero indicating superiority of the GM(1,1) model, and intervals strictly below zero indicating superiority of the AMSAA model. To convey the instances where the GM(1,1) model outperforms the AMSAA model, figures 4.1 through 4.4 show the lower confidence bounds for the various combinations of assumed FEF and number of failure modes, with results above zero indicating the statistically superior performance of the GM(1,1) model. The complete list of the lower and upper confidence bounds can be found in the tables of Appendix 4.A.

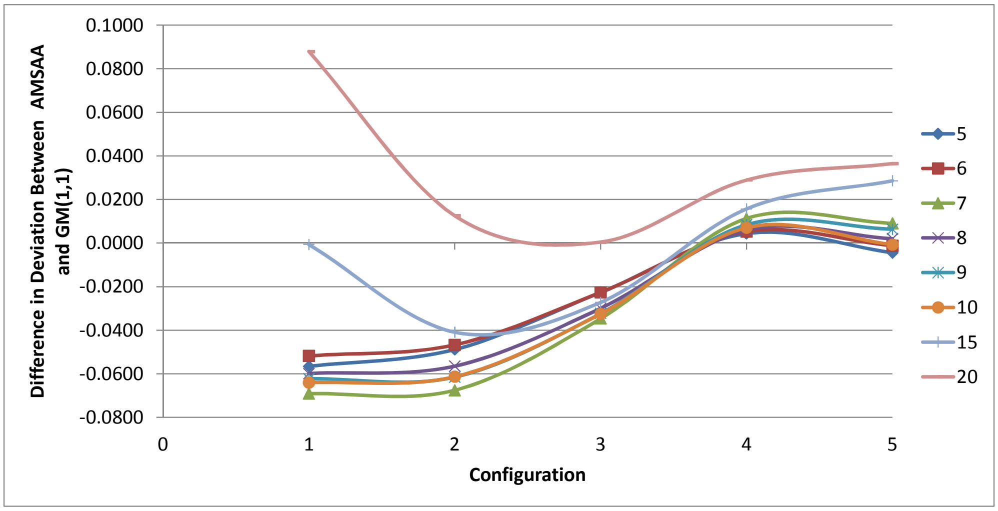

Figure 4.1: Lower Bounds for Difference Between GM(1,1) and AMSAA - 50% FEF

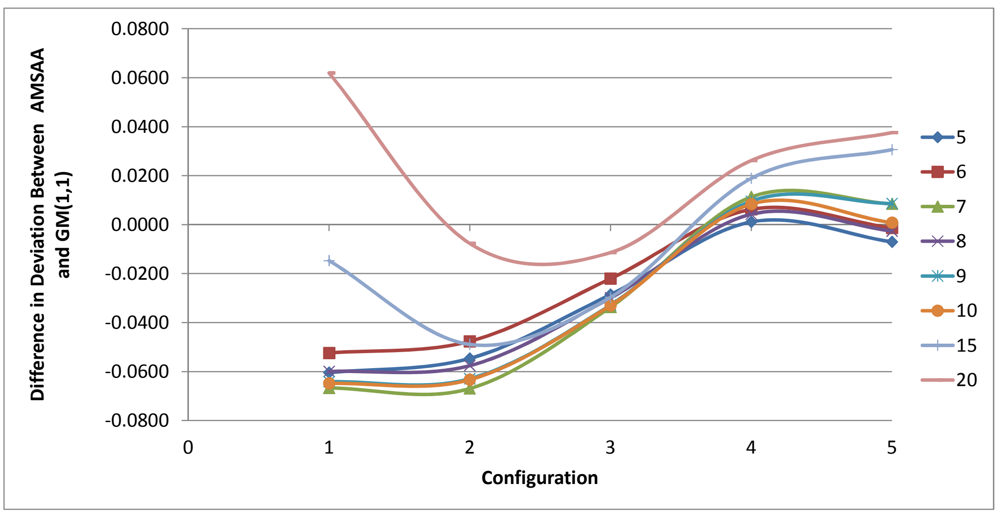

Figure 4.2: Lower Bounds for Difference Between GM(1,1) and AMSAA - 60% FEF

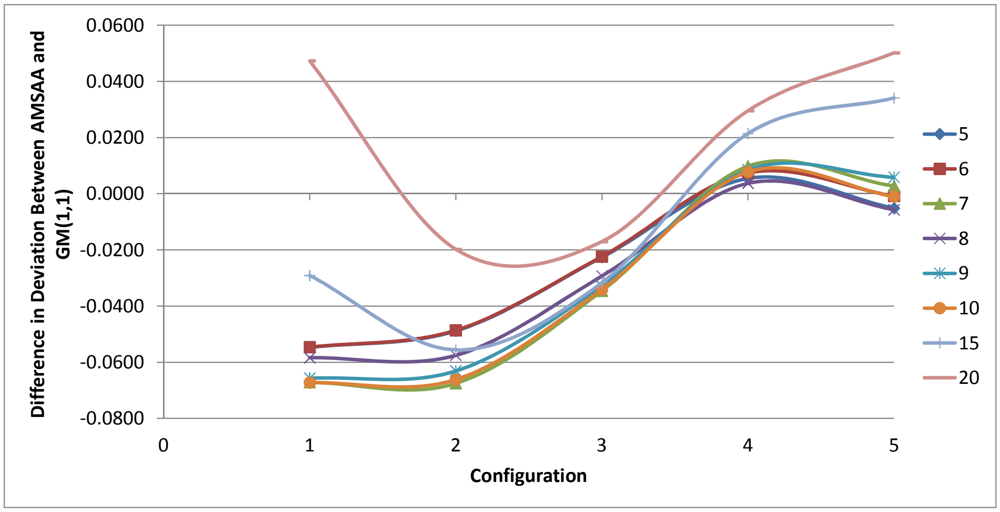

Figure 4.3: Lower Bounds for Difference Between GM(1,1) and AMSAA - 70% FEF

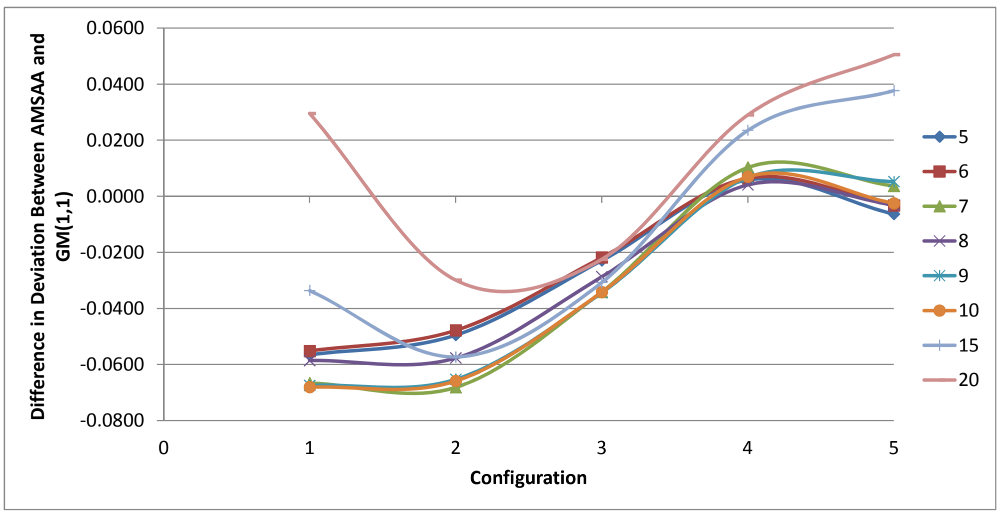

Figure 4.4: Lower Bounds for Difference Between GM(1,1) and AMSAA - 80% FEF

Analysis of the randomly generated set of failure mode parameters indicates the assumed FEF level has insignificant impact on the relative performance of the GM(1,1) model, as results are fairly consistent across all assumed FEF levels. From the confidence bounds, we see the trend of the GM(1,1) model outperforming the AMSAA model as the number of failure modes increases. We observe that the GM(1,1) model's predicted reliability for the first few configurations is inferior to that of the AMSAA, but improves in the later configurations.

If model accuracy across all testing configurations is desired, then, in general, the AMSAA model outperforms the GM(1,1) model. However, if more accuracy in later stages of testing is desired, the GM(1,1) model provides a better estimate of the true reliability, especially for systems with a larger number of failure modes, as seen in the plots of the average true system reliability and the average of each model's estimate of reliability in Figure 4.B.27. Plots for all simulation instances can be found in Appendix 4.B. We also note that across replications, the average reliability estimate for both the AMSAA and GM(1,1) models are conservative estimates to the system's true reliability for virtually all simulated instances.

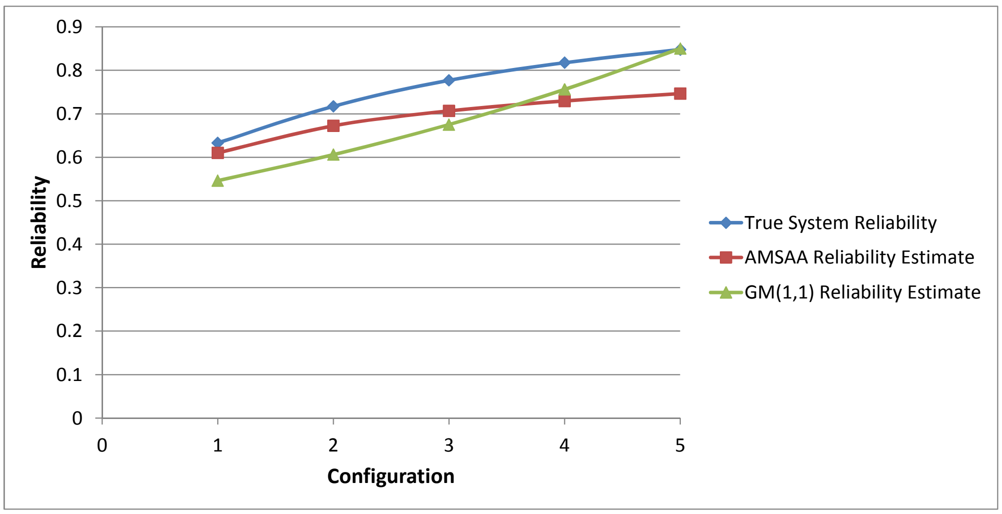

Figure 4.5: Reliability Across Configurations - 7 Failure Modes and 50% FEF

4.5 Conclusions and Future Work

In this paper, we have proposed the modified GM(1,1) model for discrete reliability growth modeling when dealing with limited failure data. To compare its effectiveness, Monte-Carlo simulation was conducted to compare prediction accuracy with the AMSAA model when handling a system whose failures follow a poly-Weibull distribution. Results of simulation indicate that the GM(1,1) model is capable of providing more accurate estimates of system reliability for complex systems with larger numbers of failure modes. While the AMSAA model's predicted reliability values in early test configurations is more accurate, the GM(1,1) model is capable of providing more accurate estimates in later stages of testing configurations, resulting in more precise estimates of reliability at the end of a test program. While beneficial that the GM(1,1) model is capable of handling as few as three cofigurations for discrete reliability growth testing, the major appeal of the GM(1,1) model stems from the relative ease of deriving parameter values via least squares, while the AMSAA model requires the more complex maximum likelihood estimation. This, combined with results showing that the GM(1,1) model is on par or superior to the AMSAA model when failure data are sparse, makes the GM(1,1) model a viable alternative to the AMSAA model for modeling the reliability growth of discrete, complex systems. Future work will seek to establish confidence bounds on the GM(1,1) estimates and investigate a larger area of the response surface to determine if more general conclusions can be drawn on the performance of the GM(1,1) model.

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Appendix

4.A Confidence Bounds for Difference Between AMSAA and GM(1,1) Monte Carlo Simulation

4.A.1 Lower Confidence Bounds

Table 4.A.1: Assumed 50% FEF

	Number of Failure Modes							
Configuration	5	6	7	8	9	10	15	20
1	-0.0566	-0.0518	-0.0691	-0.0599	-0.0623	-0.0641	-0.0007	0.0879
2	-0.0488	-0.0467	-0.0675	-0.0565	-0.0615	-0.0613	-0.0409	0.0125
3	-0.0226	-0.0226	-0.0347	-0.0299	-0.0323	-0.0325	-0.0272	0.0005
4	0.0044	0.0053	0.0113	0.0060	0.0084	0.0070	0.0157	0.0289
5	-0.0044	-0.0012	0.0091	0.0020	0.0063	-0.0007	0.0286	0.0364

Table 4.A.2: Assumed 60% FEF

	Number of Failure Modes							
Configuration	5	6	7	8	9	10	15	20
1	-0.0604	-0.0525	-0.0667	-0.0600	-0.0641	-0.0648	-0.0147	0.0619
2	-0.0546	-0.0476	-0.0669	-0.0576	-0.0629	-0.0633	-0.0489	-0.0077
3	-0.0285	-0.0221	-0.0337	-0.0299	-0.0328	-0.0330	-0.0297	-0.0114
4	0.0011	0.0062	0.0113	0.0042	0.0096	0.0084	0.0189	0.0262
5	-0.0070	-0.0014	0.0085	-0.0026	0.0086	0.0008	0.0306	0.0376

Table 4.A.3: Assumed 70% FEF

	Number of Failure Modes							
Configuration	5	6	7	8	9	10	15	20
1	-0.0547	-0.0546	-0.0671	-0.0584	-0.0657	-0.0672	-0.0291	0.0473
2	-0.0488	-0.0486	-0.0675	-0.0575	-0.0631	-0.0662	-0.0556	-0.0198
3	-0.0226	-0.0224	-0.0346	-0.0293	-0.0328	-0.0341	-0.0317	-0.0170
4	0.0054	0.0074	0.0098	0.0037	0.0085	0.0077	0.0214	0.0295
5	-0.0052	-0.0008	0.0028	-0.0057	0.0058	-0.0010	0.0341	0.0502

Table 4.A.4: Assumed 80% FEF

	Number of Failure Modes							
Configuration	5	6	7	8	9	10	15	20
1	-0.0565	-0.0552	-0.0666	-0.0586	-0.0674	-0.0681	-0.0337	0.0295
2	-0.0495	-0.0478	-0.0681	-0.0577	-0.0653	-0.0659	-0.0573	-0.0299
3	-0.0229	-0.0218	-0.0341	-0.0287	-0.0344	-0.0342	-0.0307	-0.0224
4	0.0058	0.0064	0.0103	0.0041	0.0067	0.0069	0.0235	0.0290
5	-0.0064	-0.0033	0.0036	-0.0031	0.0052	-0.0025	0.0377	0.0505

4.A.2 Upper Confidence Bounds

Table 4.A.5: Assumed 50% FEF

	Number of Failure Modes							
Configuration	5	6	7	8	9	10	15	20
1	-0.1061	-0.0965	-0.1304	-0.1122	-0.1171	-0.1203	0.0084	0.1866
2	-0.0910	-0.0867	-0.1273	-0.1053	-0.1151	-0.1145	-0.0724	0.0361
3	-0.0392	-0.0392	-0.0621	-0.0525	-0.0572	-0.0574	-0.0452	0.0120
4	0.0147	0.0166	0.0295	0.0192	0.0241	0.0217	0.0405	0.0689
5	-0.0022	0.0043	0.0258	0.0121	0.0209	0.0074	0.0673	0.0844

Table 4.A.6: Assumed 60% FEF

	Number of Failure Modes							
Configuration	5	6	7	8	9	10	15	20
1	-0.1129	-0.0979	-0.1256	-0.1124	-0.1206	-0.1217	-0.0199	0.1340
2	-0.1015	-0.0887	-0.1261	-0.1075	-0.1177	-0.1185	-0.0882	-0.0047
3	-0.0497	-0.0383	-0.0602	-0.0527	-0.0580	-0.0584	-0.0495	-0.0121
4	0.0095	0.0181	0.0293	0.0155	0.0266	0.0242	0.0472	0.0629
5	-0.0064	0.0035	0.0244	0.0028	0.0255	0.0100	0.0714	0.0865

Table 4.A.7: Assumed 70% FEF

	Number of Failure Modes							
Configuration	5	6	7	8	9	10	15	20
1	-0.1024	-0.1024	-0.1263	-0.1091	-0.1235	-0.1263	-0.0490	0.1046
2	-0.0912	-0.0907	-0.1270	-0.1074	-0.1181	-0.1240	-0.1016	-0.0290
3	-0.0397	-0.0391	-0.0619	-0.0514	-0.0580	-0.0604	-0.0537	-0.0235
4	0.0161	0.0201	0.0267	0.0142	0.0240	0.0230	0.0519	0.0695
5	-0.0046	0.0042	0.0131	-0.0037	0.0197	0.0064	0.0779	0.1113

Table 4.A.8: Assumed 80% FEF

	Number of Failure Modes							
Configuration	5	6	7	8	9	10	15	20
1	-0.1062	-0.1035	-0.1249	-0.1096	-0.1266	-0.1277	-0.0579	0.0689
2	-0.0925	-0.0891	-0.1281	-0.1077	-0.1219	-0.1231	-0.1049	-0.0494
3	-0.0404	-0.0382	-0.0608	-0.0503	-0.0609	-0.0604	-0.0518	-0.0346
4	0.0167	0.0180	0.0274	0.0147	0.0208	0.0214	0.0560	0.0682
5	-0.0074	-0.0008	0.0147	0.0010	0.0187	0.0033	0.0850	0.1115

4.B Reliability Growth Plots Across Configurations

4.B.1 Assumed 50% FEF Growth Curves

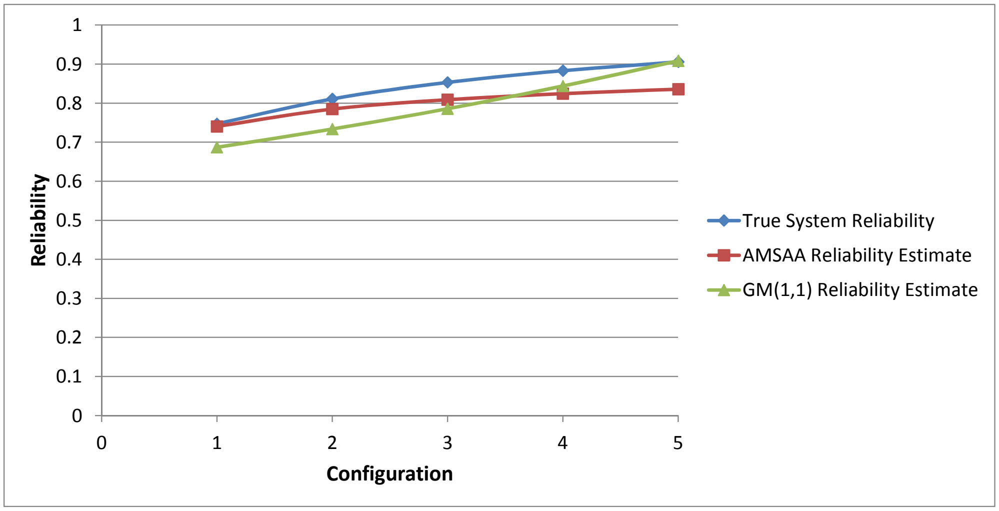

Figure 4.B.1: Reliability Across Configurations - 5 Failure Modes and 50% FEF

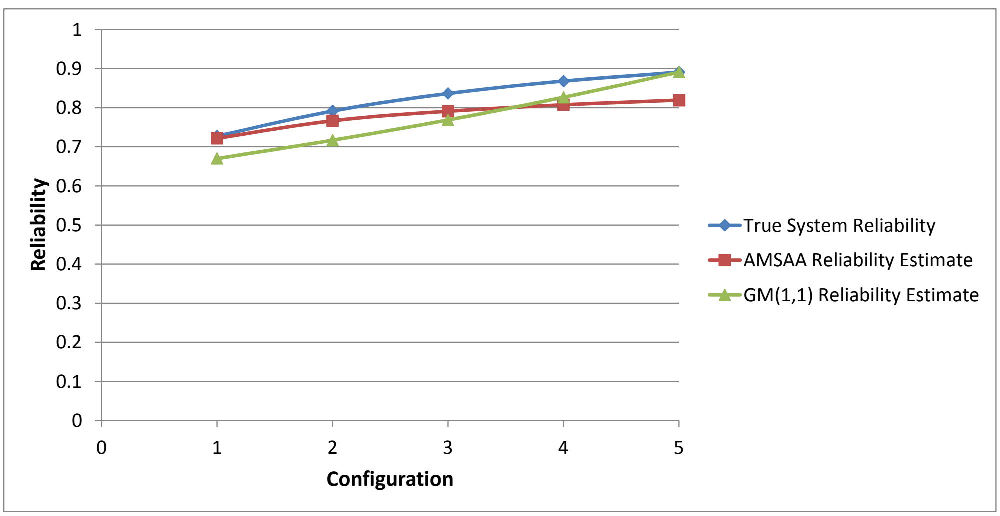

Figure 4.B.2: Reliability Across Configurations - 6 Failure Modes and 50% FEF

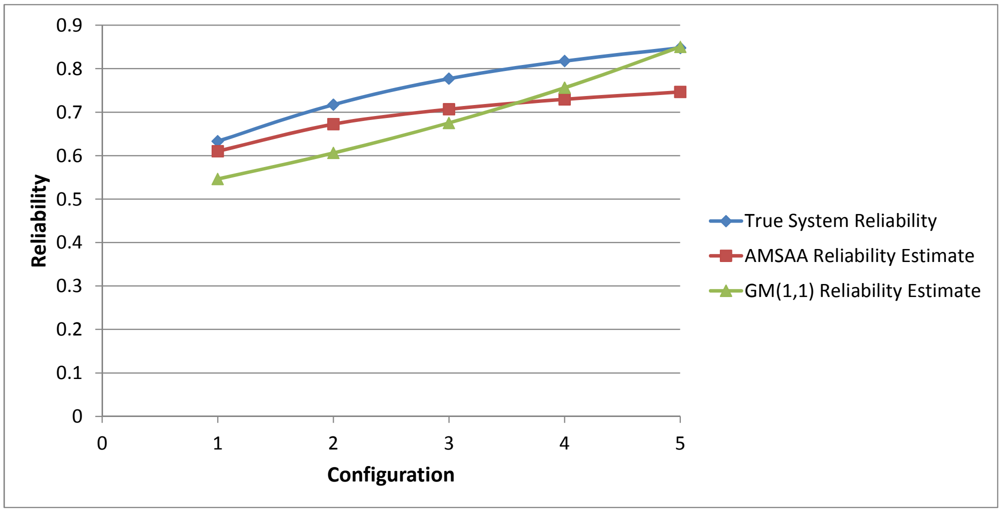

Figure 4.B.3: Reliability Across Configurations - 7 Failure Modes and 50% FEF

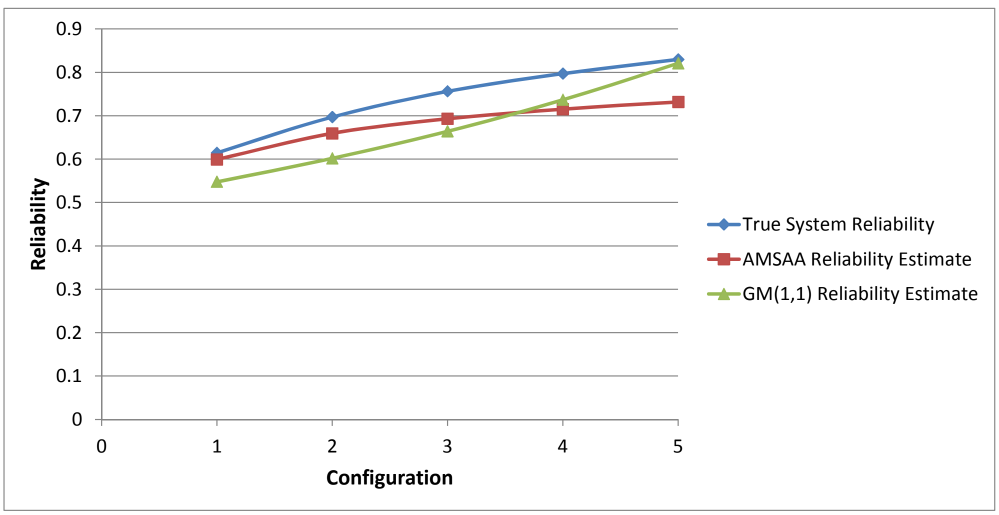

Figure 4.B.4: Reliability Across Configurations - 8 Failure Modes and 50% FEF

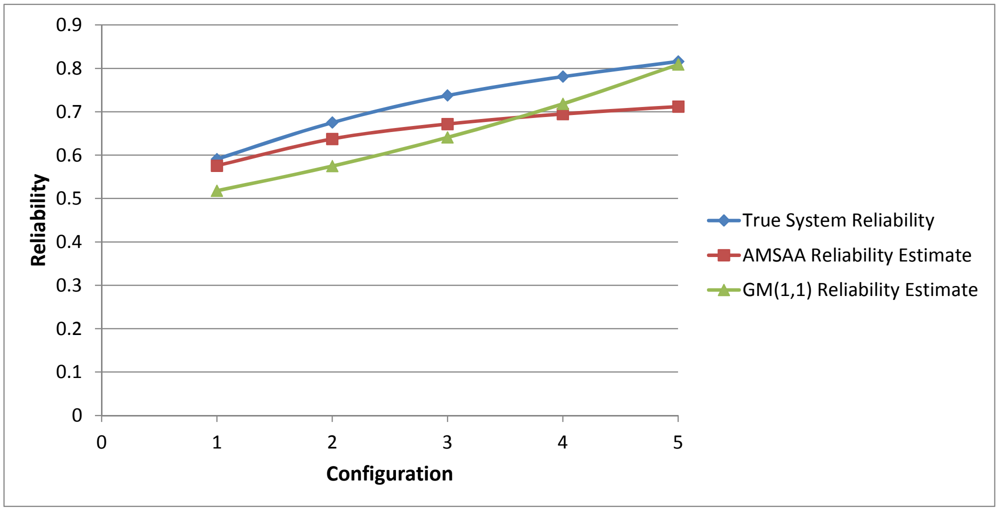

Figure 4.B.5: Reliability Across Configurations - 9 Failure Modes and 50% FEF

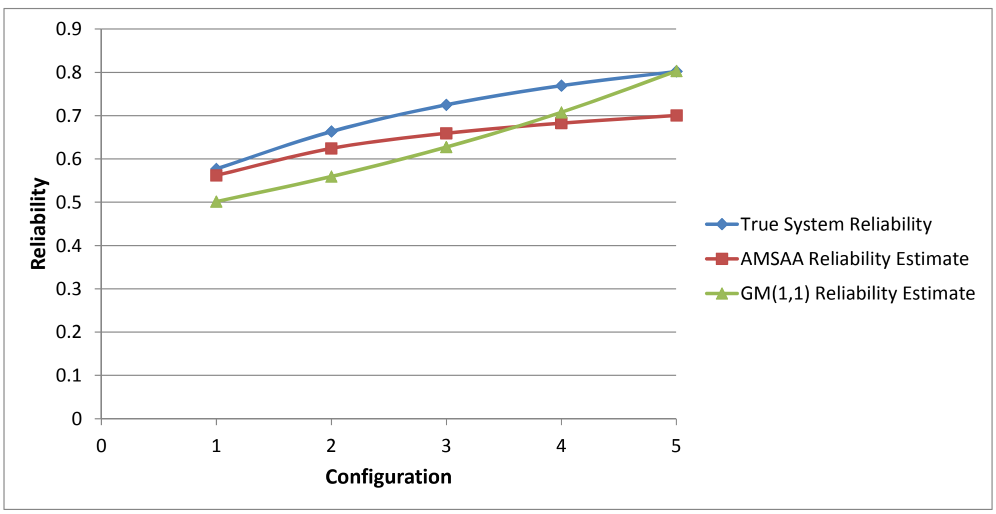

Figure 4.B.6: Reliability Across Configurations - 10 Failure Modes and 50% FEF

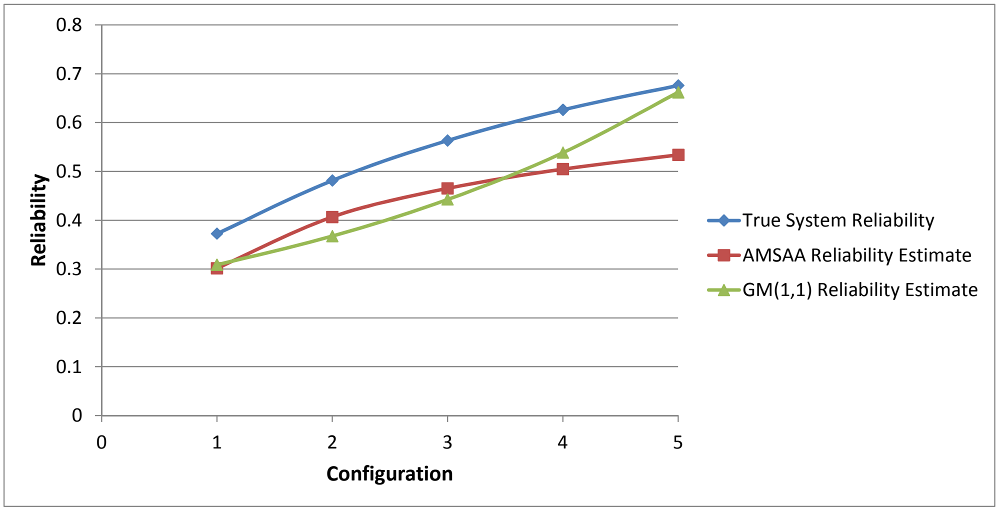

Figure 4.B.7: Reliability Across Configurations - 15 Failure Modes and 50% FEF

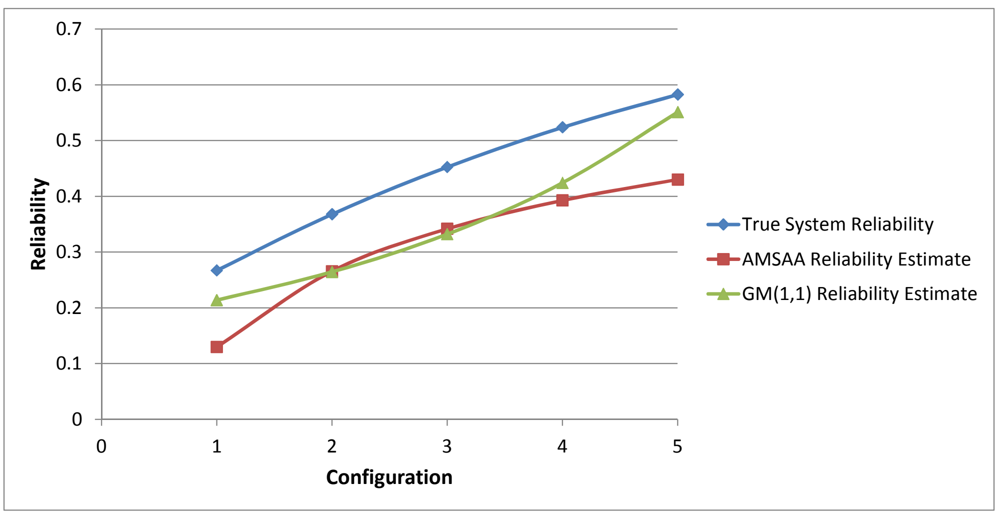

Figure 4.B.8: Reliability Across Configurations - 20 Failure Modes and 50% FEF

4.B.2 Assumed 60% FEF Growth Curves

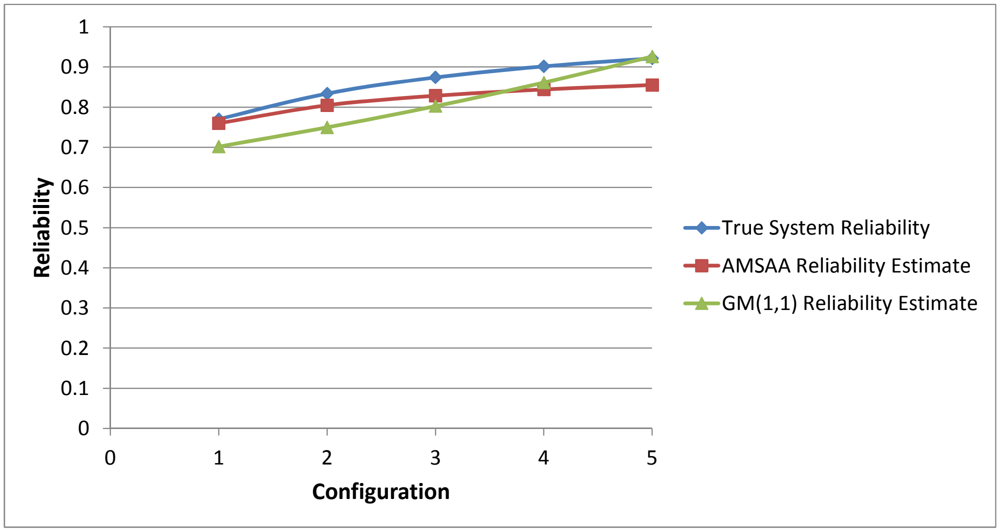

Figure 4.B.9: Reliability Across Configurations - 5 Failure Modes and 60% FEF

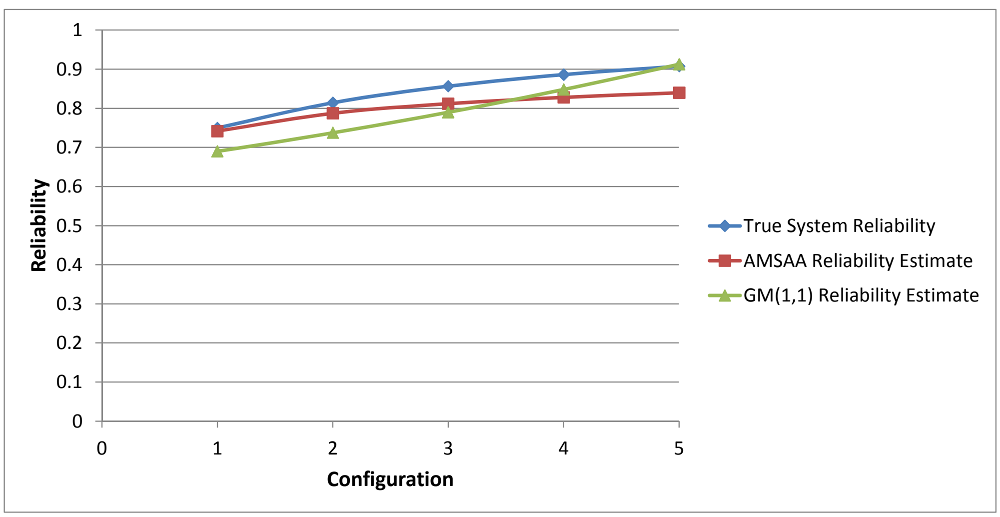

Figure 4.B.10: Reliability Across Configurations - 6 Failure Modes and 60% FEF

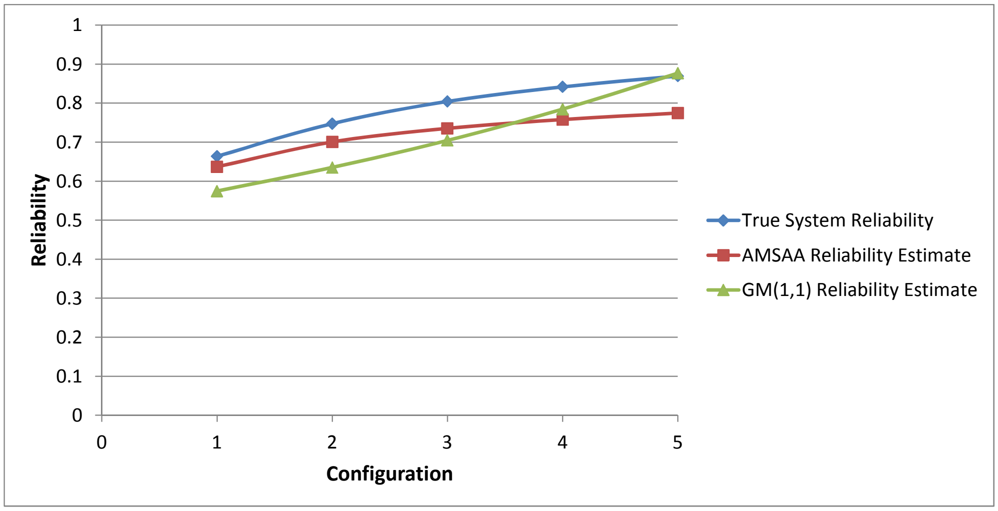

Figure 4.B.11: Reliability Across Configurations - 7 Failure Modes and 60% FEF

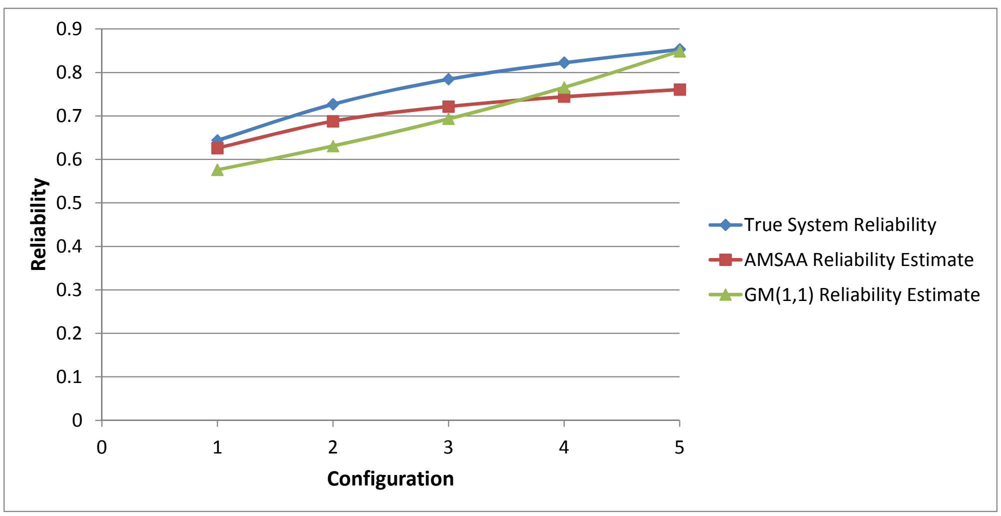

Figure 4.B.12: Reliability Across Configurations - 8 Failure Modes and 60% FEF

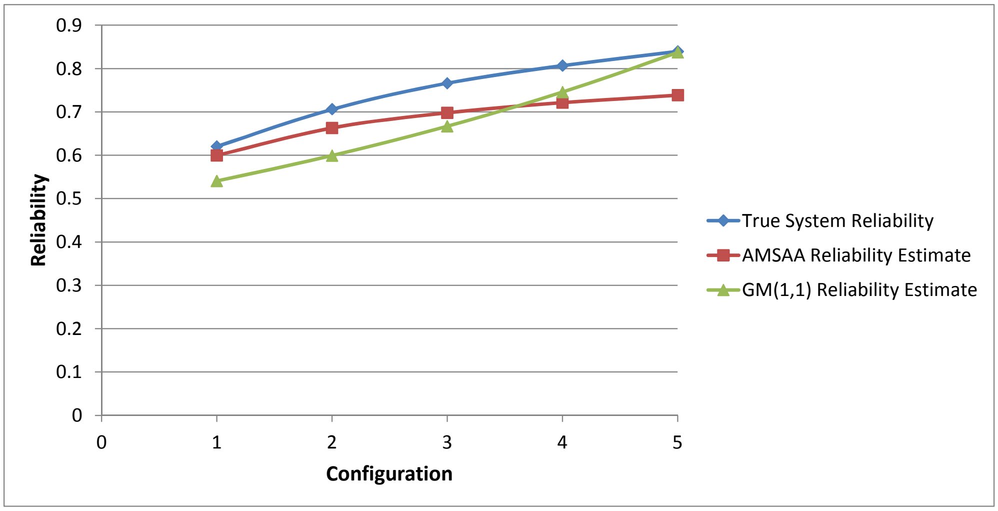

Figure 4.B.13: Reliability Across Configurations - 9 Failure Modes and 60% FEF

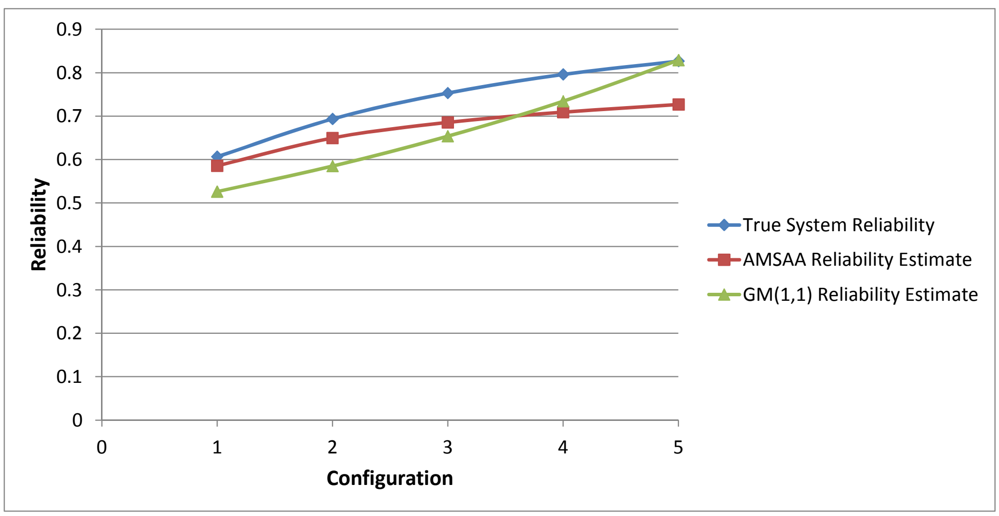

Figure 4.B.14: Reliability Across Configurations - 10 Failure Modes and 60% FEF

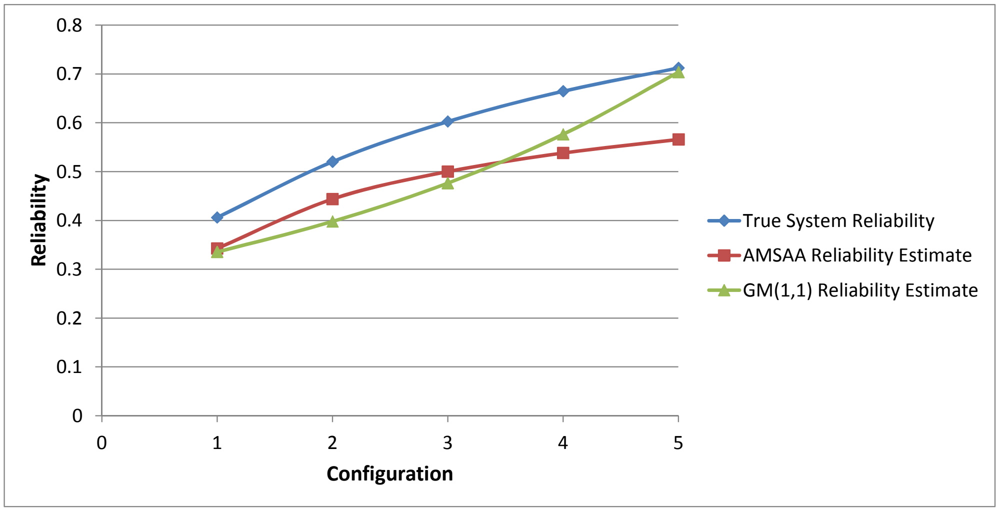

Figure 4.B.15: Reliability Across Configurations - 15 Failure Modes and 60% FEF

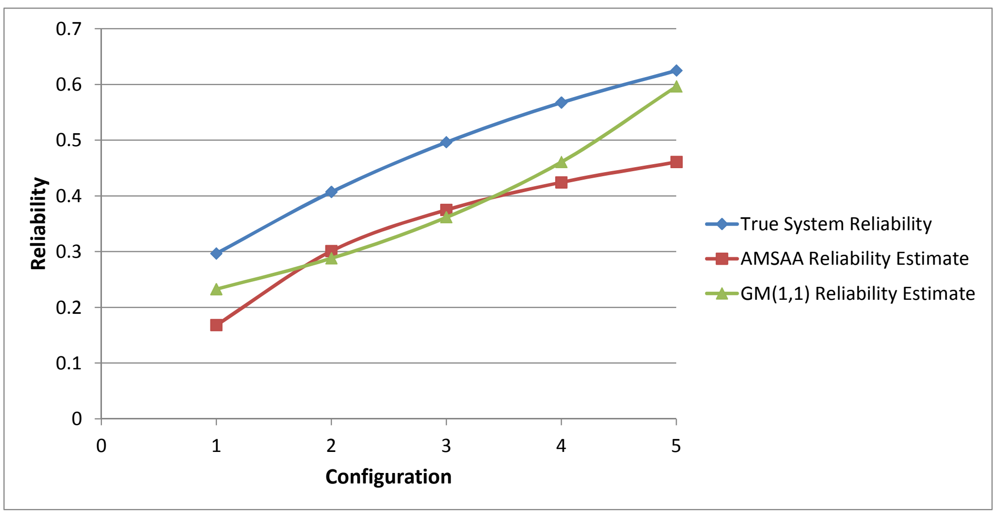

Figure 4.B.16: Reliability Across Configurations - 20 Failure Modes and 60% FEF

4.B.3 Assumed 70% FEF Growth Curves

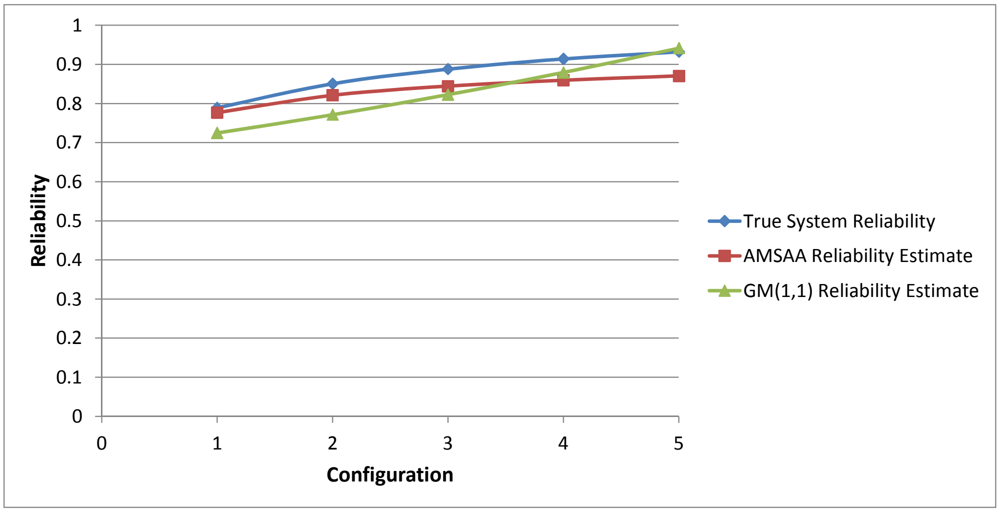

Figure 4.B.17: Reliability Across Configurations - 5 Failure Modes and 70% FEF

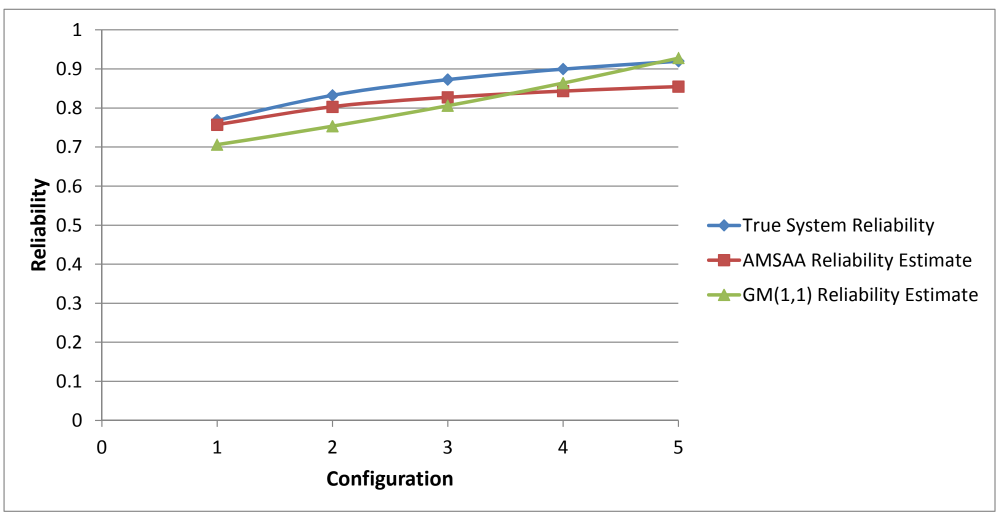

Figure 4.B.18: Reliability Across Configurations - 6 Failure Modes and 70% FEF

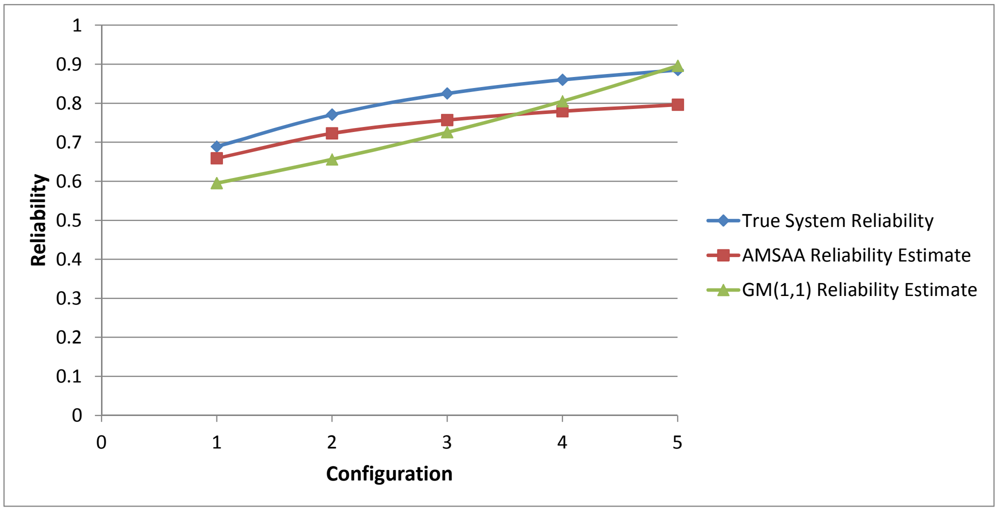

Figure 4.B.19: Reliability Across Configurations - 7 Failure Modes and 70% FEF

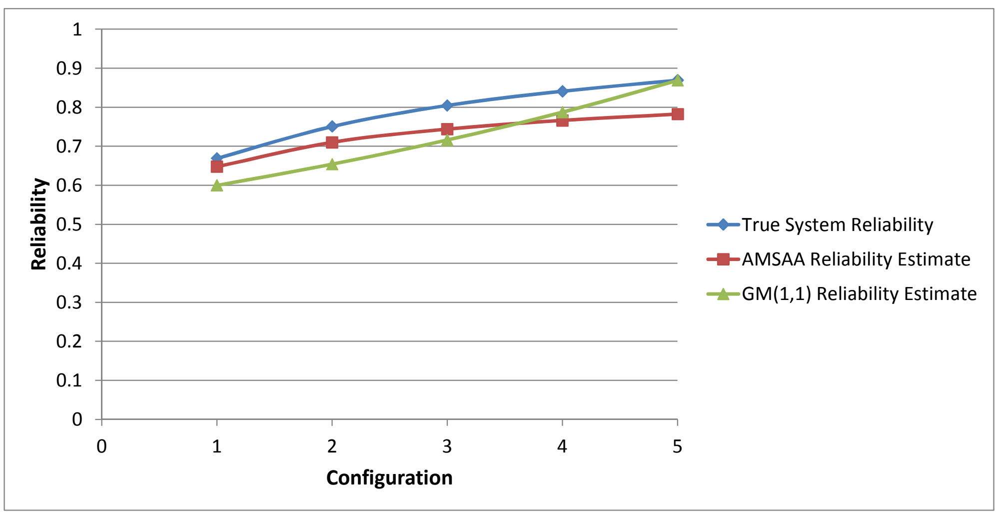

Figure 4.B.20: Reliability Across Configurations - 8 Failure Modes and 70% FEF

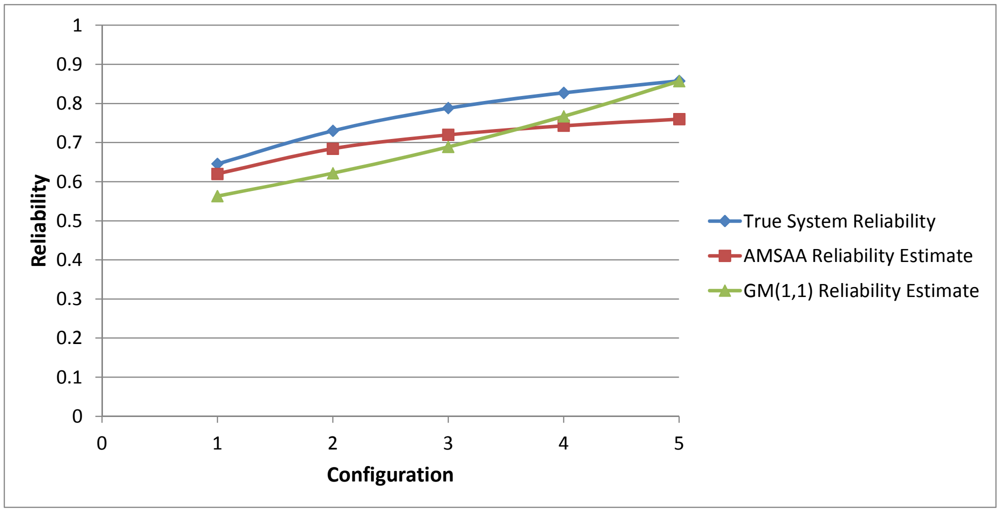

Figure 4.B.21: Reliability Across Configurations - 9 Failure Modes and 70% FEF

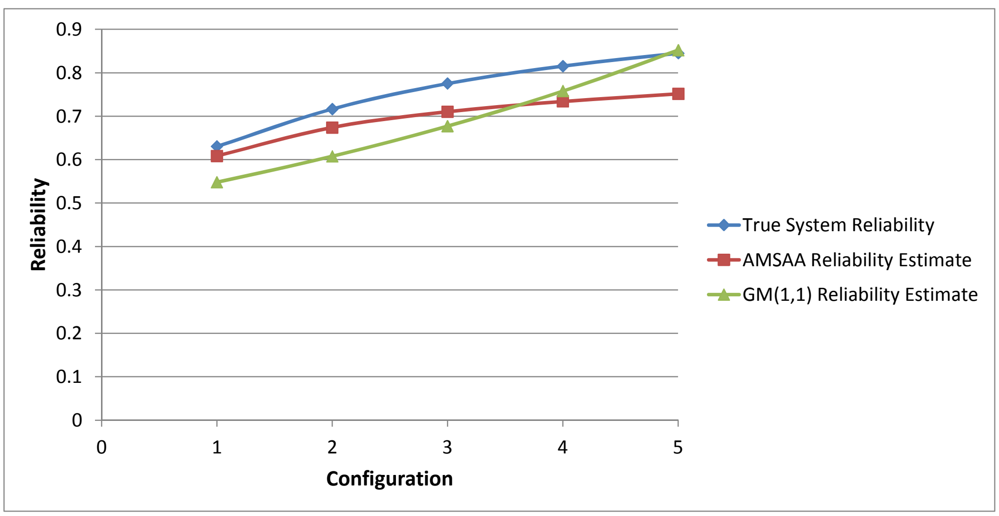

Figure 4.B.22: Reliability Across Configurations - 10 Failure Modes and 70% FEF

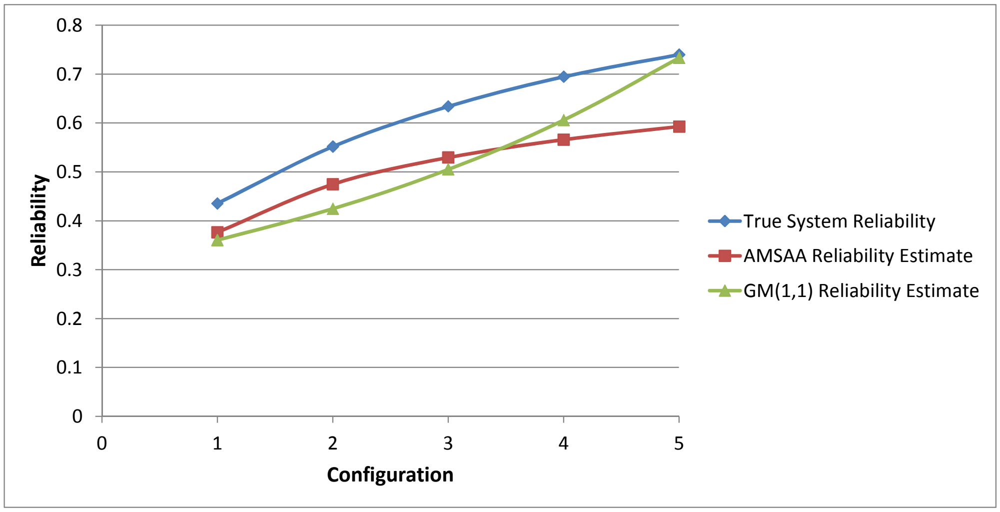

Figure 4.B.23: Reliability Across Configurations - 15 Failure Modes and 70% FEF

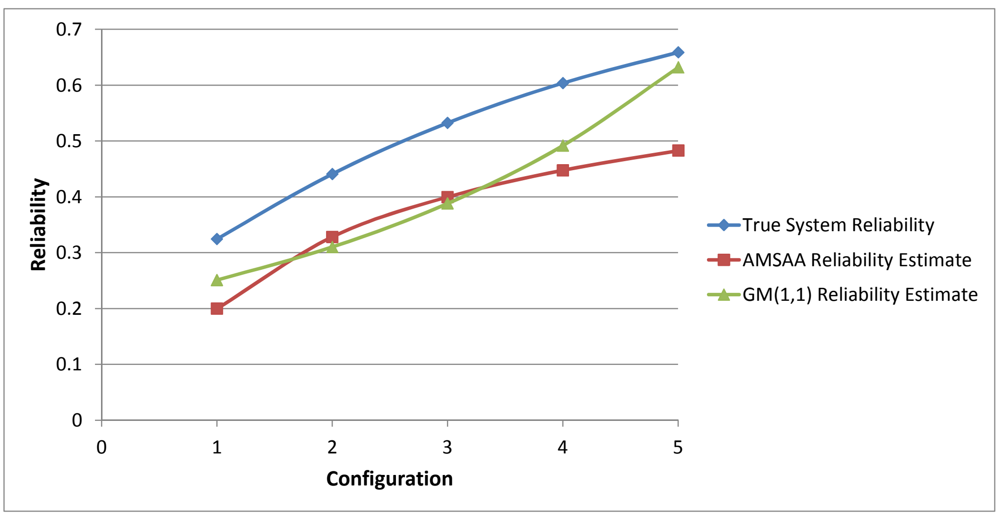

Figure 4.B.24: Reliability Across Configurations - 20 Failure Modes and 70% FEF

4.B.4 Assumed 80% FEF Growth Curves

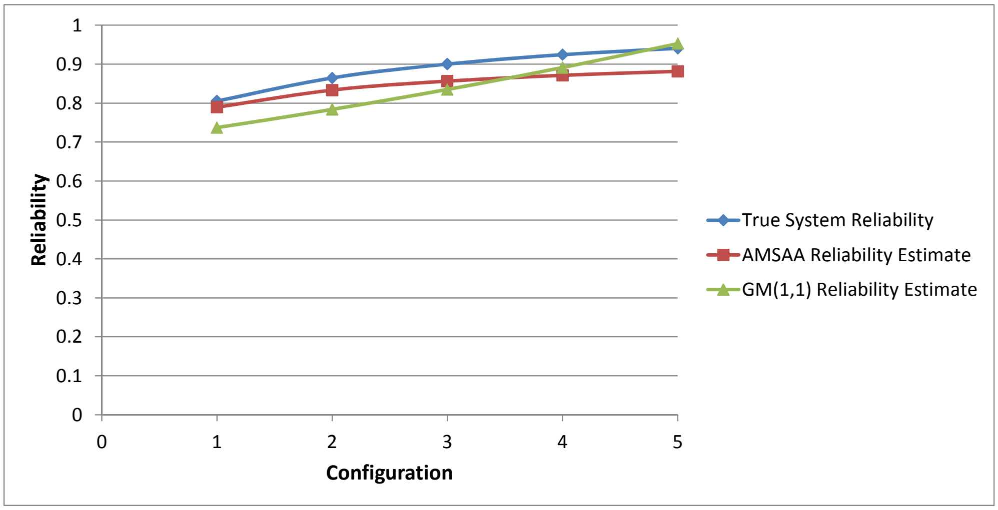

Figure 4.B.25: Reliability Across Configurations - 5 Failure Modes and 80% FEF

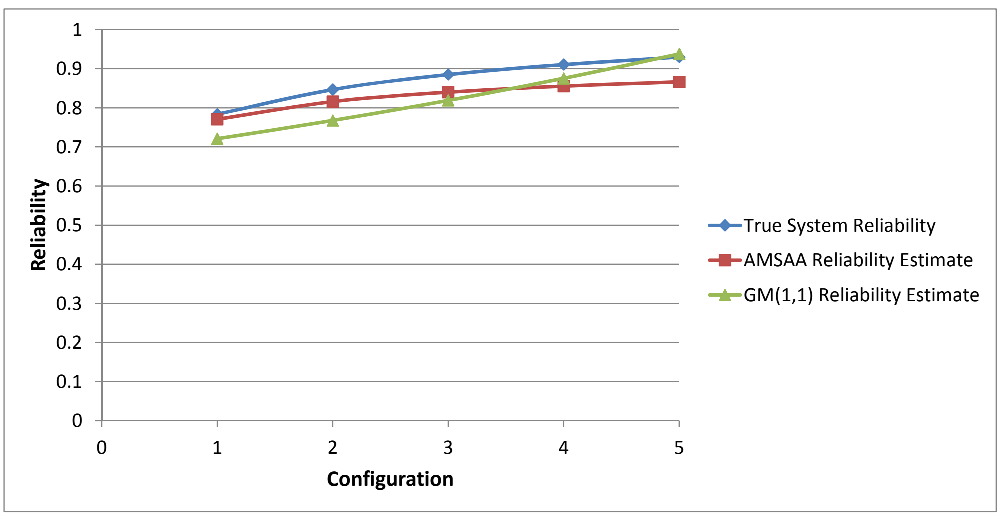

Figure 4.B.26: Reliability Across Configurations - 6 Failure Modes and 80% FEF

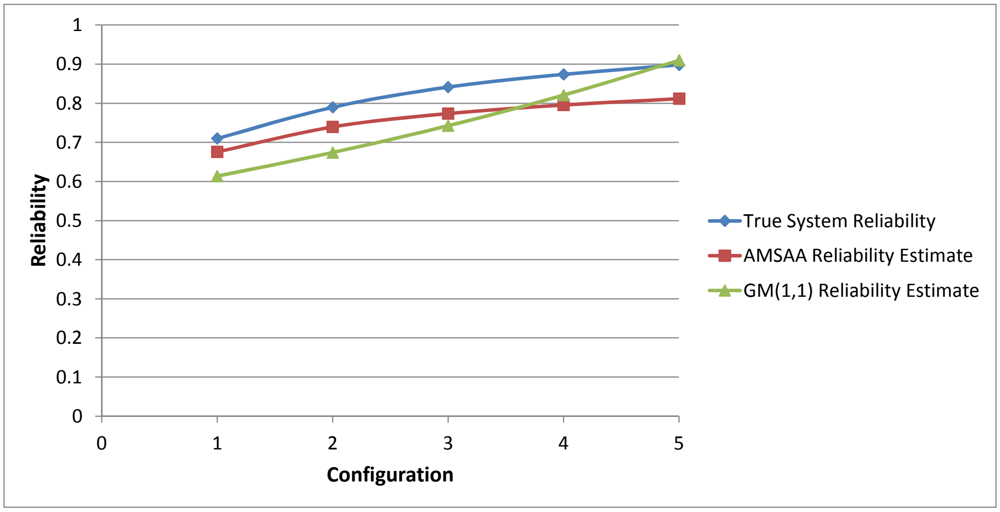

Figure 4.B.27: Reliability Across Configurations - 7 Failure Modes and 80% FEF

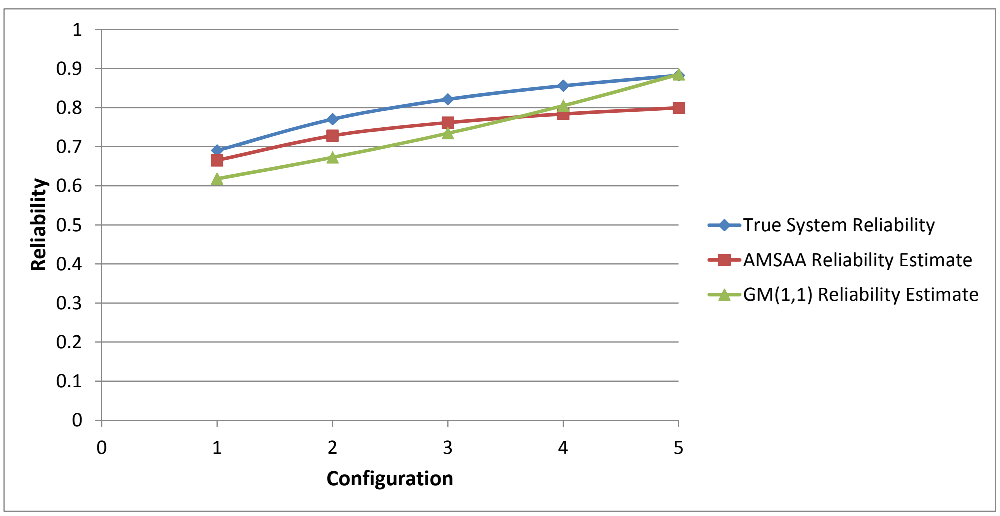

Figure 4.B.28: Reliability Across Configurations - 8 Failure Modes and 80% FEF

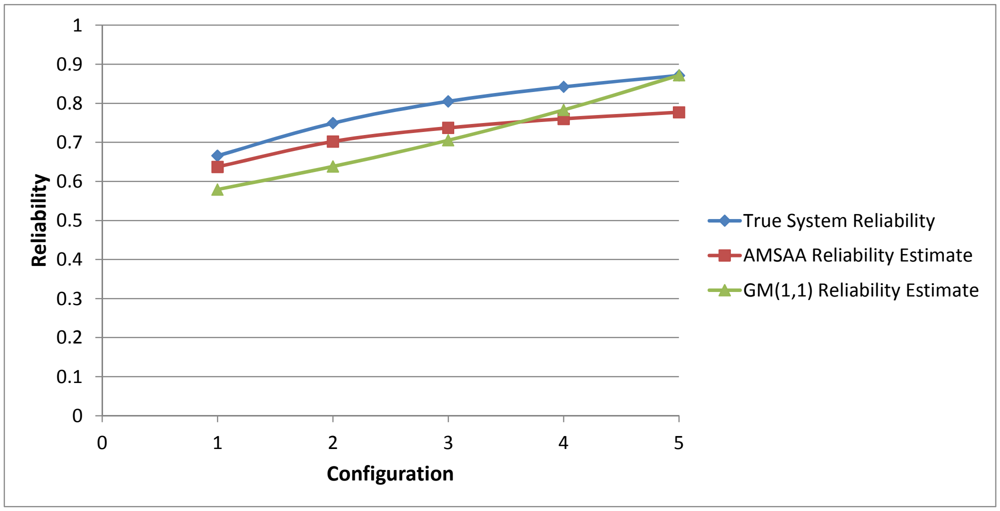

Figure 4.B.29: Reliability Across Configurations - 9 Failure Modes and 80% FEF

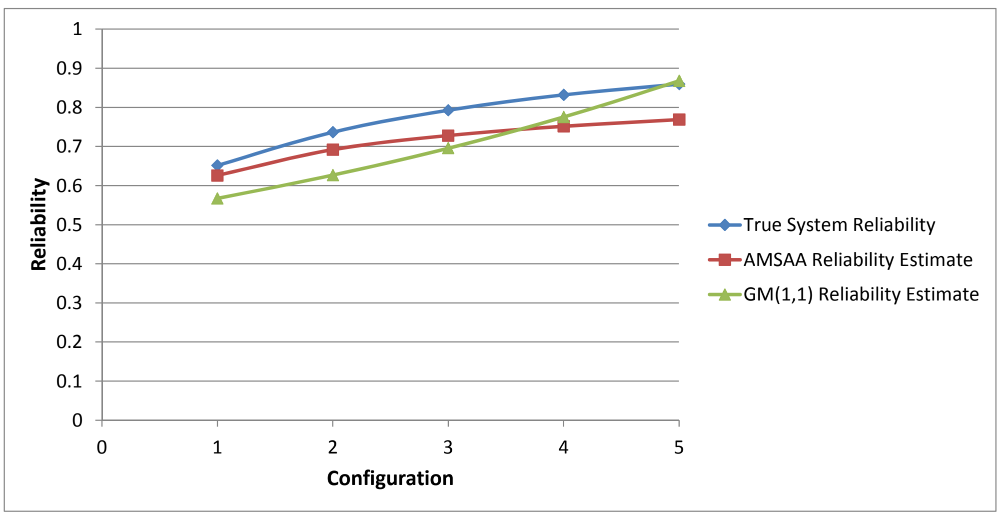

Figure 4.B.30: Reliability Across Configurations - 10 Failure Modes and 80% FEF

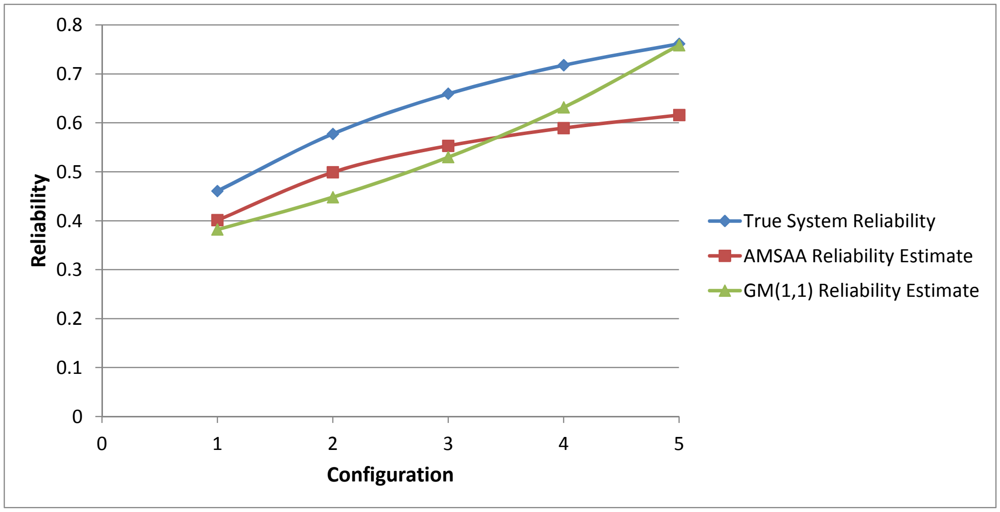

Figure 4.B.31: Reliability Across Configurations - 15 Failure Modes and 80% FEF

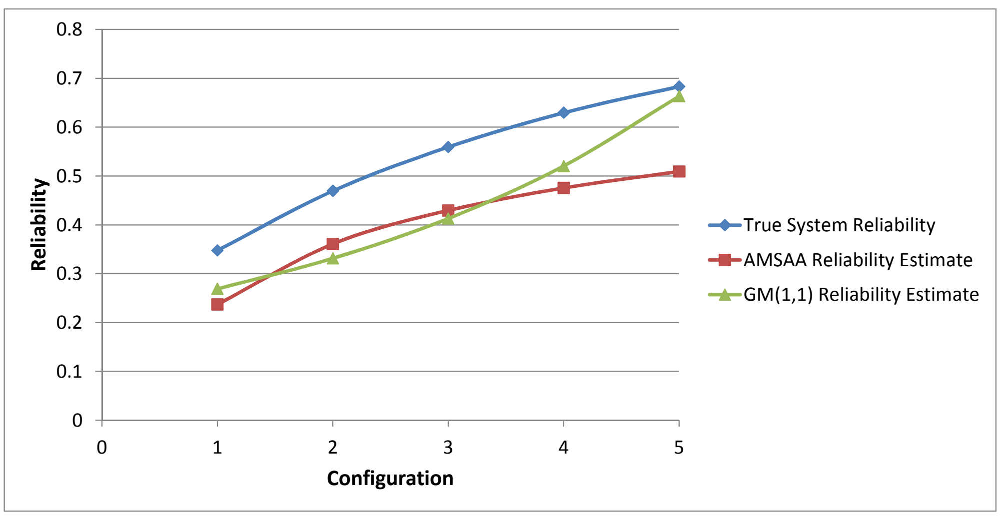

Figure 4.B.32: Reliability Across Configurations - 20 Failure Modes and 80% FEF

4.C Certification of Student Work

College of Engineering
Department of Industrial Engineering

MEMORANDUM

TO: Graduate School, University of Arkansas

FROM: Edward A. Pohl, Professor and Department Head

DATE: July 12, 2016

SUBJECT: Certification of Student Effort

I certify that greater than 51% of the work conducted for this chapter entitled “Small Sample Discrete Reliability Growth Modeling Using a Grey Systems Model” was conducted by Thomas P. Talafuse.

Sincerely,

Edward A. Pohl
ephol@uark.edu
479-575-6029
Professor and Department Head
Department of Industrial Engineering
University of Arkansas

5. A Markov Decision Process Approach for Optimizing Reliability Growth According to Reliability Growth Planning Curves

Thomas P. Talafuse   Edward A. Pohl   Shengfan Zhang

Abstract: This research effort develops methodologies to apply Markov Decision Process concepts to the field of reliability growth. In particular, we introduce a methodology for determining cost frontier policies that capture the level of corrective actions necessary to mitigate deviations from a given reliability growth planning curve. Identification of frontier policies derived from test data provides decision makers the ability to assess tradeoffs between reliability growth and corrective action expenditures. Furthermore, it provides the ability to forecast if a test program will fall short of the stated system reliability goal, as well as prevent over-designing of systems to minimize resource waste. Results demonstrate that this methodology is broadly applicable to test plan development for any system seeking to mature reliability through developmental testing.

5.1 Introduction

To mature the reliability of a complex system under development, it is important to formulate a detailed reliability growth plan. One aspect of this plan is a depiction of how the systems reliability is expected to increase over the developmental test period. The depicted growth path serves as a baseline against which reliability assessments can be compared. Baseline planning curves have frequently been developed by utilizing the assumed reliability growth pattern specified in Military Handbook 189 (MIL-HDBK-189) (Department of Defense, 2011), which employs the Crow-AMSAA planning model.

The purpose of the Crow-AMSAA planning model is to construct idealized system reliability growth curves, identify test time and growth rate required to improve system reliability, and aid in demonstrating the system reliability requirement as a point estimate. Through a series of Test-Analyze-and-Fix (TAAF) processes, system failure modes are deliberately sought out and improved/eliminated prior to full-rate production, thereby growing system reliability. Should reliability fail to grow at a rate consistent with planning curves, then additional time, resources, and testing efforts beyond those budgeted must be incorporated to meet the desired reliability objective, and is therefore undesirable. Reliability growth exceeding that which is planned may seemingly appear desirable, as it reduces the time needed to field a system, but could result in unnecessary testing, over-designing the system, and expenditure of resources that may not be needed to meet specified reliability levels within the anticipated time frame. Thus, it can be considered desirable to adhere to planned growth curves as closely as possible to ensure that a fielded system meets specified reliability requirements and a dependable system is available for the end-user without over-expenditure of resources.

Research has been done to update reliability growth planning curves based upon failure data obtained via testing (Crow, 2015). In this case, corrective action levels are assumed and are held constant, resulting in additional testing requirements should system reliability fall short of the desired level. While a beneficial approach, we propose an alternative methodology, holding the reliability growth planning curve constant and adjusting the level of corrective actions taken to improve failure modes as they are discovered. Assuming that a greater expenditure of effort and resources to reduce the effects of a discovered failure mode positively correlates with a greater Fix Effectiveness Factor (FEF), we employ Markov Decision Process (MDP) concepts to identify policies along a cost frontier for minimizing the deviation from a planned growth curve. System testing provides estimates of system failure intensity rates and an estimate of system reliability, which can be assessed with respect to the interim milestone levels of reliability identified from the idealized growth curve, allowing decisions to be made on the appropriate expenditure of resources for corrective actions to ensure system reliability is growing as planned. The probabilistic nature surrounding failure rate parameters makes this problem suitable for MDP methodologies, ensuring that optimal decisions are employed to adhere to the idealized growth plan as closely as possible and systems are more likely to meet requirements and fielded on schedule.

5.2 Background Literature

While there has been significant research done on constructing reliability growth planning curves and on tracking reliability growth, a dynamic approach focusing on the appropriate level of corrective action to take in order to adhere to a planning curve is a novel concept. Many reliability growth models have been developed to help decision makers in planning, tracking, and projecting the reliability improvement of a system. A highly detailed and comprehensive review of reliability growth planning models, tracking models, and projection models, for both continuous and discrete (one-shot) systems was recently provided by Hall (2008). His review comprises a synopsis of over 80 papers covering planning models, tracking and projection model, in addition to numerous reliability growth surveys/handbooks and 36 other papers covering theoretical results, simulation studies, real-world applications, personal perspectives, international standards, and related statistical procedures. In addition to Hall's review, we highlight pertinent literature on reliability growth planning models, along with recent work on planning considerations and modification of reliability growth planning curves.

5.2.1 MIL-HDBK-189 Planning Model

The MIL-HDBK-189 model (Department of Defense, 2011) provides decision makers with a reliability growth planning curve for developmental testing. The planning curve serves as a baseline for comparison of reliability assessments from failure data. The model is based on the Duane Postulate and consists of an idealized system reliability growth curve that portrays the profile for reliability growth throughout the developmental test period and has a constant MTBF during the initial test phase. The planning parameters that define the idealized growth curve include: (1) the initial MTBF, (2) length of the initial test phase (i.e., reliability demonstration test for the initial MTBF), (3) the final MTBF requirement, (4) the growth rate and (5) the duration of the entire growth program. The model also gives a set of expected MTBF steps during each test phase in the growth program. Corrective action periods are scheduled between each of the test phases where fixes are applied to previously observed failure modes. These improvements increase system reliability iteratively and result in an increasing sequence of MTBF steps.

5.2.2 AMSAA System-Level Planning Model

A variant of the MIL-HDBK 189 model, the system-level planning model (Ellner et al., 2000) can be used to construct system reliability growth test plans and associated idealized system reliability growth curves. The model can also prescribe the required test duration to achieve a desired point estimate for system reliability. This model provides several additional options beyond the MIL-HDBK 189 model for determining various planning parameters, which is convenient for conducting sensitivity analyses. Most often, the initial MTBF, final MTBF, growth rate, and length of the initial test phase are provided to determine the test duration required in a given development program.

5.2.3 Ellner-Hall PM2 Model

The PM2 model (Ellner and Hall, 2006) takes into consideration the lag-time due to implementation of corrective actions when constructing a reliability growth planning curve. Exact expressions are presented for the expected number of discovered failure modes and system failure intensity as functions of test time. Simulation results show that derived approximations can adequately represent the expected reliability growth for a variety of distributions for the system's initial failure rate. The main difference of this model compared to other planning models is that it is independent of the NHPP assumption and utilizes parameters directly influenced by decision makers, such as: 1) initial MTBF; 2) management strategy; 3) goal MTBF; 4) average lag-time associated with fix implementation; 5) total test time; 6) average FEF; 7) the number and placement of corrective actions, and 8) the planned test hours.

5.2.4 Other Pertinent Literature

Crow (2011) discusses how significant patterns and key parameters are used to provide a basis for general guidelines used to establish a realistic reliability growth testing program. If goals are unrealistic and set too high, assessments of progress may incorrectly indicate the program will be unsuccessful. Likewise, if the goals are set too low, problems and issues may not be uncovered in a timely manner. Crow uses historical information to assist in developing a growth plan and to evaluate the realism of a proposed reliability growth test program, enabling programs to set realistic interim reliability goals to be attained during testing to indicate that sufficient progress is being made in order to reach the final goal or requirement.

Crow (2015) also introduced a methodology to progressively update the planned reliability growth curve across future test phases based on actual test data and give projections for the expected reliability over multiple future test phases. Parameter values derived during testing are often different than the values input into the initial planning model, leading to differences between a system's reliability milestone targets on the reliability growth curve and the value assessed from testing data. The methodology proposed can be used to provide confidence that a program is on track.

Zheng (2002) discusses the optimal release time for computer software, where a conditional non-homogeneous Poisson process model is used to describe the software reliability growth behavior. A Markov decision programming formulation is used to determine a threshold-type optimal release policy, with the objective of minimizing the total discounted cost, subject to a constraint on system reliability.

5.3 Methodology

We design our MDP model with the intent of minimizing the deviation of a system's demonstrated MTBF from that of an idealized reliability growth curve. Under this construct, we make the following assumptions on the system's behavior:

- Each system is comprised of j competing failure modes whose respective fail times follow a constant failure rate (CFR).
- Failure modes are considered to be independent and in series. Thus, the system fails upon failure of any failure mode.
- Testing is conducted over k time-terminated test stages.
- Each test stage has a duration of time T.
- In each stage of testing, there are N identical systems concurrently undergoing testing.
- Upon system failure, the causal failure mode is known with certainty. The time needed for root cause analysis and corrective action is negligible compared to testing durations.
- Corrective actions reduce the hazard rate of the discovered failure mode with a known FEF that is proportional to the resources allocated (i.e. greater investment in corrective action results in greater improvement).
- Corrective actions are completed prior to the start of the next stage of testing, as described in the MIL-HDBK 189 planning model, and are always performed on the observed failure modes.
- A realistic growth plan has been established using any one of the planning models found in the reliability growth planning literature.

5.3.1 Model Formulation

The general structure of the problem is to conduct testing, derive a belief vector for system reliability from the failure data, implement a specified level of corrective action and calculate its respective expected reward, and proceed to the next stage of testing. We consider each decision epoch, t, to correspond to a stage of testing, with decisions being made following completion of a stage of testing.

To characterize the state space for decision epoch, t, we consider n discretized levels of a percent deviation from the desired MTBF, β_t. As such, the exact bounds for each state are dependent on β_t, with state $s_t(1)$ having a lower bound of zero for all decision epochs.

The action space, a_t consists of a finite set of FEF values corresponding to the level of effort invested into corrective action, and its respective cost, c_t. A decision of no corrective action has a cost of zero, while a decision of the maximum level of corrective action has a cost of one. The levels of FEF and their respective costs are constant across all decision epochs.

Since we assume failures for each failure mode follow a CFR, the system also fails according to a CFR. Then, for each stage of testing, a point estimate $\hat{\beta}$ can be derived from the observed failure data using maximum likelihood estimation, with a likelihood equation of

$$\mathcal{L}_N(\hat{\beta}) = \sum_{i=1}^{N} (\delta_i(-\log\hat{\beta} - \hat{\beta}^{-1}Y_i) - (1-\delta_i)\hat{\beta}^{-1}Y_i), \tag{5.1}$$

where Y_i is the corresponding failure time, with $Y_i = T$ for those systems that did not fail during testing, $\delta_i = 1$ if the observation is censored, and $\delta_i = 0$ if the exact failure time is observed. Since there is uncertainty surrounding $\hat{\beta}$, we derive a belief vector π_t on its true value. Under the assumption of CFR, $\beta \sim \chi^2$, we calculate the belief that the true MTBF is in state $s \in s_t$ via:

$$\begin{aligned}
\Pr(\beta \in s_t(i)) &= 1 - \chi^2\left(\frac{2N\hat{\beta}}{s_L(i+1)}, 2N\right), \quad i = 1, \\
\Pr(\beta \in s_t(i)) &= 1 - \sum_{j=1}^{i-1}\chi^2\left(\frac{2N\hat{\beta}}{s_L(j+1)}, 2N\right), \quad i = 2, \dots, n,
\end{aligned} \tag{5.2}$$

where $s_L(i+1)$ is the lower bound on the MTBF for state $s_t(i+1)$.

Transition probabilities are non-stationary, due to the nature of the state space and are defined as:

$$\Pr(s_{t+1}|\beta \in s_t, a_t) = \int_{s_L}^{s_U} f(t)dt = \int_{s_L}^{s_U} \lambda_{k+1}e^{t\lambda_{k+1}}dt, \tag{5.3}$$

where s_L and s_U are the lower and upper bounds on β for each state $s \in s_{t+1}$, respectively. Since there is a belief vector associated with the true value of β and not a point estimate, we consider the midpoint for each state s_t when deriving the transition probabilities. For purposes of deriving the midpoint for state $s_t(n)$, we consider $S_U(n) = 2 \times \beta_t$.

Rewards associated with each action are based upon the expected deviation from the desired MTBF in the next stage, β_{t+1}, after implementation of the corrective action. Rewards can be given a weight, w_t, for each decision epoch, to allow increasing weight to be given to later stages of testing. Each state can also be assigned weight w_d, to capture the reliability metric's value compared to the desired MTBF, allowing greater weight to be assigned to deviations falling below the desired MTBF than those surpassing it, giving us a reward structure of:

$$R_t(a_t) = \pi_t \times w_t \times \Big(\Pr(s_{t+1} | \beta \in s_t, a_t) \times w_d \times |E(\hat{\beta}_{t+1} | a_t) - \beta_{t+1}|\Big), \ \ \forall d \in s_t. \tag{5.4}$$

Under this construct, it is possible to evaluate a policy's impact on cost and deviation from the desired MTBF, providing decision makers with a structured methodology for determining an appropriate level of corrective action to take on identified failure modes, as well as producing a system that meets reliability requirements without being over-designed.

5.4 Numerical Experimentation and Results

Modeling corrective action decisions via MDP was explored to determine if it provided reliability growth that followed a desired growth curve effectively. For this effort, failure data for a notional system were generated, with reliability estimated at each decision epoch and compared to a notional idealized growth curve. Using enumerative policy evaluation, a cost frontier was generated, identifying those policies that produced minimal deviation from the idealized growth curve.

We consider a system undergoing $k = 5$ stages of testing and improvements. Initial MTBF is estimated to be 100, with each stage of testing having a duration of $T = 1000$, and a desired MTBF of 1028.39 upon completion of the fifth stage of testing and corrective action. The idealized growth curve with interim MTBF goals can be seen in Figure 5.1.

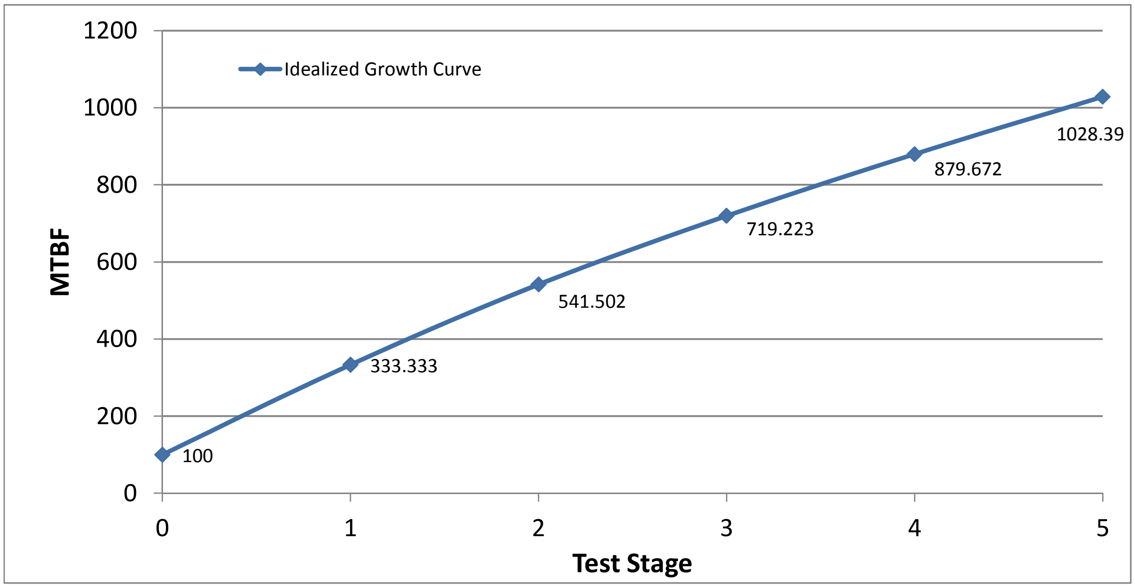

Figure 5.1: Notional Idealized Growth Curve

We discretize the state space into nine states for each decision epoch, defining $s_t = \{0, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.3\}$ as the lower bound percentage of the desired MTBF for each state. Six levels of corrective action are considered, with $a_t = \{0, 0.3, 0.5, 0.6, 0.7, 0.8\}$ representing the FEF, and respective costs for each action are $c_t = \{0, 0.2, 0.35, 0.5, 0.8, 1.0\}$. Costs do not have a linear relationship with corrective action levels and were notionally generated to follow an S-curve to reflect a marginally increasing rate of return at lower costs, and a marginally decreasing rate of return at higher costs. With five decision epochs and six possible actions for each decision epoch, there is a total of $6^5 = 7,776$ possible policies, each of which was evaluated and replicated 100 times to identify frontier policies.

The combinations of w_t and w_d investigate different combinations of weighting the decision epochs, and weighting the deviations above and below the desired MTBF level for each decision epoch, respectively and can be found in Table 5.1.

Table 5.1: Weights for Decision Epoch and Deviation From Goal MTBF

Instance	w_t	w_d
1	$1.0 \; \forall \; t$	$1 \; \forall \; s_t(d)$
2	$1.0 \; \forall \; t$	$1 \; \forall s_t(d) < \beta_t, 0.5 \; \forall \; s_t(d) > \beta_t$
3	$1.0 \; \forall \; t$	$1 \; \forall s_t(d) < \beta_t, 0.1 \; \forall \; s_t(d) > \beta_t$
4	$\{0.333, 0.5, 0.667, 0.833, 1.0\}$	$1 \; \forall \; s_t(d)$
5	$\{0.333, 0.5, 0.667, 0.833, 1.0\}$	$1 \; \forall s_t(d) < \beta_t, 0.5 \; \forall \; s_t(d) > \beta_t$
6	$\{0.333, 0.5, 0.667, 0.833, 1.0\}$	$1 \; \forall s_t(d) < \beta_t, 0.1 \; \forall \; s_t(d) > \beta_t$
7	$\{0, 0, 0, 0, 1\}$	$1 \; \forall \; s_t(d)$
8	$\{0, 0, 0, 0, 1\}$	$1 \; \forall s_t(d) < \beta_t, 0.5 \; \forall \; s_t(d) > \beta_t$
9	$\{0, 0, 0, 0, 1\}$	$1 \; \forall s_t(d) < \beta_t, 0.1 \; \forall \; s_t(d) > \beta_t$

In order to investigate the effects of the accuracy of the initial MTBF estimate of 100 on frontier policies, each instance was simulated with MTBF values that were lower, on par, and above the initial MTBF estimate. Table 5.2 lists the true system MTBF values at the start of testing.

Table 5.2: True System MTBF at Start of Testing Efforts

True System MTBF	
1	25
2	50
3	105
4	205

We illustrate the effectiveness of the MDP approach with the results from instance 5, where a decision maker is increasingly concerned with deviations from the desired reliability as testing progresses and moderately favors system over-development over a shortfall. Furthermore, to capture the idea that this MDP approach is capable of producing smaller deviations from the desired growth at a lesser cost than a constant FEF, the policies with a constant FEF are also plotted in the graph. The full tables of frontier policies and their corresponding graphs for all instances can be found in Appendix 5.A.

Table 5.3: Frontier Policies: $\beta = 25$

Reward	Cost	Policy				
2337.8653	0	1	1	1	1	1
2271.8899	0.2	1	1	1	1	2

Table 5.3 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
2227.9063	0.35	1	1	1	1	3
2205.9145	0.5	1	1	1	1	4
2205.9143	0.7	2	1	1	1	4
2183.9227	0.8	1	1	1	1	5
2161.9309	1	1	1	1	1	6
2161.9307	1.2	2	1	1	1	6
2161.9301	1.35	3	1	1	1	6
2161.9295	1.5	4	1	1	1	6
2161.9291	1.7	4	2	1	1	6
2161.9285	1.75	3	2	2	1	6
2161.9281	1.85	4	3	1	1	6
2161.9247	1.9	3	3	2	1	6
2161.9239	1.95	3	2	2	2	6
2161.9126	2.05	3	3	3	1	6
2161.9092	2.1	3	3	2	2	6
2161.8996	2.2	4	3	3	1	6
2161.8626	2.25	3	3	3	2	6
2161.7418	2.4	3	3	3	3	6
2161.6177	2.55	4	3	3	3	6
2161.4176	2.7	4	4	3	3	6
2161.0984	2.85	4	4	4	3	6
2160.6263	3	4	4	4	4	6
2159.9120	3.3	5	4	4	4	6
2158.9104	3.5	6	4	4	4	6
2158.8393	3.6	5	5	4	4	6
2157.3536	3.8	6	5	4	4	6
2157.2470	3.9	5	5	5	4	6
2155.2211	4	6	6	4	4	6
2155.0643	4.1	6	5	5	4	6
2155.0266	4.2	5	5	5	5	6

Table 5.3 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
2151.9810	4.3	6	6	5	4	6
2151.9319	4.4	6	5	5	5	6
2147.6661	4.5	6	6	6	4	6
2147.6046	4.6	6	6	5	5	6
2141.6382	4.8	6	6	6	5	6
2133.9013	5	6	6	6	6	6

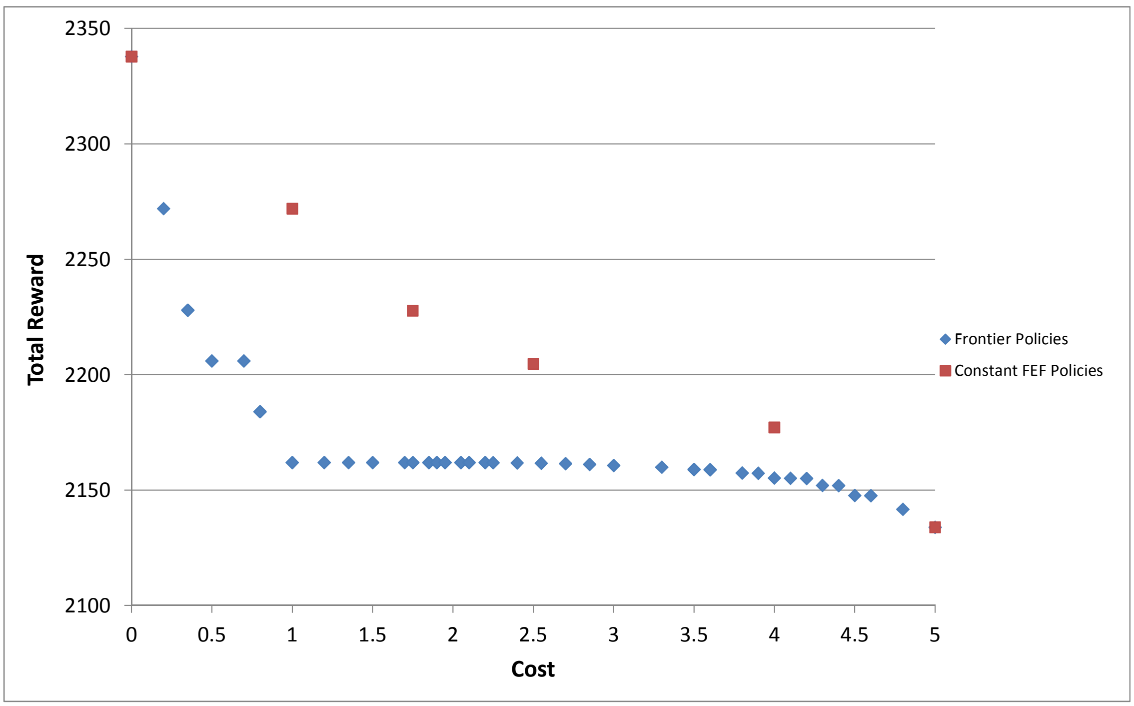

Figure 5.2: Constant FEF Policies and Policy Frontier: $\beta = 25$

Table 5.4: Frontier Policies: $\beta = 50$

Reward	Cost	Policy				
2328.9275	0	1	1	1	1	1
2262.9520	0.2	1	1	1	1	2
2218.9684	0.35	1	1	1	1	3
2196.9766	0.5	1	1	1	1	4
2196.9185	0.7	2	1	1	1	4
2174.9848	0.8	1	1	1	1	5

Table 5.4 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
2152.9930	1	1	1	1	1	6
2152.9349	1.2	2	1	1	1	6
2152.8124	1.35	3	1	1	1	6
2152.7041	1.5	4	1	1	1	6
2152.5625	1.7	3	3	1	1	6
2152.3531	1.75	3	2	2	1	6
2152.2950	1.85	4	3	1	1	6
2151.4686	1.9	3	3	2	1	6
2151.2294	1.95	3	2	2	2	6
2149.2702	2.05	3	3	3	1	6
2148.4349	2.1	3	3	2	2	6
2147.2335	2.2	4	3	3	1	6
2141.6060	2.25	3	3	3	2	6
2128.4810	2.35	4	4	4	3	4
2105.1483	2.5	4	4	4	4	4
2098.8893	2.65	5	4	4	4	3
2098.8377	2.7	6	4	4	3	3
2074.2225	2.8	5	4	4	4	4
2063.1919	2.85	6	4	4	4	3
2061.0310	2.95	5	5	4	4	3
2037.9547	3	6	4	4	4	4
2035.7590	3.1	5	5	4	4	4
2017.2551	3.15	6	5	4	4	3
2016.6661	3.2	6	6	4	3	3
2014.8825	3.25	5	5	5	4	3
1991.4017	3.3	6	5	4	4	4
1965.5649	3.35	6	6	4	4	3
1962.6803	3.45	6	5	5	4	3
1939.1774	3.5	6	6	4	4	4
1936.2650	3.6	6	5	5	4	4

Table 5.4 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
1884.1401	3.65	6	6	5	4	3
1878.7290	3.7	6	6	6	4	2
1857.3225	3.8	6	6	5	4	4
1817.8770	3.85	6	6	6	4	3
1790.8867	4	6	6	6	4	4
1749.2372	4.15	6	6	6	5	3
1743.3200	4.2	6	6	6	6	2
1722.4460	4.3	6	6	6	5	4
1680.2655	4.35	6	6	6	6	3
1654.1634	4.5	6	6	6	6	4
1632.9587	4.8	6	6	6	6	5
1611.9322	5	6	6	6	6	6

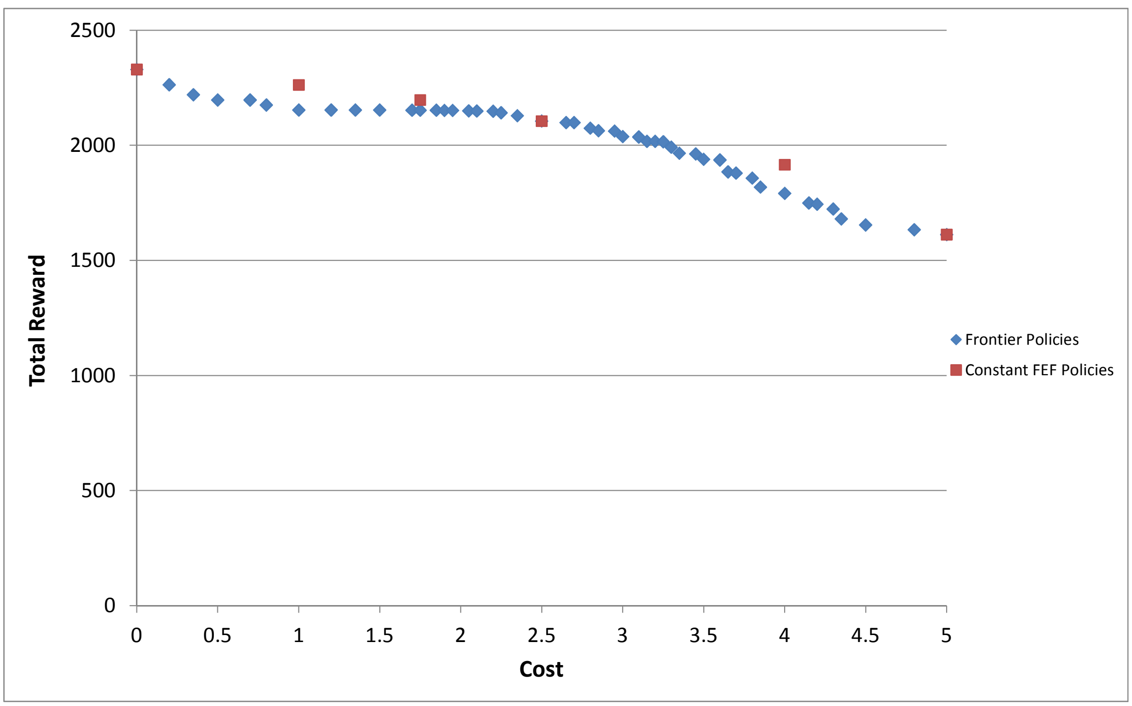

Figure 5.3: Constant FEF Policies and Policy Frontier: $\beta = 50$

Table 5.5: Frontier Policies: $\beta = 105$

Reward	Cost	Policy				
2318.3282	0	1	1	1	1	1
2252.3486	0.2	1	1	1	1	2
2208.3622	0.35	1	1	1	1	3
2186.3692	0.5	1	1	1	1	4
2180.4073	0.7	2	1	1	1	4
2164.3762	0.8	1	1	1	1	5
2142.3833	1	1	1	1	1	6
2111.7473	1.05	3	3	3	1	1
2070.1541	1.1	3	3	2	2	1
2058.4056	1.2	4	3	3	1	1
1931.8182	1.25	3	3	3	2	1
1781.5834	1.4	3	3	3	3	1
1555.5744	1.55	4	3	3	3	1
1339.8700	1.7	4	4	3	3	1
1282.2412	1.85	4	4	4	3	1
1225.2108	1.9	4	4	3	3	2
1132.9605	2	4	4	4	4	1
1108.2523	2.15	5	4	4	3	1
1074.0065	2.2	4	4	4	4	2
1048.6155	2.35	5	4	4	3	2
1041.0673	2.5	5	4	4	4	2
1014.7188	2.55	6	4	4	3	2
1007.6559	2.7	6	5	3	3	2
1006.0715	2.8	6	5	4	4	1
981.6667	2.85	6	6	4	3	1
965.7721	2.9	6	6	4	2	2
936.4994	3	6	6	4	4	1
918.7939	3.15	6	6	5	3	1
918.4003	3.3	6	6	5	4	1
901.7039	3.35	6	6	6	3	1

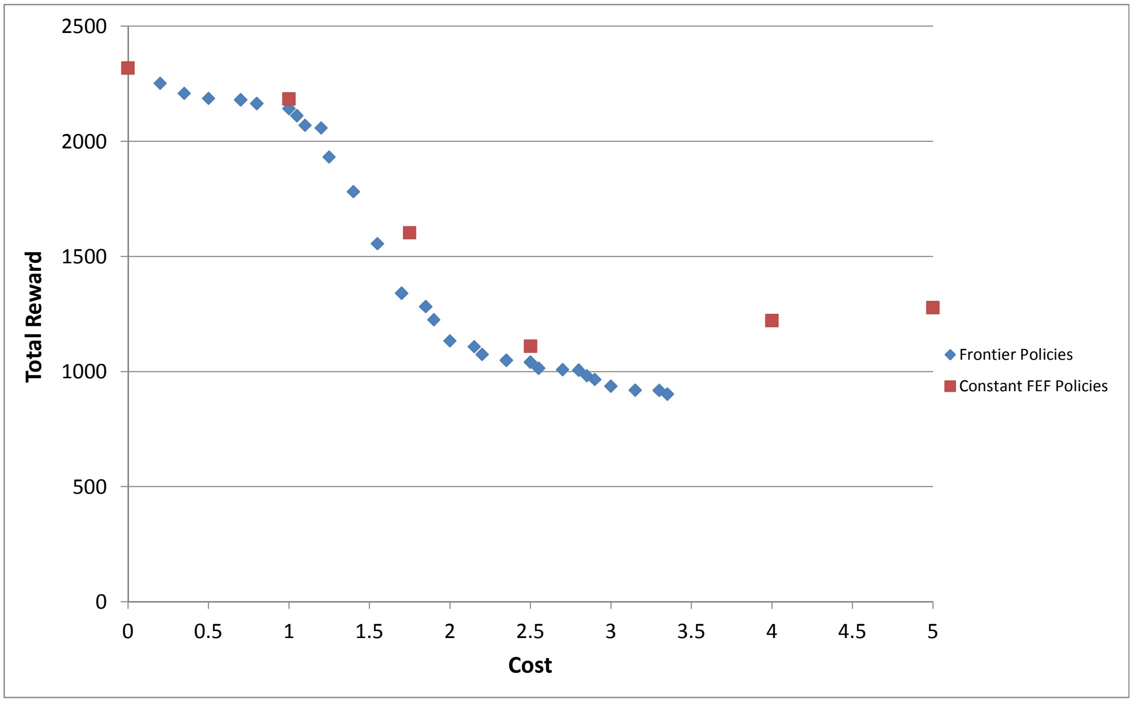

Figure 5.4: Constant FEF Policies and Policy Frontier: $\beta = 105$

Table 5.6: Frontier Policies: $\beta = 205$

Reward	Cost	Policy				
2275.7787	0	1	1	1	1	1
2196.1387	0.2	2	1	1	1	1
2102.3432	0.35	3	1	1	1	1
2038.6166	0.4	2	2	1	1	1
1888.7813	0.55	3	2	1	1	1
1810.2995	0.6	2	2	2	1	1
1682.8013	0.7	3	3	1	1	1
1581.6180	0.75	3	2	2	1	1
1403.2485	0.8	2	2	2	2	1
1062.2164	0.9	3	3	2	1	1
823.7092	1.05	3	3	3	1	1
773.0630	1.2	4	3	3	1	1
733.2781	1.25	3	3	3	2	1
707.0833	1.35	4	4	3	1	1
701.6087	1.4	4	3	3	2	1

Table 5.6 Frontier Policies: $\beta = 205$ (Cont.)

Reward	Cost	Policy				
637.5921	1.5	4	4	4	1	1
624.8224	1.65	5	4	3	1	1
623.4904	1.8	5	4	4	1	1
612.7617	1.85	6	4	3	1	1
599.9045	2.2	6	6	2	1	1

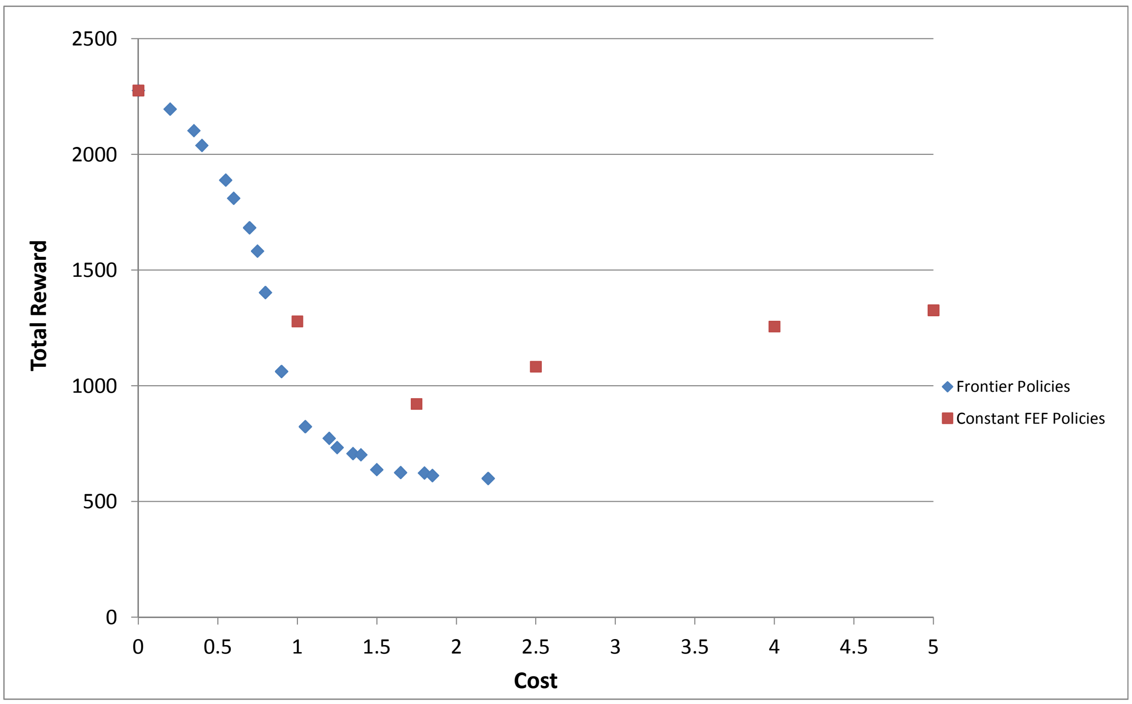

Figure 5.5: Constant FEF Policies and Policy Frontier: $\beta = 205$

As can be seen from figures 5.2-5.5, the policies along the cost frontier are capable of minimizing deviation from the desired MTBF for the specified cost. This dynamic approach for choosing levels of corrective action provide policies that dominate the deviation from any of the assumed constant FEF policies, and holds true for all other instances. Decision makers can then assess the risk of the test program meeting its stated objectives, make corrective action decisions accordingly, and determine if additional resources are needed beyond the current test plan. Additionally, for a constrained budget, the cost frontier curves identify policies that minimize deviation from the desired growth curve, while also providing justification for funding levels. We see in Figures 5.2 and 5.3 that when true system reliability is initially overestimated, a more aggressive level of corrective action at all stages is necessary to minimize the deviation from the planned growth curve. Likewise, when system reliability is initially underestimated, as seen in Figure 5.5, it is still beneficial to aggressively fix the failure modes that are discovered early in the test program, but to avoid system over-development, little or no corrective action is necessary in later stages of testing.

5.5 Conclusions and Future Work

In this effort, we proposed use of Markov Decision Process concepts to develop cost frontier policies that capture the levels of corrective actions necessary to mitigate deviations from a reliability growth planning curve. Identification of these frontier policies provides decision makers the ability to assess tradeoffs between reliability growth and corrective action expenditures. With the ability to weight rewards by each stage of testing, and by an excess or shortfall from the desired reliability metric values, this process can be utilized to support the preferences of any decision maker and can make recommendations on how aggressively corrective actions should be pursued. Furthermore, it provides the ability to forecast if a test program will fall short of the stated system reliability goal, as well as prevent over-designing of systems to minimize resource waste, and adhere to the idealized growth plan as closely as possible to meet test program requirements.

This effort also highlights the importance of developing a realistic reliability growth planning curve. A planning curve that assumes a growth rate smaller than what is achievable will result in an over-allocation of testing resources, while a planning curve that is too aggressive in its assumed level of reliability growth will fall short of achieving the desired level of demonstrated reliability, requiring additional time and resources than initially planned. An accurate initial estimate of a system's reliability is also important, but our MDP approach illustrates that a more conservative estimate of the system's initial reliability is preferred, as it is possible to not perform a corrective action if the demonstrated reliability exceeds the desired level.

Discretization of the state space with more states provides better estimates for the true deviation from the desired level of reliability, but comes at a computational cost. Since enumerative policy evaluation was used in this effort, it is essential to identify the appropriate number of states in each stage, as well as the levels of corrective action for each state, to prevent the number of possible policies from rapidly growing. Future work will investigate the sensitivity and robustness of the state space and action space to changes in their respective dimensionality. We also plan on investigating how those states should be distributed throughout the state space.

Future work also includes incorporation of a probabilistic nature to the success of a corrective action, as corrective actions are not deterministic in nature and may not always achieve the desired FEF. We also plan on relaxing the assumption that failure modes have constant failure rates, in favor of a more general Weibull distribution assumption. We also seek to identify any optimal threshold policies that may exist.

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Appendix

5.A Frontier Policies by Instance

5.A.1 Instance 1: Epochs Weighted Equally, Deviations Weighted Equally

5.A.1.1 System MTBF=25

Table 5.A.1: Frontier Policies: $\beta = 25$

Reward	Cost	Policy				
2736.8966	0	1	1	1	1	1
2670.9212	0.2	1	1	1	1	2
2626.9376	0.35	1	1	1	1	3
2604.9458	0.5	1	1	1	1	4
2604.9454	0.7	2	1	1	1	4
2582.9540	0.8	1	1	1	1	5
2560.9622	1	1	1	1	1	6
2560.9618	1.2	2	1	1	1	6
2560.9606	1.35	3	1	1	1	6
2560.9594	1.5	4	1	1	1	6
2560.9590	1.7	4	2	1	1	6
2560.9588	1.75	3	2	2	1	6
2560.9576	1.8	5	1	1	1	6
2560.9544	1.9	3	3	2	1	6
2560.9543	1.95	3	2	2	2	6
2560.9409	2.05	3	3	3	1	6
2560.9391	2.1	3	3	2	2	6
2560.9258	2.2	4	3	3	1	6
2560.8913	2.25	3	3	3	2	6
2560.7718	2.4	3	3	3	3	6
2560.6468	2.55	4	3	3	3	6
2560.4463	2.7	4	4	3	3	6
2560.1268	2.85	4	4	4	3	6
2559.6610	3	4	4	4	4	6
2559.6552	3.15	5	4	4	3	6
2558.9491	3.3	5	4	4	4	6
2558.9410	3.45	5	5	4	3	6
2557.9514	3.5	6	4	4	4	6
2557.8820	3.6	5	5	4	4	6
2557.8684	3.75	5	5	5	3	6
2556.4034	3.8	6	5	4	4	6
2556.2996	3.9	5	5	5	4	6
2554.2845	4	6	6	4	4	6
2533.3587	5	6	6	6	6	6

Table 5.A.1 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
2554.1301	4.1	6	5	5	4	6
2554.1189	4.2	5	5	5	5	6
2551.0697	4.3	6	6	5	4	6
2551.0580	4.4	6	5	5	5	6
2546.7926	4.5	6	6	6	4	6
2546.7847	4.6	6	6	5	5	6
2540.9019	4.8	6	6	6	5	6
2533.3587	5	6	6	6	6	6

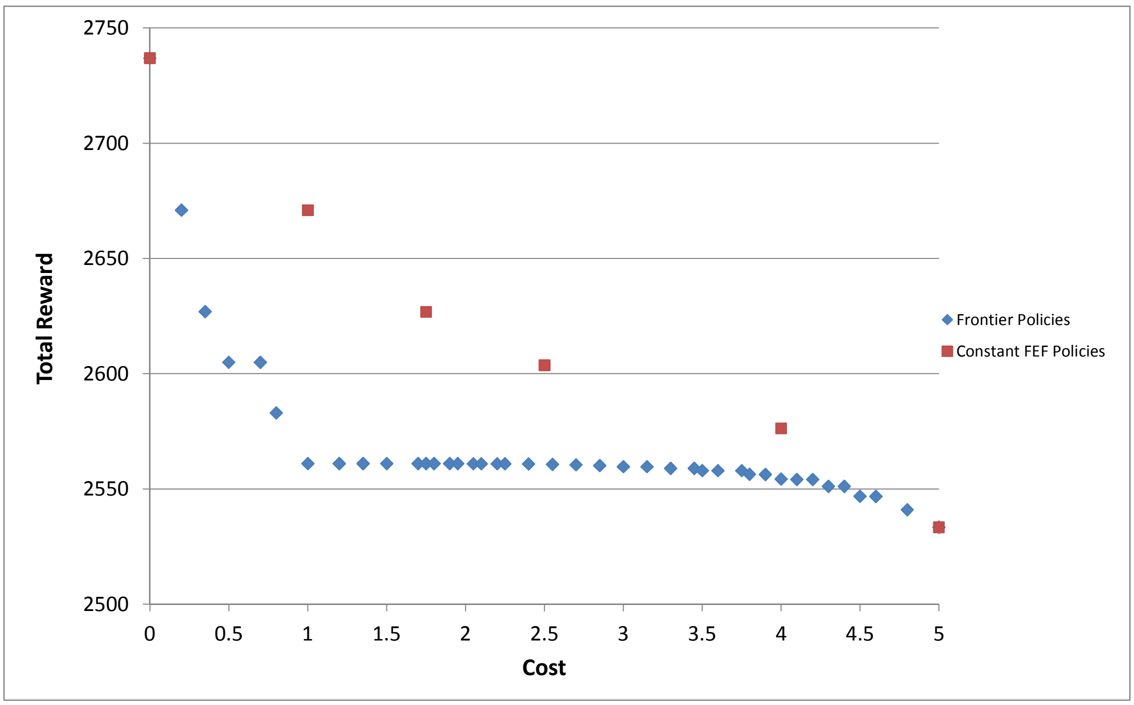

Figure 5.A.1: Constant FEF Policies and Policy Frontier: $\beta = 25$

5.A.1.2 System MTBF=50

Table 5.A.2: Frontier Policies: $\beta = 50$

Reward	Cost	Policy				
2710.7087	0	1	1	1	1	1
2644.7333	0.2	1	1	1	1	2
2600.7497	0.35	1	1	1	1	3
2578.7579	0.5	1	1	1	1	4
2578.6436	0.7	2	1	1	1	4
2556.7661	0.8	1	1	1	1	5

Table 5.A.2 Frontier Policies: β = 50 (Cont.)

Reward	Cost	Policy				
2534.7743	1	1	1	1	1	6
2534.6600	1.2	2	1	1	1	6
2534.4203	1.35	3	1	1	1	6
2534.2093	1.5	4	1	1	1	6
2534.0555	1.7	4	2	1	1	6
2533.8994	1.75	3	2	2	1	6
2533.7036	1.85	4	3	1	1	6
2532.8984	1.9	3	3	2	1	6
2532.7902	1.95	3	2	2	2	6
2530.4780	2.05	3	3	3	1	6
2529.9097	2.1	3	3	2	2	6
2528.1331	2.2	4	3	3	1	6
2522.9482	2.25	3	3	3	2	6
2509.4944	2.35	4	4	4	3	4
2486.6506	2.5	4	4	4	4	4
2479.7200	2.65	5	4	4	4	3
2478.6708	2.7	6	4	4	3	3
2455.6885	2.8	5	4	4	4	4
2443.7623	2.85	6	4	4	4	3
2441.7895	2.95	5	5	4	4	3
2419.4235	3	6	4	4	4	4
2417.4342	3.1	5	5	4	4	4
2397.8136	3.15	6	5	4	4	3
2395.8008	3.2	6	6	4	3	3
2395.7620	3.25	5	5	5	4	3
2373.2410	3.3	6	5	4	4	4
2346.2250	3.35	6	6	4	4	3
2343.5309	3.45	6	5	5	4	3
2317.8537	3.5	6	6	5	4	2
2316.2944	3.55	6	6	6	3	2
2261.9841	3.65	6	6	5	4	3
2253.6539	3.7	6	6	6	4	2
2237.6163	3.8	6	6	5	4	4
2197.0889	3.85	6	6	6	4	3
2173.3873	4	6	6	6	4	4
2132.3823	4.15	6	6	6	5	3
2123.5490	4.2	6	6	6	6	2
2110.0188	4.3	6	6	6	5	4
2068.4333	4.35	6	6	6	6	3
2048.1090	4.5	6	6	6	6	4
2034.3147	4.8	6	6	6	6	5

Table 5.A.2 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
2020.7578	5	6	6	6	6	6

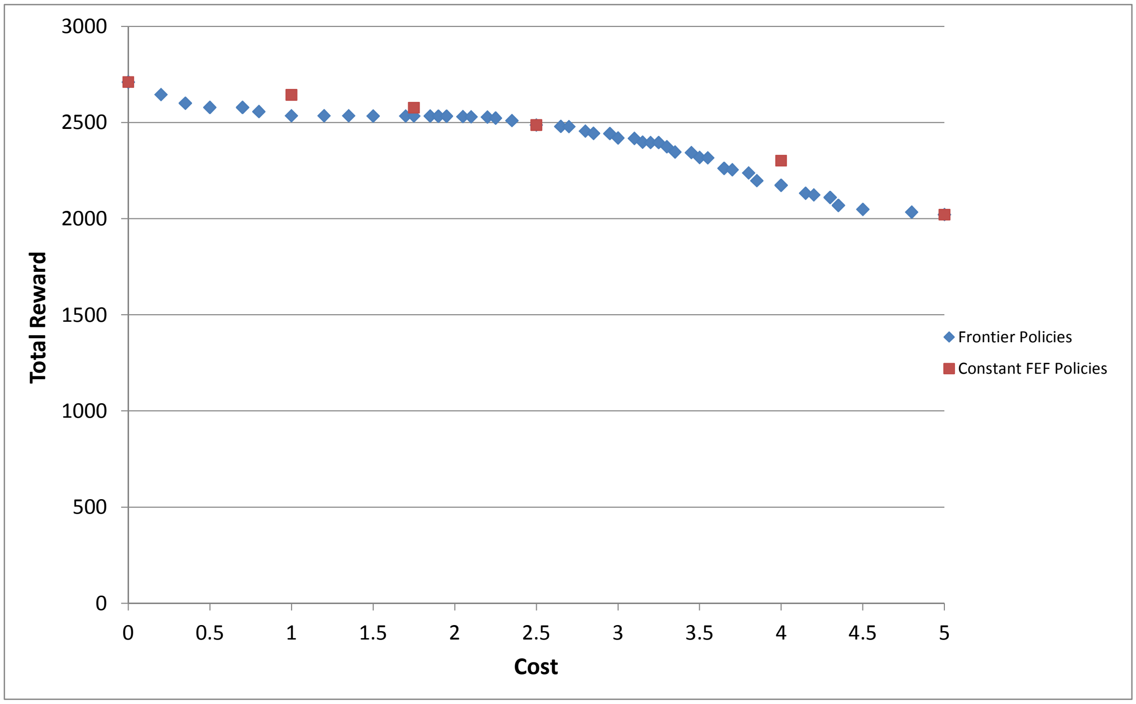

Figure 5.A.2: Constant FEF Policies and Policy Frontier: $\beta = 50$

5.A.1.3 System MTBF=105

Table 5.A.3: Frontier Policies: $\beta = 105$

Reward	Cost	Policy				
2695.2437	0	1	1	1	1	1
2629.2640	0.2	1	1	1	1	2
2585.2777	0.35	1	1	1	1	3
2563.2847	0.5	1	1	1	1	4
2552.1271	0.7	2	1	1	1	4
2541.2919	0.8	1	1	1	1	5
2537.7065	0.85	3	1	1	1	4
2534.5269	0.9	3	3	2	1	1
2519.2991	1	1	1	1	1	6
2456.6329	1.05	3	3	3	1	1
2419.8841	1.1	3	3	2	2	1
2395.5478	1.2	4	3	3	1	1
2278.4623	1.25	3	3	3	2	1

Table 5.A.3 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
2131.4150	1.4	3	3	3	3	1
1906.6531	1.55	4	3	3	3	1
1706.5149	1.7	4	4	3	3	1
1653.7497	1.85	4	4	4	3	1
1622.3755	1.9	4	4	3	3	2
1552.7024	2	4	4	4	4	1
1517.8385	2.15	5	4	4	3	1
1509.8942	2.3	5	4	4	4	1
1473.5506	2.35	6	4	4	3	1
1462.5548	2.5	6	5	3	3	1
1436.2381	2.65	6	5	4	3	1
1411.8798	2.7	6	6	4	2	1
1401.5522	2.85	6	6	4	3	1
1386.1285	3	6	6	5	2	1
1364.7712	3.2	6	6	6	2	1

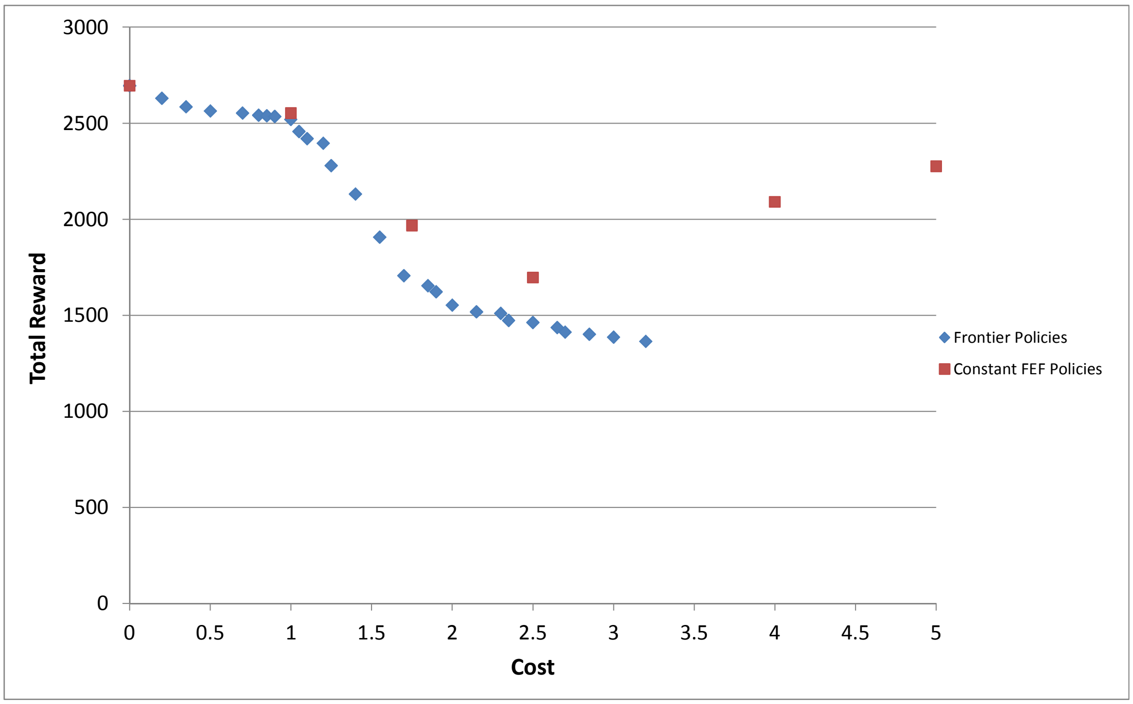

Figure 5.A.3: Constant FEF Policies and Policy Frontier: $\beta = 105$

5.A.1.4 System MTBF=205

Table 5.A.4: Frontier Policies: $\beta = 205$

Reward	Cost	Policy				
2643.8432	0	1	1	1	1	1
2528.6963	0.2	2	1	1	1	1
2410.3923	0.35	3	1	1	1	1
2344.2074	0.4	2	2	1	1	1
2166.2439	0.55	3	2	1	1	1
2107.4617	0.6	2	2	2	1	1
1933.5881	0.7	3	3	1	1	1
1852.4956	0.75	3	2	2	1	1
1715.2703	0.8	2	2	2	2	1
1336.1674	0.9	3	3	2	1	1
1154.1041	1.05	3	3	3	1	1
1110.4905	1.2	4	3	3	1	1
1067.9597	1.35	4	4	3	1	1
1063.8921	1.5	5	3	3	1	1
1044.3964	1.7	6	4	2	1	1

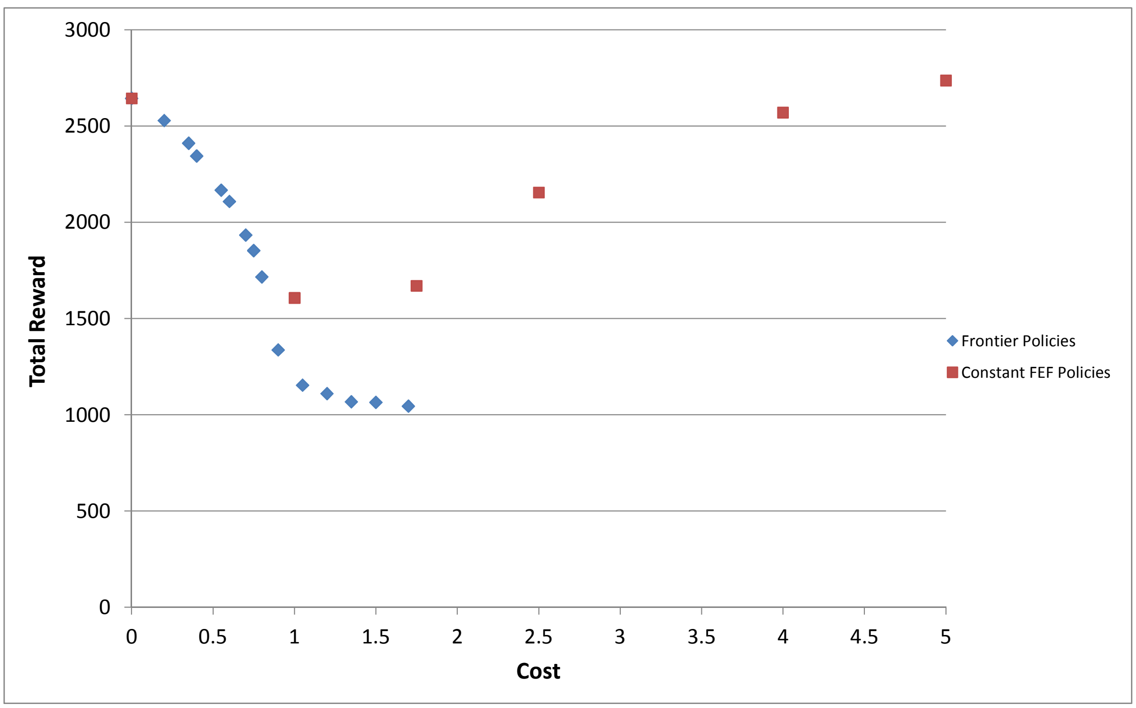

Figure 5.A.4: Constant FEF Policies and Policy Frontier: $\beta = 205$

5.A.2 Instance 2: Epoch Weights Equal, Deviations Above Curve Weighted at 0.5

5.A.2.1 System MTBF=25

Table 5.A.5: Frontier Policies: $\beta = 25$

Reward	Cost	Policy				
2736.8916	0	1	1	1	1	1
2670.9162	0.2	1	1	1	1	2
2626.9326	0.35	1	1	1	1	3
2604.9408	0.5	1	1	1	1	4
2604.9403	0.7	2	1	1	1	4
2582.9490	0.8	1	1	1	1	5
2560.9572	1	1	1	1	1	6
2560.9567	1.2	2	1	1	1	6
2560.9556	1.35	3	1	1	1	6
2560.9544	1.5	4	1	1	1	6
2560.9540	1.7	4	2	1	1	6
2560.9538	1.75	3	2	2	1	6
2560.9526	1.8	5	1	1	1	6
2560.9494	1.9	3	3	2	1	6
2560.9492	1.95	3	2	2	2	6
2560.9358	2.05	3	3	3	1	6
2560.9339	2.1	3	3	2	2	6
2560.9206	2.2	4	3	3	1	6
2560.8858	2.25	3	3	3	2	6
2560.7650	2.4	3	3	3	3	6
2560.6387	2.55	4	3	3	3	6
2560.4359	2.7	4	4	3	3	6
2560.1126	2.85	4	4	4	3	6
2559.6404	3	4	4	4	4	6
2559.6351	3.15	5	4	4	3	6
2558.9187	3.3	5	4	4	4	6
2558.9113	3.45	5	5	4	3	6
2557.9062	3.5	6	4	4	4	6
2557.8358	3.6	5	5	4	4	6
2557.8234	3.75	5	5	5	3	6
2556.3338	3.8	6	5	4	4	6
2556.2283	3.9	5	5	5	4	6
2554.1789	4	6	6	4	4	6
2554.0217	4.1	6	5	5	4	6
2554.0079	4.2	5	5	5	5	6
2550.9050	4.3	6	6	5	4	6
2550.8893	4.4	6	5	5	5	6

Table 5.A.5 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
2546.5426	4.5	6	6	6	4	6
2546.5286	4.6	6	6	5	5	6
2540.5146	4.8	6	6	6	5	6
2532.7778	5	6	6	6	6	6

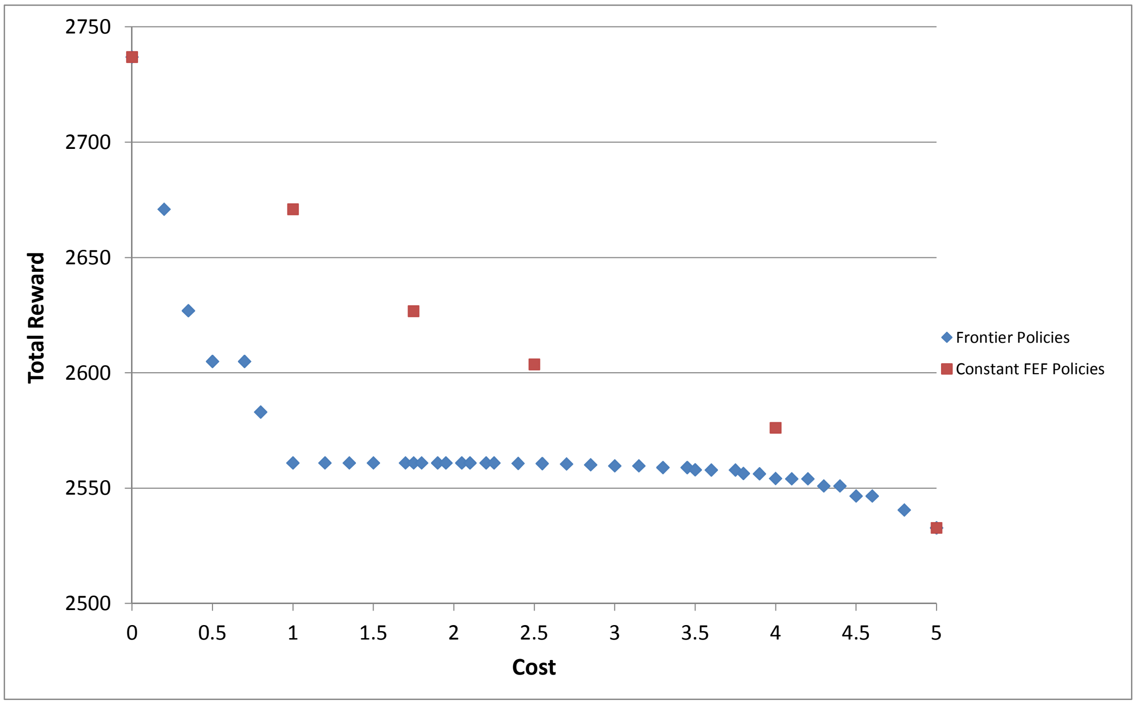

Figure 5.A.5: Constant FEF Policies and Policy Frontier: $\beta = 25$

5.A.2.2 System MTBF=50

Table 5.A.6: Frontier Policies: $\beta = 50$

Reward	Cost	Policy				
2710.0871	0	1	1	1	1	1
2644.1117	0.2	1	1	1	1	2
2600.1281	0.35	1	1	1	1	3
2578.1363	0.5	1	1	1	1	4
2578.0219	0.7	2	1	1	1	4
2556.1445	0.8	1	1	1	1	5
2534.1527	1	1	1	1	1	6
2534.0382	1.2	2	1	1	1	6
2533.7983	1.35	3	1	1	1	6
2533.5869	1.5	4	1	1	1	6

Table 5.A.6 Frontier Policies: β = 50 (Cont.)

Reward	Cost	Policy				
2533.4328	1.7	4	2	1	1	6
2533.2753	1.75	3	2	2	1	6
2533.0798	1.85	4	3	1	1	6
2532.2692	1.9	3	3	2	1	6
2532.1516	1.95	3	2	2	2	6
2529.8329	2.05	3	3	3	1	6
2529.2355	2.1	3	3	2	2	6
2527.4718	2.2	4	3	3	1	6
2522.1686	2.25	3	3	3	2	6
2507.9824	2.35	4	4	4	3	4
2484.6497	2.5	4	4	4	4	4
2477.6582	2.65	5	4	4	4	3
2476.6622	2.7	6	4	4	3	3
2452.9914	2.8	5	4	4	4	4
2441.0165	2.85	6	4	4	4	3
2438.9951	2.95	5	5	4	4	3
2415.7792	3	6	4	4	4	4
2413.7231	3.1	5	5	4	4	4
2394.0177	3.15	6	5	4	4	3
2392.1154	3.2	6	6	4	3	3
2391.8968	3.25	5	5	5	4	3
2368.1643	3.3	6	5	4	4	4
2341.0141	3.35	6	6	4	4	3
2338.2239	3.45	6	5	5	4	3
2313.4541	3.5	6	6	5	4	2
2311.9670	3.55	6	6	6	3	2
2311.8086	3.6	6	5	5	4	4
2254.4684	3.65	6	6	5	4	3
2247.6309	3.7	6	6	6	4	2
2227.6508	3.8	6	6	5	4	4
2186.7789	3.85	6	6	6	4	3
2159.7886	4	6	6	6	4	4
2118.1391	4.15	6	6	6	5	3
2112.2220	4.2	6	6	6	6	2
2091.3479	4.3	6	6	6	5	4
2049.1674	4.35	6	6	6	6	3
2023.0653	4.5	6	6	6	6	4
2001.8606	4.8	6	6	6	6	5
1980.8341	5	6	6	6	6	6

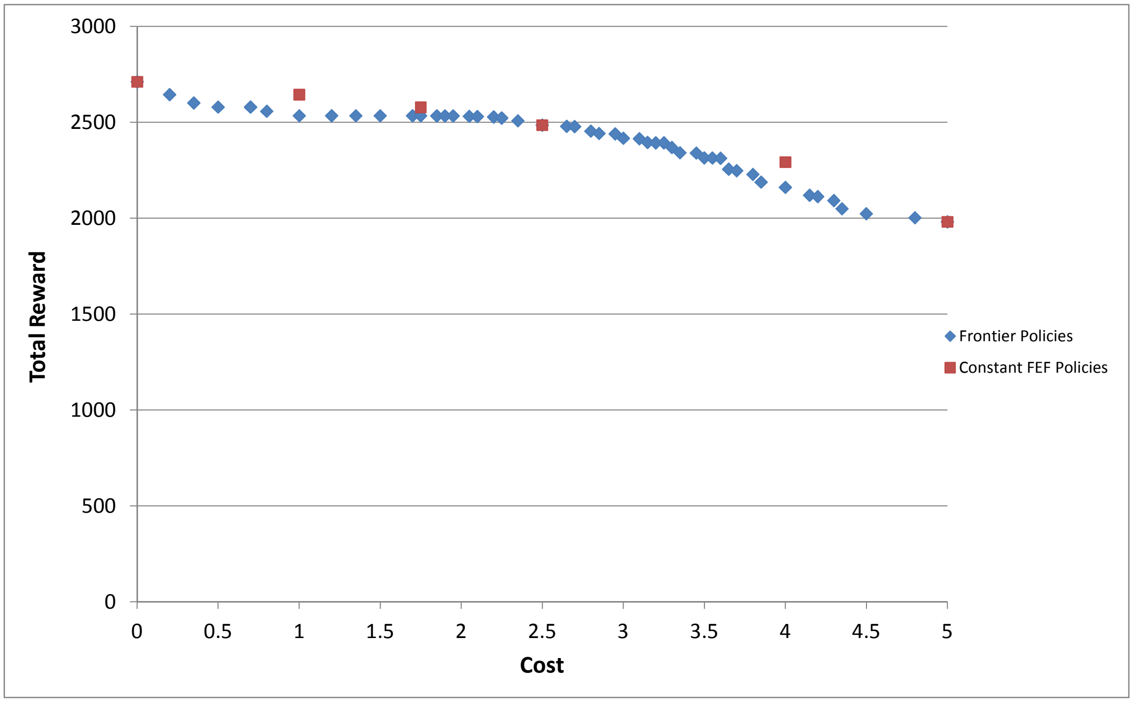

Figure 5.A.6: Constant FEF Policies and Policy Frontier: $\beta = 50$

5.A.2.3 System MTBF=105

Table 5.A.7: Frontier Policies: $\beta = 105$

Reward	Cost	Policy				
2680.1247	0	1	1	1	1	1
2614.1450	0.2	1	1	1	1	2
2570.1587	0.35	1	1	1	1	3
2548.1657	0.5	1	1	1	1	4
2536.9614	0.7	2	1	1	1	4
2526.1727	0.8	1	1	1	1	5
2522.4402	0.85	3	1	1	1	4
2518.5641	0.9	3	3	2	1	1
2504.1798	1	1	1	1	1	6
2439.9950	1.05	3	3	3	1	1
2403.2560	1.1	3	3	2	2	1
2378.2758	1.2	4	3	3	1	1
2260.0660	1.25	3	3	3	2	1
2109.8311	1.4	3	3	3	3	1
1875.4446	1.55	4	3	3	3	1
1654.3391	1.7	4	4	3	3	1
1593.5860	1.85	4	4	4	3	1
1539.6798	1.9	4	4	3	3	2

Table 5.A.7 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
1444.3053	2	4	4	4	4	1
1409.2190	2.15	5	4	4	3	1
1385.3513	2.2	4	4	4	4	2
1349.5822	2.35	5	4	4	3	2
1336.4492	2.5	6	4	4	4	1
1304.5605	2.55	6	4	4	3	2
1304.0140	2.65	6	5	4	3	1
1293.4437	2.7	6	5	3	3	2
1288.2006	2.8	6	5	4	4	1
1255.7978	2.85	6	6	4	3	1
1239.9032	2.9	6	6	4	2	2
1210.6305	3	6	6	4	4	1
1189.3713	3.15	6	6	5	3	1
1188.9777	3.3	6	6	5	4	1
1168.9438	3.35	6	6	6	3	1

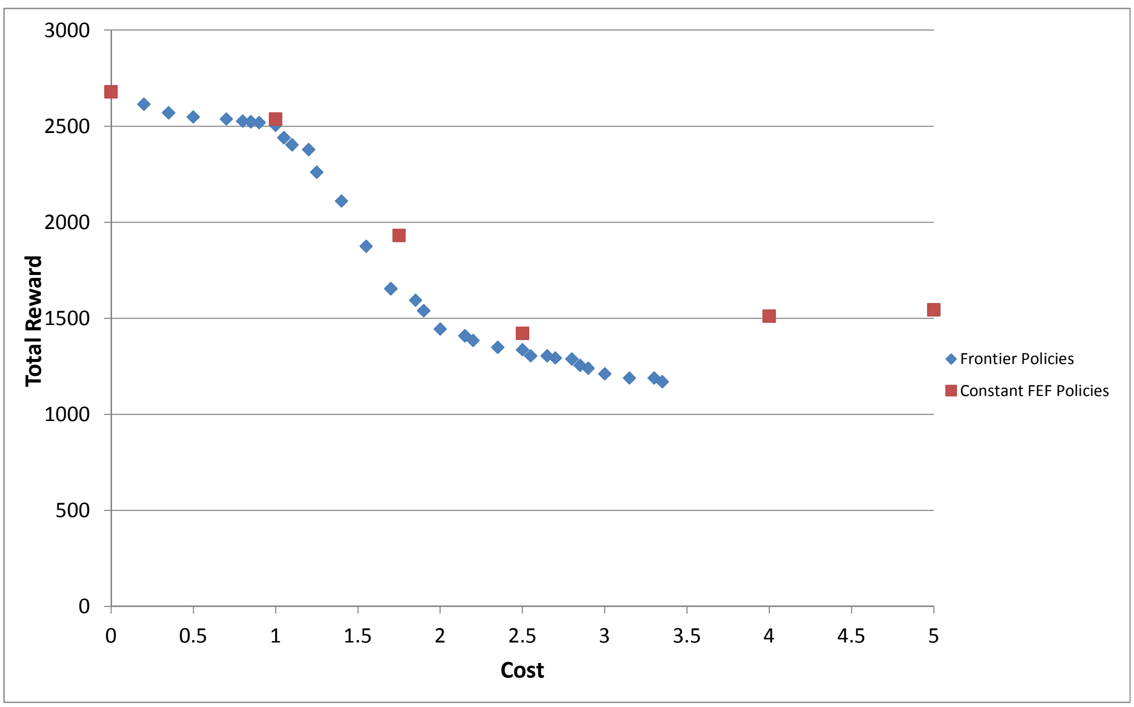

Figure 5.A.7: Constant FEF Policies and Policy Frontier: $\beta = 105$

5.A.2.4 System MTBF=205

Table 5.A.8: Frontier Policies: $\beta = 205$

Reward	Cost	Policy				
2610.9207	0	1	1	1	1	1
2492.2974	0.2	2	1	1	1	1
2368.8360	0.35	3	1	1	1	1
2306.1327	0.4	2	2	1	1	1
2121.2796	0.55	3	2	1	1	1
2065.9637	0.6	2	2	2	1	1
1880.2144	0.7	3	3	1	1	1
1800.2468	0.75	3	2	2	1	1
1658.9126	0.8	2	2	2	2	1
1244.5618	0.9	3	3	2	1	1
998.0103	1.05	3	3	3	1	1
933.6237	1.2	4	3	3	1	1
907.5792	1.25	3	3	3	2	1
854.9776	1.35	4	4	3	1	1
782.3127	1.5	4	4	4	1	1
760.9314	1.65	5	4	3	1	1
759.5319	1.8	5	4	4	1	1
741.7732	1.85	6	4	3	1	1
722.3070	2.2	6	6	2	1	1

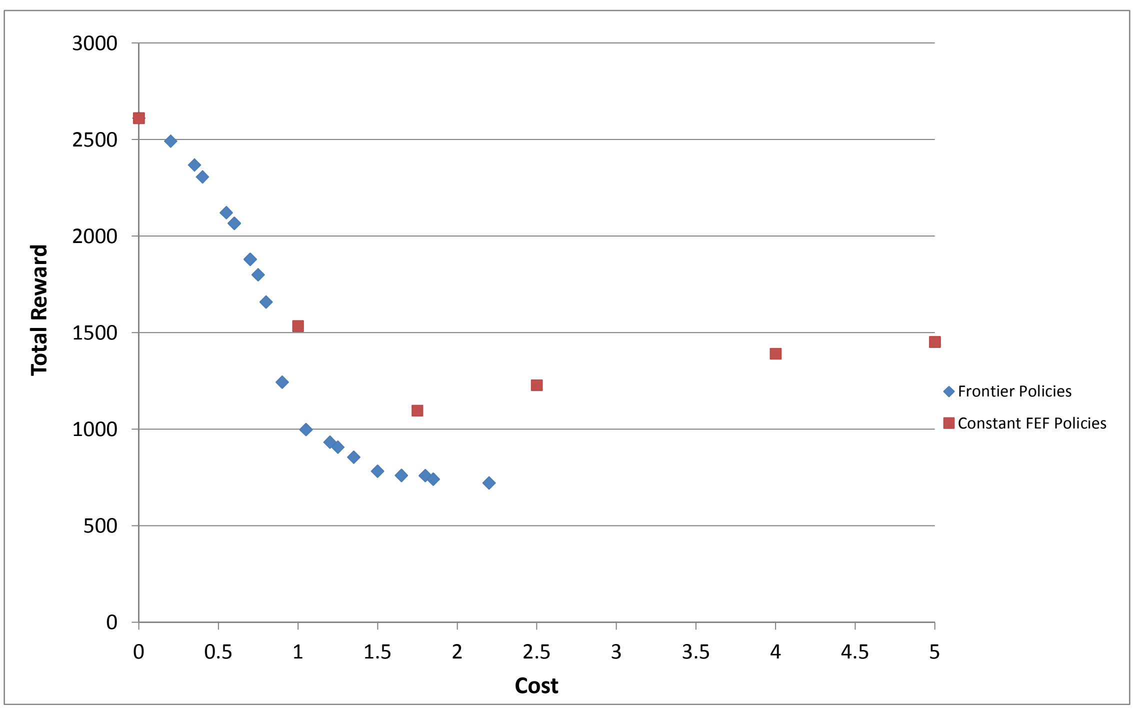

Figure 5.A.8: Constant FEF Policies and Policy Frontier: $\beta = 205$

5.A.3 Instance 3: Epoch Weights Equal, Deviations Above Curve Weighted at 0.1

5.A.3.1 System MTBF=25

Table 5.A.9: Frontier Policies: $\beta = 25$

Reward	Cost	Policy				
2736.8875	0	1	1	1	1	1
2670.9121	0.2	1	1	1	1	2
2626.9285	0.35	1	1	1	1	3
2604.9367	0.5	1	1	1	1	4
2604.9363	0.7	2	1	1	1	4
2582.9449	0.8	1	1	1	1	5
2560.9531	1	1	1	1	1	6
2560.9527	1.2	2	1	1	1	6
2560.9516	1.35	3	1	1	1	2
2560.9504	1.5	4	1	1	1	6
2560.9499	1.7	4	2	1	1	3
2560.9498	1.75	3	2	2	1	2
2560.9486	1.8	5	1	1	1	2
2560.9454	1.9	3	3	2	1	2
2560.9451	1.95	3	2	2	2	3
2560.9317	2.05	3	3	3	1	6
2560.9298	2.1	3	3	2	2	4
2560.9165	2.2	4	3	3	1	3
2560.8813	2.25	3	3	3	2	6
2560.7595	2.4	3	3	3	3	6
2560.6322	2.55	4	3	3	3	6
2560.4276	2.7	4	4	3	3	3
2560.1012	2.85	4	4	4	3	5
2559.6240	3	4	4	4	4	6
2559.6190	3.15	5	4	4	3	5
2558.8943	3.3	5	4	4	4	6
2558.8876	3.45	5	5	4	3	3
2557.8701	3.5	6	4	4	4	6
2557.7988	3.6	5	5	4	4	5
2557.7873	3.75	5	5	5	3	6
2556.2781	3.8	6	5	4	4	6
2556.1712	3.9	5	5	5	4	5
2554.0945	4	6	6	4	4	6
2553.9350	4.1	6	5	5	4	6
2553.9191	4.2	5	5	5	5	6
2550.7732	4.3	6	6	5	4	6
2550.7543	4.4	6	5	5	5	5

Table 5.A.9 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
2546.3426	4.5	6	6	6	4	6
2546.3237	4.6	6	6	5	5	6
2540.2048	4.8	6	6	6	5	6
2532.3130	5	6	6	6	6	6

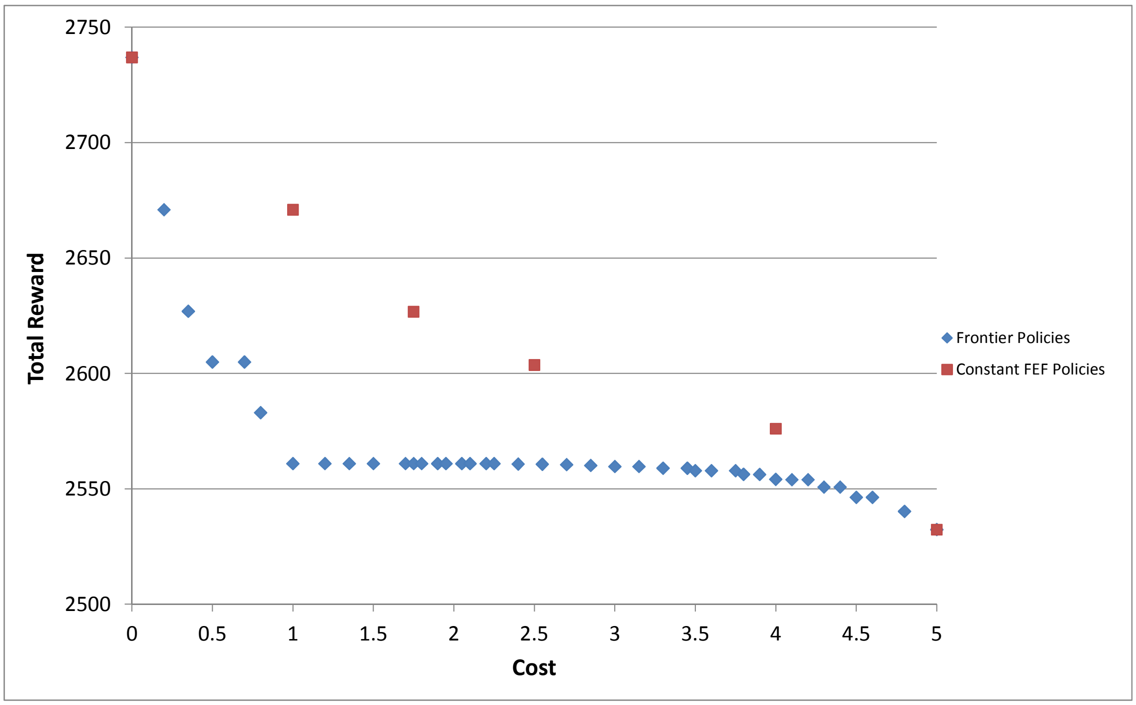

Figure 5.A.9: Constant FEF Policies and Policy Frontier: $\beta = 25$

5.A.3.2 System MTBF=50

Table 5.A.10: Frontier Policies: $\beta = 50$

Reward	Cost	Policy				
2709.5897	0	1	1	1	1	1
2643.6143	0.2	1	1	1	1	2
2599.6307	0.35	1	1	1	1	3
2577.6389	0.5	1	1	1	1	4
2577.5244	0.7	2	1	1	1	4
2555.6471	0.8	1	1	1	1	5
2533.6553	1	1	1	1	1	6
2533.5408	1.2	2	1	1	1	6
2533.3006	1.35	3	1	1	1	6
2533.0890	1.5	4	1	1	1	6

Table 5.A.10 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
2532.9345	1.7	4	2	1	1	6
2532.7759	1.75	3	2	2	1	6
2532.5807	1.85	4	3	1	1	6
2531.7658	1.9	3	3	2	1	6
2531.6407	1.95	3	2	2	2	6
2529.3167	2.05	3	3	3	1	6
2528.6961	2.1	3	3	2	2	6
2526.9427	2.2	4	3	3	1	6
2521.5450	2.25	3	3	3	2	6
2506.7728	2.35	4	4	4	3	4
2483.0489	2.5	4	4	4	4	4
2476.0088	2.65	5	4	4	4	3
2475.0553	2.7	6	4	4	3	3
2450.8338	2.8	5	4	4	4	4
2438.8197	2.85	6	4	4	4	3
2436.7597	2.95	5	5	4	4	3
2412.8638	3	6	4	4	4	4
2410.7542	3.1	5	5	4	4	4
2390.9809	3.15	6	5	4	4	3
2389.1671	3.2	6	6	4	3	3
2388.8046	3.25	5	5	5	4	3
2364.1029	3.3	6	5	4	4	4
2336.8454	3.35	6	6	4	4	3
2333.9782	3.45	6	5	5	4	3
2309.0448	3.5	6	6	4	4	4
2308.5051	3.55	6	6	6	3	2
2306.1239	3.6	6	5	5	4	4
2248.4557	3.65	6	6	5	4	3
2242.8124	3.7	6	6	6	4	2
2219.6783	3.8	6	6	5	4	4
2178.5308	3.85	6	6	6	4	3
2148.9096	4	6	6	6	4	4
2106.7444	4.15	6	6	6	5	3
2103.1603	4.2	6	6	6	6	2
2076.4112	4.3	6	6	6	5	4
2033.7546	4.35	6	6	6	6	3
2003.0303	4.5	6	6	6	6	4
1975.8973	4.8	6	6	6	6	5
1948.8951	5	6	6	6	6	6

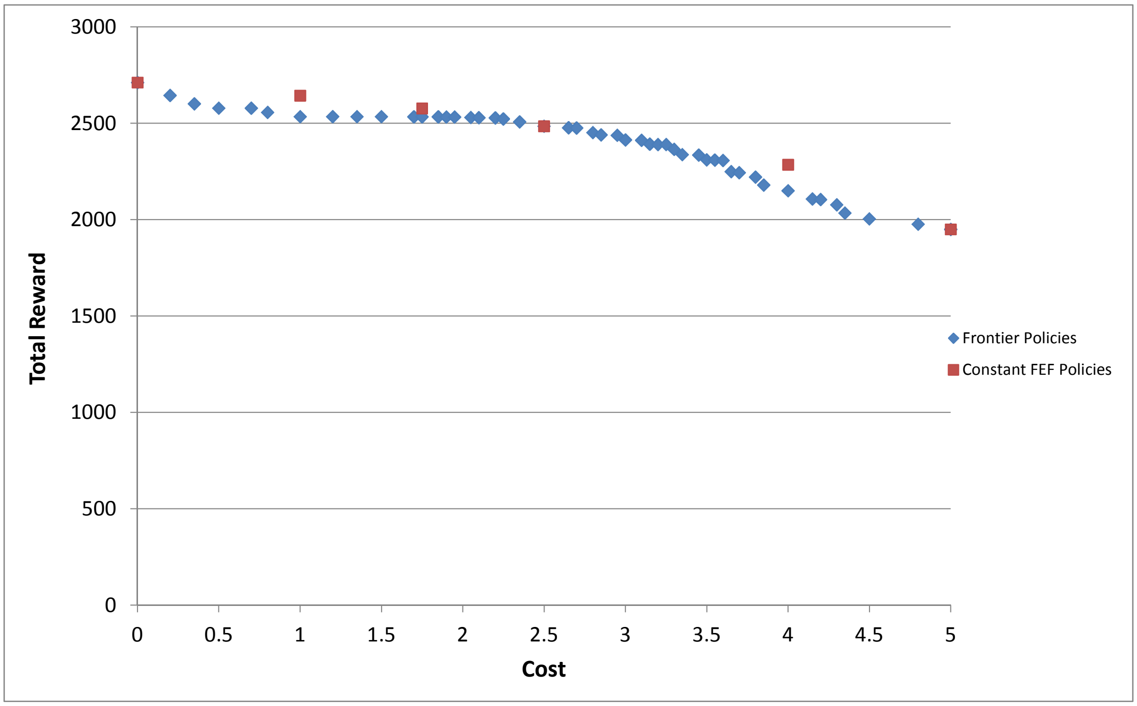

Figure 5.A.10: Constant FEF Policies and Policy Frontier: $\beta = 50$

5.A.3.3 System MTBF=105

Table 5.A.11: Frontier Policies: $\beta = 105$

Reward	Cost	Policy				
2668.0295	0	1	1	1	1	1
2602.0498	0.2	1	1	1	1	2
2558.0635	0.35	1	1	1	1	3
2536.0704	0.5	1	1	1	1	4
2524.8288	0.7	2	1	1	1	4
2514.0773	0.8	1	1	1	1	5
2510.2271	0.85	3	1	1	1	4
2505.7938	0.9	3	3	2	1	1
2492.0843	1	1	1	1	1	6
2426.6847	1.05	3	3	3	1	1
2389.9535	1.1	3	3	2	2	1
2364.4581	1.2	4	3	3	1	1
2245.3488	1.25	3	3	3	2	1
2092.5640	1.4	3	3	3	3	1
1850.4777	1.55	4	3	3	3	1
1612.5984	1.7	4	4	3	3	1
1545.4550	1.85	4	4	4	3	1
1473.5233	1.9	4	4	3	3	2

Table 5.A.11 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
1357.5876	2	4	4	4	4	1
1322.3233	2.15	5	4	4	3	1
1245.0322	2.2	4	4	4	4	2
1209.3952	2.35	5	4	4	3	2
1175.0183	2.5	5	4	4	3	3
1151.0310	2.55	6	4	4	3	2
1120.7815	2.7	6	4	4	3	3
1097.4519	2.85	6	5	4	3	2
1088.3754	2.9	6	6	3	3	2
1019.4935	3	6	6	4	4	1
997.5421	3.15	6	6	5	3	1
974.9818	3.2	6	6	4	4	2
952.5872	3.35	6	6	5	3	2
947.9767	3.5	6	6	5	4	2
926.7269	3.55	6	6	6	3	2
922.7447	3.7	6	6	6	4	2
919.5656	4	6	6	6	5	2
917.1069	4.2	6	6	6	6	2

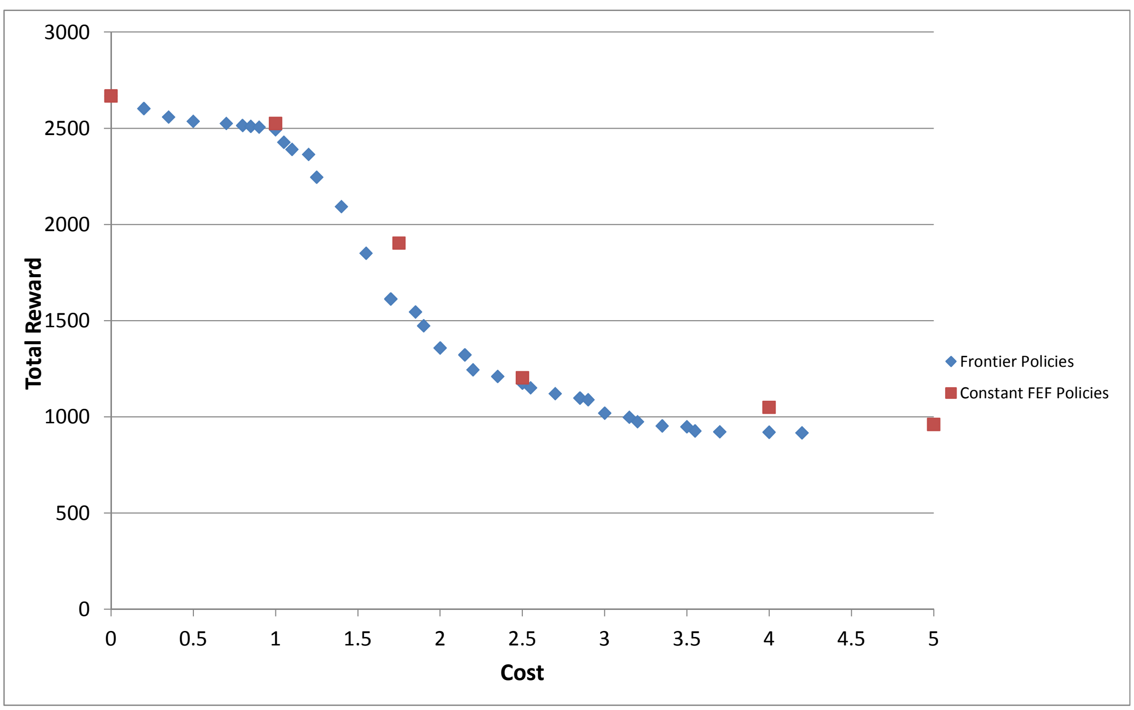

Figure 5.A.11: Constant FEF Policies and Policy Frontier: $\beta = 105$

5.A.3.4 System MTBF=205

Table 5.A.12: Frontier Policies: $\beta = 205$

Reward	Cost	Policy				
2584.5826	0	1	1	1	1	1
2463.1783	0.2	2	1	1	1	1
2335.5910	0.35	3	1	1	1	1
2275.6730	0.4	2	2	1	1	1
2085.3081	0.55	3	2	1	1	1
2032.7653	0.6	2	2	2	1	1
1837.5154	0.7	3	3	1	1	1
1758.4478	0.75	3	2	2	1	1
1613.8264	0.8	2	2	2	2	1
1171.2774	0.9	3	3	2	1	1
873.1352	1.05	3	3	3	1	1
792.1303	1.2	4	3	3	1	1
678.0219	1.25	3	3	3	2	1
613.9385	1.4	4	3	3	2	1
510.6119	1.5	4	4	4	1	1
485.1978	1.65	5	4	3	1	1
462.5652	1.7	4	4	4	1	2
423.4059	1.85	4	4	4	3	1
393.3258	2	5	4	4	2	1
370.6575	2.05	6	4	3	2	1
345.1827	2.2	6	4	4	2	1
331.3737	2.35	6	5	3	2	1
328.7227	2.5	6	5	3	3	1
316.5751	2.55	6	6	3	2	1
314.2435	2.7	6	6	3	3	1
312.0941	2.85	6	6	4	3	1
311.2379	3.15	6	6	5	3	1
310.2424	3.2	6	6	6	2	1

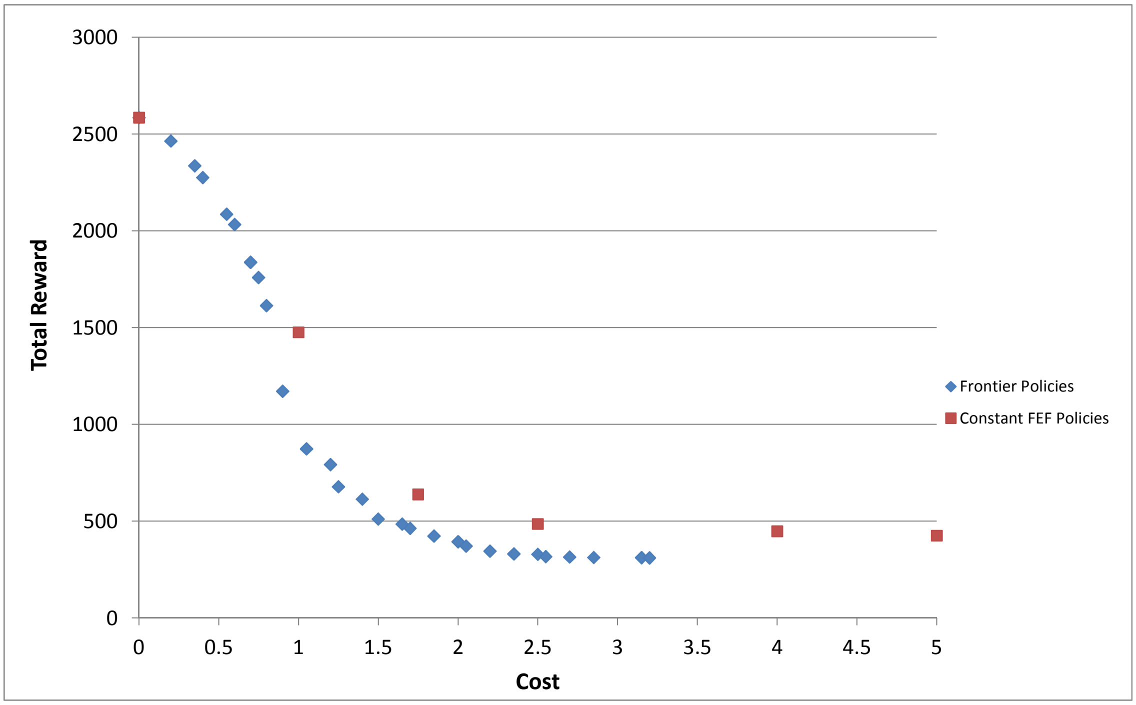

Figure 5.A.12: Constant FEF Policies and Policy Frontier: $\beta = 205$

5.A.4 Instance 4: Epoch Progressively Weighted, Deviations Weighted Equally

5.A.4.1 System MTBF=25

Table 5.A.13: Frontier Policies: $\beta = 25$

Reward	Cost	Policy				
2337.8670	0	1	1	1	1	1
2271.8916	0.2	1	1	1	1	2
2227.9080	0.35	1	1	1	1	3
2205.9162	0.5	1	1	1	1	4
2205.9159	0.7	2	1	1	1	4
2183.9244	0.8	1	1	1	1	5
2161.9326	1	1	1	1	1	6
2161.9323	1.2	2	1	1	1	6
2161.9318	1.35	3	1	1	1	6
2161.9312	1.5	4	1	1	1	6
2161.9308	1.7	4	2	1	1	6
2161.9302	1.75	3	2	2	1	6
2161.9298	1.85	4	3	1	1	6
2161.9264	1.9	3	3	2	1	6
2161.9256	1.95	3	2	2	2	6
2161.9143	2.05	3	3	3	1	6
2161.9111	2.1	3	3	2	2	6
2161.9014	2.2	4	3	3	1	6
2161.8648	2.25	3	3	3	2	6
2161.7453	2.4	3	3	3	3	6
2161.6225	2.55	4	3	3	3	6
2161.4247	2.7	4	4	3	3	6
2161.1093	2.85	4	4	4	3	6
2160.6435	3	4	4	4	4	6
2159.9390	3.3	5	4	4	4	6
2158.9522	3.5	6	4	4	4	6
2158.8822	3.6	5	5	4	4	6
2157.4198	3.8	6	5	4	4	6
2157.3150	3.9	5	5	5	4	6
2155.3233	4	6	6	4	4	6
2155.1693	4.1	6	5	5	4	6
2155.1343	4.2	5	5	5	5	6
2152.1422	4.3	6	6	5	4	6
2152.0972	4.4	6	5	5	5	6
2147.9126	4.5	6	6	6	4	6
2147.8573	4.6	6	6	5	5	6
2142.0219	4.8	6	6	6	5	6

Table 5.A.13 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
2134.4787	5	6	6	6	6	6

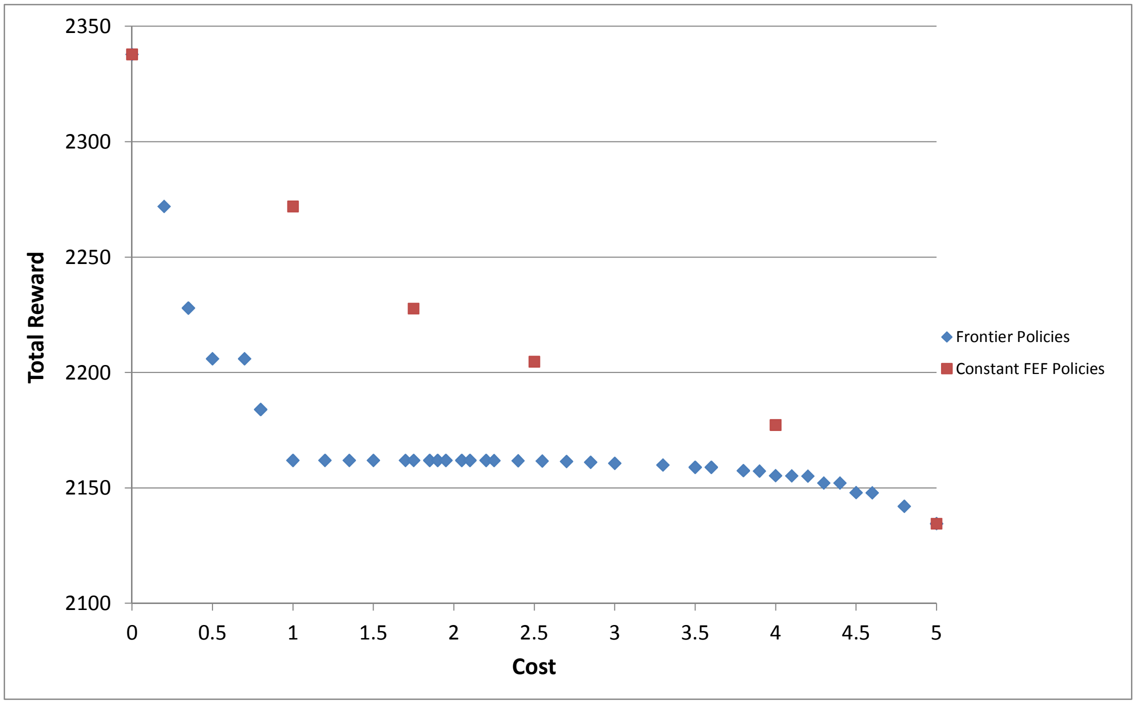

Figure 5.A.13: Constant FEF Policies and Policy Frontier: $\beta = 25$

5.A.4.2 System MTBF=50

Table 5.A.14: Frontier Policies: $\beta = 50$

Reward	Cost	Policy				
2329.1347	0	1	1	1	1	1
2263.1593	0.2	1	1	1	1	2
2219.1757	0.35	1	1	1	1	3
2197.1839	0.5	1	1	1	1	4
2197.1258	0.7	2	1	1	1	4
2175.1921	0.8	1	1	1	1	5
2153.2003	1	1	1	1	1	6
2153.1422	1.2	2	1	1	1	6
2153.0198	1.35	3	1	1	1	6
2152.9117	1.5	4	1	1	1	6
2152.7707	1.7	3	3	1	1	6
2152.5626	1.75	3	2	2	1	6
2152.5040	1.85	4	3	1	1	6

Table 5.A.14 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
2151.6830	1.9	3	3	2	1	6
2151.4533	1.95	3	2	2	2	6
2149.5002	2.05	3	3	3	1	6
2148.6943	2.1	3	3	2	2	6
2147.4791	2.2	4	3	3	1	6
2141.9704	2.25	3	3	3	2	6
2129.5754	2.35	4	4	4	3	4
2129.3204	2.4	3	3	3	3	6
2106.7316	2.5	4	4	4	4	4
2100.5315	2.65	5	4	4	4	3
2100.4237	2.7	6	4	4	3	3
2076.5000	2.8	5	4	4	4	4
2065.5152	2.85	6	4	4	4	3
2063.4030	2.95	5	5	4	4	3
2041.1764	3	6	4	4	4	4
2039.0477	3.1	5	5	4	4	4
2020.6241	3.15	6	5	4	4	3
2019.9183	3.2	6	6	4	3	3
2018.3211	3.25	5	5	5	4	3
1996.0515	3.3	6	5	4	4	4
1970.3425	3.35	6	6	4	4	3
1967.5540	3.45	6	5	5	4	3
1945.7216	3.5	6	6	4	4	4
1942.9375	3.6	6	5	5	4	4
1891.1820	3.65	6	6	5	4	3
1884.2628	3.7	6	6	6	4	2
1866.8143	3.8	6	6	5	4	4
1827.6977	3.85	6	6	6	4	3
1803.9962	4	6	6	6	4	4
1762.9911	4.15	6	6	6	5	3
1754.1578	4.2	6	6	6	6	2
1740.6276	4.3	6	6	6	5	4
1699.0421	4.35	6	6	6	6	3
1678.7179	4.5	6	6	6	6	4
1664.9235	4.8	6	6	6	6	5
1651.3666	5	6	6	6	6	6

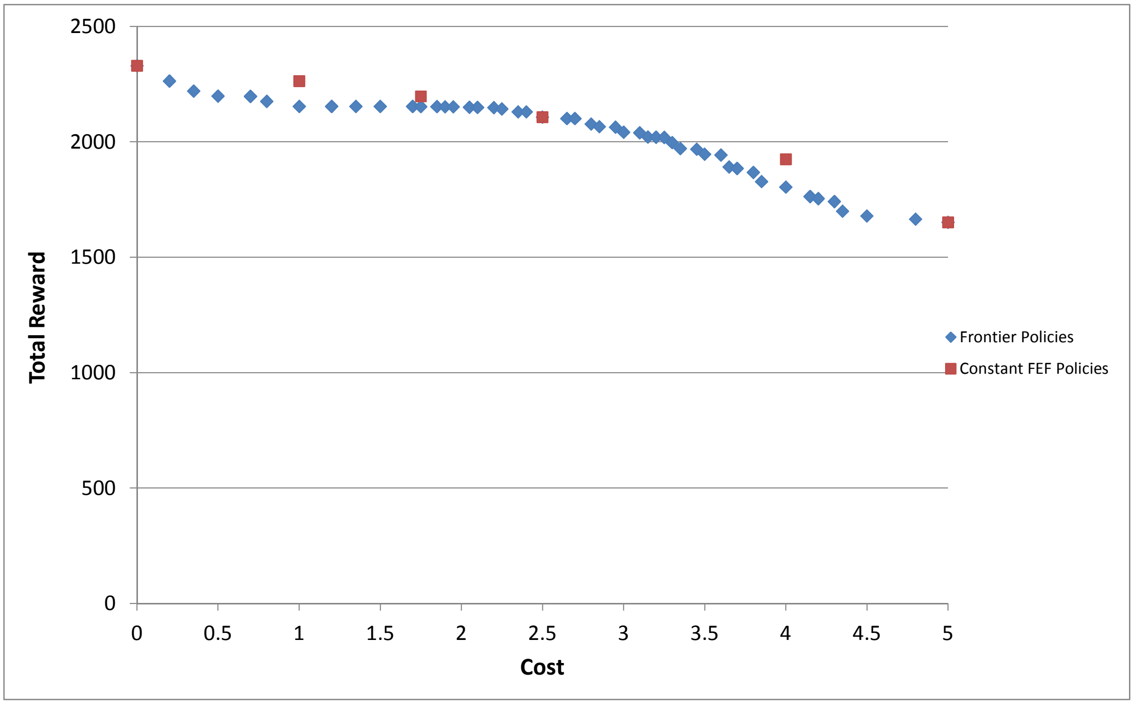

Figure 5.A.14: Constant FEF Policies and Policy Frontier: $\beta = 50$

5.A.4.3 System MTBF=105

Table 5.A.15: Frontier Policies: $\beta = 105$

Reward	Cost	Policy				
2323.3690	0	1	1	1	1	1
2257.3894	0.2	1	1	1	1	2
2213.4031	0.35	1	1	1	1	3
2191.4101	0.5	1	1	1	1	4
2185.4725	0.7	2	1	1	1	4
2169.4173	0.8	1	1	1	1	5
2147.4245	1	1	1	1	1	6
2118.0538	1.05	3	3	3	1	1
2076.5274	1.1	3	3	2	2	1
2065.2404	1.2	4	3	3	1	1
1939.8832	1.25	3	3	3	2	1
1792.8358	1.4	3	3	3	3	1
1576.3456	1.55	4	3	3	3	1
1381.5211	1.7	4	4	3	3	1
1331.7868	1.85	4	4	4	3	1
1297.3817	1.9	4	4	3	3	2
1230.7395	2	4	4	4	4	1
1206.0556	2.15	5	4	4	3	1

Table 5.A.15 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
1198.1113	2.3	5	4	4	4	1
1172.6392	2.35	6	4	4	3	1
1165.6544	2.5	6	5	3	3	1
1142.7994	2.65	6	5	4	3	1
1126.1432	2.7	6	6	4	2	1
1115.8156	2.85	6	6	4	3	1
1103.6487	3	6	6	5	2	1
1085.2781	3.2	6	6	6	2	1

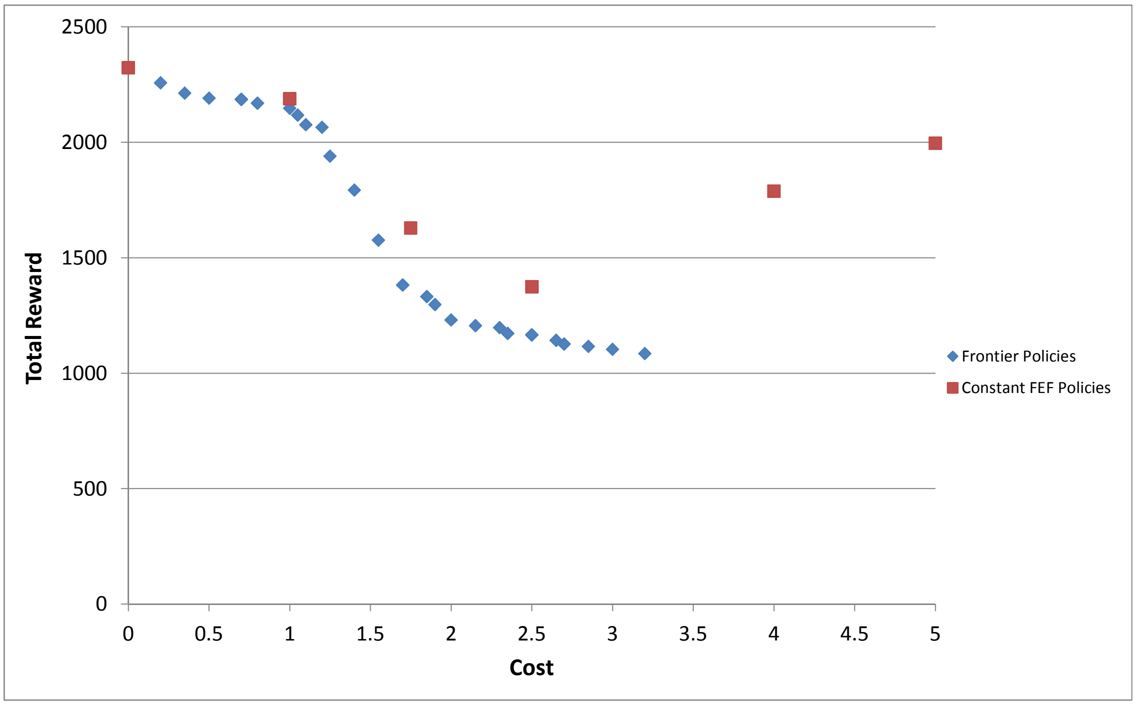

Figure 5.A.15: Constant FEF Policies and Policy Frontier: $\beta = 105$

5.A.4.4 System MTBF=205

Table 5.A.16: Frontier Policies: $\beta = 205$

Reward	Cost	Policy				
2286.9140	0	1	1	1	1	1
2209.0823	0.2	2	1	1	1	1
2118.0806	0.35	3	1	1	1	1
2052.8883	0.4	2	2	1	1	1
1907.2044	0.55	3	2	1	1	1
1827.6765	0.6	2	2	2	1	1

Table 5.A.16 Frontier Policies: $\beta = 205$ (Cont.)

Reward	Cost	Policy				
1707.5087	0.7	3	3	1	1	1
1606.7436	0.75	3	2	2	1	1
1435.4852	0.8	2	2	2	2	1
1124.1188	0.9	3	3	2	1	1
948.9158	1.05	3	3	3	1	1
916.1170	1.2	4	3	3	1	1
881.9114	1.35	4	4	3	1	1
880.1334	1.5	5	3	3	1	1
868.0933	1.7	6	4	2	1	1

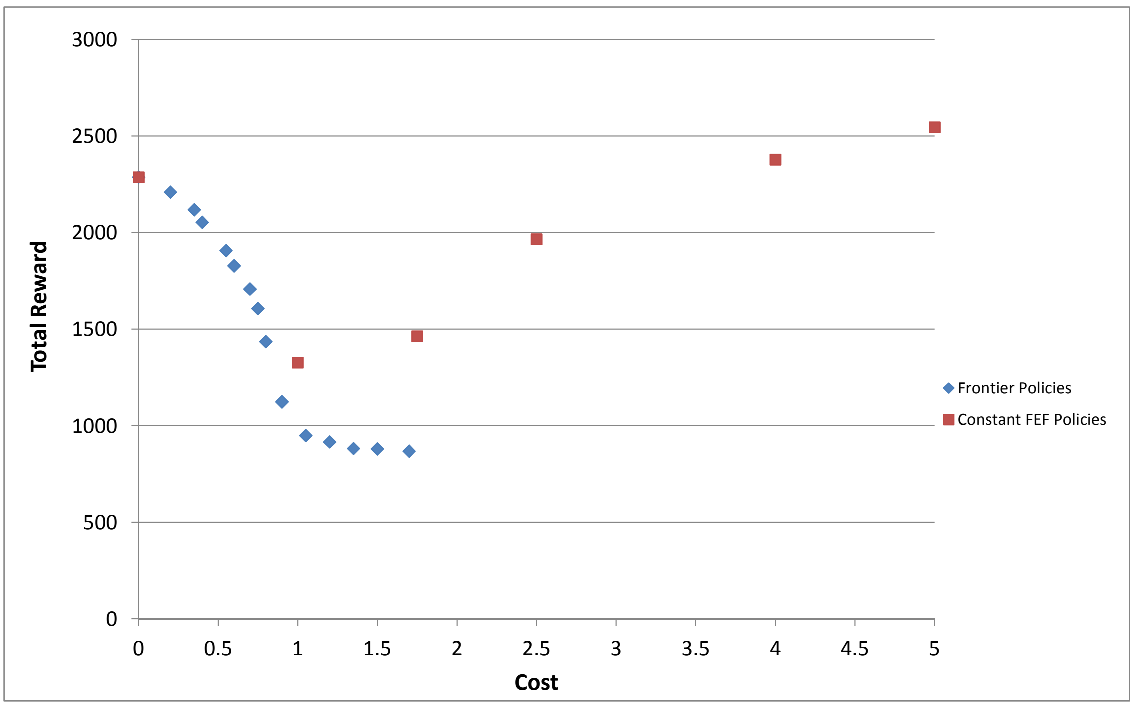

Figure 5.A.16: Constant FEF Policies and Policy Frontier: $\beta = 205$

5.A.5 Instance 6: Epoch Progressively Weighted, Deviations Above Curve Weighted at 0.1

5.A.5.1 System MTBF=25

Table 5.A.17: Frontier Policies: $\beta = 25$

Reward	Cost	Policy				
2337.8639	0	1	1	1	1	1
2271.8885	0.2	1	1	1	1	2
2227.9049	0.35	1	1	1	1	3
2205.9131	0.5	1	1	1	1	4
2205.9129	0.7	2	1	1	1	4
2183.9213	0.8	1	1	1	1	5
2161.9295	1	1	1	1	1	6
2161.9293	1.2	2	1	1	1	6
2161.9287	1.35	3	1	1	1	6
2161.9282	1.5	4	1	1	1	6
2161.9278	1.7	4	2	1	1	6
2161.9272	1.75	3	2	2	1	6
2161.9268	1.85	4	3	1	1	6
2161.9233	1.9	3	3	2	1	6
2161.9226	1.95	3	2	2	2	6
2161.9112	2.05	3	3	3	1	6
2161.9078	2.1	3	3	2	2	6
2161.8982	2.2	4	3	3	1	6
2161.8608	2.25	3	3	3	2	6
2161.7390	2.4	3	3	3	3	6
2161.6139	2.55	4	3	3	3	6
2161.4120	2.7	4	4	3	3	6
2161.0898	2.85	4	4	4	3	6
2160.6125	3	4	4	4	4	6
2159.8903	3.3	5	4	4	4	6
2158.8770	3.5	6	4	4	4	6
2158.8050	3.6	5	5	4	4	6
2157.3006	3.8	6	5	4	4	6
2157.1926	3.9	5	5	5	4	6
2155.1393	4	6	6	4	4	6
2154.9803	4.1	6	5	5	4	6
2154.9406	4.2	5	5	5	5	6
2151.8520	4.3	6	6	5	4	6
2151.7996	4.4	6	5	5	5	6
2147.4690	4.5	6	6	6	4	6
2147.4025	4.6	6	6	5	5	6
2141.3312	4.8	6	6	6	5	6

Table 5.A.17 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
2133.4394	5	6	6	6	6	6

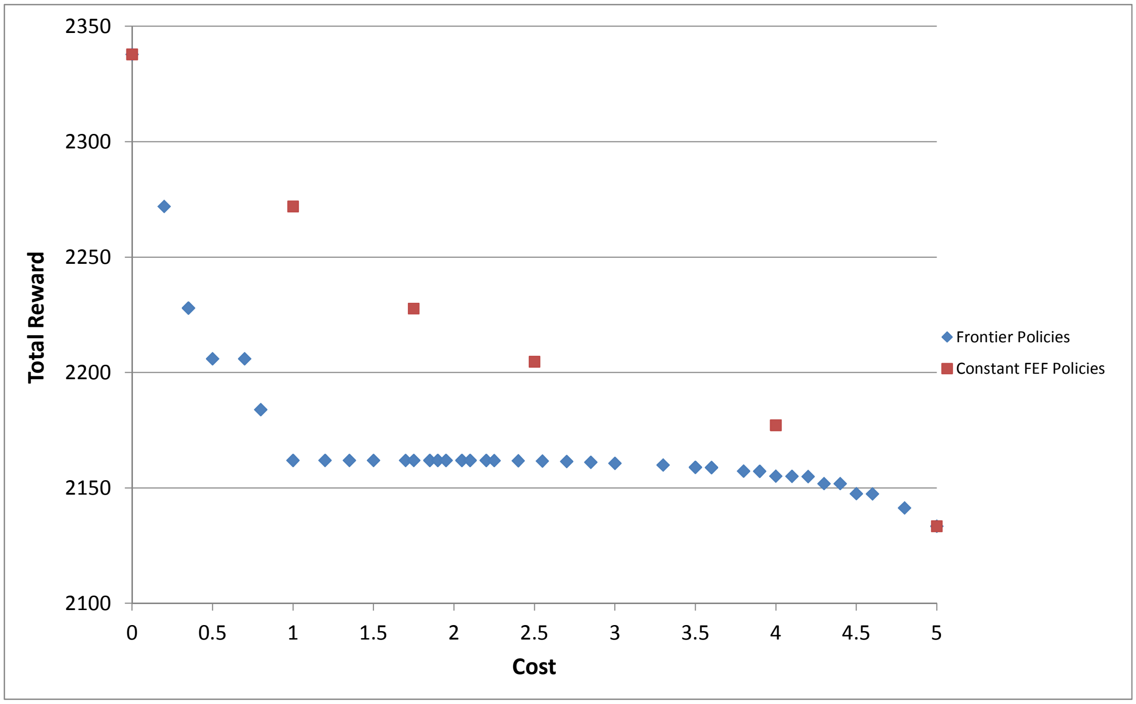

Figure 5.A.17: Constant FEF Policies and Policy Frontier: $\beta = 25$

5.A.5.2 System MTBF=50

Table 5.A.18: Frontier Policies: $\beta = 50$

Reward	Cost	Policy				
2328.7617	0	1	1	1	1	1
2262.7863	0.2	1	1	1	1	2
2218.8027	0.35	1	1	1	1	3
2196.8109	0.5	1	1	1	1	4
2196.7527	0.7	2	1	1	1	4
2174.8191	0.8	1	1	1	1	5
2152.8272	1	1	1	1	1	6
2152.7690	1.2	2	1	1	1	6
2152.6464	1.35	3	1	1	1	6
2152.5380	1.5	4	1	1	1	6
2152.3959	1.7	3	3	1	1	6
2152.1855	1.75	3	2	2	1	6
2152.1279	1.85	4	3	1	1	6

Table 5.A.18 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
2151.2970	1.9	3	3	2	1	6
2151.0503	1.95	3	2	2	2	6
2149.0863	2.05	3	3	3	1	6
2148.2274	2.1	3	3	2	2	6
2147.0371	2.2	4	3	3	1	6
2141.3145	2.25	3	3	3	2	6
2127.6055	2.35	4	4	4	3	4
2103.8817	2.5	4	4	4	4	4
2097.5756	2.65	5	4	4	4	3
2097.5688	2.7	6	4	4	3	3
2072.4006	2.8	5	4	4	4	4
2061.3332	2.85	6	4	4	4	3
2059.1335	2.95	5	5	4	4	3
2035.3773	3	6	4	4	4	4
2033.1281	3.1	5	5	4	4	4
2014.5598	3.15	6	5	4	4	3
2014.0644	3.2	6	6	4	3	3
2012.1317	3.25	5	5	5	4	3
1987.6819	3.3	6	5	4	4	4
1961.7427	3.35	6	6	4	4	3
1958.7814	3.45	6	5	5	4	3
1933.9421	3.5	6	6	4	4	4
1930.9271	3.6	6	5	5	4	4
1878.5065	3.65	6	6	5	4	3
1874.3019	3.7	6	6	6	4	2
1849.7291	3.8	6	6	5	4	4
1810.0204	3.85	6	6	6	4	3
1780.3992	4	6	6	6	4	4
1738.2340	4.15	6	6	6	5	3
1734.6498	4.2	6	6	6	6	2
1707.9008	4.3	6	6	6	5	4
1665.2442	4.35	6	6	6	6	3
1634.5198	4.5	6	6	6	6	4
1607.3869	4.8	6	6	6	6	5
1580.3846	5	6	6	6	6	6

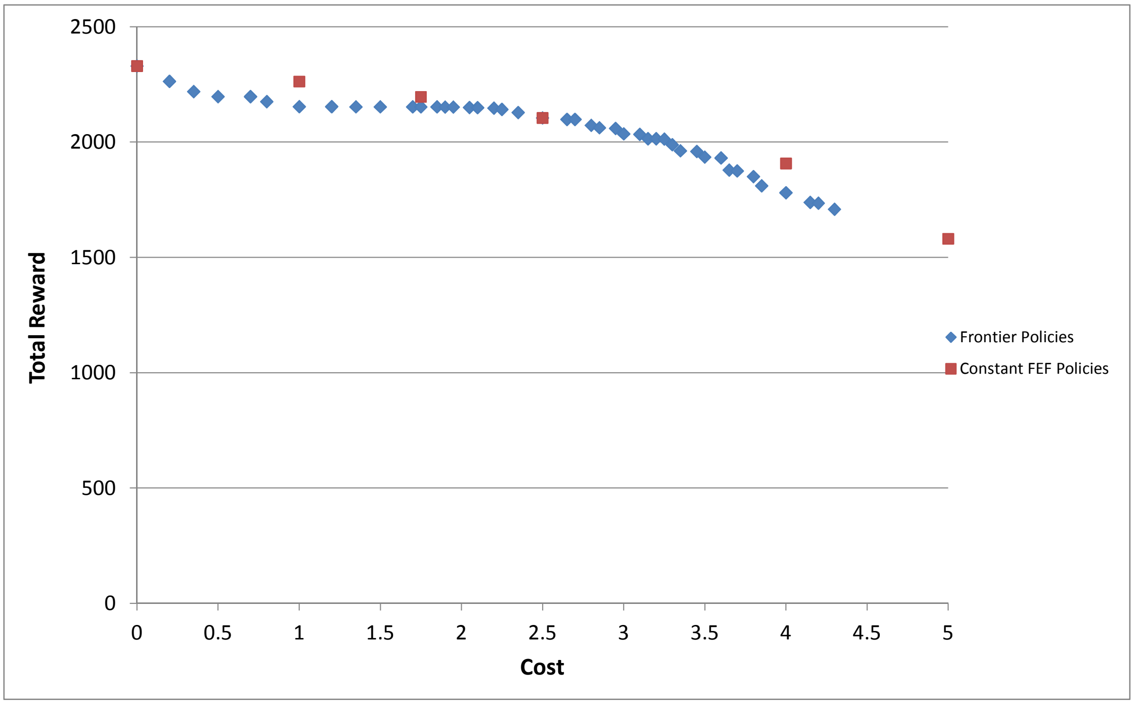

Figure 5.A.18: Constant FEF Policies and Policy Frontier: $\beta = 50$

5.A.5.3 System MTBF=105

Table 5.A.19: Frontier Policies: $\beta = 105$

Reward	Cost	Policy				
2314.2956	0	1	1	1	1	1
2248.3159	0.2	1	1	1	1	2
2204.3295	0.35	1	1	1	1	3
2182.3364	0.5	1	1	1	1	4
2176.3552	0.7	2	1	1	1	4
2160.3434	0.8	1	1	1	1	5
2138.3504	1	1	1	1	1	6
2106.7021	1.05	3	3	3	1	1
2065.0555	1.1	3	3	2	2	1
2052.9378	1.2	4	3	3	1	1
1925.3663	1.25	3	3	3	2	1
1772.5814	1.4	3	3	3	3	1
1538.9575	1.55	4	3	3	3	1
1306.5492	1.7	4	4	3	3	1
1242.6048	1.85	4	4	4	3	1
1167.4740	1.9	4	4	3	3	2
1054.7374	2	4	4	4	4	1
1030.0097	2.15	5	4	4	3	1

Table 5.A.19 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
942.1819	2.2	4	4	4	4	2
908.0878	2.35	4	4	4	4	3
882.7047	2.5	5	4	4	3	3
870.0451	2.55	6	4	4	3	2
865.7842	2.65	5	4	4	4	3
839.7956	2.7	6	4	4	3	3
824.2642	2.85	6	4	4	4	3
819.5861	2.9	6	6	3	3	2
754.6468	3	6	6	4	4	1
736.4866	3.15	6	6	5	3	1
710.1351	3.2	6	6	4	4	2
691.5317	3.35	6	6	5	3	2
686.9212	3.5	6	6	5	4	2
669.2898	3.55	6	6	6	3	2
665.3076	3.7	6	6	6	4	2
662.1284	4	6	6	6	5	2
659.6697	4.2	6	6	6	6	2

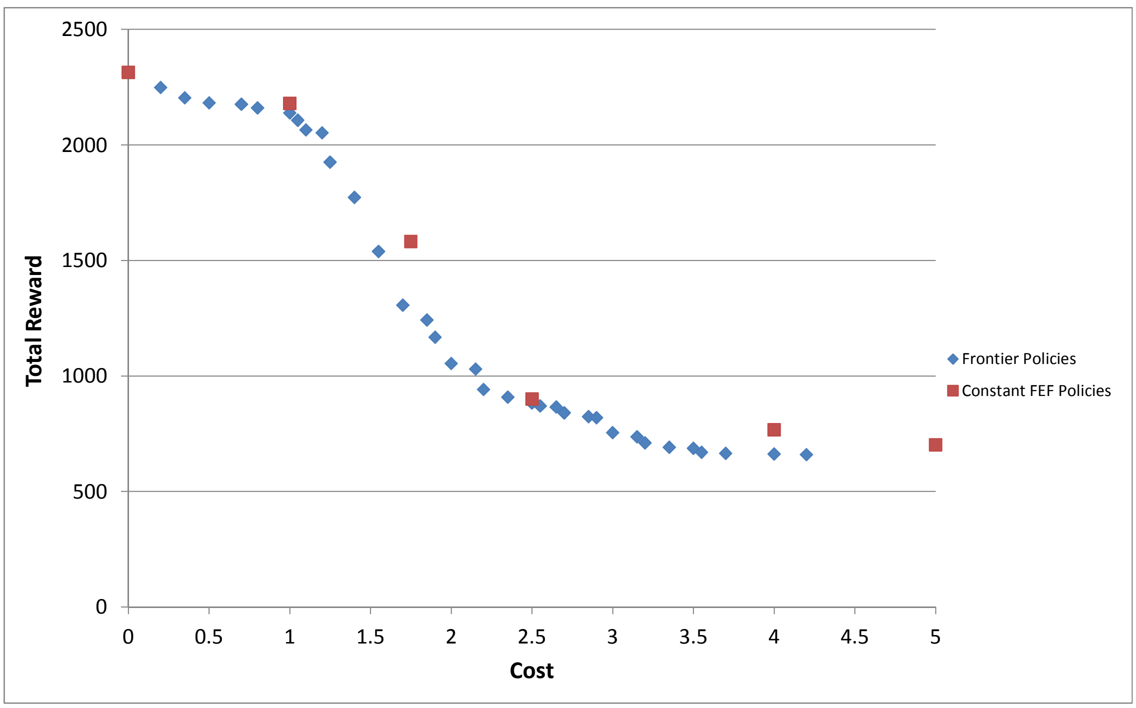

Figure 5.A.19: Constant FEF Policies and Policy Frontier: $\beta = 105$

5.A.5.4 System MTBF=205

Table 5.A.20: Frontier Policies: $\beta = 205$

Reward	Cost	Policy				
2266.8704	0	1	1	1	1	1
2185.7839	0.2	2	1	1	1	1
2089.7532	0.35	3	1	1	1	1
2027.1992	0.4	2	2	1	1	1
1874.0428	0.55	3	2	1	1	1
1796.3980	0.6	2	2	2	1	1
1663.0353	0.7	3	3	1	1	1
1561.5175	0.75	3	2	2	1	1
1377.4591	0.8	2	2	2	2	1
1012.6944	0.9	3	3	2	1	1
723.5440	1.05	3	3	3	1	1
717.5372	1.1	3	3	2	2	1
658.6198	1.2	4	3	3	1	1
528.4307	1.25	3	3	3	2	1
480.4280	1.4	4	3	3	2	1
400.7889	1.5	4	4	4	1	1
386.0955	1.65	5	4	3	1	1
352.7422	1.7	4	4	4	1	2
313.5829	1.85	4	4	4	3	1
295.1669	2	5	4	4	2	1
281.8635	2.05	6	4	3	2	1
260.3888	2.2	6	4	4	2	1
251.2301	2.35	6	5	3	2	1
248.5791	2.5	6	5	3	3	1
240.8003	2.55	6	6	3	2	1
238.4687	2.7	6	6	3	3	1
236.5124	2.85	6	6	4	3	1
235.8199	3.15	6	6	5	3	1
234.9616	3.2	6	6	6	2	1

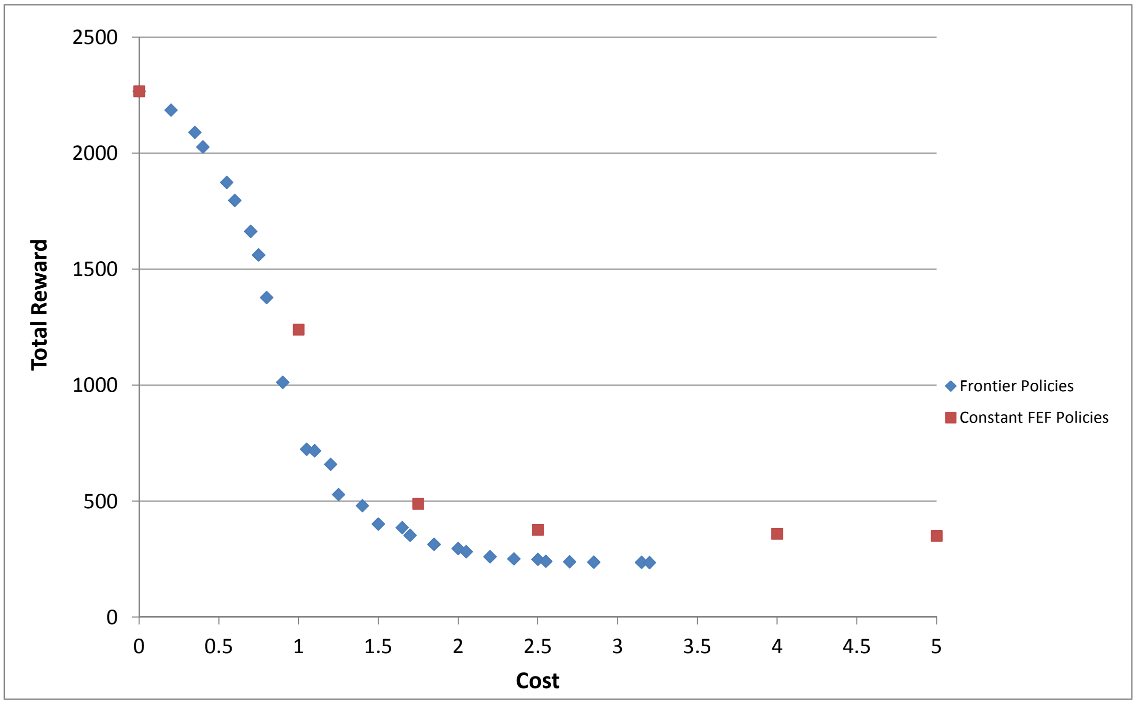

Figure 5.A.20: Constant FEF Policies and Policy Frontier: $\beta = 205$

5.A.6 Instance 7: Only Last Epoch Weighted, Deviations Weighted Equally

5.A.6.1 System MTBF=25

Table 5.A.21: Frontier Policies: $\beta = 25$

Reward	Cost	Policy				
808.4720	0	1	1	1	1	1
742.4966	0.2	1	1	1	1	2
698.5130	0.35	1	1	1	1	3
676.5212	0.5	1	1	1	1	4
654.5294	0.8	1	1	1	1	5
632.5376	1	1	1	1	1	6
632.5376	1.2	1	1	1	2	6
632.5376	1.35	1	1	1	3	6
632.5376	1.5	1	1	1	4	6
632.5376	1.8	1	1	1	5	6
632.5376	2	1	1	1	6	6
632.5376	1.55	1	1	2	3	6
632.5375	1.7	1	1	2	4	6
632.5375	2	1	1	2	5	6
632.5374	2.2	1	1	2	6	6
632.5374	1.85	1	1	3	4	6
632.5373	2.15	1	1	3	5	6
632.5370	2.35	1	1	3	6	6
632.5370	2.3	1	1	4	5	6
632.5366	2.5	1	1	4	6	6
632.5366	2.6	1	1	5	5	6
632.5359	2.8	1	1	5	6	6
632.5359	2.8	1	1	6	5	6
632.5348	3	1	1	6	6	6
632.5342	2.35	1	2	3	5	6
632.5321	2.55	1	2	3	6	6
632.5317	2.5	1	2	4	5	6
632.5281	2.7	1	2	4	6	6
632.5278	2.8	1	2	5	5	6
632.5219	3	1	2	5	6	6
632.5219	3	1	2	6	5	6
632.5124	3.2	1	2	6	6	6
632.5066	2.85	1	3	4	6	6
632.5057	2.95	1	3	5	5	6
632.4870	3.15	1	3	5	6	6
632.4870	3.15	1	3	6	5	6
632.4577	3.35	1	3	6	6	6

Table 5.A.29 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
632.4529	3.3	1	4	5	6	6
632.4529	3.3	1	4	6	5	6
632.4049	3.5	1	4	6	6	6
632.4012	3.6	1	5	5	6	6
632.4012	3.6	1	5	6	5	6
632.3256	3.8	1	5	6	6	6
632.3256	3.8	1	6	5	6	6
632.3256	3.8	1	6	6	5	6
632.2105	4	1	6	6	6	6
632.1491	3.35	2	3	5	6	6
632.1491	3.35	2	3	6	5	6
631.9449	3.55	2	3	6	6	6
631.9118	3.5	2	4	5	6	6
631.9118	3.5	2	4	6	5	6
631.5919	3.7	2	4	6	6	6
631.5683	3.8	2	5	5	6	6
631.5683	3.8	2	5	6	5	6
631.0865	4	2	5	6	6	6
631.0865	4	2	6	5	6	6
631.0865	4	2	6	6	5	6
630.3864	4.2	2	6	6	6	6
629.9843	3.85	3	4	6	6	6
629.9240	3.95	3	5	5	6	6
629.9240	3.95	3	5	6	5	6
628.7197	4.15	3	5	6	6	6
628.7197	4.15	3	6	5	6	6
628.7197	4.15	3	6	6	5	6
627.0259	4.35	3	6	6	6	6
626.7623	4.3	4	5	6	6	6
626.7623	4.3	4	6	5	6	6
626.7623	4.3	4	6	6	5	6
624.3040	4.5	4	6	6	6	6
624.1241	4.6	5	5	6	6	6
624.1241	4.6	5	6	5	6	6
624.1241	4.6	5	6	6	5	6
620.6908	4.8	5	6	6	6	6
620.6908	4.8	6	5	6	6	6
620.6908	4.8	6	6	5	6	6
620.6908	4.8	6	6	6	5	6
616.0847	5	6	6	6	6	6

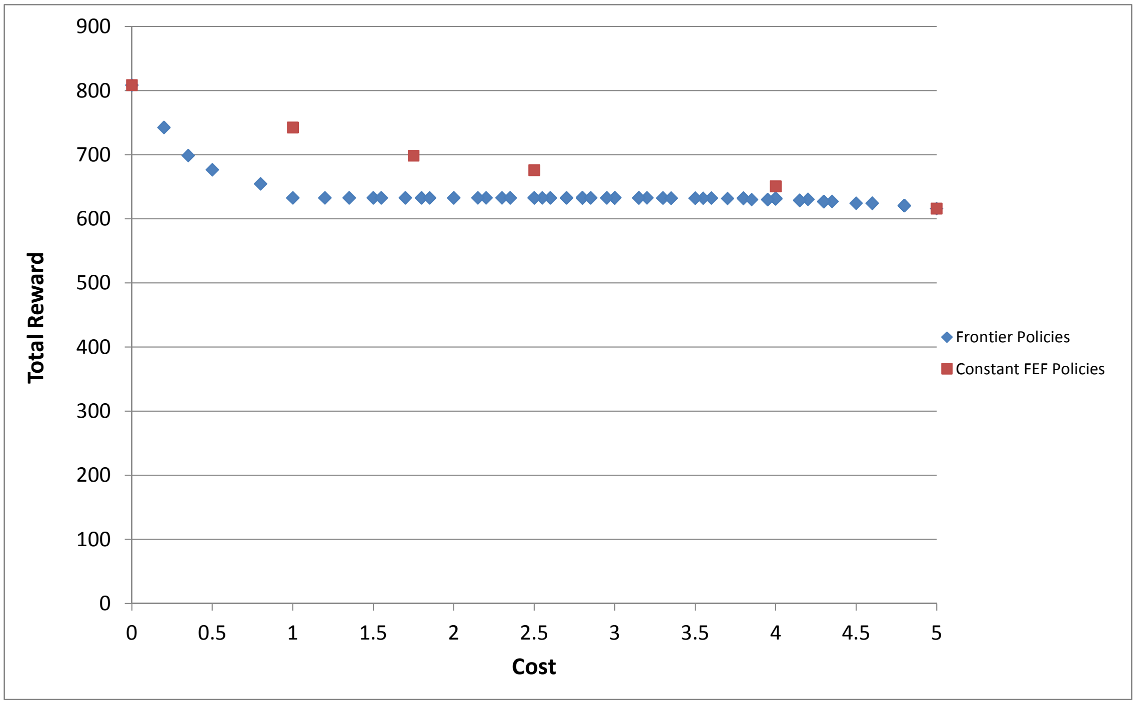

Figure 5.A.21: Constant FEF Policies and Policy Frontier: $\beta = 25$

5.A.6.2 System MTBF=50

Table 5.A.22: Frontier Policies: $\beta = 50$

Reward	Cost	Policy				
808.4720	0	1	1	1	1	1
742.4966	0.2	1	1	1	1	2
698.5130	0.35	1	1	1	1	3
676.5212	0.5	1	1	1	1	4
676.5208	0.7	1	1	1	2	4
676.5208	0.7	1	1	2	1	4
676.5208	0.7	1	2	1	1	4
676.5208	0.7	2	1	1	1	4
654.5294	0.8	1	1	1	1	5
632.5376	1	1	1	1	1	6
632.5372	1.2	1	1	1	2	6
632.5372	1.2	1	1	2	1	6
632.5372	1.2	1	2	1	1	6
632.5372	1.2	2	1	1	1	6
632.5362	1.35	1	1	1	3	6
632.5362	1.35	1	1	3	1	6
632.5362	1.35	1	3	1	1	6
632.5362	1.35	3	1	1	1	6

Table 5.A.22 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
632.5336	1.4	1	1	2	2	6
632.5336	1.4	1	2	1	2	6
632.5336	1.4	1	2	2	1	6
632.5336	1.4	2	1	1	2	6
632.5336	1.4	2	1	2	1	6
632.5336	1.4	2	2	1	1	6
632.5243	1.55	1	1	2	3	6
632.5243	1.55	1	1	3	2	6
632.5243	1.55	1	2	1	3	6
632.5243	1.55	1	2	3	1	6
632.5243	1.55	1	3	1	2	6
632.5243	1.55	1	3	2	1	6
632.5243	1.55	2	1	1	3	6
632.5243	1.55	2	1	3	1	6
632.5243	1.55	2	3	1	1	6
632.5243	1.55	3	1	1	2	6
632.5243	1.55	3	1	2	1	6
632.5243	1.55	3	2	1	1	6
632.5020	1.6	1	2	2	2	6
632.5020	1.6	2	1	2	2	6
632.5020	1.6	2	2	1	2	6
632.5020	1.6	2	2	2	1	6
632.4944	1.7	1	1	3	3	6
632.4944	1.7	1	3	1	3	6
632.4944	1.7	1	3	3	1	6
632.4944	1.7	3	1	1	3	6
632.4944	1.7	3	1	3	1	6
632.4944	1.7	3	3	1	1	6
632.4261	1.75	1	2	2	3	6
632.4261	1.75	1	2	3	2	6
632.4261	1.75	1	3	2	2	6
632.4261	1.75	2	1	2	3	6
632.4261	1.75	2	1	3	2	6
632.4261	1.75	2	2	1	3	6
632.4261	1.75	2	2	3	1	6
632.4261	1.75	2	3	1	2	6
632.4261	1.75	2	3	2	1	6
632.4261	1.75	3	1	2	2	6
632.4261	1.75	3	2	1	2	6
632.4261	1.75	3	2	2	1	6
632.2580	1.8	2	2	2	2	6

Table 5.A.22 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
632.2029	1.9	1	2	3	3	6
632.2029	1.9	1	3	2	3	6
632.2029	1.9	1	3	3	2	6
632.2029	1.9	2	1	3	3	6
632.2029	1.9	2	3	1	3	6
632.2029	1.9	2	3	3	1	6
632.2029	1.9	3	1	2	3	6
632.2029	1.9	3	1	3	2	6
632.2029	1.9	3	2	1	3	6
632.2029	1.9	3	2	3	1	6
632.2029	1.9	3	3	1	2	6
632.2029	1.9	3	3	2	1	6
631.7319	1.95	2	2	2	3	6
631.7319	1.95	2	2	3	2	6
631.7319	1.95	2	3	2	2	6
631.7319	1.95	3	2	2	2	6
631.5813	2.05	1	3	3	3	6
631.5813	2.05	3	1	3	3	6
631.5813	2.05	3	3	1	3	6
631.5813	2.05	3	3	3	1	6
630.3401	2.1	2	2	3	3	6
630.3401	2.1	2	3	2	3	6
630.3401	2.1	2	3	3	2	6
630.3401	2.1	3	2	2	3	6
630.3401	2.1	3	2	3	2	6
630.3401	2.1	3	3	2	2	6
626.9138	2.25	2	3	3	3	6
626.9138	2.25	3	2	3	3	6
626.9138	2.25	3	3	2	3	6
626.9138	2.25	3	3	3	2	6
619.1486	2.4	3	3	3	3	6
613.2519	2.55	3	3	3	4	6
613.2519	2.55	3	3	4	3	6
613.2519	2.55	3	4	3	3	6
613.2519	2.55	4	3	3	3	6
605.2319	2.7	3	3	4	4	6
605.2319	2.7	3	4	3	4	6
605.2319	2.7	3	4	4	3	6
605.2319	2.7	4	3	3	4	6
605.2319	2.7	4	3	4	3	6
605.2319	2.7	4	4	3	3	6

Table 5.A.22 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
603.7378	2.8	4	4	4	4	5
594.5910	2.85	3	4	4	4	6
594.5910	2.85	4	3	4	4	6
594.5910	2.85	4	4	3	4	6
594.5910	2.85	4	4	4	3	6
580.8515	3	4	4	4	4	6
564.7175	3.3	4	4	4	5	6
564.7175	3.3	4	4	5	4	6
564.7175	3.3	4	5	4	4	6
564.7175	3.3	5	4	4	4	6
543.5815	3.5	4	4	6	6	4
543.5815	3.5	4	6	4	6	4
543.5815	3.5	4	6	6	4	4
543.5815	3.5	6	4	4	6	4
543.5815	3.5	6	4	6	4	4
543.5815	3.5	6	6	4	4	4
542.0280	3.6	4	5	5	6	4
542.0280	3.6	4	5	6	5	4
542.0280	3.6	4	6	5	5	4
542.0280	3.6	5	4	5	6	4
542.0280	3.6	5	4	6	5	4
542.0280	3.6	5	5	4	6	4
542.0280	3.6	5	5	6	4	4
542.0280	3.6	5	6	4	5	4
542.0280	3.6	5	6	5	4	4
542.0280	3.6	6	4	5	5	4
542.0280	3.6	6	5	4	5	4
542.0280	3.6	6	5	5	4	4
537.8008	3.65	4	5	6	6	3
537.8008	3.65	4	6	5	6	3
537.8008	3.65	4	6	6	5	3
537.8008	3.65	5	4	6	6	3
537.8008	3.65	5	6	4	6	3
537.8008	3.65	5	6	6	4	3
537.8008	3.65	6	4	5	6	3
537.8008	3.65	6	4	6	5	3
537.8008	3.65	6	5	4	6	3
537.8008	3.65	6	5	6	4	3
537.8008	3.65	6	6	4	5	3
537.8008	3.65	6	6	5	4	3
535.9461	3.75	5	5	5	6	3

Table 5.A.22 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
535.9461	3.75	5	5	6	5	3
535.9461	3.75	5	6	5	5	3
535.9461	3.75	6	5	5	5	3
513.4330	3.8	4	5	6	6	4
513.4330	3.8	4	6	5	6	4
513.4330	3.8	4	6	6	5	4
513.4330	3.8	5	4	6	6	4
513.4330	3.8	5	6	4	6	4
513.4330	3.8	5	6	6	4	4
513.4330	3.8	6	4	5	6	4
513.4330	3.8	6	4	6	5	4
513.4330	3.8	6	5	4	6	4
513.4330	3.8	6	5	6	4	4
513.4330	3.8	6	6	4	5	4
513.4330	3.8	6	6	5	4	4
506.2937	3.85	4	6	6	6	3
506.2937	3.85	6	4	6	6	3
506.2937	3.85	6	6	4	6	3
506.2937	3.85	6	6	6	4	3
504.3522	3.95	5	5	6	6	3
504.3522	3.95	5	6	5	6	3
504.3522	3.95	5	6	6	5	3
504.3522	3.95	6	5	5	6	3
504.3522	3.95	6	5	6	5	3
504.3522	3.95	6	6	5	5	3
482.5922	4	4	6	6	6	4
482.5922	4	6	4	6	6	4
482.5922	4	6	6	4	6	4
482.5922	4	6	6	6	4	4
480.7066	4.1	5	5	6	6	4
480.7066	4.1	5	6	5	6	4
480.7066	4.1	5	6	6	5	4
480.7066	4.1	6	5	5	6	4
480.7066	4.1	6	5	6	5	4
480.7066	4.1	6	6	5	5	4
470.7709	4.15	5	6	6	6	3
470.7709	4.15	6	5	6	6	3
470.7709	4.15	6	6	5	6	3
470.7709	4.15	6	6	6	5	3
448.4075	4.3	5	6	6	6	4
448.4075	4.3	6	5	6	6	4

Table 5.A.22 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
448.4075	4.3	6	6	5	6	4
448.4075	4.3	6	6	6	5	4
436.4524	4.35	6	6	6	6	3
416.1282	4.5	6	6	6	6	4
402.3338	4.8	6	6	6	6	5
388.7769	5	6	6	6	6	6

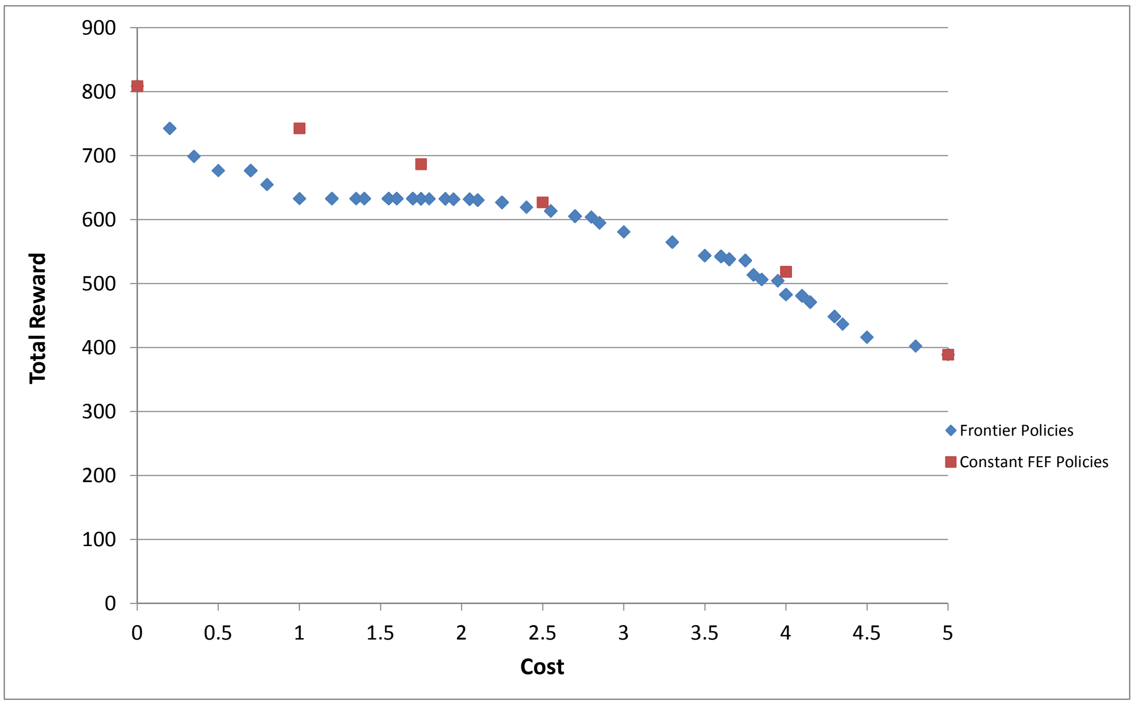

Figure 5.A.22: Constant FEF Policies and Policy Frontier: $\beta = 50$

5.A.6.3 System MTBF=105

Table 5.A.23: Frontier Policies: $\beta = 105$

Reward	Cost	Policy				
808.4576	0	1	1	1	1	1
742.4779	0.2	1	1	1	1	2
698.4916	0.35	1	1	1	1	3
676.4987	0.5	1	1	1	1	4
676.3388	0.7	1	1	1	2	4
676.3388	0.7	1	1	2	1	4
676.3388	0.7	1	2	1	1	4
676.3388	0.7	2	1	1	1	4

Table 5.A.23 Frontier Policies: β = 105 (Cont.)

Reward	Cost	Policy				
654.5058	0.8	1	1	1	1	5
632.5130	1	1	1	1	1	6
632.3394	1.2	1	1	1	2	6
632.3394	1.2	1	1	2	1	6
632.3394	1.2	1	2	1	1	6
632.3394	1.2	2	1	1	1	6
631.9570	1.35	1	1	1	3	6
631.9570	1.35	1	1	3	1	6
631.9570	1.35	1	3	1	1	6
631.9570	1.35	3	1	1	1	6
593.0880	1.4	3	3	3	3	1
570.0474	1.45	2	3	3	3	2
570.0474	1.45	3	2	3	3	2
570.0474	1.45	3	3	2	3	2
570.0474	1.45	3	3	3	2	2
489.3254	1.55	3	3	3	4	1
489.3254	1.55	3	3	4	3	1
489.3254	1.55	3	4	3	3	1
489.3254	1.55	4	3	3	3	1
483.5967	1.6	3	3	3	3	2
390.0044	1.7	3	3	4	4	1
390.0044	1.7	3	4	3	4	1
390.0044	1.7	3	4	4	3	1
390.0044	1.7	4	3	3	4	1
390.0044	1.7	4	3	4	3	1
390.0044	1.7	4	4	3	3	1
378.6796	1.75	3	3	3	4	2
378.6796	1.75	3	3	4	3	2
378.6796	1.75	3	4	3	3	2
378.6796	1.75	4	3	3	3	2
368.8638	1.85	3	4	4	4	1
368.8638	1.85	4	3	4	4	1
368.8638	1.85	4	4	3	4	1
368.8638	1.85	4	4	4	3	1
305.8650	1.9	3	3	4	4	2
305.8650	1.9	3	4	3	4	2
305.8650	1.9	3	4	4	3	2
305.8650	1.9	4	3	3	4	2
305.8650	1.9	4	3	4	3	2
305.8650	1.9	4	4	3	3	2
295.8045	2	4	4	4	4	1

Table 5.A.23 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
287.7024	2.3	4	4	4	5	1
287.7024	2.3	4	4	5	4	1
287.7024	2.3	4	5	4	4	1
287.7024	2.3	5	4	4	4	1
280.8541	2.5	4	4	4	6	1
280.8541	2.5	4	4	6	4	1
280.8541	2.5	4	6	4	4	1
280.8541	2.5	6	4	4	4	1
280.5588	2.6	4	4	5	5	1
280.5588	2.6	4	5	4	5	1
280.5588	2.6	4	5	5	4	1
280.5588	2.6	5	4	4	5	1
280.5588	2.6	5	4	5	4	1
280.5588	2.6	5	5	4	4	1
274.7531	2.8	4	4	5	6	1
274.7531	2.8	4	4	6	5	1
274.7531	2.8	4	5	4	6	1
274.7531	2.8	4	5	6	4	1
274.7531	2.8	4	6	4	5	1
274.7531	2.8	4	6	5	4	1
274.7531	2.8	5	4	4	6	1
274.7531	2.8	5	4	6	4	1
274.7531	2.8	5	6	4	4	1
274.7531	2.8	6	4	4	5	1
274.7531	2.8	6	4	5	4	1
274.7531	2.8	6	5	4	4	1
274.3885	2.9	4	5	5	5	1
274.3885	2.9	5	4	5	5	1
274.3885	2.9	5	5	4	5	1
274.3885	2.9	5	5	5	4	1
266.8586	3	4	4	6	6	1
266.8586	3	4	6	4	6	1
266.8586	3	4	6	6	4	1
266.8586	3	6	4	4	6	1
266.8586	3	6	4	6	4	1
266.8586	3	6	6	4	4	1
266.7153	3.15	3	5	6	6	1
266.7153	3.15	3	6	5	6	1
266.7153	3.15	3	6	6	5	1
266.7153	3.15	5	3	6	6	1
266.7153	3.15	5	6	3	6	1

Table 5.A.23 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
266.7153	3.15	5	6	6	3	1
266.7153	3.15	6	3	5	6	1
266.7153	3.15	6	3	6	5	1
266.7153	3.15	6	5	3	6	1
266.7153	3.15	6	5	6	3	1
266.7153	3.15	6	6	3	5	1
266.7153	3.15	6	6	5	3	1

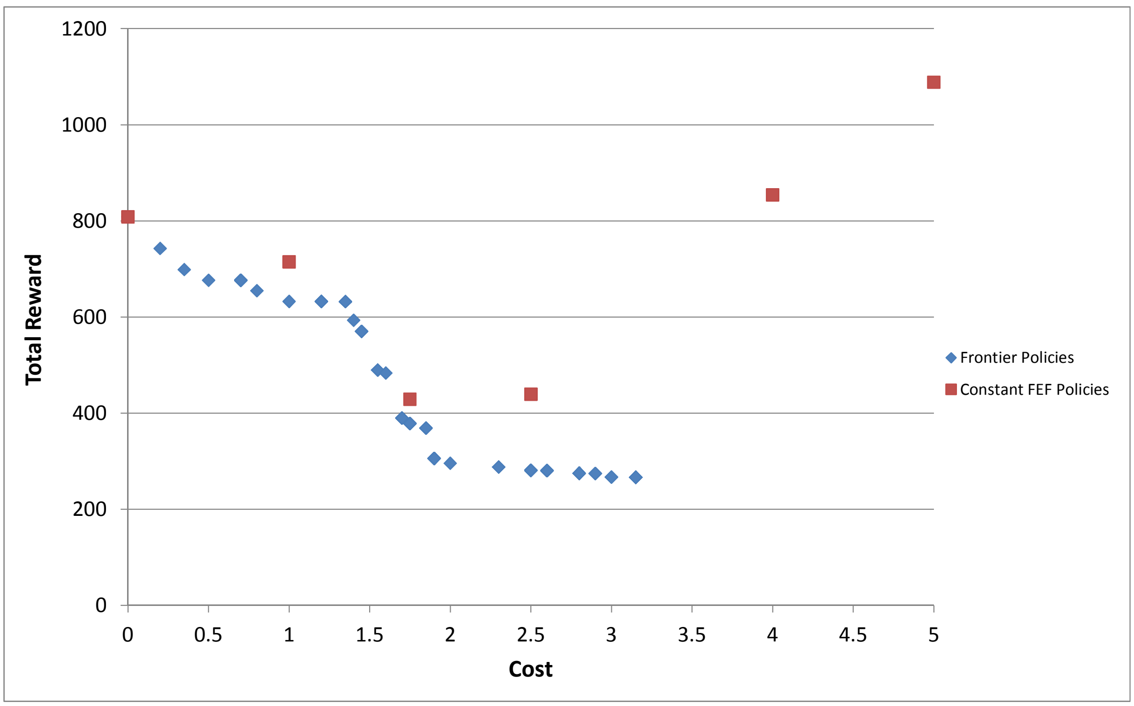

Figure 5.A.23: Constant FEF Policies and Policy Frontier: $\beta = 105$

5.A.6.4 System MTBF=205

Table 5.A.24: Frontier Policies: $\beta = 205$

Reward	Cost	Policy				
806.2611	0	1	1	1	1	1
739.6480	0.2	1	1	1	1	2
695.2842	0.35	1	1	1	1	3
673.1345	0.5	1	1	1	1	4
659.5490	0.7	1	1	1	2	4
659.5490	0.7	1	1	2	1	4
659.5490	0.7	1	2	1	1	4

Table 5.A.24 Frontier Policies: $\beta = 205$ (Cont.)

Reward	Cost	Policy				
659.5490	0.7	2	1	1	1	4
606.0077	0.75	1	2	2	3	1
606.0077	0.75	1	2	3	2	1
606.0077	0.75	1	3	2	2	1
606.0077	0.75	2	1	2	3	1
606.0077	0.75	2	1	3	2	1
606.0077	0.75	2	2	1	3	1
606.0077	0.75	2	2	3	1	1
606.0077	0.75	2	3	1	2	1
606.0077	0.75	2	3	2	1	1
606.0077	0.75	3	1	2	2	1
606.0077	0.75	3	2	1	2	1
606.0077	0.75	3	2	2	1	1
473.6156	0.8	2	2	2	2	1
392.7100	0.9	1	2	3	3	1
392.7100	0.9	1	3	2	3	1
392.7100	0.9	1	3	3	2	1
392.7100	0.9	2	1	3	3	1
392.7100	0.9	2	3	1	3	1
392.7100	0.9	2	3	3	1	1
392.7100	0.9	3	1	2	3	1
392.7100	0.9	3	1	3	2	1
392.7100	0.9	3	2	1	3	1
392.7100	0.9	3	2	3	1	1
392.7100	0.9	3	3	1	2	1
392.7100	0.9	3	3	2	1	1
354.2483	0.95	2	2	2	3	1
354.2483	0.95	2	2	3	2	1
354.2483	0.95	2	3	2	2	1
354.2483	0.95	3	2	2	2	1
295.0488	1.05	1	3	3	3	1
295.0488	1.05	3	1	3	3	1
295.0488	1.05	3	3	1	3	1
295.0488	1.05	3	3	3	1	1
280.1199	1.1	2	2	3	3	1
280.1199	1.1	2	3	2	3	1
280.1199	1.1	2	3	3	2	1
280.1199	1.1	3	2	2	3	1
280.1199	1.1	3	2	3	2	1
280.1199	1.1	3	3	2	2	1
267.8837	1.25	2	3	3	3	1

Table 5.A.24 Frontier Policies: $\beta = 205$ (Cont.)

Reward	Cost	Policy				
267.8837	1.25	3	2	3	3	1
267.8837	1.25	3	3	2	3	1
267.8837	1.25	3	3	3	2	1
267.3903	1.4	2	2	4	4	1
267.3903	1.4	2	4	2	4	1
267.3903	1.4	2	4	4	2	1
267.3903	1.4	4	2	2	4	1
267.3903	1.4	4	2	4	2	1
267.3903	1.4	4	4	2	2	1
265.8197	1.5	1	4	4	4	1
265.8197	1.5	4	1	4	4	1
265.8197	1.5	4	4	1	4	1
265.8197	1.5	4	4	4	1	1
265.7070	1.65	1	3	4	5	1
265.7070	1.65	1	3	5	4	1
265.7070	1.65	1	4	3	5	1
265.7070	1.65	1	4	5	3	1
265.7070	1.65	1	5	3	4	1
265.7070	1.65	1	5	4	3	1
265.7070	1.65	3	1	4	5	1
265.7070	1.65	3	1	5	4	1
265.7070	1.65	3	4	1	5	1
265.7070	1.65	3	4	5	1	1
265.7070	1.65	3	5	1	4	1
265.7070	1.65	3	5	4	1	1
265.7070	1.65	4	1	3	5	1
265.7070	1.65	4	1	5	3	1
265.7070	1.65	4	3	1	5	1
265.7070	1.65	4	3	5	1	1
265.7070	1.65	4	5	1	3	1
265.7070	1.65	4	5	3	1	1
265.7070	1.65	5	1	3	4	1
265.7070	1.65	5	1	4	3	1
265.7070	1.65	5	3	1	4	1
265.7070	1.65	5	3	4	1	1
265.7070	1.65	5	4	1	3	1
265.7070	1.65	5	4	3	1	1

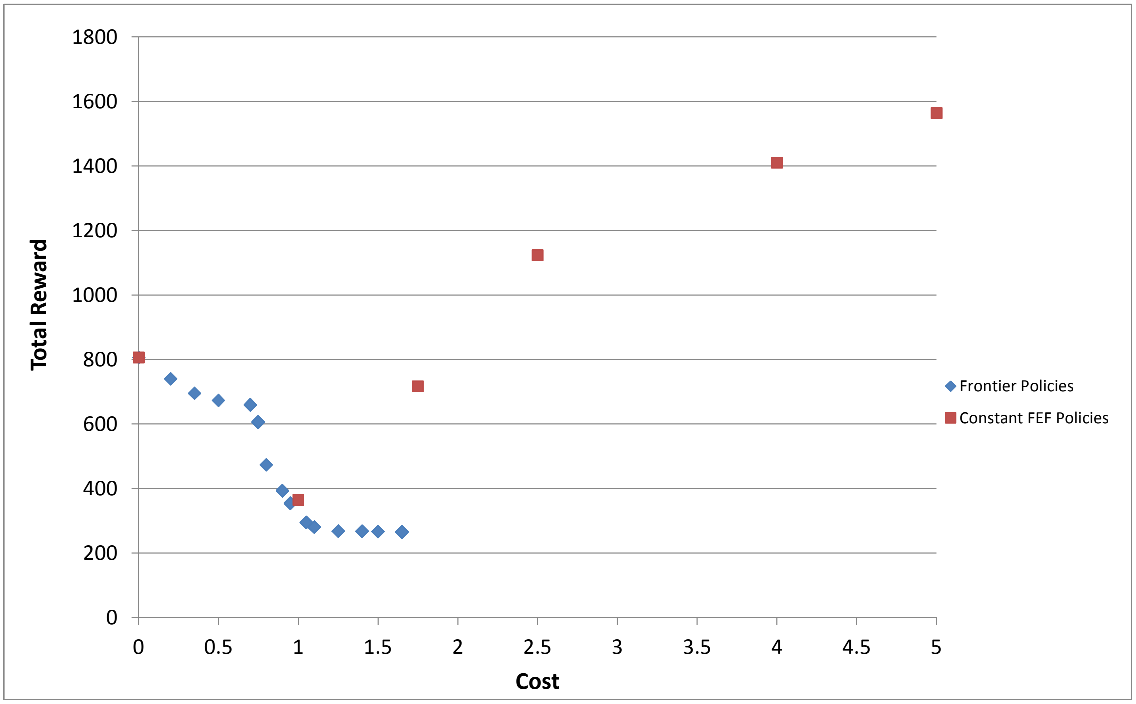

Figure 5.A.24: Constant FEF Policies and Policy Frontier: $\beta = 205$

5.A.7 Instance 8: Only Last Epoch Weighted, Deviations Above Curve Weighted at 0.5

5.A.7.1 System MTBF=25

Table 5.A.25: Frontier Policies: $\beta = 25$

Reward	Cost	Policy				
808.4720	0	1	1	1	1	1
742.4966	0.2	1	1	1	1	2
698.5130	0.35	1	1	1	1	3
676.5212	0.5	1	1	1	1	4
676.5212	0.7	1	1	1	2	4
676.5212	0.7	1	1	2	1	4
676.5212	0.7	1	2	1	1	4
676.5212	0.7	2	1	1	1	4
654.5294	0.8	1	1	1	1	5
632.5376	1	1	1	1	1	6
632.5376	1.2	1	1	1	2	6
632.5376	1.2	1	1	2	1	6
632.5376	1.2	1	2	1	1	6
632.5376	1.2	2	1	1	1	6
632.5376	1.35	1	1	1	3	6
632.5376	1.35	1	1	3	1	6
632.5376	1.35	1	3	1	1	6
632.5376	1.35	3	1	1	1	6
632.5376	1.4	1	1	2	2	6
632.5376	1.4	1	2	1	2	6
632.5376	1.4	1	2	2	1	6
632.5376	1.4	2	1	1	2	6
632.5376	1.4	2	1	2	1	6
632.5376	1.4	2	2	1	1	6
632.5376	1.55	1	1	2	3	6
632.5376	1.55	1	1	3	2	6
632.5376	1.55	1	2	1	3	6
632.5376	1.55	1	2	3	1	6
632.5376	1.55	1	3	1	2	6
632.5376	1.55	1	3	2	1	6
632.5376	1.55	2	1	1	3	6
632.5376	1.55	2	1	3	1	6
632.5376	1.55	2	3	1	1	6
632.5376	1.55	3	1	1	2	6
632.5376	1.55	3	1	2	1	6
632.5376	1.55	3	2	1	1	6
632.5375	1.6	1	2	2	2	6

Table 5.A.25 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
632.5375	1.6	2	1	2	2	6
632.5375	1.6	2	2	1	2	6
632.5375	1.6	2	2	2	1	6
632.5375	1.7	1	1	3	3	6
632.5375	1.7	1	3	1	3	6
632.5375	1.7	1	3	3	1	6
632.5375	1.7	3	1	1	3	6
632.5375	1.7	3	1	3	1	6
632.5375	1.7	3	3	1	1	6
632.5373	1.75	1	2	2	3	6
632.5373	1.75	1	2	3	2	6
632.5373	1.75	1	3	2	2	6
632.5373	1.75	2	1	2	3	6
632.5373	1.75	2	1	3	2	6
632.5373	1.75	2	2	1	3	6
632.5373	1.75	2	2	3	1	6
632.5373	1.75	2	3	1	2	6
632.5373	1.75	2	3	2	1	6
632.5373	1.75	3	1	2	2	6
632.5373	1.75	3	2	1	2	6
632.5373	1.75	3	2	2	1	6
632.5367	1.8	2	2	2	2	6
632.5364	1.9	1	2	3	3	6
632.5364	1.9	1	3	2	3	6
632.5364	1.9	1	3	3	2	6
632.5364	1.9	2	1	3	3	6
632.5364	1.9	2	3	1	3	6
632.5364	1.9	2	3	3	1	6
632.5364	1.9	3	1	2	3	6
632.5364	1.9	3	1	3	2	6
632.5364	1.9	3	2	1	3	6
632.5364	1.9	3	2	3	1	6
632.5364	1.9	3	3	1	2	6
632.5364	1.9	3	3	2	1	6
632.5344	1.95	2	2	3	2	6
632.5344	1.95	2	3	2	2	6
632.5344	1.95	3	2	2	2	6
632.5344	1.95	2	2	2	3	6
632.5336	2.05	1	3	3	3	6
632.5336	2.05	3	1	3	3	6
632.5336	2.05	3	3	1	3	6

Table 5.A.25 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
632.5336	2.05	3	3	3	1	6
632.5320	2.1	2	4	2	2	6
632.5267	2.1	3	3	2	2	6
632.5267	2.1	2	2	3	3	6
632.5267	2.1	2	3	2	3	6
632.5267	2.1	2	3	3	2	6
632.5267	2.1	3	2	2	3	6
632.5267	2.1	3	2	3	2	6
632.5189	2.25	2	2	3	4	6
632.5189	2.25	2	2	4	3	6
632.5189	2.25	2	3	2	4	6
632.5019	2.25	2	3	3	3	6
632.5019	2.25	3	2	3	3	6
632.5019	2.25	3	3	2	3	6
632.5019	2.25	3	3	3	2	6
632.4775	2.4	2	3	3	4	6
632.4775	2.4	2	3	4	3	6
632.4775	2.4	2	4	3	3	6
632.4775	2.4	3	2	3	4	6
632.4775	2.4	3	2	4	3	6
632.4775	2.4	3	3	2	4	6
632.4255	2.4	3	3	3	3	6
632.3525	2.55	3	3	3	4	6
632.3525	2.55	3	3	4	3	6
632.3525	2.55	3	4	3	3	6
632.3525	2.55	4	3	3	3	6
632.2344	2.7	3	3	4	4	6
632.2344	2.7	3	4	3	4	6
632.2344	2.7	3	4	4	3	6
632.2344	2.7	4	3	3	4	6
632.2344	2.7	4	3	4	3	6
632.2344	2.7	4	4	3	3	6
632.0462	2.85	3	4	4	4	6
632.0462	2.85	4	3	4	4	6
632.0462	2.85	4	4	3	4	6
632.0462	2.85	4	4	4	3	6
631.7489	3	4	4	4	4	6
631.3204	3.3	4	4	4	5	6
631.3204	3.3	4	4	5	4	6
631.3204	3.3	4	5	4	4	6
631.3204	3.3	5	4	4	4	6

Table 5.A.25 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
630.7215	3.5	4	4	4	6	6
630.7215	3.5	4	4	6	4	6
630.7215	3.5	4	6	4	4	6
630.7215	3.5	6	4	4	4	6
630.6782	3.6	4	4	5	5	6
630.6782	3.6	4	5	4	5	6
630.6782	3.6	4	5	5	4	6
630.6782	3.6	5	4	4	5	6
630.6782	3.6	5	4	5	4	6
630.6782	3.6	5	5	4	4	6
629.7929	3.8	4	4	5	6	6
629.7929	3.8	4	4	6	5	6
629.7929	3.8	4	5	4	6	6
629.7929	3.8	4	5	6	4	6
629.7929	3.8	4	6	4	5	6
629.7929	3.8	4	6	5	4	6
629.7929	3.8	5	4	4	6	6
629.7929	3.8	5	4	6	4	6
629.7929	3.8	5	6	4	4	6
629.7929	3.8	6	4	4	5	6
629.7929	3.8	6	4	5	4	6
629.7929	3.8	6	5	4	4	6
629.7277	3.9	4	5	5	5	6
629.7277	3.9	5	4	5	5	6
629.7277	3.9	5	5	4	5	6
629.7277	3.9	5	5	5	4	6
628.5264	4	4	4	6	6	6
628.5264	4	4	6	4	6	6
628.5264	4	4	6	6	4	6
628.5264	4	6	4	4	6	6
628.5264	4	6	4	6	4	6
628.5264	4	6	6	4	4	6
628.4318	4.1	4	5	5	6	6
628.4318	4.1	4	5	6	5	6
628.4318	4.1	4	6	5	5	6
628.4318	4.1	5	4	5	6	6
628.4318	4.1	5	4	6	5	6
628.4318	4.1	5	5	4	6	6
628.4318	4.1	5	5	6	4	6
628.4318	4.1	5	6	4	5	6
628.4318	4.1	5	6	5	4	6

Table 5.A.25 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
628.4318	4.1	6	4	5	5	6
628.4318	4.1	6	5	4	5	6
628.4318	4.1	6	5	5	4	6
628.3386	4.2	5	5	5	5	6
626.6087	4.3	4	5	6	6	6
626.6087	4.3	4	6	5	6	6
626.6087	4.3	4	6	6	5	6
626.6087	4.3	5	4	6	6	6
626.6087	4.3	5	6	4	6	6
626.6087	4.3	5	6	6	4	6
626.6087	4.3	6	4	5	6	6
626.6087	4.3	6	4	6	5	6
626.6087	4.3	6	5	4	6	6
626.6087	4.3	6	5	6	4	6
626.6087	4.3	6	6	4	5	6
626.6087	4.3	6	6	5	4	6
626.4760	4.4	5	5	5	6	6
626.4760	4.4	5	5	6	5	6
626.4760	4.4	5	6	5	5	6
626.4760	4.4	6	5	5	5	6
624.0688	4.5	4	6	6	6	6
624.0688	4.5	6	4	6	6	6
624.0688	4.5	6	6	4	6	6
624.0688	4.5	6	6	6	4	6
623.8826	4.6	5	5	6	6	6
623.8826	4.6	5	6	5	6	6
623.8826	4.6	5	6	6	5	6
623.8826	4.6	6	5	5	6	6
623.8826	4.6	6	5	6	5	6
623.8826	4.6	6	6	5	5	6
620.3239	4.8	5	6	6	6	6
620.3239	4.8	6	5	6	6	6
620.3239	4.8	6	6	5	6	6
620.3239	4.8	6	6	6	5	6
615.5326	5	6	6	6	6	6

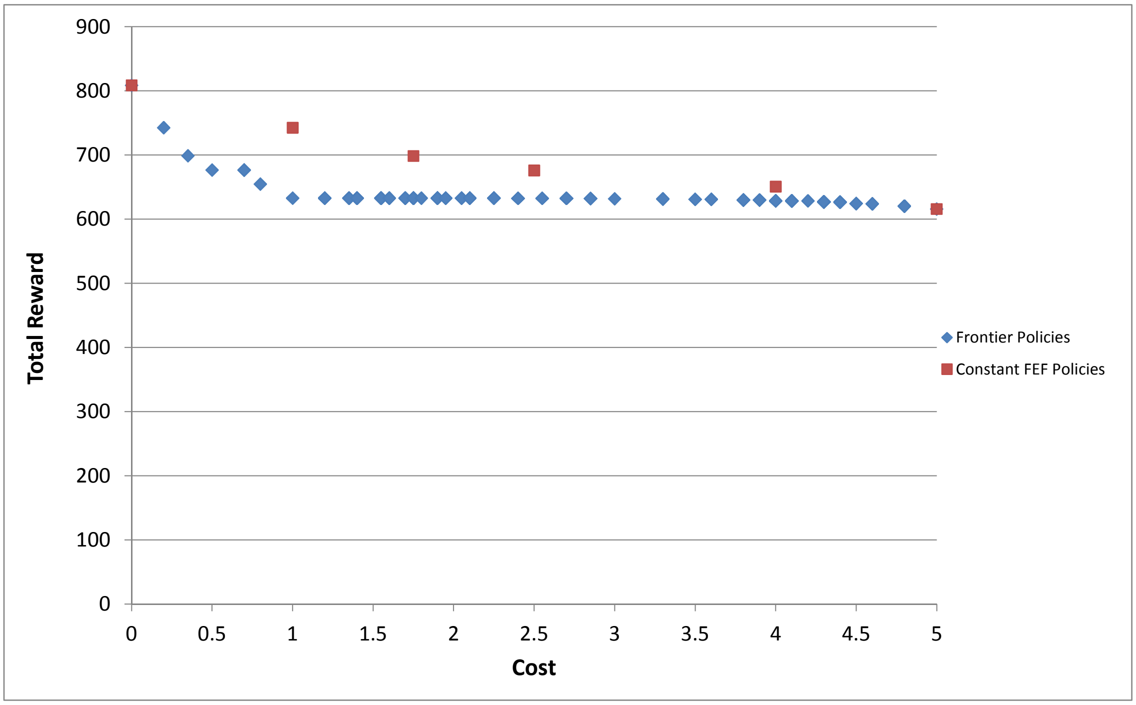

Figure 5.A.25: Constant FEF Policies and Policy Frontier: $\beta = 25$

5.A.7.2 System MTBF=50

Table 5.A.26: Frontier Policies: $\beta = 50$

Reward	Cost	Policy				
808.4720	0	1	1	1	1	1
742.4966	0.2	1	1	1	1	2
698.5130	0.35	1	1	1	1	3
676.5212	0.5	1	1	1	1	4
676.5208	0.7	1	1	1	2	4
676.5208	0.7	1	1	2	1	4
676.5208	0.7	1	2	1	1	4
676.5208	0.7	2	1	1	1	4
654.5294	0.8	1	1	1	1	5
632.5376	1	1	1	1	1	6
632.5372	1.2	1	1	1	2	6
632.5372	1.2	1	1	2	1	6
632.5372	1.2	1	2	1	1	6
632.5372	1.2	2	1	1	1	6
632.5362	1.35	1	1	1	3	6
632.5362	1.35	1	1	3	1	6
632.5362	1.35	1	3	1	1	6
632.5362	1.35	3	1	1	1	6

Table 5.A.30 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
632.5336	1.4	1	1	2	2	6
632.5336	1.4	1	2	1	2	6
632.5336	1.4	1	2	2	1	6
632.5336	1.4	2	1	1	2	6
632.5336	1.4	2	1	2	1	6
632.5336	1.4	2	2	1	1	6
632.5241	1.55	1	1	2	3	6
632.5241	1.55	1	1	3	2	6
632.5241	1.55	1	2	1	3	6
632.5241	1.55	1	2	3	1	6
632.5241	1.55	1	3	1	2	6
632.5241	1.55	1	3	2	1	6
632.5241	1.55	2	1	1	3	6
632.5241	1.55	2	1	3	1	6
632.5241	1.55	2	3	1	1	6
632.5241	1.55	3	1	1	2	6
632.5241	1.55	3	1	2	1	6
632.5241	1.55	3	2	1	1	6
632.5015	1.6	1	2	2	2	6
632.5015	1.6	2	1	2	2	6
632.5015	1.6	2	2	1	2	6
632.5015	1.6	2	2	2	1	6
632.4938	1.7	1	1	3	3	6
632.4938	1.7	1	3	1	3	6
632.4938	1.7	1	3	3	1	6
632.4938	1.7	3	1	1	3	6
632.4938	1.7	3	1	3	1	6
632.4938	1.7	3	3	1	1	6
632.4243	1.75	1	2	2	3	6
632.4243	1.75	1	2	3	2	6
632.4243	1.75	1	3	2	2	6
632.4243	1.75	2	1	2	3	6
632.4243	1.75	2	1	3	2	6
632.4243	1.75	2	2	1	3	6
632.4243	1.75	2	2	3	1	6
632.4243	1.75	2	3	1	2	6
632.4243	1.75	2	3	2	1	6
632.4243	1.75	3	1	2	2	6
632.4243	1.75	3	2	1	2	6
632.4243	1.75	3	2	2	1	6
632.2531	1.8	2	2	2	2	6

Table 5.A.30 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
632.1969	1.9	1	2	3	3	6
632.1969	1.9	1	3	2	3	6
632.1969	1.9	1	3	3	2	6
632.1969	1.9	2	1	3	3	6
632.1969	1.9	2	3	1	3	6
632.1969	1.9	2	3	3	1	6
632.1969	1.9	3	1	2	3	6
632.1969	1.9	3	1	3	2	6
632.1969	1.9	3	2	1	3	6
632.1969	1.9	3	2	3	1	6
632.1969	1.9	3	3	1	2	6
632.1969	1.9	3	3	2	1	6
631.7161	1.95	2	2	3	2	6
631.7161	1.95	2	3	2	2	6
631.7161	1.95	3	2	2	2	6
631.7161	1.95	2	2	2	3	6
631.5621	2.05	1	3	3	3	6
631.5621	2.05	3	1	3	3	6
631.5621	2.05	3	3	1	3	6
631.5621	2.05	3	3	3	1	6
630.2907	2.1	3	3	2	2	6
630.2907	2.1	2	2	3	3	6
630.2907	2.1	2	3	2	3	6
630.2907	2.1	2	3	3	2	6
630.2907	2.1	3	2	2	3	6
630.2907	2.1	3	2	3	2	6
626.7650	2.25	2	3	3	3	6
626.7650	2.25	3	2	3	3	6
626.7650	2.25	3	3	2	3	6
626.7650	2.25	3	3	3	2	6
618.7217	2.4	3	3	3	3	6
612.5770	2.55	3	3	3	4	6
612.5770	2.55	3	3	4	3	6
612.5770	2.55	3	4	3	3	6
612.5770	2.55	4	3	3	3	6
604.1763	2.7	3	3	4	4	6
604.1763	2.7	3	4	3	4	6
604.1763	2.7	3	4	4	3	6
604.1763	2.7	4	3	3	4	6
604.1763	2.7	4	3	4	3	6
604.1763	2.7	4	4	3	3	6

Table 5.A.30 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
601.8816	2.8	4	4	4	4	5
592.9596	2.85	3	4	4	4	6
592.9596	2.85	4	3	4	4	6
592.9596	2.85	4	4	3	4	6
592.9596	2.85	4	4	4	3	6
578.3627	3	4	4	4	4	6
561.0591	3.3	4	4	4	5	6
561.0591	3.3	4	4	5	4	6
561.0591	3.3	4	5	4	4	6
561.0591	3.3	5	4	4	4	6
538.0363	3.5	4	4	6	6	4
538.0363	3.5	4	6	4	6	4
538.0363	3.5	4	6	6	4	4
538.0363	3.5	6	4	4	6	4
538.0363	3.5	6	4	6	4	4
538.0363	3.5	6	6	4	4	4
536.3724	3.6	4	5	6	5	4
536.3724	3.6	4	6	5	5	4
536.3724	3.6	5	4	6	5	4
536.3724	3.6	5	6	4	5	4
536.3724	3.6	6	4	5	5	4
536.3724	3.6	6	5	4	5	4
536.3724	3.6	4	5	5	6	4
536.3724	3.6	5	4	5	6	4
536.3724	3.6	5	5	4	6	4
536.3724	3.6	5	5	6	4	4
536.3724	3.6	5	6	5	4	4
536.3724	3.6	6	5	5	4	4
532.3219	3.65	4	5	6	6	3
532.3219	3.65	4	6	5	6	3
532.3219	3.65	4	6	6	5	3
532.3219	3.65	5	4	6	6	3
532.3219	3.65	5	6	4	6	3
532.3219	3.65	5	6	6	4	3
532.3219	3.65	6	4	5	6	3
532.3219	3.65	6	4	6	5	3
532.3219	3.65	6	5	4	6	3
532.3219	3.65	6	5	6	4	3
532.3219	3.65	6	6	4	5	3
532.3219	3.65	6	6	5	4	3
530.3507	3.75	5	5	5	6	3

Table 5.A.30 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
530.3507	3.75	5	5	6	5	3
530.3507	3.75	5	6	5	5	3
530.3507	3.75	6	5	5	5	3
505.5043	3.8	4	5	6	6	4
505.5043	3.8	4	6	5	6	4
505.5043	3.8	4	6	6	5	4
505.5043	3.8	5	4	6	6	4
505.5043	3.8	5	6	4	6	4
505.5043	3.8	5	6	6	4	4
505.5043	3.8	6	4	5	6	4
505.5043	3.8	6	4	6	5	4
505.5043	3.8	6	5	4	6	4
505.5043	3.8	6	5	6	4	4
505.5043	3.8	6	6	4	5	4
505.5043	3.8	6	6	5	4	4
498.6063	3.85	4	6	6	6	3
498.6063	3.85	6	4	6	6	3
498.6063	3.85	6	6	4	6	3
498.6063	3.85	6	6	6	4	3
496.5114	3.95	5	6	6	5	3
496.5114	3.95	6	5	6	5	3
496.5114	3.95	6	6	5	5	3
496.5114	3.95	5	5	6	6	3
496.5114	3.95	5	6	5	6	3
496.5114	3.95	6	5	5	6	3
471.6160	4	4	6	6	6	4
471.6160	4	6	4	6	6	4
471.6160	4	6	6	4	6	4
471.6160	4	6	6	6	4	4
469.5206	4.1	5	5	6	6	4
469.5206	4.1	5	6	5	6	4
469.5206	4.1	5	6	6	5	4
469.5206	4.1	6	5	5	6	4
469.5206	4.1	6	5	6	5	4
469.5206	4.1	6	6	5	5	4
459.8983	4.15	5	6	6	6	3
459.8983	4.15	6	5	6	6	3
459.8983	4.15	6	6	5	6	3
459.8983	4.15	6	6	6	5	3
433.1072	4.3	5	6	6	6	4
433.1072	4.3	6	5	6	6	4

Table 5.A.30 Frontier Policies: β = 50 (Cont.)

Reward	Cost	Policy				
433.1072	4.3	6	6	5	6	4
433.1072	4.3	6	6	6	5	4
421.5534	4.35	6	6	6	6	3
395.4514	4.5	6	6	6	6	4
374.2467	4.8	6	6	6	6	5
353.2201	5	6	6	6	6	6

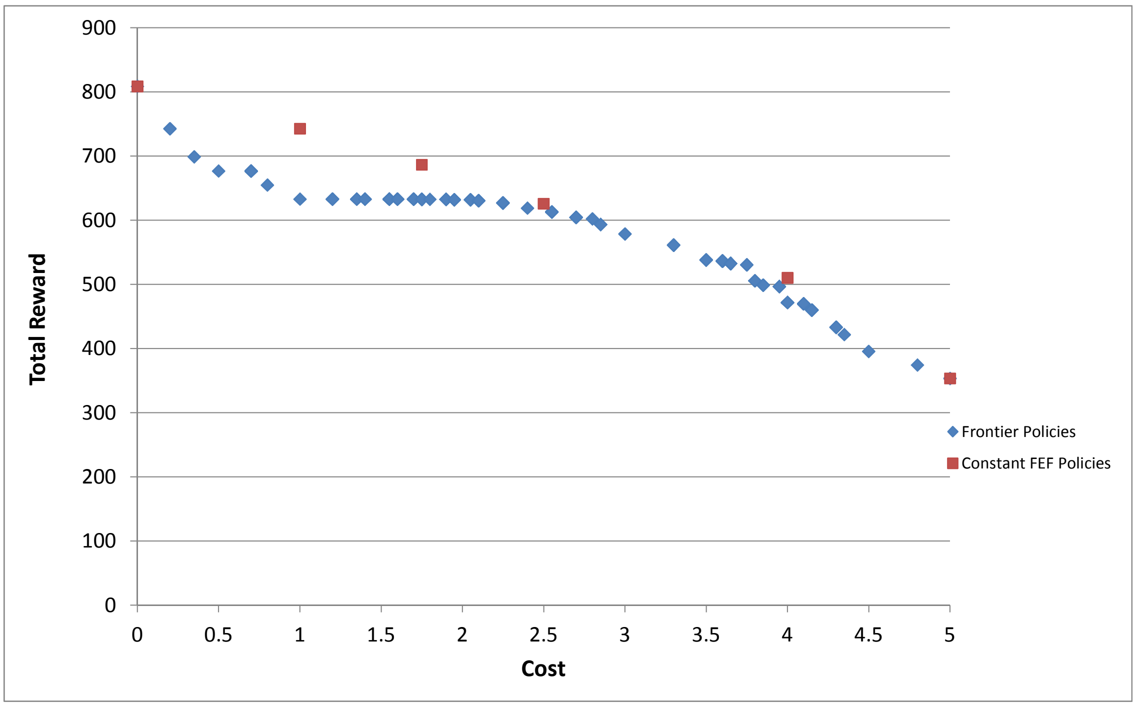

Figure 5.A.26: Constant FEF Policies and Policy Frontier: β = 50

5.A.7.3 System MTBF=105

Table 5.A.27: Frontier Policies: β = 105

Reward	Cost	Policy				
808.4576	0	1	1	1	1	1
742.4779	0.2	1	1	1	1	2
698.4916	0.35	1	1	1	1	3
676.4985	0.5	1	1	1	1	4
676.3374	0.7	1	1	1	2	4
676.3374	0.7	1	1	2	1	4
676.3374	0.7	1	2	1	1	4
676.3374	0.7	2	1	1	1	4

Table 5.A.27 Frontier Policies: β = 105 (Cont.)

Reward	Cost	Policy				
654.5056	0.8	1	1	1	1	5
632.5126	1	1	1	1	1	6
632.3361	1.2	1	1	1	2	6
632.3361	1.2	1	1	2	1	6
632.3361	1.2	1	2	1	1	6
632.3361	1.2	2	1	1	1	6
631.9461	1.35	1	1	1	3	6
631.9461	1.35	1	1	3	1	6
631.9461	1.35	1	3	1	1	6
631.9461	1.35	3	1	1	1	6
591.3836	1.4	3	3	3	3	1
566.9113	1.45	3	3	3	2	2
566.9113	1.45	2	3	3	3	2
566.9113	1.45	3	2	3	3	2
566.9113	1.45	3	3	2	3	2
484.4600	1.55	3	3	3	4	1
484.4600	1.55	3	3	4	3	1
484.4600	1.55	3	4	3	3	1
484.4600	1.55	4	3	3	3	1
476.1062	1.6	3	3	3	3	2
377.7519	1.7	3	3	4	4	1
377.7519	1.7	3	4	3	4	1
377.7519	1.7	4	3	3	4	1
377.7519	1.7	3	4	4	3	1
377.7519	1.7	4	3	4	3	1
377.7519	1.7	4	4	3	3	1
359.5862	1.75	3	3	3	4	2
359.5862	1.75	3	3	4	3	2
359.5862	1.75	3	4	3	3	2
359.5862	1.75	4	3	3	3	2
353.8602	1.85	3	4	4	4	1
353.8602	1.85	4	3	4	4	1
353.8602	1.85	4	4	3	4	1
353.8602	1.85	4	4	4	3	1
263.0926	1.9	3	3	4	4	2
263.0926	1.9	3	4	3	4	2
263.0926	1.9	4	3	3	4	2
263.0926	1.9	3	4	4	3	2
263.0926	1.9	4	3	4	3	2
263.0926	1.9	4	4	3	3	2
262.3181	2	4	4	4	4	1

Table 5.A.27 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
228.6033	2.05	3	3	4	4	3
228.6033	2.05	3	4	3	4	3
228.6033	2.05	4	3	3	4	3
228.6033	2.05	3	4	4	3	3
228.6033	2.05	4	3	4	3	3
228.6033	2.05	4	4	3	3	3
203.3641	2.2	4	4	4	4	2
201.9374	2.5	4	4	4	5	2
201.9374	2.5	4	4	5	4	2
201.9374	2.5	4	5	4	4	2
201.9374	2.5	5	4	4	4	2
201.9112	2.85	3	4	5	6	2
201.9112	2.85	3	5	4	6	2
201.9112	2.85	3	5	6	4	2
201.9112	2.85	4	3	5	6	2
201.9112	2.85	4	5	3	6	2
201.9112	2.85	4	5	6	3	2
201.9112	2.85	4	6	5	3	2
201.9112	2.85	5	3	4	6	2
201.9112	2.85	5	3	6	4	2
201.9112	2.85	5	4	3	6	2
201.9112	2.85	5	4	6	3	2
201.9112	2.85	5	6	3	4	2
201.9112	2.85	5	6	4	3	2
201.9112	2.85	6	4	5	3	2
201.9112	2.85	6	5	3	4	2
201.9112	2.85	6	5	4	3	2
201.9112	2.85	3	4	6	5	2
201.9112	2.85	3	6	4	5	2
201.9112	2.85	3	6	5	4	2
201.9112	2.85	4	3	6	5	2
201.9112	2.85	4	6	3	5	2
201.9112	2.85	6	3	4	5	2
201.9112	2.85	6	3	5	4	2
201.9112	2.85	6	4	3	5	2
201.8380	2.9	3	3	6	6	2
201.8380	2.9	3	6	3	6	2
201.8380	2.9	3	6	6	3	2
201.8380	2.9	6	3	3	6	2
201.8380	2.9	6	3	6	3	2
201.8380	2.9	6	6	3	3	2

Table 5.A.27 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
181.9913	3	4	4	6	6	1
181.9913	3	4	6	4	6	1
181.9913	3	4	6	6	4	1
181.9913	3	6	4	4	6	1
181.9913	3	6	4	6	4	1
181.9913	3	6	6	4	4	1
181.8703	3.1	4	5	6	5	1
181.8703	3.1	4	6	5	5	1
181.8703	3.1	5	4	6	5	1
181.8703	3.1	5	6	4	5	1
181.8703	3.1	6	4	5	5	1
181.8703	3.1	6	5	4	5	1
181.8703	3.1	4	5	5	6	1
181.8703	3.1	5	4	5	6	1
181.8703	3.1	5	5	4	6	1
181.8703	3.1	5	5	6	4	1
181.8703	3.1	5	6	5	4	1
181.8703	3.1	6	5	5	4	1
181.6302	3.2	5	5	5	5	1
179.0577	3.3	4	5	6	6	1
179.0577	3.3	4	6	5	6	1
179.0577	3.3	4	6	6	5	1
179.0577	3.3	5	4	6	6	1
179.0577	3.3	5	6	4	6	1
179.0577	3.3	5	6	6	4	1
179.0577	3.3	6	4	5	6	1
179.0577	3.3	6	4	6	5	1
179.0577	3.3	6	5	4	6	1
179.0577	3.3	6	5	6	4	1
179.0577	3.3	6	6	4	5	1
179.0577	3.3	6	6	5	4	1
178.9538	3.4	5	5	5	6	1
178.9538	3.4	5	5	6	5	1
178.9538	3.4	5	6	5	5	1
178.9538	3.4	6	5	5	5	1
176.6644	3.5	4	6	6	6	1
176.6644	3.5	6	4	6	6	1
176.6644	3.5	6	6	4	6	1
176.6644	3.5	6	6	6	4	1
176.4915	3.6	5	6	6	5	1
176.4915	3.6	6	5	6	5	1

Table 5.A.27 Frontier Policies: β = 105 (Cont.)

Reward	Cost	Policy				
176.4915	3.6	6	6	5	5	1
176.4915	3.6	5	5	6	6	1
176.4915	3.6	5	6	5	6	1
176.4915	3.6	6	5	5	6	1
174.5663	3.8	5	6	6	6	1
174.5663	3.8	6	5	6	6	1
174.5663	3.8	6	6	5	6	1
174.5663	3.8	6	6	6	5	1
173.0357	4	6	6	6	6	1

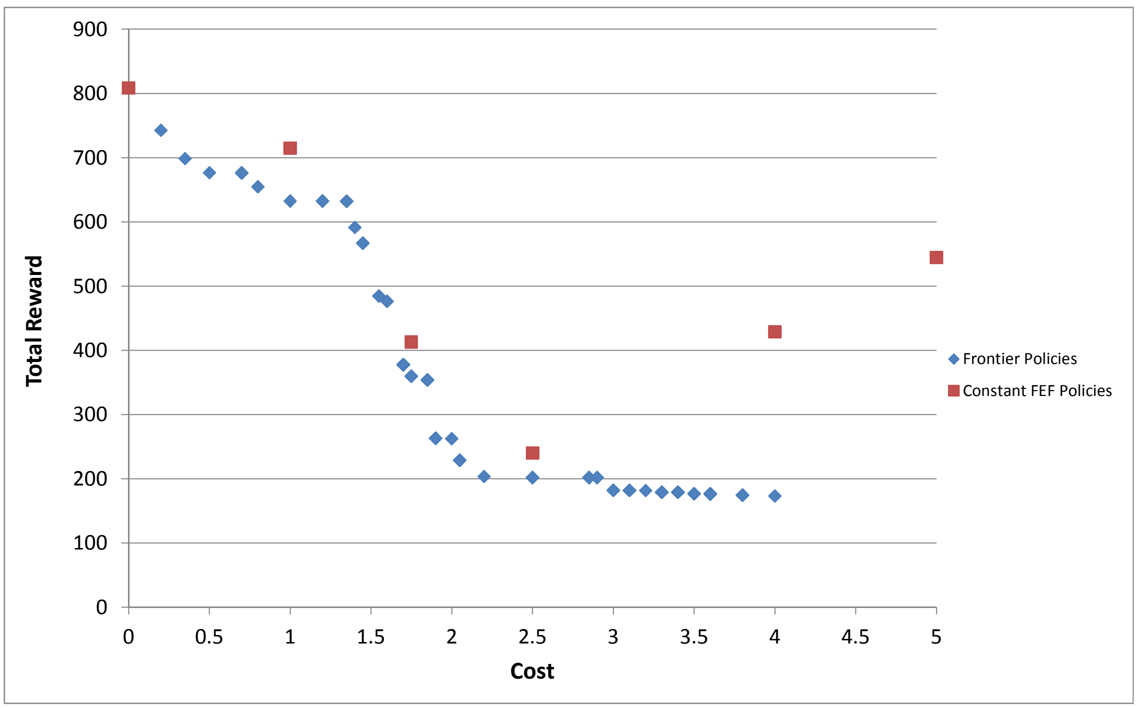

Figure 5.A.27: Constant FEF Policies and Policy Frontier: β = 105

5.A.7.4 System MTBF=205

Table 5.A.28: Frontier Policies: β = 205

Reward	Cost	Policy				
806.2600	0	1	1	1	1	1
739.6401	0.2	1	1	1	1	2
695.2606	0.35	1	1	1	1	3
673.0950	0.5	1	1	1	1	4
659.2598	0.7	1	1	1	2	4

Table 5.A.28 Frontier Policies: $\beta = 205$ (Cont.)

Reward	Cost	Policy				
659.2598	0.7	1	1	2	1	4
659.2598	0.7	1	2	1	1	4
659.2598	0.7	2	1	1	1	4
604.5359	0.75	1	2	2	3	1
604.5359	0.75	1	2	3	2	1
604.5359	0.75	1	3	2	2	1
604.5359	0.75	2	1	2	3	1
604.5359	0.75	2	1	3	2	1
604.5359	0.75	2	2	1	3	1
604.5359	0.75	2	2	3	1	1
604.5359	0.75	2	3	1	2	1
604.5359	0.75	2	3	2	1	1
604.5359	0.75	3	1	2	2	1
604.5359	0.75	3	2	1	2	1
604.5359	0.75	3	2	2	1	1
467.9788	0.8	2	2	2	2	1
380.7665	0.9	1	3	3	2	1
380.7665	0.9	3	1	3	2	1
380.7665	0.9	3	3	1	2	1
380.7665	0.9	3	3	2	1	1
380.7665	0.9	1	2	3	3	1
380.7665	0.9	1	3	2	3	1
380.7665	0.9	2	1	3	3	1
380.7665	0.9	2	3	1	3	1
380.7665	0.9	2	3	3	1	1
380.7665	0.9	3	1	2	3	1
380.7665	0.9	3	2	1	3	1
380.7665	0.9	3	2	3	1	1
336.9166	0.95	2	2	3	2	1
336.9166	0.95	2	3	2	2	1
336.9166	0.95	3	2	2	2	1
336.9166	0.95	2	2	2	3	1
261.2164	1.05	1	3	3	3	1
261.2164	1.05	3	1	3	3	1
261.2164	1.05	3	3	1	3	1
261.2164	1.05	3	3	3	1	1
237.6435	1.1	3	3	2	2	1
237.6435	1.1	2	2	3	3	1
237.6435	1.1	2	3	2	3	1
237.6435	1.1	2	3	3	2	1
237.6435	1.1	3	2	2	3	1

Table 5.A.28 Frontier Policies: β = 205 (Cont.)

Reward	Cost	Policy				
237.6435	1.1	3	2	3	2	1
234.7086	1.15	2	2	3	2	2
234.7086	1.15	2	3	2	2	2
234.7086	1.15	3	2	2	2	2
234.7086	1.15	2	2	2	3	2
180.4689	1.25	3	3	3	2	1
180.4689	1.25	2	3	3	3	1
180.4689	1.25	3	2	3	3	1
180.4689	1.25	3	3	2	3	1
174.6348	1.4	3	3	3	3	1
172.9381	1.55	3	3	3	4	1
172.9381	1.55	3	3	4	3	1
172.9381	1.55	3	4	3	3	1
172.9381	1.55	4	3	3	3	1
172.6194	2.4	2	2	6	6	1
172.6194	2.4	2	6	2	6	1
172.6194	2.4	6	2	2	6	1
172.6194	2.4	2	6	6	2	1
172.6194	2.4	6	2	6	2	1
172.6194	2.4	6	6	2	2	1
172.5348	2.8	1	5	6	6	1
172.5348	2.8	1	6	5	6	1
172.5348	2.8	1	6	6	5	1
172.5348	2.8	5	1	6	6	1
172.5348	2.8	5	6	1	6	1
172.5348	2.8	5	6	6	1	1
172.5348	2.8	6	1	5	6	1
172.5348	2.8	6	1	6	5	1
172.5348	2.8	6	5	1	6	1
172.5348	2.8	6	5	6	1	1
172.5348	2.8	6	6	1	5	1
172.5348	2.8	6	6	5	1	1

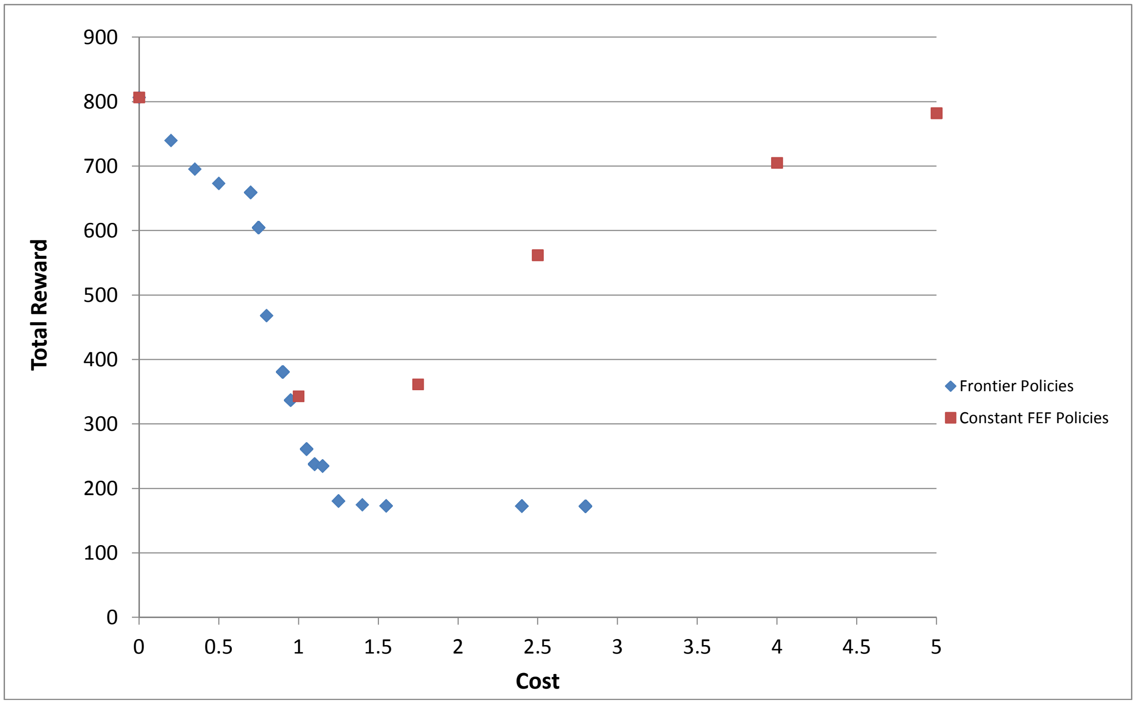

Figure 5.A.28: Constant FEF Policies and Policy Frontier: $\beta = 205$

5.A.8 Instance 9: Only Last Epoch Weighted, Deviations Above Curve Weighted at 0.1

5.A.8.1 System MTBF=25

Table 5.A.29: Frontier Policies: $\beta = 25$

Reward	Cost	Policy				
808.4720	0	1	1	1	1	1
742.4966	0.2	1	1	1	1	2
698.5130	0.35	1	1	1	1	3
676.5212	0.5	1	1	1	1	4
676.5212	0.7	1	1	1	2	4
676.5212	0.7	1	1	2	1	4
676.5212	0.7	1	2	1	1	4
676.5212	0.7	2	1	1	1	4
654.5294	0.8	1	1	1	1	5
632.5376	1	1	1	1	1	6
632.5376	1.2	1	1	1	2	6
632.5376	1.2	1	1	2	1	6
632.5376	1.2	1	2	1	1	6
632.5376	1.2	2	1	1	1	6
632.5376	1.35	1	1	1	3	6
632.5376	1.35	1	1	3	1	6
632.5376	1.35	1	3	1	1	6
632.5376	1.35	3	1	1	1	6
632.5376	1.4	1	1	2	2	6
632.5376	1.4	1	2	1	2	6
632.5376	1.4	1	2	2	1	6
632.5376	1.4	2	1	1	2	6
632.5376	1.4	2	1	2	1	6
632.5376	1.4	2	2	1	1	6
632.5376	1.55	1	1	2	3	6
632.5376	1.55	1	1	3	2	6
632.5376	1.55	1	2	1	3	6
632.5376	1.55	1	2	3	1	6
632.5376	1.55	1	3	1	2	6
632.5376	1.55	1	3	2	1	6
632.5376	1.55	2	1	1	3	6
632.5376	1.55	2	1	3	1	6
632.5376	1.55	2	3	1	1	6
632.5376	1.55	3	1	1	2	6
632.5376	1.55	3	1	2	1	6
632.5376	1.55	3	2	1	1	6
632.5375	1.6	1	2	2	2	6

Table 5.A.29 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
632.5375	1.6	2	1	2	2	6
632.5375	1.6	2	2	1	2	6
632.5375	1.6	2	2	2	1	6
632.5375	1.7	1	1	3	3	6
632.5375	1.7	1	3	1	3	6
632.5375	1.7	1	3	3	1	6
632.5375	1.7	3	1	1	3	6
632.5375	1.7	3	1	3	1	6
632.5375	1.7	3	3	1	1	6
632.5373	1.75	1	2	2	3	6
632.5373	1.75	1	2	3	2	6
632.5373	1.75	1	3	2	2	6
632.5373	1.75	2	1	2	3	6
632.5373	1.75	2	1	3	2	6
632.5373	1.75	2	2	1	3	6
632.5373	1.75	2	2	3	1	6
632.5373	1.75	2	3	1	2	6
632.5373	1.75	2	3	2	1	6
632.5373	1.75	3	1	2	2	6
632.5373	1.75	3	2	1	2	6
632.5373	1.75	3	2	2	1	6
632.5367	1.8	2	2	2	2	6
632.5364	1.9	1	2	3	3	6
632.5364	1.9	1	3	2	3	6
632.5364	1.9	1	3	3	2	6
632.5364	1.9	2	1	3	3	6
632.5364	1.9	2	3	1	3	6
632.5364	1.9	2	3	3	1	6
632.5364	1.9	3	1	2	3	6
632.5364	1.9	3	1	3	2	6
632.5364	1.9	3	2	1	3	6
632.5364	1.9	3	2	3	1	6
632.5364	1.9	3	3	1	2	6
632.5364	1.9	3	3	2	1	6
632.5343	1.95	2	2	3	2	6
632.5343	1.95	2	3	2	2	6
632.5343	1.95	3	2	2	2	6
632.5343	1.95	2	2	2	3	6
632.5336	2.05	1	3	3	3	6
632.5336	2.05	3	1	3	3	6
632.5336	2.05	3	3	1	3	6

Table 5.A.29 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
632.5336	2.05	3	3	3	1	6
632.5265	2.1	3	3	2	2	6
632.5265	2.1	2	2	3	3	6
632.5265	2.1	2	3	2	3	6
632.5265	2.1	2	3	3	2	6
632.5265	2.1	3	2	2	3	6
632.5265	2.1	3	2	3	2	6
632.5015	2.25	2	3	3	3	6
632.5015	2.25	3	2	3	3	6
632.5015	2.25	3	3	2	3	6
632.5015	2.25	3	3	3	2	6
632.4241	2.4	3	3	3	3	6
632.3500	2.55	3	3	3	4	6
632.3500	2.55	3	3	4	3	6
632.3500	2.55	3	4	3	3	6
632.3500	2.55	4	3	3	3	6
632.2302	2.7	3	3	4	4	6
632.2302	2.7	3	4	3	4	6
632.2302	2.7	3	4	4	3	6
632.2302	2.7	4	3	3	4	6
632.2302	2.7	4	3	4	3	6
632.2302	2.7	4	4	3	3	6
632.0391	2.85	3	4	4	4	6
632.0391	2.85	4	3	4	4	6
632.0391	2.85	4	4	3	4	6
632.0391	2.85	4	4	4	3	6
631.7369	3	4	4	4	4	6
631.3007	3.3	4	4	4	5	6
631.3007	3.3	4	4	5	4	6
631.3007	3.3	4	5	4	4	6
631.3007	3.3	5	4	4	4	6
630.6905	3.5	4	4	4	6	6
630.6905	3.5	4	4	6	4	6
630.6905	3.5	4	6	4	4	6
630.6905	3.5	6	4	4	4	6
630.6463	3.6	4	4	5	5	6
630.6463	3.6	4	5	4	5	6
630.6463	3.6	4	5	5	4	6
630.6463	3.6	5	4	4	5	6
630.6463	3.6	5	4	5	4	6
630.6463	3.6	5	5	4	4	6

Table 5.A.29 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
629.7431	3.8	4	4	5	6	6
629.7431	3.8	4	4	6	5	6
629.7431	3.8	4	5	4	6	6
629.7431	3.8	4	5	6	4	6
629.7431	3.8	4	6	4	5	6
629.7431	3.8	4	6	5	4	6
629.7431	3.8	5	4	4	6	6
629.7431	3.8	5	4	6	4	6
629.7431	3.8	5	6	4	4	6
629.7431	3.8	6	4	4	5	6
629.7431	3.8	6	4	5	4	6
629.7431	3.8	6	5	4	4	6
629.6765	3.9	4	5	5	5	6
629.6765	3.9	5	4	5	5	6
629.6765	3.9	5	5	4	5	6
629.6765	3.9	5	5	5	4	6
628.4488	4	4	4	6	6	6
628.4488	4	4	6	4	6	6
628.4488	4	4	6	6	4	6
628.4488	4	6	4	4	6	6
628.4488	4	6	4	6	4	6
628.4488	4	6	6	4	4	6
628.3521	4.1	4	5	5	6	6
628.3521	4.1	4	5	6	5	6
628.3521	4.1	4	6	5	5	6
628.3521	4.1	5	4	5	6	6
628.3521	4.1	5	4	6	5	6
628.3521	4.1	5	5	4	6	6
628.3521	4.1	5	5	6	4	6
628.3521	4.1	5	6	4	5	6
628.3521	4.1	5	6	5	4	6
628.3521	4.1	6	4	5	5	6
628.3521	4.1	6	5	4	5	6
628.3521	4.1	6	5	5	4	6
628.2568	4.2	5	5	5	5	6
626.4859	4.3	4	5	6	6	6
626.4859	4.3	4	6	5	6	6
626.4859	4.3	4	6	6	5	6
626.4859	4.3	5	4	6	6	6
626.4859	4.3	5	6	4	6	6
626.4859	4.3	5	6	6	4	6

Table 5.A.29 Frontier Policies: $\beta = 25$ (Cont.)

Reward	Cost	Policy				
626.4859	4.3	6	4	5	6	6
626.4859	4.3	6	4	6	5	6
626.4859	4.3	6	5	4	6	6
626.4859	4.3	6	5	6	4	6
626.4859	4.3	6	6	4	5	6
626.4859	4.3	6	6	5	4	6
626.3499	4.4	5	5	5	6	6
626.3499	4.4	5	5	6	5	6
626.3499	4.4	5	6	5	5	6
626.3499	4.4	6	5	5	5	6
623.8807	4.5	4	6	6	6	6
623.8807	4.5	6	4	6	6	6
623.8807	4.5	6	6	4	6	6
623.8807	4.5	6	6	6	4	6
623.6895	4.6	5	5	6	6	6
623.6895	4.6	5	6	5	6	6
623.6895	4.6	5	6	6	5	6
623.6895	4.6	6	5	5	6	6
623.6895	4.6	6	5	6	5	6
623.6895	4.6	6	6	5	5	6
620.0304	4.8	5	6	6	6	6
620.0304	4.8	6	5	6	6	6
620.0304	4.8	6	6	5	6	6
620.0304	4.8	6	6	6	5	6
615.0908	5	6	6	6	6	6

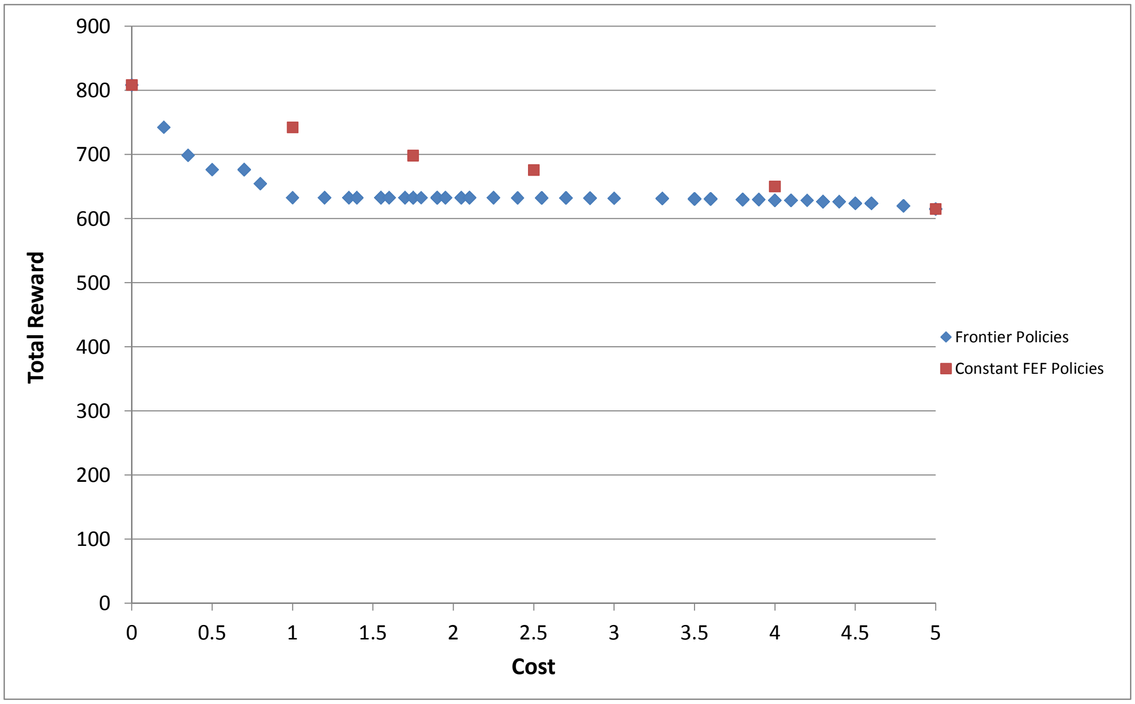

Figure 5.A.29: Constant FEF Policies and Policy Frontier: $\beta = 25$

5.A.8.2 System MTBF=50

Table 5.A.30: Frontier Policies: $\beta = 50$

Reward	Cost	Policy				
808.4720	0	1	1	1	1	1
742.4966	0.2	1	1	1	1	2
698.5130	0.35	1	1	1	1	3
676.5212	0.5	1	1	1	1	4
676.5208	0.7	1	1	1	2	4
676.5208	0.7	1	1	2	1	4
676.5208	0.7	1	2	1	1	4
676.5208	0.7	2	1	1	1	4
654.5294	0.8	1	1	1	1	5
632.5376	1	1	1	1	1	6
632.5372	1.2	1	1	1	2	6
632.5372	1.2	1	1	2	1	6
632.5372	1.2	1	2	1	1	6
632.5372	1.2	2	1	1	1	6
632.5361	1.35	1	1	1	3	6
632.5361	1.35	1	1	3	1	6
632.5361	1.35	1	3	1	1	6
632.5361	1.35	3	1	1	1	6

Table 5.A.30 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
632.5335	1.4	1	1	2	2	6
632.5335	1.4	1	2	1	2	6
632.5335	1.4	1	2	2	1	6
632.5335	1.4	2	1	1	2	6
632.5335	1.4	2	1	2	1	6
632.5335	1.4	2	2	1	1	6
632.5239	1.55	1	1	2	3	6
632.5239	1.55	1	1	3	2	6
632.5239	1.55	1	2	1	3	6
632.5239	1.55	1	2	3	1	6
632.5239	1.55	1	3	1	2	6
632.5239	1.55	1	3	2	1	6
632.5239	1.55	2	1	1	3	6
632.5239	1.55	2	1	3	1	6
632.5239	1.55	2	3	1	1	6
632.5239	1.55	3	1	1	2	6
632.5239	1.55	3	1	2	1	6
632.5239	1.55	3	2	1	1	6
632.5011	1.6	1	2	2	2	6
632.5011	1.6	2	1	2	2	6
632.5011	1.6	2	2	1	2	6
632.5011	1.6	2	2	2	1	6
632.4933	1.7	1	1	3	3	6
632.4933	1.7	1	3	1	3	6
632.4933	1.7	1	3	3	1	6
632.4933	1.7	3	1	1	3	6
632.4933	1.7	3	1	3	1	6
632.4933	1.7	3	3	1	1	6
632.4229	1.75	1	2	2	3	6
632.4229	1.75	1	2	3	2	6
632.4229	1.75	1	3	2	2	6
632.4229	1.75	2	1	2	3	6
632.4229	1.75	2	1	3	2	6
632.4229	1.75	2	2	1	3	6
632.4229	1.75	2	2	3	1	6
632.4229	1.75	2	3	1	2	6
632.4229	1.75	2	3	2	1	6
632.4229	1.75	3	1	2	2	6
632.4229	1.75	3	2	1	2	6
632.4229	1.75	3	2	2	1	6
632.2492	1.8	2	2	2	2	6

Table 5.A.30 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
632.1922	1.9	1	2	3	3	6
632.1922	1.9	1	3	2	3	6
632.1922	1.9	1	3	3	2	6
632.1922	1.9	2	1	3	3	6
632.1922	1.9	2	3	1	3	6
632.1922	1.9	2	3	3	1	6
632.1922	1.9	3	1	2	3	6
632.1922	1.9	3	1	3	2	6
632.1922	1.9	3	2	1	3	6
632.1922	1.9	3	2	3	1	6
632.1922	1.9	3	3	1	2	6
632.1922	1.9	3	3	2	1	6
631.7035	1.95	2	2	3	2	6
631.7035	1.95	2	3	2	2	6
631.7035	1.95	3	2	2	2	6
631.7035	1.95	2	2	2	3	6
631.5468	2.05	1	3	3	3	6
631.5468	2.05	3	1	3	3	6
631.5468	2.05	3	3	1	3	6
631.5468	2.05	3	3	3	1	6
630.2511	2.1	3	3	2	2	6
630.2511	2.1	2	2	3	3	6
630.2511	2.1	2	3	2	3	6
630.2511	2.1	2	3	3	2	6
630.2511	2.1	3	2	2	3	6
630.2511	2.1	3	2	3	2	6
626.6459	2.25	2	3	3	3	6
626.6459	2.25	3	2	3	3	6
626.6459	2.25	3	3	2	3	6
626.6459	2.25	3	3	3	2	6
618.3803	2.4	3	3	3	3	6
612.0371	2.55	3	3	3	4	6
612.0371	2.55	3	3	4	3	6
612.0371	2.55	3	4	3	3	6
612.0371	2.55	4	3	3	3	6
603.3318	2.7	3	3	4	4	6
603.3318	2.7	3	4	3	4	6
603.3318	2.7	3	4	4	3	6
603.3318	2.7	4	3	3	4	6
603.3318	2.7	4	3	4	3	6
603.3318	2.7	4	4	3	3	6

Table 5.A.30 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
600.3965	2.8	4	4	4	4	5
591.6544	2.85	3	4	4	4	6
591.6544	2.85	4	3	4	4	6
591.6544	2.85	4	4	3	4	6
591.6544	2.85	4	4	4	3	6
576.3717	3	4	4	4	4	6
558.1324	3.3	4	4	4	5	6
558.1324	3.3	4	4	5	4	6
558.1324	3.3	4	5	4	4	6
558.1324	3.3	5	4	4	4	6
533.6001	3.5	4	4	6	6	4
533.6001	3.5	4	6	4	6	4
533.6001	3.5	4	6	6	4	4
533.6001	3.5	6	4	4	6	4
533.6001	3.5	6	4	6	4	4
533.6001	3.5	6	6	4	4	4
531.8479	3.6	4	5	6	5	4
531.8479	3.6	4	6	5	5	4
531.8479	3.6	5	4	6	5	4
531.8479	3.6	5	6	4	5	4
531.8479	3.6	6	4	5	5	4
531.8479	3.6	6	5	4	5	4
531.8479	3.6	4	5	5	6	4
531.8479	3.6	5	4	5	6	4
531.8479	3.6	5	5	4	6	4
531.8479	3.6	5	5	6	4	4
531.8479	3.6	5	6	5	4	4
531.8479	3.6	6	5	5	4	4
527.9387	3.65	4	5	6	6	3
527.9387	3.65	4	6	5	6	3
527.9387	3.65	4	6	6	5	3
527.9387	3.65	5	4	6	6	3
527.9387	3.65	5	6	4	6	3
527.9387	3.65	5	6	6	4	3
527.9387	3.65	6	4	5	6	3
527.9387	3.65	6	4	6	5	3
527.9387	3.65	6	5	4	6	3
527.9387	3.65	6	5	6	4	3
527.9387	3.65	6	6	4	5	3
527.9387	3.65	6	6	5	4	3
525.8743	3.75	5	5	5	6	3

Table 5.A.30 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
525.8743	3.75	5	5	6	5	3
525.8743	3.75	5	6	5	5	3
525.8743	3.75	6	5	5	5	3
499.1612	3.8	4	5	6	6	4
499.1612	3.8	4	6	5	6	4
499.1612	3.8	4	6	6	5	4
499.1612	3.8	5	4	6	6	4
499.1612	3.8	5	6	4	6	4
499.1612	3.8	5	6	6	4	4
499.1612	3.8	6	4	5	6	4
499.1612	3.8	6	4	6	5	4
499.1612	3.8	6	5	4	6	4
499.1612	3.8	6	5	6	4	4
499.1612	3.8	6	6	4	5	4
499.1612	3.8	6	6	5	4	4
492.4563	3.85	4	6	6	6	3
492.4563	3.85	6	4	6	6	3
492.4563	3.85	6	6	4	6	3
492.4563	3.85	6	6	6	4	3
490.2388	3.95	5	6	6	5	3
490.2388	3.95	6	5	6	5	3
490.2388	3.95	6	6	5	5	3
490.2388	3.95	5	5	6	6	3
490.2388	3.95	5	6	5	6	3
490.2388	3.95	6	5	5	6	3
462.8351	4	4	6	6	6	4
462.8351	4	6	4	6	6	4
462.8351	4	6	6	4	6	4
462.8351	4	6	6	6	4	4
460.5717	4.1	5	5	6	6	4
460.5717	4.1	5	6	5	6	4
460.5717	4.1	5	6	6	5	4
460.5717	4.1	6	5	5	6	4
460.5717	4.1	6	5	6	5	4
460.5717	4.1	6	6	5	5	4
451.2001	4.15	5	6	6	6	3
451.2001	4.15	6	5	6	6	3
451.2001	4.15	6	6	5	6	3
451.2001	4.15	6	6	6	5	3
420.8669	4.3	5	6	6	6	4
420.8669	4.3	6	5	6	6	4

Table 5.A.30 Frontier Policies: $\beta = 50$ (Cont.)

Reward	Cost	Policy				
420.8669	4.3	6	6	5	6	4
420.8669	4.3	6	6	6	5	4
409.6342	4.35	6	6	6	6	3
378.9099	4.5	6	6	6	6	4
351.7770	4.8	6	6	6	6	5
324.7747	5	6	6	6	6	6

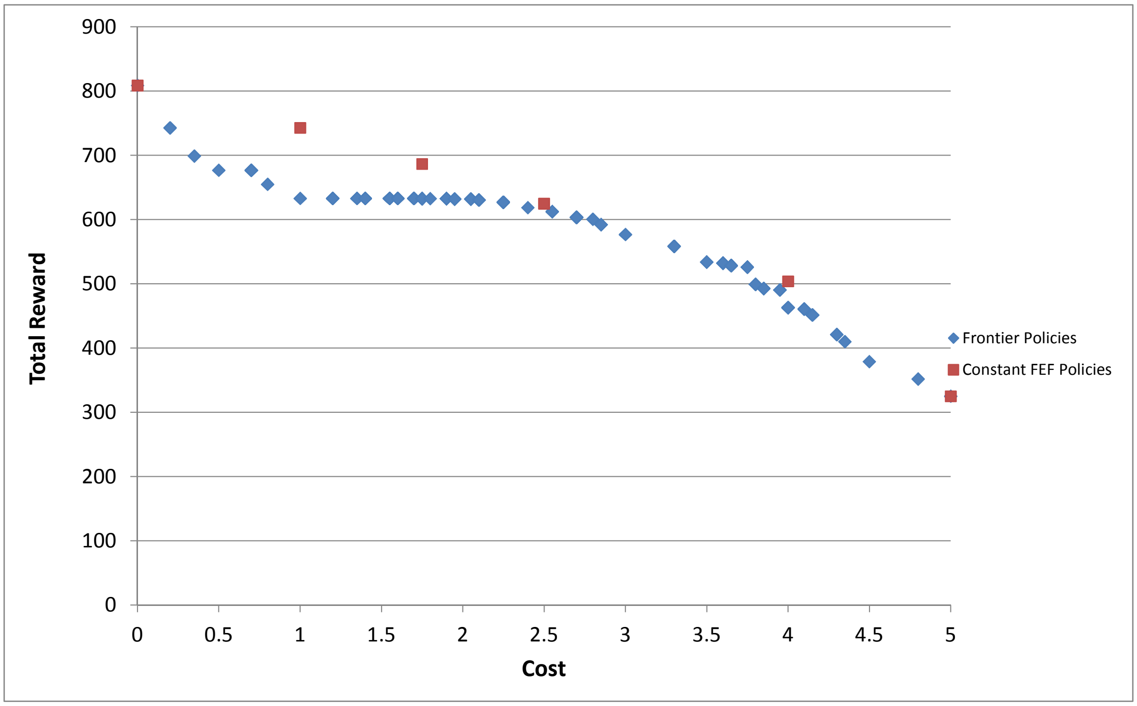

Figure 5.A.30: Constant FEF Policies and Policy Frontier: $\beta = 50$

5.A.8.3 System MTBF=105

Table 5.A.31: Frontier Policies: $\beta = 105$

Reward	Cost	Policy				
808.4576	0	1	1	1	1	1
742.4779	0.2	1	1	1	1	2
698.4915	0.35	1	1	1	1	3
676.4984	0.5	1	1	1	1	4
676.3363	0.7	1	1	1	2	4
676.3363	0.7	1	1	2	1	4
676.3363	0.7	1	2	1	1	4
676.3363	0.7	2	1	1	1	4

Table 5.A.31 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
654.5054	0.8	1	1	1	1	5
632.5124	1	1	1	1	1	6
632.3334	1.2	1	1	1	2	6
632.3334	1.2	1	1	2	1	6
632.3334	1.2	1	2	1	1	6
632.3334	1.2	2	1	1	1	6
631.9373	1.35	1	1	1	3	6
631.9373	1.35	1	1	3	1	6
631.9373	1.35	1	3	1	1	6
631.9373	1.35	3	1	1	1	6
631.1217	1.4	1	1	2	2	6
631.1217	1.4	1	2	1	2	6
631.1217	1.4	1	2	2	1	6
626.9618	1.4	1	3	3	3	3
626.9618	1.4	3	1	3	3	3
626.9618	1.4	3	3	1	3	3
626.9618	1.4	3	3	3	1	3
590.0201	1.4	3	3	3	3	1
580.0939	1.45	3	3	2	2	3
564.4024	1.45	3	3	3	2	2
564.4024	1.45	2	3	3	3	2
564.4024	1.45	3	2	3	3	2
564.4024	1.45	3	3	2	3	2
480.5676	1.55	3	3	3	4	1
480.5676	1.55	3	3	4	3	1
480.5676	1.55	3	4	3	3	1
480.5676	1.55	4	3	3	3	1
470.1138	1.6	3	3	3	3	2
367.9499	1.7	3	3	4	4	1
367.9499	1.7	3	4	3	4	1
367.9499	1.7	4	3	3	4	1
367.9499	1.7	3	4	4	3	1
367.9499	1.7	4	3	4	3	1
367.9499	1.7	4	4	3	3	1
344.3114	1.75	3	3	3	4	2
344.3114	1.75	3	3	4	3	2
344.3114	1.75	3	4	3	3	2
344.3114	1.75	4	3	3	3	2
341.8574	1.85	3	4	4	4	1
341.8574	1.85	4	3	4	4	1
341.8574	1.85	4	4	3	4	1

Table 5.A.31 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
341.8574	1.85	4	4	4	3	1
272.8683	1.9	3	3	3	4	3
272.8683	1.9	3	3	4	3	3
228.8747	1.9	3	3	4	4	2
228.8747	1.9	3	4	3	4	2
228.8747	1.9	4	3	3	4	2
228.8747	1.9	3	4	4	3	2
228.8747	1.9	4	3	4	3	2
228.8747	1.9	4	4	3	3	2
166.9215	2.05	3	3	4	4	3
166.9215	2.05	3	4	3	4	3
166.9215	2.05	4	3	3	4	3
166.9215	2.05	3	4	4	3	3
166.9215	2.05	4	3	4	3	3
166.9215	2.05	4	4	3	3	3
144.8160	2.2	3	3	4	4	4
144.8160	2.2	3	4	3	4	4
144.8160	2.2	3	4	4	3	4
144.8160	2.2	4	3	3	4	4
144.8160	2.2	4	3	4	3	4
144.8160	2.2	4	4	3	3	4
122.9736	2.2	4	4	4	4	2
88.8795	2.35	4	4	4	4	3
80.6312	2.5	4	4	4	4	4
78.5006	2.8	4	4	4	4	5
77.3021	2.8	4	4	4	5	4
77.3021	2.8	4	4	5	4	4
77.3021	2.8	4	5	4	4	4
77.3021	2.8	5	4	4	4	4
76.4728	3	4	4	4	4	6
74.9778	3	4	4	4	6	4
74.9778	3	4	4	6	4	4
74.9778	3	4	6	4	4	4
74.9778	3	6	4	4	4	4
74.8900	3.1	4	4	5	5	4
74.8900	3.1	4	5	4	5	4
74.8900	3.1	4	5	5	4	4
74.8900	3.1	5	4	4	5	4
74.8900	3.1	5	4	5	4	4
74.8900	3.1	5	5	4	4	4
69.5857	3.2	4	4	6	6	2

Table 5.A.31 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
69.5857	3.2	4	6	4	6	2
69.5857	3.2	4	6	6	4	2
69.5857	3.2	6	4	4	6	2
69.5857	3.2	6	4	6	4	2
69.5857	3.2	6	6	4	4	2
69.5312	3.3	4	5	6	5	2
69.5312	3.3	4	6	5	5	2
69.5312	3.3	5	4	6	5	2
69.5312	3.3	5	6	4	5	2
69.5312	3.3	6	4	5	5	2
69.5312	3.3	6	5	4	5	2
69.5312	3.3	4	5	5	6	2
69.5312	3.3	5	4	5	6	2
69.5312	3.3	5	5	4	6	2
69.5312	3.3	5	5	6	4	2
69.5312	3.3	5	6	5	4	2
69.5312	3.3	6	5	5	4	2
69.4235	3.4	5	5	5	5	2
68.3197	3.5	4	5	6	6	2
68.3197	3.5	4	6	5	6	2
68.3197	3.5	4	6	6	5	2
68.3197	3.5	5	4	6	6	2
68.3197	3.5	5	6	4	6	2
68.3197	3.5	5	6	6	4	2
68.3197	3.5	6	4	5	6	2
68.3197	3.5	6	4	6	5	2
68.3197	3.5	6	5	4	6	2
68.3197	3.5	6	5	6	4	2
68.3197	3.5	6	6	4	5	2
68.3197	3.5	6	6	5	4	2
68.2773	3.6	5	5	5	6	2
68.2773	3.6	5	5	6	5	2
68.2773	3.6	5	6	5	5	2
68.2773	3.6	6	5	5	5	2
67.3975	3.7	4	6	6	6	2
67.3975	3.7	6	4	6	6	2
67.3975	3.7	6	6	4	6	2
67.3975	3.7	6	6	6	4	2
67.3360	3.8	5	6	6	5	2
67.3360	3.8	6	5	6	5	2
67.3360	3.8	6	6	5	5	2

Table 5.A.31 Frontier Policies: $\beta = 105$ (Cont.)

Reward	Cost	Policy				
67.3360	3.8	5	5	6	6	2
67.3360	3.8	5	6	5	6	2
67.3360	3.8	6	5	5	6	2
66.7177	4	5	6	6	6	2
66.7177	4	6	5	6	6	2
66.7177	4	6	6	5	6	2
66.7177	4	6	6	6	5	2
66.3618	4.2	6	6	6	6	2

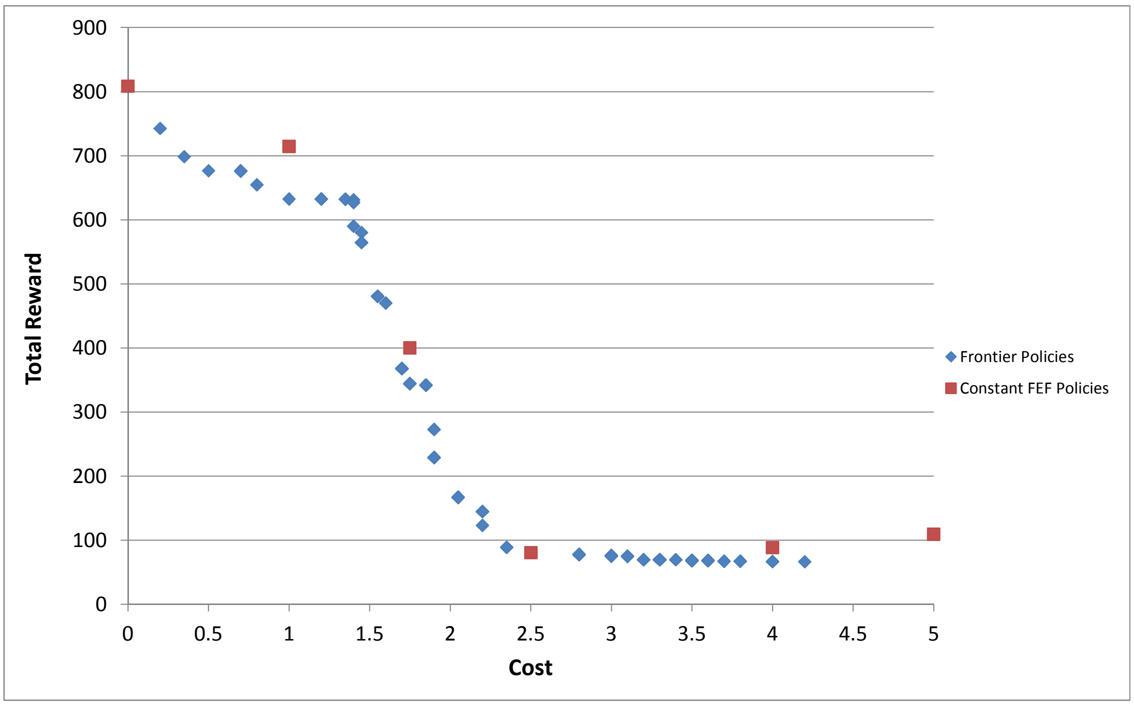

Figure 5.A.31: Constant FEF Policies and Policy Frontier: $\beta = 105$

5.A.8.4 System MTBF=205

Table 5.A.32: Frontier Policies: $\beta = 205$

Reward	Cost	Policy				
806.2591	0	1	1	1	1	1
739.6338	0.2	1	1	1	1	2
695.2418	0.35	1	1	1	1	3
673.0634	0.5	1	1	1	1	4
659.0284	0.7	1	1	1	1	4
659.0284	0.7	1	1	2	2	4

Table 5.A.32 Frontier Policies: $\beta = 205$ (Cont.)

Reward	Cost	Policy				
659.0284	0.7	1	2	1	1	4
659.0284	0.7	2	1	1	1	4
603.3585	0.75	1	2	2	2	1
603.3585	0.75	1	2	3	3	1
603.3585	0.75	1	3	2	2	1
603.3585	0.75	2	1	2	2	1
603.3585	0.75	2	1	3	3	1
603.3585	0.75	2	2	1	1	1
603.3585	0.75	2	2	3	3	1
603.3585	0.75	2	3	1	1	1
603.3585	0.75	2	3	2	2	1
603.3585	0.75	3	1	2	2	1
603.3585	0.75	3	2	1	1	1
603.3585	0.75	3	2	2	2	1
463.4694	0.8	2	2	2	2	1
371.2117	0.9	1	3	3	3	1
371.2117	0.9	3	1	3	3	1
371.2117	0.9	3	3	1	1	1
371.2117	0.9	3	3	2	2	1
371.2117	0.9	1	2	3	3	1
371.2117	0.9	1	3	2	2	1
371.2117	0.9	2	1	3	3	1
371.2117	0.9	2	3	1	1	1
371.2117	0.9	2	3	3	3	1
371.2117	0.9	3	1	2	2	1
371.2117	0.9	3	2	1	1	1
371.2117	0.9	3	2	3	3	1
323.0513	0.95	2	2	3	3	1
323.0513	0.95	2	3	2	2	1
323.0513	0.95	3	2	2	2	1
323.0513	0.95	2	2	2	2	1
234.1504	1.05	1	3	3	3	1
234.1504	1.05	3	1	3	3	1
234.1504	1.05	3	3	1	1	1
234.1504	1.05	3	3	3	3	1
203.6624	1.1	3	3	2	2	1
203.6624	1.1	2	2	3	3	1
203.6624	1.1	2	3	2	2	1
203.6624	1.1	2	3	3	3	1
203.6624	1.1	3	2	2	2	1
203.6624	1.1	3	2	3	3	1

Table 5.A.32 Frontier Policies: β = 205 (Cont.)

Reward	Cost	Policy				
188.6213	1.15	2	2	3	3	2
188.6213	1.15	2	3	2	2	2
188.6213	1.15	3	2	2	2	2
188.6213	1.15	2	2	2	2	2
110.5370	1.25	3	3	3	3	1
110.5370	1.25	2	3	3	3	1
110.5370	1.25	3	2	3	3	1
110.5370	1.25	3	3	2	2	1
104.1079	1.3	3	3	2	2	2
104.1079	1.3	2	2	3	3	2
104.1079	1.3	2	3	2	2	2
104.1079	1.3	2	3	3	3	2
104.1079	1.3	3	2	2	2	2
104.1079	1.3	3	2	3	3	2
88.3806	1.4	1	3	3	3	3
88.3806	1.4	3	1	3	3	3
88.3806	1.4	3	3	1	1	3
88.3806	1.4	3	3	3	3	3
68.9131	1.45	3	3	3	3	2
68.9131	1.45	2	3	3	3	2
68.9131	1.45	3	2	3	3	2
68.9131	1.45	3	3	2	2	2
66.7371	1.6	3	3	3	3	2
51.8175	1.7	3	3	4	4	1
51.8175	1.7	3	4	3	3	1
51.8175	1.7	4	3	3	3	1
51.8175	1.7	3	4	4	4	1
51.8175	1.7	4	3	4	4	1
51.8175	1.7	4	4	3	3	1
50.2666	1.85	3	4	4	4	1
50.2666	1.85	4	3	4	4	1
50.2666	1.85	4	4	3	3	1
50.2666	1.85	4	4	4	4	1
48.8751	2	4	4	4	4	1
47.6732	2.3	4	4	4	4	1
47.6732	2.3	4	4	5	5	1
47.6732	2.3	4	5	4	4	1
47.6732	2.3	5	4	4	4	1
46.6421	2.5	4	4	4	4	1
46.6421	2.5	4	4	6	6	1
46.6421	2.5	4	6	4	4	1

Table 5.A.32 Frontier Policies: β = 205 (Cont.)

Reward	Cost	Policy				
46.6421	2.5	6	4	4	4	1
46.5799	2.6	4	4	5	5	1
46.5799	2.6	4	5	4	4	1
46.5799	2.6	4	5	5	5	1
46.5799	2.6	5	4	4	4	1
46.5799	2.6	5	4	5	5	1
46.5799	2.6	5	5	4	4	1
41.8954	2.8	4	4	5	5	1
41.8954	2.8	4	4	6	6	1
41.8954	2.8	4	5	4	4	1
41.8954	2.8	4	5	6	6	1
41.8954	2.8	4	6	4	4	1
41.8954	2.8	4	6	5	5	1
41.8954	2.8	5	4	4	4	1
41.8954	2.8	5	4	6	6	1
41.8954	2.8	5	6	4	4	1
41.8954	2.8	6	4	4	4	1
41.8954	2.8	6	4	5	5	1
41.8954	2.8	6	5	4	4	1
41.8938	2.9	4	5	5	5	1
41.8938	2.9	5	4	5	5	1
41.8938	2.9	5	5	4	4	1
41.8938	2.9	5	5	5	5	1
41.8865	3	4	4	6	6	1
41.8865	3	4	6	4	4	1
41.8865	3	4	6	6	6	1
41.8865	3	6	4	4	4	1
41.8865	3	6	4	6	6	1
41.8865	3	6	6	4	4	1
41.8857	3.1	4	5	6	6	1
41.8857	3.1	4	6	5	5	1
41.8857	3.1	5	4	6	6	1
41.8857	3.1	5	6	4	4	1
41.8857	3.1	6	4	5	5	1
41.8857	3.1	6	5	4	4	1
41.8857	3.1	4	5	5	5	1
41.8857	3.1	5	4	5	5	1
41.8857	3.1	5	5	4	4	1
41.8857	3.1	5	5	6	6	1
41.8857	3.1	5	6	5	5	1
41.8857	3.1	6	5	5	5	1

Table 5.A.32 Frontier Policies: $\beta = 205$ (Cont.)

Reward	Cost	Policy				
41.8857	3.2	5	5	5	5	1
41.8829	3.3	4	5	6	6	1
41.8829	3.3	4	6	5	5	1
41.8829	3.3	4	6	6	6	1
41.8829	3.3	5	4	6	6	1
41.8829	3.3	5	6	4	4	1
41.8829	3.3	5	6	6	6	1
41.8829	3.3	6	4	5	5	1
41.8829	3.3	6	4	6	6	1
41.8829	3.3	6	5	4	4	1
41.8829	3.3	6	5	6	6	1
41.8829	3.3	6	6	4	4	1
41.8829	3.3	6	6	5	5	1
41.8829	3.4	5	5	5	5	1
41.8829	3.4	5	5	6	6	1
41.8829	3.4	5	6	5	5	1
41.8829	3.4	6	5	5	5	1

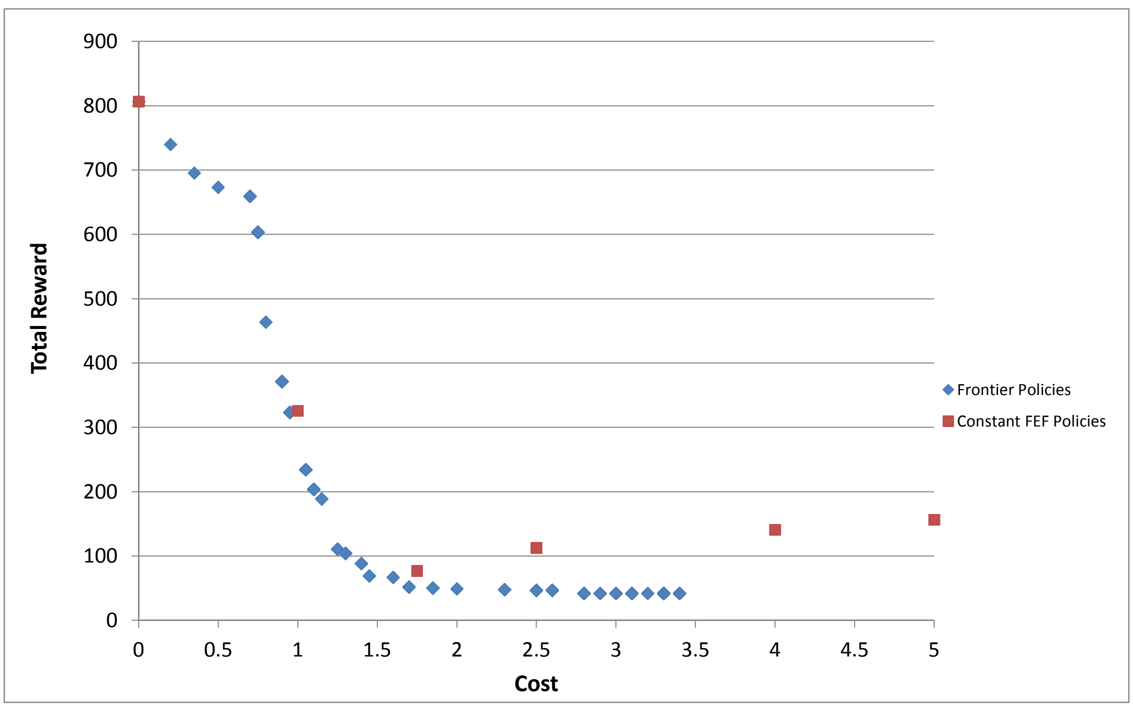

Figure 5.A.32: Constant FEF Policies and Policy Frontier: $\beta = 205$

5.B Certification of Student Work

UNIVERSITY OF ARKANSAS

College of Engineering
Department of Industrial Engineering

MEMORANDUM

TO: Graduate School, University of Arkansas

FROM: Edward A. Pohl, Professor and Department Head

DATE: July 12, 2016

SUBJECT: Certification of Student Effort

I certify that greater than 51% of the work conducted for this chapter entitled "A Markov Decision Process Approach for Optimizing Reliability Growth According to Reliability Growth Planning Curves" was conducted by Thomas P. Talafuse.

Sincerely,

Edward A. Pohl
ephol@uark.edu
479-575-6029
Professor and Department Head
Department of Industrial Engineering
University of Arkansas

6. Conclusions and Future Work

In this dissertation, we propose novel approaches for designing a complex systems, modeling the reliability growth of these systems throughout testing, and determining optimal levels of corrective action taken to improve reliability. We first introduced a meta-heuristic for determining optimal design for the redundancy allocation problem with components having deterministic reliabilities in Chapter 2. The Bat Algorithm (BA) meta-heuristic leverages the benefits of several other heuristic approaches to problems, combining them to provide optimal or near-optimal system reliability levels. Originally designed for continuous functions, we adapted the BA to handle the discrete nature of the redundancy allocation problem through manipulation of how the virtual bats flew throughout the search space. We exploited the use of a penalty function to allow search of the infeasible region to allow and encourage exploration of the feasible boundary of the search space. These efforts led to a powerful meta-heuristic, that, when applied to well known set of redundancy allocation problems, provided solutions on par or better than those found in literature. The BA can be generalized as a suitable meta-heuristic for any type of complex combinatorial problem whose solution cannot be analytically derived.

In chapters 3 and 4, we investigated a reliability growth tracking model capable of handling small sized sets of failure data. While there are many reliability growth tracking models discussed in the literature, use of these models leads to high levels of uncertainty surrounding the derived point estimates of the growth parameters when failure data are sparse. We proposed the GM(1,1) model that uses least squares approaches to derive reliability growth parameters to model a system's reliability growth, as it is more adequate for handling small sample sizes. Chapter 3 focuses on the GM(1,1) model for continuous reliability growth testing, while chapter 4 details the model for the discrete (one-shot) case. We tailored the input vectors for the GM(1,1) model to make it applicable to reliability growth testing. For complex systems whose failures follow a poly-Weibull distribution, our numerical experimentation demonstrated the superiority of the GM(1,1) model over the AMSAA model in its ability to model a system's true reliability growth.

Finally, in chapter 5, we proposed a Markov Decision Process model for modifying the level of corrective action taken to improve failure modes discovered during testing. From the failure data in each stage of testing, we derived a belief vector for the true system reliability and the expected reliability after incorporation of the corrective actions being considered. The expected reliability was then compared to the planning curve's desired level of reliability to determine the reward associated with each corrective action being considered. With a cost associated with each action, a cost frontier was established to identify those policies that minimized deviation from the desired levels taken from the reliability growth planning curve. With the ability to weight rewards by decision epoch and by a positive or negative deviation from the desired level, our MDP approach is capable of being tailored to the preferences of the decision maker(s) and can provide insights on the likelihood of a test program's success.

6.1 Future Work

Postdoctoral work relating to chapters 3 and 4 includes, but is not limited to, development of confidence intervals for parameters derived via the GM(1,1) model, for both the continuous and the discrete case. We also seek to simulate larger areas of the response surface to validate the robustness of the GM(1,1) model. We also plan on relaxing the assumption of a deterministic FEF, incorporating a probabilistic nature for the FEF level.

Future work relating to chapter 5 involves investigating the sensitivity and robustness of the state space and action space to changes in their respective dimensionality. We also plan on investigating how those states should be distributed throughout the state space. We wish to incorporate a probabilistic nature on the success of a corrective action, as corrective actions are not deterministic in nature and may not always achieve the desired FEF. We also plan on relaxing the assumption that failure modes have constant failure rates, in favor of a more general Weibull distribution assumption. Additionally we hope to identify any optimal threshold policies that may exist to reduce the action space and computational efforts.