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# Optimal Sensing and Transmission in Energy Harvesting Sensor Networks

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Optimal Sensing and Transmission in Energy Harvesting Sensor Networks

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**Abstract**

Sensor networks equipped with energy harvesting (EH) devices have attracted great attentions recently. Compared with conventional sensor networks powered by batteries, the energy harvesting abilities of the sensor nodes make sustainable and environment-friendly sensor networks possible. However, the random, scarce and non-uniform energy supply features also necessitate a completely different approach to energy management.

A typical EH wireless sensor node consists of an EH module that converts ambient energy to electrical energy, which is stored in a rechargeable battery, and will be used to power the sensing and transmission operations of the sensor. Therefore, both sensing and transmission are subject to the stochastic energy constraint imposed by the EH process. In this dissertation, we investigate optimal sensing and transmission policies for EH sensor networks under such constraints.

For EH sensing, our objective is to understand how the temporal and spatial variabilities of the EH processes would affect the sensing performance of the network, and how sensor nodes should coordinate their data collection procedures with each other to cope with the random and non-uniform energy supply and provide reliable sensing performance with analytically provable guarantees. Specifically, we investigate optimal sensing policies for a single sensor node with infinite and finite battery sizes in Chapter 2, status updating/transmission strategy of an EH Source in Chapter 3, and a collaborative sensing policy for a multi-node EH sensor network in Chapter 4.

For EH communication, our objective is to evaluate the impacts of stochastic variability of the EH process and practical battery usage constraint on the EH systems, and develop optimal transmission policies by taking such impacts into consideration. Specifically, we consider throughput optimization in an EH system under battery usage constraint in Chapter 5.

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# Dedication

To my grandparents.

## Contents

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# Chapter 1: Introduction

Sensor networks equipped with energy harvesting devices have attracted great attentions recently. Compared with conventional sensor networks powered by batteries, the energy harvesting abilities of the sensor nodes make sustainable and environment-friendly sensor networks possible. However, the random, scarce and non-uniform energy supply features also necessitate a completely different approach to energy management.

A typical EH wireless sensor node consists of an EH module that converts ambient energy to electrical energy, which is stored in a rechargeable battery, and will be used to power the sensing and transmission operations of the sensor. Therefore, both sensing and transmission are subject to the stochastic energy constraint imposed by the EH process.

EH wireless communications have attracted great attentions in academia recently. Throughput maximizing transmission policies are characterized for point-to-point channels in [1–9], for broadcast channels in [1, 10–12], for multi-access channels in [13], for interference channels in [14], for two-hop relay channels in [15–18], for systems with battery imperfections or processing costs in [16, 19–23]. Asymptotic analysis of throughput in large-scale EH communication networks is studied in [24, 25]. The optimal transmission policy for outage probability minimization in fading channels is studied in [26]. The delay minimization problem with a given energy and data arrival profile is studied in [27]. Under the assumption that a single-antenna receiver can only decode information or harvest energy from ambient radio signal at any time due to practical circuit limitations, optimal transmission and receiving policies and various trade-offs between wireless information transfer and power transfer have been characterized in different communication systems [28–43]. From an information theoretic perspective, the impact of the stochastic energy supply on channel capacity is characterized for an additive white Gaussian noise (AWGN) channel in [44–47], and a multiple-access channel in [48]. [49] describes the capacity of an EH discrete memoryless channel with finite battery using the Verdu-Han general framework [50], and [51] discusses the capacity of a noiseless binary channel with binary energy arrivals and unit-capacity battery. From the network perspective, researchers focus on routing and resource allocation problems in EH networks [52–56]. The commonly used tools include the standard dual decomposition and the subgradient methods [57], and the Lyapunov optimization technique developed in [58] and [59].

Another branch of work focuses on the sensing aspect, i.e., data collection, in EH sensor networks, and investigates energy management policies to optimize sensing and inference performance metrics, such as estimation MSE, detection delay, etc. Under an EH setting, [60, 61] propose energy-aware random sampling schemes for the recovery of sparse sensing signals using compressive sensing. [62] discusses the optimal energy allocation scheme for the "quickest detection" of change point for EH sensors. Generally speaking, the study on energy management policies for the optimization of sensing and inference performances has been limited.

In general, all of the energy management policies in EH sensor networks can be categorized as offline polices and online policies. In the offline optimization framework, it is assumed that the EH profile is predictable and known in advance for the whole duration of operation. With such assumptions, energy has been managed to optimize communication performances [1–4, 10–18, 26, 27], schedule sensing tasks [63], etc.

In contrast, in the online optimization framework, it is assumed that the system knows the past realizations of the EH process, but has only statistical knowledge of their future evolution. Besides some heuristic online algorithms [17, 18, 20, 21, 26, 64], the major approach is to formulate the optimal energy management problem as a stochastic control problem, with the objective to determine the optimal decision rules so that the expected reward of the decisions is maximized. The reward could be data throughput [5–9], channel coding rate [45, 49], sensing utility [65], etc. With this approach, the EH process and/or the data arrival process are usually modeled as Markov processes, and the online problem can be cast under the powerful framework of Markov decision processes (MDPs), which is often analytically intractable and can only be solved numerically with standard dynamic programming tools [66].

In the offline approach, non-causal knowledge of the energy and data arrival processes allows solving for the optimal policy through a one-shot optimization problem. However, in most practical scenarios, complete predictability of the EH processes is an over-simplified and optimistic assumption. On the other hand, most of the online approaches are either heuristic or numerical approaches with formidable implementation complexity and lack analytical insight.

In this dissertation, we aim to obtain optimal energy management policies to balance energy consumption and harvesting at sensor nodes, with limited EH statistics and energy state information at their disposal. We focus on both sensing and transmission aspects.

For EH sensing, our objective is to understand how the temporal and spatial variabilities of the EH processes would affect the sensing performance of the network, and how sensor nodes should coordinate their data collection procedures with each other to cope with the random and non-uniform energy supply and provide reliable sensing performance with analytically provable guarantees. Specifically, we investigate optimal sensing policies for a single sensor node with infinite and finite battery sizes in Chapter 2, status updating/transmission strategy of an EH Source in Chapter 3, and a collaborative sensing policy for a multi-node EH sensor network in Chapter 4.

In Chapter 2, we study the optimal sensing scheduling problem for an energy harvesting sensor. The objective is to strategically select the sensing time such that the long-term time average sensing performance is optimized. In the sensing system, it is assumed that the sensing performance depends on the time durations between two consecutive sensing epochs. Example applications include reconstructing a wide-sense stationary random process by using discrete-time samples collected by a sensor. We consider both scenarios where the battery size is infinite and finite, assuming the energy harvesting process is a Poisson random process. We first study the infinite battery case and identify a performance limit on the long-term time average sensing performance of the system. Motivated by the structure of the performance limit, we propose a best-effort uniform sensing policy, and prove that it achieves the limit asymptotically, thus it is optimal. We then study the finite battery case, and propose an energy-aware adaptive sensing scheduling policy. The policy dynamically chooses the next sensing epoch based on the battery level at the current sensing epoch. We show that as the battery size increases, the sensing performance under the adaptive sensing policy asymptotically converges to the limit achievable by the system with infinite battery, thus it is asymptotically optimal. The convergence rate is also analytically characterized.

In Chapter 3, we consider a scenario where an energy harvesting sensor continuously monitors a system and sends time-stamped status updates to a destination. The destination keeps track the system status through the received updates. We use the metric Age of Information (AoI), the time elapsed since the last received update was generated, to measure the "freshness" of the status information available at the destination. We assume energy arrives randomly at the sensor according to a Poisson process, and each status update consumes one unit of energy. Our objective is to design optimal online status update policies to minimize the long-term time average AoI, subject to the energy causality constraint at the sensor. We consider three scenarios, i.e., the battery size is infinite, finite, and one unit only, respectively. For the infinite battery scenario, we adopt a best-effort uniform status update policy and show that it minimizes the long-term time average AoI. For the finite battery scenario, we adopt an energy-aware adaptive status update policy, and prove that it is asymptotically optimal when the battery size goes to infinity. For the last scenario where the battery size is one, we propose a threshold based status update policy. We analytically characterize the time average AoI under this policy, and show that it outperforms any other online policy in this extreme scenario, thus it is optimal. Simulation results corroborate the theoretical bounds.

In Chapter 4, we consider a collaborative sensing scenario where sensing nodes are powered by energy harvested from ambient environment. In each time slot, an active sensor consumes one unit amount of energy to take an observation and transmit it back to a fusion center (FC). After receiving observations from all of the active sensors in a time slot, the FC aims to extract information from them. We assume that the sensing utility generated by the observations is a concave function of the number of the active sensing nodes in that slot. Our objective is to develop a sensing scheduling policy so that the time average utility generated by the sensors is maximized. We first consider an offline setting, where the energy harvesting profile over duration $[0, T-1]$ for each sensor is known beforehand. Assuming infinite battery capacity at sensors, we show that the optimal scheduling structure has a "majorization" property, and propose a procedure to construct a collaborative sensing policy with the identified structure explicitly. We then consider an online setting, under which the energy harvesting profile is available causally. Assuming the energy harvesting processes at individual sensors are independent but not necessarily identical Bernoulli processes, we show that the expected long-term time average sensing utility has an upper bound under any feasible scheduling policy satisfying the energy causality constraints. We then propose a randomized myopic policy, which aims to select a number of sensors with the highest energy levels to perform the sensing task in each slot. We show that the time average utility generated under the proposed policy converges to the upper bound almost surely as time $T$ approaches infinity, thus it is optimal. The corresponding convergence rate is also explicitly characterized.

For EH communication, our objective is to evaluate the impacts of stochastic variability of the EH process and practical battery usage constraint on the EH systems, and develop optimal transmission policies by taking such impacts into consideration. Specifically, we consider throughput optimization in an EH system under battery usage constraint in Chapter 5.

In Chapter 5, we take the impact of charging and discharging operations on battery degradation into consideration, and studies the optimal energy management policy for an energy harvesting communication system under a battery usage constraint. Specifically, in each time slot, we assume the harvested energy can be used to power the transmitter immediately without entering into the battery, or stored into the battery for now and retrieved later for transmission. Whenever the battery is charged or discharged, a cost will be incurred to account for its impact on battery degradation. We impose an long-term average cost constraint on the battery, which is translated to the average number of charge/discharge operations per unit time. The objective is to develop an online policy to maximize the long-term average throughput of the transmitter under energy causality constraint and the battery usage constraint. We first relax the energy causality constraint on the system, and impose an energy flow conservation constraint instead. We show that the optimal energy management policy has a double-threshold structure: if the amount of energy arrives in each time slot lies in between the two thresholds, it will be used immediately without involving the battery; otherwise, the battery will be charged or discharged accordingly to maintain a constant transmit power. We then modify the double-threshold policy slightly to accommodate the energy causality constraint, and analyze its long-term performance. We show that the system achieves the same long-term average performance, thus it is optimal.

# Chapter 2: Energy-aware Adaptive Sensing for EH Sensors

## 2.1 Introduction

In this chapter, we investigate the optimal online sensing scheduling of an energy harvesting sensor. Energy arrives at the sensor according to a Poisson process, and a unit amount of energy is consumed by the sensor to collect one measurement. A sensor cannot take any measurement if it does not have sufficient energy in its battery, i.e., sensing operations must satisfy the energy causality constraint. We consider an application scenario where a sensor collects measurements at discrete time epochs to estimate a time evolving physical quantity (temperature, humidity, etc). Modeling the monitored quantity as a random process, we assume that the sensing performance is a function of the discrete sensing epochs. Then, the question we aim to answer is: *Given the statistics of the energy harvesting process, how would the system strategically select the sensing epochs to optimize the long-term expected sensing performance, subject to the energy causality constraint at the sensor?* Ideally, the sensing policy should be online, lightweight, and require minimum knowledge of the energy harvesting process and/or the underlying monitored random process.

There are three dimensions of difficulty in designing such a sensing policy. First, the scarce energy supply imposes a stringent constraint on the number of measurements the sensor can take. In order to make each sample count, the sensing policy need to exploit the structural properties of the underlying monitored random process. Second, the energy harvesting process is stochastic in nature. The sensing policy should be able to cope with the fluctuations in energy supply and maintain a reliable sensing performance for almost all possible energy harvesting profiles. Third, in most practical scenario, a sensor is equipped with a finite battery, and energy overflow may happen if it is not spent in time. The sensor thus faces a dilemma of spending energy to collect less informative samples, or of saving energy for more advantageous time epochs, a step which may lead to energy loss.

In this chapter, we consider a special sensing performance function which corresponds to a random process with power-law decaying covariance [67]. We exploit the properties of the sensing performance function to devise our online sensing scheduling policy. We investigate both cases when the battery size is infinite and finite. When the battery size is infinite, we first identify a performance limit on the long-term time average sensing performance of the system. Motivated by the structure of the performance limit, we propose a best-effort uniform sensing policy, and prove that it achieves the limit asymptotically, thus it is optimal. When the battery size is finite, we aim to investigate the impact of finite battery size on the sensing performance, and bring the sensing performance as close to that of the system with infinite battery as possible. We propose an energy-aware adaptive sensing scheduling policy, which dynamically chooses the next sensing epoch based on the battery level at the current sensing epoch, and show that it is asymptotically optimal as the battery size increases. The convergence rate is also explicitly characterized. Some of the results in this chapter have been published in [68].

### 2.1.1 Main Contribution

The main contributions of this chapter are threefold:

1. First, we study an application oriented sensing scheduling for energy harvesting sensors. Different from most existing energy management schemes where the optimization objective function depends on the instantaneous power allocated to the sensor, in our formulation, the sensing performance depends on the durations between consecutive sensing epochs. Thus, instead of deciding the instantaneous power consumption over the whole operation duration, in this chapter, the objective is to decide the discrete sensing epochs for the sensor under the energy constraints. Such formulation is fundamentally different from existing works. It requires a new set of analytical tools, and results in a different type of energy management policies.

2. Second, we investigate both the infinite battery case and the finite battery case, and

propose two intuitive yet practical online sensing scheduling policies with provable performance guarantees. The proposed scheduling policies only require the instantaneous battery level to decide the sensing epochs. Thus, the sensor can be turned off between two scheduled sensing epochs to save energy. This is extremely helpful for sensors operating under stringent energy constraint. For the finite battery case, we explicitly identify the convergence rate of the proposed policy as a function of the battery size, which provides theoretical guidelines on system designs of the energy harvesting sensing system.

3. Finally, we introduce Martingale process, renewal process, and a novel virtual energy harvesting sensing system to analyze the battery level evolution under the proposed policies. Such mathematical tools are new to the area of energy harvesting communications and networks, and might be useful for related problems, especially for the construction and analysis of online scheduling policies.

### 2.1.2 Related Work

A large number of energy management schemes have been proposed to cope with the random nature of energy harvesting sensors from different perspectives. Under the infinite battery assumption, energy management schemes have been developed to optimize communication related metrics, such as channel capacity, transmission delay or network throughput [27, 44, 64], and signal processing related performance metrics, such as estimation mean squared error (MSE), detection delay, false alarm probability [60, 62].

When finite battery assumption is imposed, it changes the problem dramatically, and makes the corresponding optimal energy management much more complicated. One approach is to formulate the energy management problem as a one-shot offline optimization problem, under the assumption that the energy harvesting profile is known in advance. Examples include the throughput maximization problems studied in [1, 4, 10], where the the optimal policies are significantly different from their counterparts in an infinite battery setting [11, 27, 64]. Another approach is to formulate the optimal energy management problem as an online stochastic control problem, assuming that only the statistics and the history of the energy harvesting process are available at the controller. Modeling the energy replenishing process as a Markov process, [5] aims to maximize the time average reward by making decisions regarding whether to transmit or discard a packet based on the current energy level. The optimal policy is shown to have a threshold structure. [8] studies the performance limits of a sensing system where the battery size and the data buffer are finite and proposes an asymptotically optimal energy management scheme. The dynamic activation of sensors with unit battery in order to maximize the sensing utility is studied in [65]. In general, online optimal energy management policies under a finite battery constraint are often very difficult to characterize. Explicit solutions only exist for certain special scenarios.

The finite battery case studied in this chapter is significantly different from that in [8]. [8] considers a time-slotted system, and the objective is to adaptively vary the amount of energy spent in each time slot to optimize the system performance. However, we consider a continuous-time system in this chapter, and the proposed asymptotically optimal design varies the durations between two consecutive sensing epochs according to the instantaneous battery level. This makes the analysis of the system performance under the proposed policy much more challenging.

### 2.1.3 Chapter Outline

This chapter is organized as follows. Section 2.2 states the system model and problem formulation. Section 2.3 provides the sensing scheduling policy for the infinite battery case and proves its optimality. Section 2.4 describes an adaptive sensing scheduling policy for the finite battery case, and analytically characterizes its performance. Simulation results are provided in Section 2.5. and Section 2.6 concludes the chapter. Proofs of the main theorems are presented by the Appendix in Section 2.7.

## 2.2 System Model and Problem Formulation

### 2.2.1 Energy Harvesting Model

Consider a sensor node powered by energy harvested from the ambient environment. It is assumed that the sensor node has an energy queue, such as a rechargeable battery or a super capacitor, to store the harvested energy. The energy queue is replenished randomly and consumed by taking observations. It is assumed that a unit amount of energy is required for one sensing operation. Without loss of generality, we assume the sensor is equipped with a battery with capacity $B$, $B \geq 1$. When $B = \infty$, it corresponds to the infinite battery case.

The energy arrival follows a Poisson process with parameter 1. Hence, energy arrivals occur in discrete time instants. Specifically, we use $t_1, t_2, \ldots, t_n, \ldots$ to represent the energy arrival epochs. Then, the energy inter-arrival times $t_i - t_{i-1}$ are exponentially distributed with mean $\lambda$. We assume $\lambda = 1$ throughout this chapter for ease of exposition. If $\lambda \neq 1$, we can always normalize the time axis to make the energy arrival rate equal to one unit per unit time, and the algorithms and theoretical results presented in this chapter will still be valid on the normalized time scale. Without loss of generality, it is assumed that the system starts with an empty energy queue at time 0.

A sampling policy or sensing scheduling policy is denoted as $\{l_n\}_{n=1}^{\infty}$, where $l_n$ is the $n$-th sensing time instant. Let $l_0 = 0$, and $d_n := l_n - l_{n-1}$, for $n = 1, 2, \ldots$. Define $A(d_n)$ as the total amount of energy harvested in $[l_{n-1}, l_n)$, and $E(l_n^-)$ as the energy level of the sensor right before the scheduled sensing epoch $l_n$. Then, under any feasible sensing scheduling policy, the energy queue evolves as follows

$$E(l_{n+1}^-) = \min\{E(l_n^-) - 1 + A(d_{n+1}), B\} \tag{2.1}$$

$$E(l_n^-) \geq 1 \tag{2.2}$$

for $n = 1, 2, \ldots$. Eqn. (2.2) corresponds to the energy causality constraint in the system. Based on the Poisson arrival process assumption, $A(d_{n+1})$ is an independent Poisson random variable with parameters $d_{n+1}$.

### 2.2.2 Sensing Performance Metric

We assume the sensing performance depends on how the sensing epochs are placed in time. Given that the durations between two sensing epochs are $d_n$, $n = 1, 2, \ldots$, the sensing performance over the sensing period is measured by $\sum_n f(d_n)$. In addition, we make the following assumptions.

**Assumptions 1** *The sensing performance function $f(d)$, $d \in (0, \infty)$, has the following properties:*

*1) $f(d)$ is convex and monotonically increasing in $d$.*

*2) $f(d)/d$ is increasing in $d$.*

*3) $f(d) \leq Cd$, where $C$ is a positive constant.*

One example application that fits this model is to use samples collected at discrete time instants to estimate a time evolving physical quantity (temperature, humidity, etc), which is modeled as a random processes with power-law decaying covariance. It is shown that the linear minimum MSE (MMSE) estimation for any point on the random process only requires the two adjacent discrete-time samples bounding the point [67]. In this case, $f(d)$ can be interpreted as the total MSE over a length-$d$ interval bounded by two consecutive sensing epochs. Optimizing the overall sensing performance is equivalent to minimizing the total MSE of the linear MMSE over the whole sensing period.

Such assumptions enable us to bound the long-term average sensing performance and motivate the design of the optimal sensing policies. We point out that in this work, we require $d$ to be strictly greater than zero, i.e., we do not consider the scenario where multiple samples are collected at the same time point. This is because if multiple samples are collected at a time, in general, the long-term sensing performance will depends on the number of samples collected at individual sensing epochs, as well as the durations between them. Therefore, it may not be reasonable to assume that the sensing performance over the sensing period can be decomposed into the form of $\sum_n f(d_n)$. We will examine specific forms of sensing performance functions to accommodate such sensing operations, and explore the optimal sampling policy in this scenario in the future.

For a clear exposition of the result, we assume that two samples at time 0 and time $T$ are available at the sensor for free, i.e., no energy is used for collecting those two samples. Denote these two sampling epochs as $l_0 = 0$, $l_{N_T+1} = T$. Besides, there are $N_T$ sensing epochs placed over $(0, T)$. The overall sensing performance over the duration $[0, T]$ is then a summation of $f(d_n)$, $n = 1, 2, \ldots, N_T + 1$.

### 2.2.3 Problem Formulation

Our objective is to optimize the long-term average sensing performance by strategically selecting the sensing epochs $\{l_n\}_{n=1}^{\infty}$. We restrict to online policies, i.e., whenever the system decides a sensing epoch, its decision only depends on the energy harvesting profile up to that time, as well as previous sensing decisions. The optimization problem is formulated as

$$\begin{aligned} \min_{\{l_n\}_{n=1}^{\infty}} . \quad & \limsup_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] \\ \text{s.t.} \quad & (2.1) - (2.2) \end{aligned} \tag{2.3}$$

where the expectation in the objective function is taken over all possible energy harvesting sample paths.

This is essentially a stochastic control problem. In contrast to other discrete-time stochastic control problems where decisions need to be made at every time slot (e.g., Markov Decision Process (MDP)), in this work, we consider a continuous time setting, and decisions can be made at arbitrary time points. Actually, as we will see in Sec. 2.4, selecting the decision points could be a task for the scheduler as well. Therefore, this problem does not admit a MDP formulation in general, and it is extremely challenging to explicitly identify the optimal solution.

## 2.3 Sensing Scheduling with Infinite Battery

In this section, we will study the optimal sensing scheduling for the infinite battery case. We will show that the sensing performance (i.e., time-average MSE) in this scenario has a lower bound, which can be achieved almost surely by a best-effort uniform sensing scheduling policy. The performance limit provided in this section, and the best-effort uniform sensing algorithm will serve as a baseline for the finite battery case discussed in Section 2.4.

**Lemma 1** *Under every feasible scheduling policy, we have*

$$\limsup_{T\to+\infty} \frac{N_T}{T} \leq 1, \quad a.s. \quad \forall i \tag{2.4}$$

*where* $N_T = \sum_{n=1}^{\infty} \mathbf{1}_{l_n \leq T}$ *is the total number of samples taken in* $[0, T]$.

**Proof:** Due to the energy causality constraint (2.2), we always have $N_T \leq \sum_{n=1}^{\infty} \mathbf{1}_{t_n \leq T}$, therefore

$$\limsup_{T\to+\infty} \frac{N_T}{T} \quad \leq \quad \limsup_{T\to+\infty} \frac{\sum_{n=1}^{\infty} \mathbf{1}_{t_n \leq T}}{T} = 1 \quad a.s.$$

where the last equality follows from the strong law of large numbers. ■

**Lemma 2** *The objective function in (2.3) is lower bounded as*

$$\limsup_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] \geq f(1) \tag{2.5}$$

**Proof:**

$$\begin{aligned}
&\limsup_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] \\
&\geq \liminf_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] \\
&\geq \mathbb{E}\left[\liminf_{T\to+\infty} \frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] && (2.6) \\
&\geq \mathbb{E}\left[\liminf_{T\to+\infty} \frac{N_T+1}{T} f\left(\frac{\sum_{n=1}^{N_T+1} d_n}{N_T+1}\right)\right] && (2.7) \\
&= \mathbb{E}\left[\liminf_{T\to+\infty} \frac{N_T+1}{T} f\left(\frac{T}{N_T+1}\right)\right] \geq f(1) && (2.8)
\end{aligned}$$

where (2.6) follows from Fatou's Lemma, (2.7) follows from the convexity of $f$. The last inequality in (2.8) follows from Lemma 1 and the assumption that $f(d)/d$ is an increasing function in $d$. ■

**Definition 1 (Best-effort Uniform Sensing Scheduling)** *The sensor is scheduled to perform the sensing task at $s_n = n$, $n = 1, 2, \ldots$. The sensor performs the sensing task at $s_n$ if $E(s_n^-) \geq 1$; Otherwise, the sensor keeps silent until the next scheduled sensing epoch.*

Here we use $s_n$ to denote the $n$-th *scheduled* sensing epoch, which is in general different from the $n$-th *actual* sensing epoch $l_n$ since some of the scheduled sensing epochs may be infeasible.

**Theorem 1** *Under the best-effort uniform sensing scheduling policy, we have*

$$\lim_{T\to+\infty} \frac{N_T}{T} = 1 \quad a.s.$$

The proof of Theorem 1 is provided in Appendix 2.7.1. Theorem 1 indicates that the best-effort uniform sensing scheduling policy is asymptotically equivalent to a uniform sensing policy almost surely, i.e., the sensor has sufficient energy to perform the task for almost every scheduled sensing epoch.

**Theorem 2** *The best-effort uniform sensing scheduling policy is optimal when the battery size is infinite, i.e.,*

$$\limsup_{T\to+\infty} \frac{1}{T} \sum_{n=1}^{N_T+1} f(d_n) = f(1) \quad a.s.$$

*where* $d_n$ *is the duration between the actual sensing epochs* $l_n$ *and* $l_{n-1}$*.*

The proof of Theorem 2 is provided in Appendix 2.7.2. Theorem 2 indicates that for almost every energy harvesting sample path, the best-effort uniform sensing policy converges to the lower bound in Lemma 2 when the battery size is infinite. This is due to the fact that when the battery size is infinite, the fluctuations of the energy arrivals can be averaged out when time is sufficiently large, thus a uniform sensing scheme with sensing rate equal to the energy harvesting rate can be achieved asymptotically as $T$ is sufficiently large. Thus, the proposed best-effort uniform sensing is optimal. However, with finite battery, it may not be able to achieve the lower bound, since energy overflow is inevitable in this situation, which in turn results in more frequent infeasible sensing epochs due to battery outage.

## 2.4 Sensing Scheduling with Finite Battery

In order to optimize the sensing performance when the battery size is finite, intuitively, the sensing policy should try to prevent any battery overflow, as wasted energy leads to performance degradation. Meanwhile, the properties of the sensing performance function requires the sensing epochs to be as uniform as possible. Those two objectives are not aligned with each other, thus, the optimal scheduling policy should strike a balance between them.

In the following, we propose an energy-aware adaptive sensing scheme. Different from the best-effort uniform sensing scheduling policy that schedules the sensing epochs uniformly, the proposed sensing policy adaptively changes its sensing rate based on the instantaneous battery level. Intuitively, when the battery level is high, the sensor should sense more frequently in order to prevent battery overflow; When the battery level is low, the sensor should sense less frequently to avoid infeasible sensing epochs. Meanwhile, the sensing rate should not vary significantly so that a relatively uniform sensing scheduling can be achieved.

**Definition 2 (Energy-aware Adaptive Sensing Scheduling)** *The adaptive sensing scheduling policy defines sensing epochs $s_n$ recursively as follows*

$$s_n = s_{n-1} + \begin{cases} \frac{1}{1-\beta}, & E(s_{n-1}^-) < \frac{B}{2} \\ 1, & E(s_{n-1}^-) = \frac{B}{2} \\ \frac{1}{1+\beta}, & E(s_{n-1}^-) > \frac{B}{2} \end{cases} \tag{2.9}$$

*where $s_0 = 0$, $E(s_0^-) = 1$, and*

$$\beta := \frac{k \log B}{B} \tag{2.10}$$

*with $k$ being a positive number such that $0 < \beta < 1$. The sensor performs the sensing task at $s_n$ if $E(s_n^-) \geq 1$; Otherwise, the sensor keeps silent until the next scheduled sensing epoch.*

**Remark 1:** The policy divides the battery state space into three different regimes. At each scheduled sensing epoch, the sensor decides whether to sense according to its current battery state, and adaptively selects the next sensing epoch depending on which regime the current battery state falls in. When it is above $B/2$, the sensor senses every $\frac{1}{1+\beta}$ units of time, and when it is below $B/2$, it senses every $\frac{1}{1-\beta}$ units of time. The value of $\beta$ controls the deviation of the sensing rates. Intuitively, when the value of $\beta$ increases, the probability that the battery overflows decreases, so does the probability that a scheduled sensing epoch is infeasible. However, larger $\beta$ may also lead to larger deviations of the durations between sensing epochs, which results in sensing performance degradation.

**Remark 2:** We note that the scheduled sensing epochs are defined in a recursive fashion. At each scheduled sensing epoch, the sensor only need to check its current battery level and decide the next sensing epoch. Thus, the sensor can be turned off temporarily until the next sensing epoch. This could save a significant amount of energy of the sensor from staying awake and constantly monitoring the battery status.

**Remark 3:** As $B \to \infty$, we have $\beta \to 0$ for any fixed $k$, i.e., the adaptive sensing policy converges to the best-effort uniform sensing proposed in Section 2.3 as battery size increases. Thus, it is reasonable to expect that the adaptive sensing policy is asymptotically optimal as battery size approaches infinity.

In the following two theorems, we prove the asymptotical optimality of the adaptive sensing policy, and characterize the speed of its convergence analytically.

**Theorem 3** *Over the sensing period $(0, T)$, we denote $A(T)$ as the total amount of harvested energy, $N'_T$ as the total number of scheduled sensing epochs, and $N_T$ as the total number of actual sensing epochs as defined previously in Section 2.2. Then, under the adaptive sensing scheduling policy, the ratio of infeasible sensing epochs, denoted as $\lim_{T\to\infty} \frac{N'_T - N_T}{N'_T}$, scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right)$, and the average amount of wasted energy per unit time, denoted as $\lim_{T\to\infty} \frac{A(T)-N_T-E(T)}{T}$ scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right)$.*

Theorem 3 indicates that when $B$ is sufficiently large, both upper bounds of the battery outage and overflow probabilities decrease monotonically as $k$ increase. As the battery size $B$ increases, the upper bounds of those two probabilities decrease and eventually approaches zero. Thus, the proposed policy is asymptotically equivalent to a uniform sensing policy, similar to the best-effort uniform sensing policy for the infinite battery case.

**Theorem 4** *Under the adaptive sensing scheduling policy, the gap between the time* *average sensing performance, denoted as* $\lim_{T\to\infty}\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)$*, and the lower bound* $f(1)$ *scales in* $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}+\left(\frac{\log B}{B}\right)^2\right)$.

Theorem 4 implies that as battery size $B$ increases, the sensing performance under the adaptive sensing scheduling policy approaches the lower bound achievable for the system with infinite battery. Thus, it is asymptotically optimal. Compared to the bounds in Theorem 3, the bound in Theorem 4 has an extra term $\left(\frac{\log B}{B}\right)^2$. For a sufficiently large $B$, the bound is dominated by the first term when $k$ is small, and it is dominated by the second term when $k$ is large. Thus, it may not monotonically decrease as $k$ increase, which is consistent with the fact that the sensing performance is not only related to the battery outage and overflow probabilities, but also depends on the durations between consecutive sensing epochs.

The proofs of Theorems 3 and 4 are provided in Appendices 2.7.4 and 2.7.5, respectively. The sketch of the proof is as follows. The battery states at scheduled sensing epochs form a discrete-time random process $\{E(s_n^-)\}_{n=1}^{\infty}$. However, it differs from a conventional discrete-time random process since the duration between two consecutive time indices varies in time: it could be $\frac{1}{1-\beta}$, $\frac{1}{1+\beta}$ or 1, depending on the battery state. This makes the analysis very complicated. To simplify the analysis, in Appendix 2.7.3, we construct a "virtual" energy harvesting sensing system, whose battery state can be any integer in $(-\infty,+\infty)$. Assuming the virtual sensing system senses at a uniform rate, we analytically characterize the expected duration between two consecutive events that the virtual battery state hits a certain level. We then consider the portion of $\{E(s_n^-)\}_{n=1}^{\infty}$ lying in $(0,B/2]$ and $[B/2,B)$ separately. In Appendix 2.7.4, we show that the portion lying in each region can be mapped to a virtual system, and exploit the analytical results in Appendix 2.7.3 to prove Theorem 3. In Appendix 2.7.5, we use the results from Appendix 2.7.4 and the properties of the sensing performance function $f(d)$ to prove Theorem 4.

## 2.5 Simulation Results

The performance of the proposed sensing scheduling policies are evaluated in this section through simulations. We adopt the MSE function for random process reconstruction in [67] to measure the sensing performance under the proposed sensing scheme. Specifically, the correlation between two samples spearated by a time duration $d$ is $\rho^d$, and the average reconstruction MSE of the random field between two $d$-spaced samples is

$$f(d) = d\frac{1+\rho^{2d}}{1-\rho^{2d}} + \frac{1}{\log \rho} \tag{2.11}$$

The power-law parameter $\rho$ is set to be 0.7 in the simulations.

First, we evaluate the uniform best-effort sensing policy for the infinite battery case. We generate 1,000 energy harvesting profiles according to the Poisson random process with $\lambda = 1$, and perform the best-effort uniform sensing for each energy harvesting profile. The sensing rate, $N_T/T$, for each energy harvesting profile is tracked and recorded. One sample path and the sample average sensing rate for the 1,000 sample paths are plotted as a function of $T$ in Fig. 2.1. It is observed that the sensing rate approaches $\lambda = 1$ asymptotically as $T$ increases, as predicted in Theorem 1. Thus the best-effort sampling policy can almost surely approach the behavior of uniform sampling when $T > 400$.

The sensing performance under the best-effort uniform sensing policy is shown in Fig. 2.2. Again, we plot one sample path and the sample average over the $1,000$ sample paths of the time average sensing performance as a function of $T$ in the figure. We observe that the sensing performance curves gradually approach the lower bound $f(1)$ as $T$ increases. When $T = 500$, there is only a very small difference between the simulation results and the analytical lower bound. The results indicate that the proposed best-effort uniform sensing policy is asymptotically optimal.

Next, we evaluate the adaptive sensing scheduling policy for the finite battery case. Fixing the energy harvesting rate to be $\lambda = 1$ per unit time, and $T = 100,000$, we generate

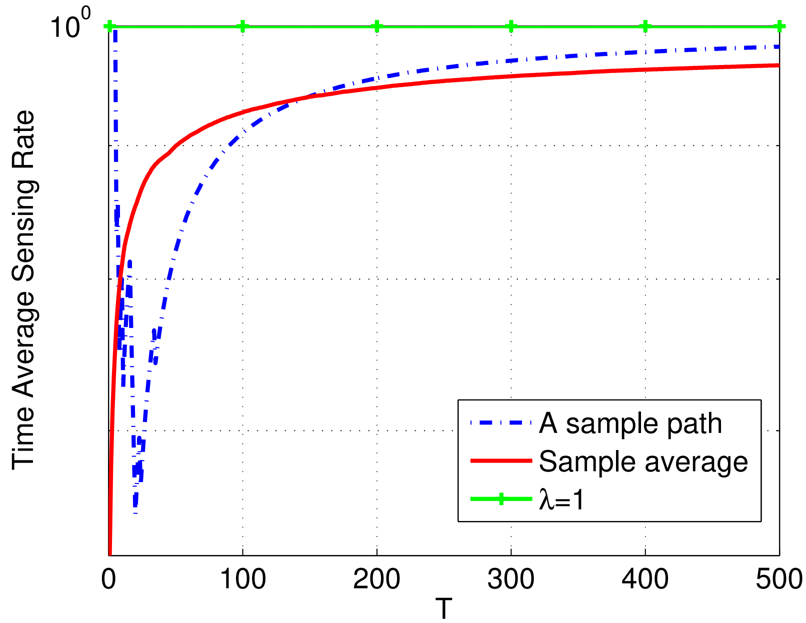



Figure 2.1: Sensing rate as a function of $T$.

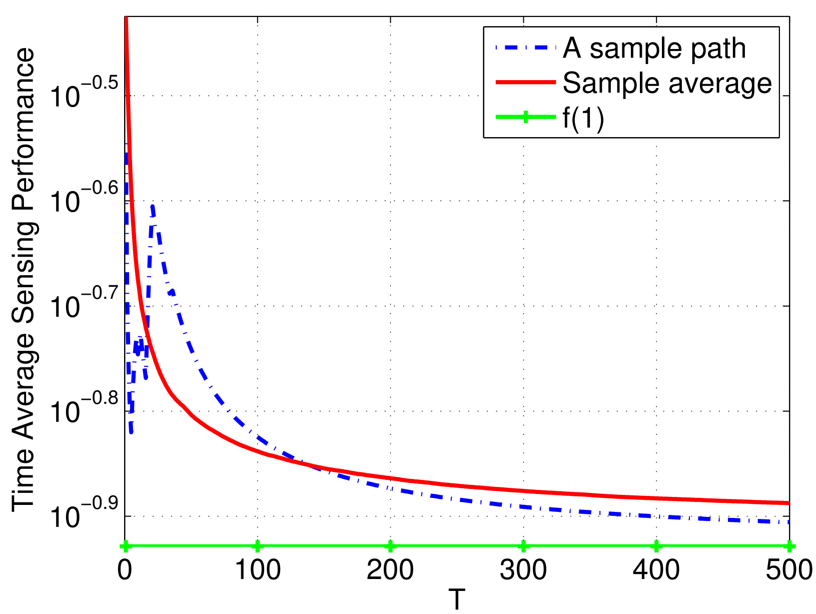



Figure 2.2: Sensing performance as a function of $T$.

a sample path for the Poisson energy harvesting process, and perform the sensing according to the policy. We keep track of the following quantities. First, we count the total number of scheduled sensing epochs under the policy. Among those scheduled sensing epochs, we count the total number of infeasible ones (i.e., the epoch $s_n$ when $E(s_n^-) < 1$), record the ratio of infeasible sensing epochs under the policy. We let $k = 0, 1, 2$, respectively, and perform the adaptive sensing according to (2.9) with battery size $B$ varying from 2 to 100. The corresponding results are plotted in Fig. 2.3. We note that for each fixed $k$, the ratio monotonically decreases as $B$ increase, and each curve is roughly convex in $B$. This is consistent with the theoretical bounds in Theorem 3. Meanwhile, for each fixed battery size, the ratio decreases as $k$ increases. This is due to the fact that the adaptive sensing policy is more conservative for larger $k$ when battery level is below $B/2$, i.e., it senses at a slower rate for larger $k$, which makes the energy level drift away from empty state with higher probability.

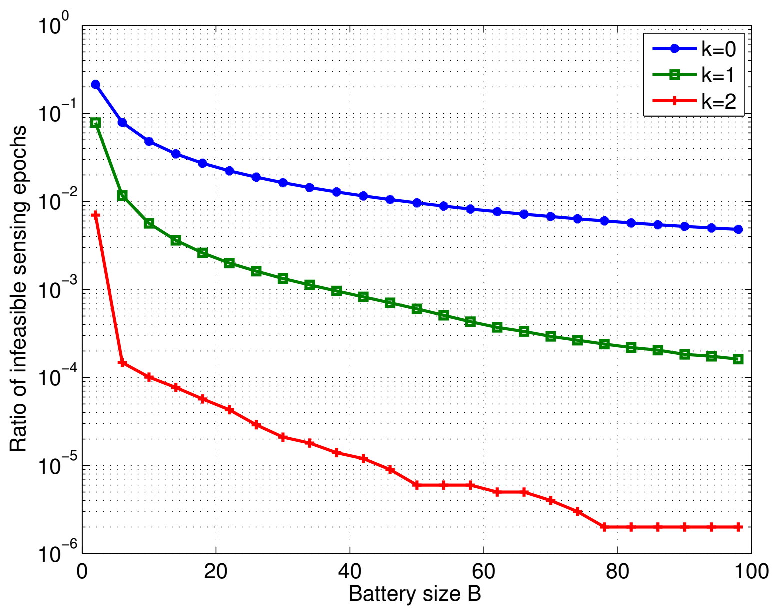



Figure 2.3: The ratio of infeasible sensing epochs.

Next, we study battery overflow under the proposed policy. We count the total number of time instants when the battery state exceeds $B$, and divide it by $T$. The average number of battery overflow events per unit time is plotted as a function of $B$ in Fig. 2.4 for $k = 0, 1, 2$, respectively. Again, we observe that for each fixed $k$, the curve is monotonically decreasing and roughly convex in $B$, as predicted by the theoretical bounds in Theorem 3. Meanwhile, for each fixed battery size, the battery overflow rate decreases as $k$ increases. This is due to the fact that the adaptive sensing policy is more aggressive for larger $k$ when battery level is above $B/2$, i.e., it senses at a faster rate for larger $k$. Thus, the energy level drifts away from full state with higher probability.

At last, we study the sensing performance in terms of the time averaged MSE. We calculate the MSE for each interval bounded by two consecutive sensing epochs as (2.11), aggregate them and divide the sum by $T$. The time averaged MSE is plotted in Fig. 2.5. We note that for each fixed $k$, the gap between the time averaged MSE and the lower bound monotonically decreases as $B$ increases, which is consistent with the theoretical result in Theorem 4. However, when $B$ is fixed, the best sensing performance is observed at $k = 1$, which is different from the results in Figs. 2.3 and 2.4. Even though the battery outage and overflow rates decrease in $k$, the average sensing performance does not exhibit such monotonicity. This is because when $k$ is large, the sensing rate varies dramatically in time. Although this leads to lower outage and overflow probabilities, it compromises the sensing performance as the sensing scheduling deviates from the desired uniform sensing scheduling. Thus, there exists a tradeoff between reducing battery outage and overflow probabilities, and equalizing the sensing rates. The optimal selection of $k$ should jointly consider those two conflicting objectives.

### 2.6 Conclusions

In this chapter, we considered the optimal online sensing scheduling policy for an energy harvesting sensing system. We first provided a lower bound on the time averaged sensing performance for the system with infinite battery, and showed that this lower bound can be achieved by a best-effort uniform sensing policy. We then investigated the finite battery

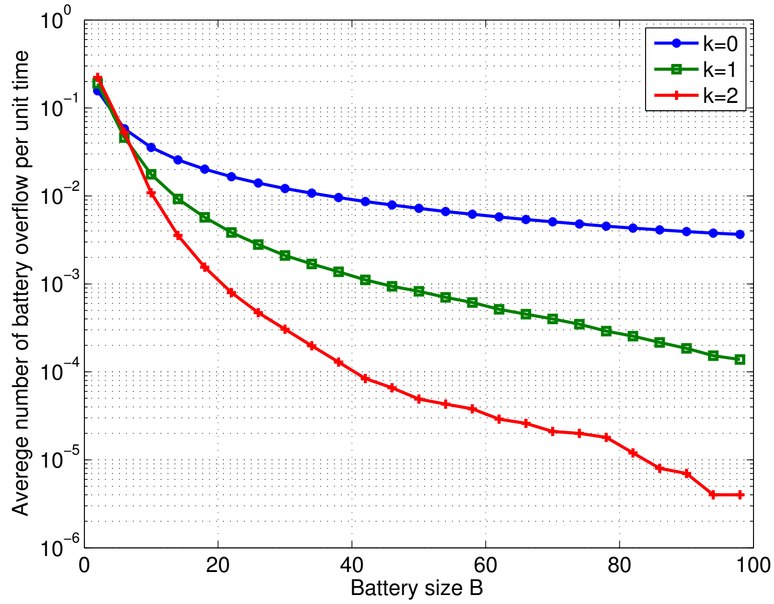



Figure 2.4: The average number of battery overflow per unit time.

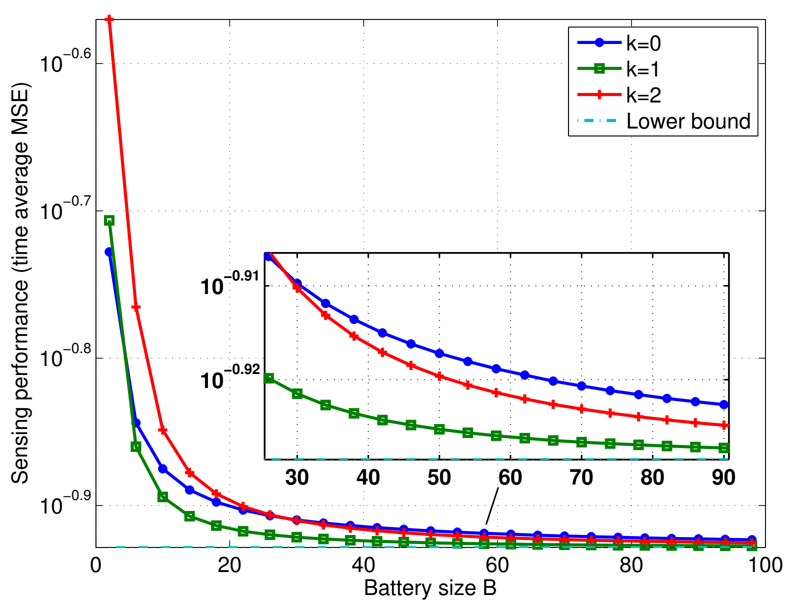



Figure 2.5: The time averaged sensing MSE.

case and proposed an energy-aware adaptive sensing scheduling policy, which dynamically varies the sensing frequency based on instantaneous energy level of the battery. We showed that the battery outage and overflow probabilities under the proposed policy approach zero as battery size goes to infinity, and the time averaged sensing performance converges to the lower bound when the battery size increases. Thus the adaptive sensing scheduling policy is asymptotically optimal. The convergence rates as a function of the battery size were also explicitly characterized. Simulation results validated the theoretical bounds.

## 2.7 Appendix

### 2.7.1 Proof of Theorem 1

The uniform best-effort sensing policy partition the time axis into slots, each with length 1. Consider the number of energy arrivals during a slot, denoted as $A$. Due to the Poisson process assumption of the energy arrival process, we have

$$\mathbb{P}\left[A = k\right] = \frac{e^{-1}}{k!}, \quad k = 0, 1, 2 \dots$$

Let $E(n)$ be the energy level of the sensor right before the scheduled sensing epoch $n$. Based on $E(n)$, we can group the time slots into segments with lengths $u_0, v_1, u_1, \dots, v_k, u_k, \dots$, where $u_i$s correspond to the segments when $E(n) = 0$ and $v_i$s correspond to the segments when $E(n) > 0$, as shown in Fig. 2.6. $E(n)$ jumps from zero to some positive value $e_i$ at the end of the segment corresponding to $u_i$. Therefore, $u_i$ follows an independent geometric distribution

$$\mathbb{P}\left[u_i = k\right] = e^{-(k-1)}(1 - e^{-1}), \quad k = 1, 2 \dots$$

and $v_i$ follows a "random walk" with increment $A - 1$ starting at some positive level $e_i$ until it hits 0. Note that $v_i$ contains a random walk $\Gamma_i$ which starts at $e_i$ and finishes at

$e_i - 1$ for the first time. Denote the duration of $\Gamma_i$ as $\tau_i$.

Let $K_T$ be the number of segments with $E(n) = 0$ during $T$. Note that $T = N_T + \sum_{i=0}^{K_T} u_i$. Therefore, to show $N_T/T \to 1$ almost surely, it suffices to show that

$$\lim_{T\to\infty} \frac{\sum_{i=0}^{K_T} u_i}{T} = 0, \quad a.s.$$

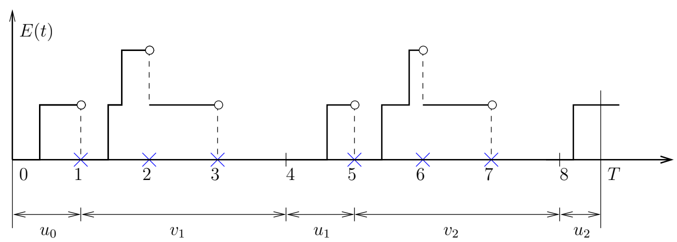



Figure 2.6: An energy level evolution sample path. Crosses represent actual sensing epochs.

Note that

$$\frac{\sum_{i=0}^{K_T} u_i}{T} = \frac{\sum_{i=0}^{K_T} u_i}{K_T}\frac{K_T}{T} \leq \frac{\sum_{i=0}^{K_T} u_i}{K_T}\frac{K_T}{\sum_{i=1}^{K_T} \tau_i}$$

As we will show in the following, $K_T \to \infty$ almost surely as $T \to \infty$. Then, by the strong law of large numbers,

$$\lim_{T\to\infty} \frac{\sum_{i=0}^{K_T} u_i}{K_T} = \frac{1}{1-e^{-1}}, \quad a.s.$$

Therefore, to prove Theorem 1, it suffices to show that

$$\lim_{T\to\infty} \frac{K_T}{\sum_{i=1}^{K_T} \tau_i} = 0, \quad a.s. \tag{2.12}$$

In the following, we will first prove that $K_T \to \infty$ almost surely as $T \to \infty$, and then show (2.12) holds.

Consider a "random walk" $\{\Omega_k\}_{k=0}^{\infty}$, which starts with 1 and increments with $A - 1$. Denote the first 0-hitting time for $\{\Omega_k\}_{k=0}^{\infty}$ as $\kappa$. Then, $\Omega_0 = 1, \Omega_\kappa = 0$. Define a random process $\{\exp(-\alpha\Omega_k - \gamma(\alpha)k)\}_{k=0}^{\infty}$ with $\alpha > 0$ and $\gamma(\alpha) = (e^{-\alpha} - (1-\alpha)) > 0$. We note that

$$\begin{aligned}
&\mathbb{E}\{\exp[-\alpha\Omega_k - \gamma(\alpha)k] | \exp(-\alpha\Omega_0), \ldots, \exp[-\alpha\Omega_{k-1} - \gamma(\alpha)(k-1)]\} \\
&= \mathbb{E}\{\exp[-\alpha(\Omega_{k-1} + A - 1) - \gamma(\alpha)(k-1+1)] | \\
&\qquad \exp(-\alpha\Omega_0), \ldots, \exp[-\alpha\Omega_{k-1} - \gamma(\alpha)(k-1)]\} \\
&= \exp[-\alpha\Omega_{k-1} - \gamma(\alpha)(k-1)] \exp[\alpha - \gamma(\alpha)] \mathbb{E}\{\exp(-\alpha A) | \\
&\quad \exp(-\alpha\Omega_0), \ldots, \exp[-\alpha\Omega_{k-1} - \gamma(\alpha)(k-1)]\} \\
&= \exp[-\alpha\Omega_{k-1} - \gamma(\alpha)(k-1)]
\end{aligned}$$

where the last equality follows from the assumption that $A$ is a Poisson random variable with parameter 1 and is independent with thee random walk prior to time slot $k$. Thus, it is a Martingale process. Based on the property of a Martingale, we have

$$\begin{aligned}
&\mathbb{E}\{\exp[-\alpha\Omega_k - \gamma(\alpha)k]\} \\
&= \mathbb{E}\{\mathbb{E}\{\exp[-\alpha\Omega_k - \gamma(\alpha)k] | \exp(-\alpha\Omega_0), \ldots, \exp[-\alpha\Omega_{k-1} - \gamma(\alpha)(k-1)]\}\} \\
&= \mathbb{E}\{\exp[-\alpha\Omega_{k-1} - \gamma(\alpha)(k-1)]\}
\end{aligned}$$

Applying this equality recursively, we have

$$\begin{aligned}
\exp(-\alpha\Omega_0) &= \mathbb{E}\{\exp[-\alpha\Omega_\kappa - \gamma(\alpha)\kappa]\} && (2.13) \\
&= \mathbb{E}\{(\mathbf{1}_{\kappa<\infty} + \mathbf{1}_{\kappa=\infty}) \cdot \exp[-\alpha\Omega_\kappa - \gamma(\alpha)\kappa]\} \\
&= \mathbb{E}\left[\mathbf{1}_{\kappa<\infty} \cdot \exp(-\alpha\Omega_\kappa - \gamma(\alpha)\kappa)\right] && (2.14)
\end{aligned}$$

where the equality in (2.14) holds due to the fact that $\exp[-\gamma(\alpha) \cdot \infty] = 0$. Let $\alpha \to 0^+$, then $\gamma(\alpha) \to 0^+$, and the equation becomes

$$1 = \mathbb{E}\left[\mathbf{1}_{\kappa<\infty}\right] = \mathbb{P}\left[\kappa < \infty\right] \tag{2.15}$$

i.e., the probability of hitting 0 in finite time is 1.

We point out that (2.14) holds for any initial state $\Omega_0$, so does (2.15). Thus, starting with any $e_i > 0$, the probability that the first 0-hitting time is finite equals 1, i.e., $\mathbb{P}[v_i < \infty] = 1$. This implies that for arbitrary time $t$, the battery will become empty within finite time after it with probability one. Thus, $\lim_{T\to\infty} \mathbb{P}[K_T < \infty] = 1$, i.e., $K_T \to \infty$ almost surely as $T \to \infty$.

Since $\Omega_\kappa = 0$, (2.13) is equivalent to

$$\mathbb{E}\left[\exp(-\gamma(\alpha)\kappa)\right] = \exp(-\alpha).$$

We note that by shifting $\Gamma_i$ to initial time index 1, it virtually follows the same random walk $\{\Omega_k\}_k$. For such $K_T$ i.i.d random walks with 0-hitting times $\tau_i$, we have

$$\mathbb{E}\left[\exp\left(-\gamma(\alpha)\left(\sum_{i=1}^{K_T}\tau_i\right)\right)\right] = \exp(-K_T\alpha), \tag{2.16}$$

Therefore,

$$\begin{aligned}
\mathbb{P}\left[\frac{K_T}{\sum_{i=1}^{K_T}\tau_i} > \epsilon\right] &= \mathbb{P}\left[\sum_{i=1}^{K_T}\tau_i < \frac{K_T}{\epsilon}\right] \\
&= \mathbb{P}\left[\exp\left(-\gamma(\alpha)\left(\sum_{i=1}^{K_T}\tau_i\right)\right) > \exp\left(-\gamma(\alpha)\frac{K_T}{\epsilon}\right)\right] \qquad (2.17)\\
&\le \frac{\exp(-K_T\alpha)}{\exp(-\gamma(\alpha)\frac{K_T}{\epsilon})} = \exp\left(-K_T\left(\alpha - \frac{\gamma(\alpha)}{\epsilon}\right)\right) \qquad (2.18)
\end{aligned}$$

where (2.17) follows from the monotonicity of $e^{-x}$ and (2.18) follows from Markov's inequality and (2.16). Since $\gamma(\alpha) = O(\alpha^2)$, for any $\epsilon > 0$, we can always find a $\alpha$ to have $\alpha - \frac{\gamma(\alpha)}{\epsilon} > 0$, and then the probability decays exponentially in $K_T$. This implies that

$$\sum_{K_T=1}^{\infty}\mathbb{P}\left[\frac{K_T}{\sum_{i=1}^{K_T}\tau_i} > \epsilon\right] < \infty.$$

According to Borel-Cantelli lemma [69], if the sum of the probabilities of a sequence of events is finite, then the probability that infinitely many of them occur is 0. Therefore,

$$\mathbb{P}\left(\limsup_{n\to\infty} \frac{K_T}{\sum_{i=1}^{K_T}\tau_i} > \epsilon\right) = 0,$$

which implies (2.12). This completes the proof.

### 2.7.2 Proof of Theorem 2

To prove Theorem 2, it suffices to show that

$$\limsup_{T\to+\infty} \frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n) \le f(1), \; a.s.$$

As illustrated in Fig. 2.6, there are $v_i$ equally spaced sensing epochs in the segment corresponding to $v_i$. Considering the duration bounded by the first and last sensing epochs in the segment, the aggregated estimation MSE equals $(v_i-1)f(1)$. The duration bounded by the last sensing epoch in the segment associated with $v_i$ and the first sensing epoch in the segment associated with $v_{i+1}$ is $f(u_i+1)$. Therefore,

$$\begin{aligned}
&\limsup_{T\to+\infty} \frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n) \\
&= \limsup_{T\to+\infty} \frac{f(u_0)+\sum_{i=1}^{K_T}\left[(v_i-1)f(1)+f(u_i+1)\right]}{T} \\
&= \limsup_{T\to+\infty} \frac{f(u_0)+\sum_{i=1}^{K_T} f(u_i+1)}{T} + \frac{T-\sum_{i=0}^{K_T}u_i - K_T}{T} f(1) \qquad (2.19)\\
&\le \limsup_{T\to+\infty} f(1) - \frac{\sum_{i=0}^{K_T}u_i}{T} f(1) - \frac{K_T}{T} f(1) + \frac{\sum_{i=0}^{K_T} Cu_i}{T} + \frac{K_T C}{T} \\
&= f(1) \qquad a.s. \qquad (2.20)
\end{aligned}$$

where (2.19) follows from the fact that $u_0+\sum_{i=1}^{K_T}(v_i+u_i)=T$, and (2.20) follows from Assumptions 1-3) and the fact that $K_T/T\to 0$ and $\sum_{i=0}^{K_T}u_i/T\to 0$ almost surely, as proved in the proof of Theorem 1.

Since

$$\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n) \le \frac{1}{T}\left(\sum_{n=1}^{N_T+1} Cd_n\right) = C,$$

it is uniformly bounded in $T$. By the Bounded Convergence Theorem [70], we have

$$\limsup_{T\to\infty} \mathbb{E}\left(\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right) = \mathbb{E}\left(\limsup_{T\to\infty} \frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right) = f(1)$$

### 2.7.3 A virtual energy harvesting sensing system

Before we define the virtual sensing system in this section, we first introduce the following Lemma 3, which will be used later to characterize the virtual battery evolution process.

**Lemma 3** *Consider a Poisson random variable $A$ with parameter $\lambda$. Given $A \ge x$ for some positive integer $x$, we have $x < \mathbb{E}[A|A \ge x] < x + \lambda$.*

**Proof:** Define $B$ as a random variable with PMF

$$\mathbb{P}[B = i] = \frac{Pb[A = x + i]}{\mathbb{P}[A \ge x]}, \quad i = 0, 1, 2, \ldots$$

Then,

$$\mathbb{E}[A|A \ge x] = \frac{\sum_{i=0}^{\infty} \mathbb{P}[A = x + i](x + i)}{\mathbb{P}[A \ge x]} \tag{2.21}$$

$$= \sum_{i=0}^{\infty} \mathbb{P}[B = i](x + i) = x + \mathbb{E}[B] \tag{2.22}$$

$$= x + \sum_{n=0}^{\infty} \mathbb{P}[B > n] > x \tag{2.23}$$

Thus, in order to prove the other inequality in Lemma 3, it suffices to prove that $\mathbb{P}[B > n] < \mathbb{P}[A > n]$ for $n = 0, 1, 2, \ldots$, which is equivalent to $\mathbb{P}[B \le n] > \mathbb{P}[A \le n]$ for

$n = 0, 1, 2, \ldots$. Based on the definition of $A$ and $B$, it then suffices to show that

$$\frac{\sum_{i=0}^{n} \lambda^{x+i}/(x+i)!}{\sum_{j=0}^{\infty} \lambda^{x+j}/(x+j)!} > \frac{\sum_{i=0}^{n} \lambda^{i}/i!}{\sum_{j=0}^{\infty} \lambda^{j}/j!} \tag{2.24}$$

i.e.,

$$\sum_{i=0}^{n} \sum_{j=0}^{\infty} \frac{\lambda^{x+i+j}}{(x+i)!j!} > \sum_{i=0}^{n} \sum_{j=0}^{\infty} \frac{\lambda^{x+i+j}}{(x+j)!i!} \tag{2.25}$$

Since

$$\sum_{i=0}^{n} \sum_{j=0}^{n} \frac{\lambda^{x+i+j}}{(x+i)!j!} > \sum_{i=0}^{n} \sum_{j=0}^{n} \frac{\lambda^{x+i+j}}{(x+j)!i!}, \tag{2.26}$$

it then suffices to show that for $i = 0, 1, \ldots, n$, $j = n+1, n+2, \ldots$, $\frac{1}{(x+i)!j!} > \frac{1}{(x+j)!i!}$. This is true since $j > i$, $x > 0$. ■

Consider an energy harvesting sensing system with a virtual battery whose state can be any integer in $(-\infty, +\infty)$. It senses every $\frac{1}{1-\beta}$ units of time, even if the battery state is zero or negative. The energy arrives at the virtual battery according to a Poisson process with parameter 1. Each sensing operation consumes one unit of energy. We use $E_\beta(n)$ to denote the battery state right before the $n$-th sensing epoch, i.e., at time $\frac{n}{1-\beta}$. Assume the system starts with initial energy level $x$, then, the battery status evolves according to

$$E_\beta(0) = x \tag{2.27}$$

$$E_\beta(n) = E_\beta(n-1) + A\left(\frac{1}{1-\beta}\right) - 1, \quad n = 1, 2, \ldots \tag{2.28}$$

where $A\left(\frac{1}{1-\beta}\right)$ is a Poisson random variable with parameter $\frac{1}{1-\beta}$. Thus,

$$\mathbb{E}[E_\beta(n)] = x + \frac{\beta}{1-\beta} n \tag{2.29}$$

Therefore, when $0 < \beta < 1$, the energy level drifts up in expectation; Otherwise, when

$\beta < 0$, it drifts down.

Define

$$\Lambda_\beta(\alpha) := \log \mathbb{E}\left[e^{-\alpha\left(A\left(\frac{1}{1-\beta}\right)-1\right)}\right] = \frac{e^{-\alpha}-1}{1-\beta} + \alpha \tag{2.30}$$

We note that $\Lambda_\beta(\alpha)$ is convex in $\alpha$, $\Lambda_\beta(0) = 0$, and $\Lambda'_\beta(\alpha) = -\frac{e^{-\alpha}}{1-\beta} + 1$. Thus, equation $\Lambda_\beta(\alpha) = 0$ has another root besides 0, denoted as $\alpha_0$. We have

$$\frac{e^{-\alpha_0}-1}{1-\beta} + \alpha_0 = 0, \quad \Lambda'_\beta(0) = -\frac{\beta}{1-\beta} \tag{2.31}$$

When $\alpha_0$ is sufficiently small, we have

$$\beta = \frac{\alpha_0}{2} + o(\alpha_0) \tag{2.32}$$

Assume $x \in (0, M)$, where $M$ is a positive integer. We are interested in the event that the random process $\{E_\beta(n)\}_{n=0}^\infty$ hits or exceeds one of the two boundary levels 0 and $M$ for the first time. We have the following observations.

**Lemma 4** *Consider the random process $\{E_\beta(n)\}_{n=0}^\infty$ defined in (2.27)-(2.28). Let $\kappa$ be the smallest $n$ such that $E_\beta(n) \geq M$ or $E_\beta(n) = 0$, and $\tau_x := \mathbb{E}[\kappa]$. Define $P_{x,M}$ as the probability that $E_\beta(\kappa) \geq M$, and $P_{x,0}$ as the probability that $E_\beta(\kappa) = 0$. Then,*

$$P_{x,M} = \frac{1-e^{-\alpha_0 x}}{1-e^{-\alpha_0(M+\theta_x)}} \tag{2.33}$$

$$P_{x,0} = \frac{e^{-\alpha_0 x} - e^{-\alpha_0(M+\theta_x)}}{1-e^{-\alpha_0(M+\theta_x)}} \tag{2.34}$$

$$\tau_x = \frac{1-\beta}{\beta}((M+\phi_x)P_{x,M} - x) \tag{2.35}$$

*where $0 \leq \theta_x \leq \frac{1}{1-\beta}$, $0 \leq \phi_x \leq \frac{1}{1-\beta}$.*

**Proof:** Define $\Omega_n := \exp(-\alpha(E_\beta(n) + \Lambda_\beta(\alpha)n))$. Then, $\{\Omega_n\}_{n=0}^\infty$ is a martingale process with initial state $\Omega_0 = \exp\{-\alpha x\}$. Based on the definition, we have

$$\mathbb{E}[\Omega_n] = \mathbb{E}[\mathbb{E}[\Omega_n|\Omega_0, \ldots, \Omega_{n-1}]] = \mathbb{E}[\Omega_{n-1}] = \ldots = \mathbb{E}[\Omega_0] = \exp(-\alpha x) \tag{2.36}$$

Taking derivative of both sides with respect to $\alpha$, we have

$$\mathbb{E}[(E_\beta(n) + \Lambda'_\beta(\alpha)n)\Omega_n] = x\exp(-\alpha x) \tag{2.37}$$

Letting $\alpha \to 0$ in (2.36) for $n = \kappa$, we have

$$\begin{aligned} &\text{LHS} = \mathbb{E}[\Omega_\kappa] \\ &= \mathbb{E}[\Omega_\kappa|\text{first hits } M]P_{x,M} + \mathbb{E}[\Omega_\kappa|\text{first hits } 0]P_{x,0} \\ &= P_{x,M} + P_{x,0} = 1 = \text{RHS} \end{aligned} \tag{2.38}$$

Similarly, letting $\alpha \to \alpha_0$ in (2.36) for $n = \kappa$, we have

$$\text{LHS} = \mathbb{E}[\Omega_\kappa|\text{first hits } M]P_{x,M} + P_{x,0} = \exp(-\alpha_0 x) = \text{RHS} \tag{2.39}$$

We note

$$\begin{aligned} &\mathbb{E}[\Omega_\kappa|\text{first hits } M] \\ &= \mathbb{E}[\exp(-\alpha_0(E_\beta(\kappa) + \Lambda_\beta(\alpha_0)\kappa))|E_\beta(\kappa) \geq M] \\ &= \mathbb{E}[\exp(-\alpha_0 E_\beta(\kappa))|E_\beta(\kappa) \geq M] \\ &\leq e^{-\alpha_0 M} \end{aligned} \tag{2.40}$$

On the other hand, we have

$$\begin{aligned} &\mathbb{E}[\exp(-\alpha_0 E_\beta(\kappa))|E_\beta(\kappa) \geq M] \\ &\geq \exp(-\alpha_0 \mathbb{E}[E_\beta(\kappa)|E_\beta(\kappa) \geq M]) \end{aligned} \tag{2.41}$$

$$\geq e^{-\alpha_0\left(M+\frac{1}{1-\beta}\right)} \tag{2.42}$$

where (2.41) follows from Jensen's inequality, and (2.42) follows from Lemma 3.

Combining (2.39), (2.40) and (2.42), we have

$$P_{x,M}e^{-\alpha_0(M+\theta_x)} + P_{x,0} = e^{-\alpha_0 x}, \tag{2.43}$$

where $0 \leq \theta_x \leq \frac{1}{1-\beta}$.

Solving (2.38) and (2.43), we obtain (2.33)-(2.34).

Let $\alpha \to 0$ in (2.37) for $n = \kappa$, we have

$$\begin{aligned}
\text{LHS} &= \mathbb{E}[(E_\beta(\kappa) + \Lambda'_\beta(\alpha)\kappa)\exp(-\alpha)] \\
&= \mathbb{E}\left[E_\beta(\kappa) - \left(\frac{1}{1-\beta} - 1\right)\kappa\right] \\
&= (M + \phi_x)P_{x,M} - \frac{\beta}{1-\beta}\tau_x = x = \text{RHS}
\end{aligned}$$

where $0 \leq \phi_x \leq \frac{1}{1-\beta}$. Thus, we have (2.35) established. ■

**Lemma 5** *Consider the random process $\{E_\beta(n)\}_{n=0}^{\infty}$ defined in (2.27)-(2.28). Define $S^-_{x,M}$ as the expected time index $n$ when $\{E_\beta(n)\}_{n=0}^{\infty}$ with $\alpha_0 = -\frac{k\log M}{M} + o\left(\frac{\log M}{M}\right) < 0$ hits boundary level $M$ for the first time, and $S^+_{x,0}$ as the expected time index $n$ when $\{E_\beta(n)\}_{n=0}^{\infty}$ with $\alpha_0 = \frac{k\log M}{M} + o\left(\frac{\log M}{M}\right) > 0$ hits boundary level $0$ for the first time. Then, $S^-_{M,M} = \Omega\left(\frac{M^{k+1}}{k(\log M)^2}\right)$, $S^+_{0,0} = \Omega\left(\frac{M^{k+1}}{k(\log M)^2}\right)$.*

**Proof:** First, let us consider the case when $\alpha_0 = -\frac{k\log M}{M} + o\left(\frac{\log M}{M}\right) < 0$. We use superscript $-$ to indicate that $\alpha_0$ involved in the corresponding quantities is negative. Applying Lemma 4 for $x = 1$ and $x = M - 1$, we have

$$
\begin{aligned}
P^-_{1,M} &= \frac{1-e^{-\alpha_0}}{1-e^{-\alpha_0(M+\theta^-_1)}} = \frac{\alpha_0(1+O(\alpha_0))}{-M^k(1+O(\alpha_0+M^{-k})} \\
&= \frac{\alpha_0}{-M^k}(1+O(\alpha_0+M^{-k}))
\end{aligned}
$$

$$
\begin{aligned}
P^-_{M-1,0} &= \frac{e^{-\alpha_0(M-1)}-e^{-\alpha_0(M+\theta^-_{M-1})}}{1-e^{-\alpha_0(M+\theta^-_{M-1})}} = \frac{e^{\alpha_0}-e^{-\alpha_0\theta^-_{M-1}}}{e^{\alpha_0 M}-e^{-\alpha_0\theta^-_{M-1}}} \\
&= \frac{\alpha_0(1+\theta^-_{M-1}(1+O(\alpha_0))}{-1+O(\alpha_0+M^{-k})} \\
&= -\alpha_0(1+\theta^-_{M-1})(1+O(\alpha_0+M^{-k}))
\end{aligned}
$$

For the corresponding expected first hitting time, we have

$$
\begin{aligned}
\tau^-_1 &= \frac{1-\beta}{\beta}\left(\left(M+\phi^-_1\right)P^-_{1,M}-1\right) \\
&= \frac{1-\beta}{\beta}(-1+o(1)) \\
&= -\frac{2}{\alpha_0}(1+o(1))
\end{aligned}
\tag{2.44}
$$

and

$$
\begin{aligned}
\tau^-_{M-1} &= \frac{1-\beta}{\beta}\left(\left(M+\phi^-_{M-1}\right)P^-_{M-1,M}-(M-1)\right) \\
&= \frac{1-\beta}{\beta}\left[\left(M+\phi^+_{M-1}\right)(1-M^{-k}2\alpha_0(1+o(1))-(M-1)\right] \\
&= \frac{1-\beta}{\beta}(\phi^-_{M-1}+1)\alpha_0(1+o(1)) \\
&= 2(M+\phi^-_{M-1})(1+o(1))
\end{aligned}
\tag{2.45}
$$

We note that

$$
S^-_{M-1,M} = \tau^-_{M-1} + P^-_{M-1,0}\cdot S^-_{0,M} \tag{2.46}
$$

$$
S^-_{0,M} \geq \sum_{x=0}^{M} q_{0,x}\left(\tau_x + P^-_{x,0}S^-_{0,M}\right) \tag{2.47}
$$

where $q_{0,x}$ is the probability that given the random process $\{E_\beta(n)\}_{n=0}^{\infty}$ first hits boundary

0, it re-enters the range $[0, M]$ with state $x$. Thus,

$$S^-_{0,M} \geq \frac{\sum_{x=0}^{M} q_{0,x}\tau_x}{1-\sum_{x=0}^{M} q_{0,x}P^-_{x,0}} = \frac{\sum_{x=0}^{M} q_{0,x}\tau_x}{\sum_{x=0}^{M} q_{0,x}P^-_{x,M}} \tag{2.48}$$

According to (2.33), when $\alpha_0 x$ is sufficiently small, we have

$$P^-_{x,M} = \frac{1-e^{-\alpha_0 x}}{1-e^{-\alpha_0(M+\theta_x)}} = \frac{x\alpha_0}{-M^k}(1+O(\alpha_0+M^{-k})) \tag{2.49}$$

Pick the smallest positive integer $K$ such that $\frac{1}{K!} < \frac{1}{M^{k+2}}$. Hence $K = O(\log M)$ and $\alpha_0 K = o(1)$. For sufficiently large $M$, $P^-_{x,M} \leq P^-_{K,M}$. Thus, we have

$$\begin{aligned}
\sum_{x=0}^{M} q_{0,x}P^-_{x,M} &\leq \sum_{x=0}^{K} q_{0,x}P^-_{x,M} + \sum_{x=K+1}^{M} q_{0,x} \\
&\leq \left(\sum_{x=0}^{K} q_{0,x}\right) P^-_{K,M} + \sum_{x=K+1}^{M} q_{0,x} \\
&= (1-q)P^-_{K,M} + q
\end{aligned}$$

where $q := \sum_{x=K+1}^{M} q_{0,x}$. By induction, we can show that $q = O\left(\frac{1}{(K-1)!}\right)$. Therefore,

$$S^-_{0,M} \geq \frac{q_{0,1}\tau^-_1}{P^-_{K,M}(1+O(\alpha_0+M^{-k}))} \tag{2.50}$$

Plugging (2.50) in (2.46), we have

$$\begin{aligned}
S^-_{M-1,M} &\geq \tau^-_{M-1} + \frac{P^-_{M-1,0}q_{0,1}\tau^-_1}{P^-_{K,M}(1+O(\alpha_0+M^{-k}))} && (2.51)\\
&\geq 2M + \frac{M^k}{K} q_{0,1} \frac{2M}{k\log M}(1+O(\alpha_0+M^{-k})) && (2.52)\\
&\sim \Omega\left(\frac{M^{k+1}}{k(\log M)^2}\right) && (2.53)
\end{aligned}$$

Since $S^-_{M-1,M} > (1+S^-_{M-1,M})\mathbb{P}\left[A\left(\frac{1}{1-\beta}\right)=0\right]$, we have $S^-_{M,M} = \Omega\left(\frac{M^{k+1}}{k(\log M)^2}\right)$.

Next, let us consider the case when $\alpha_0 = \frac{k \log M}{M} + o\left(\frac{\log M}{M}\right) > 0$. In the following, we use superscript + to indicate that $\alpha_0$ involved in the corresponding quantities is positive.

Applying Lemma 4 for $x = 1$ and $x = M - 1$, we have

$$\begin{aligned} P^+_{1,M} &= \frac{1 - e^{-\alpha_0}}{1 - e^{-\alpha_0(M+\theta^+_1)}} = \frac{\alpha_0(1 + O(\alpha_0))}{1 + O(M^{-k})} \\ &= \alpha_0(1 + O(\alpha_0 + M^{-k})) \end{aligned}$$

and

$$\begin{aligned} P^+_{M-1,0} &= \frac{e^{-\alpha_0(M-1)} - e^{-\alpha_0(M+\theta^+_{M-1})}}{1 - e^{-\alpha_0(M+\theta^+_{M-1})}} \\ &= \frac{e^{-\alpha_0 M}\left(e^{\alpha_0} - e^{-\alpha_0\theta^+_{M-1}}\right)}{1 - e^{-\alpha_0(M+\theta^+_{M-1})}} \\ &= \frac{M^{-k}\alpha_0(1 + \theta^+_{M-1} + O(\alpha_0))}{1 + O(M^{-k})} \\ &\leq M^{-k} \cdot 2\alpha_0(1 + O(\alpha_0 + M^{-k})) \end{aligned}$$

where the inequality follows from the fact that $\theta^+_{M-1} \leq \frac{1}{1-\beta} = 1 + O(\alpha_0)$. Thus,

$$\frac{P^+_{1,M}}{P^+_{M-1,0}} \geq \frac{M^k}{2}(1 + O(\alpha_0 + M^{-k})) \tag{2.54}$$

For the corresponding expected first hitting time, we have

$$\begin{aligned} \tau^+_1 &= \frac{1-\beta}{\beta}\left(\left(M + \phi^+_1\right) P^+_{1,M} - 1\right) \\ &= \frac{1-\beta}{\beta}\left(\left(M + \phi^+_1\right) \alpha_0(1 + o(1)) - 1\right) \\ &= 2\left(M + \phi^+_1\right)(1 + o(1)) \end{aligned} \tag{2.55}$$

and

$$\tau^+_{M-1} = \frac{1-\beta}{\beta}\left(\left(M + \phi^+_{M-1}\right) P^+_{M-1,M} - (M-1)\right)$$

$$= \frac{1-\beta}{\beta}\Big( \big(M + \phi^+_{M-1}\big)\left(1 - M^{-k} 2\alpha_0 (1+o(1)\right) - (M-1)\Big)$$
$$= \frac{1-\beta}{\beta}(\phi^+_{M-1} + 1)(1+o(1))$$
$$= \frac{2(1+\phi^+_{M-1})}{\alpha_0}(1+o(1)) \tag{2.56}$$

Following similar arguments as in (2.46)-(2.50), we have

$$S^+_{1,0} \geq \tau^+_1 + \frac{P^+_{1,M}}{P^+_{M-1,0}} \cdot \tau^+_{M-1}$$
$$\geq 2M + \frac{M^k}{2}(1 + O(\alpha_0 + M^{-k})) \cdot \frac{2M}{k \log M} \tag{2.57}$$
$$\sim \frac{M^{k+1}}{k \log M} \tag{2.58}$$

where (2.57) follows from (2.54), (2.55) and (2.56). ■

### 2.7.4 Proof of Theorem 3

Now consider the energy state evolution process $\{E(s^-_n)\}^\infty_{n=1}$ under the proposed adaptive sensing scheduling policy. We focus on the portion of the random process lying in ranges $[0, B/2)$ and $(B/2, B]$, respectively. Comparing the random process $\{E(s^-_n)\}^\infty_{n=1}$ with the virtual battery evolution process defined in (2.27)-(2.28), we note that each portion can be treated as part of $\{E_\beta(n)\}^\infty_{n=0}$ lying in the corresponding range. Therefore, the characterization of $\{E_\beta(n)\}^\infty_{n=0}$ in Lemma 4 and Lemma 5 can be slightly modified to characterize $\{E(s^-_n)\}^\infty_{n=1}$.

Specifically, for the portion lying in $[0, B/2)$, we let $M = B/2$, $\beta = \frac{k \log B}{B} > 0$, then, the expected number of epochs between two consecutive battery outage events, i.e., $E(s^-_n) = 0$, can be bounded below by $S^+_{0,0}$. Thus, based on law of large numbers, the probability that a sensing epoch is infeasible is bounded above by $1/S^+_{0,0}$. Therefore, it scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right)$.

Similarly, for the portion lying in $[B/2, B)$, we map $B \to M$, $B/2 \to 0$,

$\beta = -\frac{k \log B}{B} < 0$, then, the expected number of epochs between two consecutive battery overflow events, i.e., $E(s_n^-) = B$, can be bounded below by $S_{M,M}^-$. Again, based on law of large numbers, the rate of battery overflow scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right)$. Due to the properties of Poisson process, we can show that the amount of wasted energy per unit time is bounded by twice of the battery overflow rate, thus it scales in the same order.

### 2.7.5 Proof of Theorem 4

Consider the first $n$ scheduled sensing epochs under the proposed adaptive sensing scheduling policy. Let $n_+$ denote the number of intervals between two scheduled sensing epochs with duration $\frac{1}{1-\beta}$, $n_-$ be that with duration $\frac{1}{1+\beta}$, and $n_0$ be that with duration 1. Let $\bar{n}$ be the number of sensing epochs the battery overflows, and $\underline{n}$ be the number of infeasible sensing epochs. Then, the $n$-th scheduled sensing epoch happens at time $T_n := \frac{n_+}{1-\beta} + n_0 + \frac{n_-}{1+\beta}$. Let $A_n^+$ be the total amount of energy wasted. Then,

$$E(S_n^-) = (A(T_n) - A_n^+) - (n - \underline{n}) \tag{2.59}$$

where $A(T_n)$ is a Poisson random variable with parameter $T_n$. Dividing both sides by $n$ and taking the limit as $n$ goes to $+\infty$, we have

$$\lim_{n\to\infty} \frac{E(n)}{n} = \lim_{n\to\infty} \frac{A(T_n)}{T_n} \cdot \frac{T_n}{n} - \lim_{n\to\infty} \frac{A_n^+}{n} - \left(1 - \lim_{n\to\infty} \frac{\underline{n}}{n}\right)$$

Therefore,

$$\lim_{n\to\infty} \frac{T_n}{n} = 1 + O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right) \tag{2.60}$$

Based on Taylor expansion and (2.60), we have

$$\lim_{n\to\infty} \frac{n_+ f\left(\frac{1}{1-\beta}\right) + n_0 f(1) + n_- f\left(\frac{1}{1+\beta}\right)}{T_n}$$
$$= f(1) + O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}} + \left(\frac{\log B}{B}\right)^2\right)$$

On the other hand, due to the existence of infeasible sensing epochs, we have

$$\lim_{n\to\infty} \frac{\sum_n f(d_n) - \left[n_+ f\left(\frac{1}{1-\beta}\right) + n_0 f(1) + n_- f\left(\frac{1}{1+\beta}\right)\right]}{T_n}$$
$$\leq \lim_{n\to\infty} \frac{\sum_{d_n : d_n \geq \frac{1}{1-\beta}} f(d_n)}{T_n} \tag{2.61}$$
$$\leq \lim_{n\to\infty} \frac{\sum_{d_n : d_n \geq \frac{1}{1-\beta}} C d_n}{T_n} \tag{2.62}$$
$$\leq \lim_{n\to\infty} \frac{2C\underline{n}}{T_n} = O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right) \tag{2.63}$$

where (2.61) follows from the fact that the difference between the actual sensing performance and scheduled sensing performance is due to the infeasible sensing epochs. (2.62) follows from the property of $f(d)$, and (2.63) follows from Theorem 3 and (2.60). Thus,

$$\lim_{n\to\infty} \frac{\sum_n f(d_n)}{T_n} = f(1) + O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}} + \left(\frac{\log B}{B}\right)^2\right)$$

# Chapter 3: Optimal Status Updating to Minimize Age of Information with an Energy Harvesting Source

## 3.1 Introduction

Enabled by the widespread wireless communications and the proliferation of ultra-low power sensors, ubiquitous sensing has profoundly changed almost every aspect of our daily lives. In many applications, such as environment monitoring [71], vechicle tracking [72], sensors are deployed to monitor the status of sensing objects, and communicate the status information to a fusion center (FC). To keep track of the status, the FC requires status updates as timely as possible. However, this is often constrained by limited physical resources, such as energy and bandwidth. In order to measure the timeliness of the status updates at the FC, a metric called "Age of Information" (AoI) has been introduced in recent literature [73]. Specifically, AoI is defined as the time elapsed since the last received update was generated.

With this definition, AoI in various queueing systems has been analyzed, such as single-source single-server queues [73], the $M/M/1$ Last-Come First-Served (LCFS) queue with preemption in service [74], and the $M/M/1$ First-Come First-Served (FCFS) system with multiple sources [75, 76]. AoI with out-of-order packet delivery has been evaluated in [77–79]. A related metric, Peak Age of Information (PAoI), is introduced in [80], and has been studied in [81, 82]. Most recently, optimality properties of a Last Generated First Served (LGFS) service discipline when updates arrive out of order are identified in [83], packet deadlines are found to improve AoI in [84], AoI in the presence of errors is evaluated in [85], and LCFS with non-memoryless gamma-distributed service times is considered in [86]. Optimal status update policy with knowledge of the server state has been studied in [87]. Under an energy harvesting setting, [88, 89] investigate various status update polices assuming the battery equipped with the source is sufficiently large. It has been shown in [88] that with knowledge of the system state, updates should be submitted only when the server is free to avoid queueing delay. Moreover, a greedy policy that submits a fresh update as the system becomes idle is shown to be inefficient; a *lazy* update policy that introduces inter-update delays is better. The optimal update policy remains open in this setting. In [89], under the assumption that a status update packet can be generated and served (transmitted) instantly, the authors investigate optimal offline and online policies. The optimal offline policy is to equalize the inter-update delays as much as possible, subject to the energy constraint imposed by the energy harvesting source. The online problem is cast as a Markov Decision Process in a discrete-time setting, and solved through dynamic programming. Although it is analytically intractable, the optimal policy is shown to have a threshold structure.

In this chapter, we investigate optimal *online* status update policies for an energy harvesting source with various battery sizes. We consider a setting similar to [89], where a status update packet can be generated by the source at any time and transmitted to a FC instantly, given sufficient energy is available at the source. We assume that the energy unit is normalized so that each status update requires one unit of energy. This energy unit represents the cost of both measuring and transmitting a status packet. We assume energy arrives at the sensor according to a Poisson process, and the sensor only has causal information of the energy arrival profile. Our objective is then to determine the sequence of update instants so that the time average AoI at the FC is minimized, subject to the energy causality constraints at the source.

We first study the properties of AoI as a function of inter-update delays, and establish a connection between this problem and the optimal sensing problem studied in Chapter 2. This motivates us to adopt the (asymptotically) optimal sensing policies in Chapter 2 for AoI minimization, namely, a best-effort uniform status update policy for the infinite battery case, and an energy-aware adaptive status update policy for the finite battery case. Since the AoI function does not have all the properties required to establish the optimality of those policies in Chapter 2, we revise the proofs accordingly to re-establish their (asymptotic) optimality. We then study a special case where the battery size is one unit, and propose a threshold based status update policy. We analytically characterize the time average AoI under this policy, and show that it outperforms any other online policy in this special scenario. This chapter has been submitted to IEEE International Conference on Communications 2017 for possible publication.

## 3.2 System Model and Problem Formulation

Consider a scenario where an energy harvesting sensor continuously monitors a system and sends time-stamped status updates to a destination. The destination keeps track the system status through the received updates. We use the metric Age of Information (AoI) to measure the "freshness" of the status information available at the destination.

We assume the time used to collect and transmit a status update is negligible compared with the time scale of inter-update delays. Therefore, given sufficient energy is available at the source, a status update can be generated by the source at any time and transmitted to a FC instantly. In this case, a status update is transmitted immediately after it is generated to avoid unnecessary queueing delay.

We assume that the energy unit is normalized so that each status update requires one unit of energy. This energy unit represents the cost of both measuring and transmitting a status update. Assuming energy arrives at the sensor according to a Poisson process with parameter $\lambda$. Hence, energy arrivals occur in discrete time instants $t_1, t_2, \ldots$. We assume $\lambda = 1$ throughout this paper for ease of exposition. The sensor is equipped with a battery with capacity $B$, $B \geq 1$. When $B = \infty$, it corresponds to the infinite battery case.

A status update policy is denoted as $\{l_n\}_{n=1}^{\infty}$, where $l_n$ is the $n$-th sampling instant. Let $l_0 = 0$, and $d_n := l_n - l_{n-1}$, for $n = 1, 2, \ldots$. Define $A(d_n)$ as the total amount of energy harvested in $[l_{n-1}, l_n)$, and $E(l_n^-)$ as the energy level of the sensor right before the scheduled sensing epoch $l_n$. Then, under any feasible status update policy, the energy queue evolves as follows

$$E(l_{n+1}^-) = \min\{E(l_n^-) - 1 + A(d_{n+1}), B\} \tag{3.1}$$

$$E(l_n^-) \geq 1 \tag{3.2}$$

for $n = 1, 2, \ldots$. Eqn. (3.2) corresponds to the energy causality constraint in the system. Based on the Poisson arrival process assumption, $A(d_{n+1})$ is an independent Poisson random variable with parameters $d_{n+1}$.

Under any feasible status update policy, the AoI as a function of time is shown in Figure 3.1. For a clear exposition of the results, we assume that two samples at time 0 and time $T$ are available at the sensor for free, i.e., no energy is used for collecting those two samples. Denote these two sampling epochs as $l_0 = 0$, $l_{N_T+1} = T$. Besides, there are $N_T$ sensing epochs placed over $(0, T)$. Then. the time average AoI over the duration $[0, T]$ can be expressed as $\sum_{n=1}^{N_T+1} f(d_n)$, where $f(d_n) = d_n^2/2$.

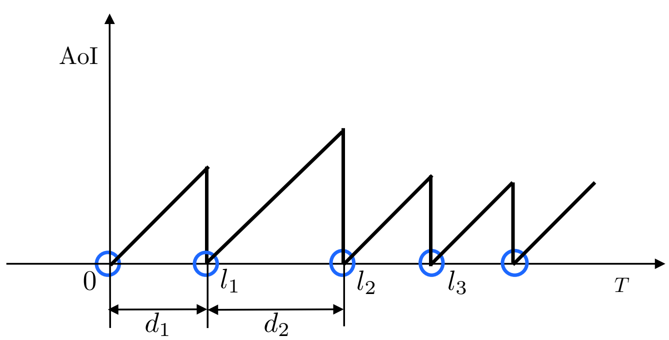



Figure 3.1: AoI as a function of $T$. Circles represent status update instants.

With causal information of the energy arrival profile, our objective is to determine the sequence of update instants $l_1, l_2, \ldots$, so that the time average AoI at the FC is minimized, subject to the energy causality constraint. The optimization problem can formulated as

$$\begin{aligned} \min_{\{l_n\}_{n=1}^{\infty}} . \quad & \limsup_{T \to +\infty} \mathbb{E}\left[\frac{1}{T}\sum_{n=1}^{N_T+1} f(d_n)\right] \\ \text{s.t.} \quad & (3.1) - (3.2) \end{aligned} \tag{3.3}$$

where the expectation in the objective function is taken over all possible energy harvesting sample paths. Under the continuous time setting, the sensor can sample the system status at arbitrary time points. Therefore, this problem does not admit a MDP formulation in general, and it is extremely challenging to explicitly identify the optimal solution.

## 3.3 Optimal Status Update Policies

In Chapter 2, we studied an optimal sensing scheduling problem. Our objective was to strategically select the sensing epochs, so that the long-term sensing performance can be optimized. We assumed that the sensing performance function can be decomposed as a summation of $f(d_n)$, where $d_n$ is the $n$-th inter-sensing duration. The optimization problem was in the same form of (3.3). Under the assumption that 1) $f(d)$ is convex and monotonically increasing in $d$; 2) $f(d)/d$ is increasing in $d$; and 3) $f(d)/d$ is upper bounded by a positive constant, we proposed two sensing policies, for the infinite and finite battery cases, respectively, and proved their (asymptotic) optimality.

We note that the AoI minimization problem can be treated as a particularized case of the optimal sensing scheduling problem studied in Chapter 2, by replacing the general sensing performance metric with AoI. The only challenge is that in this case, $f(d) = d^2/2$. While this satisfies the first two assumptions required for the optimality of the proposed sensing scheduling policies, it does not satisfy the last one, since $f(d)/d = d/2$, and it is unbounded. Therefore, the optimality of the policies need to be re-justified.

For the completeness of this Chapter, in this section, we adapt the major results and policies in Chapter 2 for the AoI minimization setup. We leave out the proofs that are unaffected by the third assumption, and provide necessary new proofs only.

### 3.3.1 Status Update with Infinite Battery

When the battery size is infinite, no energy overflow will happen. Thus, the maximum achievable long time average status update rate is one update per unit time. If we drop the energy causality constraint, and replace it with this long-term average status update rate constraint, we obtain a lower bound on the long-term average AoI as follows:

**Lemma 6** *The long-term average AoI is lower bounded by 1/2.*

This lower bound corresponds to a uniform status update policy which updates once per unit time. This motivates us to propose the following policy.

**Definition 3 (Best-effort Uniform Status Update Policy)** *The sensor is scheduled to update the status at $s_n = n$, $n = 1, 2, \ldots$. The sensor performs the task at $s_n$ if $E(s_n^-) \geq 1$; Otherwise, the sensor keeps silent until the next scheduled status update epoch.*

Here we use $s_n$ to denote the $n$-th *scheduled* status update epoch, which is in general different from the $n$-th *actual* status update epoch $l_n$ since some of the scheduled status update epochs may be infeasible.

**Theorem 5** *The best-effort uniform status update policy is optimal when the battery size is infinite, i.e.,*

$$\limsup_{T\to+\infty} \frac{1}{T} \sum_{n=1}^{N_T+1} f(d_n) = 1/2 \quad a.s.$$

The proof of Theorem 5 is provided in Appendix 3.6.1. Intuitively, when the battery size is infinite, the fluctuations of the energy arrivals can be averaged out when $T$ is sufficiently large, thus the uniform status update policy can be achieved asymptotically.

### 3.3.2 Status Update with Finite Battery

In order to minimize long-term average AoI when the battery size is finite, intuitively, the status update policy should try to prevent any battery overflow, as wasted energy leads to performance degradation. Meanwhile, the properties of AoI function requires the status update epochs to be as uniform as possible. Those two objectives are not aligned with each other, thus, the optimal status update policy should strike a balance between them.

In the following, we propose an energy-aware adaptive status update policy, which adaptively changes its update rate based on the instantaneous battery level. When the battery level is high, the sensor updates more frequently in order to prevent battery overflow; When the battery level is low, the sensor updates less frequently to avoid infeasible status update epochs. Meanwhile, the update rate does not vary significantly in order to achieve a relatively uniform status update.

**Definition 4 (Energy-aware Adaptive Status Update Policy)** *The adaptive status update policy defines status update epochs $s_n$ recursively as follows*

$$s_n = s_{n-1} + \begin{cases} \frac{1}{1-\beta}, & E(s_{n-1}^-) < \frac{B}{2} \\ 1, & E(s_{n-1}^-) = \frac{B}{2} \\ \frac{1}{1+\beta}, & E(s_{n-1}^-) > \frac{B}{2} \end{cases} \tag{3.4}$$

*where $s_0 = 0$, $E(s_0^-) = 1$, and $\beta := \frac{k \log B}{B}$, with $k$ being a positive number such that $0 < \beta < 1$. The sensor samples and updates the status at $s_n$ if $E(s_n^-) \geq 1$; Otherwise, the sensor keeps silent until the next scheduled status update epoch.*

As $B \to \infty$, we have $\beta \to 0$ for any fixed $k$, i.e., the adaptive updating policy converges to the best-effort uniform updating policy as battery size increases. Thus, it is reasonable to expect that the adaptive status update policy is asymptotically optimal as battery size approaches infinity.

**Theorem 6** *Under the adaptive status update policy, the gap between the long-term average AoI and its lower bound $1/2$ scales in $O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}} + \left(\frac{\log B}{B}\right)^2\right)$.*

Theorem 6 implies that as battery size $B$ increases, the long-term average AoI under the adaptive status update policy approaches the lower bound achievable for the system with infinite battery. Thus, it is asymptotically optimal. The proof of Theorem 6 is provided in Appendix 3.6.2.

### 3.4 A Special Case: $B = 1$

In the previous section, we investigate the optimal and asymptotically optimal status update policies when battery size $B$ is infinite, or finite but sufficiently large, respectively. However, when the battery size is so small that the asymptotics cannot kick in, those policies may not perform very well. That motivates us to investigate other status update policies when battery size $B$ is small. One extreme case for this scenario is when $B = 1$, i.e., the battery can only store the energy for one status update operation. In this case, the battery only has two states: empty, or full. When it is empty, obviously, any status update should not be scheduled. When one unit amount of energy arrives, the battery jumps to the other state, and it then need to decides when to spend the energy for status update. Intuitively, it is still desirable to update as uniform as possible. Thus, we propose the following policy.

**Definition 5 (Threshold-based Status Update Policy)** *When an energy unit enters an empty battery, the sensor performs a status update immediately if the AoI at the FC is greater than a threshold $\tau$; Otherwise, it holds its operation until the AoI is exactly equal to $\tau$.*

The long-term average AoI under this policy can be analytically characterized based on the memoryless property of the exponentially distributed inter-arrival times of energy units. We summarize the result in the following theorem.

**Theorem 7** *Under the threshold-based status update policy, the long-term average AoI is $h(\tau) := \frac{2\tau e^{-\tau}+2e^{-\tau}+\tau^2}{2(e^{-\tau}+\tau)}$.*

The proof of Theorem 7 is provided in Appendix 3.6.3. Moreover, we can show that $h(\tau)$ is first decreasing, then increasing in $\tau$. Therefore, the optimal $\tau$ corresponds to the point where $h'(\tau) = 0$. Solving the equation, we have $\tau^* = 0.901$, and the corresponding long-term average AoI is 0.9012.

**Theorem 8** *When $B = 1$, the threshold-based status update policy achieves the minimum long-term average AoI among all online policies, thus it is optimal.*

The proof of Theorem 8 is sketched as follow. We model the the status update instants under any online policy as a renewal process. Then, the long-term average AoI under the policy is equal to the expected average AoI over one renewal interval. Next, we focus on one renewal interval, and show that the optimal policy should depend on the first energy arrival time in that interval. Through functional analysis, we show that the threshold based policy always outperforms any other online policy.

### 3.5 Simulation Results

The performances of the proposed status update policies are evaluated in this section through simulations.

First, we fix the battery size $B = \infty$. We generate sample paths for the Poisson energy harvesting process, and perform status updating according to the best-effort uniform status update policy. The time average AoI as a function of $T$ is shown in Fig. 3.2. We plot one sample path and the sample average over $1,000$ sample paths in the figure. We observe that the time average AoI curves gradually approach the lower bound $1/2$ as $T$ increases. When $T = 500$, there is only a very small difference between the simulation results and the analytical lower bound. The results indicate that the proposed best-effort uniform status update policy is optimal.

Next, we study the time average AoI under the adaptive status update policy with finite battery sizes. We fix $T = 100,000$ and plot the average AoI over 1,000 sample paths in Fig. 3.3. We note that for each fixed $k$, the gap between the time average AoI and the lower bound $1/2$ monotonically decreases as $B$ increases, which is consistent with the theoretical result in Theorem 6.

Last, we compare the performances of the three policies for $B = 1$. For a fair comparison, we optimize the parameters for the best-effort uniform policy and adaptive

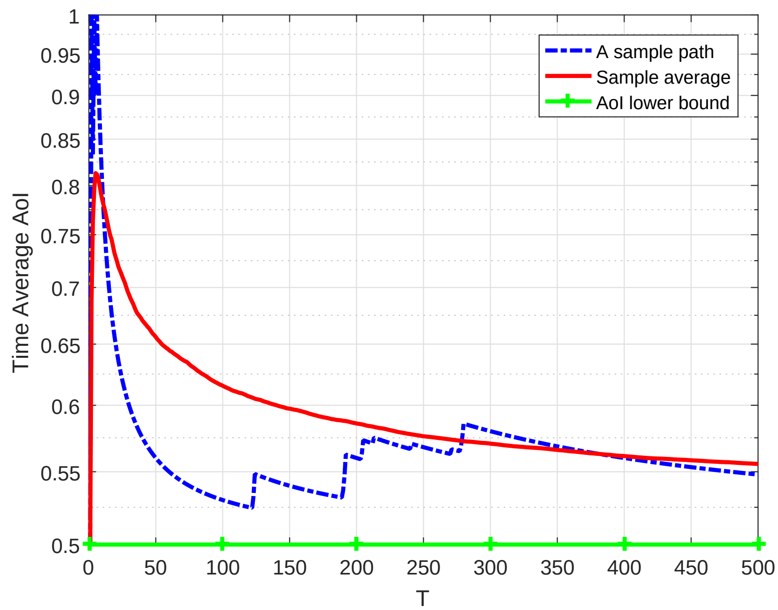


Figure 3.2: Time average AoI under best-effort uniform status update policy.

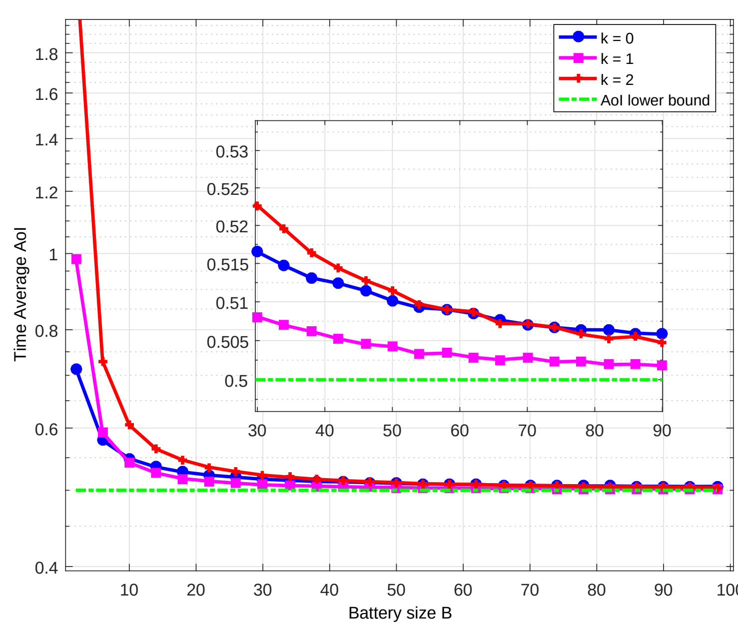


Figure 3.3: Time average AoI under adaptive status update policy.

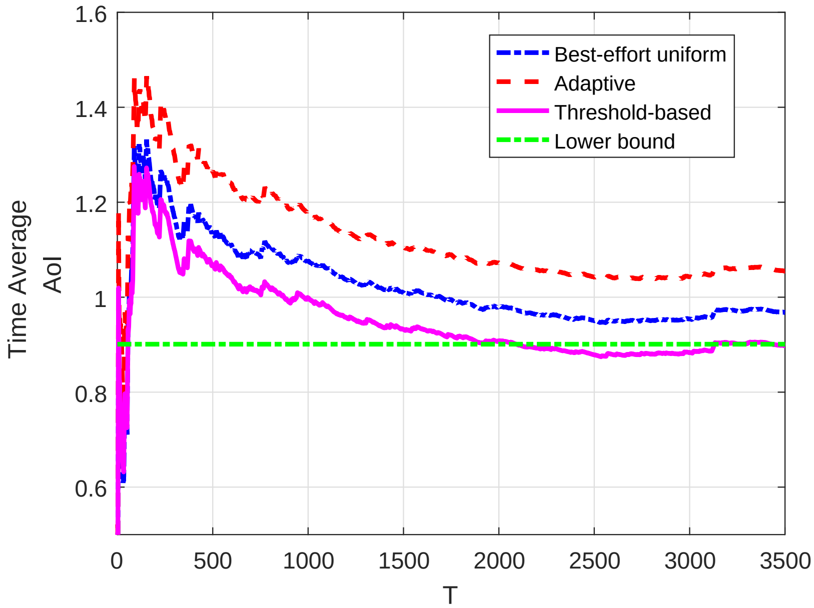



Figure 3.4: Performance comparison when $B = 1$.

state update policy numerically before we perform the comparison. We note that the optimal update rate for the best-effort uniform policy is once every 0.43 unit of time. We also modify the adaptive status update policy to make it applicable for the case $B = 1$. Specifically, we schedule the next update $\frac{1}{1+\beta}$ away if the battery level is full right before the current update; otherwise, we schedule it in time $\frac{1}{1-\beta}$. We numerically search for the optimal value of $\beta$, and it turns out that when $\beta = -0.145$, the time average AoI is minimized. This is opposite to the case when $B$ is large but finite. Although it is a bit counter intuitive, it is due to the fact that when $B = 1$, then sensor will become empty immediately after it updates the status, and the AoI will linearly grow from zero; While for the other case where the battery is empty at a scheduled update epoch, the AoI has a positive value already, and will grow with the same rate. The memoryless property of the inter-arrival time indicates that the expected waiting time for the next energy arrival would be the same after the current scheduled update epoch. The convexity of the AoI function $f(d) = d^2/2$ implies that the system should be more aggressive to update if the battery is empty for the current scheduled update in order to minimize the time average

AoI. We then generate a sample path and plot the time average AoI as a function of time $T$ under each policy, as shown in Fig. 3.4. As we expect, the threshold based updating policy outperforms the other two, and approaches its limit as $T$ gets sufficiently large.

## 3.6 Appendix

### 3.6.1 Proof of Theorem 5

To prove Theorem 5, it suffices to show that

$$\limsup_{T\to+\infty} \frac{1}{T} \sum_{n=1}^{N_T+1} f(d_n) \leq 1/2,\ a.s.$$

The uniform best-effort status update policy partition the time axis into slots, each with length 1. Let $E(n)$ be the energy level of the sensor right before the scheduled sensing epoch $n$. Based on $E(n)$, we can group the time slots into segments with lengths $u_0, v_1, u_1, \ldots, v_k, u_k, \ldots$, where $u_i$s correspond to the segments when $E(n) = 0$ and $v_i$s correspond to the segments when $E(n) > 0$. We note that there are $v_i$ equally spaced sensing epochs in the segment corresponding to $v_i$. Besides, $E(n)$ jumps from zero to some positive value $e_i$ at the end of the segment corresponding to $u_i$. Therefore, $u_i$ follows an independent geometric distribution

$$\mathbb{P}[u_i = k] = e^{-(k-1)}(1 - e^{-1}), \quad k = 1, 2\ldots \tag{3.5}$$

Considering the duration bounded by the first and last sensing epochs in the segment, the aggregated AoI equals $(v_i - 1)f(1)$, where $f(x) = x^2/2$. The duration bounded by the last sensing epoch in the segment associated with $v_i$ and the first sensing epoch in the segment associated with $v_{i+1}$ is $f(u_i + 1)$. Let $K_T$ be the number of segments with

$E(n) = 0$ over $[0, T]$. Note that $T = N_T + \sum_{i=0}^{K_T} u_i$. Therefore,

$$\begin{aligned}
&\limsup_{T\to+\infty} \frac{1}{T} \sum_{n=1}^{N_T+1} f(d_n) \\
&= \limsup_{T\to+\infty} \frac{f(u_0) + \sum_{i=1}^{K_T} [(v_i - 1)f(1) + f(u_i + 1)]}{T} \\
&= \limsup_{T\to+\infty} \frac{f(u_0) + \sum_{i=1}^{K_T} f(u_i + 1)}{T} + \frac{T - \sum_{i=0}^{K_T} u_i - K_T}{T} f(1) && (3.6) \\
&= \limsup_{T\to+\infty} \frac{u_0^2 + \sum_{i=1}^{K_T} (u_i + 1)^2}{2T} + \frac{1}{2} - \frac{\sum_{i=0}^{K_T} u_i}{2T} - \frac{K_T}{2T} \\
&= \limsup_{T\to+\infty} \frac{u_0^2}{2T} + \left( \frac{\sum_{i=1}^{K_T} u_i^2}{2K_T} + \frac{\sum_{i=1}^{K_T} u_i}{K_T} + 1 \right) \frac{K_T}{T} + \frac{1}{2} && (3.7)
\end{aligned}$$

where (2.19) follows from the fact that $u_0 + \sum_{i=1}^{K_T} (v_i + u_i) = T$, (3.7) follows from the fact that $f(x) = x^2/2$, and $K_T/T \to 0$ and $\sum_{i=0}^{K_T} u_i/T \to 0$ almost surely, as proved in the proof of Theorem 1 in [68]. Since $u_i$'s are i.i.d. geometric random variables, $\frac{\sum_{i=1}^{K_T} u_i}{K_T}$ and $\frac{\sum_{i=1}^{K_T} u_i^2}{K_T}$ converges to the first and second moments of the geometric distribution specified in (3.5), which are finite constants. Therefore, we have (3.7) converges to 1/2 almost surely.

### 3.6.2 Proof of Theorem 6

Consider the first $n$ scheduled status update epochs under the proposed adaptive status update policy. Let $n_+$ denote the number of intervals between two scheduled sensing epochs with duration $\frac{1}{1-\beta}$, $n_-$ be that with duration $\frac{1}{1+\beta}$, and $n_0$ be that with duration 1. Let $\bar{n}$ be the number of sensing epochs the battery overflows, and $\underline{n}$ be the number of infeasible status update epochs. Then, the $n$-th scheduled status update epoch happens at time $T_n := \frac{n_+}{1-\beta} + n_0 + \frac{n_-}{1+\beta}$. Let $A_n^+$ be the total amount of energy wasted. Then,

$$E(S_n^-) = (A(T_n) - A_n^+) - (n - \underline{n}) \tag{3.8}$$

where $A(T_n)$ is a Poisson random variable with parameter $T_n$. Dividing both sides by $n$ and taking the limit as $n$ goes to $+\infty$, we have

$$\lim_{n\to\infty} \frac{E(n)}{n} = \lim_{n\to\infty} \frac{A(T_n)}{T_n} \cdot \frac{T_n}{n} - \lim_{n\to\infty} \frac{A_n^+}{n} - \left(1 - \lim_{n\to\infty} \frac{\underline{n}}{n}\right)$$

Therefore,

$$\lim_{n\to\infty} \frac{T_n}{n} = 1 + O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}}\right) \tag{3.9}$$

Based on Taylor expansion and (3.9), we have

$$\begin{aligned} &\lim_{n\to\infty} \frac{n_+ f\left(\frac{1}{1-\beta}\right) + n_0 f(1) + n_- f\left(\frac{1}{1+\beta}\right)}{T_n} \\ &= f(1) + O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}} + \left(\frac{\log B}{B}\right)^2\right) \end{aligned}$$

On the other hand, due to the existence of infeasible status update epochs, we have

$$\begin{aligned} &\lim_{n\to\infty} \frac{\sum_n f(d_n) - \left[n_+ f\left(\frac{1}{1-\beta}\right) + n_0 f(1) + n_- f\left(\frac{1}{1+\beta}\right)\right]}{T_n} \\ &\leq \lim_{n\to\infty} \frac{\sum_{d_n : d_n > \frac{1}{1-\beta}} f(d_n)}{T_n} \qquad (3.10) \\ &= \lim_{n\to\infty} \frac{\sum_{d_n : d_n > \frac{1}{1-\beta}} d_n^2}{2n'} \frac{n'}{T_n} \qquad (3.11) \end{aligned}$$

where $n'$ in (3.11) denote the number of $d_n$'s with $d_n > \frac{1}{1-\beta}$. (3.10) follows from the fact that the difference between the actual time average AoI and that with scheduled status update epochs is due to the infeasible status update epochs. (3.11) follows from the fact that $f(x) = x^2/2$.

We note that for all $d_n \geq \frac{1}{1-\beta}$, $d_n' := d_n(1-\beta)$ follows a geometric distribution with parameter $p = 1 - e^{-\frac{1}{1-\beta}}$, and its second moment is $\frac{2-p}{p^2}$. Then,

$$\lim_{n\to\infty} \frac{\sum_{d_n:d_n>\frac{1}{1-\beta}} d_n^2}{2n'} = \lim_{n\to\infty} \frac{\sum_{d_n:d_n>\frac{1}{1-\beta}} (d'_n)^2}{2(1-\beta)^2 n'} \tag{3.12}$$

$$= \frac{2-p}{2p^2(1-\beta)^2} \qquad \text{a.s.} \tag{3.13}$$

Meanwhile, we have $\lim_{n\to\infty} \frac{n'}{T_n} \le \frac{n}{T_n}$. Thus, based on Theorem 3 in [68] and (3.9), we have

$$\lim_{n\to\infty} \frac{\sum_n f(d_n)}{T_n} = \frac{1}{2} + O\left(\frac{2^{k+1}k(\log B)^2}{B^{k+1}} + \left(\frac{\log B}{B}\right)^2\right)$$

### 3.6.3 Proof of Theorem 7

Define $X_n, n = 1, 2, \ldots$ as the duration between the $n$-th and $(n-1)$-th status update instances under the threshold-based status update policy. Then, the status update instants forms a renewal process, and $X_n$s are i.i.d random variables. Denote the time difference between the $n$-th status update instance $l_n$ and the first energy arrival time after $l_n$ as $Y_n$. Thus, $X_n$ equals $\tau$ if $Y_n \le \tau$, and it equals $Y_n$ if $Y_n > \tau$. Based on the memoryless property of the inter-arrival time for Poisson process, $Y_n$ is an exponential random variable with parameter 1. Therefore,

$$\begin{aligned}
\mathbb{E}[X_n] &= \mathbb{E}[X_n|Y_n > \tau]\mathbb{P}[Y_n > \tau] + \mathbb{E}[X_n|Y_n \le \tau]\mathbb{P}[Y_n \le \tau] \\
&= \int_\tau^{+\infty} ye^{-y}dy + \tau(1-e^{-\tau}) \\
&= (1+\tau)e^{-\tau} + \tau(1-e^{-\tau}) = e^{-\tau} + \tau \qquad (3.14) \\
\mathbb{E}[X_n^2] &= \mathbb{E}[X_n^2|Y_n > \tau]\mathbb{P}[Y_n > \tau] + \mathbb{E}[X_n^2|Y_n \le \tau]\mathbb{P}[Y_n \le \tau] \\
&= \int_\tau^{+\infty} y^2e^{-y}dy + \tau^2(1-e^{-\tau}) \\
&= (\tau^2 + 2\tau + 2)e^{-\tau} + \tau^2(1-e^{-\tau}) \qquad (3.15)
\end{aligned}$$

Thus, based on the property of renewal process [90], we have

$$\lim_{T \to +\infty} \frac{\sum_{n=1}^{N_T} X_n^2}{2\sum_{n=1}^{N_T} X_n} = \frac{\mathbb{E}[X_n^2]}{2\mathbb{E}[X_n]} = \frac{(2\tau + 2)e^{-\tau} + \tau^2}{2(e^{-\tau} + \tau)} \tag{3.16}$$

# Chapter 4: Collaborative Sensing in Energy Harvesting Sensor Networks

## 4.1 Introduction

Sensor networks equipped with energy harvesting devices have attracted great attentions recently. Compared with conventional sensor networks powered by batteries, the energy harvesting abilities of the sensor nodes make sustainable and environment-friendly sensor networks possible. However, the random, scarce and non-uniform energy supply features also necessitate a completely different approach to energy management.

Different energy management schemes have been proposed to cope with the random nature of energy harvesting sensors from different perspectives. In general, they can be categorized as offline polices and online policies. In the offline optimization framework, it is assumed that the energy harvesting profile over the whole duration of operation is known in advance at the sensor nodes. With such assumptions, energy has been managed to optimize communication performances [1, 4, 27], schedule sensing tasks [63], etc. In contrast, in the online optimization framework, the energy harvesting profile becomes available at sensor nodes causally. Besides some heuristic online algorithms [17, 18, 20, 64], the major approach is to formulate the optimal energy management problem as a stochastic control problem, with the objective to maximize the expected reward, such as data throughput [5, 7, 8, 91–94], channel coding rate [45, 49], sensing utility [65], etc.

In this chapter, we focus on the design of a collaborative sensing scheduling scheme in an energy harvesting sensor network under both offline and online settings. Our motivation is a collaborative sensing scenario where multiple sensors are deployed to monitor the status of a phenomenon in a region. Our objective is to coordinate the sensing actions among multiple sensor nodes in a way so that the time average sensing utility is optimized. Our primary constraint is the energy causality constraint at each sensor. Specifically, we assume that a sensor takes a unit of energy to sense the nature and send its observation to a fusion center (FC). A sensor cannot perform the sensing task when there is not sufficient energy in its battery. The FC combines the observations collected from sensors and extracts information from them. We assume that in each slot, the sensing *utility* generated by those observations is a function of the set of active sensors in that slot. Our objective is to select a subset of sensors to perform the sensing task in each time slot, such that the time average sensing utility is optimized, while the energy constraint at each individual sensor is satisfied at every time slot. The problem has a combinatorial nature and is hard to solve in general. The randomness of the energy harvesting processes at sensors makes the problem even more challenging. To make the problem tractable, as a first step, we assume that the sensing utility function is symmetric with respect to sensors, i.e., it is a concave function of the total number of active sensors in each slot. In addition, we assume that the battery size at each sensor is infinite.

Under such assumptions, we first consider an offline setting, where the energy harvesting profile over duration $[0, T-1]$ for each sensor is known beforehand. We show that the optimal offline sensing scheduling has a "majorization" structure, i.e., the number of active sensors in each slot should be as even as possible, subject to the energy causality constraints at individual sensors. We propose an algorithm to identify the optimal number of active sensors in each slot, and construct a sensing scheduling with the identified subset sizes. With the insight gained from the offline setting, we then study the corresponding online sensing scheduling where the energy harvesting profile is available causally. We first show that the expected time average sensing utility has an upper bound under any feasible sensing scheduling policy satisfying the energy causality constraint. We then propose a myopic policy, which aims to select a fixed number of active sensors with the *longest energy queues* in each slot to perform the sensing actions. Under the assumptions that the energy harvesting process is Bernoulli and the battery capacity is infinite at each individual sensor, we show the expected time average utility generated under the myopic policy converges to the upper bound as time $T$ approaches infinity, thus the myopic policy is optimal. Moreover, we explicitly characterize the convergence rates. The near exponential convergence rate indicates that the time average sensing utility generated under the myopic policy gets close to the upper bound within a short period of time. The myopic sensing scheduling policy relies on the current energy queue lengths to make the sensing decisions, and it resolves the computational complexity issue caused by the combinatorial optimization. Besides, it does not require the parameters of the energy harvesting processes at individual sensors to be the same. Therefore, it is extremely suitable for large-scale energy harvesting sensor networks with non-uniform energy supplies at sensors. Part of this work has been published in [95].

## 4.2 Related Work

For the offline sensing scheduling, a similar "majorization" scheduling structure has been observed in throughput optimization problems with energy harvesting transmitters [11, 13, 27]. In [27], the optimal transmission policy for a single transmitter under the given energy causality constraint is to equalize the transmit power as much as possible. The "majorization" structure of the solution is due to the concavity of the function $r = \frac{1}{2}\log(1 + P)$. However, there are fundamental differences between the problem studied in this chapter and [27]. The optimization problem in this chapter is to select a subset of sensors in each slot, and each selected sensor consumes a unit of energy for sensing, while in [27], the objective is to vary the power to maximize the throughput. The latter is formulated as a convex optimization problem, while the former has a combinatorial nature, and in general cannot be solved through convex optimization.

For the online sensing scheduling, similar problems have been studied in [91–94] for throughput maximization in energy harvesting communication networks. Without knowledge of the instantaneous states of the nodes' batteries, the access point needs to allocate $K$ orthogonal channels to $K$ out of the $N$ nodes in the network. The corresponding throughput maximization problems are formulated with partially observable

Markov decision processes (POMDP) and cast into a restless multi-armed bandit. [91] and [92] show the optimality of a Round-Robin based myopic policy that schedules the $K$ nodes with the largest beliefs to maximize the immediate reward under different system models. In [93, 94], under the infinite battery assumption, a uniformizing random ordered policy that selects the sensors based on a predefined random priority list and the outcome of transmissions in the previous time slot is shown to be asymptotically optimal in infinite horizon for a broad class of energy harvesting process. The proposed queue-length based myopic policy is also very similar to the *longest-connected-queue* server allocation policy studied in [96, 97], etc. The problem studied in this chapter is different from these work from the following aspects. First, we do not have restrictions on the number of active sensors in each slot. Our myopic policy selects a fixed number of active sensors due to the properties of the sensing utility function, rather than a hard constraint assumed in the system model. Second, we assume that the statuses of batteries at sensor nodes are available at the fusion center, thus the optimization problem is actually a Markov decision process rather than a POMDP. Third, the optimality of the myopic policies in [91, 92, 96, 97] requires the energy harvesting processes to be uniform at sensors, while such assumption is not required for the optimality of our policy. Fourth, utilizing large deviation theory, we explicitly characterize the convergence rate of our policy. To the best of our knowledge, such characterization is not available for similar scheduling policies proposed in the literature.

## 4.3 System Model

In this chapter, we consider a sensor network consisting of $N$ sensors (randomly) distributed in an area. Each sensor node is powered by energy harvested from ambient environment. We assume that each sensor node has a battery to store the harvested energy, and it is replenished randomly and consumed by taking observations and transmitting them to a fusion center (FC). We assume the battery size is infinite, and the instantaneous battery statuses at the sensors are available at the FC to make sensing scheduling decisions. We consider a time-slotted system. In time slot $t$, a subset of sensors, denoted as $\mathcal{C}_t$, is selected to sense the environment, and transmit their observations to the FC. We assume that a unit amount of energy is required for one sense-and-transmit operation, and a sensor can make at most one sense-and-transmit operation in each slot.

Let $E_i(t)$ denote the amount of energy remaining in the battery of node $i$ at the beginning of time slot $t$, $A_i(t)$ be the amount of harvested energy at node $i$ during slot $t$. Assume the system starts with an empty state. Then, the energy queue evolves according to

$$
\begin{aligned}
E_i(0) &= 0, \forall i \\
E_i(t+1) &= E_i(t) - \mathbf{1}_{i \in \mathcal{C}_t} + A_i(t), \quad t = 0, 1, 2, \ldots, \forall i
\end{aligned} \tag{4.1}
$$

where $\mathbf{1}_x$ is an indicator function, i.e., it equals one if $x$ is true, and it equals zero otherwise. Since an observation cannot be made if $E_i(t) < 1$, we impose the following energy constraint

$$
E_i(t) \geq \mathbf{1}_{i \in \mathcal{C}_t}, \qquad \forall i, t. \tag{4.2}
$$

In each time slot, the FC receives the measurements taken from the active sensors and extracts the information from them. We assume the sensing utility generated by those measurements is a function of $\mathcal{C}_t$, denoted as $f(\mathcal{C}_t)$. The total sensing utility over duration $[1, T]$ is simply the sum of the utilities generated in each slot in $[1, T]$. We make the following assumptions on the utility function $f(\mathcal{C}_t)$.

**Assumption 1**

*(i)* $f(\mathcal{C})$ *is a function of the size of* $\mathcal{C}$*, i.e.,* $f(\mathcal{C}) = f(|\mathcal{C}|)$.

*(ii)* $f(x)$ *is monotonically increasing in* $x \in \mathbb{Z}_+$.

*(iii)* $f(x+1) + f(x-1) < 2f(x)$ *for* $x \in \mathbb{Z}_+$.

Assumption 1-(i) implies that $f(\mathcal{C})$ is symmetric with respect to sensor nodes. By imposing this assumption, we essentially ignore the differences in contributions from different sensing nodes, and focus on the impact of the total number of collected observations on the sensing performance. Assumption 1-(ii) means that the utility function increases as more observations are collected. Assumption 1-(iii) essentially means that $f(x)$ is a concave function defined over $\mathbb{Z}_+$. These assumptions are quite general and reasonable. Below we give two examples where the assumptions are satisfied.

**Example 1**: Throughput maximization in a symmetric Gaussian multiple-access channel. Consider a scenario where the sensing utility completely depends on the total number of information bits extracted from the messages received at the fusion center. Under the assumption that each sensor has a fixed transmit power $P$ and the channel is a symmetric Gaussian multiple-access channel, the maximum sum-rate in each slot equals $\frac{1}{2}\log(1 + xP)$, where $x$ is the number of sensors transmitting simultaneously. Apparently, the sum-rate function satisfies Assumption 1.

**Example 2**: Variance minimization for maximum likelihood estimation (MLE). Consider the case where the samples collected by sensors in each time slot are i.i.d. and can be received perfectly at the fusion center. An MLE is then performed to estimate the quantity of interest. Under mild regularity conditions, the MLE has an asymptotically Gaussian distribution, whose mean equals the true value of the quantity, and variance scales in $\frac{1}{n}$. In order to minimize the time average variance of the MLE, we can define $f(x) = -\frac{1}{x}$, which satisfies the properties in Assumption 1.

Our objective in this chapter is to develop a sensing scheduling scheme, such that the time average sensing utility under the scheduling is optimized, subject to the energy causality constraints at individual sensors. We consider both offline and online settings, and study them in Section 4.4 and Section 4.5, respectively.

## 4.4 Optimal Offline Sensing Scheduling

We start with a finite-horizon offline formulation, where the energy harvesting profile up to time slot $T-1$, i.e., $\{A_i(t)\}_{t=0}^{T-1}$, $\forall i$, is known beforehand. Our objective is to select the subset of sensors $\mathcal{C}_t$ to perform the sensing task in each time slot $t$, such that the time average utility generated over $[1, T]$ is maximized. Such scheduling must satisfy the energy constraint for each individual sensor at every time slot. Thus, the optimization problem is formulated as

$$\max_{\{\mathcal{C}_t\}} \quad \frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t) \quad \text{s.t. } (4.1) - (4.2) \tag{4.3}$$

The optimization problem in (4.3) has a combinatorial nature, and is in general hard to solve. However, with Assumption 1, we show that the optimal solution has a "majorization" structure, which can be exploited to obtain the optimal sensing scheduling explicitly. In this section, we first describe a procedure to determine the structure of the optimal scheduling, and then construct a scheduling policy explicitly with the obtained structure.

### 4.4.1 Identify a Majorization Scheduling Structure

First, since each sense-and-transmit operation costs one unit of energy, the energy harvesting profile $\{A_i(t)\}_{t=0}^{T-1}$ for sensor $i$ imposes constraints on the total number of time slots that a sensor be active up to time slot $t$, for $t = 1, 2, \ldots, T$. Let $B_i(t) = \sum_{j=0}^{t-1} A_i(j)$ be the total amount of energy harvested up to the beginning of time slot $t$. Apparently, $B_i(t)$ is an upper bound on the total number of time slots that a sensor can be *active* up to time slot $t$. However, since at most one unit of energy can be spent in each slot, $B_i(t)$ might not be tight. To provide a tighter upper bound on the total number of active times slots for sensors, we introduce another quantity $S_i(t)$, which is defined recursively as follows:

$$S_i(0) = 0, \quad \forall i \tag{4.4}$$

$$S_i(t) = \min\{S_i(t-1) + 1, B_i(t)\}, \quad \forall i, t \tag{4.5}$$

Based on this definition, we have

$$\sum_{j=1}^{t} \mathbf{1}_{i \in \mathcal{C}_j} \leq S_i(t), \quad \forall i \tag{4.6}$$

Sum up the inequalities in (4.6) over $i$, we get

$$\sum_{i=1}^{N} \sum_{j=1}^{t} \mathbf{1}_{i \in \mathcal{C}_j} \leq \sum_{i=1}^{N} S_i(t) := S(t) \tag{4.7}$$

which is equivalent to

$$\sum_{j=1}^{t} |\mathcal{C}_j| \leq S(t), \quad \forall t \tag{4.8}$$

Eqn. (4.8) imposes a constraint on the cumulative number of observations the FC can collect up to time slot $t$. Due to the concavity of the utility function $f(\mathcal{C}_t)$ in $|\mathcal{C}_t|$, intuitively, to maximize the objective function in (4.3), we should equalize $\{|\mathcal{C}_t|\}_{t=1}^{T}$ as much as possible, under the constraints in (4.6) for each individual sensor. While handling $N$ individual constraints simultaneously is too complicated, in the following, we equalize $\{|\mathcal{C}_t|\}_{t=1}^{T}$ under the sum constraint (4.8) only. In general, the solution obtained with such relaxation may not be feasible when individual constraints are imposed. However, as we will show in Sec. 4.4.2, the $\{|\mathcal{C}_t|\}_{t=1}^{T}$ obtained under constraint (4.8) is always feasible.

The procedure to obtain the optimal $\{|\mathcal{C}_t|\}_{t=1}^{T}$ is equivalent to identifying the time slots in which the equality in (4.8) is met (denoted as $t_1$, $t_2$, ...,etc). We summarize the procedure in Algorithm 1. It works in a progressive fashion. Starting with $t_0 = 0$, Eqn. (4.9) calculates the average number of active nodes in each slot over $[1, t]$ assuming $t_1 = t$, i.e., constraint (4.8) is tight at $t$, for $t = 1, 2, \dots, T$. Then, $t_1$ is identified as the time slot $t$ associated with the minimum average number of active nodes. The procedure then proceeds with the new starting point $t_1 + 1$ to identify $t_2$. The procedure continues until the right hand side of Eqn. (4.9) equals $T$. The way we obtain the sequence $t_1$, $t_2$, ... implies that the average number of active sensor nodes in each slot over $[t_{n-1}+1, t_n]$ is $\frac{S(t_n)-S(t_{n-1})}{t_n-t_{n-1}}$. Since this may be an non-integer, in order to obtain a valid scheduling, we determine $|\mathcal{C}_t|$ according to (4.10)-(4.11). In this way, we keep the total number of observations collected over $[t_{n-1}+1, t_n]$ to be $S(t_n) - S(t_{n-1})$, and ensure that the number of active nodes in each slot is an integer. Intuitively, this is the most equalized valid scheduling structure we can have. The optimality of the scheduling structure will be proved later in Theorem 10.

**Algorithm 1** An algorithm to equalize $\{|\mathcal{C}_t|\}_{t=1}^T$

Input: $\{S(t)\}_{t=1}^T$.
Initialization: $n = 0$, $t_0 = 0$.
**while** $t_n < T$ **do**
    $n = n + 1$;
    Let

$$t_n = \arg \min_{t_{n-1} < t \leq T} \left\{ \frac{S(t) - S(t_{n-1})}{t - t_{n-1}} \right\} \tag{4.9}$$

$$r = S(t_n) - S(t_{n-1}) - (t_n - t_{n-1}) \left\lfloor \frac{S(t_n) - S(t_{n-1})}{t_n - t_{n-1}} \right\rfloor \tag{4.10}$$

$$c_t = \begin{cases} \left\lfloor \frac{S(t_n)-S(t_{n-1})}{t_n-t_{n-1}} \right\rfloor, & t_{n-1} < t \leq t_n - r \\ \left\lceil \frac{S(t_n)-S(t_{n-1})}{t_n-t_{n-1}} \right\rceil, & t_n - r < t \leq t_n \end{cases} \tag{4.11}$$

**end while**
Output: $\{c_t\}_{t=1}^T$.

### 4.4.2 Construct a Sensing Scheduling with $\{c_t\}_{t=1}^T$

With the scheduling structure $\{c_t\}_{t=1}^T$ obtained in Algorithm 1, we aim to construct a sensing policy, under which the number of active nodes in slot $t$ equals $c_t$ exactly, and each individual energy constraint in (4.6) is satisfied.

The algorithm to construct the sensing scheduling is summarized in Algorithm 2. It starts with an initial scheduling where each sensor perform sensing in a greedy fashion. Specifically, we let each sensor node spend one unit of energy to take an observation whenever it has sufficient energy. By designing the sensing policy in this way, sensor $i$ senses in time slot $t$ whenever $S_i(t) - S_i(t-1) = 1$. Thus, we have exact $S(t) - S(t-1)$ active sensor nodes in slot $t$. We use $\mathcal{S}(t)$ to track the set of active nodes in slot $t$, and define $|\mathcal{S}(t)| := s_t$. Initially, $\mathcal{S}(t)$ includes all sensors with at least one unit of energy at the beginning of slot $t$ under the greedy sensing policy.

Then, Algorithm 2 adjusts the initial scheduling by letting a subset of sensors postpone their sensing actions scheduled in certain time slots and sense with the saved energy in some subsequent time slots. The rescheduling procedure is carried out iteratively. In each iteration, we define $\hat{t}$ as the first $t$ such that $s_t < c_t$, and $\bar{t}$ as the first $t$ such that $s_t > c_t$. As we will see in Lemma 9, we always have $\bar{t} < \hat{t}$.

**Algorithm 2** Sensing scheduling construction

Input: $\{S_i(t)\}_{t=1}^T$, $\forall i$; $\{c_t\}_{t=1}^T$.

Initialization: for $t = 1, 2, \dots, T$,

$$\mathcal{S}(t) = \{i | S_i(t) - S_i(t-1) = 1\}, \quad s_t = |\mathcal{S}(t)|.$$

$\bar{t} = \hat{t} = 1$.

**while** $\bar{t} \le T$ **do**

  **while** $s_{\hat{t}} \ge c_{\hat{t}}$ & $\hat{t} < T$ **do**

    $\hat{t} = \hat{t} + 1$;

  **end while**

  **while** $s_{\bar{t}} \le c_{\bar{t}}$ **do**

    $\bar{t} = \bar{t} + 1$;

  **end while**

  $\delta = \min(s_{\bar{t}} - c_{\bar{t}}, c_{\hat{t}} - s_{\hat{t}})$;

  Randomly remove $\delta$ sensors from $\mathcal{S}(\bar{t}) \backslash \mathcal{S}(\hat{t})$ and add them to $\mathcal{S}(\hat{t})$;

  Update $\mathcal{S}(\bar{t})$, $\mathcal{S}(\hat{t})$, $s_{\bar{t}}$, $s_{\hat{t}}$.

**end while**

Output: $\{\mathcal{S}(t)\}_{t=1}^T$.

Recall that we assume each sensor can take at most one observation in each slot. Let

$\delta = \min(s_{\bar{t}} - c_{\bar{t}}, c_{\hat{t}} - s_{\hat{t}})$. We then randomly remove $\delta$ sensors from $\mathcal{S}(\bar{t})\backslash\mathcal{S}(\hat{t})$ and add them to $\mathcal{S}(\hat{t})$. By doing this, we let the corresponding subset of sensors keep silent in time slot $\bar{t}$ and be active in $\hat{t}$. Since $\bar{t} < \hat{t}$, this does not violate the individual energy causality constraints in (4.6). After updating $\mathcal{S}_{\hat{t}}$, $\mathcal{S}_{\bar{t}}$ and $s_{\hat{t}}$, $s_{\bar{t}}$, the algorithm repeats the procedure with another iteration. As we will prove in the proof of Lemma 9, the new $\hat{t}$ and $\bar{t}$ are always greater than or equal to the $\hat{t}$ and $\bar{t}$ in the previous iteration, respectively. Thus, when sweeping for the new $\hat{t}$ and $\bar{t}$ in each iteration, the procedure only needs to start with the $\hat{t}$ and $\bar{t}$ in the previous iteration. The rescheduling process completes until all time slots are swept. The rescheduling is coordinated in a way that exact $c_t$ sensors are scheduled for sensing in time slot $t$.

In order to prove that Algorithm 2 returns with a sensing scheduling with the desired sensing structure $\{c_t\}_{t=1}^{T}$, we introduce the following lemmas. The first two lemmas can be easily proved based on Algorithm 1.

**Lemma 7** $\sum_{j=1}^{t} c_j \leq S(t)$ *for* $1 \leq t \leq T$*. The equality holds if* $t \in \{t_n\}$*, where* $t_n$ *is defined in (4.9).*

**Lemma 8** *If* $t_{n-1} < t_1 < t_2 \leq t_n$*, we must have either* $c_{t_1} = c_{t_2}$*, or* $c_{t_1} = c_{t_2} - 1$*.*

**Lemma 9** *In each iteration of Algorithm 2, we must have a)* $\bar{t} < \hat{t}$*, and b)* $\sum_{t=1}^{\tau} s_t = S(\tau)$*,* $\forall \tau \geq \hat{t}$ *after rescheduling.*

**Proof:** We prove the lemma through induction. First, we prove that it is true in the first iteration with the initial scheduling. We then assume that it is true for the current iteration, and prove that it still holds in the next iteration.

Part a) in the first iteration can be proved through contradiction. If $\bar{t} > \hat{t}$, based on the definition of $\bar{t}$ and $\hat{t}$, we have $\sum_{t=1}^{\hat{t}} s_t = \sum_{t=1}^{\hat{t}-1} c_t + s_t < \sum_{t=1}^{\hat{t}} c_t \leq S(\hat{t})$ where the last inequality follows from Lemma 7. This contradicts with the fact that $\sum_{t=1}^{\tau} s_t = S(\tau)$, $\forall \tau$ in the initial scheduling. Thus, we must have $\bar{t} < \hat{t}$. Since the rescheduling only involves $\mathcal{S}(\bar{t})$ and $\mathcal{S}(\hat{t})$, and $\bar{t} < \hat{t}$, we have Part b) hold after the rescheduling in the first iteration.

We then assume the lemma is true for the current iteration. After rescheduling and updating $\mathcal{S}(\bar{t})$ and $\mathcal{S}(\hat{t})$, we still have $s_t \geq c_t$ for $t < \hat{t}$. Therefore, the new $\hat{t}$ in the next iteration, denoted as $\hat{t}'$, can only be greater than or equal to the current $\hat{t}$. Based on Part b), we have $\sum_{t=1}^{\hat{t}'} s_t = S(\hat{t}')$ prior to the rescheduling in the next iteration. Following similar arguments as in the first iteration, we can prove that the new $\bar{t}$ in the next iteration must be smaller than $\hat{t}'$, and $\sum_{t=1}^{\tau} s_t = S(\tau)$, $\forall \tau \geq \hat{t}'$ after the rescheduling. ■

**Theorem 9** *Algorithm 2 always finishes with a valid sensing scheduling with scheduling structure* $\{c_t\}_{t=1}^{T}$.

**Theorem 10** *The obtained sensing scheduling with the structure* $\{c_t\}_{t=1}^{T}$ *determined by Algorithm 1 maximizes the sensing utility generated over* $[0, T]$ *under Assumption 1.*

The proofs of Theorem 9 and Theorem 10 are provided in Appendix 4.8.1 and Appendix 4.8.2, respectively.

### 4.5 Optimal Online Sensing Scheduling

In this section, we consider an online setting, where energy arrives randomly at sensors in each time slot. Assuming the statistics of the energy harvesting processes are known at the FC, our objective is to design an online collaborative sensing scheduling $\{\mathcal{C}_t\}_{t=1}^{\infty}$, such that the expected long-term time average sensing utility is maximized.

Specifically, for every sensor node $i$, we assume the energy arrival process is a Bernoulli process with parameter $\lambda_i$, $0 \leq \lambda_i \leq 1$, i.e., $\mathbb{E}[A_i(t)] = \lambda_i$. The arrival processes are independent and may not be identical across sensors. We consider a general case where $\sum_{i=1}^{N} \lambda_i$ is a non-integer. The online optimization problem is formulated as

$$\begin{aligned} \max_{\{\mathcal{C}_t\}} \quad & \liminf_{T \to +\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t)\right] \\ \text{s.t.} \quad & (4.1) - (4.2) \end{aligned} \tag{4.12}$$

where the expectation in the objective function is taken with respect to all possible energy harvesting sample paths. The optimization problem in (4.12) is stochastic and has a combinatorial nature, thus it is in general hard to solve. However, with Assumption 1, we first show that the optimal solution has an upper bound, which corresponds to a scheduling policy with a fixed number of active sensors in every slot. Motivated by this observation, we then propose a myopic policy, which greedily selects a subset of sensors with the longest energy queues to perform the sensing task in each slot. We prove its optimality by showing that the myopic policy asymptotically achieves the upper bound.

### 4.5.1 An upper bound

**Definition 6** *A sensing scheduling policy $\{\mathcal{C}_t\}_{t=1}^{\infty}$ is feasible if $E_i(t) \geq 1$, for every $i \in \mathcal{C}_t$, $\forall t$, i.e., the energy causality constraint (4.2) is always satisfied for every $i, t$.*

**Lemma 10** *Under every feasible scheduling policy, we have*

$$\limsup_{T \to +\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbf{1}_{i \in \mathcal{C}_t} \leq \lambda_i, \quad a.s. \quad \forall i \tag{4.13}$$

**Proof:** Lemma 10 can be proved based on the energy queue evolution described in (4.1) and the definition of feasible scheduling policy. Since $E_i(t) - \mathbf{1}_{i \in \mathcal{C}_t} \geq 0$ for every $t \geq 1$, we have $\sum_{t=1}^{T} \mathbf{1}_{i \in \mathcal{C}_t} \leq \sum_{t=0}^{T-1} A_i(t)$. Therefore,

$$\limsup_{T \to +\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbf{1}_{i \in \mathcal{C}_t} \leq \limsup_{T \to +\infty} \frac{1}{T} \sum_{t=0}^{T-1} A_i(t) = \lambda_i, \quad a.s. \tag{4.14}$$

where the last equality follows from the strong law of large numbers. ■

Lemma 10 implies that for any feasible scheduling policy $\{\mathcal{C}_t\}_{t=1}^{\infty}$, the long-term fraction of time slots that a sensor is active must be upper bounded by the energy arrival rate at that sensor. This is an intuitive result due to the energy causality constraint. Lemma 10 motivates us to obtain an upper bound on the objective function in (4.12) by removing the energy causality constraint in (4.2), and impose a relaxed energy constraint, i.e., the average energy constraint in (4.13) instead.

**Lemma 11** *The objective function in (4.12) is upper bounded as*

$$\max_{\{\mathcal{C}_t\}} \liminf_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t)\right] \leq (1-q)f\left(\lfloor\Lambda\rfloor\right) + qf\left(\lceil\Lambda\rceil\right), \tag{4.15}$$

*where* $\Lambda = \sum_{i=1}^{N}\lambda_i$, $q = \sum_{i=1}^{N}\lambda_i - \left\lfloor\sum_{i=1}^{N}\lambda_i\right\rfloor$.

**Proof:** First, we extend the domain of $f$ from $\mathbb{Z}_+$ to $\mathbb{R}_+$. Specifically, for $x \notin \mathbb{Z}_+$, we let

$$f(x) \triangleq (1-q)f\left(\lfloor x\rfloor\right) + qf\left(\lceil x\rceil\right) \tag{4.16}$$

where $q = x - \lfloor x\rfloor$. Be defining $f(x)$ in this way, we extend $f(x)$ from a discrete function to a piecewise linear continuous function. It is straightforward to verify that $f(x)$ is a monotonically increasing and concave function over $\mathbb{R}_+$, based on which Lemma 11 can be proved.

Specifically, we have

$$\begin{aligned}
&\max_{\{\mathcal{C}_t\}} \liminf_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t)\right] \\
&\leq \max_{\{\mathcal{C}_t\}} \limsup_{T\to+\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t)\right] && (4.17)\\
&\leq \max_{\{\mathcal{C}_t\}} \mathbb{E}\left[\limsup_{T\to+\infty} \frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t)\right] && (4.18)\\
&\leq \max_{\{\mathcal{C}_t\}} \mathbb{E}\left[f\left(\limsup_{T\to+\infty} \frac{1}{T}\sum_{t=1}^{T} |\mathcal{C}_t|\right)\right] && (4.19)\\
&\leq f\left(\sum_{i=1}^{N}\lambda_i\right) && (4.20)\\
&\triangleq (1-q)f\left(\lfloor\Lambda\rfloor\right) + qf\left(\lceil\Lambda\rceil\right) && (4.21)
\end{aligned}$$

where (4.18) follows from Fatou's Lemma, and (4.19) follows from the concavity and monotonicity of function $f$, (4.20) follows from Lemma 10, and the last equality follows from the extended definition of $f(x)$ in (4.16). ■

Extending the domain of $f(x)$ from $\mathbb{Z}_+$ to $\mathbb{R}_+$ enables us to obtain the upper bound in Lemma 11. However, the extension so far does not have any physical meaning. Observing the upper bound in Lemma 11, we note that the upper bound obtained by the extension actually implies the desired structure of the optimal scheduling policy. Specifically, in order to maximize the long-term average utility, on average, we should have $\left\lceil \sum_{i=1}^{N} \lambda_i \right\rceil$ active sensors for $q$ of the time slots, and $\left\lfloor \sum_{i=1}^{N} \lambda_i \right\rfloor$ active sensors for $1-q$ of the time slots. The selection should be coordinated in a way to ensure that, with high probability, there exists sufficient sensor nodes with non-empty energy queues (i.e., $E_i(t) \geq 1$) in every time slot. The randomness of the energy arrival processes makes such coordination non-trivial.

Let us start with a special case when $\sum_{i=1}^{N} \lambda_i$ is an integer. For this case, the upper bound becomes $f\left(\sum_{i=1}^{N} \lambda_i\right)$. Thus, to achieve the upper bound, the scheduler should select $\sum_{i=1}^{N} \lambda_i$ sensor nodes to perform the sensing task for almost every time slot. For a network with identical energy harvesting statistics for all sensors (i.e., $\lambda_i$s are equal), the optimal scheduling is quite intuitive: Sensor nodes with higher energy level should be utilized in the current slot, since their probabilities to become empty in future slots are relative low. Thus, $\sum_{i=1}^{N} \lambda_i$ sensor nodes with the longest energy queues should be selected in each slot. However, when $\lambda_i$s are not equal, the optimal scheduling is not quite straightforward. There are possibilities that sensors have larger $\lambda_i$ may have shorter queue lengths in certain time slots, due to fluctuations in the energy harvesting processes. For this case, the probability that a sensor will become empty in the future does not only depend on the current queue length, but the energy arrival rate as well. In general, the sensor selection should jointly consider the current energy queue length information as well as the energy arrival rate for each sensor, which makes the problem very complicated.

When $\sum_{i=1}^{N} \lambda_i$ is not an integer, the situation becomes even more complicated. The form of the upper bound in Lemma 11 motivates us to propose a *randomized* myopic policy, which selects a *random* number of active sensors in a greedy fashion at the beginning of each slot. As we will show in the following section, the randomized myopic policy asymptotically achieves the upper bound for a general setup, thus it is optimal. When $\sum_{i=1}^{N} \lambda_i$ is an integer, it reduces to a deterministic myopic policy which greedily selects $\sum_{i=1}^{N} \lambda_i$ sensor nodes with the longest energy queues in each slot.

### 4.5.2 A randomized myopic policy

Motivated by the upper bound in Lemma 11, and the intuition to balance the energy queue lengths for the purpose of reducing the probability that energy queues become empty in the future, we propose a randomized myopic policy as follows.

Let $q = \sum_{i=1}^{N} \lambda_i - \lfloor \sum_{i=1}^{N} \lambda_i \rfloor$. Define $m_t$ to be an i.i.d random variable taking value $\lceil \sum_{i=1}^{N} \lambda_i \rceil$ with probability $q$ and $\lfloor \sum_{i=1}^{N} \lambda_i \rfloor$ with probability $1-q$.

At the beginning of time slot $t$, the system first selects $m_t$ nodes with the longest energy queues and form a candidate set of active sensors, denoted as $\mathcal{C}'_t$. Then, the scheduling policy $\{\mathcal{C}^*_t\}$ is determined as

$$\mathcal{C}^*_t = \{i | i \in \mathcal{C}'_t, E_i(t) \geq 1\}. \tag{4.22}$$

Such selection guarantees that the randomized myopic policy is always feasible.

**Theorem 11** *The randomized myopic policy $\{\mathcal{C}^*_t\}_{t=1}^{\infty}$ achieves the upper bound on the long-term average sensing utility, i.e.,*

$$\liminf_{T \to +\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}^*_t)\right] = f\left(\sum_{i=1}^{N} \lambda_i\right) \tag{4.23}$$

*Therefore, it is optimal.*

**Theorem 12** *Under the randomized myopic scheduling policy, for any sufficiently large $T$,* *we have*

$$\mathbb{P}\left[\frac{1}{T}\sum_{t=1}^{T}\mathbf{1}_{|\mathcal{C}_t^*|\neq m_t} \geq \epsilon\right] \leq (T+1)^2 \exp\left(-\frac{T\epsilon^2}{12(N+1)\Lambda^2}\right) \tag{4.24}$$

$$\mathbb{P}\left[\left|\frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t^*) - f(\Lambda)\right| \geq \epsilon\right] \leq 2T^2 \exp\left(-\frac{T\epsilon^2}{12(N+1)^2 M^2 \Lambda^2}\right). \tag{4.25}$$

*where* $\Lambda = \sum_{i=1}^{N}\lambda_i$, $M := f\left(\sum_{i=1}^{N}\lambda_i\right) - f(0)$.

The proofs of Theorem 11 and Theorem 12 are provided in Appendix 4.8.3 and Appendix 4.8.4, respectively.

Theorem 11 indicates that the expected average utility generated under the myopic policy converges to the upper bound, thus it is optimal. Theorem 12 implies that in almost every time slot, we have $|\mathcal{C}_t^*| = m_t$, and the time average utility generated under the myopic policy converges to its upper bound almost surely. The corresponding convergence rates are explicitly characterized.

We note that although the randomized myopic policy only relies on the current queue lengths to make the sensing scheduling decisions, it is still optimal. This is true even for the case where the energy harvesting processes at sensors are not uniform. Although this is counterintuitive, it can be explained as follows: the randomized myopic policy achieves the upper bound in Lemma 11 since $\lim_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbf{1}_{|\mathcal{C}_t^*|\neq m_t} = 0$ almost surely. However, it does not imply that $\mathbb{P}\left[\frac{1}{T}\sum_{t=1}^{T}\mathbf{1}_{|\mathcal{C}_t^*|\neq m_t} \geq \epsilon\right]$ is minimized under the myopic policy. There may exist other policies that converge to the upper bound at a faster rate if the statistics of the energy harvesting processes could be utilized to make the sensing scheduling decisions.

## 4.6 Numerical Results

In this section, we evaluate the performances of the proposed scheduling algorithms under the offline and online settings through numerical examples.

### 4.6.1 Offline Results

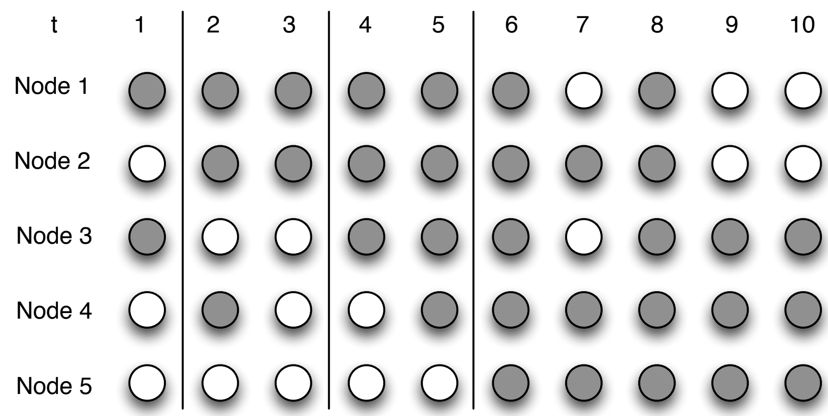
In this section, we use a numerical example to illustrate our scheduling algorithm under an offline setting. We consider a sensor network with 5 sensor nodes. The amount of energy harvested at each sensor node in slot $t-1$, $t \in \{1, 2, \dots, 10\}$ is provided in the following table.

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Node 1 | 4 | | | 2 | | | | 1 | | |
| Node 2 | | 5 | | | | 2 | | | | |
| Node 3 | 1 | | | 3 | | | | 7 | | |
| Node 4 | | 1 | | | 5 | | | | 2 | |
| Node 5 | | | | | | 7 | | | | |
| $c_t$ | 2 | 2 | 3 | 3 | 4 | 3 | 4 | 4 | 4 | 4 |

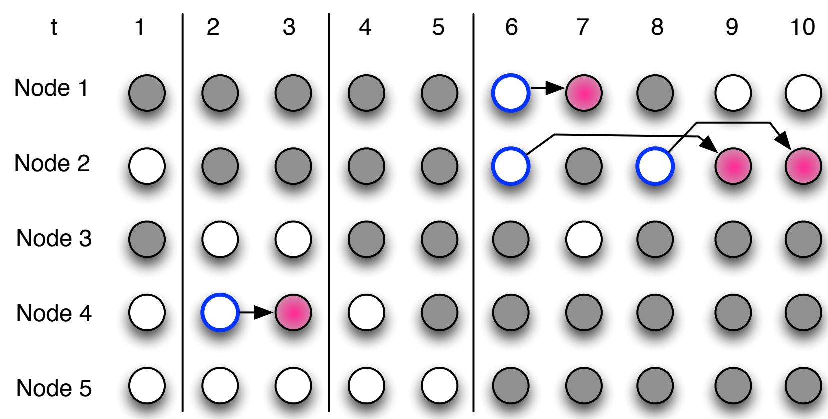
Table 4.1: The energy harvesting profile for sensors over duration $[1, 10]$. The last line represents the number of active sensors in each slot obtained by Algorithm 1.

We then illustrate the procedure to obtain a feasible scheduling with the given scheduling structure $\{c_t\}_{t=1}^{10}$. The initial greedy scheduling is illustrated in Fig. 4.1(a), where we use a dot and a circle to represent the *active* and *idle* status of a node in a given time slot, respectively.

We then perform the rescheduling according to the procedure described in Algorithm 2, and obtain the final scheduling in Fig. 4.1(b). We note that a subset of sensor nodes change their statuses from *busy* to *idle* in certain time slots, and the saved energy is used in a time slot later. The final scheduling has exact $c_t$ active sensors in slot $t$. We point out that the final scheduling is not unique in general. For example, at time $t = 7$, in order to have $c_7 = 4$, we can let node 1 keep silent in slot 6 and be active in slot 7, as indicated in Fig 4.1(b). We could also let node 3 be active in slot 7 with the energy saved by keeping silent in slot 6. The rescheduling in the remaining time slots will be adjusted accordingly to obtain a feasible scheduling with the same structure $\{c_t\}_{t=1}^{10}$.




(a)




(b)

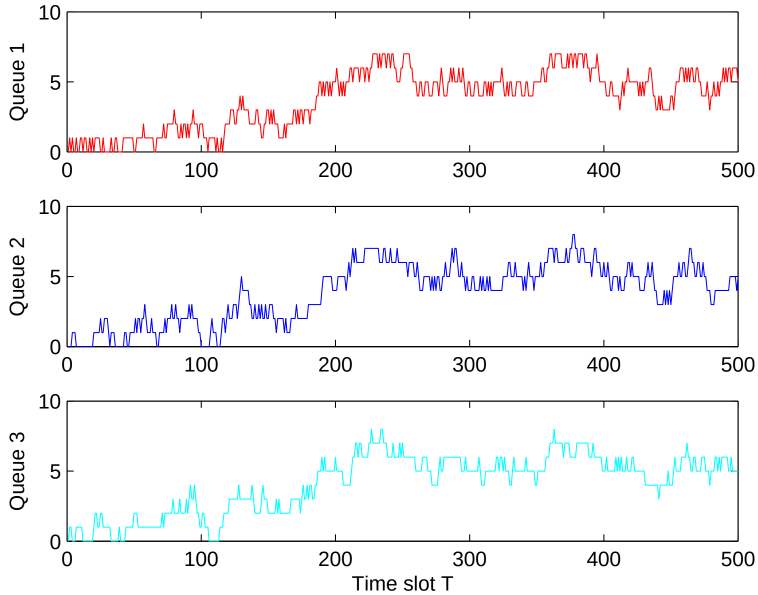
Figure 4.1: Dots represent active sensors in each time slot under the initial scheduling and final scheduling in Fig. 4.1(a) and Fig. 4.1(b), respectively. Arrows connecting a circle and a dot in Fig. 4.1(b) indicates the scheduling adjustments upon the initialization.

### 4.6.2 Online Results

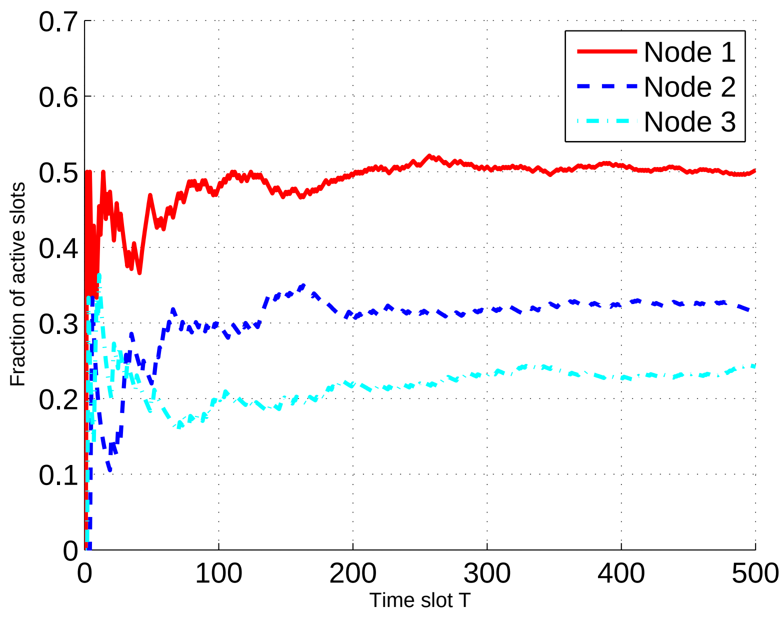
In this section, we evaluate the performances of the proposed myopic scheduling policy through simulations. In order to illustrate the performance of our policy, we assume the utility function $f(x) = \log(1+x)$ for $x \in \mathbb{Z}_+$, and for non-integer $x \in \mathbb{R}_+$, $f(x)$ is defined according to (4.16).

To illustrate the temporal evolution of the energy queue lengths and the scheduling procedure under the randomized myopic policy, we first consider a small sensor network consisting of 3 sensor nodes. The energy arrival rates for sensors are $\lambda_1 = 1/2$, $\lambda_2 = 1/3$, $\lambda_3 = 1/4$. The randomized myopic policy is thus to select one or two sensors with the longest energy queue lengths to perform the sensing task in each time slot. Starting with an empty initial state, one sample path of the energy queue evolution for those sensors is plotted in Fig. 4.2(a). We observe that the energy queue lengths of those three sensor nodes are closely coupled together. The differences in queue lengths are small for most of the time slots, and the queue lengths fluctuate in the same manner in time. This coincides with our objective to balance the queue lengths through the randomized myopic scheduling policy. The fraction of time slots when a sensor is active is plotted as a function of $T$ in Fig. 4.2(b). We observe that the sample path-wise fraction of active time slots for sensor $i$ approaches its upper bound $\lambda_i$, $i = 1, 2, 3$ as $T$ increases. The time average utility generated under the randomized myopic policy is plotted as a function of $T$ in Fig. 4.3. Although this curve fluctuates significantly when $T$ is small, it becomes smooth as $T$ increases, and gradually approaches $f(13/12)$. This indicates that under the randomized myopic scheduling policy, the sample path-wise time average utility asymptotically achieves its upper bound as $T$ increases, which validates its optimality. The result in Fig. 4.3 implies the effectiveness of balancing energy queues in maximizing the time-average utility function.

Then, we fix the energy arrival rate at each sensor node to be $1/3$, and vary the size of the sensor networks to be $N = 30, 60, 120$. Under this setup, the randomized myopic policy




(a)




(b)

Figure 4.2: A sensor network with $N = 3$, $\lambda_1 = 1/2$, $\lambda_2 = 1/3$, $\lambda_3 = 1/4$. Fig. 4.2(a) plots a sample path of the energy queue lengths. Fig. 4.2(b) shows the corresponding fraction of active time slots for each sensor as a function of $T$.

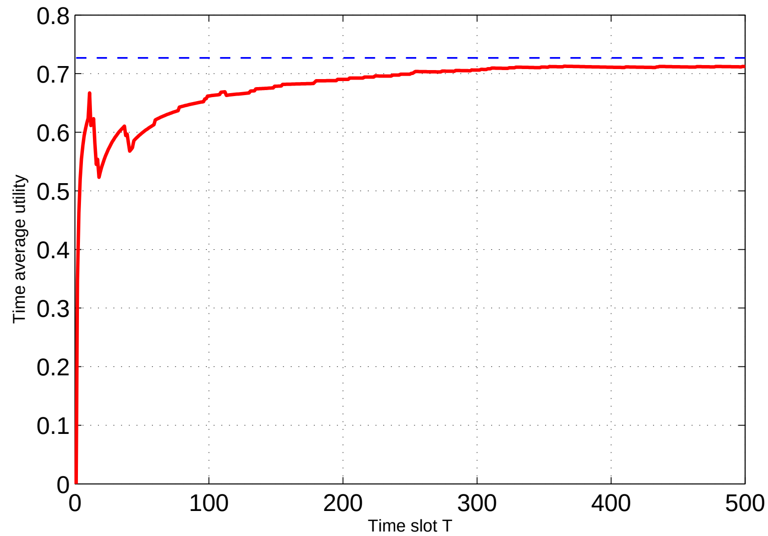



Figure 4.3: A sample path of the time average utility generated under the randomized myopic policy.

becomes a deterministic myopic policy, i.e., for any time slot $t$, $m_t = N/3$. For a time slot with $|\mathcal{C}_t^*| \neq m_t$, we call it an *unsaturated* time slot; otherwise, we call it a *saturated* time slot. We run 1000 samples paths for each setup, and plot the average fraction of saturated time slots under the myopic policy in Fig. 4.4. We observe that among those three curves, the curve corresponding to $N = 120$ is always at the bottom, while the curve corresponding to $N = 30$ is always on the top. This is consistent with the theoretical results in Theorem 12, i.e., for a fixed $T$, the fraction of unsaturated time slots increases in $N$.

The sample average of $\frac{1}{T}\sum_{t=1}^{T} f(|\mathcal{C}_t^*|)$ generated under the myopic policy is plotted in Fig. 4.5 for each setup, where we use the vertical bars to represent the 95% confidence intervals. The results indicate that for a majority of the 1000 sample paths, the time average utility generated under the myopic policy converges to their corresponding upper limits quickly.

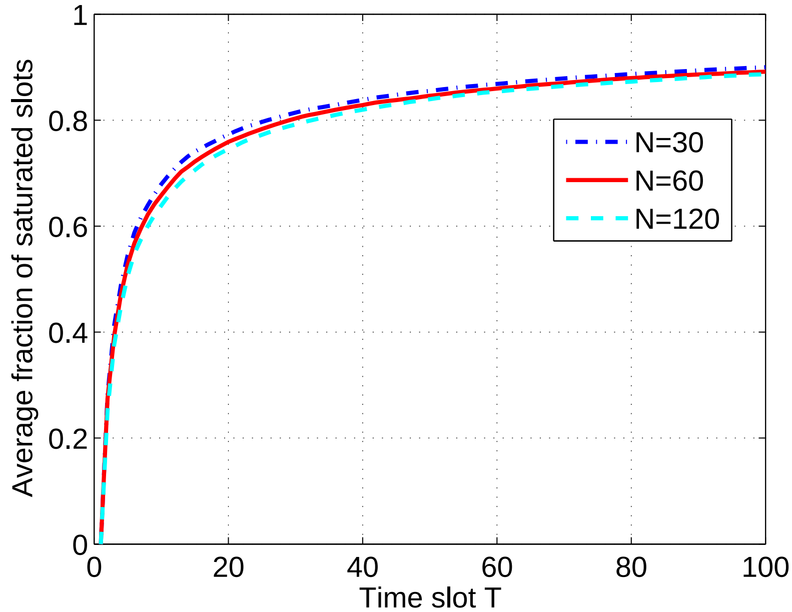



Figure 4.4: The sample average fraction of saturated time slots as a function of time index $T$. The energy harvesting rate $\lambda_i = 1/3, \forall i$, and the network sizes $N = 30$, 60, 120, respectively.

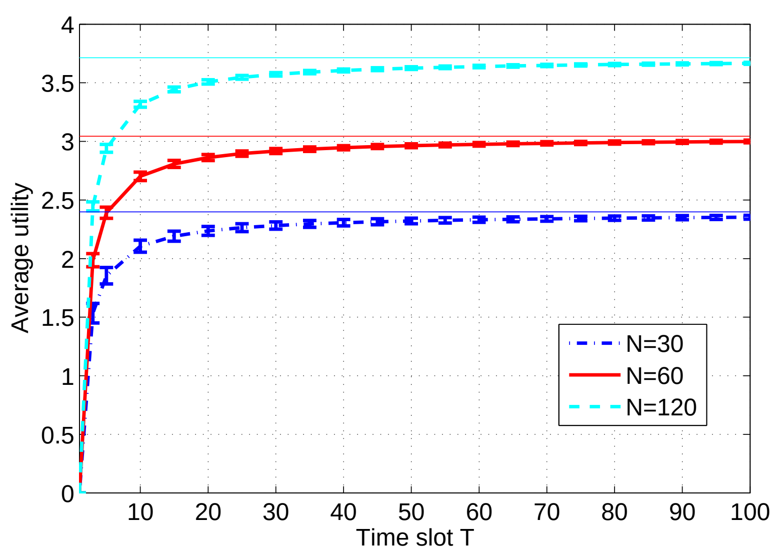



Figure 4.5: The sample average of $\frac{1}{T}\sum_{t=1}^{T} f(|\mathcal{C}_t^*|)$ as a function of time index $T$. Vertical bars represent 95% confidence intervals. Horizontal lines indicate $f\left(\sum_{i=1}^{N} \lambda_i\right)$. The energy harvesting rates $\lambda_i = 1/3, \forall i$, and the network sizes $N = 30$, 60, 120, respectively.

### 4.7 Conclusions

In this chapter, we considered optimal collaborative sensing scheduling policies in a sensor network powered by energy harvested from the nature. The objective is to maximize the time average utility generated by the sensors under the energy causality constraints at individual sensors, where the utility function is assumed to be concave and monotonically increasing in the number of active sensors in each slot. We considered both offline and online settings. Under the offline setting, we first noted that the optimal sensing policy has a "majorization" structure, and then proposed an algorithm to identify the scheduling policy satisfying the energy causality constraints at individual sensors and the structural requirements of the optimal policy. Under the online setting, we first proved that the long-term expected time average utility generated under any feasible policy has an upper bound. Then, we proposed a randomized myopic policy, and showed that as $T$ approaches infinity, the expected time average utility generated under the policy converges to the upper bound almost surely, thus it is optimal. The corresponding convergence rate was also explicitly characterized.

### 4.8 Appendix

#### 4.8.1 Proof of Theorem 9

The proof of the feasibility of the rescheduling procedure includes two parts: First, we prove that in each iteration, we must have $t_{n-1} < \bar{t} < \hat{t} \leq t_n$ for some $n$. Second, we prove that with obtained $\bar{t}$ and $\hat{t}$ in each iteration, we can always find $\delta$ active sensors from $\mathcal{S}(\bar{t})\backslash\mathcal{S}(\hat{t})$ for the rescheduling.

The first part can then be proved through contradiction. According to Lemma 9, we always have $\bar{t} < \hat{t}$. Assume $\bar{t} \leq t_{n-1} < \hat{t} \leq t_n$ for some $n$. Since $s_{\bar{t}} > c_{\bar{t}}$, we must have $\sum_{j=1}^{\bar{t}} s_j > \sum_{j=1}^{\bar{t}} c_j$. Therefore, $\sum_{j=1}^{t_{n-1}} s_j > \sum_{j=1}^{t_{n-1}} c_j = S(t_{n-1})$, where the last equality follows from Lemma 7. This implies that the energy causality constraint (4.8) is violated at

$t_{n-1}$, which contradicts with the fact the energy causality constraint is always satisfied in each iteration. Thus, we must have $t_{n-1} < \bar{t} < \hat{t} \leq t_n$.

To prove the second part, we note since $s_{\bar{t}} > c_{\bar{t}}$, $s_{\hat{t}} < c_{\hat{t}}$, and $t_{n-1} < \bar{t} < \hat{t} \leq t_n$, applying Lemma 8, we have $s_{\bar{t}} > s_{\hat{t}}$. Therefore,

$$|\mathcal{S}(\bar{t})\backslash\mathcal{S}(\hat{t})| \geq s_{\bar{t}} - s_{\hat{t}} \geq c_{\hat{t}} - s_{\hat{t}} \tag{4.26}$$

which ensures that we can always select $\delta$ active sensors from $\mathcal{S}(\bar{t})\backslash\mathcal{S}(\hat{t})$.

Since both parts hold in every iteration of Algorithm 2, the rescheduling procedure continues until $s_t$ equals $c_t$ for all $t$. Thus, the algorithm returns a valid sensing scheduling with scheduling structure $\{c_t\}_{t=1}^T$.

### 4.8.2 Proof of Theorem 10

First, based on the definition of $S_i(t)$ in (4.4)-(4.5), $S_i(t)$ is the maximum number of time slots that sensor $i$ can be active over $[1, t]$, while $S(t)$ is the maximum number of observations that the FC can collect from active sensors over $[1, t]$. Algorithm 1 ensures that at time $T$ the total number of observations taken by sensors equals $S(T)$, which implies that every sensor senses exactly $S_i(T)$ times over $[1, T]$. Any other policy that collects less than $S(T)$ sensing measurements over $[1, T]$ can always be improved by letting at least one of the sensors make one more observation in its *idle* slot during $[1, T]$ without violating its energy causality constraint. Therefore, in the following, we focus on the set of policies where each sensor $i$ senses exactly $S_i(T)$ out of $T$ time slots. We prove that the policy obtained by Algorithm 1 achieves the maximum sum utility among those policies.

We prove the optimality through contradiction. Let $\{c_t\}_{t=1}^T$ be the sizes of subsets determined by Algorithm 1. Let $\{c'_t\}_{t=1}^T$ be the *optimal* set of subset sizes satisfying the sum causality constraints:

$$\sum_{j=1}^{t} c'_j \leq S(t), \quad t = 1, 2, \ldots, T-1 \tag{4.27}$$

$$\sum_{j=1}^{T} c'_j = S(T) \tag{4.28}$$

We assume $\{c'_t\}_{t=1}^T$ is *strictly better* than $\{c_t\}_{t=1}^T$.

Let $i$ be the first time slot that $c_i \neq c'_i$. We assume that $t_{n-1} < i \leq t_n$. There are two possible cases:

**a)** $c_i < c'_i$**.** According to Lemma 7, we must have another time slot $j$, $t_{n-1} < i \leq j \leq t_n$, such that $c_j > c'_j$.

If $c_i = c_j$, we have $c'_j < c_j = c_i < c'_i$. Due to Assumption 1-(iii),

$$f(c'_i) + f(c'_j) < f(c'_i - 1) + f(c'_j + 1). \tag{4.29}$$

Therefore, without violating energy constraints (4.27)-(4.28), the scheduling with structure $\{c'_t\}_{t=1}^T$ can always be improved by replacing $c'_i$, $c'_j$ with $c'_i - 1$ and $c'_j + 1$, respectively. Thus, $\{c'_t\}_{t=1}^T$ cannot be optimal.

If $c_j = c_i + 1$, $c'_i = c_j$, $c'_j = c_i$, then $f(c'_i) + f(c'_j) = f(c_i) + f(c_j)$. If $c'_i \geq c_j > c_i \geq c'_j$, let $\delta = \min(c'_i - c_i, c_j - c'_j)$, we have

$$f(c'_i) + f(c'_j) < f(c'_i - \delta) + f(c'_j + \delta) \tag{4.30}$$

based on Assumption 1-(iii). Therefore, $\{c'_t\}_{t=1}^T$ cannot be optimal.

**b)** $c_i > c'_i$**.** There must exist a time slot $j > i$ with $c_j < c'_j$. Algorithm 1 implies that $c_i \leq c_j$ or $c_i = c_j + 1$. For the former case, we let $\delta = \min(c_i - c'_i, c'_j - c_j)$. Assumption 1-(iii) implies that

$$f(c'_i) + f(c'_j) < f(c'_i + \delta) + f(c'_j - \delta) \tag{4.31}$$

Therefore, $\{c'_t\}_{t=1}^T$ cannot be optimal. For the latter case, if $c'_i = c_j$, $c'_j = c_i$, then both policies give the same utility; otherwise, we can always let $\delta = \min(c_i - c'_i, c'_j - c_j)$ and improve the policy by replacing $c'_i$ and $c'_j$ with $c'_i + \delta$ and $c'_j - \delta$, respectively.

In summary, we cannot find a different policy that is strictly better than $\{c_t\}_{t=1}^T$. Thus, $\{c_t\}_{t=1}^T$ is optimal.

### 4.8.3 Proof of Theorem 11

Before we proceed, we first introduce Hoeffding's inequality, which will be used repeatedly in the proof.

**Theorem 13 (Hoeffding's inequality [98])** *Let $X_1$, $X_2$, ..., $X_n$ be independent bounded random variables such that $X_i \in [a_i, b_i]$ with probability 1. Let $S_n = \sum_{i=1}^n X_i$. Then for any $\epsilon > 0$, we have*

$$\mathbb{P}\left(|S_n - \mathbb{E}(S_n)| \geq \epsilon\right) \leq 2\exp\left(-\frac{2\epsilon^2}{\sum_{i=1}^n (b_i - a_i)^2}\right).$$

By Fatou's lemma, in order to prove Theorem 11, it suffices to prove that

$$\mathbb{E}\left[\liminf_{T\to+\infty} \frac{1}{T}\sum_{t=1}^{T} f(\mathcal{C}_t^*)\right] = f\left(\sum_{i=1}^{N}\lambda_i\right) \tag{4.32}$$

where $f\left(\sum_{i=1}^N \lambda_i\right)$ is defined according to (4.16).

The definition of $\mathcal{C}_t^*$ implies that $\mathcal{C}_t^* \subseteq \mathcal{C}'_t$, $|\mathcal{C}_t^*| \leq |\mathcal{C}'_t| = m_t$. Due to Assumption 1, when $|\mathcal{C}_t^*| = |\mathcal{C}'_t|$, $f(\mathcal{C}_t^*) = f(m_t)$; when $|\mathcal{C}_t^*| < |\mathcal{C}'_t|$, $f(\mathcal{C}_t^*) < f(m_t)$. Thus, in order to prove (4.32), it suffices to prove that

$$\limsup_{T\to+\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}_{|\mathcal{C}_t^*|<m_t} = 0, \quad a.s. \tag{4.33}$$

At each time slot $t$, we reorder $E_i(t)$, $i = 1, 2, \ldots, N$ according to their values, and denote $E_{(i)}(t)$ as the $i$-th largest one among them. For a given $T$, we define $T_1$ as the largest time index $t$, $t \leq T$, such that $E_{(m_t)}(t) = 0$, i.e., $T_1$ is the last time slot prior to $T$

such that the $m_t$-th longest energy queue is zero. Thus for any $t \in (T_1, T]$, we have $E_{(m_t)}(t) \geq 1$, which implies $|\mathcal{C}_t^*| = m_t$. Assuming the system starts with empty energy queues, $T_1$ always exists.

When $E_{(1)}(T_1) > 0$, we define $T_0$ as the smallest time index $t$ such that $E_{(1)}(t+1) = E_{(1)}(T_1)$, i.e., $T_0$ is the time slot right before the longest energy queue reaches $E_{(1)}(T_1)$ for the first time. Thus, $T_0 < T_1$. For any energy queue, the Bernoulli arrival assumption ensures that the queue length in a slot deviates at most by one from its previous slot. This observation together with the empty initial state assumption implies that $E_{(1)}(T_0) = E_{(1)}(T_0 + 1) - 1$. Then, at time $T_0$, we must have

$$E_{(1)}(T_0) = \ldots = E_{(m_{T_0}+1)}(T_0) = E_{(1)}(T_1) - 1 \tag{4.34}$$

This is due to the fact that in order to have a jump for the longest queue length at time $T_0 + 1$, the associated sensor should have the same amount of energy as $E_{(1)}(T_0)$ at time $T_0$, and does not sense in slot $T_0$. At the same time, there must exist additional $m_{T_0}$ sensors with the same energy level to sense in slot $T_0$. Therefore, we have

$$\sum_{i=1}^{N} E_i(T_0) \geq (m_{T_0} + 1)[E_{(1)}(T_1) - 1]. \tag{4.35}$$

On the other hand, based on the definition of $T_1$, we have

$$\sum_{i=1}^{N} E_i(T_1) \leq (m_{T_1} - 1)E_{(1)}(T_1). \tag{4.36}$$

Combining (4.35) and (4.36), we have

$$\begin{aligned}
&\sum_{i=1}^{N} E_i(T_1) - \sum_{i=1}^{N} E_i(T_0) \\
&\leq (-2 + m_{T_1} - m_{T_0})E_{(1)}(T_1) + m_{T_0} + 1 \qquad (4.37)\\
&\leq -E_{(1)}(T_1) + m_{T_0} + 1 \qquad (4.38)
\end{aligned}$$

where (4.38) is due to the fact that $-1 \leq m_{T_1} - m_{T_0} \leq 1$.

Based on the definition of $E_i(t)$ in (4.1), we have

$$\sum_{i=1}^{N} E_i(T_1) - \sum_{i=1}^{N} E_i(T_0) \geq \sum_{t=T_0}^{T_1-1} \left( \sum_{i=1}^{N} A_i(t) - m_t \right) \tag{4.39}$$

$$\sum_{i=1}^{N} E_i(T) - \sum_{i=1}^{N} E_i(T_1) \leq \sum_{t=T_1}^{T-1} \left( \sum_{i=1}^{N} A_i(t) - m_t \right) + m_{T_1} \tag{4.40}$$

To simplify the notation, we let $A(t) := \sum_{i=1}^{N} A_i(t)$, $\Lambda = \sum_{i=1}^{N} \lambda_i$, $\bar{\Lambda} = \lceil \Lambda \rceil$.Then,

$$\begin{aligned}
&\mathbb{P}\left[\sum_{i=1}^{N} E_i(T) > T\epsilon\right] \\
&\leq \mathbb{P}\left[\sum_{i=1}^{N} E_i(T_1) + \sum_{t=T_1}^{T-1} (A(t) - m_t) + m_{T_1} > T\epsilon\right] &(4.41)\\
&\leq \mathbb{P}\left[m_{T_1} E_{(1)}(T_1) + \sum_{t=T_1}^{T-1} (A(t) - m_t) + m_{T_1} > T\epsilon\right] &(4.42)\\
&\leq \mathbb{P}\left[m_{T_1} E_{(1)}(T_1) + \sum_{t=T_1}^{T-1} (A(t) - m_t) > T\epsilon - m_{T_1}, E_{(1)}(T_1) \leq \frac{T\epsilon}{2m_{T_1}}\right] \\
&+ \mathbb{P}\left[E_{(1)}(T_1) > \frac{T\epsilon}{2m_{T_1}}\right] &(4.43)\\
&\leq \mathbb{P}\left[\sum_{t=T_1}^{T-1} (A(t) - m_t) > \frac{T\epsilon}{2} - m_{T_1}\right] + \mathbb{P}\left[E_{(1)}(T_1) > \frac{T\epsilon}{2m_{T_1}}\right] &(4.44)
\end{aligned}$$

where (4.41) follows from (4.40), (4.42) follows from (4.36). Note

$$\begin{aligned}
&\mathbb{P}\left[\sum_{t=T_1}^{T-1} (A(t) - m_t) > \frac{T\epsilon}{2} - m_{T_1}\right] \\
&= \sum_{t_1=1}^{T-1} \mathbb{P}\left[\sum_{t=T_1}^{T-1} (A(t) - m_t) > \frac{T\epsilon}{2} - m_{T_1}, T_1 = t_1\right] &(4.45)\\
&\leq \sum_{t_1=1}^{T-1} \mathbb{P}\left[\sum_{t=t_1}^{T-1} (A(t) - m_t) > \frac{T\epsilon}{2} - m_{t_1}\right] &(4.46)
\end{aligned}$$

$$\leq \sum_{t_1=1}^{T-1} 2\exp\left(-\frac{\left(T\epsilon - 2\bar{\Lambda}\right)^2}{2(T-t_1-1)(N+1)^2}\right) \tag{4.47}$$

$$\leq 2(T-1)\exp\left(-\frac{\left(T\epsilon - 2\bar{\Lambda}\right)^2}{2T(N+1)^2}\right) \tag{4.48}$$

where (4.47) follows from Hoeffding's inequality [98], the i.i.d. assumption on $A(t)$ and the definition of $m_t$. Besides,

$$\mathbb{P}\left[E_{(1)}(T_1) > \frac{T\epsilon}{2m_{T_1}}\right]$$

$$= \mathbb{P}\left[-E_{(1)}(T_1) + m_{T_0} + 1 < -\frac{T\epsilon}{2m_{T_1}} + m_{T_0} + 1\right] \tag{4.49}$$

$$\leq \mathbb{P}\left[\sum_{t=T_0}^{T_1-1}(A(t) - m_t) < -\frac{T\epsilon}{2m_{T_1}} + m_{T_0} + 1\right] \tag{4.50}$$

$$\leq \sum_{t_0=1}^{T-1}\sum_{t_1=t_0+1}^{T-1}\mathbb{P}\left[\sum_{t=t_0+1}^{t_1}(A(t) - m_t) < -\frac{T\epsilon}{2m_{T_1}} + m_{T_0} + 1\right] \tag{4.51}$$

$$\leq \sum_{t_0=1}^{T-1}\sum_{t_1=t_0+1}^{T-1} 2\exp\left(-\frac{(T\epsilon - 2\bar{\Lambda}(\bar{\Lambda}+1))^2}{2(t_1-t_0)(N+1)^2\bar{\Lambda}^2}\right) \tag{4.52}$$

$$\leq (T-1)(T-2)\exp\left(-\frac{(T\epsilon - 2\bar{\Lambda}(\bar{\Lambda}+1))^2}{2T(N+1)^2\bar{\Lambda}^2}\right) \tag{4.53}$$

where (4.50) follows from (4.38) and (4.39), (4.52) follows from Hoeffding's inequality. When $T$ is sufficiently large, we have

$$(4.48) \leq 2(T-1)\exp\left(-\frac{T\epsilon^2}{3(N+1)^2}\right) \tag{4.54}$$

$$(4.53) \leq (T-1)(T-2)\exp\left(-\frac{T\epsilon^2}{3(N+1)^2\Lambda^2}\right) \tag{4.55}$$

Combining (4.54) and (4.55), we have

$$\mathbb{P}\left[\sum_{i=1}^{N} E_i(T) > T\epsilon\right] \leq T(T-1)\exp\left(-\frac{T\epsilon^2}{3(N+1)^2\Lambda^2}\right) \tag{4.56}$$

Since

$$\sum_{T=1}^{\infty} T(T-1)\exp\left(-\frac{T\epsilon^2}{3(N+1)^2\Lambda^2}\right) < \infty, \tag{4.57}$$

according to Borel-Cantelli lemma [69], we have $\mathbb{P}\left[\limsup_{T\to+\infty}\sum_{i=1}^{N} E_i(T)/T > \epsilon\right] = 0$, i.e.,

$$\lim_{T\to+\infty}\frac{1}{T}\sum_{i=1}^{N} E_i(T) = 0, \quad a.s. \tag{4.58}$$

Based on (4.1), under the randomized myopic policy, we have

$$\lim_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T}|\mathcal{C}_t^*| = \lim_{T\to+\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{i=1}^{N} A_i(t) \tag{4.59}$$

On the other hand, the strong law of large numbers indicates that

$$\lim_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T} m_t = \lim_{T\to+\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{i=1}^{N} A_i(t) = \sum_{i=1}^{N}\lambda_i, \quad a.s. \tag{4.60}$$

Thus,

$$\lim_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T}|\mathcal{C}_t^*| = \lim_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T} m_t, \quad a.s. \tag{4.61}$$

Therefore,

$$\limsup_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbf{1}_{|\mathcal{C}_t^*|<m_t} \le \lim_{T\to+\infty}\frac{1}{T}\sum_{t=1}^{T}(m_t - |\mathcal{C}_t^*|) = 0, \quad a.s. \tag{4.62}$$

which implies (4.33) and completes the proof.

### 4.8.4 Proof of Theorem 12

Based on $|\mathcal{C}_t^*|$, we partition the set of time indices up to time $T$ into two subsets, $\mathcal{T}_T$ and $(\mathcal{T}_T)^c$, where

$$\mathcal{T}_T := \{t : |\mathcal{C}_t^*| < m_t, 1 \le t \le T\}. \tag{4.63}$$

Then,

$$\begin{aligned}
&\mathbb{P}\left[\frac{1}{T}\sum_{t=1}^{T}\mathbf{1}_{|\mathcal{C}_t^*|\neq m_t} > \epsilon\right] \\
&\le \mathbb{P}\left[\left|\frac{1}{T}\sum_{t=1}^{T}|\mathcal{C}_t^*| - m_t\right| > \epsilon\right] && (4.64)\\
&= \mathbb{P}\left[\left|\frac{1}{T}\sum_{t=0}^{T}(A(t)-m_t) - \frac{1}{T}\sum_{i=1}^{N}E_i(T+1)\right| > \epsilon\right] && (4.65)\\
&\le \mathbb{P}\left[\left|\frac{1}{T}\sum_{t=0}^{T}(A(t)-m_t)\right| > \frac{\epsilon}{2}\right] + \mathbb{P}\left[\left|\frac{1}{T}\sum_{i=1}^{N}E_i(T+1)\right| > \frac{\epsilon}{2}\right] && (4.66)\\
&\le 2\exp\left(-\frac{T^2\epsilon^2}{2(T+1)(N+1)^2}\right) + (T+1)T\exp\left(-\frac{T\epsilon^2}{12(N+1)^2\Lambda^2}\right) && (4.67)\\
&\le (T+1)^2\exp\left(-\frac{T\epsilon^2}{12(N+1)^2\Lambda^2}\right) && (4.68)
\end{aligned}$$

where (4.67) follows from (4.56) and Hoeffding's inequality.

Let $M := f\left(\left\lceil\sum_{i=1}^{N}\lambda_i\right\rceil\right) - f(0)$. Due to the monotonicity of $f$, we have

$$f\left(\left\lceil\sum_{i=1}^{N}\lambda_i\right\rceil\right) - f(\mathcal{C}_t^*) \le M, \quad \forall t \tag{4.69}$$

We observe that

$$\begin{aligned}
&\mathbb{P}\left[\left|\frac{1}{T}\sum_{t=1}^{T}f(\mathcal{C}_t^*) - f\left(\sum_{i=1}^{N}\lambda_i\right)\right| > \epsilon\right] \\
&\le \mathbb{P}\left[\left|\frac{1}{T}\sum_{t=1}^{T}\left[f(\mathcal{C}_t^*) - f(m_t)\right]\right| > \frac{\epsilon}{2}\right] + \mathbb{P}\left[\left|\frac{1}{T}\sum_{t=1}^{T}f(m_t) - f\left(\sum_{i=1}^{N}\lambda_i\right)\right| > \frac{\epsilon}{2}\right] && (4.70)
\end{aligned}$$

where

$$
\begin{aligned}
&\mathbb{P}\left[\left|\frac{1}{T}\sum_{t=1}^{T}\left[f(\mathcal{C}_t^*)-f\left(m_t\right)\right]\right|>\frac{\epsilon}{2}\right] \\
&=\mathbb{P}\left[\left|\frac{1}{T}\sum_{t\in\mathcal{T}_T}\left[f(\mathcal{C}_t^*)-f(m_t)\right]\right|>\frac{\epsilon}{2}\right] && (4.71)\\
&\leq\mathbb{P}\left[\frac{1}{T}\sum_{t=1}^{T}\mathbf{1}_{|\mathcal{C}_t^*|\neq m_t}>\frac{\epsilon}{2M}\right] && (4.72)\\
&\leq (T+1)^2\exp\left(-\frac{T\epsilon^2}{12(N+1)^2M^2\Lambda^2}\right) && (4.73)
\end{aligned}
$$

and

$$
\begin{aligned}
&\mathbb{P}\left[\left|\frac{1}{T}\sum_{t=1}^{T}f(m_t)-f\left(\sum_{i=1}^{N}\lambda_i\right)\right|>\frac{\epsilon}{2}\right] \\
&=\mathbb{P}\left[\left|f\left(\frac{1}{T}\sum_{t=1}^{T}m_t\right)-f\left(\sum_{i=1}^{N}\lambda_i\right)\right|>\frac{\epsilon}{2}\right] && (4.74)\\
&\leq\mathbb{P}\left[\left[f(1)-f(0)\right]\left|\frac{1}{T}\sum_{t=1}^{T}m_t-\sum_{i=1}^{N}\lambda_i\right|>\frac{\epsilon}{2}\right] && (4.75)\\
&\leq 2\exp\left(-\frac{T\epsilon^2}{2M^2}\right) && (4.76)
\end{aligned}
$$

where (4.74) follows from the fact that $m_t=\lceil\Lambda\rceil$ or $\lfloor\Lambda\rfloor$ and the definition of $f(x)$ in (4.16) for $x\in\mathbb{R}_+$, (4.75) follows from Hoeffding's inequality. Therefore, when $T$ is sufficiently large, we have

$$
\mathbb{P}\left[\left|\frac{1}{T}\sum_{t=1}^{T}f(\mathcal{C}_t^*)-f\left(\Lambda\right)\right|\geq\epsilon\right]\leq 2T^2\exp\left(-\frac{T\epsilon^2}{12(N+1)^2M^2\Lambda^2}\right). \tag{4.77}
$$

# Chapter 5: Optimal Energy Management for Energy Harvesting Transmitters under Battery Usage Constraint

## 5.1 Introduction

The random and intermittent nature of harvested energy imposes critical challenges on the design of sustainable and reliable energy harvesting wireless sensor networks. Rechargeable batteries are usually employed as an energy buffer to filter out the fluctuations in the energy harvesting process and maintain a continuous and stable energy output. A large number of energy management schemes have been proposed to optimize the performances of such systems.

Modeling the battery as an ideal energy buffer for energy storage and retrieval, researchers have developed various energy management schemes to optimize different performance metrics under infinite battery setting [27, 44, 64] and finite battery setting [1, 4, 5, 8, 10, 65]. The performance metrics include channel capacity [44], transmission delay [27], throughput [1, 4, 10], etc.

However, modeling batteries as perfect energy buffers may not be realistic, since battery operations involve very complicated mechanisms, which lead to inevitable energy storage imperfections and battery degradation. In this context, some works aim to take more practical battery characteristics into the optimization framework, and investigate their impacts on the optimal energy management policies and system performances.
In [16], the authors consider battery storage imperfections where stored energy leaks in time, and the battery degrades at the same time. An optimal throughput maximization policy is proposed under an offline setting. Reference [99] proposes a battery health model to capture the dependency of battery degradation on its discharge depth, and investigates degradation-aware policy to improve the lifetime of the battery while guaranteeing the minimum QoS requirement. The problem is casted into the framework of Markov Decision

Processes, and solved independently for each health state by exploiting the timescale separation between the communication time-slot and the battery degradation process. [100] investigates the scenario where a portion of energy is lost instantaneously when it enters the battery, and proposes optimal offline transmission policies under various settings. The optimal policy has a double-threshold structure, where the battery charges/discharges when the harvested energy is above/below the thresholds and transmits with the corresponding threshold.

It has been shown that the battery lifetime is closely related to its charge/discharge cycles. Frequent battery charge/discharge operations result in irreversible battery capacity degradation and jeopardize its battery lifetime. In this chapter, we take the impact of charge/discharge operations on battery lifetime into consideration, and study the optimal energy management policy for an energy harvesting communication system under a battery usage constraint. Specifically, in each time slot, we assume the harvested energy can be used to power the transmitter immediately without entering into the battery, or stored into the battery for now and retrieved later for transmission. Besides the energy causality constraints, we impose a battery cost constraint, which is translated into the average number of charge/discharge operations per unit time. The objective is to maximize the long-term average throughput of the transmitter under energy causality constraint and the battery usage constraint. We do not consider battery degradation explicitly in this setup, as we assume that the aging process happens over a time scale that is much longer than the communication period we consider about, and the battery storage capacity is always sufficiently large to prevent any energy overflow in our setting.

We first relax the energy causality constraint on the system, and impose a long-term energy flow conservation constraint instead. We show that the optimal energy management policy has a double-threshold structure: if the amount of energy arrives in each time slot lies in between the two thresholds, it will be used immediately without involving the battery; otherwise, the battery will be charged or discharged accordingly to maintain a constant transmit power. We then modify the two-threshold policy slightly to accommodate the energy causality constraint, and analyze its long-term performance. We show that the system achieves the same long-term average performance, thus it is optimal. We have presented part of this work in [101].

Despite a similar double-threshold structure, our policy is fundamentally different from that studied in [100] due to different constraints we impose on the system. Essentially, under the battery inefficiency assumption that a ratio of the saved energy will be lost in [100], the *amount* of energy to be saved in the battery is the key factor, which can be identified by solving the standard convex optimization problem. While under the battery usage constraint, the *number* of charge/discharge operations matters. Thus, our optimization problem has a combinatorial flavor, which cannot be solved straightforwardly via convex optimization. As a result, under our policy, the transmitter always tries to equalize the transmit power whenever it charges or discharges, while in [100], the transmitter transmits with the corresponding thresholds.

## 5.2 System Model and Problem Formulation

Consider a time slotted energy harvesting communication system. Let $A_t$ be the energy harvested from the ambient environment in time slot $t$, $t = 1, 2, \ldots, T$. $A_t$s are i.i.d random variables with known probability density function (pdf) $p_A(\cdot)$. Energy can be used to transmit data from a backlogged buffer, or stored in a battery for later use, as shown in Fig. 5.1. Let $B_t$ be the amount of energy that enters the battery in time $t$, and $C_t$ be the remaining amount from $A_t$. Then,

$$A_t = B_t + C_t \tag{5.1}$$

Let $D_t$ be the energy drawn from the battery in time $t$. The total amount of energy used for transmission in time slot $t$ is then equal to $P_t := D_t + C_t$. Then, the battery level evolves according to

$$E_{t+1} = E_t - D_t + B_t, \quad D_t \le E_t \tag{5.2}$$

with $E_0 = 0$.

Assume the transmission rate is a concave function of $P_t$, denoted as $R(P_t)$. Our objective is to optimize the long-term average transmission rate under the energy causality constraint and the battery usage constraint, which is denoted as the expected number of charge/discharge operations per time slot. Then, the optimization problem is formulated as

$$\max_{\{C_t, D_t\}} \quad \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[R(P_t)] \tag{5.3}$$

$$\text{s.t.} \quad (5.1) - (5.2) \tag{5.4}$$

$$\lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\left(\mathbf{1}_{D_t} + \mathbf{1}_{B_t}\right) \le \rho \tag{5.5}$$

The expectations in the objective function and the constraint are taken over all possible energy harvesting sample paths. The optimization problem has a combinatorial flavor, as we need to decide in which time slots the system should charge or discharge the battery. Thus, to make the problem tractable, in the following, we will first relax the energy causality constraint and study the problem with a relaxed long-term energy flow conservation constraint for the battery. With the structured optimal energy management policy obtained for this case, we will propose a best-effort transmission policy which obeys the energy causality constraint and prove that it achieves the same performance as time $T$ goes to infinity. Therefore, it is optimal.

### 5.3 Optimal Policy Without Causality Constraints

In the following, we will first consider a relaxed optimization problem, where we replace the energy causality constraint in (5.2) with the following long-term energy flow conservation

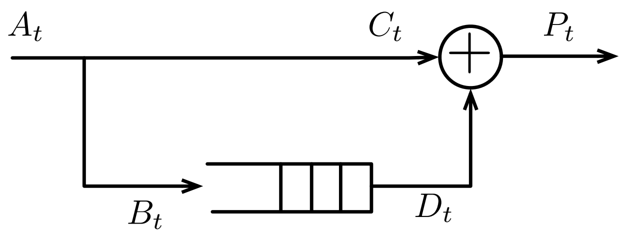



Figure 5.1: System model

constraint for the battery:

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}D_t \le \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}B_t \tag{5.6}$$

Assume $\mathcal{Q}$ is the optimal policy satisfying the battery usage constraint in (5.5) and the energy flow conservation constraint in (5.6). In general, under $\mathcal{Q}$, the transmit power $P_t$ may depend on the current energy arrival $A_t$, as well as the energy arrival and departure history up to $t-1$, denoted as $\mathcal{H}^{t-1}$. With a little abuse of notation, in this section, we use $P_t$ to denote the transmit power in time slot $t$ under policy $\mathcal{Q}$. We assume $P_t$ is a deterministic function of $A_t$ and $\mathcal{H}^{t-1}$, denoted as $P_t = Q(A_t, \mathcal{H}^{t-1})$. In the following, we will identify the structural properties of $\mathcal{Q}$, and show that it can be explicitly obtained using a simple approach. Our analysis can be directly extended to handle any randomized policy as well.

Define $\mathcal{A}_t := \{(A_t, \mathcal{H}^{t-1})|A_t \neq Q(A_t, \mathcal{H}^{t-1})\}$, $t = 1, 2, \ldots$, i.e., the set of states in which the battery charges or discharges in time slot $t$ under $\mathcal{Q}$. Define

$$P_0 = \lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\mathbb{E}[A_t|(A_t, \mathcal{H}^{t-1}) \in \mathcal{A}_t], \tag{5.7}$$

i.e., the average amount of energy harvested during the states included in $\cup_{t=1}^{\infty}\mathcal{A}_t$. We assume the limit exists. Then, we have the following observations.

**Lemma 12** *Under the optimal policy $\mathcal{Q}$, $B_t$ and $D_t$ cannot be positive in the same slot $t$.*

This is obvious due to the fact that if $B_t$ and $D_t$ are both positive, we can always adjust the values of $B_t$ and $D_t$ to make one of them to be zero, and achieve the same transmit power $P_t$ with a reduced battery usage cost.

**Lemma 13** *Under the optimal policy $\mathcal{Q}$, whenever the battery charges or discharges, the transmit power $P_t$ should be a constant and equal to $P_0$.*

Lemma 13 can be proved by Jensen's inequality. Based on Lemmas 12 and 13, we have the following theorem.

**Theorem 14** *The optimal policy under the relaxed long-term energy flow conservation constraint depends on the instantaneous energy arrival only, and has a double-threshold structure, i.e., if $A_t < \tau_1$, we must have $D_t = P_0 - A_t$, $P_t = P_0$; if $A_t > \tau_2$, we must have $B_t = A_t - P_0$, $P_t = P_0$, where $P_0$, $\tau_1$ and $\tau_2$ are the solution to the following optimization problem*

$$\max_{P_0,\tau_1,\tau_2} \quad R(P_0)\rho + \int_{\tau_1}^{\tau_2} R(x)p_A(x)dx \tag{5.8}$$

$$s.t. \quad \mathbb{P}[A_t > \tau_2] + \mathbb{P}[A_t < \tau_1] = \rho \tag{5.9}$$

$$\mathbb{E}[A_t - P_0 | A_t > \tau_2] = \mathbb{E}[P_0 - A_t | A_t < \tau_1] \tag{5.10}$$

$$\tau_1 \leq P_0 \leq \tau_2 \tag{5.11}$$

Theorem 14 can be proved through contradiction. Assume that $\mathcal{Q}$ does not have such double-threshold structure. Then, we can always construct another policy to outperform it without violating the constraints in (5.5) and (5.6). The detailed proof is provided in the Appendix in Section 5.6.

Theorem 14 provides an upper bound on any energy management policy satisfying the energy causality constraint and the battery usage constraint.

**Theorem 15** *The objective function (5.8) can be reduced to a function with a single variable $\tau_1$. Moreover, it first increases then decrease in $\tau_1$, and the maximum point* *corresponds to the optimal solution satisfying (5.9)-(5.11).*

**Proof:** Since

$$\rho = \int_0^{\tau_1} p_A(x)dx + \int_{\tau_2}^{\infty} p_A(x)dx \tag{5.12}$$

$$P_0 = \frac{1}{\rho}\left(\int_0^{\tau_1} xp_A(x)dx + \int_{\tau_2}^{\infty} xp_A(x)dx\right) \tag{5.13}$$

Both $P_0$ and $\tau_2$ can be treated as functions of $\tau_1$. Taking derivative of (5.12) with respect to $\tau_1$, we have

$$\frac{d\tau_2}{d\tau_1} = \frac{p_A(\tau_1)}{p_A(\tau_2)} > 0 \tag{5.14}$$

Taking derivative of (5.13), we have

$$\frac{dP_0}{d\tau_1} = \frac{1}{\rho}\left(\tau_1 p_A(\tau_1) - \tau_2 p_A(\tau_2)\frac{d\tau_2}{d\tau_1}\right) \tag{5.15}$$

$$= \frac{\tau_2 - \tau_1}{\rho} p_A(\tau_1) < 0 \tag{5.16}$$

Therefore, $\tau_2$ is increasing in $\tau_1$ while $P_0$ is decreasing in $\tau_1$.

The objective function is equivalent to

$$F(\tau_1) := \int_0^{\tau_1} [R(P_0) - R(x)]p_A(x)dx + \int_{\tau_2}^{\infty} [R(P_0) - R(x)]p_A(x)dx \tag{5.17}$$

Thus,

$$\begin{aligned} F'(\tau_1) &= \frac{dF}{d\tau_1} + \frac{dF}{d\tau_2}\frac{d\tau_2}{d\tau_1} + \frac{dF}{dP_0}\frac{dP_0}{d\tau_1} \\ &= (R(\tau_2) - R(\tau_1))\, p_A(\tau_1) + R'(P_0)(\tau_1 - \tau_2)p_A(\tau_1) \\ &= (\tau_2 - \tau_1)p_A(\tau_1)\left(\frac{R(\tau_2) - R(\tau_1)}{\tau_2 - \tau_1} - R'(P_0)\right) \end{aligned}$$

Based on the property of concave functions, we can show that $\frac{R(\tau_2)-R(\tau_1)}{\tau_2-\tau_1}$ is decreasing in $\tau_1$ while $R'(P_0)$ is increasing in $\tau_1$. When $\tau_1 = 0$, $P_0 > \tau_2$, thus, $R'(P_0) < \frac{R(\tau_2)-R(\tau_1)}{\tau_2-\tau_1}$, and $F'(\tau_1) > 0$; When $\tau_1$ is sufficiently large, $\tau_2 = \infty$ and $P_0 < \tau_1$, thus $R'(P_0) > \frac{R(\tau_2)-R(\tau_1)}{\tau_2-\tau_1}$ and $F'(\tau_1) < 0$. Therefore, when we gradually increase $\tau_1$, $F'(\tau_1)$ is positive at first and then become negative, which implies that $F(\tau_1)$ is first increasing then decreasing in $\tau_1$. At the maximum point, we have $\frac{R(\tau_2)-R(\tau_1)}{\tau_2-\tau_1} = R'(P_0)$. Since $R'(\tau_2) < \frac{R(\tau_2)-R(\tau_1)}{\tau_2-\tau_1} < R'(\tau_1)$, we must have $P_0$ lying between $\tau_1$ and $\tau_2$. ■

Theorem 15 suggest a computationally efficient way to solve the optimization problem described in Theorem 14. Starting with $\tau_1 = 0$, we first solve (5.9)(5.10) to get $\tau_2$ and $P_0$ and evaluate the objective function. We gradually increase $\tau_1$, repeat the process, and keep track of the objective function value until we observe a decrease. The turning point corresponds the optimal solution.

### 5.4 Optimal Policy under Causality Constraints

Let $\tau_1, \tau_2, P_0$ be the optimal solution to the optimization problem described in Theorem 14. Let $\mathcal{B} = [0, \tau_1] \cup [\tau_2, \infty]$. Then, we define a best-effort transmission policy as follows.

**Definition 7 (Best-effort transmission policy)** *In each time slot $t$, if $A_t \notin \mathcal{B}$, the transmitter transmits with the harvested energy $A_t$. Otherwise, if $A_t > \tau_2$, the battery is charged with amount $A_t - P_0$, and transmitter transmits with $P_0$; if $A_t < \tau_1$ and $E_t \neq 0$, the battery is discharged with amount* $\min\{E_t, P_0 - A_t\}$*, and the transmitter transmits with* $\min\{E_t, P_0 - A_t\} + A_t$.

We note that the energy causality constraint is always satisfied under the proposed best-effort transmission policy. Besides, the battery usage constraint is satisfied as well. Due to the energy causality constraint, the transmitter may not be able to transmit with power $P_0$ if $A_t < \tau_1$ and $E_t$ is not sufficiently large. This may result in some performance degradation. However, as we will show in the following theorem, the probability of such scenario will decrease exponentially fast as $T$ increases. Thus, the long-term average throughput will converge to that upper bound exponentially, which indicates the optimality of the proposed best-effort policy.

Define the planned charge/discharge process as

$$A_t^* = \begin{cases} A_t - P_0 & A_t \in \mathcal{B} \\ 0 & A_t \notin \mathcal{B} \end{cases} \tag{5.18}$$

Then, under the proposed best effort policy, we have $E_{t+1} = \max\{E_t + A_t^*, 0\}$, and the energy spent at $t$ is

$$P_t = A_t + E_t - E_{t+1} \tag{5.19}$$

We define

$$Q_t = \begin{cases} P_0 & A_t \in \mathcal{B} \\ A_t & A_t \notin \mathcal{B} \end{cases} \tag{5.20}$$

Thus, $P_t \neq Q_t$ if and only if $E_t + A_t < P_0$, and $P_t \leq Q_t$, $\forall t$. Note that $Q_t$ is exactly the optimal policy defined in Theorem 14.

**Theorem 16** *Assume $|A_t| \leq M$ and $R(\cdot)$ is Lipschitz. Under the best-effort transmission policy,*

$$\lim_{T \to +\infty} \frac{1}{T} \sum_{t=1}^{T} (Q_t - P_t) = 0, \quad a.s. \tag{5.21}$$

**Proof:** First, we note that

$$\begin{aligned} E_{T+1} &= \sum_{t=1}^{T} A_t - \sum_{t=1}^{T} P_t = \sum_{t=1}^{T} (A_t - Q_t) + \sum_{t=1}^{T} (Q_t - P_t) \\ &= \sum_{t=1}^{T} A_t^* + \sum_{t=1}^{T} (Q_t - P_t) \end{aligned}$$

Thus,

$$
\begin{aligned}
&\mathbb{P}\left(\frac{1}{T}\sum_{t=1}^{T}(Q_t - P_t) \geq \epsilon\right) \\
&\leq \mathbb{P}\left(\sum_{t=1}^{T}(Q_t - P_t) \geq T\epsilon, E_{T+1} \geq T\epsilon/2\right) + \mathbb{P}\left(\sum_{t=1}^{T}(Q_t - P_t) \geq T\epsilon, E_{T+1} < T\epsilon/2\right) \\
&\leq \sum_{T_1=1}^{T}\mathbb{P}\left(\sum_{t=T_1}^{T} A_t^* \geq T\epsilon/2\right) + \mathbb{P}\left(\sum_{t=1}^{T} A_t^* < -T\epsilon/2\right) \qquad (5.22) \\
&\leq T\exp\left(-\frac{T\epsilon^2}{2M^2}\right) + \exp\left(-\frac{T\epsilon^2}{2M^2}\right) \qquad (5.23) \\
&= (T+1)\exp\left(-\frac{T\epsilon^2}{2M^2}\right)
\end{aligned}
$$

where $T_1$ in (5.22) is the largest time index such that (5.23) follows from Hoeffding's inequality [98]. ■

Theorem 16 indicates that the best-effort transmission policy converges to the optimal policy described in Theorem 14 almost surely. Therefore, we have the following observation.

**Theorem 17** *The best-effort transmission policy achieves the upper bound on the long-term expected throughput characterized in Theorem 14 almost surely. Therefore, it is optimal.*

## 5.5 Numerical Results

In this section, we use numerical results to illustrate the proposed best-effort transmission policy and evaluate its performance.

We assume the energy arrivals are i.i.d. random variables uniformly distributed over $[0, 6]$. We let $\rho = 0.3$, i.e., the battery can only be charged or discharged for 30% of the time, and the rate function $R(x) = \frac{1}{2}\log(1 + x)$. We first numerical solve the equations in Theorem 14, and identify the corresponding thresholds $\tau_1 = 1.0158$, $\tau_2 = 5.2158$, and $P_0 = 2.7298$. The corresponding time-average transmission rate is 0.6761, which is the upper bound for any online policy.

We then plot one sample path of the energy arrivals for the first 20 time slots in Fig. 5.2, and indicate the corresponding transmit power under the proposed policy. As expected, the transmit power equals $A_t$ if $A_t$ falls between those two thresholds, and equals $P_0$ otherwise, except when $t = 9, 14$. In those time slots, the battery does not have sufficient energy to meet the power demand $P_0$, and the transmitter transmits with all the power the system has at that time.

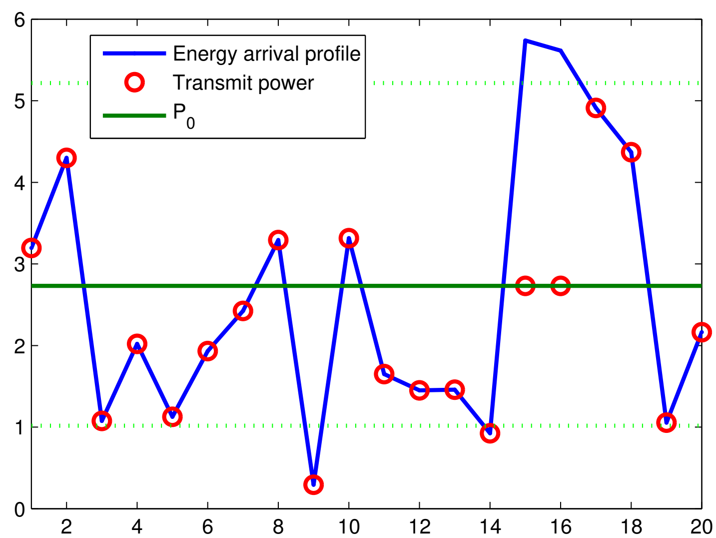



Figure 5.2: A sample path of the energy arrivals and the transmit policy.

We then evaluate the time-average transmission rate and the average number of battery charge/discharge operations per time slot. We plot a sample path in Fig. 5.3. We observe that both curves fluctuate at the beginning, and become stable after about 250 time slots. This corroborates with our theoretical results that the performance of the best-effort transmission policy converges to the upper bound almost surely. Finally, we run the simulation 1000 times, and plot the sample average of $\frac{1}{T}\sum_{t=1}^{T} R(P_t)$ as a function of $T$ in Fig. 5.4. The sample average of battery charge/discharge rate is also plotted in the same figure. We observe that the sample average of $\frac{1}{T}\sum_{t=1}^{T} R(P_t)$ converges to the upper bound as expected. The sample average of battery charge/discharge rate is very close to the battery usage constraint after a short time period. This implies that the desired battery usage constraint is satisfied under the proposed policy.

## 5.6 Appendix

Assume under the optimal policy $\mathcal{Q}$, the transmit power does not obey the double-threshold structure. Define

$$\mathcal{A}_t^- = \{(A_t, \mathcal{H}^{t-1}) | (A_t, \mathcal{H}^{t-1}) \in \mathcal{A}_t, A_t < P_0, \}, t = 1, 2, \ldots$$
$$\mathcal{A}_t^+ = \{(A_t, \mathcal{H}^{t-1}) | (A_t, \mathcal{H}^{t-1}) \in \mathcal{A}_t, A_t \geq P_0, \}, t = 1, 2, \ldots$$

and

$$\mathbb{P}[\mathcal{A}^-] = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{P}[\mathcal{A}_t^-], \quad \mathbb{P}[\mathcal{A}^+] = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{P}[\mathcal{A}_t^+].$$

Then, we define

$$\underline{\mathcal{A}}_t^- = \{(x, \mathcal{H}^{t-1}) | \mathbb{P}[0 \leq A_t \leq x] \leq \mathbb{P}[\mathcal{A}^-]\}, t = 1, 2, \ldots$$
$$\bar{\mathcal{A}}_t^+ = \{(x, \mathcal{H}^{t-1}) | \mathbb{P}[A_t \geq x] \leq \mathbb{P}[\mathcal{A}^+]\}, t = 1, 2, \ldots$$

Denote $\underline{\mathcal{A}}_t = \underline{\mathcal{A}}_t^- \cup \mathcal{A}_t^+$, $\bar{\mathcal{A}}_t = \mathcal{A}_t^- \cup \bar{\mathcal{A}}_t^+$. Define

$$\underline{P}_0 := \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[A_t | (A_t, \mathcal{H}^{t-1}) \in \underline{\mathcal{A}}_t] \tag{5.24}$$
$$\bar{P}_0 := \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[A_t | (A_t, \mathcal{H}^{t-1}) \in \bar{\mathcal{A}}_t] \tag{5.25}$$

We define two policies $\underline{\mathcal{Q}}$ and $\bar{\mathcal{Q}}$ under which in each time slot $t$, the transmitter power

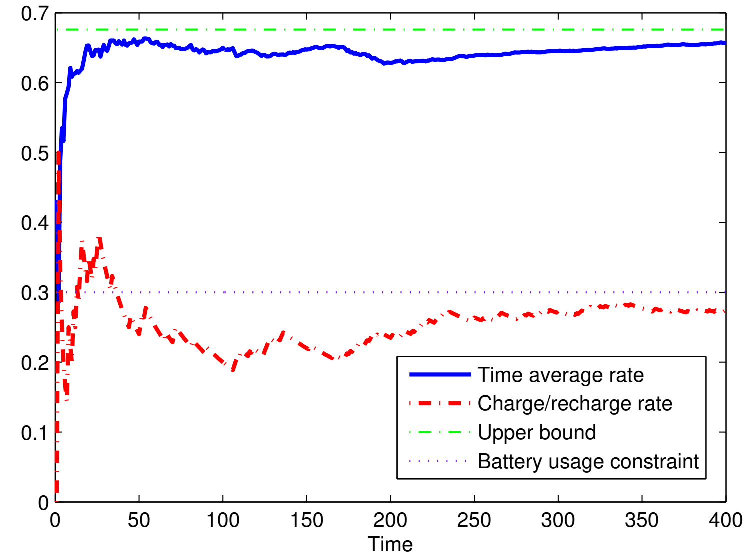



Figure 5.3: A sample path of the time-average transmit rate and the battery charge/discharge rate.

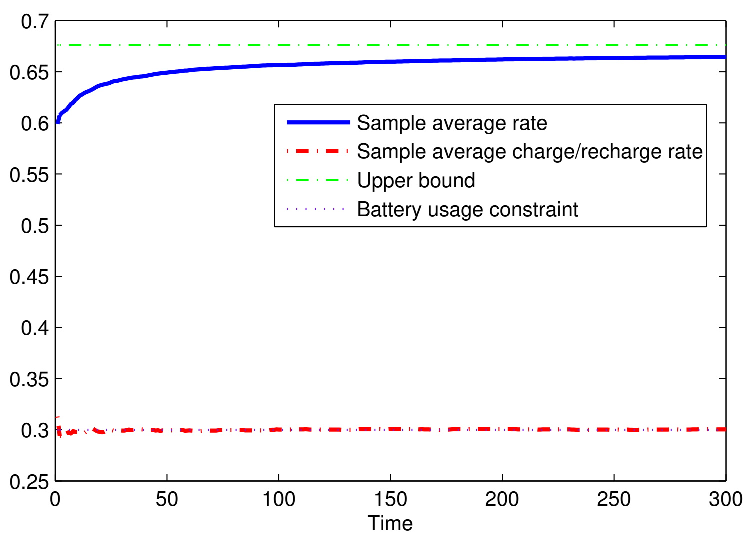



Figure 5.4: Sample path of the energy arrivals and the transmit policy.

is defined as follows respectively.

$$\underline{P}_t = \begin{cases} A_t, & (A_t, \mathcal{H}^{t-1}) \notin \underline{\mathcal{A}}_t \\ \underline{P}_0, & (A_t, \mathcal{H}^{t-1}) \in \underline{\mathcal{A}}_t \end{cases} \tag{5.26}$$

$$\bar{P}_t = \begin{cases} A_t, & (A_t, \mathcal{H}^{t-1}) \notin \bar{\mathcal{A}}_t \\ \bar{P}_0, & (A_t, \mathcal{H}^{t-1}) \in \bar{\mathcal{A}}_t \end{cases} \tag{5.27}$$

Denote

$$R(\mathcal{Q}) := \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} R(P_t)$$
$$R(\underline{\mathcal{Q}}) := \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} R(\underline{P}_t)$$
$$R(\bar{\mathcal{Q}}) := \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} R(\bar{P}_t).$$

We aim to show that

$$\mathbb{E}[R(\mathcal{Q})] \leq \mathbb{E}[R(\underline{\mathcal{Q}})], \quad \mathbb{E}[R(\mathcal{Q})] \leq \mathbb{E}[R(\bar{\mathcal{Q}})],$$

based on which we can claim that a necessary condition for $\mathcal{Q}$ to be optimal is, in each time slot $t$,

$$\underline{\mathcal{A}}_t^- = \mathcal{A}_t^-, \quad \bar{\mathcal{A}}_t^+ = \mathcal{A}_t^+,$$
$$P_t = P_0, \quad \forall (A_t, \mathcal{H}^{t-1}) \in \mathcal{A}_t,$$

i.e., a double-threshold structure.

**Definition 8** *Let $f$, $g$ be two increasing functions defined over $I_c := [0, c]$. We say $f \prec g$ if*

1. $\forall t \in I_c$, $\int_0^t f(s)ds \geq \int_0^t g(s)ds$.

*2.* $\int_0^c f(s)ds = \int_0^c g(s)ds.$

**Lemma 14** *If $f \prec g$, then for any concave function $r(\cdot)$,*

$$\int_0^c r(f(s))ds \geq \int_0^c r(g(s))ds$$

**Proof:** Partition $I_c$ to $n$ equal length segment, and let $x_0 = 0$, $x_1 = \frac{c}{n}$, $\ldots$, $x_{n-1} = \frac{n-1}{n}c$, $x_n = c$. Define $a_1 = f(x_1)$, $a_2 = f(x_2)$, $\ldots$, $a_n = f(x_n)$, and $b_1 = g(x_0)$, $b_2 = g(x_1)$, $\ldots$, $b_n = g(x_{n-1})$. Then,

$$\begin{aligned}
&\frac{c}{n}(a_1 + a_2 + \ldots + a_k) && (5.28)\\
&= \frac{c}{n}(f(x_1) + f(x_2) + \ldots + f(x_k)) && (5.29)\\
&\geq \int_0^{kc/n} f(x)dx && (5.30)\\
&\geq \int_0^{kc/n} f(x)dx && (5.31)\\
&\geq \frac{c}{n}(g(x_0) + g(x_1) + \ldots + g(x_{k-1})) && (5.32)\\
&= \frac{c}{n}(b_1 + b_2 + \ldots + b_k) && (5.33)
\end{aligned}$$

for $k = 1, 2, \ldots, n$.

Let $b_n^* = \sum_{i=1}^n a_i - \sum_{i=1}^{n-1} b_i$. Note that $b_n^* \geq b_n$. We have $\{a_1, a_2, \ldots, a_n\} \succ \{b_1, b_2, \ldots, b_{n-1}, b_n^*\}$. Therefore,

$$\begin{aligned}
\frac{c}{n}\sum_{i=1}^n r(a_i) &\geq \frac{c}{n}\left(\sum_{i=1}^n r(b_i) + r(b_n^*) - r(b_n)\right) && (5.34)\\
&\geq \frac{c}{n}\sum_{i=1}^n r(b_i) && (5.35)
\end{aligned}$$

Let $n \to \infty$ on both sides of (5.35) , we have

$$\lim_{n\to\infty} \frac{c}{n}\sum_{i=1}^n r(a_i) \geq \lim_{n\to\infty} \frac{c}{n}\sum_{i=1}^n r(b_i) \quad (5.36)$$

which is equivalent to

$$\int_0^c r(f(x))dx \geq \int_0^c r(g(x))dx \tag{5.37}$$

■

Given $\mathcal{A}_t$, $t = 1, 2, \ldots$ define the sub-level function

$$\phi_{\mathcal{A}}(x) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{P}[A_t \leq x, (A_t, \mathcal{H}^{t-1}) \in \mathcal{A}_t]$$

Take one of its quasi-inverse, denote as $x_{\mathcal{A}}(\phi)$. Note $\phi_{\mathcal{A}}(x_{\mathcal{A}}(\phi_{\mathcal{A}}(x))) = \phi_{\mathcal{A}}(x)$. Assume both are increasing. Then,

$$\phi_{\mathcal{A}}(\infty) = \lim_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{P}[\mathcal{A}_t] := \mathbb{P}[\mathcal{A}]$$

We note that

$$\begin{aligned} \phi_{\underline{\mathcal{A}}}(x) &\geq \phi_{\mathcal{A}}(x), \quad \text{if } x \in [0, P_0) \\ \phi_{\underline{\mathcal{A}}}(x) &= \phi_{\mathcal{A}}(x), \quad \text{if } x \in [P_0, \infty) \end{aligned}$$

Thus,

$$\begin{aligned} x_{\underline{\mathcal{A}}}(\phi) &\leq x_{\mathcal{A}}(\phi), \quad \text{if } \phi \in [0, \phi_{\mathcal{A}}(P_0)) \\ x_{\underline{\mathcal{A}}}(\phi) &= x_{\mathcal{A}}(\phi), \quad \text{if } \phi \in [\phi_{\mathcal{A}}(P_0), \phi_{\mathcal{A}}(\infty)) \end{aligned}$$

Define

$$f(\phi) = \begin{cases} x_{\mathcal{A}}(\phi) & \phi \in [0, \phi_{\mathcal{A}}(\underline{P}_0)) \\ \underline{P}_0 & \phi \in [\phi_{\mathcal{A}}(\underline{P}_0), \phi_{\mathcal{A}}(\underline{P}_0) + \phi_{\mathcal{A}}(\infty)] \\ x_{\mathcal{A}}(\phi - \phi_{\mathcal{A}}(\infty)) & \phi \in (\phi_{\mathcal{A}}(\underline{P}_0) + \phi_{\mathcal{A}}(\infty), 2\phi_{\mathcal{A}}(\infty)] \end{cases}$$

$$g(\phi) = \begin{cases} x_{\underline{\mathcal{A}}}(\phi) & \phi \in [0, \phi_{\underline{\mathcal{A}}}(P_0)) \\ P_0 & \phi \in [\phi_{\underline{\mathcal{A}}}(P_0), \phi_{\underline{\mathcal{A}}}(P_0) + \phi_{\underline{\mathcal{A}}}(\infty)] \\ x_{\underline{\mathcal{A}}}(\phi - \phi_{\underline{\mathcal{A}}}(\infty)) & \phi \in (\phi_{\underline{\mathcal{A}}}(P_0) + \phi_{\underline{\mathcal{A}}}(\infty), 2\phi_{\underline{\mathcal{A}}}(\infty)] \end{cases}$$

Then, we have the following Lemmas.

**Lemma 15** $f \prec g$ *on* $[0, 2\phi_{\mathcal{A}}(\infty)]$.

**Proof:**

$$\int_0^{2\phi_{\mathcal{A}}(\infty)} f(\phi) d\phi \tag{5.38}$$

$$= \int_0^{\phi_{\mathcal{A}}(\infty)} x_{\underline{\mathcal{A}}}(\phi) d\phi + P_0 \phi_{\mathcal{A}}(\infty) \tag{5.39}$$

$$= \int_0^{\infty} x \mathbf{1}_{\mathcal{A}}(x) p(x) dx + P_0 \phi_{\mathcal{A}}(\infty) \tag{5.40}$$

$$= \mathbb{E}(x|\mathcal{A}) \mathbb{P}[\mathcal{A}] + P_0 \mathbb{P}[\mathcal{A}] \tag{5.41}$$

and

$$\int_0^{2\phi_{\mathcal{A}}(\infty)} g(t) dt = \mathbb{E}(x|\mathcal{A}) \mathbb{P}[\mathcal{A}] + \underline{P}_0 \mathbb{P}[\mathcal{A}] \tag{5.42}$$

and by definition

$$\mathbb{E}(x|\underline{\mathcal{A}}) = \underline{P}_0, \quad \mathbb{E}(x|\mathcal{A}) = P_0 \tag{5.43}$$

Since $f(\phi) \geq g(\phi)$ for $\phi \in [0, \phi_{\underline{\mathcal{A}}}(\underline{P}_0)]$ and $f(\phi) \leq g(\phi)$ for $\phi \in [\phi_{\mathcal{A}}(P_0), 2\phi_{\mathcal{A}}(\infty)]$. Thus,

$$\int_0^{\phi} f(s) ds \geq \int_0^{\phi} g(s) ds \tag{5.44}$$

for the above $\phi$. For $\phi \in [\phi_{\underline{\mathcal{A}}}(\underline{P}_0), \phi_{\mathcal{A}}(P_0)]$, we want to show

$$\int_0^{\phi} x_{\underline{\mathcal{A}}}(s)ds \leq \int_0^{\phi_{\mathcal{A}}(\underline{P}_0)} x_{\mathcal{A}}(s)ds + \underline{P}_0(\phi - \phi_{\mathcal{A}}(\underline{P}_0)) \tag{5.45}$$

which is equivalent to

$$\int_0^{\phi} x_{\underline{\mathcal{A}}}(s)ds + \int_{\phi_{\mathcal{A}}(P_0)}^{\phi_{\mathcal{A}}(\infty)} x_{\underline{\mathcal{A}}}(s)ds - \underline{P}_0(\phi - \phi_{\mathcal{A}}(\underline{P}_0)) \tag{5.46}$$

$$\leq \int_0^{\phi_{\mathcal{A}}(\underline{P}_0)} x_{\mathcal{A}}(s)ds + \int_{\phi_{\mathcal{A}}(P_0)}^{\phi_{\mathcal{A}}(\infty)} x_{\mathcal{A}}(s)ds \tag{5.47}$$

We note that

$$LHS \leq \underline{P}_0\left(\phi_{\mathcal{A}}(\infty) - \phi_{\mathcal{A}}(P_0) + \phi\right) - \underline{P}_0(\phi - \phi_{\mathcal{A}}(P_0)) \tag{5.48}$$

$$= \underline{P}_0\left(\phi_{\mathcal{A}}(\infty) - \phi_{\mathcal{A}}(P_0) + \phi_{\mathcal{A}}(\underline{P}_0)\right) \tag{5.49}$$

since $x_{\underline{\mathcal{A}}}(\phi) \geq P_0$ for $\phi \in [t_{\underline{\mathcal{A}}}(\underline{p_0}), t_{\mathcal{A}}(p_0)]$ and $\mathbb{E}(x|\mathcal{A}) = \underline{P}_0$.

Meanwhile,

$$RHS \geq P_t^0\left(\phi_{\mathcal{A}}(\infty) - \phi_{\mathcal{A}}(P_0) + \phi_{\underline{\mathcal{A}}}(\underline{P}_0)\right) \tag{5.50}$$

since $x_{\mathcal{A}}(\phi) \leq P_0$ for $\phi \in [\phi_{\underline{\mathcal{A}}}(\underline{P}_0), \phi_{\mathcal{A}}(P_0)]$ and $\mathbb{E}(x|\underline{\mathcal{A}}) = \underline{P}_0$. Therefore $f \prec g$.

■

**Lemma 16** *Denote* $\int_{\mathcal{A}} R(\cdot) = \mathbb{E}[R(\cdot)|\mathcal{A}] \cdot \mathbb{P}[\mathcal{A}]$. *We have*

$$\int_0^{2\phi_{\mathcal{A}}(\infty)} R(f(t))dt = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}\left(\int_{\underline{\mathcal{A}}_t} R(\underline{P}_0) + \int_{\mathcal{A}_t} R(A_t)\right)$$
$$\int_0^{2\phi_{\mathcal{A}}(\infty)} R(g(t))dt = \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T}\left(\int_{\mathcal{A}_t} R(P_0) + \int_{\underline{\mathcal{A}}_t} R(A_t)\right)$$

**Proof:** First, the integrals are

$$\int_0^{2\phi_{\underline{\mathcal{A}}}(\infty)} R(f(\phi))d\phi \tag{5.51}$$

$$= \int_0^{\phi_{\underline{\mathcal{A}}}(P_0)} R(x_{\underline{\mathcal{A}}}(\phi))d\phi + \int_{\phi_{\underline{\mathcal{A}}}(P_t^0)}^{\phi_{\underline{\mathcal{A}}}(P_0)+\phi_{\underline{\mathcal{A}}}(\infty)} R(P_0)d\phi + \int_{\phi_{\underline{\mathcal{A}}}(P_0)+\phi_{\underline{\mathcal{A}}}(\infty)}^{2\phi_{\underline{\mathcal{A}}}(\infty)} R(x_{\underline{\mathcal{A}}}(\phi - \phi_{\underline{\mathcal{A}}}(\infty)))dt \tag{5.52}$$

$$= \int_0^{\phi_{\underline{\mathcal{A}}}(P_0)} R(x_{\underline{\mathcal{A}}}(\phi))d\phi + R(P_0)\phi_{\underline{\mathcal{A}}}(\infty) + \int_{\phi_{\underline{\mathcal{A}}}(P_t^0)}^{\phi_{\underline{\mathcal{A}}}(\infty)} R(x_{\underline{\mathcal{A}}}(\phi))dt \tag{5.53}$$

$$= \int_0^{\phi_{\underline{\mathcal{A}}}(\infty)} R(x_{\underline{\mathcal{A}}}(\phi))d\phi + R(P_t^0)\phi_{\underline{\mathcal{A}}}(\infty) \tag{5.54}$$

$$= \int_0^{\infty} R(x)\mathbf{1}_{\underline{\mathcal{A}}}(x)p_A(x)dx + \int_0^{\infty} R(P_0)\mathbf{1}_{\mathcal{A}}(x)p(x)dx \tag{5.55}$$

where $x_{\underline{\mathcal{A}}}(\phi) = x_{\mathcal{A}}(\phi)$ when $\phi \geq \phi_{\mathcal{A}}(P_t^0)$.

Similarly,

$$\int_0^{2\phi_{\mathcal{A}}(\infty)} R(g(\phi))d\phi = \int_0^{\infty} R(x)\mathbf{1}_{\mathcal{A}}(x)p_A(x)dx + \int_0^{\infty} R(\underline{P}_0)\mathbf{1}_{\underline{\mathcal{A}}}(x)p_A(x)dx \tag{5.56}$$

■

In order to show $\mathbb{E}(R(\mathcal{Q})) \leq \mathbb{E}(R(\underline{\mathcal{Q}}))$, it suffices to show that

$$\mathbb{E}[R(\mathcal{Q})] - \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}[R(A_t)] \leq \mathbb{E}[R(\underline{\mathcal{Q}})] - \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}[R(A_t)] \tag{5.57}$$

We note that under $\mathcal{Q}$, we have $P_t = A_t$ if $(A_t, H^{t-1}) \notin \mathcal{A}_t$; Similarly, under $\underline{\mathcal{Q}}$, we have $\underline{P}_t = A_t$ if $(A_t, H^{t-1}) \notin \underline{\mathcal{A}}_t$. Thus, (5.57) is equivalent to

$$\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \int_{\mathcal{A}_t} R(P_t) - R(A_t) \leq \lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \int_{\underline{\mathcal{A}}_t} R(\underline{P}_t) - R(A_t)$$

i.e.,

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\left(\int_{\mathcal{A}_t}R(P_t)+\int_{\underline{\mathcal{A}}_t}R(A_t)\right)\leq\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\left(\int_{\underline{\mathcal{A}}_t}R(\underline{P}_t)+\int_{\mathcal{A}_t}R(A_t)\right)$$

which is then true due to Lemma 13, the definition of $\underline{P}_t$, Lemma 15 and Lemma 16.

Similarly, we can show that $\mathbb{E}(R(\mathcal{Q}))\leq\mathbb{E}(R(\bar{\mathcal{Q}}))$. Therefore, the optimal policy must have the double-threshold structure specified in Theorem 14.

# Chapter 6: Conclusions

In this dissertation, we investigated optimal sensing and transmission policies for EH sensor networks under stochastic energy constraints imposed by the EH processes at the sensors.

We first considered the optimal online sensing scheduling policy for an energy harvesting sensing system. We first provided a lower bound on the time averaged sensing performance for the system with infinite battery, and showed that this lower bound can be achieved by a best-effort uniform sensing policy. We then investigated the finite battery case and proposed an energy-aware adaptive sensing scheduling policy, which dynamically varies the sensing frequency based on instantaneous energy level of the battery. We showed that the battery outage and overflow probabilities under the proposed policy approach zero as battery size goes to infinity, and the time averaged sensing performance converges to the lower bound when the battery size increases. Thus the adaptive sensing scheduling policy is asymptotically optimal. The convergence rates as a function of the battery size were also explicitly characterized. Simulation results validated the theoretical bounds.

We then consider the optimal status updating to minimize age of information with an energy harvesting source. The objective is to minimize the long-term time average AoI within the energy causality constraint at the sensor. We considered sensor's battery size with three different scenarios. For the infinite battery, we showed a best-effort uniform status update policy is optimal. For the finite battery, we proposed an energy-aware adaptive state update policy, and we proved that it is asymptotically optimal. When the battery size is one unit, we proposed a threshold-based status update policy. We derived the analytic result for the time average AoI under this policy, and proved that it outperforms any other online status update policy in this extreme scenario, thus it is optimal.

Next, we considered optimal collaborative sensing scheduling policies in a sensor network powered by energy harvested from the nature. The objective is to maximize the time average utility generated by the sensors under the energy causality constraints at individual sensors, where the utility function is assumed to be concave and monotonically increasing in the number of active sensors in each slot. We considered both offline and online settings. Under the offline setting, we first noted that the optimal sensing policy has a "majorization" structure, and then proposed an algorithm to identify the scheduling policy satisfying the energy causality constraints at individual sensors and the structural requirements of the optimal policy. Under the online setting, we first proved that the long-term expected time average utility generated under any feasible policy has an upper bound. Then, we proposed a randomized myopic policy, and showed that as $T$ approaches infinity, the expected time average utility generated under the policy converges to the upper bound almost surely, thus it is optimal. The corresponding convergence rate was also explicitly characterized.

Finally, we studied the optimal energy management policy for an EH communication system under a battery usage constraint. We imposed an long-term average cost constraint on the battery, which is translated to the average number of charge/discharge operations per unit time. The objective was to develop an online policy to maximize the long-term average throughput of the transmitter under energy causality constraint and the battery usage constraint. We first relaxed the energy causality constraint on the system, and imposed an energy flow conservation constraint instead. We showed that under the relaxed setting, the optimal energy management policy has a double-threshold structure. We then modified the double-threshold policy slightly to accommodate the energy causality constraint, and analyzed its long-term performance. We showed that the system achieves the same long-term average performance, thus it is optimal.

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