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Configuring Traditional Multi-Dock, Unit-Load Warehouses

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Configuring Traditional Multi-Dock, Unit-Load Warehouses

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by

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Abstract

The development of expected-distance formulas for multi-dock-door, unit-load warehouse configurations is the focus of the dissertation. From formulations derived, the width-to-depth ratios minimizing expected distances are obtained for rectangle-shaped, unit-load warehouse configurations. Partitioning the storage region in the warehouse into three classes, the performance of a multi-dock-door, unit-load warehouse is studied when storage regions can be either rectangle-shaped or contour-line-shaped.

Our first contribution is the development of formulas for expected distance traveled in storing and retrieving unit loads in a rectangle-shaped warehouse having multiple dock doors along one warehouse wall and storage racks aligned perpendicular to that wall. Two formulations of the optimization problem of minimizing expected distance are considered: a discrete formulation and a continuous formulation with decision variables being the width and depth of the warehouse for single- and dual-command travel. Based on dock door configurations treated in the literature and used in practice, three scenarios are considered for the locations of dock doors: 1) uniformly distributed over the entire width of a wall; 2) centrally located on a wall with a fixed distance between adjacent dock doors; and 3) not centrally located on a wall, but with a specified distance between adjacent dock doors.

Our second contribution is the investigation of the effect on the optimal width-to-depth ratio (shape factor) of the number and locations of dock doors located along one wall or two adjacent walls of the warehouse. Inserting a middle-cross-aisle in the storage area, storage racks are aligned either perpendicular or parallel to warehouse walls containing dock doors. As with the warehouse having storage racks aligned perpendicular to the warehouse wall, discrete and

continuous formulations of the optimization problem are developed for both single- and dual-command travel and three scenarios for dock-door locations are investigated.

Our final contribution is the analysis of the performance of a unit-load warehouse when a storage region or storage regions can be either rectangle-shaped or contour-line-shaped. Particularly, we consider two cases for the locations of dock doors: equally spaced over an entire wall of the warehouse and centrally located on a wall, but with a specified distance between adjacent dock doors. Minimizing expected distance, the best rectangle-shaped configuration is determined and its expected distance is compared with the expected distance in its counterpart contour-line-shaped configuration.

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Dedication

To my beloved parents

Mr. Hakki Tutam & Mrs. Vildan Tutam

and

To my dear sisters

Miss. Sumeyya Tutam & Mrs. Melek Tutam

and

To my little brother & my best friend

Mr. M. Emin Tutam

for their enormous personal sacrifices and unconditional loves.

This humble study is a sign of my love to you!

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Abbreviations

SC	=	single-command travel
$E[SC]$	=	expected single-command distance
TB	=	travel-between-distance
$E[TB]$	=	expected travel-between distance
DC	=	dual-command travel ($DC = SC + TB$)
$E[DC]$	=	expected dual-command distance ($E[DC] = E[SC] + E[TB]$)
S/R	=	Storage/retrieval
AS/RS	=	automated storage and retrieval systems
I/O	=	input/output
P&D	=	pickup and deposit

List of Publications

Tutam, M. and White, J. A. (in press) A multi-dock unit-load warehouse design, *IISE Transactions*. doi: 10.1080/24725854.2018.1488307.

Chapter 1

Introduction

In supply chain and logistics systems, unit-load warehouses have played a critical role for decades in decreasing costs and reducing response times for demands. Although unit-load warehouses typically have multiple dock doors for receiving and shipping, most researchers have based their calculations on an assumption of a single dock door located at the centerline of one wall of a rectangle-shaped warehouse. Relaxing the single-dock-door assumption results in more realism to the research. Likewise, relaxing the centrally-located-dock-door(s) assumption provides flexibility for the locations of dock doors when additional space is needed.

In incorporating multiple dock doors in the design of the warehouse, designers need to understand the impact of having more dock doors than necessary. Not only does having more than the necessary number of dock doors increases equipment costs, it also increases the expected distance traveled in storing and retrieving unit loads. Therefore, the analytical models we develop for multi-dock-door, unit-load warehouses should provide beneficial insights for designers.

Relaxing assumptions to produce a more accurate representation of reality can reveal new design opportunities. Recent studies show innovative aisle designs improve the performance of a rectangle-shaped, unit-load warehouse by reducing expected distance traveled. Rectangle-shaped warehouse design is another implicit assumption used in warehouse design. Developing formulas for a contour-line-shaped warehouse provides a lower bound for expected-distance calculations and reveals the penalty of requiring the unspoken design rule that the shape be rectangular.

This research focuses on developing expected-distance formulations for single- and dual command travel in traditional unit-load warehouse designs having multiple dock doors along one wall or two adjacent walls of the warehouse. Defining shape factor as the width-to-depth ratio for a unit-load warehouse, from the formulas derived, shape factor values minimizing expected distances are obtained in Chapters 2 and 3. Moreover, Chapter 3 compares the performance of three traditional layout designs; also, features of two of the three designs are combined to obtain a fourth layout design. In Chapter 4, the performances of rectangle-shaped warehouses are analyzed and compared with contour-line shaped warehouses considering randomized and class-based storage policies. Computational results are provided in each chapter. In Chapter 5, research finding are summarized, design conclusions are drawn, recommendations for further research are given and suggestions are provided concerning the application of the research results in designing unit-load warehouses.

In Chapter 2, single- and dual- command expected-distance formulas are developed for a traditional warehouse design having storage racks aligned perpendicular to the warehouse wall on which *k* dock doors are located. Based on dock door configurations treated in the literature and used in practice, three scenarios are considered for the locations of *k* dock doors: 1) dock doors are dispersed over an entire warehouse wall; 2) dock doors are symmetrically located about the centerline of a warehouse wall with a specified distance between adjacent dock doors; and 3) dock doors are not centrally located, but a specified distance exists between the leftmost wall and the nearest dock door and a fixed distance exists between adjacent dock doors. In developing discrete formulations for expected-distances traveled, a formulation of a nonlinear-integer-programming optimization problem is presented. Moreover, in order to obtain closed-form expressions facilitating sensitivity analyses and to avoid the use of a specialized software package, a general formulation of the nonlinear, convex-programming optimization problem is provided by employing expected-distance approximations. Theorems, propositions and corollaries are included for continuous approximations. Optimization formulations are solved using specified values of parameters, and results are provided. In addition, a given set of parameter values are tested to examine the percentage error for continuous approximations. Because continuous formulations provide reliable results for both single- and dual-command travel, optimal shape factor values are determined for each scenario by using continuous approximations.

In Chapter 3, optimization problems are considered similar to those of Chapter 2, but for three additional layout configurations. The first design is obtained by inserting a cross aisle in the "middle" of the design described in Chapter 2. Rotating the storage racks and middle-cross-aisle in the first design, the second design is obtained. Moreover, the optimal shape factor formulations for two designs including a middle-cross-aisle are provided. Investigating the effect on the optimal shape factor of the number and locations of dock doors located along two adjacent warehouse walls, the third design is introduced by combining features of the first and second designs. As with Chapter 2, formulations of optimization problems are developed for the same dock-door-location scenarios and solved for both single- and dual-command travel. Comparing all designs, the performances of warehouse designs are compared, based on an equal number of S/R locations. Allowing shipping dock doors to be located along one wall and receiving dock doors to be located along an adjacent wall of the warehouse, results are provided for expected distance and the optimal shape factor for the fourth design. Additionally, considering a mixture of single-command, dual-command and cross-docking travel, three scenarios are considered: 1) single-command focused warehouse, 2) dual-command focused warehouse and 3) cross-docking focused warehouse.

Comparisons of the performance of a rectangle-shaped, unit-load warehouse with a contour-line-shaped unit-load warehouse under a randomized storage policy are provided in Chapter 4. Furthermore, dividing the unit-load warehouse into three different storage regions (ABC class-based storage policy) and using continuous formulations from Chapter 2, expected single-command-distance formulas are derived for each region and the best rectangle-shaped configuration is determined. Because contour lines determine the shape of each storage region, expected-distance formulas are developed for contour-line-shaped storage regions by using a special case of the Neyman-Pearson Lemma employed by Francis (1967). Therefore, the expected distance in the best rectangle-shaped configuration is compared with the expected distance in its counterpart contour-line-shaped configuration and the penalty of requiring the storage regions to be rectangle-shaped is calculated. Different skewness levels are examined by using Bender’s formulation (Bender, 1981) to illustrate the effect of ABC curve shapes on the penalty of requiring the warehouse to be rectangle-shaped.

Chapter 5 includes a summary of the dissertation and design conclusions drawn, as well as suggestions regarding the use of the research results in designing unit-load warehouses. Recommendations for further study are also provided.

Bibliography

Bender, P. S. (1981) Mathematical modeling of the 20/80 rule: Theory and practice. *Journal of Business Logistics*, 2(2), 139-157.

Francis, R. L. (1967) Sufficient conditions for some optimum property facility designs. *Operations Research*, 15(3), 448-466.

Chapter 2

Contribution 1: A Paper on, “A Multi-Dock, Unit-Load Warehouse Design”

Abstract

Expected-distance formulations are developed for a rectangle-shaped, unit-load warehouse having dock doors along one warehouse wall. Based on dock-door configurations treated in the literature and/or used in practice, three scenarios are considered: 1) equally spaced dock doors spanning a wall, 2) equally spaced dock doors with a specified distance between adjacent dock doors, and an equal number of dock doors located on each side of the wall’s centerline, and 3) equally spaced dock doors with a specified distance between adjacent dock doors and the first dock door located a given distance to the right of the left wall. Defining shape factor as the warehouse width divided by its depth, the shape factor minimizing expected distance is determined. Single- and dual-command travel results from discrete formulations are compared with results from closed-form expressions using continuous approximations. The optimal shape factor depends on the number and locations of dock doors. When the distance between adjacent dock doors is a function of the warehouse’s width, previous research results are confirmed. However, when distances between adjacent dock doors are specified, our results differ from a commonly held belief the optimal shape factor is always less than or equal to 2.0.

Keywords: Multiple dock doors, Shape factor, Unit-load, Single-command, Dual-command.

2.1. Introduction

In today's business environment, to increase profit margins, companies are reluctant to increase prices in order to improve service levels for their customers. Additionally, customers demand next-day or same-day delivery of orders placed. Therefore, companies focus on decreasing costs to gain competitive advantage and reduce response times in order to provide better service. Both conditions result in a need to reduce the time to store and retrieve products in warehouses. Material flow is a primary consideration in designing warehouses.

A variety of facilities with a common identification, *warehouse* or *distribution center*, play a critical role in today's supply and distribution networks by facilitating and speeding up movements of products between manufacturers and customers, as well as reducing costs of operations. The design of the network includes decisions regarding the number, sizes and locations of distribution centers. Due to a vast number of design alternatives and uncertainty of demands, designing and managing a distribution center or warehouse can be a complex task with multiple conflicting objectives such as minimizing operating cost and minimizing capital investment. Alternatively, depending on the warehouse mission, the design objective can be the minimization of the maximum time required to retrieve products in the warehouse or the maximization of the probability the time to store or retrieve a unit load is less than an aspiration level.

Activities most commonly performed in a warehouse include receiving, staging, storing, retrieving, order picking, and shipping. Because 20-50% of total operating cost consists of transporting products, operating cost can be reduced by 10-30% by minimizing expected distance (Bartholdi and Hackman, 2014). Including storage and retrieval operations, the storage function is a key component of warehouses. Because much of a warehouse worker's time is spent traveling between dock doors and storage/retrieval (S/R) locations, the storage function is one of the most labor intensive and costly material flow activities.

As noted in the title of the chapter, we limit our attention to the design of a facility for storing and retrieving unit loads of product(s): a *unit-load warehouse*. Specifically, we limit the storage of unit loads to selective single-deep pallet rack (Tompkins *et al*., 2010) installed perpendicular to the wall containing the dock door(s). Consistent with the research literature treating the design of unit-load warehouses, we employ the design objective of minimizing expected distance traveled between dock doors and storage locations.

Francis (1967a) showed that locating a single dock door at the centerline of the wall containing the dock door will minimize expected distance between the dock door and uniformly distributed S/R locations in the rectangular storage region. Thereafter, researchers studying unit-load warehouses having traditional layouts with storage racks installed perpendicular to a given warehouse wall have tended to limit their studies to having a single dock door located at the centerline of a warehouse wall. However, warehouses typically have multiple dock doors.

Bassan *et al*. (1980) concluded dock doors should be located as near as possible to the centerline of the warehouse if a unit-load warehouse has multiple dock doors. Apparently, increasing the number of dock doors results in locating them farther from the centerline of the warehouse when dock doors either equally spaced over an entire wall (Scenario 1) or equally spaced with a specified distance between adjacent dock doors (Scenario 2); therefore, it increases expected distance between dock doors and S/R locations. With the objective of minimizing expected distance, using multiple dock doors is not a good choice. An important question arises as to what would be the advantage of using multiple dock doors.

Because the cost of installing a dock door in the wall of a warehouse when it is built is substantially less than the cost of adding a dock door after the warehouse is built, it is quite common for warehouse designers to space dock doors over an entire wall of the warehouse. While it might be less expensive to construct warehouses in this way, it can result in significantly greater travel distances if, in fact, the throughput requirements for the warehouse do not justify having the number of dock doors provided. Specifically, the required number of dock doors is determined by the time between truck arrivals, the number of trucks served over a period of time (day, week or season) and the average time for loading or unloading. Although having fewer dock doors than the required number results in decreasing expected distance traveled, it creates congestion; therefore, extra waiting time for S/R equipment results. However, little research has been performed regarding the degree to which expected distance increases when dock doors are added to meet the throughput requirements of the warehouse. Likewise, the impact on expected distance of various locations of dock doors has not been well-studied.

From an expected-distance perspective, an optimal number of dock doors can be determined when dock doors are equally spaced with a specified distance between adjacent dock doors and the first dock door located a given distance to the right of the left wall (Scenario 3). As noted previously, because the number of dock doors is generally based on throughput requirements, the number of dock doors is a parameter, not a decision variable. (If it were a decision variable, a single-dock-door warehouse would be recommended, assuming throughput requirements are met.)

Francis (1967a) showed, to minimize expected rectilinear distance, the width of the warehouse wall containing the dock doors should be twice the depth of the warehouse. Interestingly, warehouse designers have tended to employ a “rule of thumb” that the warehouse shape factor (width-to-depth ratio) should be equal to 2.0, regardless of the number of dock doors located along the warehouse wall.

Given widespread industry practice to design warehouses twice as wide as they are deep and to have dock doors over an entire warehouse wall, we sought to answer the following questions:

1. What impact does warehouse shape factor have on expected distance between dock doors and S/R locations in a unit-load warehouse?
2. What impact does the number of dock doors have on expected distance?
3. What impact does the number of dock doors have on the optimal warehouse shape?
4. What impact does the location of dock doors have on expected distance?
5. What impact does the location of dock doors have on the optimal warehouse shape?

To answer our questions, first, we develop a formulation of the optimization problem with discrete formulations by considering the number of S/R aisles and the number of S/R locations along one side and one level of an S/R aisle as decision variables. The formulation includes discrete formulas of distances between dock doors and S/R locations, as well as between S/R locations; travel is restricted to an orthogonal set of S/R aisles and cross-aisles. Thereafter, because the optimal shape factor with the discrete formulations cannot be easily determined, we obtain closed-form formulas by employing a continuous approximation with decision variables being the width and depth of the warehouse. Particularly, the warehouse is treated as a continuous region; expected distance is measured rectilinearly between dock doors and S/R locations, and the locations of S/R racks and aisles are ignored for single-command travel. However, a continuous approximation of the discrete formulation for travel-between distance is employed in expected dual-command distance formulations because continuous space formulas underestimate expected distance between two S/R locations when two S/R locations are in different S/R aisles, resulting in an error of approximately 31.69% for a particular set of parameter values (given in Section 2.7).

In storing and retrieving unit loads, single- and dual-command travel can occur. Single-command travel occurs when S/R equipment transports a unit load from a dock door to a storage location and returns (empty) to the dock door or S/R equipment travels (empty) from a dock door to a retrieval location and transports a unit load to the dock door. Dual-command travel occurs when S/R equipment transports a unit load from a dock door to a storage location, travels (empty) to a retrieval location, and transports a unit load to the dock door. (The distance between storage and retrieval locations is called *travel-between distance*.)

We limit our analysis to planar travel; hence, distances to S/R positions in upper levels of the S/R racks are not included. Therefore, in developing formulations, two dimensions of planar travel are considered: *horizontal travel* and *vertical travel*. Horizontal travel occurs when S/R equipment travels parallel to the wall containing dock doors. Vertical travel occurs when S/R equipment travels perpendicular to the wall containing dock doors.

We assume dock doors are equally likely to be selected for travel to or from S/R locations and S/R locations are equally likely to be visited within the storage region. Expected distance for S/R equipment traveling along the orthogonal set of S/R aisles and cross-aisles is the sum of expected vertical and horizontal roundtrip-distances. Notice, because dock doors are located along a single wall, neither the number nor the locations of dock doors affects expected vertical distance or expected travel-between distance. S/R aisles are used to access S/R locations; cross-aisles are used to move between S/R aisles. We assume S/R aisles are wide enough for 2-way travel to occur and for S/R equipment to access either side of the aisle in storing or retrieving a unit load.

The remainder of the chapter is organized as follows. In the subsequent section, unit-load warehouse design literature is reviewed. In Section 2.3, the notation employed in discrete and continuous formulations is provided. Removing the single-dock-door constraint, Section 2.4 addresses three basic scenarios regarding the number and locations of dock doors. Section 2.5 provides discrete formulations for expected distance and develops integer-programming models for the scenarios. In Section 2.6, expected-distance approximations are developed and closed-form expressions for the optimal shape factor are provided. (Both single-command and dual-command operations are considered in Sections 2.5 and 2.6). In section 2.7, the accuracy of the continuous approximations is tested based on a set of parameter values and the effects of the scenarios on expected distance and optimal shape factor are examined and compared for a particular set of parameter values. In Section 2.8, findings from the research are summarized, conclusions are drawn and recommendations for future research are provided. Finally, proofs of theorems, corollaries, and propositions, as well as tables of computational results, are provided in the Appendix.

2.2. Literature Review

A vast body of research exists addressing how to design a warehouse with specific assumptions and limitations. Earlier studies focused on two well-known warehouse types: *unit-load warehouses* and *order-picking warehouses.* Our focus is on unit-load warehouses. Furthermore, we limit our review to literature treating traditional aisle structures (an orthogonal set of S/R aisles and cross-aisles).

The first formulation of single-command travel for a unit-load warehouse was provided by Francis (1967a). He concluded a shape factor of 2.0 minimizes expected single-command distance when a single dock door is located at the mid-point of a wall. Subsequently, Francis

(1967b) provided sufficient conditions for warehouse designs having a single dock door to minimize expected rectilinear distance between the dock door and uniformly distributed S/R locations.

Mallette and Francis (1972) extended Francis' earlier studies to include discrete space formulations by treating the facility design problem as a generalized assignment problem with storage areas in the plane being composed of *n* grid squares. Francis and White (1974) employed contour lines to obtain warehouse designs when travel is based on rectilinear, Euclidean, Chebyshev, and squared-Euclidean metrics. Treating the warehouse as a continuous space, they developed formulations to minimize expected single-command distance and determined the optimal shape of the storage region. Our research extends the work of Francis and Mallette to include multiple dock doors and a variety of locations of the dock doors along a single wall. In addition, rather than allow the storage region to be contour-line shaped, we limit our attention to rectangle-shaped warehouses.

Assuming unit loads are received on one side of the warehouse while shipping occurs on the opposite side of the warehouse, Bassan *et al*. (1980) considered storage racks and determined the best alignment of S/R aisles. They concluded a multi-dock-door, unit-load warehouse should have its dock doors located as near as possible to the centerline of the warehouse. However, they did not indicate how multiple dock doors and their locations affect the optimal shape of the warehouse; our research addresses both the number and locations of dock doors.

Mayer (1961) is credited with coining the term, dual-command. He evaluated the performance of a single-dock-door warehouse with dual-command travel and found it increases output per unit time. He concluded the optimal depth of a warehouse is less than the width of the warehouse when dual-command travel is used. We develop both single-command and dual-command formulations of expected distance when multiple dock doors are included in the warehouse design and determine the width-to-depth ratio of the storage area that minimizes expected distances.

Most studies related to dual-command travel have focused on analyzing automated storage and retrieval systems (AS/RS) with interleaving which combines a storage operation with a retrieval operation in a dual-command operation cycle. For a detailed survey of literature on AS/R systems with interleaving, see Malmborg and AlTassan (2000) and Roodbergen and Vis (2009). In contrast to the AS/RS related literature, our research does not employ Chebyshev distance metrics; likewise, we do not limit our research to a single dock door or input/output (I/O) point.

Pohl *et al*. (2009) appear to be the first to analyze dual-command travel in traditional unit-load warehouse layouts. Assuming a centrally located dock door and defining distance between two random points in the warehouse as travel-between (*TB*), they developed expected dual-command distance formulas. They also confirmed the conclusions of Francis (1967a) and Bassan *et al*. (1980) regarding the optimal location of a single dock door with single-command travel. Pohl *et al*. (2009) acknowledged the optimal shape factor is approximately the same for both single- and dual-command travel for the layout in Figure 2.1 (left). Drawing on their recommendations for future research, the influence multiple dock doors and dock-door locations have on expected distance and the optimal layout is examined in this chapter.

Considering a single shipping and a single receiving dock door, Ang *et al*. (2012) developed a robust optimization model for the storage assignment problem in a unit-load warehouse. Particularly, they considered a factor-based demand model in which demand of each product in each period depends on uncertain factors. Taking into account the variability of product flow and the capacity constraints of storage classes, they obtained a storage-retrieval policy for a moderate-size problem under a restricted linear decision rule.

Thomas and Meller (2014) investigated the impact on optimal shape factor of dock doors being uniformly distributed across an entire wall of the warehouse. They concluded the optimal shape factor is 1.5 when an infinite number of dock doors are located over the entire wall, but the optimal shape factor is 2.0 with a single centrally located dock doors. As illustrated in Figure 2.1 (right), our research extends their work by considering a specified number of dock doors and/or fixed distances between adjacent dock doors. In addition, we do not require dock doors to be located symmetrically with respect to the centerline of the wall containing dock doors.

Recently, a different version of the expected-distance formulation was introduced by Tutam and White (2015). Specifically, the number of dock doors and the distances between adjacent dock doors were specified. They showed the effect on expected distance of having multiple dock doors considering multiple scenarios for single-command travel. Without taking into account the width constraint, they showed the impact a limited but feasible number of dock doors has on the optimal shape factor. They derived expected dual-command distance formulas for Scenario 2. Using their expressions and introducing space and width constraints, we develop discrete formulation of the optimization problem and closed-form expressions for single-command travel. Unlike Tutam and White (2015), we develop dual-command travel formulas for all scenarios under space and width constraints. Moreover, among the contributions of this chapter, theorems and propositions are included. Therefore, our study extends those of Francis (1967a), Pohl *et al*. (2009), Thomas and Meller (2014), and Tutam and White (2015).

In summary, the major contributions of this chapter are a) formulations for three scenarios of dock-door locations and single- and dual-command travel of a nonlinear discrete optimization problem, and b) closed-form expressions for the optimal shape factor for continuous formulations. We address the five questions previously posed and show the effect on expected distance of shape factor, number of dock doors, and locations of dock doors for single- and dual-command travel.

2.3. Notation

The notation depicted in Figure 2.1 (right) and/or employed in this chapter is defined as follows:

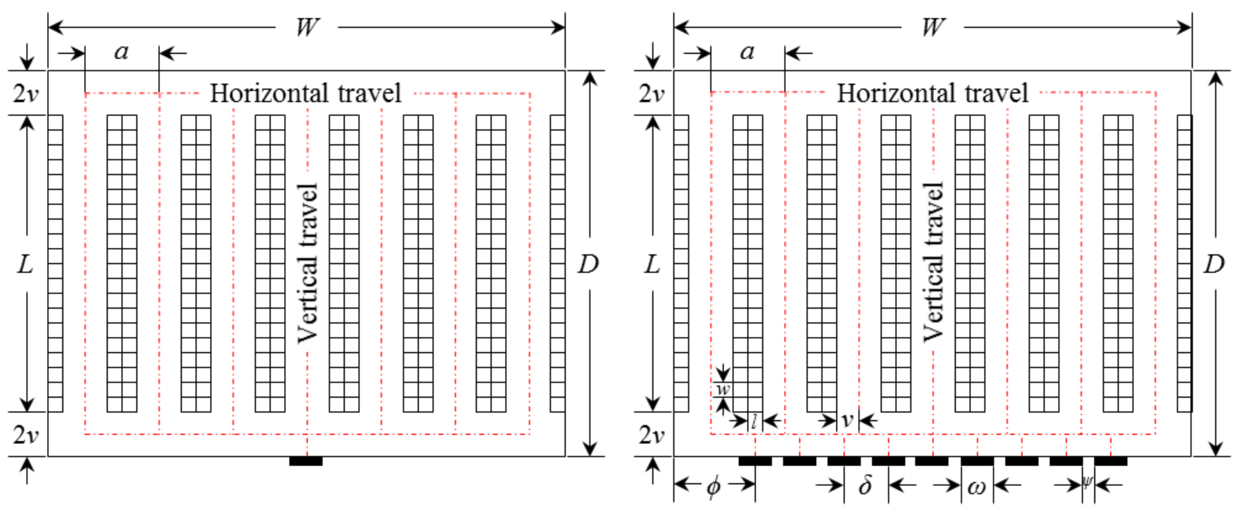

Figure 2.1: Single-dock (left) and multi-dock (right) unit-load warehouses and notation

l	=	the length or depth of an S/R location
w	=	the width of an S/R location
m	=	number of S/R locations along one side and one level of an S/R aisle
L	=	length of an S/R aisle ($L = w\,m$)
v	=	half the width of a cross-aisle
D	=	depth of the warehouse ($D = L + 4v = w\,m + 4v$)
a	=	distance between centerlines of adjacent aisles ($a = 2\,(l + v)$)

n = number of S/R aisles

W = width of the warehouse ($W = n\,a$)

A = the minimum total storage area required ($A \leq W\,D$ for discrete formulations, $A = W\,D$ for continuous formulations)

S = shape factor ($S = W / D$)

k = number of dock doors

d_i = the horizontal distance between dock door i and the left wall

t_i = the horizontal distance between the left end of the wall containing dock doors and the centerline of the back-to-back rack closest to dock door i ($t_i = a \times ROUND\,[d_i / a, 0]$)

ω = the width of a dock door

ψ = the clearance between adjacent dock doors

δ = the distance between centerlines of two adjacent dock doors (i.e. i^{th} and $(i+1)^{th}$ dock doors) ($\delta = \omega + \psi$)

ϕ = the distance between the left end of the wall and the leftmost dock door

c_i = i^{th} constant value

$E\,[SC]$ = expected single-command distance

$E\,[TB]$ = expected travel-between distance

$E\,[DC]$ = expected dual-command distance ($E\,[DC] = E\,[SC] + E\,[TB]$)

Superscripts D and C denote expected distance for discrete formulations and continuous approximations, respectively. Subscripts h and v denote expected distance for horizontal and vertical travel, respectively.

2.4. Basic Scenarios

Allowing multiple dock doors to be located along a given wall leads to numerous scenarios regarding the number and locations of dock doors. We consider three scenarios (see Figure 2.2) based on the literature and/or existing warehouse designs. Although 3 dock doors are shown in Figure 2.2, our formulations are valid for both an even and an odd number of dock doors.

In the first scenario (see Figure 2.2.*a*.), dock doors are equally spaced over an entire wall of the warehouse; the scenario is commonly treated in the research literature, but is not commonly incorporated in the design of unit-load warehouses. We consider the first scenario in order to compare the results of our research with the results of previous studies.

In the second scenario (see Figure 2.2.*b*.), dock doors are located with a fixed distance between adjacent dock doors. In addition, dock doors are located symmetrically about the centerline of one wall of the warehouse; locating dock doors with a specified separation distance occurs commonly in practice. A motivation for Scenario 2 is that clustering dock doors in the center of the warehouse wall is the best location for dock doors in terms of minimizing distance between dock doors and S/R locations (Bassan *et al*., 1980). Also, using Scenario 2 “frees up” larger sections of space along each end of the wall for other purposes, such as providing ground level access to the facility, providing access for first responders, and having dock doors specifically used for waste removal, equipment delivery, and receipt of products from other than over-the-road trailers. Spreading dock doors out more than necessary increases expected distance between dock doors and S/R locations; it also can result in operational inefficiencies and duplication of equipment in loading and unloading over-the-road trailers.

In Scenario 3 (see Figure 2.2.*c*.), the first dock door is located a given distance to the right of the left wall and a fixed distance exists between adjacent dock doors. The third scenario relaxes the centrally located dock door(s) assumption and provides more flexibility for the locations of dock doors. The third scenario can occur when additional storage space is needed without requiring the addition of dock doors and an existing warehouse is expanded by extending its width in one direction. In addition, site topography might preclude having adequate apron and staging space for trucks across the entire width of the warehouse, necessitating a concentration of dock doors toward the end of the warehouse wall.

Another situation that can result in Scenario 3 is the conversion to a storage facility of a building originally used for other purposes; the dock doors are already in place and the number is adequate for the throughput requirement. In such a case, our formulations can be used to determine the optimum shape factor for the storage area within the existing building.

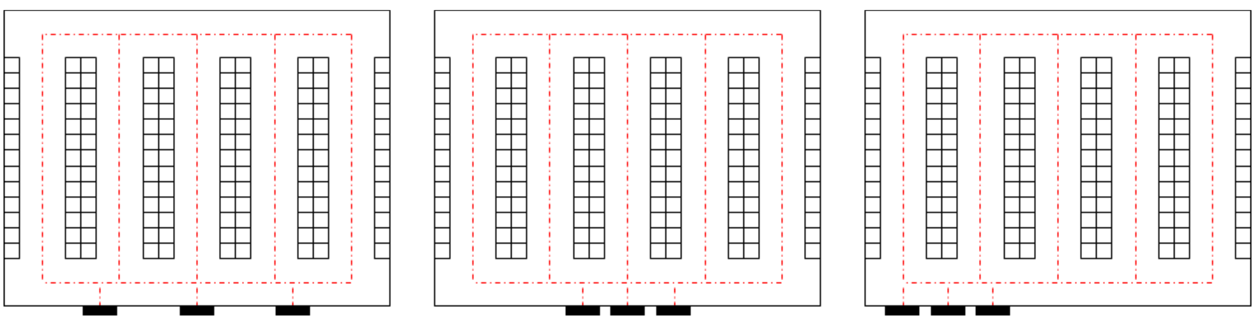

Figure 2.2: *a*) k = 3 dock doors are equally spaced along one wall of the warehouse, *b*) k = 3 dock doors are centrally located on the wall with a specified distance between adjacent dock doors, and *c*) k = 3 dock doors are not centrally located along a wall with a specified distance between adjacent dock doors

2.5. Discrete Formulations

In this section, we develop discrete expected-distance formulations by measuring the distance between the centerline of a dock door and the centerline of an S/R location and between centerlines of two S/R locations. The optimal number of S/R aisles (n^*) and S/R locations in each S/R aisle (m^*) are determined. Hereafter, the location of a dock door and the location of an

S/R aisle refer to the locations of the centerline of a dock door and the centerline of an S/R aisle, respectively.

Because the separation between adjacent dock doors and the alignment of storage aisles with dock doors can vary, depending on the width of S/R aisles and the distance between adjacent dock doors, we measure the distance from the left-end of the wall containing dock doors. Hence d_i is the horizontal distance between dock door i and the left-end of the wall. We number dock doors from left to right, with dock door i being the i^{th} dock door to the right of the left-end of the wall. Because d_i differs among scenarios, different equations are used to calculate its value for each scenario.

Obtaining the spacing between adjacent dock doors for Scenario 1, with k dock doors, the width of the warehouse is divided into $(k + 1)$ equal-sized segments. Therefore, the distance between the left-end of the wall and the leftmost dock door (d_1) is $W / (k + 1)$ and the distance between adjacent dock doors is $W / (k + 1)$. Hence, the distance between the left-end of the wall and dock door i for Scenario 1 is

$$d_i = W / (k + 1) + [W (i - 1)] / (k + 1) = (i\, W) / (k + 1). \tag{2.1}$$

Because the spacing between adjacent dock doors is a fixed distance (δ) for Scenario 2, the distance from the left-end of the wall to the leftmost dock door is $[W - (k - 1)\, \delta] / 2$. Hence, the distance between the left-end of the wall and dock door i is

$$d_i = [W - (k - 1)\, \delta] / 2 + (i - 1)\, \delta. \tag{2.2}$$

Relaxing the centrally located dock-door assumption and letting the distance between the left end of the wall point and the leftmost dock door be ϕ for Scenario 3, the distance from the left-end of the wall to the i^{th} dock door is

$$d_i = \phi + (i - 1)\, \delta. \tag{2.3}$$

Expected horizontal roundtrip-distance formulations are developed by measuring the distance between a dock door and the nearest S/R aisle. To obtain the distance, t_i is used to measure the horizontal distance between the left-end of the wall and the centerline of the back-to-back rack closest to dock door *i*. Because *a* denotes the distance between centerlines of adjacent aisles, the distance between the left-end of the wall and t_i is a multiple of *a*. Therefore, from the relationship between d_i and *a*, the value of t_i is calculated by rounding d_i to the nearest multiple of *a*. Hence, $t_i = a \times ROUND\,[d_i / a, 0]$.

As shown in Figure 2.3, in calculating the distance between a dock door and the nearest S/R aisle, four cases occur: *a*) d_i is smaller than t_i, *b*) d_i equals t_i, *c*) d_i is greater than t_i, and *d*) $|d_i - t_i|$ equals half the distance between adjacent S/R aisles ($a / 2$).

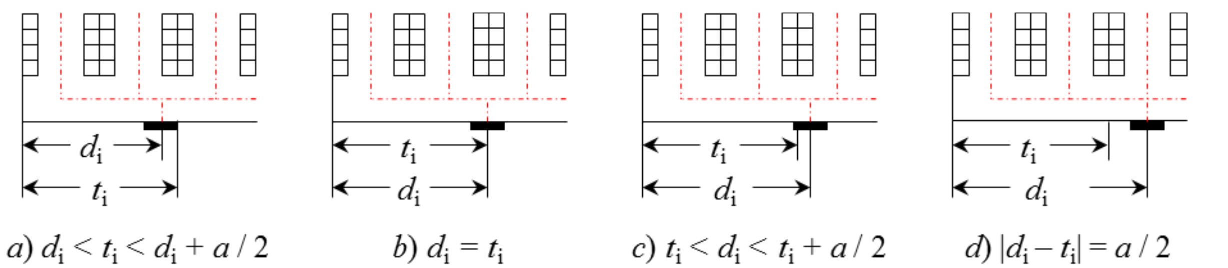

Figure 2.3: Cases for dock-door locations

<u>Proposition 2.1:</u> There are t_i / a and $n - t_i / a$ S/R aisles to the left and to the right of dock door *i*, respectively. Because the distance between dock door *i* and the nearest S/R aisle is $a / 2 - |d_i - t_i|$, the distance between dock door *i* and S/R aisle *j* equals $|d_i - (j - 1 / 2)\, a|$ for $j = 1, 2, \ldots, n$.

Proposition 2.1 applies for all cases. (Proof of Proposition 2.1 is provided in the Appendix).

2.5.1. Single-command travel

With each S/R location equally likely to be selected, the expected horizontal roundtrip-distance to and from dock door *i* is obtained by doubling the sum of the expected distance to the left and to the right of dock door *i*. Summing the results over all dock doors and dividing by the number of dock doors, the expected horizontal roundtrip-distance for *k* dock doors is

$$E\left[SC_h^D\right] = \frac{2}{n\,k}\sum_{i=1}^{k}\sum_{j=1}^{n}\left|d_i - \left(j - 1/2\right)a\right|. \tag{2.4}$$

As noted, increasing the number of dock doors or changing the location of a dock door does not affect expected vertical distances. Therefore, the expected vertical roundtrip-distance is

$$E\left[SC_v^D\right] = \frac{2}{m}\sum_{j=1}^{m}\left(jw - w/2 + 2v\right) = wm + 4v = D. \tag{2.5}$$

Summing Equations (2.4) and (2.5), the expected single-command distance is

$$E\left[SC^D\right] = E\left[SC_h^D\right] + E\left[SC_v^D\right] = \frac{2}{n\,k}\sum_{i=1}^{k}\sum_{j=1}^{n}\left|d_i - \left(j - 1/2\right)a\right| + D. \tag{2.6}$$

2.5.2. Dual-command travel

To calculate expected dual-command distance, we add the expected distance between two random S/R locations and the expected single-command distance. Although all S/R locations in an S/R aisle are equally likely to be chosen, the probability of two S/R locations being either in the same aisle or in different aisles must be taken into account.

When two S/R locations are in the same aisle, there is no travel in the horizontal direction. Visiting the same location for both storage and retrieval operations in the same trip is not practical. However, it is practical to store a unit load on one side of the aisle and retrieve another unit load on the opposite side of the aisle; likewise, it is practical to store a unit load at a particular level of the storage rack and retrieve a unit load from the same floor location, but a different level of the storage rack. However, because we ignore travel between different levels of the storage rack, the latter possibility is not factored into our calculations.

Admittedly, even by allowing a storage and a retrieval to occur at the same floor location an approximation continues to exist by assuming each storage location is equally likely to be visited. To eliminate the approximation, we do not include the occurrence of a storage and retrieval from the same storage location in our calculations. Therefore, with probability 1 / *n*, expected vertical distance between two S/R locations in the same aisle (*sa*) is

$$E\left[TB_{sa}^{D}\right]=\frac{4w}{2m\left(2m-1\right)}\sum_{i=1}^{m}\sum_{j=1}^{m}\left|i-j\right|=\frac{2w\left(m^{2}-1\right)}{3\left(2m-1\right)}. \qquad (2.7)$$

When two S/R locations are in different aisles, the expected horizontal travel-between distance is provided by Pohl *et al*. (2009) as $a\,(n^2 - 1) / (3n)$. Numbering S/R locations from the bottom to the top, the shortest distance between S/R locations *i* and *j* is min $(i + j - 1,$ $2m - i - j + 1) + 2v$. The probability of traveling from one aisle to another aisle is $1 - 1 / n$. Summing distances over all possible combinations of S/R locations, dividing by the number of combinations, and multiplying by the width of S/R locations; the expected vertical distance between two S/R locations in different aisles (*da*) is

$$E\left[TB_{da}^{D}\right]=\frac{w}{m^{2}}\left[\sum_{i=1}^{m}\sum_{j=1}^{m}\min\left(i+j-1,\ 2m-i-j+1\right)\right]+2v=\frac{w}{3m}\left(2m^{2}+1\right)+2v. \qquad (2.8)$$

Incorporating probabilities, combining Equations (2.7) and (2.8) and adding the expected horizontal travel-between, the expected travel-between distance becomes

$$E\left[TB^{D}\right]=\frac{1}{n}\left\{\frac{2w\left(m^{2}-1\right)}{3\left(2m-1\right)}+\left(n-1\right)\left[\frac{w}{3m}\left(2m^{2}+1\right)+2v\right]\right\}+\frac{a\left(n^{2}-1\right)}{3n}. \qquad (2.9)$$

Combining Equations (2.6) and (2.9), the expected dual-command distance is

$$E\left[DC^{D}\right]=\left[2/(n\,k)\right]\sum_{i=1}^{k}\sum_{j=1}^{n}\left|d_i-(j-1/2)a\right|+D$$
$$+(1/n)\left\{2w\left(m^2-1\right)/(6m-3)+(n-1)\left[\left(2m^2w+w\right)/3m+2v\right]\right\}+a\left(n^2-1\right)/3n. \tag{2.10}$$

2.5.3. Discrete optimization problem

Based on the expected-distance formulations obtained, the following discrete model of the optimization problem is used to determine the number and length of S/R aisles:

Minimize : $E\,[SC^D]$ or $E\,[DC^D]$

Subject to : $n\,a\,(w\,m + 4v) \geq A$

1) $n\,a \geq (k + 1)\,(\omega + \psi)$, 2) $n\,a \geq k\,\delta$ or 3) $n\,a \geq \phi + (k - 0.5)\,\delta$

n and m integers greater than zero.

The first constraint in the optimization model assures the space requirement is met. Its pre-determined value is given as A. Assuring the width of the warehouse allows k dock doors to be located on one wall of the warehouse; the constraints for the width of the warehouse in the optimization model are specific to a scenario. Obtaining the optimal shape factor, the optimization model is solved for the optimum number of S/R aisles (n*) and the optimum number S/R locations (m*). The resulting optimal shape factor is

$$S^* = (a\,n^*) / (w\,m^* + 4v). \tag{2.11}$$

The nonlinear-integer-programming optimization problem is implemented using *Couenne* (2006) in *AMPL* (2013) software package. *Couenne* (2006) is an open source code to solve Mixed-Integer Nonlinear Programming (MINLP) formulations by implementing linearization, bound reduction and branching methods within a branch and bound algorithm (Belotti, 2009; Belotti *et al*. 2009). Computational results from *Couenne* (2006) are provided in Section 2.7. The optimality of solutions is tested either by using *Mathematica* (2015) software package or by enumerating in *Microsoft Excel* (2013).

2.6. Continuous Approximations

To eliminate the need for specialized software to solve the optimization model and to facilitate sensitivity analyses, we develop closed-form expressions of expected distances and optimal shape factors by employing continuous approximations. The continuous approximations presented in this section provide useful insights regarding the design of multi-dock-door, unit-load warehouses. For single-command travel, the interior of the warehouse is treated as a continuous region by ignoring storage racks, S/R aisles and cross-aisles by assuming S/R locations are uniformly distributed over a rectangular storage region. For dual-command travel, a result from the discrete formulation is used to approximate expected travel-between distance.

2.6.1. Single-command travel

To illustrate the procedure used to calculate expected distance with continuous approximation, let a single dock door be located on the centerline of a warehouse wall having width *W*. From Tutam and White (2015), expected single-command distance for a centrally located dock door is

$$E\,[SC^{C}] \approx W / 2 + D. \tag{2.12}$$

Axiom 2.1: Expressing expected single-command distance as a function of the warehouse's width, taking the first derivative with respect to the warehouse's width, setting it equal to zero, and solving for the warehouse's width, stationary points are obtained for expected single-command distance. If a single stationary point exists, taking the second derivative of expected single-command distance and finding the second derivative is greater than zero for all values of the warehouse's width establishes expected single-command distance is a convex function of the warehouse's width and the stationary point is the optimal width of the warehouse.

Lemma 2.1: When expected single-command distance is expressed as $E\,[SC^C] \approx c_1\, W + c_2\, W^{-1} + c_3$, then expected single-command distance is a convex function of W with stationary point $W \approx (c_2 / c_1)^{1/2}$.

Corollary 2.1: Expected single-command distance for a single centrally located dock door is a convex function of the warehouse's width with stationary point $W \approx (2\,A)^{1/2}$ and corresponding shape factor $S \approx 2.0$ (The same result was obtained by Francis (1967a)).

When k dock doors are equally spaced over an entire wall of the warehouse, the expected horizontal roundtrip-distance to the left of dock door i is $i\,W / (k + 1)$ and to the right of dock door i is $[(k + 1 - i)\,W] / (k + 1)$. The probabilities of traveling to the left and right of dock door i are $i / (k+1)$ and $(k + 1 - i) / (k+1)$, respectively. As before, the expected vertical roundtrip-distance is D. Therefore, expected single-command distance for k dock doors is

$$E\left[SC^C\right] \approx \frac{1}{k}\sum_{i=1}^{k}\left\{\left(\frac{i}{k+1}\right)\left(\frac{i\,W}{k+1}\right)+\left(\frac{k+1-i}{k+1}\right)\left(\frac{(k+1-i\,)W}{k+1}\right)\right\}+D \approx \frac{(2k+1)W}{3(k+1)}+D. \quad (2.13)$$

When k dock doors are located centrally along one wall of the warehouse with a specified distance (δ) between adjacent dock doors, the expected horizontal roundtrip-distance to the left of dock door i is $\{W - [k - (2i - 1)]\,\delta\} / 2$ and to the right of dock door i is $\{W + [k - (2i - 1)]\,\delta\} / 2$; also, the probability of traveling to the left of dock door i is $\{W - [k - (2i - 1)]\,\delta\} / 2W$ and the probability of traveling to the right of dock door i is $\{W + [k - (2i - 1)]\,\delta\} / 2W$. Therefore, the expected single-command distance is

$$E\left[SC^C\right] \approx \frac{1}{4Wk}\sum_{i=1}^{k}\left(\left\{W-\left[k-(2i-1)\right]\delta\right\}^2+\left\{W+\left[k-(2i-1)\right]\delta\right\}^2\right)+D$$
$$\approx \frac{W}{2}+\frac{\delta^2(k^2-1)}{6W}+D. \qquad (2.14)$$

When k dock doors are not centrally located on the wall containing dock doors, a fixed distance of δ exists between adjacent dock doors, and the leftmost dock door is located a distance of ϕ from the left-end of the wall, the expected horizontal roundtrip-distance to the left of dock door i is $[\phi + (i - 1)\,\delta]$ and the probability of traveling to the left of dock door i is $[\phi + (i - 1)\,\delta] / W$; also, the expected horizontal roundtrip-distance to the right of dock door i is $[W - \phi - (i - 1)\,\delta]$ and the probability of traveling to the right of dock door i is $[W - \phi - (i - 1)\,\delta] / W$. Therefore, the expected single-command distance for k dock doors is

$$E\left[SC^C\right] \approx \frac{1}{k}\sum_{i=1}^{k}\left\{\frac{\left[\phi+(i-1)\delta\right]^2}{W}+\frac{\left[W-\phi-(i-1)\delta\right]^2}{W}\right\}+D$$
$$\approx W+\frac{6\phi^2+6(k-1)\phi\delta+\left(2k^2-3k+1\right)\delta^2}{3W}-\left(2\phi+(k-1)\delta\right)+D. \qquad (2.15)$$

Corollary 2.2: For k dock doors, expected single-command distance for Scenarios 1, 2 and 3 is a convex function of the width of the warehouse with stationary points $W \approx [3A(k+1) / (2k+1)]^{1/2}$, $W \approx [2A + [\delta^2 (k^2 - 1)] / 3]^{1/2}$ and $W \approx \{[3A + 6\phi^2 + 6\phi\,\delta\,(k - 1) + (2k^2 - 3k + 1)\,\delta^2] / 3\}^{1/2}$ and corresponding shape factors of $S \approx 3(k+1) / (2k+1)$, $S \approx 2 + [\delta^2 (k^2 - 1)] / 3A$ and $S \approx 1 + [6\phi^2 + 6\phi\,\delta\,(k - 1) + (2k^2 - 3k + 1)\,\delta^2] / 3A$, respectively.

For Scenario 1, taking the limit of S as k approaches infinity yields an optimal shape factor of 1.5. The same result is obtained by Thomas and Meller (2014) with a uniformly distributed dock-door assumption. Although having an infinite number of dock doors is impractical, the result provides a lower bound for the optimal shape factor under a uniformly distributed dock-door assumption. Thomas and Meller (2014) only considered the cases of a single dock door and an infinite number of dock doors. Our formulation holds for any number of dock doors and provides the exact optimal shape factor values when the number of dock doors and/or the spacing between adjacent dock doors are/is specified.

With a continuous approximation, we must also include the following width constraints for Scenarios 1, 2, and 3: $W \geq (k + 1)(\omega + \psi)$, $W \geq k\,\delta$, and $W \geq \phi + (k - 0.5)\,\delta$. Because expected distance is a convex function of warehouse width, if the unconstrained optimal width violates the constraint, then the width (and corresponding shape factor) will be determined by the width constraint.

Proposition 2.2: For Scenario 1, $S^*_{SC} \approx 3(k+1) / (2k+1)$ if $S \geq [(k + 1)^2 (\omega + \psi)^2] / A$; otherwise, $S^*_{SC} \approx [(k + 1)^2 (\omega + \psi)^2] / A$. For Scenario 2, $S^*_{SC} \approx 2 + [\delta^2 (k^2 - 1)] / 3A$ if $S \geq k^2 \delta^2 / A$; otherwise, $S^*_{SC} \approx k^2 \delta^2 / A$. For Scenario 3, $S^*_{SC} \approx 1 + [6\phi^2 + 6\phi\delta (k - 1) + (2k^2 - 3k + 1)\,\delta^2] / 3A$ if $S \geq [\phi + (k - 0.5)\,\delta]^2 / A$; otherwise, $S^*_{SC} \approx [\phi + (k - 0.5)\,\delta]^2 / A$.

2.6.2. Dual-command travel

From Figure 2.4, rectilinear distance between two S/R locations in different S/R aisles underestimates travel-between distance. To facilitate calculations in obtaining the optimal shape factor for dual-command travel, we introduce a new approximation for expected travel-between distance and modify Equation (2.9) in the previous section.

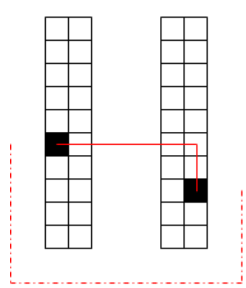

Figure 2.4: Rectilinear (solid) and actual (dashed) distances between two S/R locations

Specifically, $(m^2 - 1) / (2m - 1)$ and $(2m^2 + 1) / m$ in Equation (2.9) are replaced with 0.5 m and $2m$, respectively. The resulting approximation for expected travel-between distance is

$$E\left[TB^C\right] \approx \frac{1}{n}\left[\frac{wm}{3} + (n-1)\left(\frac{2wm}{3} + 2v\right)\right] + \frac{a\left(n^2 - 1\right)}{3n}. \tag{2.16}$$

Although arrived at in a different way, by letting $w\ m$ equal L, Equation (2.16) is identical to the expected travel-between distance formula in Pohl *et al*. (2009). To obtain expected dual-command distance, Equation (2.16) is combined with the appropriate expected single-command distance equation.

Combining Equations (2.12) and (2.16), expected dual-command distance for a single-dock, unit-load warehouse is

$$\begin{aligned} E\left[DC^C\right] \approx E\left[SC^C\right] + E\left[TB^C\right] &\approx \frac{W}{2} + D + \frac{1}{n}\left[\frac{wm}{3} + (n-1)\left(\frac{2wm}{3} + 2v\right)\right] + \frac{a\left(n^2 - 1\right)}{3n} \\ &\approx \frac{5W^3 - 4vW^2 + \left(10A - 2a^2 - 4av\right)W - 2aA}{6W^2}. \end{aligned} \tag{2.17}$$

<u>Axiom 2.2</u>: Expressing expected dual-command distance as a function of the warehouse's width and taking the first derivative with respect to the warehouse's width, a cubic equation is obtained. For reasonable parameter values (the necessary condition for each scenario is provided in the proof of Corollary 2.4), the discriminant of the cubic equation is greater than zero. Therefore, the cubic equation has three distinct real roots, but there exist no rational roots because the cubic equation is *irreducible polynomial* (from Galois Theory). Solving an irreducible cubic equation requires taking the roots of complex quantities. Therefore, reducing the cubic equation to *depressed form*, setting the depressed cubic equation equal to zero and solving for the warehouse's width, the viable root can be obtained using *Viète's trigonometric solution* (Nickalls, 2006). The viable root is the first root because results with the second and third roots are infeasible (the value of expected distance is negative for the second root and the width of the warehouse is zero for the third root). Taking the second derivative of expected dual-command distance with respect to the warehouse's width and finding the second derivative is greater than zero for all reasonable values of the warehouse's width establishes expected dual-command distance is a convex function of the warehouse's width and the viable root is the optimal width of the warehouse.

<u>Lemma 2.2</u>: Expressing expected dual-command distance as $E[DC] \approx (c_1 W^3 + c_2 W^2 + c_3 W + c_4) / (c_5 W^2)$, expected dual-command distance is a convex function of the warehouse's width with stationary point $W \approx 2 (c_3 / 3c_1)^{1/2} \cos \{\arccos [c_4 c_1^{1/2} (3 / c_3)^{3/2}] / 3\}$.

<u>Corollary 2.3</u>: Expected dual-command distance for a centrally located dock door is a convex function of the warehouse's width with stationary point $W \approx 2 (c_3 / 3c_1)^{1/2} \cos \{\arccos [c_4 c_1^{1/2} (3 / c_3)^{3/2}] / 3\}$ and corresponding shape factor $S \approx 4c_3 (\cos \{\arccos [c_4 c_1^{1/2} (3 / c_3)^{3/2}] / 3\})^2 / (3A c_1)$ where $c_1 = 5$, $c_3 = 10A - 2a^2 - 4av$ and $c_4 = -2aA$.

Combining Equation (2.16) with Equations (2.13), (2.14) and (2.15), expected dual-command distance for the various scenarios is obtained as follows

Scenario 1:
$$E\left[DC^C\right] \approx \frac{(3k+2)W^3 - 2(k+1)vW^2}{3(k+1)W^2} + \frac{\left[(k+1)\left(5A - a^2 - 2av\right)\right]W - (k+1)aA}{3(k+1)W^2}, \tag{2.18}$$

Scenario 2:
$$E\left[DC^C\right] \approx \frac{5W^3 - 4vW^2 + \left[10A - 2a^2 - 4av + \left(k^2 - 1\right)\delta^2\right]W - 2aA}{6W^2}, \tag{2.19}$$

Scenario 3:
$$E\left[DC^C\right] \approx \frac{4W^3 - \left[2v + 3(k-1)\delta + 6\phi\right]W^2}{3W^2} + \frac{\left[5A - a^2 - 2av + (2k^2 - 3k + 1)\delta^2 + 6(k-1)\phi\delta + 6\phi^2\right]W - aA}{3W^2}. \tag{2.20}$$

Corollary 2.4: For k dock doors, expected dual-command distance for Scenarios 1, 2 and 3 is a convex function of the width of the warehouse with stationary points $W \approx 2\,(c_3 / 3c_1)^{1/2}$ $\cos\{\arccos[c_4\, c_1^{1/2}\,(3 / c_3)^{3/2}] / 3\}3\})^2 / (3A\, c_1)$ and corresponding shape factors of $S \approx 4c_3\,(\cos\{\arccos[c_4\, c_1^{1/2}\,(3 / c_3)^{3/2}] / 3\})^2 / (3A\, c_1)$ where $c_1 = (3k + 2)$, $c_3 = (k + 1)\,(5A - a^2 - 2a\,v)$ and $c_4 = -\,(1 + k)\,a\,A$ for Scenario 1; $c_1 = 5$, $c_3 = 10A - 2a^2 - 4a\,v + \delta^2\,(k^2 - 1)$ and $c_4 = -\,2a\,A$ for Scenario 2; and $c_1 = 4$, $c_3 = 5A - a^2 - 2a\,v + 6\phi^2 + 6\phi\,\delta\,(k - 1)$ $+ (2k^2 - 3k + 1)\,\delta^2$ and $c_4 = -\,a\,A$ for Scenario 3.

As with single-command travel, the width (and corresponding shape factor) will be determined by the width constraint when the unconstrained optimal width violates the constraint.

Proposition 2.3: When the width constraint is satisfied, the optimal shape factor is $S^*_{DC} \approx 4c_3$ $(\cos\{\arccos[c_4\, c_1^{1/2}\,(3 / c_3)^{3/2}] / 3\})^2 / (3A\, c_1)$ where $c_1 = (2 + 3k)$, $c_3 = (1 + k)\,(5A - a^2 - 2a\,v)$ and $c_4 = -\,(1 + k)\,a\,A$ for Scenario 1; $c_1 = 5$, $c_3 = 10A - 2a^2 - 4a\,v + \delta^2\,(k^2 - 1)$ and $c_4 = -\,2a\,A$ for Scenario 2; and $c_1 = 4$, $c_3 = 5A - a^2 - 2a\,v + 6\phi^2 + 6\phi\,\delta\,(k - 1) + (2k^2 - 3k + 1)\,\delta^2$ and $c_4 = -\,a\,A$ for Scenario 3. Otherwise, the optimal shape factor for each scenario is $S^*_{DC} \approx (k + 1)^2\,\delta^2 / A$, $S^*_{DC} \approx k^2\,\delta^2 / A$ and $S^*_{DC} \approx [\phi + (k - 0.5)\,\delta]^2 / A$, respectively.

Proposition 2.4: For Scenario 1, a *balanced warehouse* (expected horizontal roundtrip-distance equals expected vertical roundtrip-distance) exists for single-and dual-command travel when a warehouse is optimally configured and its width is equal to or greater than $(k + 1)\,(\omega + \psi)$. For Scenario 2, a warehouse is an *unbalanced warehouse* (expected horizontal roundtrip-distance is greater than expected vertical roundtrip-distance) for single- and dual-command travel when the warehouse is configured optimally. For Scenario 3, depending on the number of dock doors, the expected horizontal roundtrip-distance can be less than or greater than the expected vertical roundtrip-distance for single- and dual-command travel when the warehouse is optimally configured.

2.7. Computational Results

In this section, we provide results for both discrete formulations and continuous approximations by using specified values for the parameters in the formulations. The computational results for both single-command and dual-command travel are tabulated and provided for each scenario in the Appendix. All calculations are conducted on a PC with Intel i7-4600M 2.90GHz processor and 16 GB of memory. For the stated parameter values, the computational time is less than ten seconds for any number of dock doors. In addition, in the section, we address the accuracy of the continuous approximations for each scenario. Examining the percentage error for continuous approximations ($\{|E\ [SC_{\text{Discrete}}] - E\ [SC_{\text{Continuous}}]| / E\ [SC_{\text{Discrete}}]\} \times 100$ or $\{|E\ [DC_{\text{Discrete}}] - E\ [DC_{\text{Continuous}}]| / E\ [DC_{\text{Discrete}}]\} \times 100$), the following set of parameter values are tested. (The most common set of values are chosen based on data obtained after visiting several unit-load warehouses.)

- A = 150,000, 250,000 and 350,000 ft^2,
- k = 1, 16, 31, 46 and 61 dock doors,
- v = 5 and 6 ft,
- w = 3 and 4 ft,
- l = 3 and 4 ft,
- ω = 9 ft and ψ = 1, 2 and 3 ft (δ = 10, 11 and 12 ft),
- ϕ = 10, 20, 30, 40 and 50 ft.

As illustrated in Table 2.1, the minimum, maximum and average approximation errors for single-command travel with Scenario 1 are 0.00%, 1.28%, and 0.23%, respectively. Similarly, the minimum, maximum and average approximation errors for dual-command travel are 0.01%, 1.24% and 0.20%, respectively. Based on the computational results for Scenario 2, using a continuous approximation for single-command travel, the percentage error varies from 0.00% to 0.60%, with an average value of 0.14%. The percentage error for dual-command travel varies from 0.00% to 0.65%, with an average value of 0.16%.

Table 2.1: The percentage errors of continuous approximations for scenarios

	E [*SC*]			*E* [*DC*]		
	Minimum	Maximum	Average	Minimum	Maximum	Average
Scenario 1	0.00%	1.28%	0.23%	0.01%	1.24%	0.20%
Scenario 2	0.00%	0.60%	0.14%	0.00%	0.65%	0.16%
Scenario 3	0.00%	0.57%	0.15%	0.00%	0.60%	0.17%

From the computational results for Scenario 3, the percentage error resulting from the use of the continuous approximation ranges from 0.00% to 0.57% for single-command travel, with an average value of 0.15%. For dual-command travel, the percentage error ranges from 0.00% to 0.60%, with an average value of 0.17%. Therefore, the continuous approximation appears to provide reliable results for both single- and dual-command travel.

Solving the optimization model, the optimum number of aisles (n^*) and the optimum number of S/R locations in each S/R aisle (m^*) are determined, such that the optimal shape factor is obtained for a warehouse having k equally spaced dock doors over an entire wall. Moreover, the optimal width (W^*) and the optimal depth (D^*) of the warehouse are approximated for single-and dual-command travel by using closed-form expression given in Sections 2.6.1 and 2.6.2 for single- and dual-command expressions, respectively. Ranging the number of dock doors from 1 to 60 for Scenario 1 and from 1 to 75 for Scenarios 2 and 3, we employ the following parameter values: w = 4 ft, l = 4 ft, v = 6 ft, $a = 2(l + v)$ = 20 ft, ω = 9 ft, ψ = 3 ft, $\delta = \omega + \psi$ = 12 ft, ϕ = 30 ft, and A = 250,000 ft^2. For ease of computation, the continuous approximation is used to produce shape-factor-figures in the following sub-sections for each scenario, unless stated otherwise.

2.7.1. Single-command travel

For the stated parameter values, Figure 2.5 illustrates the impact of the number of dock doors on expected distance (left) and the optimal shape factor (right) for single-command travel for the three scenarios. Increasing the number of dock doors increases expected single-command distance for Scenarios 1 and 2 because dock doors are located farther from the centerline of the warehouse. Unlike Scenarios 1 and 2, expected single-command distance may increase or decrease for Scenario 3 as the number of dock doors increases. When the width of the warehouse is governed by the width constraint, expected single-command distance is approximately the same for all scenarios.

To understand why, with Scenario 3, expected single-command distance decreases and, then, increases as the number of dock doors increase, recall dock doors are not centrally located and a fixed distance of ϕ exists between the leftmost dock door and the left wall. Therefore, increasing the number of dock doors results in dock doors, initially, being located nearer the centerline of the warehouse. Then, dock doors are being located farther from the centerline of the warehouse. For the stated parameter values, increasing the number of dock doors increases expected distance when there exist more than 37 dock doors for single-command travel. If the first dock door is located on the right side of the warehouse's wall ($\phi > W / 2$), increasing the number of dock doors will increase the expected single-command distance for any value of k.

For Scenario 1, increasing the number of dock doors decreases the optimal shape factor for single-command travel when the width constraint is not violated ($\omega + \psi \geq 12$ ft). As noted previously, the lower bound for the optimal shape factor is 1.5. For Scenario 2, increasing the number of dock doors increases the optimal shape factor and the optimal shape factor is equal to or greater than 2.0 for any value of k. Likewise, increasing the number of dock doors increases the optimal shape factor for Scenario 3 and the optimal shape factor is greater than 1.0 for any value of k. When the width constrained is violated, increasing the number of dock doors will increase the optimal shape factor for all scenarios.

Among the scenarios, which performs best? From Figure 2.5, with the exception of a single-dock-door warehouse, Scenario 2 performs better than either Scenario 1 or Scenario 3. However, the relative ranking of Scenarios 1 and 3 changes as the number of dock doors increases. In comparing Scenarios 1 and 3, when the number of dock doors is small, with Scenario 3 they are clustered toward the end of the wall; whereas, with Scenario 1 they are distributed across the wall and symmetrically around the centerline of the wall. Thus, for a small number of dock doors, Scenario 1 outperforms Scenario 3.

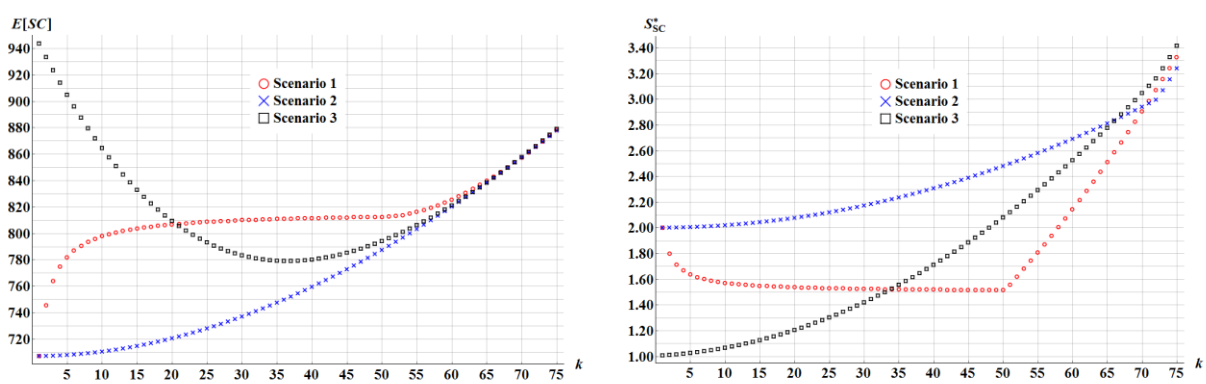

Figure 2.5: E [SC] (left) and S^*_{SC} (right) comparison of scenarios

When the width constraint is satisfied, the warehouse for Scenario 2 is wider than the warehouses for Scenario 1 and 3. For a small number of dock doors, the width of the warehouse with Scenario 1 is greater than the width of the warehouse with Scenario 3. For a large number of dock doors, the warehouse with Scenario 3 becomes wider. When the width constraint is violated, the warehouse with Scenario 3 becomes the widest warehouse because of the fixed distance from the left wall.

For Scenario 2, the requirement for adjacent dock doors to be δ feet apart results in the optimal shape factor increasing with an increasing number of dock doors. Hence, the warehouse is wider and shallower than occurs with Scenario 1. For the warehouse to be *balanced* the depth of the warehouse must increase and the width must decrease, resulting in an increase in expected distance. However, a relatively small increase occurs. Specifically, the maximum percentage difference in expected single-command distance is 0.04%. As shown in Figure 2.6 (left), when the width constraint is violated, the warehouse is forced to be an unbalanced warehouse because of the width constraint.

For Scenario 3, the optimally configured warehouse can be (and most likely is) *unbalanced* regarding horizontal and vertical roundtrip-distances. Specifically, for a small number of dock doors, vertical roundtrip-distance is greater than horizontal roundtrip-distance; however, for a large number of dock doors, horizontal roundtrip-distance is greater than vertical roundtrip-distance. The maximum percentage difference in the expected distance for single-command travel is 0.02%. As illustrated in Figure 2.6 (left), a relatively small expected-distance penalty results from forcing the warehouse to be balanced for single-command travel.

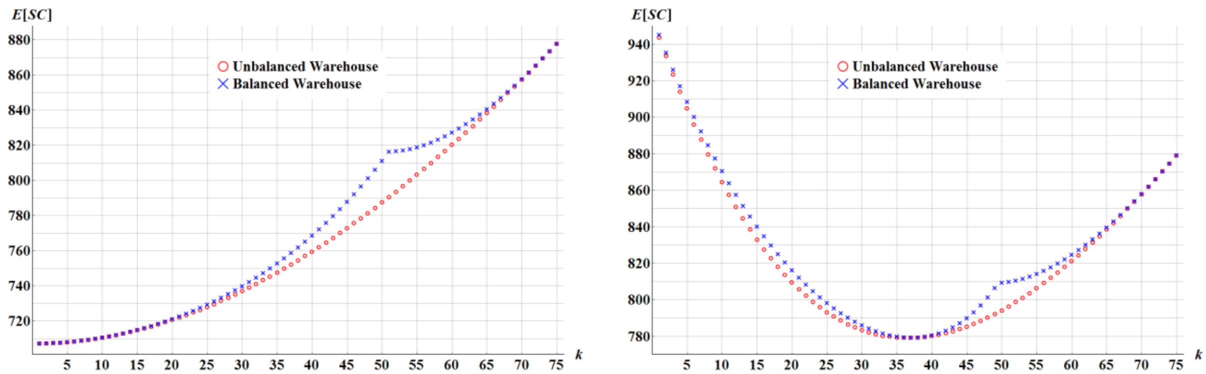

Figure 2.6: Comparison of balanced and unbalanced warehouses for single-command travel with Scenario 2 (left) and Scenario 3 (right)

As illustrated in Figure 2.7 (right), forcing a warehouse to be balanced can result in a shape factor significantly different than the optimal shape factor with Scenarios 2 and 3. The width constraint for the balanced warehouse is active when the number of dock doors exceeds 51 and 49 for Scenarios 2 and 3, respectively. When the number of dock doors is greater than 38, the horizontal distance becomes greater than the vertical distance. Therefore, the warehouse is forced to be narrower; hence, the horizontal distance and the shape factor decrease.

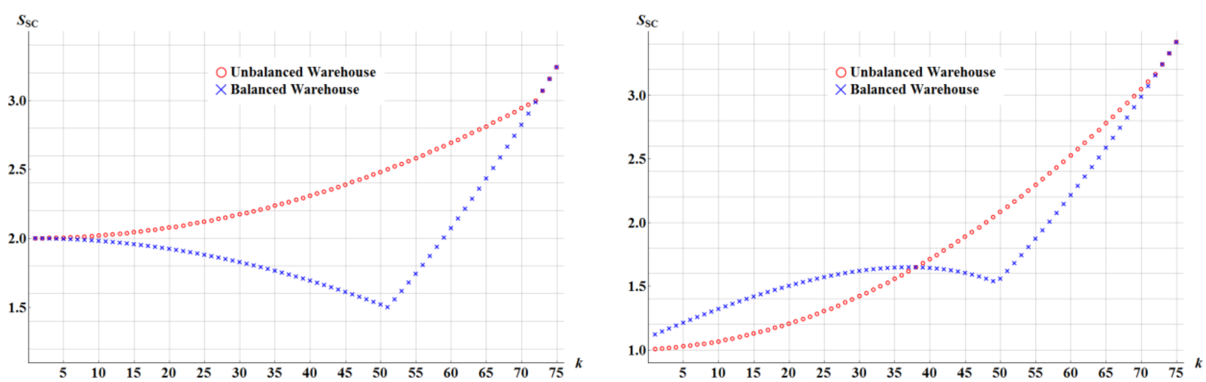

Figure 2.7: Shape factor comparison of balanced and unbalanced warehouses for single-command travel with Scenario 2 (left) and Scenario 3 (right)

Figure 2.8 examines the effect of δ on the expected single-command distance for Scenario 2 (left) and Scenario 3 (right). As anticipated, for Scenario 2, increasing the distance between adjacent dock doors increases expected distance because dock doors are located farther from the centerline of the warehouse. As the number of dock doors increases, the impact of δ on the expected distance increases significantly. Unlike Scenario 2, as the distance between adjacent dock doors increases, expected distance either increases or decreases depending on the number of dock doors and the offset distance from the left wall for Scenario 3. With the stated parameter values, increasing the value of δ decreases expected distance for a small number of dock doors; whereas, expected distance increases as the distance between adjacent dock doors increases for a large number of dock doors.

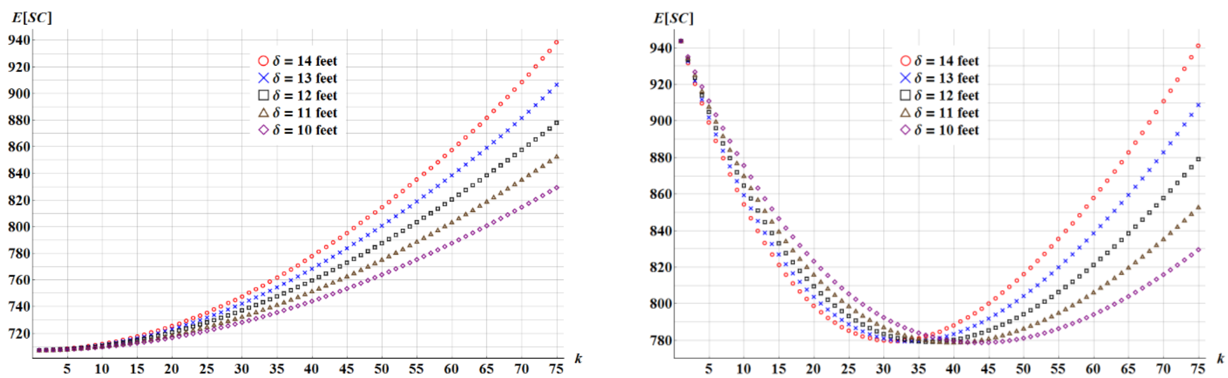

Figure 2.8: The effect of δ on $E\,[SC]$ for Scenario 2 (left) and Scenario 3 (right)

As illustrated in Figure 2.9, increasing the distance between adjacent dock doors increases the optimal shape factor for both Scenario 2 (left) and Scenario 3 (right). As the value of δ increases, the warehouse is forced to be wider. Furthermore, larger δ values cause the width constraint to be violated for smaller values of k.

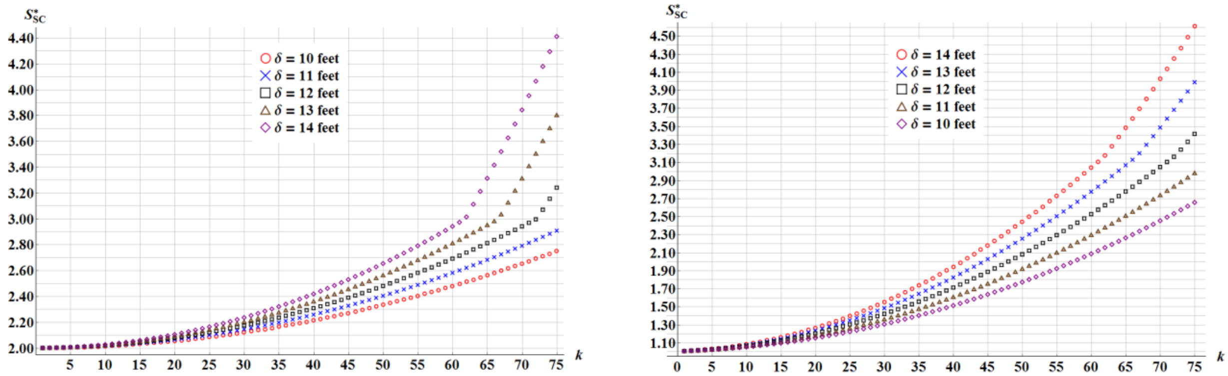

Figure 2.9: The effect of δ on the optimal shape factor for single-command travel with Scenario 2 (left) and Scenario 3 (right)

2.7.2. Dual-command travel

Figure 2.10 illustrates the impact of the number of dock doors on the expected distance (left) and the optimal shape factor (right) for dual-command travel with three scenarios. As with single-command travel, increasing the number of dock doors increases expected dual-command distance for Scenarios 1 and 2. However, increasing the number of dock doors may increase or decrease expected dual-command distance for Scenario 3.

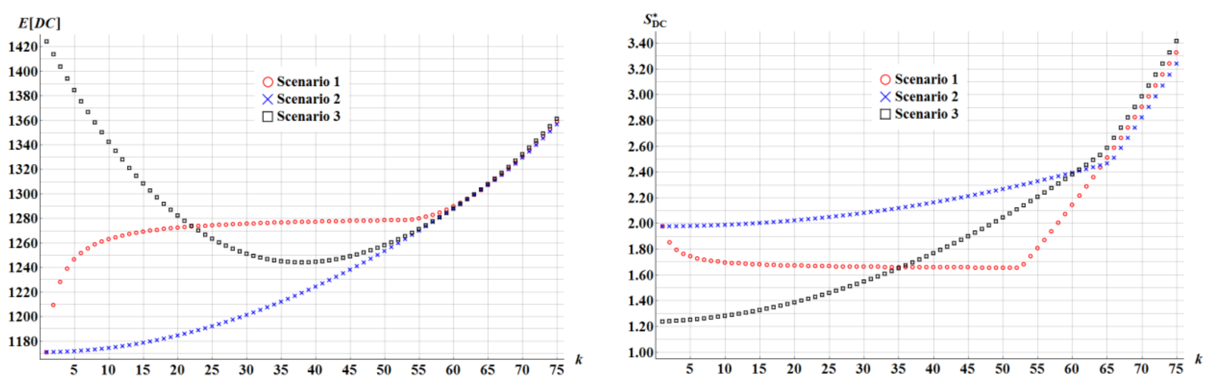

Figure 2.10: E [DC] (left) and S^*_{DC} (right) comparison of scenarios

With the stated parameter values, expected dual-command distance increases when $k > 38$. If the offset distance is greater than the half-width of the warehouse, the first dock door is located on the right side of the warehouse's wall ($\phi > W / 2$); hence, increasing the number of dock doors always results in increasing the expected dual-command distance. For all scenarios, the optimal shape factor for travel-between is 1.94 for a number of dock doors satisfying the width constraint because the number and locations of dock doors do not affect expected travel-between distance. If the width constraint is violated, the optimal shape factor is governed by the width constraint; hence, increasing the number of dock doors increases the width of the warehouse for all scenarios and increases the optimal shape factor for dual-command travel and travel-between.

When the width constraint is not violated, the optimal shape factor for dual-command travel decreases as the number of dock doors increases. The minimum value of the optimal shape factor is determined by the parameter values for dual-command travel (the minimum optimal shape factor value is 1.65 with the stated parameter values). For Scenario 2, increasing the number of dock doors increases the optimal shape factor. The optimal shape factor is less than 2.0 for a small number of dock doors (a minimum value of 1.97 with the stated parameter values); whereas, it is greater than 2.00 for a medium or a large number of dock doors ($k \geq 15$). For Scenario 3, increasing the number of dock doors increases the optimal shape factor. The optimal shape factor is greater than 1.0 for any value of k (a minimum value of 1.24 with the stated parameters).

Comparing the expected dual-command distance performances of scenarios, the same conclusions hold for all scenarios. Therefore, the optimal shape factor results for dual-command travel are compared to those for single-command travel instead of repeating the same conclusions from the previous subsection. For Scenario 1, except for the single-dock-door case, the optimal shape factor for dual-command travel is greater than the corresponding optimal shape factor for single-command travel. Notice, for the single-dock-door case, the optimal shape factor for travel-between is less than the optimal shape factor for single- and dual-command travel. In contrast to Scenario 1 (except for the single-dock-door case), with Scenario 2, the optimal shape factor for single-command travel is greater than the corresponding optimal shape factor for dual-command travel regardless of the value of *k*, because the optimal shape factor for travel-between is smaller than the optimal shape factor value for single-command travel. For Scenario 3, depending on the number of dock doors and the offset distance from the left wall, the optimal shape factor for single-command travel could be less than or greater than the corresponding optimal shape factor for dual-command travel.

To obtain a *balanced* warehouse when performing dual-command operations with Scenario 2, the width of the warehouse must decrease because the horizontal distance is greater than the vertical distance for any number of dock doors. By doing so, a relatively small increase occurs in expected dual-command travel and the maximum percentage difference is 0.02%. As with single-command travel, vertical roundtrip-distance is greater than horizontal roundtrip-distance for a small number of dock doors; whereas, horizontal roundtrip-distance is greater than vertical roundtrip-distance for a large number of dock doors. Forcing the warehouse to be balanced with Scenario 3 results in a maximum percentage difference of 0.01% in the expected distance for dual-command travel. From Figures 2.6 and 2.11, a relatively small expected-distance penalty results from forcing the warehouse to be balanced for either single-command travel or dual-command travel with Scenarios 2 and 3. Therefore, for practical purposes, designing a balanced warehouse is a reasonable design goal.

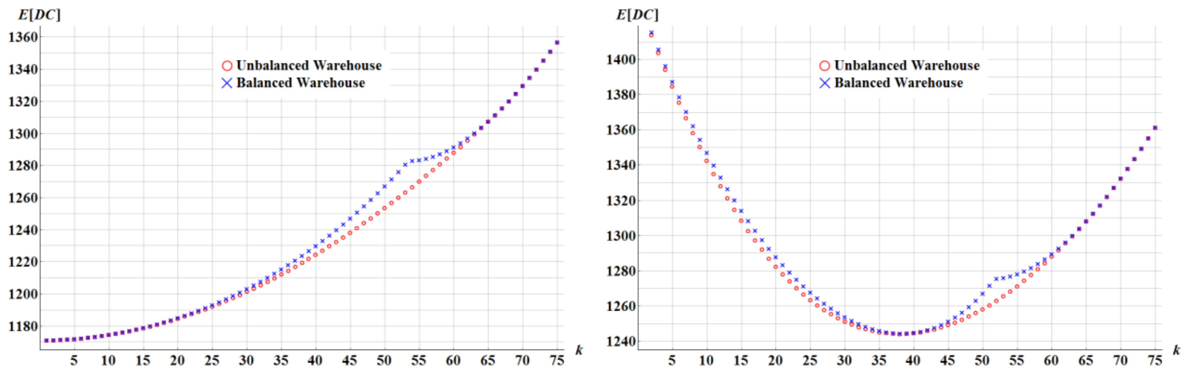

Figure 2.11: Comparison of balanced and unbalanced warehouses for dual-command travel with Scenario 2 (left) and Scenario 3 (right)

Figure 2.12 compares the optimal shape factor results with the shape factor results for a balanced warehouse. The shape factor of a warehouse forced to be balanced is significantly different from the optimal shape factor for both Scenarios 2 and 3. The width constraint for the balanced warehouse is active when the number of dock doors exceeds 53 and 51 for Scenarios 2 and 3, respectively.

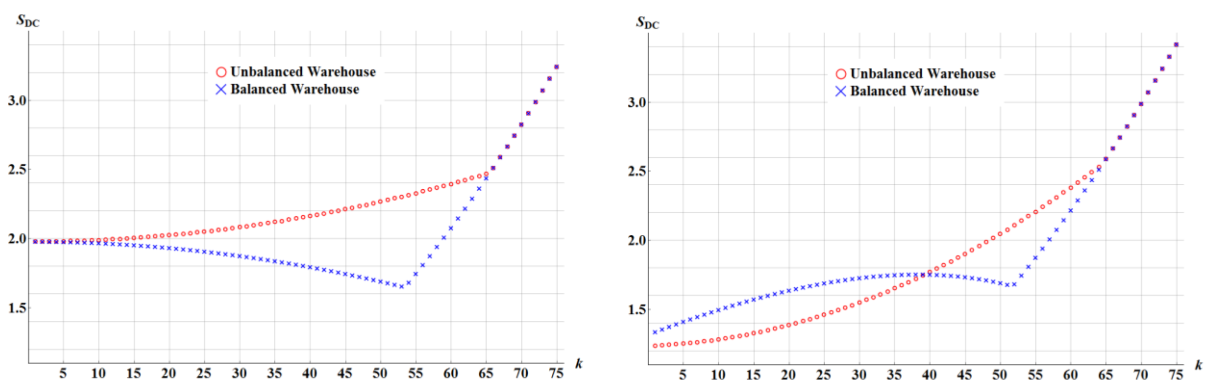

Figure 2.12: Shape factor comparison of balanced and unbalanced warehouses for dual-command travel with Scenario 2 (left) and Scenario 3 (right)

As with single-command travel, Figure 2.13 illustrates the effect of δ on the expected dual-command distance for Scenarios 2 (left) and 3 (right). Expected dual-command distance increases when the distance between adjacent dock doors increases for Scenario 2; whereas, it may increase or decrease for Scenario 3 depending on the number of dock doors and/or the offset distance from the left wall. The impact of δ on the expected distance increases significantly as the number of dock doors increases,

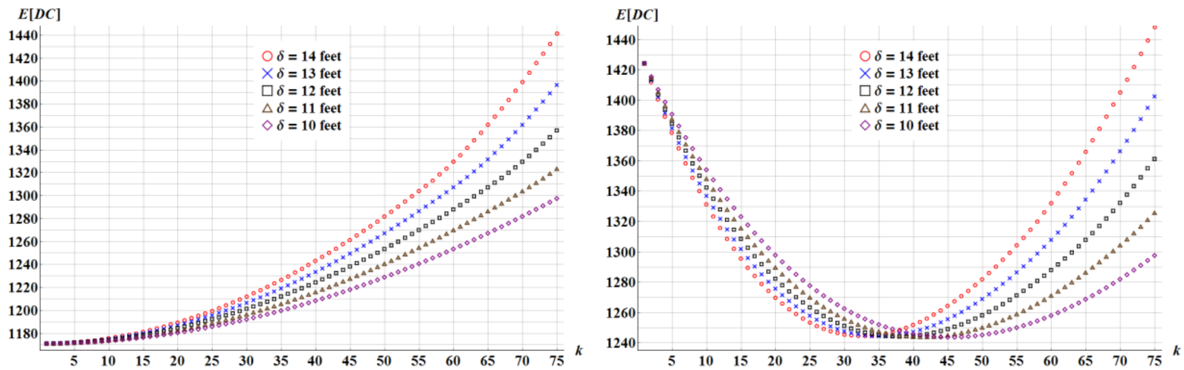

Figure 2.13: The effect of δ on $E\,[DC]$ for Scenario 2 (left) and Scenario 3 (right)

As shown in Figure 2.14, increasing the distance between adjacent dock doors increases the optimal shape factor for both Scenarios 2 (left) and 3 (right). As the value of δ increases, the warehouse is forced to be wider and the width constraint is violated for smaller values of k.

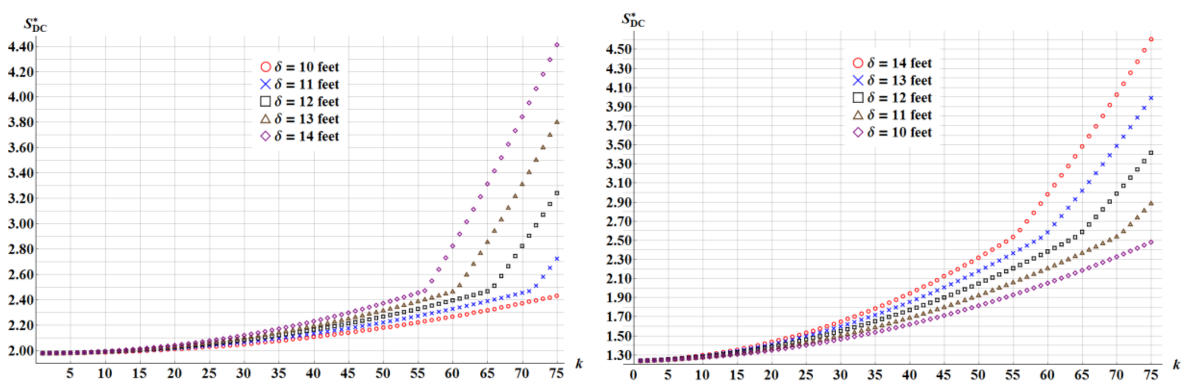

Figure 2.14: The effect of δ on the optimal shape factor for dual-command travel with Scenario 2 (left) and Scenario 3 (right)

Because the optimal shape factor for single-command travel with a large number of dock doors is greater than that for dual-command travel, the width constraint is active for dual-command travel with fewer dock doors.

2.8. Summary, Conclusions and Recommendations

Although warehouses typically have multiple dock doors for receiving and shipping, previous research on traditional layouts of unit-load warehouses focused primarily on a single, centrally located dock door. Because the number and locations of dock doors significantly affect expected distance, we extended previous studies by considering multiple dock doors and different dock-door locations for a unit-load warehouse having storage racks aligned perpendicular to the wall containing dock doors.

Discrete formulations of the optimization problem were employed to determine the optimum number of S/R aisles and S/R locations for single- and dual-command travel. Similarly, continuous formulations were employed to determine the optimal width and depth of the warehouse. The optimal shape factor was determined for both single- and dual-command travel.

> *For both single- and dual-command travel, increasing the number of dock doors will always increase expected distance when dock doors are centrally located; however, expected distance may increase or decrease depending the number of dock doors when they are non-centrally located.*

Because spacing dock doors over an entire wall of the warehouse when it is built is less expensive than adding dock doors after the warehouse is built, designers tend to install dock doors over an entire wall of the warehouse. Our results proved having too many dock doors can inhibit throughput when throughput is defined as the reciprocal of expected distance. However, having fewer dock doors than the required number creates congestion and results in additional idleness of S/R equipment. If an existing warehouse is occupied by a new tenant and the number of dock doors exceeds the number required to meet the throughput requirement, our research results can be used to determine which dock doors to close and how to configure the storage region within the facility.

Dock doors should be located as near as possible to the centerline of the warehouse.

Locating dock doors farther from the centerline of the warehouse increases the expected horizontal distance between dock doors and S/R locations. Therefore, when designing a new warehouse, once the number of dock doors required has been determined, they should be centrally located along a wall; when occupying an existing warehouse having more dock doors than needed, dock doors located farthest from the centerline of the warehouse wall should be closed.

The optimal shape factor depends on the number and locations of dock doors. When dock doors are spread over an entire wall of the warehouse, the distance between adjacent dock doors is a function of the warehouse's width; the optimal shape factor is between 1.5 and 2.0. However, when dock doors are distributed about the centerline of a warehouse wall and distances between adjacent dock doors are specified, the optimal shape factor is equal to or greater than 2.0. When dock doors are clustered toward the end of a wall, the optimal shape factor can be less than 1.5, between 1.5 and 2.0, or greater than 2.0, depending on the number of dock doors and the distance from the leftmost end of the wall and the nearest dock door.

After determining the required number of dock doors, the optimal shape can be determined for any number and any location of dock doors over an entire of the warehouse wall by using the formulations developed. More importantly, closed-form expression will eliminate the requirement of using a specialized software package or generating an extensive set of tables to determine the optimal shape. With the closed-form expressions, the penalty for a non-optimal design can be calculated easily and sensitivity analyses can be performed quickly.

> *Configuring a warehouse optimally results in a balanced warehouse when dock doors are equally distributed over an entire warehouse wall; whereas, it results in an unbalanced warehouse when the distance between adjacent dock doors is specified.*

Forcing a warehouse to be balanced can result in a significantly different shape factor than for an optimally designed warehouse. However, the difference in expected distance for an optimally designed warehouse and a balanced warehouse is relatively small for both single- and dual-command travel. Therefore, for practical purposes, designing a balanced warehouse is a reasonable design goal.

Our research showed a rule of thumb among warehouse designers of the warehouse width being twice the warehouse depth does not hold for multiple dock doors. However, designing a warehouse having a width-to-depth ratio greater than 2.0 results in a relatively small expected-distance penalty. We concluded, the rule of thumb performs very well even when multiple dock doors are installed along one of the warehouse walls.

Insofar as future research is concerned, other layout configurations having multiple dock doors can be considered. Likewise, because we assumed a random storage policy, consideration of class-based and turnover-based storage policies would be welcome. Finally, having unequal probabilities of dock usage appears to be a subject worthy of future research.

Bibliography

AMPL (2013) AMPL: A Modeling Language for Mathematical Programming. AMPL Optimization, Inc.

Ang, M., Lim, Y. F. and Sim, M. (2012) Robust storage assignment in unit-load warehouses, *Management Science*, 58 (11), 2114-2130.

Bartholdi, J. J. and Hackman, S. T. (2014) *Warehouse & Distribution Science*, Version 0.96, available at http://warehouse-science.com, accessed on January 05, 2016.

Bassan, Y., Roll, Y. and Rosenblatt, M. J. (1980) Internal layout design of a warehouse. *AIIE Transactions*, 12(4), 317-322.

Belotti, P. (2009) Couenne: a users manual. Technical report, Lehigh University.

Belotti, P., Lee, J., Liberti, L., Margot, F. and Wächter, A. (2009) Branching and bounds tightening techniques for non-convex MINLP. *Optimization Methods and Software*, 24 (4-5), 597-634.

Couenne (2006) Couenne: Convex Over and Under Envelopes for Nonlinear Estimation, an exact solver for nonconvex MINLPs, IBM and Carnegie Mellon University, available at https://projects.coin-or.org/Couenne/, accessed on May 04, 2016.

Francis, R. L. (1967a) On some problems of rectangular warehouse design and layout. *The Journal of Industrial Engineering*, 18(10), 595-604.

Francis, R. L. (1967b) Sufficient conditions for some optimum property facility designs. *Operations Research*, 15(3), 448-466.

Francis, R. L. and White, J. A. (1974) *Facility Layout and Location: An Analytic Approach*. Prentice-Hall, New Jersey, NJ.

Mallette, A. J. and Francis, R. L. (1972) A generalized assignment approach to optimal facility layout. *AIIE Transactions*, 4 (2), 144-147.

Mayer Jr., H. E. (1961) Storage and retrieval of material. *The Western Electric Engineer*, 5(1), 42-48.

Malmborg, C. J. and AlTassan, K. (2000) An integrated performance model for order picking systems with randomized storage. *Applied Mathematical Modelling*, 24(2), 95-111.

Mathematica (2015) Mathematica 10.2 for Students, Microsoft Windows Version. Wolfram Research, Inc.

Nickalls, R. W. D. (2006) Viète, Descartes and the Cubic Equation. *The Mathematical Gazette*, 90(518), 203-208, doi:10.1017/S0025557200179598.

Pohl, L. M., Meller, R. D. and Gue, K. R. (2009) An analysis of dual command travel in common warehouse designs. *Transportation Research, Part E: Logistics and Transportation Review*, 45(3), 367-379.

Roodbergen, K. J. and Vis, I. F. A. (2009) A survey of literature on automated storage and retrieval systems. *European Journal of Operational Research*, 194(2), 343-362.

Thomas, L. M. and Meller, R. D. (2014) Analytical Formulations for warehouse configuration. *IIE Transactions*, 46(9), 928-947.

Tompkins, J. A., White, J. A., Bozer, Y. A. and Tanchoco, J. M. A. (2010) *Facilities Planning*, fourth edition, Wiley, New York, NY.

Tutam, M. and White, J. A. (2015) A conventional warehouse design with multiple docks, in *Proceedings of the 2015 Industrial and Systems Engineering Research Conference*, Nashville, Tennessee.

Appendix

Proof of Proposition 2.1

Case 1: when d_i is smaller than t_i, the closest S/R aisle to dock door i is located to the left of dock door i. Therefore, the distance between dock door i and the nearest S/R aisle is $a / 2 - t_i + d_i$. Because there are t_i / a S/R aisles to the left of dock door i, the distance between dock door i and S/R aisle j located to the left of dock door i equals $(t_i / a - j)\, a + a / 2 - t_i + d_i = d_i - (j - 1 / 2)\, a$ for $j = 1, 2, \ldots, t_i / a$. Similarly, the distance between dock door i and the nearest S/R aisle located to the right of dock door i equals $a / 2 + t_i - d_i$. Because there are $n - t_i / a$ S/R aisles to the right of dock door i, the distance between dock door i and S/R aisle j located to the right of dock door i equals $(j - t_i / a - 1)\, a + a / 2 + t_i - d_i = (j - 1 / 2)\, a - d_i$ for $j = t_i / a + 1, t_i / a + 2, \ldots, n$. Therefore, the distance between dock door i and the S/R aisle j equals $|d_i - (j - 1 / 2)\, a|$ for $j = 1, 2, \ldots, n$.

Case 2: dock door i coincides with a back-to-back rack location. Therefore, in traveling to the S/R aisle nearest dock door i, the distances to the right and to the left of dock door i are identical and equal one half of the distance between two adjacent S/R aisles ($a / 2$). As before, there are $n - t_i / a$ and t_i / a S/R aisles to the right and to the left of dock door i, respectively. Therefore, the equations given for Case 1 are valid, because $t_i - d_i$ equals zero.

Case 3: when d_i is greater than t_i, the closest S/R aisle to dock door i is located to the right of dock door i. Even though the closest S/R aisle is located to the right of dock door i; the distance between dock door i and the nearest S/R aisle located to the right of dock door i still equals $a / 2 + t_i - d_i$, and the distance between dock door i and the nearest S/R aisle located to the left of dock door i still equals $a / 2 - t_i + d_i$. Again, there exist t_i / a S/R aisles to the left of dock door i and

$n - t_i / a$ aisles to the right of dock door i. Therefore, the equations given for Case 1 apply for Case 3.

Case 4: the absolute value of d_i minus t_i equals one-half the distance between two adjacent S/R aisles; movement does not exist in the parallel direction to reach the closest S/R aisle to dock door i because dock door i coincides with an S/R aisle. Therefore, equations derived for Case 1 apply for Case 4 with the absolute difference between d_i and t_i equaling $a / 2$.

Proof of Lemma 2.1

Suppose expected distance is expressed as

$$E\,[SC] \approx c_1\, W + c_2\, W^{-1} + c_3 \qquad \text{(A.1)}$$

Taking the first derivative of Equation (A.1) with respect to the warehouse's width

$$\partial\, E\,[SC] \,/\, \partial\, W \approx c_1 + c_2\, W^{-2} \qquad \text{(A.2)}$$

Setting Equation (A.2) equal to zero and solving for the warehouse's width, the stationary point is $W \approx (c_2 / c_1)^{1/2}$.

Taking the second derivative of Equation (A.1) with respect to the warehouse's width gives

$$\partial^2\, E\,[SC] \,/\, \partial\, W^2 \approx 2c_2\, W^{-3}, \qquad \text{(A.3)}$$

which is greater than zero for values of c_2 greater than zero. Because c_2 is greater than zero, Equation (A.3) is positive for all values of W. Therefore, expected single-command roundtrip-distance is a convex function of the warehouse's width and the stationary point, $W \approx (c_2 / c_1)^{1/2}$, is the optimal width.

Proof of Corollary 2.1

From Equation 2.12, the expected single-command distance for a single-dock-door is

$$E\,[SC] \approx W / 2 + \mathrm{A} / W. \qquad \text{(A.4)}$$

Therefore, from Lemma 2.1, $E\,[SC]$ is a convex function of W with stationary point $W \approx (2\,A)^{1/2}$.By definition, $S = W / D$ and $A = W\,D$. Therefore, $S = W^2 / A$. Hence, the shape factor for the optimal warehouse width is $S \approx 2.0$.

Proof of Corollary 2.2

Equations (2.13), (2.14), and (2.15) have the form

$$E\,[SC] \approx c_1\,W + c_2\,W^{-1} + c_3 \qquad \text{(A.5)}$$

Therefore, from Lemma 2.1 they are convex functions of W (c_2 is greater than zero for all scenarios) with stationary points $W \approx [3A\,(k+1) / (2k+1)]^{1/2}$, $W \approx [2A + [\delta^2\,(k^2 - 1)] / 3]^{1/2}$ and $W \approx \{[3A + 6\phi^2 + 6\phi\,\delta\,(k-1) + (2k^2 - 3k + 1)\,\delta^2] / 3\}^{1/2}$, respectively. Therefore, the shape factors for the optimal warehouse widths are $S \approx 3(k+1) / (2k+1)$, $S \approx 2 + [\delta^2\,(k^2 - 1)] / 3A$ and $S \approx 1 + [6\phi^2 + 6\phi\,\delta\,(k-1) + (2k^2 - 3k + 1)\,\delta^2] / 3A$, respectively.

Proof of Proposition 2.2

When $S \geq [(k+1)^2\,(\omega + \psi)^2] / A$, the warehouse width constraint is satisfied. From Lemma 2.1, $c_1 = (2k+1) / [3\,(k+1)]$ and $c_2 = A$ and $c_2 = 0$. Because expected roundtrip-distance is a convex function of W (from Corollary 2.1), the stationary point $S^*_{SC} \approx c_2 / (c_1\,A) \approx 3(k+1) / (2k+1)$ is the optimal shape factor. When $S < [(k+1)^2\,(\omega + \psi)^2] / A$, the width constraint is violated. Therefore, the optimum shape factor is determined by the width constraint: $S^*_{SC} \approx [(k+1)^2\,(\omega + \psi)^2] / A$.

The proof provided for Scenario 1 can be applied for Scenarios 2 and 3.

Proof of Lemma 2.2

Suppose expected distance is expressed as

$$E\,[DC] \approx (c_1\,W^3 + c_2\,W^2 + c_3\,W + c_4) / (c_5\,W^2) \qquad \text{(A.6)}$$

Taking the first derivative of Equation (A.6) with respect to the warehouse's width

$$\partial E\,[DC] \,/\, \partial W \approx (c_1 W^3 - c_3 W - 2c_4) \,/\, (c_5 W^3) \tag{A.7}$$

Equation (A.7) is an irreducible polynomial. Therefore, depressing the cubic equation and using *Viète's trigonometric solution*, the stationary point is $W \approx 2\,(c_3 / 3c_1)^{1/2} \cos\{ \arccos[c_4\, c_1^{1/2}\,(3 / c_3)^{3/2}] / 3\}$.

Taking the second derivative of Equation (A.6) with respect to the warehouse's width gives

$$\partial^2 E\,[DC] \,/\, \partial W^2 \approx (2c_3 W + 6c_4) \,/\, (c_5 W^4) \tag{A.8}$$

Evaluating Equation (A.8) yields a value greater than zero for reasonable parameter values (necessary conditions are provided in the proofs of Corollaries 2.3 and 2.4). Therefore, expected roundtrip-distance is a convex function of the warehouse's width and the stationary point, $W \approx 2\,(c_3 / 3c_1)^{1/2} \cos\{\arccos[c_4\, c_1^{1/2}\,(3 / c_3)^{3/2}] / 3\}$, is the optimal width.

Proof of Corollary 2.3

From Equation 2.17, the expected dual-command distance for a single dock door is

$$E[DC] \approx \frac{5W^3 - 4vW^2 + \left(10A - 2a^2 - 4av\right)W - 2aA}{6W^2} \tag{A.9}$$

Therefore, from Lemma 2.2, $E\,[DC]$ is a convex function of W with stationary point $W \approx 2\,(c_3 / 3c_1)^{1/2} \cos\{\arccos[c_4\, c_1^{1/2}\,(3 / c_3)^{3/2}] / 3\}$ where $c_1 = 5$, $c_3 = 10A - 2a^2 - 4av$ and $c_4 = -2\,a\,A$. By definition, $S = W / D$ and $A = W\,D$. Therefore, $S = W^2 / A$. Hence, the shape factor for the optimal warehouse width is $S \approx 4c_3\,(\cos\{\arccos[c_4\, c_1^{1/2}\,(3 / c_3)^{3/2}] / 3\})^2 / (3A\,c_1)$ where $c_1 = 5$, $c_3 = 10\,A - 2\,a^2 - 4\,a\,v$ and $c_4 = -2\,a\,A$.

Taking the second derivative of Equation (A.9) with respect to the width of the warehouse gives

$$\partial^2 E\,[DC] \,/\, \partial W^2 \approx (10A\,W - 2a^2\,W - 4a\,v\,W - 6a\,A) \,/\, (3W^4) \tag{A.10}$$

Evaluating (A.10) yields a value which is greater than zero for all $W > (3a\ A) / (5A - a^2 - 2a\ v)$ with reasonable parameter values (e.g., $10A\ W - 2a^2\ W - 4a\ v\ W - 6a\ A > 0$ for all $W > 12.0062$ ft when $A = 250{,}000$ ft^2, $a = 20$ ft and $v = 6$ ft).

Proof of Corollary 2.4

Equations (2.18), (2.19) and (2.20) have the form

$$E\,[DC] \approx (c_1\,W^3 + c_2\,W^2 + c_3\,W + c_4) / (c_5\,W^2) \qquad \text{(A.11)}$$

Therefore, from Lemma 2.2 they are convex functions of W with stationary points $W \approx 2\,(c_3 / 3c_1)^{1/2} \cos\{\arccos[c_4\,c_1^{\,1/2}\,(3 / c_3)^{3/2}] / 3\}3\})^2 / (3A\,c_1)$ where $c_1 = (2 + 3k)$, $c_3 = (1 + k)\,(5A - a^2 - 2a\,v)$ and $c_4 = -\,(1 + k)\,a\,A$ for Scenario 1; $c_1 = 5$, $c_3 = 10A - 2a^2 - 4a\,v + \delta^2\,(k^2 - 1)$ and $c_4 = -\,2a\,A$ for Scenario 2; and $c_1 = 4$, $c_3 = 5A - a^2 - 2a\,v + 6\phi^2 + 6\phi\,\delta\,(k - 1) + (2k^2 - 3k + 1)\,\delta^2$ and $c_4 = -\,a\,A$ for Scenario 3.

The second derivatives of Equations (2.18), (2.19) and (2.20) with respect to the warehouse width are

Scenario 1: $(10A\,W - 2a^2\,W - 4a\,v\,W - 6a\,A) / (3W^4)$

Scenario 2: $[10A\,W - 2a^2\,W - 4a\,v\,W - 6a\,A + (k^2 - 1)\,\delta^2\,W] / (3W^4)$

Scenario 3: $[10A\,W - 2a^2\,W - 4a\,v\,W - 6a\,A + 2(2k^2 - 3k + 1)\,\delta^2\,W + 12\phi\,\delta\,(k - 1)\,W + 12\phi^2\,W) / (3W^4)$

Finding the second derivative is greater than zero, the necessary condition for each scenario is

Scenario 1: $W > (3a\,A) / (5A - a^2 - 2a\,v)$.

(e.g. $10A\,W - 2a^2\,W - 4a\,v\,W - 6a\,A > 0$ for all $W > 12.0062$ ft when $A = 250{,}000$ ft^2, $a = 20$ ft and $v = 6$ ft)

Scenario 2: $W > (6a\,A) / [10A - 2a^2 - 4a\,v + (k^2 - 1)\,\delta^2]$

(e.g. $10A\,W - 2a^2\,W - 4a\,v\,W - 6a\,A + 2(2k^2 - 3k + 1)\,\delta^2\,W + 12\phi\,\delta\,(k - 1)\,W + 12\phi^2\,W > 0$ for all $W > 12.01$ ft when $A = 250{,}000$ ft^2, $a = 20$ ft, $v = 6$ ft, $\delta = 12$ ft and $k = 1$). Increasing the value k decreases the lower bound for W.

Scenario 3: $W > (3a\,A) / [5A - a^2 - 2a\,v + (2k^2 - 3k + 1)\,\delta^2 + 6\phi\,\delta\,(k - 1) + 6\phi^2]$

(e.g. $10A\,W - 2a^2\,W - 4a\,v\,W - 6a\,A + (k^2 - 1)\,\delta^2\,W > 0$ for all $W > 11.96$ ft when $A = 250{,}000$ ft^2, $a = 20$ ft, $v = 6$ ft, $\delta = 12$ ft, $\phi = 30$ ft and $k = 1$). Increasing the value k decreases the lower bound for W.

Proof of Proposition 2.3

Using Lemma 2.2 and Corollary 2.3, the proof of Proposition 2.2 can be applied to Proposition 2.3.

Proof of Proposition 2.4

When dock doors are equally spaced along the wall containing dock doors, the expected single-command distance (Equation (2.14)) for k dock doors is given by

$$E\,[SC] \approx [(2k + 1)\,W] / [3(k + 1)] + D. \qquad \text{(A.12)}$$

Using the relationship between a given area ($A = W^*\,D^*$) and the optimal shape factor ($S^* = W^* / D^*$), the width and depth of an optimally designed warehouse as functions of shape factor and a given area are $W^* = \sqrt{AS^*}$ and $D^* = \sqrt{A/S^*}$, respectively. Rewriting Equation (A.12) as a function of the optimal shape factor and a given area, the expected roundtrip single-command distance for Scenario 1 is

$$E\,[SC] \approx [(2k + 1)\sqrt{AS^*}\,] / [3(k + 1)] + \sqrt{A/S^*}\,. \qquad \text{(A.13)}$$

Substituting the optimal shape factor expression for Scenario 1, $S^*_{SC} \approx 3(k+1) / (2k+1)$, into Equation (A.13), the minimum expected single-command distance is $\{[A\,(2k + 1)] / [3\,(k + 1)]\}^{1/2} + \{[A\,(2k + 1)] / [3\,(k + 1)]\}^{1/2}$. Therefore, the expected horizontal roundtrip-distance equals the expected vertical roundtrip-distance when $S \geq [(k + 1)^2 (\omega + \psi)^2] / A$.

Following similar steps, we can show the expected horizontal distance also equals the expected vertical distance for dual-command travel.

Using the appropriate Equations, the proof for Scenario 1 can be applied to Scenarios 2 and 3.

Table 2.2: Discrete formulation results for *SC* with Scenario 1

k	*E*[SC]*	*n**	*m**	*W**	*D**	*A*	S^*_{SC}
1	708.00	34	86	680	368	250240	1.85
2	745.65	34	86	680	368	250240	1.85
3	764.47	34	86	680	368	250240	1.85
4	775.87	31	95	620	404	250480	1.53
5	782.75	31	95	620	404	250480	1.53
6	787.69	31	95	620	404	250480	1.53
7	791.37	31	95	620	404	250480	1.53
8	794.25	31	95	620	404	250480	1.53
9	796.54	31	95	620	404	250480	1.53
10	798.43	31	95	620	404	250480	1.53
11	799.99	31	95	620	404	250480	1.53
12	801.32	31	95	620	404	250480	1.53
13	802.45	31	95	620	404	250480	1.53
14	803.44	31	95	620	404	250480	1.53
15	804.30	31	95	620	404	250480	1.53
16	805.06	31	95	620	404	250480	1.53
17	805.74	31	95	620	404	250480	1.53
18	806.34	31	95	620	404	250480	1.53
19	806.89	31	95	620	404	250480	1.53
20	807.38	31	95	620	404	250480	1.53
21	807.83	31	95	620	404	250480	1.53
22	808.24	31	95	620	404	250480	1.53
23	808.61	31	95	620	404	250480	1.53
24	808.95	31	95	620	404	250480	1.53
25	809.27	31	95	620	404	250480	1.53
26	809.57	31	95	620	404	250480	1.53
27	809.84	31	95	620	404	250480	1.53
28	810.10	31	95	620	404	250480	1.53
29	810.33	31	95	620	404	250480	1.53
30	810.67	31	95	620	404	250480	1.53
31	810.76	31	95	620	404	250480	1.53
32	810.96	31	95	620	404	250480	1.53
33	811.14	31	95	620	404	250480	1.53
34	811.32	31	95	620	404	250480	1.53
35	811.48	31	95	620	404	250480	1.53
36	811.64	31	95	620	404	250480	1.53
37	811.78	31	95	620	404	250480	1.53
38	811.92	31	95	620	404	250480	1.53
39	812.06	31	95	620	404	250480	1.53
40	812.18	31	95	620	404	250480	1.53
41	812.30	31	95	620	404	250480	1.53
42	812.42	31	95	620	404	250480	1.53
43	812.53	31	95	620	404	250480	1.53
44	812.63	31	95	620	404	250480	1.53
45	812.73	31	95	620	404	250480	1.53

Table 2.2: Discrete formulation results for *SC* with Scenario 1 (Cont.)

k	***E*[SC]***	***n****	***m****	***W****	***D****	***A***	S^*_{SC}
46	812.83	31	95	620	404	250480	1.53
47	812.92	31	95	620	404	250480	1.53
48	813.01	31	95	620	404	250480	1.53
49	813.09	31	95	620	404	250480	1.53
50	813.17	31	95	620	404	250480	1.53
51	814.46	32	92	640	392	250880	1.63
52	814.54	32	92	640	392	250880	1.63
53	815.82	33	89	660	380	250800	1.74
54	815.90	33	89	660	380	250800	1.74
55	817.19	34	86	680	368	250240	1.85
56	822.48	35	84	700	360	252000	1.94
57	822.55	35	84	700	360	252000	1.94
58	823.84	36	81	720	348	250560	2.07
59	823.91	36	81	720	348	250560	2.07
60	829.20	37	79	740	340	251600	2.18

Table 2.3: Discrete formulation results for *DC* with Scenario 1

k	*E*[DC]*	*n**	*m**	*W**	*D**	*A*	S^*_{DC}
1	1172.11	34	86	680	368	250240	1.85
2	1209.76	34	86	680	368	250240	1.85
3	1228.58	34	86	680	368	250240	1.85
4	1240.00	34	86	680	368	250240	1.85
5	1247.56	34	86	680	368	250240	1.85
6	1252.95	34	86	680	368	250240	1.85
7	1256.99	34	86	680	368	250240	1.85
8	1260.15	34	86	680	368	250240	1.85
9	1262.67	34	86	680	368	250240	1.85
10	1264.73	34	86	680	368	250240	1.85
11	1266.44	34	86	680	368	250240	1.85
12	1267.90	34	86	680	368	250240	1.85
13	1269.15	34	86	680	368	250240	1.85
14	1270.23	34	86	680	368	250240	1.85
15	1271.17	34	86	680	368	250240	1.85
16	1272.11	34	86	680	368	250240	1.85
17	1272.75	34	86	680	368	250240	1.85
18	1273.41	34	86	680	368	250240	1.85
19	1274.01	34	86	680	368	250240	1.85
20	1274.55	34	86	680	368	250240	1.85
21	1275.04	34	86	680	368	250240	1.85
22	1275.49	34	86	680	368	250240	1.85
23	1275.90	34	86	680	368	250240	1.85
24	1276.28	34	86	680	368	250240	1.85
25	1276.62	31	95	620	404	250480	1.53
26	1276.91	31	95	620	404	250480	1.53
27	1277.19	31	95	620	404	250480	1.53
28	1277.44	31	95	620	404	250480	1.53
29	1277.68	31	95	620	404	250480	1.53
30	1277.96	32	92	640	392	250880	1.63
31	1278.11	31	95	620	404	250480	1.53
32	1278.31	31	95	620	404	250480	1.53
33	1278.49	31	95	620	404	250480	1.53
34	1278.66	31	95	620	404	250480	1.53
35	1278.83	31	95	620	404	250480	1.53
36	1278.98	31	95	620	404	250480	1.53
37	1279.13	31	95	620	404	250480	1.53
38	1279.27	31	95	620	404	250480	1.53
39	1279.40	31	95	620	404	250480	1.53
40	1279.53	31	95	620	404	250480	1.53
41	1279.65	31	95	620	404	250480	1.53
42	1279.76	31	95	620	404	250480	1.53
43	1279.87	31	95	620	404	250480	1.53
44	1279.98	31	95	620	404	250480	1.53
45	1280.08	31	95	620	404	250480	1.53

Table 2.3: Discrete formulation results for *DC* with Scenario 1 (Cont.)

k	$E^*[DC]$	n^*	m^*	W^*	D^*	A	S^*_{DC}
46	1280.17	31	95	620	404	250480	1.53
47	1280.26	31	95	620	404	250480	1.53
48	1280.35	31	95	620	404	250480	1.53
49	1280.44	31	95	620	404	250480	1.53
50	1280.52	31	95	620	404	250480	1.53
51	1280.74	32	92	640	392	250880	1.63
52	1280.82	32	92	640	392	250880	1.63
53	1281.03	33	89	660	380	250800	1.74
54	1281.11	33	89	660	380	250800	1.74
55	1281.30	34	86	680	368	250240	1.85
56	1288.11	35	84	700	360	252000	1.94
57	1288.18	35	84	700	360	252000	1.94
58	1288.35	36	81	720	348	250560	2.07
59	1288.43	36	81	720	348	250560	2.07
60	1295.21	37	79	740	340	251600	2.18

Table 2.4: Continuous approximation results for *SC* with Scenario 1

k	*E*[SC]*	*n**	*m**	*W**	*D**	*A*	S^*_{SC}
1	707.11	35.36	82.39	707.11	353.55	250000	2.00
2	745.36	33.54	87.17	670.82	372.68	250000	1.80
3	763.76	32.73	89.47	654.65	381.88	250000	1.71
4	774.60	32.27	90.82	645.50	387.30	250000	1.67
5	781.74	31.98	91.72	639.60	390.87	250000	1.64
6	786.80	31.77	92.35	635.49	393.40	250000	1.62
7	790.57	31.62	92.82	632.46	395.28	250000	1.60
8	793.49	31.51	93.19	630.13	396.75	250000	1.59
9	795.82	31.41	93.48	628.28	397.91	250000	1.58
10	797.72	31.34	93.72	626.78	398.86	250000	1.57
11	799.31	31.28	93.91	625.54	399.65	250000	1.57
12	800.64	31.22	94.08	624.50	400.32	250000	1.56
13	801.78	31.18	94.22	623.61	400.89	250000	1.56
14	802.77	31.14	94.35	622.84	401.39	250000	1.55
15	803.64	31.11	94.45	622.17	401.82	250000	1.55
16	804.40	31.08	94.55	621.58	402.20	250000	1.55
17	805.08	31.05	94.63	621.06	402.54	250000	1.54
18	805.68	31.03	94.71	620.59	402.84	250000	1.54
19	806.23	31.01	94.78	620.17	403.11	250000	1.54
20	806.72	30.99	94.84	619.80	403.36	250000	1.54
21	807.16	30.97	94.90	619.45	403.58	250000	1.53
22	807.57	30.96	94.95	619.14	403.79	250000	1.53
23	807.95	30.94	94.99	618.85	403.97	250000	1.53
24	808.29	30.93	95.04	618.59	404.15	250000	1.53
25	808.61	30.92	95.08	618.35	404.30	250000	1.53
26	808.90	30.91	95.11	618.12	404.45	250000	1.53
27	809.17	30.90	95.15	617.91	404.59	250000	1.53
28	809.43	30.89	95.18	617.72	404.71	250000	1.53
29	809.66	30.88	95.21	617.54	404.83	250000	1.53
30	809.89	30.87	95.24	617.37	404.94	250000	1.52
31	810.09	30.86	95.26	617.21	405.05	250000	1.52
32	810.29	30.85	95.29	617.07	405.14	250000	1.52
33	810.47	30.85	95.31	616.93	405.24	250000	1.52
34	810.64	30.84	95.33	616.79	405.32	250000	1.52
35	810.81	30.83	95.35	616.67	405.40	250000	1.52
36	810.96	30.83	95.37	616.55	405.48	250000	1.52
37	811.11	30.82	95.39	616.44	405.55	250000	1.52
38	811.25	30.82	95.41	616.34	405.62	250000	1.52
39	811.38	30.81	95.42	616.24	405.69	250000	1.52
40	811.50	30.81	95.44	616.14	405.75	250000	1.52
41	811.62	30.80	95.45	616.05	405.81	250000	1.52
42	811.74	30.80	95.47	615.96	405.87	250000	1.52
43	811.84	30.79	95.48	615.88	405.92	250000	1.52
44	811.95	30.79	95.49	615.80	405.97	250000	1.52
45	812.05	30.79	95.51	615.73	406.02	250000	1.52

Table 2.4: Continuous approximation results for *SC* with Scenario 1 (Cont.)

k	***E*[SC]***	***n****	***m****	***W****	***D****	***A***	S^*_{SC}
46	812.14	30.78	95.52	615.66	406.07	250000	1.52
47	812.23	30.78	95.53	615.59	406.12	250000	1.52
48	812.32	30.78	95.54	615.52	406.16	250000	1.52
49	812.40	30.77	95.55	615.46	406.20	250000	1.52
50	812.48	30.77	95.56	615.40	406.24	250000	1.51
51	812.64	31.20	94.16	624.00	400.64	250000	1.56
52	813.08	31.80	92.27	636.00	393.08	250000	1.62
53	813.80	32.40	90.45	648.00	385.80	250000	1.68
54	814.79	33.00	88.70	660.00	378.79	250000	1.74
55	816.02	33.60	87.01	672.00	372.02	250000	1.81
56	817.50	34.20	85.37	684.00	365.50	250000	1.87
57	819.20	34.80	83.80	696.00	359.20	250000	1.94
58	821.11	35.40	82.28	708.00	353.11	250000	2.01
59	823.22	36.00	80.81	720.00	347.22	250000	2.07
60	825.53	36.60	79.38	732.00	341.53	250000	2.14

Table 2.5: Continuous approximation results for *DC* with Scenario 1

k	***E*[DC]***	***n****	***m****	***W****	***D****	***A***	S^*_{DC}
1	1170.86	35.14	82.92	702.89	355.67	250000	1.98
2	1209.27	34.02	85.85	680.44	367.41	250000	1.85
3	1228.03	33.50	87.29	669.99	373.14	250000	1.80
4	1239.14	33.20	88.13	663.94	376.54	250000	1.76
5	1246.50	33.00	88.70	660.00	378.79	250000	1.74
6	1251.72	32.86	89.10	657.23	380.38	250000	1.73
7	1255.63	32.76	89.39	655.17	381.58	250000	1.72
8	1258.66	32.68	89.63	653.59	382.51	250000	1.71
9	1261.08	32.62	89.81	652.32	383.24	250000	1.70
10	1263.05	32.56	89.96	651.30	383.85	250000	1.70
11	1264.70	32.52	90.09	650.45	384.35	250000	1.69
12	1266.09	32.49	90.19	649.73	384.78	250000	1.69
13	1267.28	32.46	90.28	649.12	385.14	250000	1.69
14	1268.31	32.43	90.36	648.58	385.45	250000	1.68
15	1269.21	32.41	90.43	648.12	385.73	250000	1.68
16	1270.00	32.39	90.49	647.71	385.97	250000	1.68
17	1270.71	32.37	90.55	647.35	386.19	250000	1.68
18	1271.34	32.35	90.60	647.03	386.38	250000	1.67
19	1271.90	32.34	90.64	646.74	386.56	250000	1.67
20	1272.42	32.32	90.68	646.48	386.71	250000	1.67
21	1272.88	32.31	90.71	646.24	386.85	250000	1.67
22	1273.31	32.30	90.75	646.02	386.98	250000	1.67
23	1273.70	32.29	90.78	645.82	387.10	250000	1.67
24	1274.06	32.28	90.80	645.64	387.21	250000	1.67
25	1274.39	32.27	90.83	645.47	387.32	250000	1.67
26	1274.69	32.27	90.85	645.31	387.41	250000	1.67
27	1274.98	32.26	90.87	645.17	387.50	250000	1.66
28	1275.24	32.25	90.89	645.03	387.58	250000	1.66
29	1275.49	32.25	90.91	644.91	387.65	250000	1.66
30	1275.72	32.24	90.93	644.79	387.72	250000	1.66
31	1275.94	32.23	90.95	644.68	387.79	250000	1.66
32	1276.14	32.23	90.96	644.58	387.85	250000	1.66
33	1276.33	32.22	90.98	644.48	387.91	250000	1.66
34	1276.51	32.22	90.99	644.39	387.97	250000	1.66
35	1276.68	32.22	91.00	644.30	388.02	250000	1.66
36	1276.85	32.21	91.02	644.22	388.07	250000	1.66
37	1277.00	32.21	91.03	644.14	388.11	250000	1.66
38	1277.14	32.20	91.04	644.07	388.16	250000	1.66
39	1277.28	32.20	91.05	644.00	388.20	250000	1.66
40	1277.41	32.20	91.06	643.93	388.24	250000	1.66
41	1277.54	32.19	91.07	643.87	388.28	250000	1.66
42	1277.66	32.19	91.08	643.81	388.31	250000	1.66
43	1277.77	32.19	91.09	643.75	388.35	250000	1.66
44	1277.88	32.18	91.10	643.70	388.38	250000	1.66
45	1277.98	32.18	91.10	643.64	388.41	250000	1.66

Table 2.5: Continuous approximation results for *DC* with Scenario 1 (Cont.)

k	$E^*[DC]$	n^*	m^*	W^*	D^*	A	S^*_{DC}
46	1278.08	32.18	91.11	643.59	388.44	250000	1.66
47	1278.18	32.18	91.12	643.54	388.47	250000	1.66
48	1278.27	32.17	91.13	643.50	388.50	250000	1.66
49	1278.35	32.17	91.13	643.45	388.53	250000	1.66
50	1278.44	32.17	91.14	643.41	388.55	250000	1.66
51	1278.52	32.17	91.14	643.37	388.58	250000	1.66
52	1278.60	32.17	91.15	643.33	388.60	250000	1.66
53	1278.71	32.40	90.45	648.00	385.80	250000	1.68
54	1279.16	33.00	88.70	660.00	378.79	250000	1.74
55	1280.03	33.60	87.01	672.00	372.02	250000	1.81
56	1281.29	34.20	85.37	684.00	365.50	250000	1.87
57	1282.91	34.80	83.80	696.00	359.20	250000	1.94
58	1284.89	35.40	82.28	708.00	353.11	250000	2.01
59	1287.19	36.00	80.81	720.00	347.22	250000	2.07
60	1289.81	36.60	79.38	732.00	341.53	250000	2.14

Table 2.6: Discrete formulation results for *SC* with Scenario 2

k	***E*[SC]***	***n****	***m****	***W****	***D****	***A***	***S*$_{SC}$***
1	708.00	34	86	680	368	250240	1.85
2	708.00	34	86	680	368	250240	1.85
3	708.15	36	81	720	348	250560	2.07
4	708.44	36	81	720	348	250560	2.07
5	708.71	36	81	720	348	250560	2.07
6	709.04	36	81	720	348	250560	2.07
7	709.52	36	81	720	348	250560	2.07
8	710.00	36	81	720	348	250560	2.07
9	710.57	36	81	720	348	250560	2.07
10	711.20	36	81	720	348	250560	2.07
11	711.92	36	81	720	348	250560	2.07
12	712.67	36	81	720	348	250560	2.07
13	713.50	36	81	720	348	250560	2.07
14	714.41	36	81	720	348	250560	2.07
15	715.38	36	81	720	348	250560	2.07
16	716.39	36	81	720	348	250560	2.07
17	717.52	36	81	720	348	250560	2.07
18	718.67	36	81	720	348	250560	2.07
19	719.91	36	81	720	348	250560	2.07
20	721.20	36	81	720	348	250560	2.07
21	722.58	36	81	720	348	250560	2.07
22	724.00	36	81	720	348	250560	2.07
23	725.51	36	81	720	348	250560	2.07
24	727.07	36	81	720	348	250560	2.07
25	728.71	36	81	720	348	250560	2.07
26	730.39	36	81	720	348	250560	2.07
27	732.18	36	81	720	348	250560	2.07
28	734.00	36	81	720	348	250560	2.07
29	735.91	36	81	720	348	250560	2.07
30	737.87	36	81	720	348	250560	2.07
31	739.91	36	81	720	348	250560	2.07
32	742.00	36	81	720	348	250560	2.07
33	744.18	36	81	720	348	250560	2.07
34	746.41	36	81	720	348	250560	2.07
35	748.71	36	81	720	348	250560	2.07
36	751.06	36	81	720	348	250560	2.07
37	753.51	36	81	720	348	250560	2.07
38	756.00	36	81	720	348	250560	2.07
39	758.58	36	81	720	348	250560	2.07
40	761.20	36	81	720	348	250560	2.07
41	763.91	36	81	720	348	250560	2.07
42	766.67	36	81	720	348	250560	2.07
43	769.51	36	81	720	348	250560	2.07
44	772.40	36	81	720	348	250560	2.07
45	775.38	36	81	720	348	250560	2.07

Table 2.6: Discrete formulation results for *SC* with Scenario 2 (Cont.)

k	$E^*[SC]$	n^*	m^*	W^*	D^*	A	S^*_{SC}
46	778.40	36	81	720	348	250560	2.07
47	781.51	36	81	720	348	250560	2.07
48	784.60	37	79	740	340	251600	2.18
49	787.70	38	77	760	332	252320	2.29
50	790.81	39	75	780	324	252720	2.41
51	793.90	39	75	780	324	252720	2.41
52	797.00	40	73	800	316	252800	2.53
53	800.10	41	71	820	308	252560	2.66
54	803.20	42	69	840	300	252000	2.80
55	806.31	43	67	860	292	251120	2.95
56	809.42	43	67	860	292	251120	2.95
57	812.56	43	67	860	292	251120	2.95
58	815.78	43	67	860	292	251120	2.95
59	819.04	43	67	860	292	251120	2.95
60	822.36	43	67	860	292	251120	2.95
61	825.73	43	67	860	292	251120	2.95
62	829.17	43	67	860	292	251120	2.95
63	832.65	43	67	860	292	251120	2.95
64	836.20	43	67	860	292	251120	2.95
65	839.80	43	67	860	292	251120	2.95
66	843.46	43	67	860	292	251120	2.95
67	847.16	43	67	860	292	251120	2.95
68	850.94	43	67	860	292	251120	2.95
69	854.76	43	67	860	292	251120	2.95
70	858.64	43	67	860	292	251120	2.95
71	862.57	43	67	860	292	251120	2.95
72	867.13	46	62	920	272	250240	3.38
73	870.92	46	62	920	272	250240	3.38
74	874.75	46	62	920	272	250240	3.38
75	878.64	46	62	920	272	250240	3.38

Table 2.7: Discrete formulation results for *DC* with Scenario 2

k	*E*[DC]*	*n**	*m**	*W**	*D**	*A*	S^*_{DC}
1	1172.11	34	86	680	368	250240	1.85
2	1172.11	34	86	680	368	250240	1.85
3	1172.27	34	86	680	368	250240	1.85
4	1172.58	34	86	680	368	250240	1.85
5	1172.87	34	86	680	368	250240	1.85
6	1173.21	34	86	680	368	250240	1.85
7	1173.73	34	86	680	368	250240	1.85
8	1174.23	34	86	680	368	250240	1.85
9	1174.83	34	86	680	368	250240	1.85
10	1175.50	34	86	680	368	250240	1.85
11	1176.26	34	86	680	368	250240	1.85
12	1177.05	34	86	680	368	250240	1.85
13	1177.94	34	86	680	368	250240	1.85
14	1178.90	34	86	680	368	250240	1.85
15	1179.89	36	81	720	348	250560	2.07
16	1180.90	36	81	720	348	250560	2.07
17	1182.03	36	81	720	348	250560	2.07
18	1183.18	36	81	720	348	250560	2.07
19	1184.42	36	81	720	348	250560	2.07
20	1185.72	36	81	720	348	250560	2.07
21	1187.10	36	81	720	348	250560	2.07
22	1188.52	36	81	720	348	250560	2.07
23	1190.02	36	81	720	348	250560	2.07
24	1191.59	36	81	720	348	250560	2.07
25	1193.23	36	81	720	348	250560	2.07
26	1194.91	36	81	720	348	250560	2.07
27	1196.70	36	81	720	348	250560	2.07
28	1198.52	36	81	720	348	250560	2.07
29	1200.42	36	81	720	348	250560	2.07
30	1202.38	36	81	720	348	250560	2.07
31	1204.43	36	81	720	348	250560	2.07
32	1206.52	36	81	720	348	250560	2.07
33	1208.69	36	81	720	348	250560	2.07
34	1210.92	36	81	720	348	250560	2.07
35	1213.23	36	81	720	348	250560	2.07
36	1215.58	36	81	720	348	250560	2.07
37	1218.03	36	81	720	348	250560	2.07
38	1220.52	36	81	720	348	250560	2.07
39	1223.09	36	81	720	348	250560	2.07
40	1225.72	36	81	720	348	250560	2.07
41	1228.43	36	81	720	348	250560	2.07
42	1231.18	36	81	720	348	250560	2.07
43	1234.02	36	81	720	348	250560	2.07
44	1236.92	36	81	720	348	250560	2.07
45	1239.89	36	81	720	348	250560	2.07

Table 2.7: Discrete formulation results for *DC* with Scenario 2 (Cont.)

k	***E**[*DC*]**	***n****	***m****	***W****	***D****	***A***	S^*_{DC}
46	1242.91	36	81	720	348	250560	2.07
47	1246.03	36	81	720	348	250560	2.07
48	1249.18	36	81	720	348	250560	2.07
49	1252.42	36	81	720	348	250560	2.07
50	1255.72	36	81	720	348	250560	2.07
51	1259.10	36	81	720	348	250560	2.07
52	1262.52	36	81	720	348	250560	2.07
53	1266.03	36	81	720	348	250560	2.07
54	1269.59	36	81	720	348	250560	2.07
55	1273.23	36	81	720	348	250560	2.07
56	1276.91	36	81	720	348	250560	2.07
57	1280.69	36	81	720	348	250560	2.07
58	1284.52	36	81	720	348	250560	2.07
59	1288.43	36	81	720	348	250560	2.07
60	1292.38	36	81	720	348	250560	2.07
61	1296.56	37	79	740	340	251600	2.18
62	1300.77	38	77	760	332	252320	2.29
63	1304.73	38	77	760	332	252320	2.29
64	1308.91	39	75	780	324	252720	2.41
65	1312.87	39	75	780	324	252720	2.41
66	1317.03	40	73	800	316	252800	2.53
67	1321.20	41	71	820	308	252560	2.66
68	1325.16	41	71	820	308	252560	2.66
69	1329.32	42	69	840	300	252000	2.80
70	1333.29	42	69	840	300	252000	2.80
71	1337.42	43	67	860	292	251120	2.95
72	1348.22	44	66	880	288	253440	3.06
73	1352.18	44	66	880	288	253440	3.06
74	1356.32	45	64	900	280	252000	3.21
75	1360.28	45	64	900	280	252000	3.21

Table 2.8: Continuous approximation results for *SC* with Scenario 2

k	$E^*[SC]$	n^*	m^*	W^*	D^*	A	S^*_{SC}
1	707.11	35.36	82.39	707.11	353.55	250000	2.00
2	707.21	35.36	82.38	707.21	353.50	250000	2.00
3	707.38	35.37	82.35	707.38	353.42	250000	2.00
4	707.62	35.38	82.32	707.62	353.30	250000	2.00
5	707.92	35.40	82.29	707.92	353.15	250000	2.00
6	708.29	35.41	82.24	708.29	352.96	250000	2.01
7	708.73	35.44	82.19	708.73	352.74	250000	2.01
8	709.24	35.46	82.12	709.24	352.49	250000	2.01
9	709.82	35.49	82.05	709.82	352.20	250000	2.02
10	710.46	35.52	81.97	710.46	351.89	250000	2.02
11	711.17	35.56	81.88	711.17	351.53	250000	2.02
12	711.94	35.60	81.79	711.94	351.15	250000	2.03
13	712.79	35.64	81.68	712.79	350.74	250000	2.03
14	713.69	35.68	81.57	713.69	350.29	250000	2.04
15	714.67	35.73	81.45	714.67	349.81	250000	2.04
16	715.71	35.79	81.33	715.71	349.30	250000	2.05
17	716.82	35.84	81.19	716.82	348.76	250000	2.06
18	717.99	35.90	81.05	717.99	348.20	250000	2.06
19	719.22	35.96	80.90	719.22	347.60	250000	2.07
20	720.52	36.03	80.74	720.52	346.97	250000	2.08
21	721.89	36.09	80.58	721.89	346.31	250000	2.08
22	723.31	36.17	80.41	723.31	345.63	250000	2.09
23	724.81	36.24	80.23	724.81	344.92	250000	2.10
24	726.36	36.32	80.05	726.36	344.18	250000	2.11
25	727.98	36.40	79.85	727.98	343.42	250000	2.12
26	729.66	36.48	79.66	729.66	342.63	250000	2.13
27	731.40	36.57	79.45	731.40	341.81	250000	2.14
28	733.20	36.66	79.24	733.20	340.97	250000	2.15
29	735.06	36.75	79.03	735.06	340.11	250000	2.16
30	736.99	36.85	78.80	736.99	339.22	250000	2.17
31	738.97	36.95	78.58	738.97	338.31	250000	2.18
32	741.02	37.05	78.34	741.02	337.37	250000	2.20
33	743.12	37.16	78.11	743.12	336.42	250000	2.21
34	745.28	37.26	77.86	745.28	335.45	250000	2.22
35	747.50	37.37	77.61	747.50	334.45	250000	2.24
36	749.77	37.49	77.36	749.77	333.43	250000	2.25
37	752.11	37.61	77.10	752.11	332.40	250000	2.26
38	754.50	37.72	76.84	754.50	331.35	250000	2.28
39	756.94	37.85	76.57	756.94	330.28	250000	2.29
40	759.44	37.97	76.30	759.44	329.19	250000	2.31
41	762.00	38.10	76.02	762.00	328.09	250000	2.32
42	764.61	38.23	75.74	764.61	326.97	250000	2.34
43	767.27	38.36	75.46	767.27	325.83	250000	2.35
44	769.99	38.50	75.17	769.99	324.68	250000	2.37
45	772.76	38.64	74.88	772.76	323.52	250000	2.39

Table 2.8: Continuous approximation results for *SC* with Scenario 2 (Cont.)

k	***E**[*SC*]**	***n****	***m****	***W****	***D****	***A***	S^*_{SC}
46	775.58	38.78	74.59	775.58	322.34	250000	2.41
47	778.45	38.92	74.29	778.45	321.15	250000	2.42
48	781.37	39.07	73.99	781.37	319.95	250000	2.44
49	784.35	39.22	73.68	784.35	318.74	250000	2.46
50	787.37	39.37	73.38	787.37	317.51	250000	2.48
51	790.44	39.52	73.07	790.44	316.28	250000	2.50
52	793.56	39.68	72.76	793.56	315.03	250000	2.52
53	796.73	39.84	72.45	796.73	313.78	250000	2.54
54	799.95	40.00	72.13	799.95	312.52	250000	2.56
55	803.21	40.16	71.81	803.21	311.25	250000	2.58
56	806.52	40.33	71.49	806.52	309.97	250000	2.60
57	809.88	40.49	71.17	809.88	308.69	250000	2.62
58	813.28	40.66	70.85	813.28	307.40	250000	2.65
59	816.73	40.84	70.53	816.73	306.10	250000	2.67
60	820.21	41.01	70.20	820.21	304.80	250000	2.69
61	823.75	41.19	69.87	823.75	303.49	250000	2.71
62	827.32	41.37	69.54	827.32	302.18	250000	2.74
63	830.94	41.55	69.22	830.94	300.86	250000	2.76
64	834.60	41.73	68.89	834.60	299.54	250000	2.79
65	838.30	41.92	68.56	838.30	298.22	250000	2.81
66	842.05	42.10	68.22	842.05	296.90	250000	2.84
67	845.83	42.29	67.89	845.83	295.57	250000	2.86
68	849.65	42.48	67.56	849.65	294.24	250000	2.89
69	853.51	42.68	67.23	853.51	292.91	250000	2.91
70	857.41	42.87	66.89	857.41	291.58	250000	2.94
71	861.35	43.07	66.56	861.35	290.24	250000	2.97
72	865.32	43.27	66.23	865.32	288.91	250000	3.00
73	869.36	43.80	65.35	876.00	285.39	250000	3.07
74	873.50	44.40	64.38	888.00	281.53	250000	3.15
75	877.75	45.00	63.44	900.00	277.78	250000	3.24

Table 2.9: Continuous approximation results for *DC* with Scenario 2

k	***E*[DC]***	***n****	***m****	***W****	***D****	***A***	S^*_{DC}
1	1170.86	35.14	82.92	702.89	355.67	250000	1.98
2	1170.96	35.15	82.91	702.95	355.64	250000	1.98
3	1171.13	35.15	82.90	703.05	355.59	250000	1.98
4	1171.37	35.16	82.88	703.20	355.52	250000	1.98
5	1171.67	35.17	82.86	703.38	355.42	250000	1.98
6	1172.05	35.18	82.83	703.61	355.31	250000	1.98
7	1172.49	35.19	82.79	703.88	355.18	250000	1.98
8	1173.00	35.21	82.75	704.19	355.02	250000	1.98
9	1173.58	35.23	82.71	704.54	354.84	250000	1.99
10	1174.23	35.25	82.66	704.93	354.65	250000	1.99
11	1174.95	35.27	82.61	705.36	354.43	250000	1.99
12	1175.73	35.29	82.55	705.83	354.19	250000	1.99
13	1176.58	35.32	82.48	706.34	353.94	250000	2.00
14	1177.49	35.34	82.41	706.90	353.66	250000	2.00
15	1178.48	35.37	82.34	707.49	353.36	250000	2.00
16	1179.53	35.41	82.26	708.12	353.05	250000	2.01
17	1180.65	35.44	82.18	708.80	352.71	250000	2.01
18	1181.83	35.48	82.09	709.51	352.35	250000	2.01
19	1183.08	35.51	81.99	710.27	351.98	250000	2.02
20	1184.40	35.55	81.90	711.06	351.59	250000	2.02
21	1185.78	35.59	81.79	711.90	351.17	250000	2.03
22	1187.23	35.64	81.69	712.77	350.74	250000	2.03
23	1188.75	35.68	81.57	713.68	350.29	250000	2.04
24	1190.33	35.73	81.46	714.64	349.83	250000	2.04
25	1191.97	35.78	81.34	715.63	349.34	250000	2.05
26	1193.68	35.83	81.21	716.66	348.84	250000	2.05
27	1195.45	35.89	81.08	717.73	348.32	250000	2.06
28	1197.29	35.94	80.95	718.84	347.78	250000	2.07
29	1199.19	36.00	80.81	719.99	347.23	250000	2.07
30	1201.16	36.06	80.66	721.17	346.66	250000	2.08
31	1203.19	36.12	80.52	722.40	346.07	250000	2.09
32	1205.28	36.18	80.37	723.66	345.47	250000	2.09
33	1207.43	36.25	80.21	724.96	344.85	250000	2.10
34	1209.65	36.31	80.05	726.29	344.21	250000	2.11
35	1211.92	36.38	79.89	727.67	343.56	250000	2.12
36	1214.26	36.45	79.72	729.08	342.90	250000	2.13
37	1216.66	36.53	79.55	730.53	342.22	250000	2.13
38	1219.13	36.60	79.38	732.01	341.53	250000	2.14
39	1221.65	36.68	79.20	733.53	340.82	250000	2.15
40	1224.23	36.75	79.02	735.09	340.10	250000	2.16
41	1226.87	36.83	78.84	736.68	339.36	250000	2.17
42	1229.57	36.92	78.65	738.31	338.61	250000	2.18
43	1232.33	37.00	78.46	739.97	337.85	250000	2.19
44	1235.15	37.08	78.27	741.67	337.08	250000	2.20
45	1238.03	37.17	78.07	743.41	336.29	250000	2.21

Table 2.9: Continuous approximation results for *DC* with Scenario 2 (Cont.)

k	***E**[*DC*]**	***n****	***m****	***W****	***D****	***A***	S^*_{DC}
46	1240.96	37.26	77.87	745.18	335.49	250000	2.22
47	1243.95	37.35	77.67	746.98	334.68	250000	2.23
48	1247.00	37.44	77.46	748.82	333.86	250000	2.24
49	1250.11	37.53	77.26	750.69	333.03	250000	2.25
50	1253.27	37.63	77.05	752.60	332.18	250000	2.27
51	1256.49	37.73	76.83	754.53	331.33	250000	2.28
52	1259.76	37.83	76.62	756.51	330.47	250000	2.29
53	1263.08	37.93	76.40	758.51	329.59	250000	2.30
54	1266.47	38.03	76.18	760.55	328.71	250000	2.31
55	1269.90	38.13	75.95	762.62	327.82	250000	2.33
56	1273.39	38.24	75.73	764.72	326.92	250000	2.34
57	1276.93	38.34	75.50	766.86	326.01	250000	2.35
58	1280.52	38.45	75.27	769.02	325.09	250000	2.37
59	1284.17	38.56	75.04	771.22	324.16	250000	2.38
60	1287.87	38.67	74.81	773.45	323.23	250000	2.39
61	1291.62	38.79	74.57	775.71	322.29	250000	2.41
62	1295.42	38.90	74.33	778.00	321.34	250000	2.42
63	1299.27	39.02	74.10	780.32	320.38	250000	2.44
64	1303.17	39.13	73.86	782.67	319.42	250000	2.45
65	1307.12	39.25	73.61	785.05	318.45	250000	2.47
66	1311.14	39.60	72.91	792.00	315.66	250000	2.51
67	1315.37	40.20	71.74	804.00	310.95	250000	2.59
68	1319.83	40.80	70.59	816.00	306.37	250000	2.66
69	1324.50	41.40	69.48	828.00	301.93	250000	2.74
70	1329.39	42.00	68.40	840.00	297.62	250000	2.82
71	1334.47	42.60	67.36	852.00	293.43	250000	2.90
72	1339.75	43.20	66.34	864.00	289.35	250000	2.99
73	1345.20	43.80	65.35	876.00	285.39	250000	3.07
74	1350.84	44.40	64.38	888.00	281.53	250000	3.15
75	1356.64	45.00	63.44	900.00	277.78	250000	3.24

Table 2.10: Discrete formulation results for *SC* with Scenario 3

k	***E*[SC]***	***n****	***m****	***W****	***D****	***A***	S^*_{SC}
1	943.20	25	119	500	500	250000	1.00
2	933.12	25	119	500	500	250000	1.00
3	923.25	25	119	500	500	250000	1.00
4	913.76	25	119	500	500	250000	1.00
5	904.67	25	119	500	500	250000	1.00
6	895.89	25	119	500	500	250000	1.00
7	887.57	25	119	500	500	250000	1.00
8	879.60	25	119	500	500	250000	1.00
9	872.02	25	119	500	500	250000	1.00
10	864.83	25	119	500	500	250000	1.00
11	857.99	25	119	500	500	250000	1.00
12	851.57	25	119	500	500	250000	1.00
13	845.53	25	119	500	500	250000	1.00
14	839.72	27	110	540	464	250560	1.16
15	833.96	27	110	540	464	250560	1.16
16	828.52	27	110	540	464	250560	1.16
17	823.46	27	110	540	464	250560	1.16
18	818.75	27	110	540	464	250560	1.16
19	814.39	27	110	540	464	250560	1.16
20	810.40	27	110	540	464	250560	1.16
21	806.74	27	110	540	464	250560	1.16
22	803.46	27	110	540	464	250560	1.16
23	800.22	29	102	580	432	250560	1.34
24	796.99	29	102	580	432	250560	1.34
25	794.10	29	102	580	432	250560	1.34
26	791.52	29	102	580	432	250560	1.34
27	789.29	29	102	580	432	250560	1.34
28	787.39	29	102	580	432	250560	1.34
29	785.82	29	102	580	432	250560	1.34
30	784.58	29	102	580	432	250560	1.34
31	783.30	31	95	620	404	250480	1.53
32	781.98	31	95	620	404	250480	1.53
33	780.98	31	95	620	404	250480	1.53
34	780.28	31	95	620	404	250480	1.53
35	779.90	31	95	620	404	250480	1.53
36	779.81	31	95	620	404	250480	1.53
37	780.05	31	95	620	404	250480	1.53
38	780.45	32	92	640	392	250880	1.63
39	780.87	33	89	660	380	250800	1.74
40	781.24	33	89	660	380	250800	1.74
41	781.60	34	86	680	368	250240	1.85
42	782.16	34	86	680	368	250240	1.85
43	783.01	34	86	680	368	250240	1.85
44	784.14	34	86	680	368	250240	1.85
45	785.55	34	86	680	368	250240	1.85

Table 2.10: Discrete formulation results for *SC* with Scenario 3 (Cont.)

k	***E**[*SC*]**	***n****	***m****	***W****	***D****	***A***	S^*_{SC}
46	787.24	34	86	680	368	250240	1.85
47	789.22	34	86	680	368	250240	1.85
48	791.06	36	81	720	348	250560	2.07
49	792.80	36	81	720	348	250560	2.07
50	794.80	36	81	720	348	250560	2.07
51	797.06	36	81	720	348	250560	2.07
52	799.60	36	81	720	348	250560	2.07
53	802.40	36	81	720	348	250560	2.07
54	805.47	36	81	720	348	250560	2.07
55	808.67	37	79	740	340	251600	2.18
56	811.85	37	79	740	340	251600	2.18
57	814.99	38	77	760	332	252320	2.29
58	818.21	39	75	780	324	252720	2.41
59	821.35	39	75	780	324	252720	2.41
60	824.52	40	73	800	316	252800	2.53
61	827.76	40	73	800	316	252800	2.53
62	830.87	41	71	820	308	252560	2.66
63	834.06	42	69	840	300	252000	2.80
64	837.26	42	69	840	300	252000	2.80
65	840.39	43	67	860	292	251120	2.95
66	843.68	43	67	860	292	251120	2.95
67	847.20	43	67	860	292	251120	2.95
68	850.94	43	67	860	292	251120	2.95
69	854.90	43	67	860	292	251120	2.95
70	860.28	46	62	920	272	250240	3.38
71	863.62	46	62	920	272	250240	3.38
72	867.16	46	62	920	272	250240	3.38
73	870.92	46	62	920	272	250240	3.38
74	874.89	46	62	920	272	250240	3.38
75	881.55	47	61	940	268	251920	3.51

Table 2.11: Discrete formulation results for *DC* with Scenario 3

k	*E*[DC]*	*n**	*m**	*W**	*D**	*A*	S^*_{DC}
1	1426.21	27	110	540	464	250560	1.16
2	1415.99	27	110	540	464	250560	1.16
3	1405.96	27	110	540	464	250560	1.16
4	1396.28	27	110	540	464	250560	1.16
5	1386.96	29	102	580	432	250560	1.34
6	1377.74	29	102	580	432	250560	1.34
7	1368.90	29	102	580	432	250560	1.34
8	1360.38	29	102	580	432	250560	1.34
9	1352.19	29	102	580	432	250560	1.34
10	1344.34	29	102	580	432	250560	1.34
11	1336.79	29	102	580	432	250560	1.34
12	1329.60	29	102	580	432	250560	1.34
13	1322.73	29	102	580	432	250560	1.34
14	1316.19	29	102	580	432	250560	1.34
15	1309.99	29	102	580	432	250560	1.34
16	1304.10	29	102	580	432	250560	1.34
17	1298.57	29	102	580	432	250560	1.34
18	1293.35	29	102	580	432	250560	1.34
19	1288.47	29	102	580	432	250560	1.34
20	1283.92	29	102	580	432	250560	1.34
21	1279.69	29	102	580	432	250560	1.34
22	1275.81	29	102	580	432	250560	1.34
23	1272.25	29	102	580	432	250560	1.34
24	1268.53	31	95	620	404	250480	1.53
25	1265.05	31	95	620	404	250480	1.53
26	1261.87	31	95	620	404	250480	1.53
27	1259.01	31	95	620	404	250480	1.53
28	1256.45	31	95	620	404	250480	1.53
29	1254.21	31	95	620	404	250480	1.53
30	1252.28	31	95	620	404	250480	1.53
31	1250.64	31	95	620	404	250480	1.53
32	1249.33	31	95	620	404	250480	1.53
33	1248.32	31	95	620	404	250480	1.53
34	1247.63	31	95	620	404	250480	1.53
35	1247.18	32	92	640	392	250880	1.63
36	1246.73	33	89	660	380	250800	1.74
37	1246.22	33	89	660	380	250800	1.74
38	1245.71	34	86	680	368	250240	1.85
39	1245.43	34	86	680	368	250240	1.85
40	1245.43	34	86	680	368	250240	1.85
41	1245.71	34	86	680	368	250240	1.85
42	1246.28	34	86	680	368	250240	1.85
43	1247.12	34	86	680	368	250240	1.85
44	1248.25	34	86	680	368	250240	1.85
45	1249.67	34	86	680	368	250240	1.85

Table 2.11: Discrete formulation results for *DC* with Scenario 3 (Cont.)

k	$E^*[DC]$	n^*	m^*	W^*	D^*	A	S^*_{DC}
46	1251.36	34	86	680	368	250240	1.85
47	1253.33	34	86	680	368	250240	1.85
48	1255.58	36	81	720	348	250560	2.07
49	1257.31	36	81	720	348	250560	2.07
50	1259.32	36	81	720	348	250560	2.07
51	1261.58	36	81	720	348	250560	2.07
52	1264.11	36	81	720	348	250560	2.07
53	1266.91	36	81	720	348	250560	2.07
54	1269.98	36	81	720	348	250560	2.07
55	1273.32	36	81	720	348	250560	2.07
56	1276.91	36	81	720	348	250560	2.07
57	1280.78	36	81	720	348	250560	2.07
58	1284.91	36	81	720	348	250560	2.07
59	1288.96	37	79	740	340	251600	2.18
60	1293.11	38	77	760	332	252320	2.29
61	1297.15	38	77	760	332	252320	2.29
62	1301.24	39	75	780	324	252720	2.41
63	1305.36	39	75	780	324	252720	2.41
64	1309.39	40	73	800	316	252800	2.53
65	1313.52	41	71	820	308	252560	2.66
66	1317.55	41	71	820	308	252560	2.66
67	1321.63	42	69	840	300	252000	2.80
68	1325.74	42	69	840	300	252000	2.80
69	1329.76	43	67	860	292	251120	2.95
70	1340.51	44	66	880	288	253440	3.06
71	1344.54	44	66	880	288	253440	3.06
72	1348.60	45	64	900	280	252000	3.21
73	1352.71	45	64	900	280	252000	3.21
74	1356.72	46	62	920	272	250240	3.38
75	1367.46	47	61	940	268	251920	3.51

Table 2.12: Continuous approximation results for *SC* with Scenario 3

k	***E*[SC]***	***n****	***m****	***W****	***D****	***A***	S^*_{SC}
1	943.59	25.09	118.55	501.80	498.21	250000	1.01
2	933.31	25.13	118.34	502.66	497.36	250000	1.01
3	923.41	25.19	118.08	503.71	496.32	250000	1.01
4	913.89	25.25	117.78	504.94	495.10	250000	1.02
5	904.73	25.32	117.43	506.37	493.71	250000	1.03
6	895.95	25.40	117.04	507.98	492.15	250000	1.03
7	887.54	25.49	116.60	509.77	490.42	250000	1.04
8	879.48	25.59	116.13	511.74	488.53	250000	1.05
9	871.79	25.69	115.62	513.89	486.48	250000	1.06
10	864.45	25.81	115.07	516.22	484.29	250000	1.07
11	857.46	25.94	114.49	518.73	481.95	250000	1.08
12	850.81	26.07	113.87	521.41	479.47	250000	1.09
13	844.50	26.21	113.22	524.25	476.87	250000	1.10
14	838.53	26.36	112.54	527.26	474.15	250000	1.11
15	832.88	26.52	111.83	530.44	471.31	250000	1.13
16	827.56	26.69	111.09	533.78	468.36	250000	1.14
17	822.55	26.86	110.33	537.27	465.31	250000	1.15
18	817.85	27.05	109.54	540.93	462.17	250000	1.17
19	813.45	27.24	108.74	544.73	458.95	250000	1.19
20	809.36	27.43	107.91	548.68	455.64	250000	1.20
21	805.55	27.64	107.07	552.77	452.26	250000	1.22
22	802.03	27.85	106.21	557.01	448.82	250000	1.24
23	798.78	28.07	105.33	561.39	445.32	250000	1.26
24	795.81	28.30	104.44	565.90	441.77	250000	1.28
25	793.10	28.53	103.54	570.55	438.17	250000	1.30
26	790.65	28.77	102.63	575.33	434.54	250000	1.32
27	788.46	29.01	101.72	580.23	430.87	250000	1.35
28	786.50	29.26	100.79	585.25	427.17	250000	1.37
29	784.79	29.52	99.86	590.40	423.44	250000	1.39
30	783.32	29.78	98.93	595.66	419.70	250000	1.42
31	782.06	30.05	97.99	601.03	415.95	250000	1.44
32	781.04	30.33	97.05	606.52	412.19	250000	1.47
33	780.22	30.61	96.11	612.11	408.42	250000	1.50
34	779.62	30.89	95.16	617.81	404.66	250000	1.53
35	779.22	31.18	94.22	623.61	400.89	250000	1.56
36	779.02	31.48	93.28	629.51	397.14	250000	1.59
37	779.01	31.78	92.35	635.50	393.39	250000	1.62
38	779.18	32.08	91.41	641.59	389.66	250000	1.65
39	779.54	32.39	90.48	647.77	385.94	250000	1.68
40	780.08	32.70	89.56	654.04	382.24	250000	1.71
41	780.79	33.02	88.64	660.39	378.56	250000	1.74
42	781.66	33.34	87.73	666.83	374.91	250000	1.78
43	782.70	33.67	86.82	673.35	371.28	250000	1.81
44	783.89	34.00	85.92	679.95	367.68	250000	1.85
45	785.24	34.33	85.03	686.62	364.10	250000	1.89

Table 2.12: Continuous approximation results for *SC* with Scenario 3 (Cont.)

k	***E*[SC]***	***n****	***m****	***W****	***D****	***A***	S^*_{SC}
46	786.74	34.67	84.14	693.37	360.56	250000	1.92
47	788.38	35.01	83.26	700.19	357.05	250000	1.96
48	790.16	35.35	82.39	707.08	353.57	250000	2.00
49	792.07	35.70	81.53	714.04	350.12	250000	2.04
50	794.12	36.05	80.68	721.06	346.71	250000	2.08
51	796.30	36.41	79.83	728.15	343.34	250000	2.12
52	798.60	36.76	79.00	735.30	340.00	250000	2.16
53	801.02	37.13	78.17	742.51	336.70	250000	2.21
54	803.56	37.49	77.36	749.78	333.43	250000	2.25
55	806.21	37.86	76.55	757.11	330.21	250000	2.29
56	808.97	38.22	75.75	764.49	327.02	250000	2.34
57	811.84	38.60	74.97	771.92	323.87	250000	2.38
58	814.82	38.97	74.19	779.41	320.76	250000	2.43
59	817.90	39.35	73.42	786.95	317.68	250000	2.48
60	821.07	39.73	72.66	794.54	314.65	250000	2.53
61	824.34	40.11	71.91	802.17	311.65	250000	2.57
62	827.71	40.49	71.17	809.85	308.70	250000	2.62
63	831.16	40.88	70.44	817.58	305.78	250000	2.67
64	834.71	41.27	69.73	825.35	302.90	250000	2.72
65	838.33	41.66	69.01	833.17	300.06	250000	2.78
66	842.05	42.05	68.31	841.02	297.26	250000	2.83
67	845.84	42.45	67.62	848.92	294.49	250000	2.88
68	849.71	42.84	66.94	856.85	291.76	250000	2.94
69	853.66	43.24	66.27	864.83	289.07	250000	2.99
70	857.68	43.64	65.61	872.84	286.42	250000	3.05
71	861.77	44.04	64.95	880.89	283.81	250000	3.10
72	865.94	44.45	64.31	888.97	281.22	250000	3.16
73	870.18	45.00	63.44	900.00	277.78	250000	3.24
74	874.52	45.60	62.53	912.00	274.12	250000	3.33
75	878.95	46.20	61.64	924.00	270.56	250000	3.42

Table 2.13: Continuous approximation results for *DC* with Scenario 3

k	*E*[DC]*	*n**	*m**	*W**	*D**	*A*	S^*_{DC}
1	1424.20	27.80	106.40	556.05	449.60	250000	1.24
2	1413.75	27.83	106.28	556.64	449.12	250000	1.24
3	1403.65	27.87	106.14	557.35	448.55	250000	1.24
4	1393.88	27.91	105.97	558.20	447.87	250000	1.25
5	1384.46	27.96	105.77	559.17	447.09	250000	1.25
6	1375.38	28.01	105.55	560.27	446.21	250000	1.26
7	1366.63	28.08	105.31	561.50	445.23	250000	1.26
8	1358.21	28.14	105.04	562.86	444.16	250000	1.27
9	1350.13	28.22	104.75	564.34	443.00	250000	1.27
10	1342.38	28.30	104.44	565.94	441.74	250000	1.28
11	1334.95	28.38	104.10	567.67	440.40	250000	1.29
12	1327.85	28.48	103.74	569.52	438.97	250000	1.30
13	1321.06	28.57	103.36	571.49	437.46	250000	1.31
14	1314.60	28.68	102.97	573.57	435.86	250000	1.32
15	1308.44	28.79	102.55	575.78	434.19	250000	1.33
16	1302.60	28.91	102.11	578.11	432.45	250000	1.34
17	1297.06	29.03	101.66	580.54	430.63	250000	1.35
18	1291.83	29.15	101.19	583.10	428.74	250000	1.36
19	1286.89	29.29	100.70	585.76	426.79	250000	1.37
20	1282.25	29.43	100.20	588.54	424.78	250000	1.39
21	1277.90	29.57	99.68	591.42	422.71	250000	1.40
22	1273.83	29.72	99.14	594.42	420.58	250000	1.41
23	1270.05	29.88	98.60	597.52	418.40	250000	1.43
24	1266.54	30.04	98.04	600.72	416.17	250000	1.44
25	1263.30	30.20	97.47	604.03	413.89	250000	1.46
26	1260.34	30.37	96.89	607.43	411.57	250000	1.48
27	1257.63	30.55	96.30	610.94	409.21	250000	1.49
28	1255.19	30.73	95.70	614.54	406.81	250000	1.51
29	1253.00	30.91	95.09	618.24	404.37	250000	1.53
30	1251.07	31.10	94.48	622.03	401.91	250000	1.55
31	1249.37	31.30	93.85	625.92	399.41	250000	1.57
32	1247.92	31.49	93.22	629.90	396.89	250000	1.59
33	1246.71	31.70	92.59	633.96	394.35	250000	1.61
34	1245.73	31.91	91.95	638.11	391.78	250000	1.63
35	1244.97	32.12	91.30	642.35	389.20	250000	1.65
36	1244.44	32.33	90.65	646.67	386.60	250000	1.67
37	1244.13	32.55	90.00	651.07	383.98	250000	1.70
38	1244.03	32.78	89.34	655.56	381.35	250000	1.72
39	1244.15	33.01	88.68	660.12	378.72	250000	1.74
40	1244.46	33.24	88.02	664.76	376.08	250000	1.77
41	1244.98	33.47	87.36	669.47	373.43	250000	1.79
42	1245.70	33.71	86.69	674.26	370.78	250000	1.82
43	1246.61	33.96	86.03	679.12	368.12	250000	1.84
44	1247.71	34.20	85.37	684.06	365.47	250000	1.87
45	1248.99	34.45	84.70	689.06	362.82	250000	1.90

Table 2.13: Continuous approximation results for *DC* with Scenario 3 (Cont.)

k	***E*[DC]***	***n****	***m****	***W****	***D****	***A***	S^*_{DC}
46	1250.46	34.71	84.04	694.12	360.17	250000	1.93
47	1252.10	34.96	83.38	699.26	357.52	250000	1.96
48	1253.91	35.22	82.72	704.46	354.88	250000	1.99
49	1255.90	35.49	82.06	709.72	352.25	250000	2.01
50	1258.05	35.75	81.41	715.05	349.63	250000	2.05
51	1260.36	36.02	80.75	720.43	347.01	250000	2.08
52	1262.83	36.29	80.10	725.87	344.41	250000	2.11
53	1265.45	36.57	79.46	731.38	341.82	250000	2.14
54	1268.23	36.85	78.81	736.94	339.24	250000	2.17
55	1271.15	37.13	78.17	742.55	336.68	250000	2.21
56	1274.22	37.41	77.53	748.22	334.13	250000	2.24
57	1277.43	37.70	76.90	753.94	331.59	250000	2.27
58	1280.78	37.99	76.27	759.71	329.07	250000	2.31
59	1284.26	38.28	75.64	765.53	326.57	250000	2.34
60	1287.88	38.57	75.02	771.40	324.08	250000	2.38
61	1291.62	38.87	74.40	777.32	321.62	250000	2.42
62	1295.50	39.16	73.79	783.29	319.17	250000	2.45
63	1299.49	39.47	73.18	789.31	316.73	250000	2.49
64	1303.60	39.77	72.58	795.36	314.32	250000	2.53
65	1307.85	40.20	71.74	804.00	310.95	250000	2.59
66	1312.30	40.80	70.59	816.00	306.37	250000	2.66
67	1316.97	41.40	69.48	828.00	301.93	250000	2.74
68	1321.84	42.00	68.40	840.00	297.62	250000	2.82
69	1326.92	42.60	67.36	852.00	293.43	250000	2.90
70	1332.19	43.20	66.34	864.00	289.35	250000	2.99
71	1337.64	43.80	65.35	876.00	285.39	250000	3.07
72	1343.27	44.40	64.38	888.00	281.53	250000	3.15
73	1349.07	45.00	63.44	900.00	277.78	250000	3.24
74	1355.03	45.60	62.53	912.00	274.12	250000	3.33
75	1361.14	46.20	61.64	924.00	270.56	250000	3.42

Certification of Student Work

UNIVERSITY OF ARKANSAS

College of Engineering
Department of Industrial Engineering
308 John A. White, Jr. Engineering Hall

MEMORANDUM

TO: University of Arkansas Graduate School

FROM: John A. White, Distinguished Professor

DATE: June 27, 2018

SUBJECT: Certification of Student Effort and Contribution

I certify Mr. Mahmut Tutam contributed more than 51 percent of the work included in the chapter entitled, "Contribution 1: Revisions to a Submitted Paper on, 'A Multi-Dock, Unit-Load Warehouse Design'" contained in the doctoral dissertation entitled, "Configuring Traditional Multi-Dock, Unit-Load Warehouses".

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Chapter 3

Contribution 2: A Working Paper on, "Multi-Dock Unit-Load Warehouse Designs with a Cross-Aisle"

Abstract

Defining shape factor as the width-to-depth ratio of a rectangle-shaped warehouse, we determine the shape factor that minimizes expected distance traveled in a unit load warehouse with a cross-aisle. We investigate the effect on the optimal shape factor of the number and locations of multiple dock doors located along one wall or two adjacent warehouse walls. Storage/retrieval aisles, aligned perpendicular to the wall containing k_1 dock doors or/and parallel to the warehouse wall on which k_2 dock doors are located, include a cross-aisle centrally located in the storage area. Both single- and dual-command travel are considered. Because of the importance of how dock doors are located along one or two adjacent walls, three scenarios for the locations of dock doors are investigated: 1) equally-spaced dock doors over an entire warehouse wall; 2) a specified distance between adjacent dock doors located symmetrically about the mid-point of a warehouse wall; and 3) a specified distance between adjacent dock doors, with the leftmost dock door located a specified distance from the leftmost storage/retrieval location.

Keywords: Multiple dock doors, Shape factor, Cross-aisle, Single-Command, Dual-command.

3.1. Introduction

In traditional warehouse layout configurations, a single, centrally located dock door along one wall is often assumed. Then, all subsequent calculations or comparisons of configurations are made based on this assumption. Pohl *et al*. (2009) examined single-dock-door versions of three traditional layout configurations (called Layouts A, B and C). Recognizing warehouses typically have multiple dock doors for receiving and shipping, Tutam and White (in press) developed discrete and continuous multi-dock-door formulations of the optimization problem for a unit-load warehouse having storage racks aligned perpendicular to the wall containing dock doors (Layout A). By inserting a cross aisle in the "middle" of the storage area of Layout A, Layout B is obtained (see Figure 3.1.*a*). By rotating the storage racks and cross aisle in Layout B, Layout C is obtained (see Figure 3.1.*b*). By combining features of Layouts B and C, we obtain Layout D (see Figure 3.1.*c*).

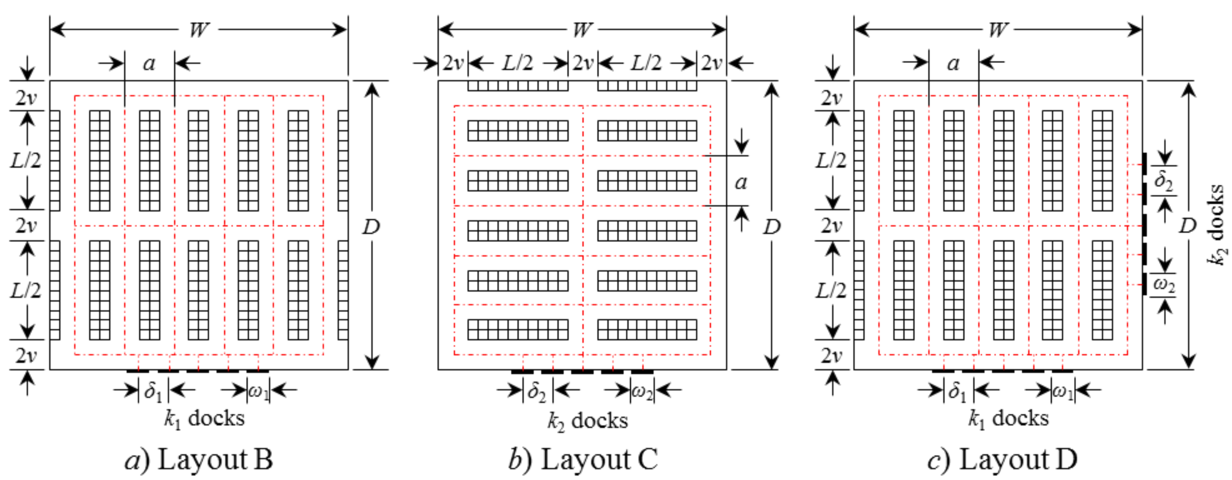

Figure 3.1: Warehouse S/R aisle configurations

Incorporating the nomenclature of Pohl *et al*. (2009) and using the procedure developed by Tutam and White (in press), we examine multi-dock-door versions of Layouts B, C and D.

Therefore, designing three multi-dock-door unit-load warehouses (Layouts B, C and D) to minimize expected distance for single- and dual-command travel is the focus of the chapter.

Layout B is composed of several storage/retrieval (S/R) aisles aligned perpendicular to the wall containing dock doors, with a cross-aisle located in the middle of the warehouse (middle-cross-aisle). A motivation for Layout B is the middle-cross-aisle decreases expected distance significantly when dual-command travel occurs, because movement between S/R aisles is more efficient when performed in the middle of the warehouse (Pohl *et al*., 2009).

The design for Layout C differs from that considered by Pohl et al. (2009) by removing storage/retrieval (S/R) locations along the wall containing dock doors. They noted expected dual-command travel with Layout C is less than that for Layout A when both warehouses have similar shapes. We examine the impact on expected distance for both single- and dual-command travel and show Layout C has disadvantages in a multi-dock-door unit-load warehouse when compared with Layout A.

A motivation for Layout D is to separate shipping and receiving dock doors by locating them on adjacent walls. Unit loads enter the warehouse along one wall and depart along an adjacent wall. After unit loads are received, they can be stored and then retrieved for shipping. Alternatively, unit loads can enter along one wall and be delivered directly to shipping dock doors (cross-docking).

As in Chapter 2, discrete and continuous formulations of the optimization problem are developed for both single- and dual-command travel and three scenarios for dock-door-locations are investigated. With single-command travel, either S/R equipment transports a unit load from a dock door to an S/R location, places the unit-load in a storage location and returns (empty) to the dock door or S/R equipment travels (empty) from a dock door to a retrieval location, retrieves a unit load and transports the unit-load to the dock door. With dual-command travel, S/R equipment transports a unit load from a dock door to a storage location, stores the unit-load, travels (empty) to a retrieval location, retrieves a unit load and transports the unit-load to the dock door. With dual-command travel, the empty travel from a storage location to a retrieval location is called travel-between.

Warehouse shape factor is an important design parameter because the shape of the warehouse directly affects the number and length of S/R aisles. Because of the single-dock-door assumption in earlier studies, the research literature did not address the impact of the number and location of dock doors on the optimal shape factor. To the best of our knowledge, Tutam and White (in press) are the first to determine the relation between the optimal shape factor and the number and locations of dock doors, albeit their study is limited to Layout A. Therefore, an objective of this research is the determination of the optimal shape factor for three common warehouse configurations (Layouts B, C and D) involving different scenarios for the number of dock doors, the spacing between adjacent dock doors and the locations of dock doors along the warehouse wall(s).

In general, we make the following assumptions when developing expected-distance expressions:

- Travel is limited to the floor of the warehouse. Vertical travel to access S/R locations above floor-level is ignored.
- S/R aisles have the same width and are wide enough for two-way travel, such that S/R equipment can access S/R locations on both sides of an aisle.
- S/R equipment travels at a constant velocity.
- Storage and retrieval times are ignored because they do not affect distance.

Two dimensions of planar travel are of interest. The first dimension, *horizontal travel*, is performed by S/R equipment traveling parallel to the “bottom wall” along which either k_1 or k_2 dock doors are located. The second dimension, *vertical travel*, is performed by S/R equipment traveling perpendicular to the “bottom wall”. Because the number and locations of dock doors do not affect vertical roundtrip-distances (Tutam and White, in press), we focus on horizontal roundtrip-distance for Layouts B and C.

In this chapter, we employ detailed discrete formulations to obtain values for expected distances in the warehouse. Specifically, Layout A formulations by Tutam and White (in press) are modified to account for additional travel created by the middle-cross-aisle and a new constraint is employed to require an equal number of storage locations on each side of the middle-cross-aisle. Because the S/R equipment follows the shortest path between dock-door-locations and storage locations or between S/R locations, discrete distance expressions for Layout C are developed by employing a similar approach.

As noted in Tutam and White (in press), optimal shape factor calculations require solutions of nonlinear, integer programming problems when using discrete formulations. Consequently, we develop very accurate continuous approximations in determining the optimal shape factor for the various warehouse configurations considered.

The three scenarios in Tutam and White (in press) are considered: 1) dock doors are uniformly dispersed along the entire width of the wall(s), 2) dock doors are centrally dispersed with a specified distance between adjacent dock doors, and 3) the leftmost dock door is located to the right of the leftmost storage location with a specified offset distance and a fixed distance between adjacent dock doors. For all cases, the optimal shape factor depends on the number and locations of dock doors; further, the optimal shape factor can differ significantly for the scenarios.

The remainder of the chapter is organized as follows. First, we review the literature of traditional unit-load warehouse layouts for both single- and dual-command travel in Section 3.2. Section 3.3 introduces the notation used throughout the chapter. Sections 3.4 and 3.5 include discrete and continuous expected-distance formulations for both single and dual-command travel in Layouts B and C, respectively. Section 3.6 provides a comparison of traditional warehouse designs with an equal number of S/R locations. In Section 3.7, we introduce Layout D by combining features of Layouts B and C, and present discrete and continuous expected-distance formulations for both single- and dual-command operations. Finally, Section 3.8 summarizes the results of the chapter and provides suggestions for future research.

3.2. Literature Review

A wide range of topics related to the warehouse design problem are addressed in the research literature. Reviewing the literature of warehouse design optimization, Ashayeri and Gelders (1985) proposed a two-step warehouse design approach: first analytical models are considered to reduce alternative design configurations, then simulation models are used to provide a general solution procedure. A review paper by Cormier and Gunn (1992) addresses the literature associated with the optimization of warehouse design and operations; they concluded warehouse design is a strategic decision and has a significant impact on profitability of facilities. Review papers by Rouwenhorst *et al*. (2000), de Koster *et al*. (2007), Gu *et al*. (2007 and 2010) and Karásek (2013) provide an overview of research on designing and controlling warehousing systems. An extensive identification of warehouse related literature can be found in Roodbergen (2007) including books, Ph.D. theses and scientific articles.

Francis (1967a) studied the problem of rectangle-shaped warehouse design to minimize total cost of traveling between S/R locations and a single-dock-door location. He concluded the optimal warehouse shape for the warehouse is twice as wide as it is deep; from this result, the optimal warehouse shape factor is widely accepted as 2:1 for Layout A with single-command travel. (Bassan *et al*., 1980; Pohl *et al*., 2009).

Thomas and Meller (2014) concluded warehouse shape factor is sensitive to the number of dock doors. Removing the assumption of a single dock door and the fixed distance between adjacent dock doors, when dock doors are equally likely to be used and random storage is used, they proved the optimal one-sided warehouse shape factor approaches 1.5:1 as the number of dock doors approaches infinity.

Tutam and White (in press) provided early formulations of single- and dual-command travel for a variety of dock-door locations in a multi-dock-door, unit-load, rectangle-shaped warehouse having storage racks aligned perpendicular to the wall containing dock doors (Layout A). They developed discrete and continuous formulations. After demonstrating the accuracy of their continuous approximations, they used a continuous approximation to determine the optimal shape factor for Layout A. Confirming previous research results, they showed the optimal shape factor is between 1.5 and 2.0 when the distance between adjacent dock doors is a function of the warehouse’s width. However, their results showed the optimal shape factor is greater than 2.0 when the distance between adjacent dock doors is specified.

Tutam and White (2016) developed expected-distance formulations for Layouts B, C and D with a limited but feasible number of dock doors when the distance between adjacent dock doors is specified. Their results indicated the optimal shape factor for Layout B with single-command travel can be greater than 2.00. Based on computational results, they asserted the optimal shape factor for Layout C is less than 2.00 without proving their assertion. They concluded the optimal shape factor for Layout D ranges from 1.00 to greater than 2.00 depending on the combination of single- and dual-command operations. Our research extends their results by considering general formulation of discrete and continuous versions of the optimization problem. Including theorems, propositions and corollaries for continuous approximations, we compare the performance of Layouts A, B and C. Considering a mixture of single-command, dual-command and cross-docking travel, we provide the results for expected distance and the optimal shape factor for Layout D.

Bassan *et al*. (1980) provided cost models for Layouts A and C taking into account the costs for material handling, warehouse space and warehouse perimeters. They developed expressions for optimal design parameters such as the optimal number of S/R aisles and the optimal number of S/R locations in each S/R aisle. Comparing the alignment of S/R aisles (parallel versus perpendicular to the bottom wall on which dock doors are located), they concluded operating cost is significantly impacted by the alignment of S/R aisles. They also analyzed optimal locations of dock doors and concluded all dock doors should be located as near as possible to the center of a warehouse wall. Extending their studies and using their expressions for optimal design parameters, Rosenblatt and Roll (1984) proposed a twelve-step simulation-based procedure to find the optimal warehouse design considering costs associated with the warehouse area and storage policies.

Two early papers by Mayer (1961) and Malmborg and Krishnakumar (1987) considered dual-command travel for Layout A. Pohl *et al*. (2009) were the first to model the expected single- and dual-command travel in Layouts A, B and C under the assumption of a centrally located dock door. They determined the optimal number of aisles minimizing single- and dual-command travel in the three layouts; they also noted expected travel-between distance is not a function of a dock door's location.

Inserting a middle-cross-aisle in Layout A, expected distance can be significantly decreased for multiple picks (Roodbergen and de Koster, 2001). Pohl *et al*. (2009) confirmed the conclusion of Roodbergen and de Koster (2001) and acknowledged establishing a middle-cross-aisle is only useful for travel between S/R locations. Inserting a middle-cross-aisle (Layout B) increases the expected distance for single-command travel, while decreasing the expected distance for dual-command travel. They also showed the optimal placement for the middle-cross-aisle is between the center of the warehouse and the top-cross-aisle of the warehouse. Distinct from earlier studies, Vaughan and Petersen (1999) and Roodbergen *et al*. (2008) examined the effects of additional cross-aisles in a warehouse; they concluded having sufficient cross-aisles may result in smaller travel distances because of efficient travel routing options.

3.3. Notation

The notation in Figure 3.1 is defined as follows:

a = distance between centerlines of adjacent aisles

c_i = i^{th} constant value

n = number of S/R aisles

w = the width of an S/R location

m = number of S/R locations along one side and one level of an S/R aisle, which is even ($Mod\ [m, 2] = 0$)

L = length of S/R aisles ($L = wm$)

v = half the width of a cross-aisle

W = width of the warehouse ($W = a\ n$ in Layout B, $W = L + 6v$ in Layout C, and $W = a\,n + 0.5a + v$ in Layout D)

D = depth of the warehouse ($D = L + 6v$ in Layouts B and D, and $D = a\,n + 0.5a + v$ in Layout C)

A = total warehouse area ($A = W\,D$)

S = shape factor ($S = W / D$)

k_j = number of dock doors located on the wall of side j (j = 1, 2) of the warehouse

ω_j = the width of a dock door located on the wall of side j (j = 1, 2) of the warehouse

δ_j = the distance between centerlines of two adjacent dock doors located on the wall of side j (j = 1, 2) of the warehouse (i.e. i^{th} and $(i+1)^{th}$ dock doors) ($\delta_j > \omega_j$)

ϕ_j = the distance between the wall of side j (j = 1, 2) and the leftmost storage location

d_i = the distance between the "leftmost storage location" and the centerline of the i^{th} dock door

t_i = the distance between the back-to-back rack location closest to dock door i and the leftmost storage location (*Round* [d_i, a] for Layout B and *Round* [d_i, w] for Layout C)

$E\,[SC]$ = expected single-command distance

$E\,[TB]$ = expected travel-between distance

$E\,[DC]$ = expected dual-command distance ($E\,[DC] = E\,[SC] + E\,[TB]$)

$E\,[MC]$ = expected mixed-command distance

3.4. Layout B

Pohl *et al*. (2009) defined Layout B as a layout design with a middle-cross-aisle of width $2v$, located halfway between the top-cross-aisle and bottom-cross-aisle. As shown in Figure 3.1.*a*, S/R aisles continue to be perpendicular to the wall containing dock doors. Extending the work of

Tutam and White (2016 and in press), in this section, we develop a multi-dock-door formulation of expected distance for Layout B.

3.4.1. Discrete Formulations

Single-command travel

Inserting a middle-cross-aisle does not affect horizontal roundtrip-distance. Hence, using Equation 2.2 in Chapter 2, the expected horizontal roundtrip-distance ($E[SC_h]$) for k_1 dock door is

$$E\left[SC_h\right] = \frac{2}{n\,k_1}\sum_{i=1}^{k_1}\sum_{j=1}^{n}\left|d_i - \left(j - 1/2\right)a\right|. \tag{3.1}$$

With additional travel because of the middle-cross-aisle, the expected vertical roundtrip-distance ($E[SC_v]$) becomes

$$E\left[SC_v\right] = \frac{2}{m}\left\{\sum_{j=1}^{m/2}\left(jw - w/2 + 2v\right) + \sum_{j=m/2+1}^{m}\left(jw - w/2 + 4v\right)\right\} = wm + 6v = D\,. \tag{3.2}$$

Summing Equations 3.1 and 3.2, expected single-command travel for Layout B is

$$E\left[SC\right] = E\left[SC_h\right] + E\left[SC_v\right] = \frac{2}{n\,k_1}\sum_{i=1}^{k_1}\sum_{j=1}^{n}\left|d_i - \left(j - 1/2\right)a\right| + D\,. \tag{3.3}$$

Dual-command travel

Determining the expected dual-command travel, the expected travel-between distance is added to the expected single-command travel. The expected horizontal travel-between distance is identical to the expected horizontal travel-between distance in Layout A provided by Pohl *et al*. (2009), $E\,[TB_h] = a\,(n^2 - 1) / 3n$. Although all S/R locations are equally likely to be chosen, there exist four possibilities for two S/R locations: 1) both S/R locations are in the same aisle and on the same side of the middle-cross-aisle, denoted *ss*; 2) both S/R locations are in the same aisle, but on different sides of the middle-cross-aisle, denoted *sd*; 3) S/R locations are in different aisles, but on the same side of the middle-cross-aisle, denoted *ds*; and 4) S/R locations are in different aisles and on different sides of the middle-cross-aisle, denoted *dd*.

When both S/R locations are in the same aisle and on the same side of the middle-cross-aisle or both S/R locations are in the same aisle but on different sides of the middle-cross-aisle, there is no travel in the parallel direction.

The expected vertical distance between two S/R locations in the same aisle and on the same side of the middle-cross-aisle is

$$E[TB_{ss}] = \frac{4w}{m^2}\sum_{i=1}^{m/2}\sum_{j=1}^{m/2}|i-j| = \frac{4w}{m^2}\sum_{i=m/2+1}^{m}\sum_{j=m/2+1}^{m}|i-j| = \frac{w(m^2-4)}{6m}, \tag{3.4}$$

and the expected vertical distance between two S/R locations in the same aisle but on different sides of the middle-cross-aisle is

$$E[TB_{sd}] = \frac{4w}{m^2}\sum_{i=1}^{m/2}\sum_{j=m/2+1}^{m}(j-i)+2v = \frac{4w}{m^2}\sum_{i=m/2+1}^{m}\sum_{j=1}^{m/2}(i-j)+2v = \frac{wm}{2}+2v\,. \tag{3.5}$$

Therefore, if two S/R locations are in the same aisle (*sa*), the expected vertical travel-between-distance is

$$E[TB_{sa}] = \frac{1}{2}\left(E[TB_{ss}]+E[TB_{sd}]\right) = \frac{1}{2}\left[\frac{w(m^2-4)}{6m}+\frac{wm}{2}+2v\right] = \frac{w(m^2-1)}{3m}+v\,. \tag{3.6}$$

The expected vertical distance between two S/R locations in different aisles but on the same side of the middle-cross-aisle is

$$E[TB_{ds}] = \frac{4w}{m^2}\left[2\sum_{i=1}^{m/2}\sum_{j=i}^{m/2} j-\left(\frac{m}{2}\right)^2\right]+2v = \frac{w(m^2+2)}{3m}+2v, \tag{3.7}$$

and the expected vertical distance between two S/R locations in different aisles and different sides of the middle-cross-aisle is

$$E\left[TB_{dd}\right]=\frac{4w}{m^2}\sum_{i=1}^{m/2}\sum_{j=m/2+1}^{m}\left(j-i\right)+2v=\frac{4w}{m^2}\sum_{i=m/2+1}^{m}\sum_{j=1}^{m/2}\left(i-j\right)+2v=\frac{wm}{2}+2v\,. \qquad (3.8)$$

When two S/R locations are in different aisles (*da*), the expected vertical distance for travel-between is

$$E\left[TB_{da}\right]=\frac{1}{2}\left(E\left[TB_{ds}\right]+E\left[TB_{dd}\right]\right)=\frac{1}{2}\left[\frac{w\left(m^2+2\right)}{3m}+2v+\frac{wm}{2}+2v\right]=\frac{w\left(5m^2+4\right)}{12m}+2v\,. \qquad (3.9)$$

The probability of two S/R locations being in the same aisle is 1 / *n* and the probability of two S/R locations being in different aisles is 1–1 / *n*. Combining Equations (3.6) and (3.9), incorporating probabilities and adding the expected horizontal distance for travel-between, the expected distance for travel-between is

$$E\left[TB\right]=\frac{1}{n}\left[\frac{w\left(m^2-1\right)}{3m}+v\right]+\frac{n-1}{n}\left[\frac{w\left(5m^2+4\right)}{12m}+2v\right]+\frac{a\left(n^2-1\right)}{3n} \qquad (3.10)$$

To obtain the expected distance for dual-command travel, we add the expected distance between two random S/R locations and the expected single-command travel. Thus, combining Equations (3.3) and (3.10), the expected-distance formulation for dual-command travel is

$$\begin{aligned} E\left[DC\right]&=\frac{2}{n\,k_1}\sum_{i=1}^{k_1}\sum_{j=1}^{n}\left|d_i-\left(j-1/2\right)a\right|+D \\ &+\frac{1}{n}\left[\frac{w\left(m^2-1\right)}{3m}+v+\left(n-1\right)\left(\frac{w\left(5m^2+4\right)}{12m}+2v\right)\right]+\frac{a\left(n^2-1\right)}{3n}. \end{aligned} \qquad (3.11)$$

Discrete optimization problem

Adding the constraint, *Mod* [*m*, 2] = 0, to Formulation 1 in Chapter 2, we determine the number and length of S/R aisles for Layout B for each expected-distance formulation. The additional constraint assures there are an even number of S/R locations on each side of the cross aisle. As with Chapter 2, *Couenne* (2006) in *AMPL* (2013) is used to implement the nonlinear-integer-programming optimization problem for the same scenarios (see Section 2.4 for scenarios). Computational results are provided in Section 3.8.

As before, using discrete formulations are tedious and time-consuming, as well as requiring the use of a specialized software. To obtain useful insights regarding the design of multi-dock-door, unit-load warehouses having a middle-cross-aisle, we employ continuous approximations.

3.4.2. Continuous Approximations

Single-command travel

Continuous approximations of single-command travel for Layout B are almost identical to those developed in Chapter 2 for Layout A. The only difference is vertical travel where $D = wm + 6v$ for Layout B; whereas, $D = wm + 4v$ for Layout A. Hereafter, identifying formulas developed by using a continuous approximation, we use an approximate sign (≈) in the equations. The expected single-command travel expressions for Layout B, based on the three scenarios, are

Scenario 1: $$E[SC] \approx \frac{(2k_1+1)W}{3(k_1+1)} + wm + 6v, \tag{3.12}$$

Scenario 2: $$E[SC] \approx \frac{W}{2} + \frac{\delta_1^2(k_1^2-1)}{6W} + wm + 6v, \tag{3.13}$$

Scenario 3: $$E\left[SC\right]=W+\frac{6\phi_1^{\,2}+6\left(k_1-1\right)\phi_1\,\delta_1+\left(2k_1^{\,2}-3k_1+1\right)\delta_1^{\,2}}{3W}-\left[2\phi_1+\left(k_1-1\right)\delta_1\right]+D. \quad (3.14)$$

Adding the constant term (2v) does not change the corollaries, propositions and theorems included in Chapter 2. However, including a constraint for an even number of S/R locations produces different results than obtained in Chapter 2.

Dual-command travel

To develop a continuous approximation for travel-between distance, (m^2 – 1) / m is replaced with m in Equation (3.6) and ($5m^2$ + 4) / m is replaced with 5m in Equation (3.9). The resulting approximation for expected travel-between distance is

$$E[TB]\approx\frac{1}{n}\left[\frac{wm}{3}+v+\left(n-1\right)\left(\frac{5wm}{12}+2v\right)\right]+\frac{a\left(n^2-1\right)}{3n}. \quad (3.15)$$

Although obtained using a different approach, Equation (3.15) is identical to that obtained by Pohl, *et al*. (2009).

Combining Equation (3.15) with Equations (3.12), (3.13) and (3.14), the following expected-distance expressions for dual-command travel are obtained:

Scenario 1: $$E\left[DC\right]\approx D\left(\frac{17n-1}{12n}\right)-v\left(\frac{n+1}{2n}\right)+a\left(\frac{n^2\left(3k_1+2\right)-\left(k_1+1\right)}{3n\left(k_1+1\right)}\right), \quad (3.16)$$

Scenario 2: $$E\left[DC\right]\approx D\left(\frac{17n-1}{12n}\right)-v\left(\frac{n+1}{2n}\right)+a\left(\frac{5n^2-2}{6n}\right)+\frac{\delta_1^{\,2}(k_1^{\,2}-1)}{6an}, \quad (3.17)$$

Scenario 3: $$E\left[DC\right]\approx D\left(\frac{17n-1}{12n}\right)-v\left(\frac{n+1}{n}\right)+a\left(\frac{4n^2-1}{3n}\right)+\frac{\delta_1^{\,2}(2k_1^{\,2}-3k_1+1)+6\left(k_1-1\right)\phi_1\,\delta_1+6\phi_1^{\,2}-3an\left[\left(k_1-1\right)\delta_1+2\phi_1\right]}{3an}. \quad (3.18)$$

As with single-command travel, the corollaries, propositions and theorems included in Chapter 2 apply for Layout B. However, changing the travel-between expression and adding the even number of storage spaces constraint will produce different results than obtained in Chapter 2.

3.4.3. Optimal Shape Factor

Single-command travel

By including the space required for the middle-cross-aisle in the area of the warehouse, the optimal shape factor formulas for single-command travel for Layout B are identical to those developed in Chapter 2. Therefore, the optimal shape factor for Layout B is obtained using Lemma 2.1 and Corollary 2.2 in Chapter 2. Modifying the results in Chapter 2 by incorporating changes in notation, we obtain the following:

Proposition 3.1: With Scenario 1, $S^*_{SC} \approx 3(k_1+1) / (2k_1+1)$ if $S \geq [(k_1 + 1)^2 \, \delta_1{}^2] / A$. Otherwise, $S^*_{SC} \approx [(k_1 + 1)^2 \, \delta_1{}^2] / A$.

Proposition 3.2: With Scenario 2, $S^*_{SC} \approx 2 + [\delta_1{}^2 \, (k_1{}^2 - 1)] / 3A$ if $S \geq k_1{}^2 \, \delta_1{}^2 / A$. Otherwise, $S^*_{SC} \approx k_1{}^2 \, \delta_1{}^2 / A$.

Proposition 3.3: With Scenario 3, $S^*_{SC} \approx 1 + [6\phi_1{}^2 + 6\phi_1\delta_1 \, (k_1 - 1) + (2k_1{}^2 - 3 \, k_1 + 1) \, \delta_1{}^2] / 3A$ if $S \geq [\phi_1 + (k_1 - 0.5) \, \delta_1]^2 / A$. Otherwise, $S^*_{SC} \approx [\phi_1 + (k - 0.5) \, \delta_1]^2 / A$.

Dual-command travel

Because the travel-between expression for Layout B is different than that for Layout A, Lemma 2.2 and Corollary 2.4 are used to obtain the optimal shape factor, resulting in the following:

Proposition 3.4: With Scenario 1, $S^*_{DC} \approx 4c_3 (\cos\{\arccos[c_4 c_1^{1/2} (3 / c_3)^{3/2}] / 3\})^2 / (3A c_1)$ if $S \geq [(k_1 + 1)^2 \delta_1^2] / A$ where $c_1 = 4 (2 + 3k_1)$, $c_3 = (1 + k_1) (17A - 4a^2 - 8a v)$ and $c_4 = -(1 + k_1) a A$. Otherwise, $S^*_{DC} \approx [(k_1 + 1)^2 \delta^2] / A$.

Proposition 3.5: With Scenario 2, $S^*_{DC} \approx 4c_3 (\cos\{\arccos[c_4 c_1^{1/2} (3 / c_3)^{3/2}] / 3\})^2 / (3A c_1)$ if $S \geq k_1^2 \delta_1^2 / A$ where $c_1 = 10$, $c_3 = 17A - 4a^2 - 8a v + 2\delta_1^2 (k_1^2 - 1)$ and $c_4 = -a A$. Otherwise, $S^*_{DC} \approx k_1^2 \delta_1^2 / A$.

Proposition 3.6: With Scenario 3, $S^*_{DC} \approx 4c_3 (\cos\{\arccos[c_4 c_1^{1/2} (3 / c_3)^{3/2}] / 3\})^2 / (3A c_1)$ if $S \geq [\phi_1 + (k_1 - 0.5) \delta_1]^2 / A$ where $c_1 = 16$, $c_3 = 17A - 4a^2 - 8a v + 24\phi_1^2 + 24\phi_1 \delta_1 (k_1 - 1) + 4 (2k_1^2 - 3k_1 + 1) \delta_1^2$ and $c_4 = -a A$. Otherwise, $S^*_{DC} \approx [\phi_1 + (k_1 - 0.5) \delta_1]^2 / A$.

3.4.4. Computational Results

As with Chapter 2, this section provides results for both discrete formulations and continuous approximations by using the following specified values for the parameters $w = 4$ ft, $v = 6$ ft, $a = 20$ ft, $\delta_1 = 12$ ft, $\phi_1 = 30$ ft, $A = 250{,}000$ ft^2, k_1 ranging from 1 to 60 for Scenario 1 and from 1 to 75 for Scenarios 2 and 3. Adjusting space and width constraints for Layout B, Formulations 1 and 2 are solved by using *Couenne* (2006) in *AMPL* (2013). *Mathematica* (2015) is used to produce figures based on the continuous approximation results.

As seen in Figure 3.2, with Scenario 1, increasing the number of dock doors decreases the optimal shape factor for both single- and dual-command travel if the width constraint is satisfied ($\delta_1 \geq 12$ ft). Otherwise, increasing the number of dock door increases the width of the warehouse and increases the optimal shape factor. The optimal shape factor for travel-between is 1.22 when the width constraint is not violated. As expected, the optimal shape factor for single-command travel is greater than the corresponding optimal shape factor for dual-command travel. When the width constraint is satisfied, the optimal shape factor for single-command travel is greater than the corresponding optimal shape factor for both travel-between and dual-command travel. Similarly, the optimal shape factor for dual-command travel is greater than the optimal shape factor for travel-between. Therefore, for the same number of dock doors, the width constraint comes into play at k_1 = 51 for single-command travel, at k_1 = 49 for dual-command travel and at k_1 = 46 for travel-between.

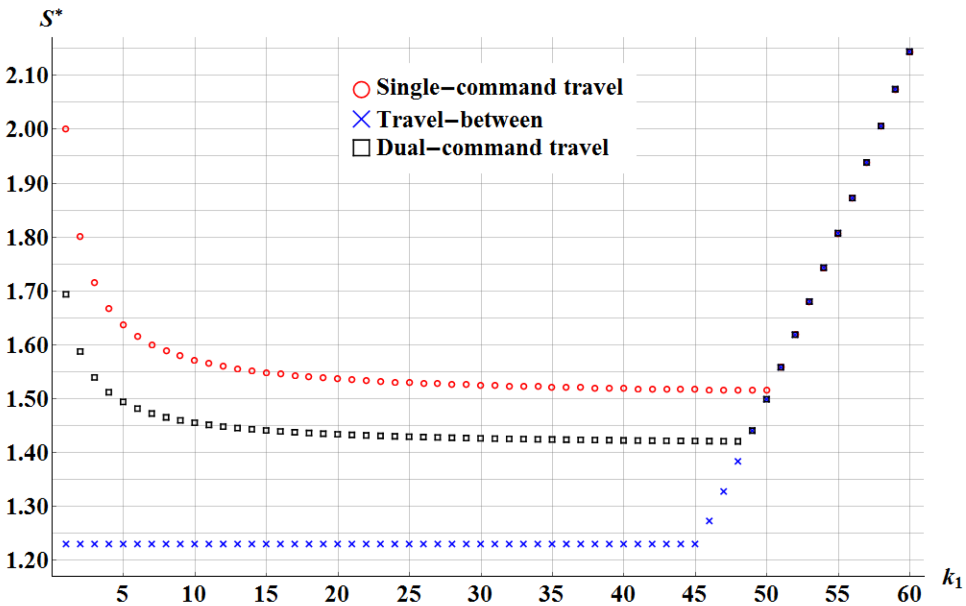

Figure 3.2: Optimal shape factor for k_1 dock doors with Scenario 1

As shown in Figure 3.3, with Scenario 2, increasing the number of dock doors increases the optimal shape factor for any value of k_1. The optimal shape factor for single-command travel is greater than 2.0. As before for Scenario 1, the optimal shape factor for single-command travel is greater than the corresponding optimal shape factor for dual-command travel and travel-between with any number of dock doors. Therefore, the layout configuration for single-command travel is wider than the corresponding layout configurations for travel-between and dual-command travel. Notice the optimal shape factor patterns change at k_1 = 73 for single-command travel, at k_1 = 61 for dual-command travel and at k_1 = 47 for travel-between.

From Figure 3.4, with Scenario 3, increasing the number of dock doors increases the optimal shape factor. The optimal shape factor for both single- and dual-command travel is greater than 1.0. The optimal shape factor for single-command travel can be less than or greater than the corresponding optimal shape factor for dual-command travel, depending on the number of dock doors. When dock doors are clustered on the left side of the warehouse (for a small number of dock doors), the optimal shape factor is slightly greater than 1.0.

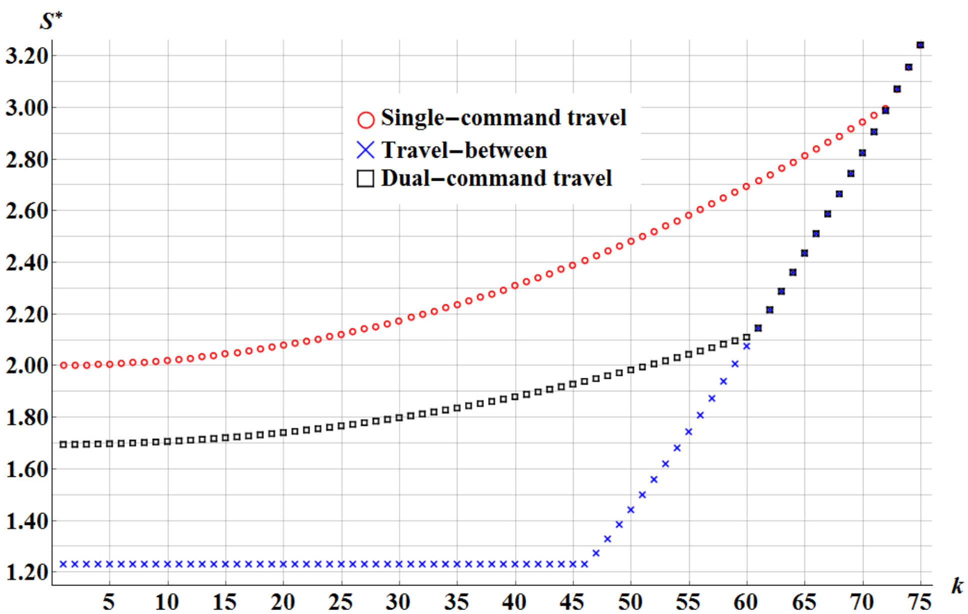

Figure 3.3: Optimal shape factor for k_1 dock doors with Scenario 2

Because the optimal shape factor for travel-between is 1.22 regardless the number of dock doors, the optimal shape factor for dual-command travel is greater than the corresponding optimal shape factor for single-command travel. Increasing the number of dock doors increases the optimal shape factor for single-command travel, but it does not affect the optimal shape factor for travel-between. Therefore, the optimal shape factor for dual-command travel is affected less by the number of dock doors. For a large number of dock doors, the warehouse with single-command travel is wider than the warehouse with dual-command travel. Because the width constraint comes into play for a large number of dock doors, the optimal shape factor patterns change at $k_1 = 73$ for single-command travel, at $k_1 = 53$ for dual-command travel and at $k_1 = 45$ for travel-between.

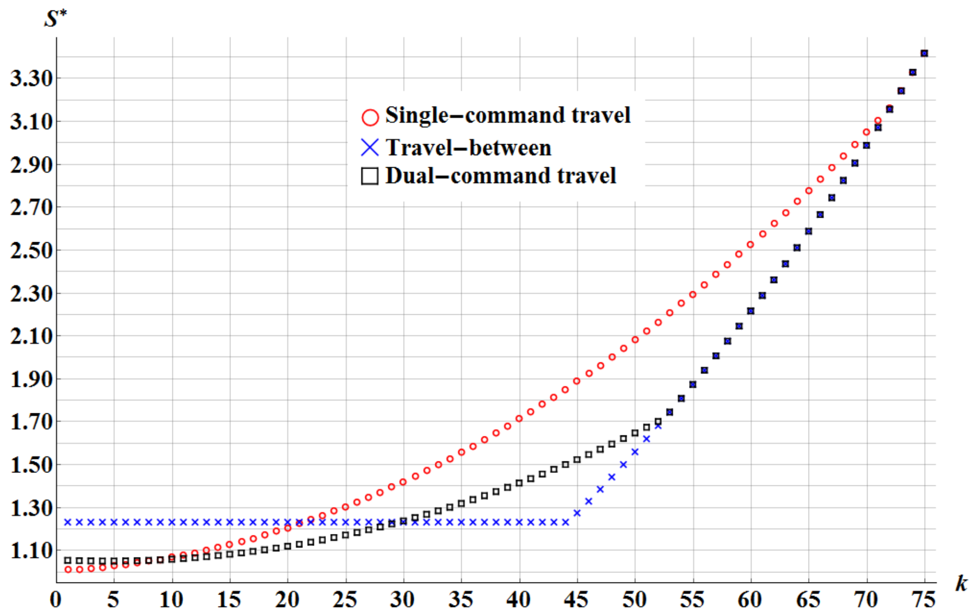

Figure 3.4: Optimal shape factor for k_1 dock doors with Scenario 3

3.5. Layout C

Layout C is similar to Layout B, except the S/R aisles are parallel to the wall containing the dock doors. As noted, our Layout C differs from Layout C considered by Pohl *et al*. (2009); we include an additional S/R aisle along the wall containing the dock doors. As shown in Figure 3.1, the additional S/R aisle has storage positions on only one side of the aisle. In determining expected distance, the S/R equipment follows the shortest path between dock-door locations and storage locations. As with travel-between, using a continuous formulation is not realistic; it under-estimates the exact distance traveled, because S/R locations can be obstacles for vertical travel to the wall containing dock doors.

Because dock doors are located symmetrically about the middle-cross-aisle, in developing formulas for expected distances for Scenarios 1 and 2, we only consider dock doors located on the left half of the warehouse. Therefore, with Scenarios 1 and 2, unlike Layouts A and B, we must consider whether the number of dock doors is odd or even for Layout C. In the case of Scenario 3, we divide the warehouse wall with the dock doors into four regions and determine the number of dock doors located in each region. Then, we calculate total distance for dock doors located in each region. Summing total distances for each region and dividing by the number of dock doors, we obtain expected distance for Scenario 3. Calculation details are provided in the following sections.

3.5.1. Discrete Formulations

Single-command travel

In developing discrete expressions, we apply an approach similar to that used in Chapter 2 for Layout A. *An initial point* (the leftmost storage location) is used to measure the horizontal distance between dock doors and storage locations. Based on the initial point, we measure the distance between the centerline of a dock door and the centerline of the storage location nearest to the dock door. Hereafter, locations of a dock door and a storage position refer to the locations of the centerline of a dock door and a storage position. As defined, d_i and t_i are used to obtain a distance, depending on the dock-door location. Dock doors and storage locations are numbered sequentially from left to the right. To round numbers to the nearest even integer value when using *Couenne* (2006) in *AMPL* (2013), we add a constraint and introduce a new variable.

There are four cases for dock-door locations, as shown in Figure 3.5: 1) the nearest back-to-back storage location is to the left of the dock door, 2) a back-to-back storage location coincides with a dock-door location, 3) the nearest back-to-back storage location is to the right of the dock-door location and 4) a storage location coincides with a dock-door location.

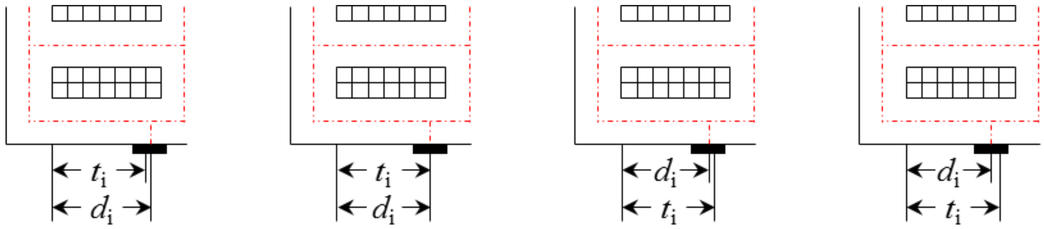

Figure 3.5: Cases for dock-door locations

Proposition 3.1: For a one-sided S/R aisle, there are t_i / w S/R locations to the left of dock door i, $m / 2 - t_i / w$ S/R locations to the right of dock door i on the left side of the warehouse and $m / 2$ S/R locations to the right of dock door i on the right side of the warehouse. The distance between dock door i and S/R aisle j located to the left of dock door i equals $d_i - j\,w + w / 2$ for $j = 1, 2, \ldots, t_i / w$ (see storage locations 1 thru 5 in Figure 3.6). The distance between dock door i and S/R aisle j to the right of dock door i on the left of the warehouse equals $j\,w - d_i - w / 2$ for $j = t_i / w + 1, t_i / w + 2, \ldots, m / 2$ (see storage locations 6 thru 8 in Figure 3.6). The distance between dock door i and S/R aisle j to the right of dock door i on the right of the warehouse equals $j\,w - d_i - w / 2 + 2v$ for $j = m / 2 + 1, m / 2 + 2, \ldots, m$ (see storage locations 9 thru 16 in Figure 3.6). For two-sided S/R aisles, first, the shortest path between dock door i and storage location j is determined. There are $m / 2 - t_i / w$ S/R locations visited by traveling to the left of dock door i, t_i / w S/R locations visited by traveling to the right of dock door i on the left side of the warehouse, and $m / 2$ S/R locations visited by traveling to the right of dock door i on the right side of the warehouse. The shortest-path distance between dock door i and S/R aisle j visited by traveling to the left of dock door i equals $d_i + j\,w - w / 2 + 2v$ for $j = 1, 2, \ldots, m /2 - t_i / w$. (see storage locations 1 thru 3 in Figure 3.6). The shortest-path distance between dock door i and S/R aisle j visited by traveling to the right of dock door i on the left of the warehouse equals $m\,w - d_i - j\,w + w / 2 + 2v$ for $j = m / 2 - t_i / w + 1, m / 2 - t_i / w + 2, \ldots, m / 2$ (see storage locations 4 thru

8 in Figure 3.6). The shortest-path distance between dock door i and S/R aisle j visited by traveling to the right of dock door i on the right of the warehouse equals $j\,w - d_i - w / 2 + 2v$ for $j = m / 2 + 1,\ m / 2 + 2, \ldots, m$ (see storage locations 9 thru 16 in Figure 3.6).

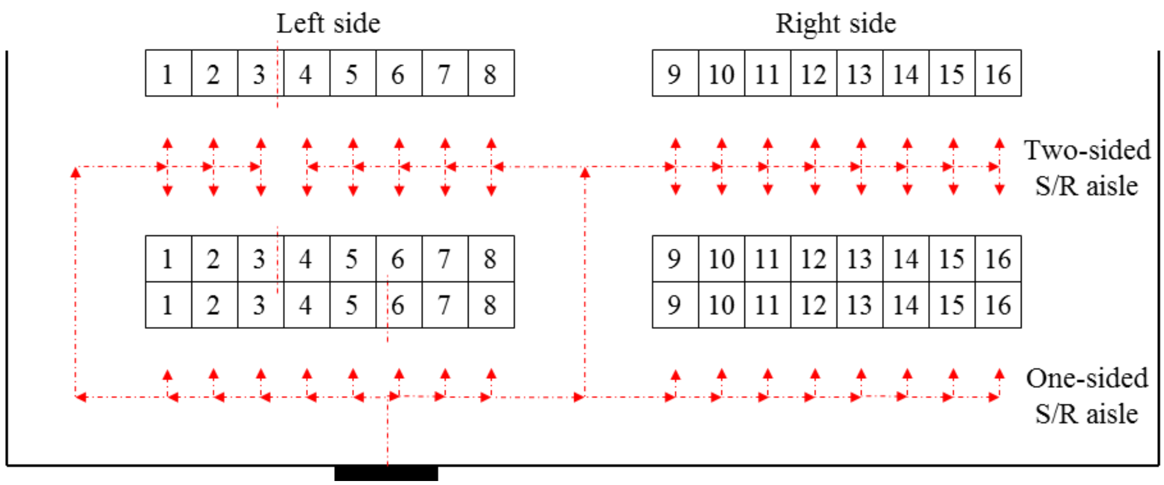

Figure 3.6: Storage locations

Proposition 3.1 applies for all cases (Proof of Proposition 3.1 is provided in Appendix).

Scenarios 1 and 2

Probabilities of traveling to the one- and two-sided aisles are $1 / (2n + 1)$ and $2 / (2n + 1)$, respectively. Summing the expected distance to the left and right (both sides) of dock door i and doubling the results, the expected distance for dock door i is obtained. Summing the results for dock doors located on the left side, and dividing by the number of dock doors located on the left side, the expected horizontal roundtrip-distance with an even number of dock doors is

$$E\left[SC_h\right]=\frac{4}{\left(2n+1\right)m\,k_2}\sum_{i=1}^{k_2/2}\left\{\sum_{j=1}^{t_i/w}\left(d_i-j\,w+w/2\right)+\sum_{j=t_i/w+1}^{m/2}\left(j\,w-d_i-w/2\right)\right.$$
$$+\sum_{j=m/2+1}^{m}\left(j\,w-d_i-w/2+2v\right)+2n\sum_{j=1}^{m/2-t_i/w}\left(d_i+j\,w-w/2+2v\right)\tag{3.19}$$
$$\left.+2n\left[\sum_{j=m/2-t_i/w+1}^{m/2}\left(mw-d_i-j\,w+w/2+2v\right)+\sum_{j=m/2+1}^{m}\left(j\,w-d_i-w/2+2v\right)\right]\right\}.$$

Equation (3.19) reduces to

$$E\left[SC_h\right]=mw+\frac{2v\left(4n+1\right)}{2n+1}+4\sum_{i=1}^{k_2/2}\left\{\frac{t_i\left(2n-1\right)\left[t_i-2d_i\right]-d_iwm}{w\left(2n+1\right)m\,k_2}\right\}\;.\tag{3.20}$$

Because increasing the number of dock doors does not affect expected vertical roundtrip-distance, the expected vertical roundtrip-distance is

$$E\left[SC_v\right]=\frac{1}{2n+1}\left(2v\right)+\frac{2}{2n+1}\sum_{j=1}^{n}\left(2\,ja+2v\right)=\frac{2\,n\,a\left(n+1\right)}{2n+1}+2v\,.\tag{3.21}$$

Summing Equations (3.20) and (3.21), the expected roundtrip-distance for single-command travel for Layout C with an even number of dock doors is

$$E\left[SC_{even}\right]=mw+4v\left(\frac{3n+1}{2n+1}\right)+\frac{2\,n\,a\left(n+1\right)}{2n+1}+4\sum_{i=1}^{k_2/2}\left\{\frac{t_i\left(2n-1\right)\left[t_i-2d_i\right]-d_iwm}{w\left(2n+1\right)m\,k_2}\right\}\;.\tag{3.22}$$

In case of an odd number of dock doors, the middle dock door is located on the centerline of the warehouse for both Scenarios 1 and 2. Adjusting Equation (3.22) for (k_2 – 1) dock doors, adding the distance for centrally located dock door ($w\,m$ / 2 + 2v) and dividing the resulting equation by the total number of dock doors, the expected roundtrip-distance for single-command travel for Layout C with an odd number of dock doors is

$$E\left[SC_{odd}\right]=mw\left(\frac{2k_2-1}{2k_2}\right)+4v\left(\frac{3nk_2-n+k_2}{k_2\left(2n+1\right)}\right)+\frac{2na\left(n+1\right)}{2n+1}$$
$$+4\sum_{i=1}^{(k_2-1)/2}\left\{\frac{t_i\left(2n-1\right)\left[t_i-2d_i\right]-d_i wm}{w\left(2n+1\right)mk_2}\right\}. \tag{3.23}$$

Notice $d_i = i\ (w\ m + 2v) / (k_2 + 1)$ and $d_i = [w\ m + 2v - \delta_2\ (k_2 + 1)] / 2 + \delta_2\ (i - 1)$ for Scenarios 1 and 2, respectively.

Scenario 3

Because dock doors are no longer located symmetrically about the middle-cross-aisle for Scenario 3, first, the number and locations of dock doors must be determined. Depending on the locations of dock doors, four different expressions are developed using Proposition 3.1.

Proposition 3.2: If a dock door is located in Region 1 (R1, see docks 1, 2 and 3 in Figure 3.7), then the expected horizontal roundtrip-distance for dock door i is $w\ m + 2v\ (4n + 1) / (2n + 1) - 2d_i / (2n + 1) + \{t_i\ (4n - 2)\ [t_i - 2d_i]\} / [w\ m\ (2n + 1)]$. When a dock door is located in Region 2 (R1, see dock door 4 in Figure 3.7), the expected horizontal-roundtrip distance for dock door i is $\{4v\ (4n + 1) + w\ m\ (6n + 1) - 8n\ d_i\} / \{2\ (2n + 1)\}$. When a dock door is located in Region 3 (R3, see dock door 5 in Figure 3.7), the expected horizontal roundtrip-distance for dock door i is $\{4v - w\ m\ (2n - 1) + 8n\ d_i\} / \{2\ (2n + 1)\}$. When a dock door is located in Region 4 (R4, see dock door 6 in Figure 3.7), the expected horizontal roundtrip-distance for dock door i is $\{(2n - 1)\ [8v\ (d_i - v) + 2t_i\ (t_i - 2d_i) - w^2\ m^2]\} / [w\ m\ (2n + 1)] + [2d_i\ (4n - 1) - 2v\ (4n - 3)] / (2n + 1)$. Regardless of the location of dock door i the expected vertical roundtrip-distance is $[2n\ a\ (n + 1)] / (2n + 1) + 2v$.

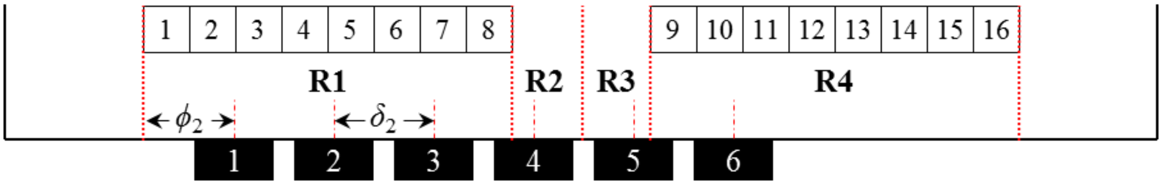

Figure 3.7: Locations of dock doors

Using Proposition 3.2 and determining the locations of dock doors ($d_i = \phi_2 + \delta_2\,(i-1)$), the following conditional expression is developed

If $d_{k_2} \le w\,m/2$

$$E[SC] = mw + 4v\left(\frac{3n+1}{2n+1}\right) + \frac{2na(n+1)}{(2n+1)} + \sum_{i=1}^{k_2}\left\{\frac{t_i(4n-2)[t_i-2d_i]-2d_i wm}{w(2n+1)mk_2}\right\}. \tag{3.24}$$

If $w\,m/2 < d_{k_2} \le w\,m/2 + v$

$$\begin{aligned} E[SC] = &\sum_{i=1}^{\left\lceil \frac{wm-2\phi_2}{2\delta_2}\right\rceil}\left[\frac{wm\left[2v(4n+1)+mw(2n+1)-2d_i\right]+t_i(4n-2)[t_i-2d_i]}{w(2n+1)mk_2}\right] \\ &+\sum_{i=\left\lceil \frac{wm-2\phi_2}{2\delta_2}\right\rceil+1}^{k_2}\left[\frac{4v(4n+1)+mw(6n+1)-8nd_i}{2(2n+1)k_2}\right]+\frac{2na(n+1)}{(2n+1)}+2v. \end{aligned} \tag{3.25}$$

If $w\,m/2 + v < d_{k_2} \le w\,m/2 + 2v$

$$\begin{aligned} E[SC] = &\sum_{i=1}^{\left\lceil \frac{wm-2\phi_2}{2\delta_2}\right\rceil}\left[\frac{wm\left[2v(4n+1)+mw(2n+1)-2d_i\right]+t_i(4n-2)[t_i-2d_i]}{w(2n+1)mk_2}\right] \\ &+\sum_{i=\left\lceil \frac{wm-2\phi_2}{2\delta_2}\right\rceil+1}^{\left\lceil \frac{wm-2\phi_2+2v}{2\delta_2}\right\rceil}\left[\frac{4v(4n+1)+mw(6n+1)-8nd_i}{2(2n+1)k_2}\right] \\ &+\sum_{i=\left\lceil \frac{wm-2\phi_2+2v}{2\delta_2}\right\rceil+1}^{k_2}\left[\frac{4v-mw(2n-1)-8nd_i}{2(2n+1)k_2}\right]+\frac{2na(n+1)}{(2n+1)}+2v. \end{aligned} \tag{3.26}$$

If $w\,m\,/\,2 + 2v < d_{k_2}$

$$E[SC] = \sum_{i=1}^{\left\lceil \frac{wm-2\phi_2}{2\delta_2} \right\rceil} \left[\frac{wm\left[2v(4n+1)+mw(2n+1)-2d_i\right]+t_i(4n-2)\left[t_i-2d_i\right]}{w(2n+1)m\,k_2} \right]$$
$$+ \sum_{i=\left\lceil \frac{wm-2\phi_2}{2\delta_2} \right\rceil+1}^{\left\lceil \frac{wm-2\phi_2+2v}{2\delta_2} \right\rceil} \left[\frac{4v(4n+1)+mw(6n+1)-8nd_i}{2(2n+1)k_2} \right] + \sum_{i=\left\lceil \frac{wm-2\phi_2+2v}{2\delta_2} \right\rceil+1}^{\left\lceil \frac{wm-2\phi_2+4v}{2\delta_2} \right\rceil} \left[\frac{4v-mw(2n-1)-8nd_i}{2(2n+1)k_2} \right] \tag{3.27}$$
$$+ \sum_{i=\left\lceil \frac{wm-2\phi+4v}{2\delta} \right\rceil+1}^{k_2} \left[\frac{(2n-1)\left[8v(d_i-v)+2t_i(t_i-2d_i)-w^2m^2\right]}{w(2n+1)m\,k_2} + \frac{2d_i(4n-1)-2v(4n-3)}{(2n+1)k_2} \right]$$
$$+ \frac{2\,n\,a(n+1)}{(2n+1)} + 2v.$$

Travel-between

As with Layout B, we consider horizontal and vertical travel separately in developing travel-between formulas for Layout C. Because there are two types of S/R aisles (one- and two-sided aisles), four travel types can occur: 1) traveling between two locations in the one-sided S/R aisle, denoted *oo*; 2) traveling between a location in the one-sided S/R aisle to another location in a two-sided S/R aisle, denoted *ot;* 3) traveling between two locations in two-sided S/R aisles on the same side of the middle-cross-aisle, denoted *ts*; and 4) traveling between two locations in two-sided S/R aisles on different sides of the middle-cross-aisle, denoted *td*. Because there exist *n* two-sided S/R aisles and a single one-sided S/R aisle, there are $(2n + 1)^2$ ways to travel between S/R aisles.

The probability of traveling between two locations in the one-sided aisle is $1 / (2n + 1)^2$. From Equation (3.6), the expected horizontal distance for *oo* is

$$E\left[TB_{oo}\right]=\frac{1}{\left(2n+1\right)^2}\left[\frac{w\left(m^2-1\right)}{3m}+v\right]. \tag{3.28}$$

The probability of traveling between a location in a one-sided S/R aisle and another location in a two-sided S/R aisle is $4n / (2n + 1)^2$. From Equation (3.9), the expected horizontal distance for *ot* is

$$E\left[TB_{ot}\right]=\frac{4n}{\left(2n+1\right)^2}\left[\frac{w\left(5m^2+4\right)}{12m}+2v\right]. \tag{3.29}$$

The probability of traveling between two locations in two-sided S/R aisles on the same side of the middle-cross-aisle is $4n / (2n + 1)^2$. Therefore, from Equation (3.6), the expected horizontal distance for *ts* is

$$E\left[TB_{ts}\right]=\frac{4n}{\left(2n+1\right)^2}\left[\frac{w\left(m^2-1\right)}{3m}+v\right]. \tag{3.30}$$

From Equation (3.9), with the probability of traveling between two locations in two-sided S/R aisles and on the different side of the middle-cross-aisle, $[4n\,(n - 1)] / (2n + 1)^2$, the expected horizontal distance for *td* is

$$E\left[TB_{td}\right]=\frac{4n^2-4n}{\left(2n+1\right)^2}\left[\frac{w\left(5m^2+4\right)}{12m}+2v\right]. \tag{3.31}$$

Summing Equations (3.28-3.31) and reducing the resulting equation, the expected horizontal travel-between distance ($E[TB_h]$) for Layout C is

$$E\left[TB_h\right]=\frac{3m\left(1+4n+8n^2\right)v}{3m\left(2n+1\right)^2}+\frac{w\left(-1-4n+4n^2\right)+m^2w\left(1+4n+5n^2\right)}{3m\left(2n+1\right)^2}. \tag{3.32}$$

When two locations are on the same S/R aisle, there is no vertical travel. The expected vertical travel between a location in the one-sided S/R aisle to another location in two-sided S/R aisles is a (n + 1) / 2 and the expected vertical travel between two locations in two-sided S/R aisles and on different sides of the middle-cross-aisle is a (n + 1) / 3.Therefore, the expected vertical travel-between distance for Layout C is

$$E\left[TB_v\right]=\frac{4n}{\left(2n+1\right)^2}\left[\frac{a\left(n+1\right)}{2}\right]+\frac{4n^2-4n}{\left(2n+1\right)^2}\left[\frac{a\left(n+1\right)}{3}\right]=\frac{2na\left(n+1\right)}{3+6n}. \tag{3.33}$$

Combining Equations (3.32) and (3.33) and reducing the resulting equation, the expected travel-between distance for Layout C is

$$E\left[TB\right]=\frac{2na\left(n+1\right)}{3+6n}+\frac{\left(8n^2+4n+1\right)v}{\left(2n+1\right)^2}+\frac{w\left(4n^2-4n-1\right)+m^2w\left(1+4n+5n^2\right)}{3m\left(2n+1\right)^2}. \tag{3.34}$$

Dual-command travel

The expected distance for dual-command travel is the sum of expected distances for single-command travel and travel-between. Because expected-distance expressions for single-command travel and travel-between are provided in Section 3.5.2, the interested reader can refer to those sections to obtain the formulas for dual-command travel. Specifically, obtaining dual-command expressions for Scenarios 1 and 2, Equations (3.22), (3.23) and (3.32) are modified by including the corresponding equation for the parameter d_i. Because there are four equations (3.24-3.27), for Scenario 3, Equation (3.32) is added to the appropriate equation, depending on the locations of dock doors.

3.5.2. Continuous Approximations

In this section, expected-distance formulations are developed for Layout C using continuous approximations. Specifically, the number of S/R aisles is discrete; whereas, the number of S/R locations in each aisle is assumed to be continuous. Because the expected horizontal roundtrip-distance from a dock door to S/R locations in the one-sided aisle includes small numbers compared to the numbers for two-sided aisles, we ignore S/R locations in the one-sided aisle. As with Layout B, an approximate sign (≈) is used for continuous formulations.

Single-command travel

For simplicity, the expected horizontal roundtrip-distances for different cases are summarized in Table 3.1, as well as the probabilities of traveling to corresponding direction. Those summarized expressions are used in the following two subsections in order to obtain expected-distance expressions.

Scenarios 1 and 2

Because we only consider dock doors located on the left half of the warehouse for Scenarios 1 and 2, equations for R1 in Table 3.1 are used to develop expected horizontal roundtrip expressions.

Table 3.1: Horizontal roundtrip-distances and probabilities from dock door *i* to a two-sided aisle

Dock-door Location	Aisle Type	Travel Direction	Warehouse Side	Distance	Probability
R1	Two-sided	Left	Left	$(L + 2d_i + 8v) / 2$	$(L - 2d_i) / 2L$
	Two-sided	Right	Left	$L - d_i + 4v$	d_i / L
	Two-sided	Right	Right	$(3L - 4d_i + 8v) / 2$	$1 / 2$
R2	Two-sided	Right	Left (Right)	$(3L - 4d_i + 8v) / 2$	$1 / 2$ $(1 / 2)$
R3	Two-sided	Left	Left (Right)	$(4d_i - L) / 2$	$1 / 2$ $(1 / 2)$
R4	Two-sided	Left	Left	$(4d_i - L) / 2$	$1 / 2$
	Two-sided	Left	Right	$d_i + 2v$	$(L - d_i + 2v) / L$
	Two-sided	Right	Right	$(3L - 2d_i + 12v) / 2$	$(2d_i - L - 4v) / 2L$

Multiplying distance by the probability and summing the results, the expected distance for dock door *i* is obtained.

Summing the results for dock doors located on the left side, dividing by the number of dock doors, and adding vertical distance, the expected single-command travel with an even number of dock doors is

$$E\left[SC_{even}\right] \approx 2\sum_{i=1}^{k_2/2}\left[\frac{L\left(L+4v\right)-2d_i^{\,2}}{k_2\,L}\right]+\frac{2\,n\,a\left(n+1\right)}{\left(2n+1\right)}+2v\,. \tag{3.35}$$

As with the discrete formulation, the middle dock door is located on the centerline of the warehouse for both Scenarios 1 and 2 if the number of dock doors is odd. Adjusting Equation (3.35) for (k_2 – 1) dock doors, adding the distance for the middle dock door (L / 2 + 2v) and dividing by the total number of dock doors, the expected single-command travel with an odd number of dock doors is

$$E\left[SC_{odd}\right] \approx 2\sum_{i=1}^{(k_2-1)/2}\left[\frac{L\left(L+4v\right)-2d_i^{\,2}}{k_2\,L}\right]+\frac{L+4v}{2k_2}+\frac{2\,n\,a\left(n+1\right)}{2n+1}+2v \tag{3.36}$$

Notice $d_i = i\,(L + 2v) / (k_2 + 1)$ and $d_i = [L + 2v - \delta_2\,(k_2 + 1)] / 2 + \delta_2\,(i - 1)$ for Scenarios 1 and 2, respectively.

<u>*Scenario 3*</u>

The number and locations of dock doors are calculated, based on the first dock door being located a given distance from the leftmost storage location and with a fixed distance between adjacent dock doors. Using Table 3.1 and determining the locations of dock doors ($d_i = \phi_2 + \delta_2\,(i - 1)$), the following conditional expressions are developed

If $d_{k_2} \le L/2$

$$E[SC] \approx \sum_{i=1}^{k_2}\left[\frac{L(L+4v)-2d_i^{\,2}}{k_2\,L}\right]+\frac{2\,n\,a(n+1)}{2n+1}+2v\,. \tag{3.37}$$

If $L/2 < d_{k_2} \le L/2 + v$

$$E[SC] \approx \sum_{i=1}^{\left\lceil \frac{L-2\phi_2}{2\delta_2}\right\rceil}\left[\frac{L(L+4v)-2d_i^{\,2}}{k_2\,L}\right]+\sum_{i=\left\lceil \frac{L-2\phi_2}{2\delta_2}\right\rceil+1}^{k_2}\left[\frac{3L-4d_i+8v}{2k_2}\right]+\frac{2\,n\,a(n+1)}{2n+1}+2v\,. \tag{3.38}$$

If $L/2 + v < d_{k_2} \le L/2 + 2v$

$$E[SC] \approx \sum_{i=1}^{\left\lceil \frac{L-2\phi_2}{2\delta_2}\right\rceil}\left[\frac{L(L+4v)-2d_i^{\,2}}{k_2\,L}\right]+\sum_{i=\left\lceil \frac{L-2\phi_2}{2\delta_2}\right\rceil+1}^{\left\lceil \frac{L-2\phi_2+2v}{2\delta_2}\right\rceil}\left[\frac{3L-4d_i+8v}{2k_2}\right]+\sum_{i=\left\lceil \frac{L-2\phi_2+2v}{2\delta_2}\right\rceil+1}^{k_2}\left[\frac{4d_i-L}{2k_2}\right]+\frac{2\,n\,a(n+1)}{2n+1}+2v. \tag{3.39}$$

If $L/2 + 2v < d_{k_2}$

$$E[SC] \approx \sum_{i=1}^{\left\lceil \frac{L-2\phi_2}{2\delta_2}\right\rceil}\left[\frac{L(L+4v)-2d_i^{\,2}}{k_2\,L}\right]+\sum_{i=\left\lceil \frac{L-2\phi_2}{2\delta_2}\right\rceil+1}^{\left\lceil \frac{L-2\phi_2+2v}{2\delta_2}\right\rceil}\left[\frac{3L-4d_i+8v}{2k_2}\right]+\sum_{i=\left\lceil \frac{L-2\phi_2+2v}{2\delta_2}\right\rceil+1}^{\left\lceil \frac{L-2\phi_2+4v}{2\delta_2}\right\rceil}\left[\frac{4d_i-L}{2k_2}\right]$$
$$+\sum_{i=\left\lceil \frac{L-2\phi_2+4v}{2\delta_2}\right\rceil+1}^{k_2}\left[\frac{4d_i(L+2v)-L(L+4v)-8v^2-2d_i^{\,2}}{k_2\,L}\right]+\frac{2\,n\,a(n+1)}{2n+1}+2v. \tag{3.40}$$

Travel-between

Because the one-sided aisle is ignored in approximating horizontal distance, the expected horizontal travel-between distance is similar to Equation (3.15) for Layout B. Using Equation (3.33) to calculate the expected vertical travel-between distance, the expected travel-between distance for Layout C with the continuous approximation is

$$E[TB] \approx \frac{1}{n}\left[\frac{L}{3} + v + (n-1)\left(\frac{5L}{12} + 2v\right)\right] + \frac{2na(n+1)}{3+6n}. \tag{3.41}$$

As with the discrete formulations, the interested reader can obtain dual-command expressions for Scenarios 1 and 2 by summing Equation (3.35) (3.36) and (3.41) for an even (odd) number of dock doors. A conditional expression for Scenario 3 with dual-command travel can be obtained by summing Equation (3.41) and the appropriate equation (Equations 3.37-3.40), depending on the locations of dock doors.

Although expected single- or dual-command travel can be transformed into a convenient closed-from expression, deriving the optimal shape factor in closed-form is not analytically tractable because closed-form expressions for expected distance with respect to the width (depth) of the warehouse and the given area are quite complicated. Therefore, optimal shape factor values are obtained by employing optimization software to solve the optimization problems.

3.5.3. Computational Results

Our computational results are based on parameter values employed previously. In addition, the following parameter values are used: δ_2 = 12 ft, ϕ_2 = 30 ft, k_2 varying from 1 – 60 for Scenario 1 and from 1 – 75 for Scenarios 2 and 3. As with Layout B, optimum solutions are obtained by using *Couenne* (2006) in *AMPL* (2013). Notice the space constraint for Layout C is $(a\,n + 0.5a + v)\,(w\,m + 6v) \geq A$; also, the width constraints for Scenarios 1-3 are $w\,m + 2v \geq (k_2 + 1)\,\delta_2$, $w\,m + 2v \geq k_2\,\delta_2$ and $w\,m + 2v \geq \phi_2 + (k_2 - 0.5)\,\delta_2$. Optimal shape factor results depicted in figures are based on continuous approximation results.

Figure 3.8 compares the results of Formulations 1 and 2 for single- and dual-command travel with Scenario 1. Continuous approximation underestimates the expected distance (except k_2 = 56 and k_2 = 59 for single-command travel and k_2 = 56 for dual-command travel). When the width constraint is satisfied, the average percentage errors of continuous approximations are 0.34% and 0.69% for single- and dual-command travel, respectively. Because of integer values for the variables in discrete formulations, the average percentage errors increase when the width constraint is violated (0.43% and 0.92% for single- and dual-command, respectively).

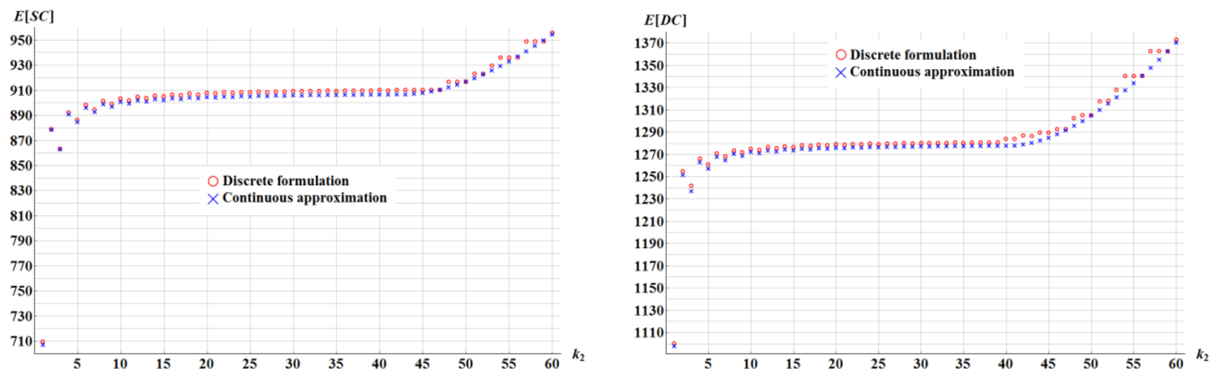

Figure 3.8: Comparison of expected-distance results of Formulations 1 and 2 with Scenario 1

Having two dock doors increases expected distance 24% and 14% for single- and dual command, respectively. This occurs because dock doors are located farther apart. Because the location of the middle dock door coincides with the centerline of the warehouse, having an odd number of dock doors dampens the expected distance for small values of k_2. When the number of dock doors is large, increasing the number of dock doors increases expected distance for single- and dual-command travel.

As seen in Figure 3.9, the optimal shape factor for travel-between is 0.81 when the width constraint is not violated. For any given number of dock doors, the optimal shape factor for dual-command operations is less than the corresponding optimal shape factor for single-command travel. For a small number of dock doors, the optimal shape factor fluctuates depending on the number of dock doors being either odd or even. For a large number of dock doors, increasing the number of dock doors slightly decreases the optimal shape factor for both single- and dual-command travel when the width constraint is satisfied. Otherwise, as stated previously, the width constraint governs the optimal shape factor; hence, increasing the number of dock doors increases the optimal shape factor.

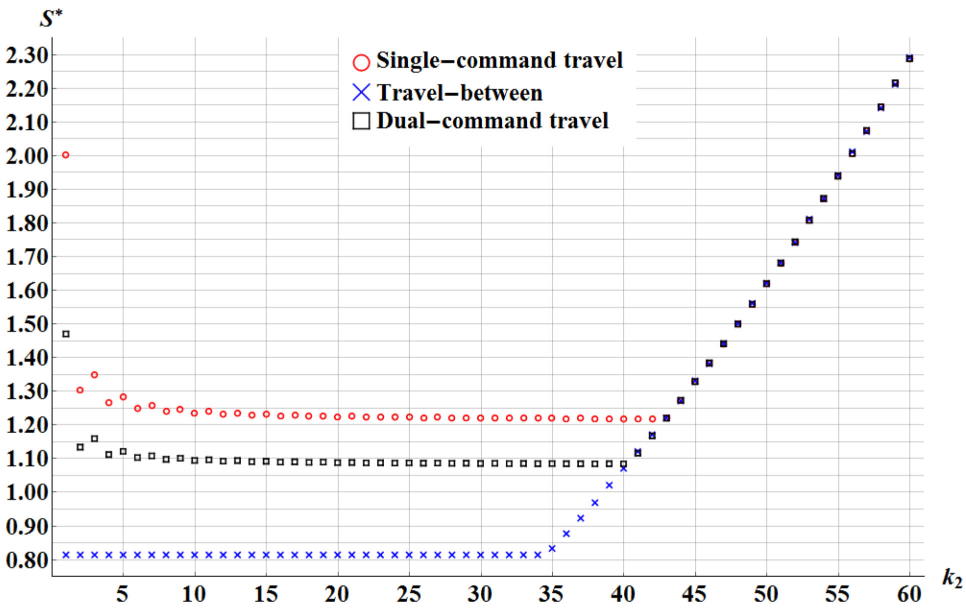

Figure 3.9: Optimal shape factor for k_2 dock doors with Scenario 1

Figure 3.10 compares the optimal shape factor values for discrete formulations and continuous approximations. Although, the same insights can be drawn using either discrete formulations or continuous approximations, optimal shape factor values are noticeably different because of the constraints for the required area and an even number of storage locations.

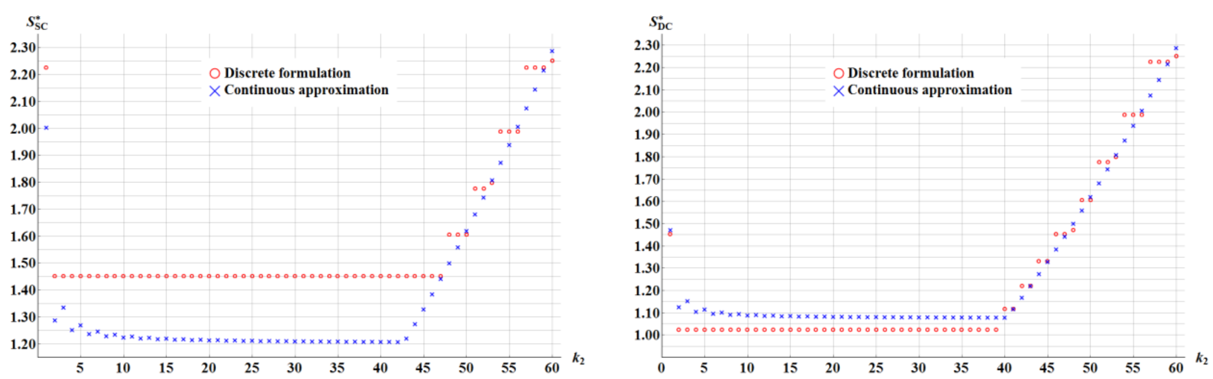

Figure 3.10: Comparison of optimal shape factor results of Formulations 1 and 2

From Figure 3.11, increasing the number of dock doors increases the expected distance for single- and dual-command travel when the distance between adjacent dock doors is specified. Because the one-sided aisle is ignored, a continuous approximation overestimates or underestimates the expected distance, depending on the number of dock doors. If the width constraint is satisfied, the average percentage errors for single- and dual-command travel are 0.17% and 0.07%, respectively. Otherwise, the percentage error is 0.23% for both single- and dual-command travel.

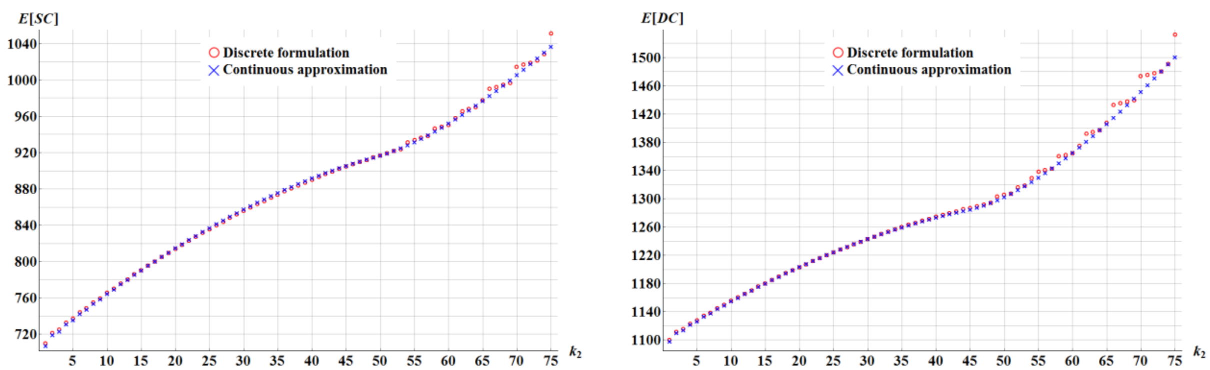

Figure 3.11: Comparison of expected-distance results of Formulations 1 and 2 with Scenario 2

As illustrated in Figure 3.12, increasing the number of dock doors decreases the optimal shape factor when the width constraint is satisfied; otherwise, the width constraint forces the warehouse to be wider. Regardless of the number of dock doors, as with Scenario 1, the optimal shape factor for travel between is 0.81. The warehouse optimized for single-command travel is wider than the warehouse optimized for dual-command travel.

Comparisons of the optimal shape factors for single- and dual-command travel are provided in Figure 3.13. Although the constraints for the required area and an even number of S/R locations results in different optimal shape factor values for discrete formulations, the same insights can be drawn using continuous approximations. From the computational results for Scenario 3, the percentage errors for single-and dual command travel are 0.12% and 0.19%, respectively, when the width constraint is satisfied. If the width constraint is violated, the average percentage errors are 0.17% and 0.25% for single- and dual-command travel, respectively. Recalling Figure 3.7, when dock doors are located in Region 1, increasing the number of dock doors decreases expected distance for both single- and dual-command travel.

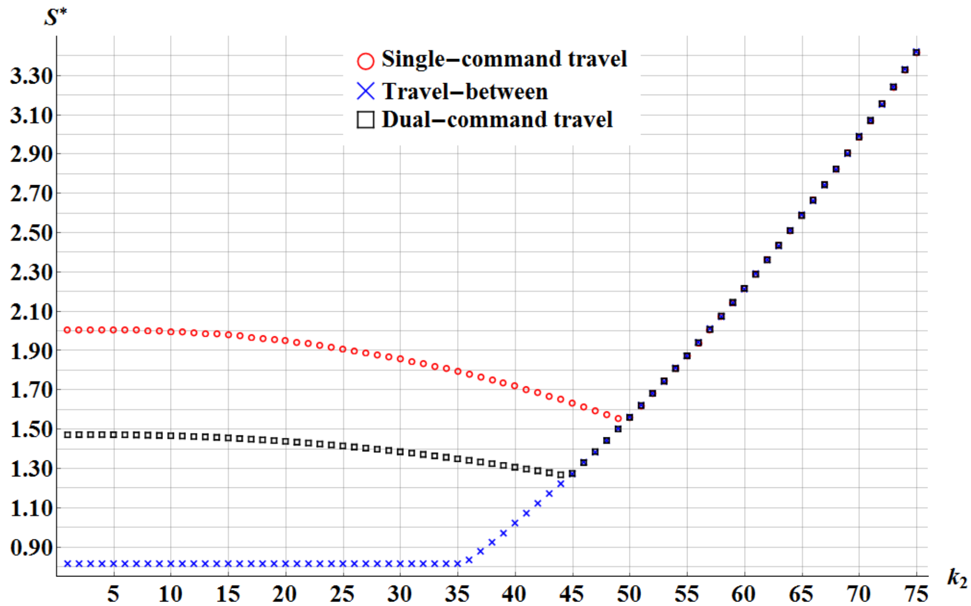

Figure 3.12: Optimal shape factor for k_2 dock doors with Scenario 2

Locating dock doors in Regions 2-4 dampens the decrement on expected distance (after 17-26 and 17-28 dock doors for single- and dual command travel, respectively). After locating dock doors in Region 4 (6 and 8 dock doors for single- and dual-command travel, respectively), increasing the number of dock doors increases expected single- and dual-command distance.

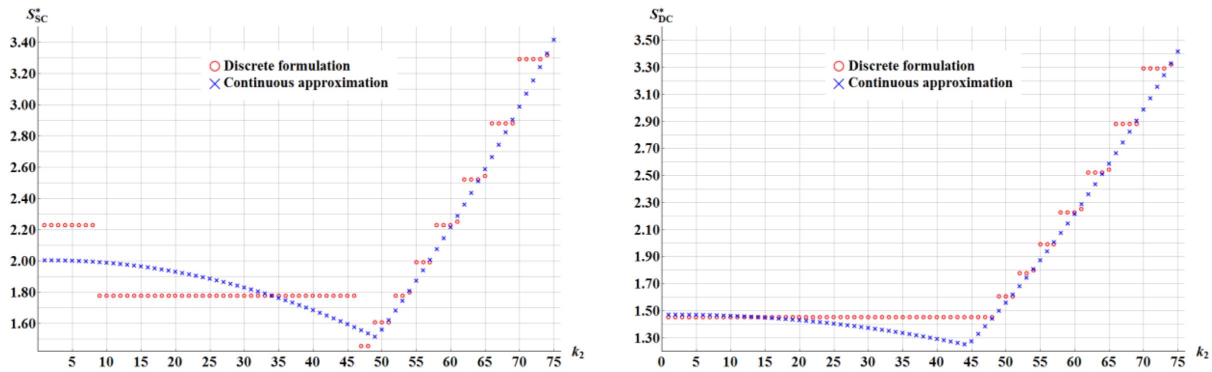

Figure 3.13: Comparison of optimal shape factor results of Formulations 1 and 2

As illustrated in Figure 3.14, expected distance with discrete formulation fluctuates when the width constraint is violated because of the constraints for space and an even number of S/R locations. A continuous approximation appears to provide reliable results for both single- and dual-command travel.

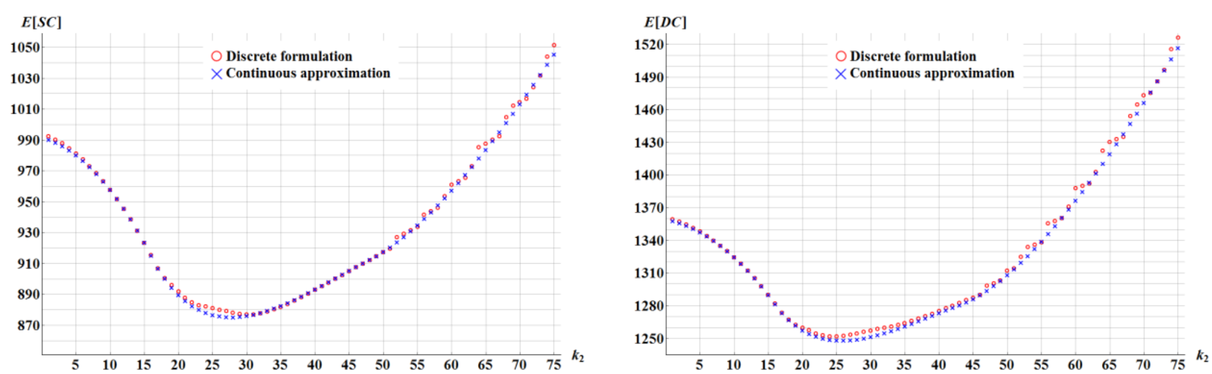

Figure 3.14: Comparison of expected-distance results of Formulations 1 and 2 with Scenario 3

From Figure 3.15, increasing the number of dock doors decreases the optimal shape factor when dock doors are located in Region 1 (after 17 dock doors for both single- and dual-command travel). When dock doors are located in Regions 2-4, increasing the number of dock doors increases the optimal shape factor for single- and dual command travel.

Notice the optimal shape factor fluctuates after 17 dock doors in Figure 3.15 because increasing the number of dock doors changes the number of dock doors located in each region.

The optimized warehouse with single-command travel is wider than the optimized warehouse with dual-command travel because travel-between dampens the optimal shape factor for dual-command travel.

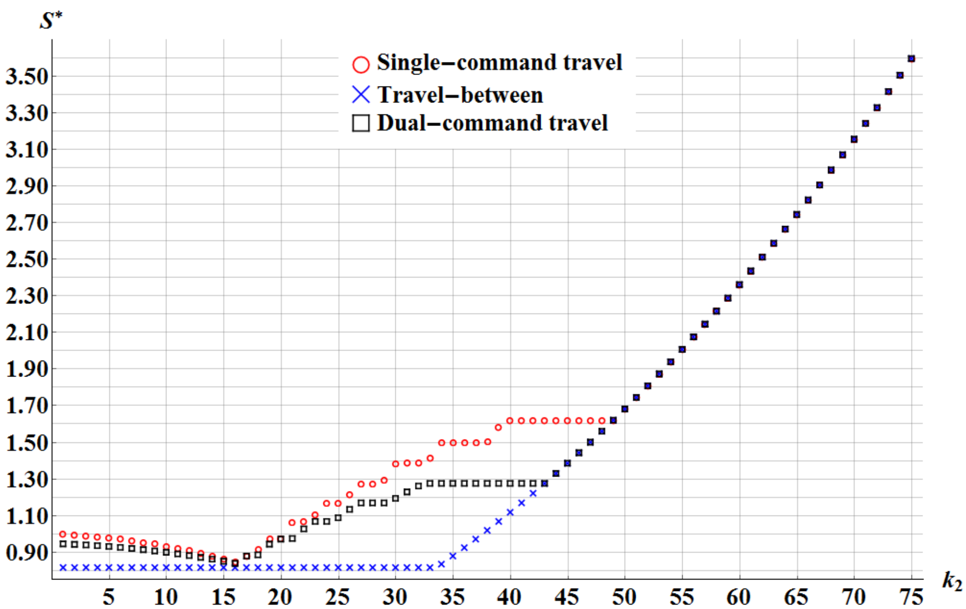

Figure 3.15: Optimal shape factor for k_2 dock doors with Scenario 3

In contrast to Scenarios 1 and 2, a continuous approximation appears to provide reliable values of the optimal shape factor for single- and dual-command travel. From Figure 3.16, the same insights can be drawn using either discrete formulations or continuous approximations.

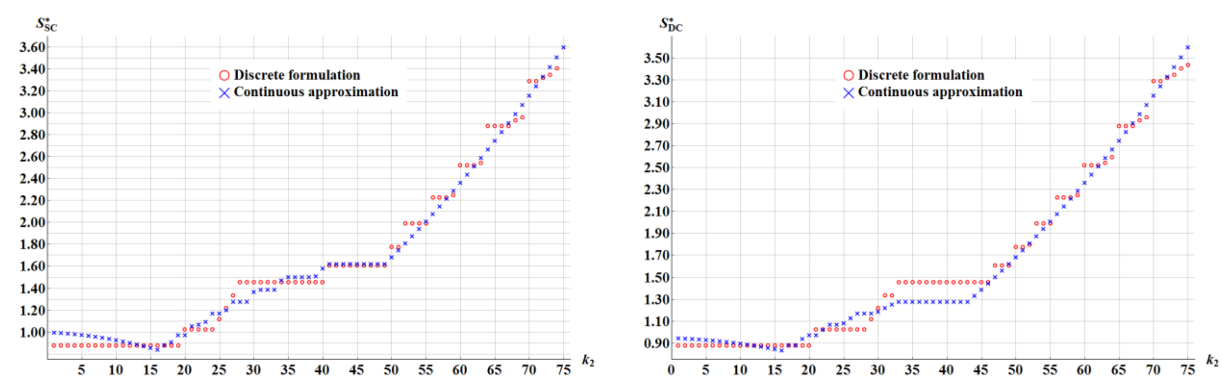

Figure 3.16: Comparison of optimal shape factor results of Formulations 1 and 2

3.6. Comparison of Traditional Warehouses

In this section, we compare traditional layout configurations by employing continuous approximations of expected distance. We continue to use the same parameter values as in previous sections for Layouts B and C. Retaining a consistent comparison, we used formulations given in Chapter 2 to provide results for Layout A. Comparing configurations with the same number of S/R locations; we remove the space constraint from the optimization models and employ a constraint on the number of S/R locations. The S/R location constraint assures each configuration has the same number of S/R locations; it is $2m\,n = 6{,}000$ for Layouts A and B and $m\,(2n + 0.5) = 6{,}000$ for Layout C ($A \geq 250{,}000$ ft^2).

3.6.1. Scenario 1

For single-command travel, Layout A outperforms Layouts B and C as illustrated in Figure 3.17 (except for the single-dock-door case for Layout C). The expected distance for Layouts A and C are approximately the same when a single dock door is located on the centerline of the warehouse. Because of middle-cross aisle travel for S/R locations above the middle-cross aisle, Layout B has the greatest expected-distance value. In contrast to single-command travel, Layout B always outperforms Layouts A and C for dual-command travel. Although Layout C performs well for a single dock door, its expected distance is greater than those for Layouts A and B for both single- and dual-command travel. When the width constraint governs the shape of the warehouse, the expected dual-command distance for Layouts B and C increase dramatically as the number of dock doors increases because of travel-between distance. The expected distance for all configurations increases with an increasing number of dock doors.

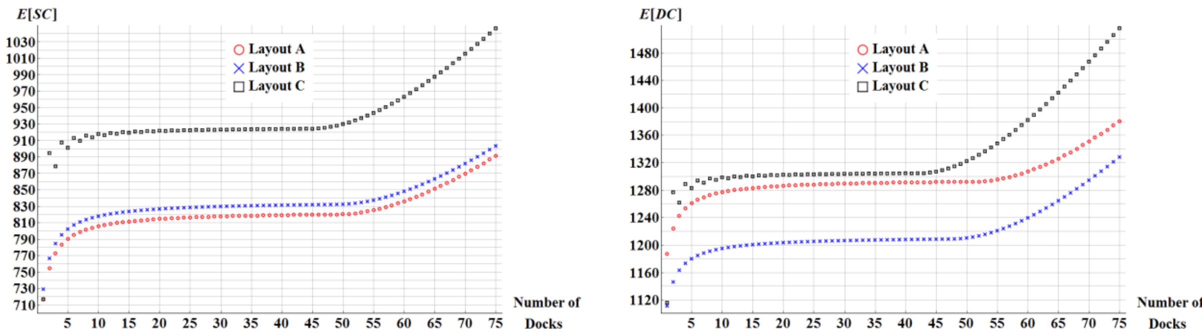

Figure 3.17: Expected-distance comparison of traditional configurations with Scenario 1

When a dock door is centrally located, the optimally designed Layout C is wider than the optimally designed Layouts A and B. Notice this result is different from that obtained by Pohl et al. (2009) because the S/R locations along the wall containing k_2 dock doors are removed. From Figure 3.18, an optimally configured Layout A is always wider and shorter than an optimally configured Layout B for both single-and dual-command travel. Increasing the number of dock doors decreases optimal shape factor values for Layouts A and B when the width constraint is satisfied.

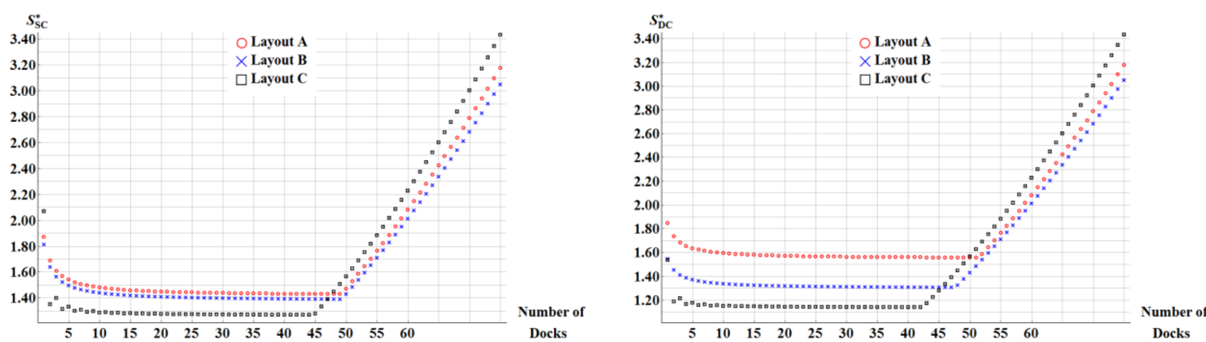

Figure 3.18: Optimal shape factor comparison of traditional configurations with Scenario 1

3.6.2. Scenario 2

When dock doors are located with a specified distance between adjacent dock doors, increasing the number of dock doors increases dramatically expected distance for Layout C (see Figure 3.19). Except for the single-dock-door case, Layout A outperforms Layouts B and C with single-command travel. Layout B always performs better than Layouts A and C for dual-command travel. With a small number of dock doors, Layout C dominates Layout A; whereas, Layout A outperforms Layout C when the number of dock doors is large or when the width constraint is violated. As with Scenario 1, increasing the number of dock doors always increases expected distance for all configurations.

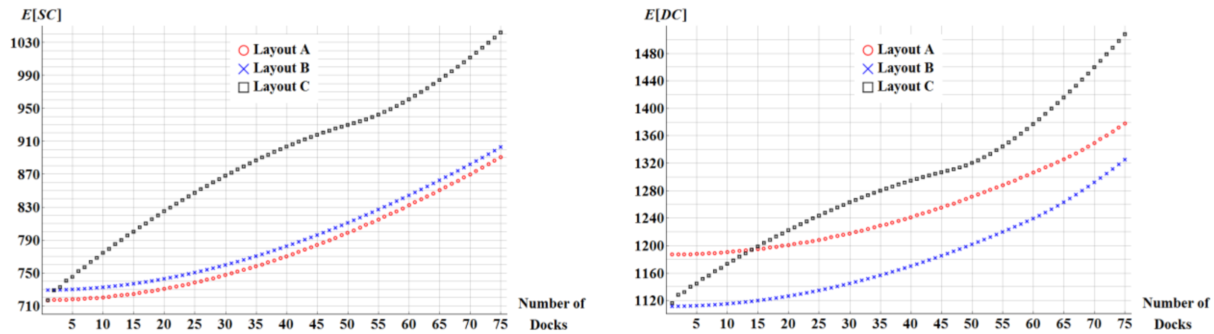

Figure 3.19: Expected-distance comparison of traditional configurations with Scenario 2

As illustrated in Figure 3.20, increasing the number of dock doors increases the optimal shape factor for Layouts A and B; whereas, the optimal shape factor decreases with an increasing number of dock doors for Layout C when the width constraint is satisfied. However, an optimally configured Layout A is wider and smaller than optimally configured Layouts B and C for dual-command travel.

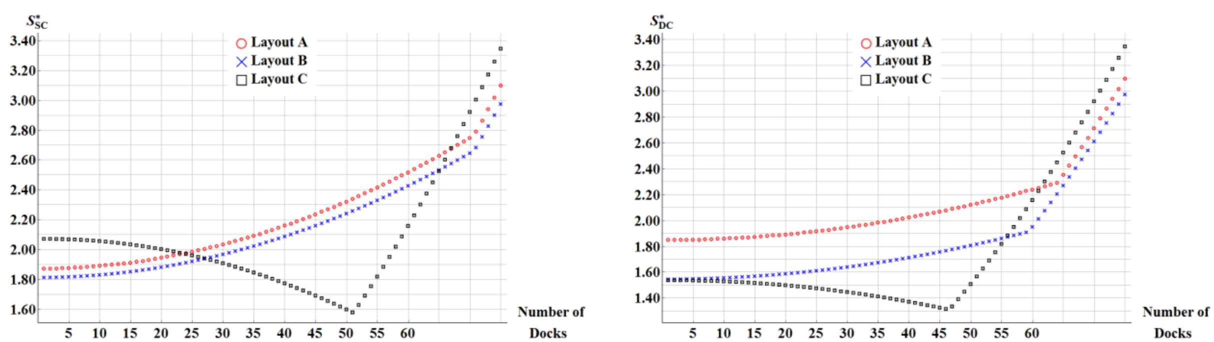

Figure 3.20: Optimal shape factor comparison of traditional configurations with Scenario 2

3.6.3. Scenario 3

In contrast to Scenarios 1 and 2, increasing the number of dock doors decreases the expected distance for a small number of dock doors (see Figure 3.21). Layout A always outperforms Layouts B and C for single-command travel; whereas, Layout B always performs better than Layouts A and C for dual-command travel. For dual-command travel, the performances of Layouts A and C are the same when dock doors are located close to the centerline of the warehouse.

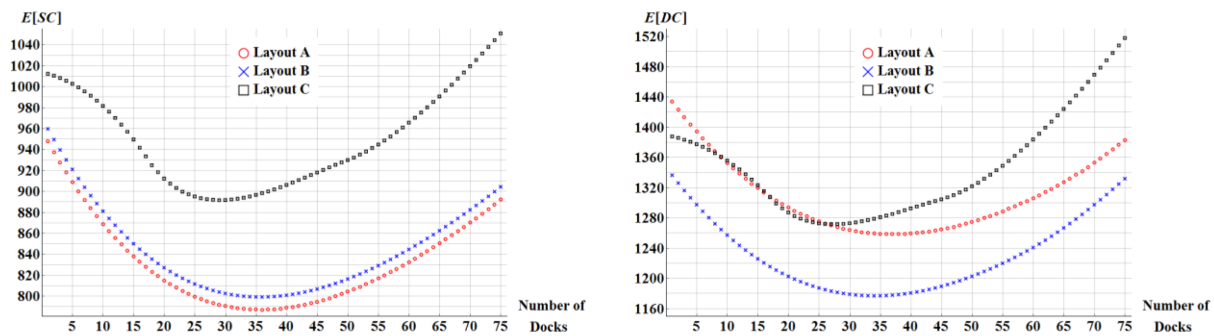

Figure 3.21: Expected-distance comparison of traditional configurations with Scenario 3

Increasing the number of dock doors always increases the optimal shape factor for Layouts A and B; whereas, the optimal shape factor may increase or decrease for Layout C (see Figure 3.22). Having a large number of dock doors results in the optimal shape factor fluctuating for Layout C because of dock doors being located on different sides of the warehouse.

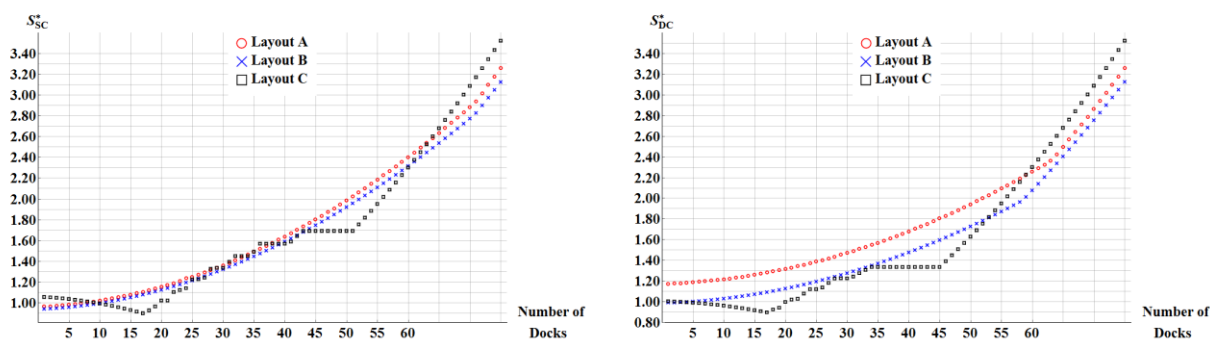

Figure 3.22: Optimal shape factor comparison of traditional configurations with Scenario 3

3.7. Layout D

Locating shipping and receiving dock doors on adjacent walls, we allow unit loads to be received from k_1 dock doors located along one wall and to be shipped from k_2 dock doors located on an adjacent wall. We consider a mixture of single-command, dual-command and cross-docking travel. We let p_i denote the probability of travel type i occurring, where $i = 1$ for single-command and $i = 2$ for dual-command. Therefore, the probability of cross-docking travel is $1 - p_1 - p_2$.

The aisle configuration of Layout D is identical to that of Layout C. However, Layout D differs from Layout B by including an additional one-sided aisle and a half cross-aisle to locate k_2 dock doors. Therefore, in developing formulas for expected distance, we adjust equations in Section 3.4 according to the new configuration, we use equations given in Section 3.5 by changing the parameter d_i, and we develop formulations for cross-docking travel.

We assume the number of storage operations is equal to the number of retrieval operations because, in the long run, the number of unit loads received equals the number of unit loads shipped. In performing single-command operations, two types of moves occur: transporting a unit-load from a receiving dock door to a storage location ($E\ [SC_B]$) and transporting a unit-load from a retrieval location to a shipping dock door ($E\ [SC_C]$). Based on the aforementioned assumption, the expected single-command for a unit-load is ($E\ [SC_B] + E\ [SC_C]$) / 2. In dual-command operations, S/R equipment transports a unit-load from receiving to storage ($E\ [SC_B]$ / 2), travels empty from the storage location to the retrieval location ($E\ [TB_C]$), transports a unit-load to shipping ($E\ [SC_C]$ / 2), and travels empty from shipping to receiving ($E\ [CD]$ / 2). Cross-docking operations include two moves: transporting a unit-load from receiving to shipping and traveling empty from shipping to receiving ($E\ [CD]$). Therefore, the overall expected distance traveled for a unit-load is

$$E[MC] = p_1\left(E[SC_B] + E[SC_C]\right) + p_2\left(E[SC_B]/2 + E[TB_C] + E[SC_C]/2 + E[CD]/2\right) \\ + \left(1 - p_1 - p_2\right) E[CD]. \tag{3.42}$$

Reducing Equation (3.42), we obtain

$$E[MC] = \left(p_1 + 0.5p_2\right)\left\{E[SC_B] + E[SC_C]\right\} + p_2 E[TB_C] + \left(1 - p_1 - 0.5p_2\right) E[CD]. \tag{3.43}$$

3.7.1. Discrete Formulations

In the new configuration, there are n two-sided aisles and a single one-sided aisle. Because inserting a one-sided aisle does not affect vertical roundtrip-distance for Layout B, we only adjust horizontal roundtrip-distance. The distance between dock door i and the one-sided aisle equals

$n\ a + a\ /\ 2 - d_i$. Notice the probabilities of traveling to a one-sided and a two-sided aisle are $1\ /\ (2n + 1)$ and $2\ /\ (2n + 1)$, respectively. Adjusting Equation (3.1) for the new aisle configuration, the expected horizontal roundtrip-distance ($E[SC_h]$) for the adjusted Layout B with k_1 dock door is

$$E\left[SC_h\right] = \frac{4}{(2n+1)k_1}\sum_{i=1}^{k_1}\sum_{j=1}^{n}\left|d_i - \left(j - 1/2\right)a\right| + \frac{2}{(2n+1)k_1}\sum_{i=1}^{k_1}\left(na + a/2 - d_i\right). \tag{3.44}$$

Notice we choose the bottom right corner as the *initial point*. Therefore, d_{1i} denotes the distance between the centerline of the i^{th} dock door located on the wall containing k_1 dock doors and the wall containing k_2 dock doors. Substituting $d_i = W - d_{1i}$ in Equation (3.44) and adding the expected vertical distance, the expected single-command travel for the adjusted Layout B becomes

$$E[SC_B] = D + \frac{1}{a(2n+1)k_1} \sum_{i=1}^{k_1} \left[a^2 \left(1 + 2n + 2n^2\right) - 2\left(W - d_{1i}\right)\left(a + 2an - 4t_{1i}\right) - 4t_{1i}^{\;2} \right]. \qquad (3.45)$$

where t_{1i} = *Round* [$W - d_{1i}$, a]. Note that $d_{1i} = [W(k_1 - i + 1)] / (k_1 + 1)$ for Scenario1, $d_{1i} = [W + (k_1 - 1)\,\delta_1] / 2 - (i - 1)\,\delta_1$ for Scenario 2 and $d_{1i} = \phi_1 + (k_1 - i)\,\delta_1$ for Scenario 3.

To avoid confusion of dock-door locations for receiving and shipping, we change parameter d_i to d_{2j} for shipping dock doors (similarly, t_i is changed to t_{2j}). Modifying Equations (3.22) and (3.23) for an even and an odd number of dock doors, respectively, we can obtain formulations for shipping dock doors with Scenarios 1 and 2. A conditional expression for Scenario 3 can be obtained by using d_{2j} instead of d_i in Equations (3.24-3.27) for shipping dock doors (similarly, t_{2j} instead of t_i).

The distance between the i^{th} dock door located on the wall containing k_1 dock doors and the j^{th} dock door located on the wall containing k_2 dock doors is $\phi_1 + (k_1 - i)\,\delta_1 + \phi_2 + (j - 1)\,\delta_2$. Notice dock doors are numbered in ascending order from left to right or from bottom to top. Summing the distance between all pairs of dock doors, multiplying by two and dividing by the number of pairs, the expected cross-docking roundtrip-distance is

$$E[CD] = \frac{2}{k_1 k_2} \sum_{i=1}^{k_1} \sum_{j=1}^{k_2} \left[d_{1i} + d_{2j} + 4v \right]. \qquad (3.46)$$

3.7.2. Continuous Approximations

Equations in Section 3.4.2 (Section 3.5.2) are used to develop continuous formulations for receiving (shipping) dock doors because the locations of S/R racks and aisles are ignored in calculating expected single-command distance. Specifically, Equations (3.12-3.14) are used to obtain formulations for receiving dock doors depending on the scenarios. However, for Scenarios 1 and 2, Equations (3.35) and (3.36) are used to develop expected-distance expressions for shipping dock doors with an even and an odd number of dock doors, respectively. In addition, the appropriate equation (Equations 3.37-3.40) is used for Scenario 3 depending on the locations of dock doors and Equation (3.46) holds for the calculations of expected cross-docking distance. The formulations for each scenario are provided in Appendix B.

3.7.3. Computational Results

In previous sections, we presented results for single-command and dual-command travel individually. Locating shipping dock doors along one wall and receiving dock doors along an adjacent wall of the warehouse, we provide the results for expected distance and the optimal shape factor considering a mixture of single-command, dual-command and cross-docking travel. In doing so, we consider three scenarios: 1) the warehouse is more focused on single-command operations ($p_1 = 0.6$ and $p_2 = 0.2$), 2) the warehouse is more focused on dual-command operations ($p_1 = 0.2$ and $p_2 = 0.6$) and 3) the warehouse is more focused on cross-docking operations ($p_1 = 0.2$ and $p_2 = 0.2$).

As before, *Couenne* (2006) in *AMPL* (2013) is used to obtain computational results based on parameter values employed previously. The space constraint for Layout D is $(a\,n+0.5a + v)\,(w\,m + 6v) \geq A$. However, the width constraints are $w\,m + 2v \geq (k_2 + 1)\,\delta_2$ and $a\,n + w \geq (k_1 + 1)\,\delta_1$ for Scenario 1; $w\,m + 2v \geq k_2\,\delta_2$ and $a\,n + w \geq k_1\,\delta_1$ for Scenario 2; and $w\,m + 2v \geq \phi_2 + (k_2 - 0.5)\,\delta_2$ and $a\,n+0.5a + v \geq \phi_1 + (k_1 - 0.5)\,\delta_1$ for Scenario 3. Assuming $\phi_1 \geq 2v$ for Scenario 3, we enforce a minimum separation between the closest shipping and receiving dock doors. Expected distance and optimal shape factor tables provided in the following sections are based on continuous approximations.

Scenario 1

The first observation from Table 3.2 is that increasing the number of receiving dock doors will always increase the expected distance traveled regardless of the focus of the warehouse. Conversely, increasing the number of shipping dock doors alternately increases and decreases the expected distance when the width constraint is satisfied because the midmost dock door with an odd number of dock doors coincides with the middle-cross-aisle (e.g. increasing the shipping dock doors from 1 to 2 increases expected distance from 1425.1 ft to 1516.3 ft for a single-command focused warehouse; whereas, increasing the shipping dock doors from 2 to 3 decreases expected distance from 1516.3 ft to 1507.5). Locating an odd number of dock doors dampens expected distance because the midmost dock door coincides with the middle-cross-aisle. However, because shipping dock doors are aligned parallel to S/R locations, increasing the number of shipping dock doors has a greater impact on expected distance than does increasing the number of receiving dock doors.

Due to the space constraint and the constraint on the distance between adjacent dock doors ($\delta_1 \geq 12$ ft and $\delta_2 \geq 12$ ft), a limited number of dock doors can be located along the adjacent walls. The maximum number of receiving and shipping dock doors is a function of the storage area and width constraints; for example, based on the parameters used throughout the chapter locating 41 shipping and 41 receiving dock doors simultaneously is infeasible.

Examining the percentage error due to the use of continuous approximations for the parameter values employed, the average percentage error in expected distance is approximately 0.45%.

Table 3.2: Expected-distance values for Scenario 1

$p_1 = 0.6$ and $p_2 = 0.2$

k_1 dock doors (receiving)	k_2 dock doors (shipping)								
	1	2	3	4	5	11	21	31	41
1	1425.1	1516.3	1507.5	1523.4	1519.9	1528.3	1531.2	1532.1	1546.4
2	1443.9	1536.4	1527.4	1543.5	1540.0	1548.6	1551.5	1552.4	1564.2
3	1453.2	1546.3	1537.3	1553.5	1549.9	1558.6	1561.5	1562.4	1573.1
4	1458.7	1552.2	1543.2	1559.5	1555.9	1564.6	1567.5	1568.4	1578.5
5	1462.4	1556.2	1547.1	1563.4	1559.8	1568.5	1571.5	1572.4	1582.0
11	1471.6	1566.0	1556.8	1573.3	1569.7	1578.4	1581.4	1582.3	1590.9
21	1475.7	1570.4	1561.2	1577.7	1574.1	1582.9	1585.8	1586.8	1595.0
31	1477.3	1572.1	1562.9	1579.4	1575.8	1584.6	1587.5	1588.5	1596.5
41	1481.1	1572.9	1563.8	1580.3	1576.6	1585.4	1588.4	1589.3	*inf.*

$p_1 = 0.2$ and $p_2 = 0.6$

k_1 dock doors (receiving)	k_2 dock doors (shipping)								
	1	2	3	4	5	11	21	31	41
1	1478.6	1543.7	1537.4	1548.8	1546.3	1552.4	1554.5	1555.1	1567.1
2	1492.2	1558.0	1551.5	1563.1	1560.6	1566.7	1568.8	1569.5	1579.8
3	1498.9	1565.0	1558.6	1570.2	1567.6	1573.8	1575.9	1576.6	1586.1
4	1503.0	1569.3	1562.8	1574.4	1571.9	1578.1	1580.2	1580.9	1589.9
5	1505.6	1572.1	1565.6	1577.3	1574.7	1580.9	1583.0	1583.7	1592.5
11	1512.3	1579.1	1572.5	1584.3	1581.7	1588.0	1590.1	1590.7	1598.8
21	1515.3	1582.2	1575.7	1587.5	1584.9	1591.1	1593.2	1593.9	1601.7
31	1516.5	1583.4	1576.9	1588.7	1586.1	1592.3	1594.4	1595.1	1602.8
41	1518.5	1584.0	1577.5	1589.3	1586.7	1593.0	1595.1	1595.8	*inf.*

$p_1 = 0.2$ and $p_2 = 0.2$

k_1 dock doors (receiving)	k_2 dock doors (shipping)								
	1	2	3	4	5	11	21	31	41
1	1237.9	1277.6	1273.7	1280.8	1279.2	1283.0	1284.2	1284.6	1291.1
2	1246.0	1286.0	1282.1	1289.2	1287.6	1291.4	1292.6	1293.1	1298.7
3	1250.1	1290.2	1286.2	1293.3	1291.8	1295.6	1296.8	1297.3	1302.6
4	1252.5	1292.7	1288.7	1295.8	1294.3	1298.1	1299.3	1299.8	1304.8
5	1254.1	1294.3	1290.4	1297.5	1295.9	1299.7	1301.0	1301.4	1306.4
11	1258.1	1298.5	1294.5	1301.6	1300.1	1303.9	1305.2	1305.6	1310.2
21	1259.9	1300.3	1296.4	1303.5	1301.9	1305.8	1307.0	1307.5	1311.9
31	1260.5	1301.0	1297.1	1304.2	1302.6	1306.5	1307.8	1308.2	1312.6
41	1262.1	1301.5	1297.6	1304.6	1303.1	1306.9	1308.1	1308.6	*inf.*

Comparing results for different focused warehouses, notice the expected distance for a single-command focused warehouse is less than that for the corresponding dual-command focused warehouse because returning S/R equipment to the receiving dock-door locations in dual-command operations results in traveling an additional distance (equivalent to the one-way cross-docking distance); the additional distance is greater than the distance reduced by performing a dual-command operation.

However, as the number of dock doors increases, the difference in expected distance between the single-command focused warehouse and the dual-command focused warehouse decreases. As expected, a cross-docking focused warehouse outperforms warehouses more focused on either single-command operations or dual-command operations.

Table 3.3 provides the optimal shape factor values for the three ratios of p_1 and p_2. When the width constraint is satisfied, increasing the number of shipping dock doors may increase or decrease the optimal shape factor depending on the number of shipping dock doors being either odd or even. In contrast to receiving dock doors, the optimal shape factor decreases as the number of receiving dock doors increases. When the width constraint is violated, the optimal shape factor is governed by the width constraint. Relative to the wall containing the most dock doors, as the number of dock doors increases, the relative width of the warehouse increases in order to have enough room to locate all dock doors. Specifically, although the optimal shape factor decreases for an increasing number of receiving dock doors, it increases when the width constraint comes into play.

Table 3.3: Optimal shape factor values for Scenario 1

$p_1 = 0.6$ and $p_2 = 0.2$

k_1 dock doors (receiving) \ k_2 dock doors (shipping)	1	2	3	4	5	11	21	31	41
1	1.011	1.148	1.134	1.159	1.153	1.167	1.171	1.173	0.897
2	0.984	1.117	1.104	1.128	1.123	1.135	1.140	1.141	0.897
3	0.971	1.103	1.089	1.113	1.108	1.120	1.125	1.126	0.897
4	0.963	1.094	1.081	1.104	1.099	1.112	1.116	1.117	0.897
5	0.958	1.088	1.075	1.099	1.093	1.106	1.110	1.111	0.897
11	0.946	1.074	1.061	1.084	1.079	1.092	1.096	1.097	0.897
21	0.940	1.068	1.055	1.078	1.073	1.085	1.089	1.091	0.897
31	0.938	1.066	1.053	1.076	1.071	1.083	1.087	1.088	0.897
41	1.065	1.065	1.065	1.075	1.069	1.082	1.086	1.087	*Inf.*

$p_1 = 0.2$ and $p_2 = 0.6$

k_1 dock doors (receiving) \ k_2 dock doors (shipping)	1	2	3	4	5	11	21	31	41
1	1.031	1.127	1.118	1.135	1.131	1.140	1.143	1.144	0.897
2	1.012	1.106	1.097	1.114	1.110	1.119	1.122	1.123	0.897
3	1.002	1.096	1.086	1.103	1.099	1.108	1.111	1.112	0.897
4	0.997	1.090	1.080	1.097	1.093	1.102	1.105	1.106	0.897
5	0.993	1.086	1.076	1.093	1.089	1.098	1.101	1.102	0.897
11	0.984	1.076	1.067	1.083	1.079	1.088	1.091	1.092	0.897
21	0.980	1.071	1.062	1.079	1.075	1.084	1.087	1.088	0.897
31	0.979	1.070	1.060	1.077	1.073	1.082	1.085	1.086	0.897
41	1.065	1.069	1.065	1.076	1.072	1.081	1.084	1.085	*Inf.*

$p_1 = 0.2$ and $p_2 = 0.2$

k_1 dock doors (receiving) \ k_2 dock doors (shipping)	1	2	3	4	5	11	21	31	41
1	1.012	1.081	1.074	1.086	1.084	1.090	1.093	1.093	0.897
2	0.999	1.066	1.060	1.072	1.069	1.076	1.078	1.078	0.897
3	0.992	1.059	1.052	1.065	1.062	1.068	1.071	1.071	0.897
4	0.988	1.055	1.048	1.060	1.058	1.064	1.066	1.067	0.897
5	0.985	1.052	1.045	1.057	1.055	1.061	1.063	1.064	0.897
11	0.979	1.045	1.039	1.050	1.048	1.054	1.056	1.057	0.897
21	0.976	1.042	1.035	1.047	1.045	1.051	1.053	1.054	0.897
31	0.975	1.041	1.034	1.046	1.043	1.050	1.052	1.053	0.897
41	1.065	1.065	1.065	1.065	1.065	1.065	1.065	1.065	*Inf.*

Scenario 2

In contrast to Scenario 1, increasing the number of dock doors will always increase expected distance regardless of warehouse type because dock-door locations are specified. As expected, expected distances for Scenario 2 are smaller than for Scenario 1 because dock doors are clustered around the centerlines of walls. Further, increasing the number of dock doors from 1 to 6 with Scenario 1 results in a significantly greater increase in expected distance than occurs with Scenario 2. Thereafter, increasing the number of dock doors from 6 to 11 results in a smaller increase in expected distance with Scenario 1 than occurs with Scenario 2 (12.88). This occurs because increasing the number of dock doors with Scenario 2 results in decreasing the distance between adjacent dock doors and resulting in innermost dock doors being located closer to the centerlines of walls.

As with Scenario 1, the single-command focused warehouse outperforms the dual-command focused warehouse, because the expected distance to return S/R equipment to receiving dock doors diminishes the improvement inherent in dual-command operations. Further, increasing the number of dock doors decreases the difference in expected distance between the single-command focused warehouse and the dual-command focused warehouse. (The average percentage error resulting from continuous approximations is 0.38%.)

As with Scenario 1, locating 41 shipping and 41 receiving dock doors simultaneously is infeasible because of the space constraint and the constraint on the distance between adjacent dock doors. As with Scenario 1, the cross-docking focused warehouse performs the best among the warehouses considered.

Table 3.4: Expected-distance values for Scenario 2

$p_1 = 0.6$ and $p_2 = 0.2$

k_1 dock doors (receiving)	k_2 dock doors (shipping) 1	2	3	4	5	11	21	31	41
1	1425.1	1433.5	1436.2	1441.6	1444.8	1467.6	1499.1	1523.0	1543.5
2	1425.2	1433.6	1436.3	1441.7	1444.9	1467.7	1499.2	1523.1	1543.6
3	1425.3	1433.7	1436.5	1441.9	1445.1	1467.9	1499.4	1523.3	1543.8
4	1425.6	1434.0	1436.7	1442.2	1445.3	1468.1	1499.7	1523.5	1544.1
5	1425.9	1434.3	1437.0	1442.5	1445.6	1468.4	1500.0	1523.8	1544.4
11	1429.2	1437.6	1440.3	1445.8	1448.9	1471.7	1503.2	1527.1	1547.8
21	1440.2	1448.6	1451.3	1456.7	1459.9	1482.7	1514.1	1537.8	1559.3
31	1457.8	1466.2	1468.9	1474.4	1477.5	1500.3	1531.5	1554.9	1577.9
41	1481.9	1490.3	1493.0	1498.4	1501.6	1524.2	1555.3	1578.4	*inf.*

$p_1 = 0.2$ and $p_2 = 0.6$

k_1 dock doors (receiving)	k_2 dock doors (shipping) 1	2	3	4	5	11	21	31	41
1	1478.6	1484.6	1486.5	1490.4	1492.7	1508.9	1531.4	1548.4	1564.1
2	1478.7	1484.7	1486.6	1490.5	1492.7	1509.0	1531.5	1548.5	1564.2
3	1478.8	1484.8	1486.7	1490.6	1492.9	1509.1	1531.6	1548.6	1564.3
4	1478.9	1484.9	1486.9	1490.8	1493.0	1509.3	1531.8	1548.8	1564.5
5	1479.2	1485.2	1487.1	1491.0	1493.3	1509.5	1532.0	1549.0	1564.7
11	1481.5	1487.5	1489.5	1493.3	1495.6	1511.9	1534.3	1551.3	1567.2
21	1489.3	1495.3	1497.2	1501.1	1503.4	1519.6	1542.0	1558.9	1575.4
31	1501.8	1507.8	1509.8	1513.7	1515.9	1532.1	1554.5	1571.2	1588.7
41	1519.0	1525.0	1526.9	1530.8	1533.1	1549.3	1571.5	1588.1	*inf.*

$p_1 = 0.2$ and $p_2 = 0.2$

k_1 dock doors (receiving)	k_2 dock doors (shipping) 1	2	3	4	5	11	21	31	41
1	1237.9	1241.5	1242.7	1245.0	1246.4	1256.2	1269.7	1280.0	1289.2
2	1238.0	1241.6	1242.8	1245.1	1246.4	1256.2	1269.7	1280.0	1289.2
3	1238.1	1241.7	1242.8	1245.2	1246.5	1256.3	1269.8	1280.1	1289.3
4	1238.2	1241.8	1242.9	1245.3	1246.6	1256.4	1269.9	1280.2	1289.4
5	1238.3	1241.9	1243.1	1245.4	1246.7	1256.5	1270.1	1280.4	1289.5
11	1239.7	1243.3	1244.5	1246.8	1248.2	1257.9	1271.5	1281.8	1291.0
21	1244.4	1248.0	1249.2	1251.5	1252.9	1262.6	1276.2	1286.4	1295.9
31	1252.0	1255.6	1256.8	1259.1	1260.5	1270.3	1283.7	1293.9	1303.9
41	1262.5	1266.1	1267.3	1269.6	1271.0	1280.7	1294.1	1304.3	*inf.*

As illustrated in Table 3.5, increasing the number of dock doors increases the optimal shape factor when the width constraint is satisfied. In contrast to Scenario 1, the optimal shape factor increases as the number of shipping dock doors increases. Based on observations regarding Layout B, this is an expected result. As before, when it is violated, the width constraint determines the value of the optimal shape factor. For a large number of dock doors ($k_1 \geq 26$ and $k_2 \geq 26$), the single-command focused warehouse is wider than the dual-command focused warehouse because of the additional travel to return S/R equipment to the dock-door locations.

Table 3.5: Optimal shape factor values for Scenario 2

$p_1 = 0.6$ and $p_2 = 0.2$	k_1 dock doors (receiving)		k_2 dock doors (shipping)								
			1	2	3	4	5	11	21	31	41
		1	1.011	1.011	1.011	1.011	1.011	1.016	1.032	1.062	0.939
		2	1.011	1.011	1.011	1.011	1.011	1.016	1.032	1.062	0.939
		3	1.011	1.011	1.011	1.011	1.012	1.016	1.033	1.062	0.939
		4	1.011	1.011	1.011	1.012	1.012	1.016	1.033	1.062	0.939
		5	1.012	1.012	1.012	1.012	1.012	1.017	1.033	1.063	0.939
		11	1.017	1.017	1.017	1.017	1.017	1.022	1.038	1.068	0.939
		21	1.033	1.033	1.033	1.033	1.033	1.038	1.055	1.085	0.939
		31	1.059	1.059	1.059	1.059	1.060	1.064	1.082	1.113	0.939
		41	1.095	1.095	1.095	1.096	1.096	1.101	1.119	1.151	*Inf.*

$p_1 = 0.2$ and $p_2 = 0.6$	k_1 dock doors (receiving)		k_2 dock doors (shipping)								
			1	2	3	4	5	11	21	31	41
		1	1.031	1.031	1.031	1.032	1.032	1.035	1.047	1.067	0.939
		2	1.031	1.031	1.032	1.032	1.032	1.035	1.047	1.067	0.939
		3	1.032	1.032	1.032	1.032	1.032	1.035	1.047	1.068	0.939
		4	1.032	1.032	1.032	1.032	1.032	1.035	1.047	1.068	0.939
		5	1.032	1.032	1.032	1.032	1.033	1.036	1.048	1.068	0.939
		11	1.036	1.036	1.036	1.036	1.036	1.039	1.051	1.072	0.939
		21	1.047	1.047	1.047	1.047	1.047	1.051	1.062	1.083	0.939
		31	1.065	1.065	1.065	1.065	1.066	1.069	1.081	1.102	0.939
		41	1.091	1.091	1.091	1.091	1.091	1.094	1.107	1.129	*Inf.*

Table 3.5: Optimal shape factor values for Scenario 2 (Cont.)

$p_1 = 0.2$ and $p_2 = 0.2$	k_1 dock doors (receiving)	k_2 dock doors (shipping): 1	2	3	4	5	11	21	31	41
	1	1.012	1.012	1.012	1.013	1.013	1.015	1.023	1.037	0.939
	2	1.012	1.012	1.013	1.013	1.013	1.015	1.023	1.037	0.939
	3	1.013	1.013	1.013	1.013	1.013	1.015	1.023	1.037	0.939
	4	1.013	1.013	1.013	1.013	1.013	1.015	1.023	1.038	0.939
	5	1.013	1.013	1.013	1.013	1.013	1.016	1.024	1.038	0.939
	11	1.015	1.015	1.015	1.016	1.016	1.018	1.026	1.040	0.939
	21	1.024	1.024	1.024	1.024	1.024	1.026	1.034	1.049	0.939
	31	1.037	1.037	1.037	1.037	1.037	1.039	1.048	1.062	0.939
	41	1.055	1.055	1.055	1.055	1.055	1.057	1.066	1.081	*Inf.*

Scenario 3

In contrast to Scenarios 1 and 2, the expected distance for the dual-command focused warehouse is smaller than the single-command focused warehouse because the two sets of dock doors are located closer together (see Table 3.6). The additional travel of S/R equipment returning to receiving dock-door locations is less with Scenario 3 than with Scenario 1 or 2. Compared to a single-command focused warehouse, the minimum, maximum and average percentage savings for a cross-docking focused warehouse are 19.1%, 91.3% and 35.0%, respectively. Similarly, compared to a more dual-command focused warehouse, the minimum, maximum and average percentage savings for a cross-docking forced warehouse are 20.1%, 64.8% and 30.6%, respectively.

Table 3.6: Expected-distance values for Scenario 3

$p_1 = 0.6$ and $p_2 = 0.2$

k_1 dock doors (receiving)	k_2 dock doors (shipping)								
	1	2	3	4	5	11	21	31	41
1	1466.3	1474.8	1475.7	1468.8	1465.5	1479.0	1501.0	1527.0	1553.6
2	1451.2	1459.6	1460.4	1453.4	1450.5	1464.4	1486.8	1512.8	1539.3
3	1442.9	1451.1	1451.8	1444.4	1442.3	1456.9	1479.9	1506.1	1532.6
4	1441.0	1449.1	1449.4	1441.7	1440.8	1456.6	1480.0	1506.5	1533.4
5	1445.3	1453.2	1453.2	1444.9	1446.0	1462.8	1486.9	1513.5	1541.8
11	1455.5	1463.2	1462.7	1453.7	1457.0	1475.2	1500.2	1526.9	1557.6
21	1471.2	1478.5	1477.5	1468.4	1473.9	1493.7	1519.2	1546.0	1581.0
31	1491.9	1498.9	1497.3	1488.5	1496.3	1517.3	1543.5	1570.6	1611.8
41	1517.2	1523.9	1521.7	1513.1	1523.5	1546.2	1572.8	1601.9	*inf.*

$p_1 = 0.2$ and $p_2 = 0.6$

k_1 dock doors (receiving)	k_2 dock doors (shipping)								
	1	2	3	4	5	11	21	31	41
1	1263.4	1286.5	1304.1	1316.2	1331.5	1359.1	1392.8	1428.8	1464.9
2	1269.7	1292.7	1310.3	1322.2	1337.8	1365.8	1399.6	1435.6	1471.9
3	1280.7	1303.7	1321.2	1332.9	1349.1	1377.5	1411.5	1447.6	1484.3
4	1296.4	1319.2	1336.5	1348.0	1365.1	1394.1	1428.4	1464.5	1502.0
5	1316.5	1339.2	1356.3	1367.4	1385.7	1415.3	1450.1	1486.3	1525.1
11	1340.8	1363.4	1380.2	1390.9	1410.5	1441.1	1476.2	1512.5	1553.5
21	1369.1	1391.5	1407.9	1418.6	1439.7	1471.1	1506.7	1542.9	1587.4
31	1401.1	1423.3	1439.4	1450.2	1472.8	1505.0	1540.9	1577.6	1626.6
41	1436.5	1458.5	1474.2	1485.1	1509.3	1542.7	1578.8	1617.0	*inf.*

$p_1 = 0.2$ and $p_2 = 0.2$

k_1 dock doors (receiving)	k_2 dock doors (shipping)								
	1	2	3	4	5	11	21	31	41
1	766.4	804.3	839.0	870.3	903.3	943.6	987.5	1033.0	1078.6
2	794.2	832.1	866.7	897.9	931.1	971.5	1015.7	1061.2	1106.8
3	824.9	862.7	897.2	928.3	961.8	1002.6	1046.9	1092.5	1138.2
4	858.3	896.1	930.5	961.4	995.5	1036.7	1081.1	1126.8	1172.9
5	894.4	932.1	966.4	997.1	1031.9	1073.6	1118.3	1164.0	1210.7
11	933.1	970.6	1004.7	1035.2	1070.9	1113.1	1158.1	1203.9	1251.8
21	974.0	1011.5	1045.4	1075.7	1112.4	1155.2	1200.5	1246.2	1296.1
31	1017.2	1054.6	1088.2	1118.7	1156.3	1199.6	1245.1	1291.1	1343.6
41	1062.4	1099.6	1133.0	1163.6	1202.2	1246.2	1291.9	1338.7	*inf.*

For a single-command focused warehouse, increasing the number of dock doors decreases expected distance because increasing the number of dock doors results in locating dock doors closer to the centerlines of walls. However, increasing the number of dock doors increases expected distance for a dual-command focused warehouse because the additional travel to return S/R equipment to the receiving dock-door locations increases with an increasing number of dock doors.

Table 3.7 contains the optimal shape factor values for Scenario 3 with three ratios of p_1 and p_2. Therefore, shipping dock doors force the warehouse to be wider. Thereafter, increasing the number of dock doors decreases the optimal shape factor for a large number of dock doors because some shipping dock doors are located above the middle-cross aisle and the warehouse tends to be narrower. For a large number of receiving dock doors ($k_1 \geq 31$) and a small number of shipping dock doors ($k_2 \leq 18$), the single-command focused warehouse is narrower than the dual-command focused warehouse.

Moreover, for a small number of receiving and shipping dock doors ($k_1 \leq 31$ and $k_1 \leq 21$), the single-command focused warehouse is narrower than the dual-command focused warehouse and the expected distance for a single-command focused warehouse is larger than that for a cross-dock dooring focused warehouse.

Table 3.7: Optimal shape factor values for Scenario 3

$p_1 = 0.6$ and $p_2 = 0.2$

k_1 dock doors (receiving)	k_2 dock doors (shipping): 1	2	3	4	5	11	21	31	41
1	1.018	1.032	1.059	1.100	0.996	0.912	0.857	0.846	0.856
2	1.031	1.045	1.072	1.113	1.011	0.927	0.857	0.857	0.857
3	1.053	1.068	1.096	1.138	1.033	0.939	0.884	0.859	0.857
4	1.086	1.101	1.129	1.173	1.033	0.949	0.927	0.907	0.857
5	1.128	1.144	1.173	1.218	1.068	1.000	0.942	0.939	0.857
11	1.179	1.196	1.227	1.268	1.127	1.033	1.010	1.006	0.857
21	1.240	1.258	1.291	1.275	1.145	1.092	1.050	1.057	0.857
31	1.311	1.330	1.365	1.351	1.225	1.141	1.141	1.085	0.857
41	1.391	1.411	1.449	1.417	1.268	1.231	1.209	1.085	*Inf.*

$p_1 = 0.2$ and $p_2 = 0.6$

k_1 dock doors (receiving)	k_2 dock doors (shipping): 1	2	3	4	5	11	21	31	41
1	1.045	1.058	1.081	1.117	1.029	0.939	0.900	0.881	0.857
2	1.056	1.069	1.093	1.129	1.033	0.939	0.914	0.897	0.857
3	1.076	1.089	1.113	1.150	1.033	0.957	0.939	0.924	0.857
4	1.104	1.117	1.143	1.181	1.050	0.991	0.941	0.939	0.857
5	1.140	1.155	1.181	1.220	1.091	1.033	0.988	0.982	0.857
11	1.186	1.200	1.227	1.268	1.141	1.049	1.033	1.033	0.857
21	1.239	1.255	1.283	1.269	1.157	1.112	1.079	1.085	0.857
31	1.301	1.317	1.347	1.335	1.225	1.148	1.141	1.085	0.857
41	1.371	1.388	1.420	1.410	1.268	1.230	1.212	1.085	*Inf.*

$p_1 = 0.2$ and $p_2 = 0.2$

k_1 dock doors (receiving)	k_2 dock doors (shipping): 1	2	3	4	5	11	21	31	41
1	1.030	1.044	1.069	1.108	1.011	0.933	0.867	0.857	0.857
2	1.042	1.056	1.081	1.121	1.025	0.939	0.882	0.859	0.857
3	1.063	1.077	1.104	1.144	1.033	0.939	0.910	0.889	0.857
4	1.094	1.108	1.135	1.177	1.034	0.968	0.939	0.933	0.857
5	1.133	1.149	1.176	1.219	1.079	1.015	0.963	0.954	0.857
11	1.182	1.198	1.227	1.268	1.133	1.033	1.026	1.024	0.857
21	1.240	1.257	1.287	1.273	1.150	1.101	1.063	1.071	0.857
31	1.307	1.324	1.357	1.344	1.225	1.141	1.141	1.085	0.857
41	1.382	1.401	1.436	1.417	1.268	1.231	1.211	1.085	*Inf.*

3.8. Conclusion

The expected distance traveled in a warehouse is impacted by the layout configuration, the arrangement of S/R locations, S/R aisles and cross-aisles, and the number and locations of dock doors. Extending previous studies by considering multiple dock doors, we analyzed three unit-load warehouses (Layouts B, C and D) with a middle-cross aisle for single- and dual-command travel. Defining shape factor as the width-to-depth ratio of a warehouse, we presented optimal shape factor results for different locations and number of dock doors along either one wall or two adjacent walls. Modifying formulations proposed by Tutam and White (in press) for Layout A, discrete and continuous formulations of the optimization problem were developed for the aforementioned configurations.

For Layout B, the following insights were obtained from the research:

- When the width constraint is satisfied, increasing the number of dock doors decreases the optimal shape factor for both single- and dual-command travel with Scenario 1; whereas increasing the number of dock doors increases the optimal shape factor for any value of k_1 with Scenarios 2 and 3. Otherwise, increasing the number of dock door increases the width of the warehouse and increases the optimal shape factor for all scenarios.
- The optimal shape factor for single-command travel is greater than the corresponding optimal shape factor for dual-command travel with any number of dock doors for Scenarios 1 and 2; whereas, for Scenario 3, the optimal shape factor for single-command travel can be less than or greater than the corresponding optimal shape factor for dual-command travel, depending on the number of dock doors.
- For Scenario 3, the optimal shape factor is slightly greater than 1.0 for a small number of dock doors.

For Layout C, our research yielded the following insights:

- For Scenario 1,
  - having two dock doors increases expected distance dramatically for single- and dual command because dock doors are located farther apart;
  - having an odd number of dock doors dampens the expected distance for small values of k_2 because the location of the middle dock door coincides with the centerline of the warehouse;
  - for a small number of dock doors, the optimal shape factor fluctuates depending on the number of dock doors being either odd or even; and
  - for a large number of dock doors, increasing the number of dock doors increases expected distance for single- and dual-command travel.
- For Scenario 2, increasing the number of dock doors decreases the optimal shape factor when the width constraint is satisfied.
- For Scenario 3, increasing the number of dock doors decreases the optimal shape factor when dock doors are located in Region 1; whereas, increasing the number of dock doors increases the optimal shape factor for single- and dual command travel when dock doors are located in Regions 2-4.
- For Scenarios 2 and 3, the optimized warehouse for single-command travel is wider than the warehouse optimized for dual-command travel because travel-between dampens the optimal shape factor for dual-command travel.

Comparing Layout configurations, the following insights were obtained:

- For Scenario 1,

    - Layout A outperforms Layouts B and C for single-command travel (except for the single-dock-door case, Layouts A and C are approximately the same when a single dock door is present);
    - In contrast to single-command travel, Layout B always outperforms Layouts A and C for dual-command travel; and
    - When a dock door is centrally located, an optimally designed Layout C is wider than optimally designed Layouts A and B. Notice this result differs from that obtained by Pohl et al. (2009) because the S/R locations along the wall containing k_2 dock doors are removed.
- For Scenario 2,
    - when the width constraint is satisfied, an optimally configured Layout A is wider and shorter than optimally configured Layouts B and C for dual-command travel;
    - increasing the number of dock doors decreases optimal shape factor values for Layouts A and B when the width constraint is satisfied;
    - increasing the number of dock doors increases dramatically expected distance for Layout C;
    - except for the single-dock-door case, Layout A outperforms Layouts B and C with single-command travel;
    - Layout B always performs better than Layouts A and C for dual-command travel;
    - with a small number of dock doors, Layout C dominates Layout A for Scenario 2; whereas, Layout A outperforms Layout C when the number of dock doors is large or when the width constraint is violated; and

    - when the width constraint is satisfied, increasing the number of dock doors increases the optimal shape factor for Layouts A and B; whereas, the optimal shape factor decreases with an increasing number of dock doors for Layout C.
- For Scenarios 1 and 2, expected distance for all configurations increases with an increasing number of dock doors.
- For Scenario 3,
    - increasing the number of dock doors decreases the expected distance for a small number of dock doors;
    - Layout A always outperforms Layouts B and C for single-command travel; whereas, Layout B always performs better than Layouts A and C for dual-command travel;
    - for dual-command travel, the performances of Layouts A and C are the same when dock doors are located close to the centerline of the warehouse;
    - increasing the number of dock doors always increases the optimal shape factor for Layouts A and B; whereas, the optimal shape factor may increase or decrease for Layout C; and
    - having a large number of dock doors results in the optimal shape factor fluctuating for Layout C because of dock doors being located on different sides of the centerline of the wall.

<u>Observations from Layout D:</u>

- For Scenario 1,
    - increasing the number of receiving dock doors will always increase the expected distance regardless of the focus of the warehouse;

  - increasing the number of shipping dock doors alternately increases (even number of dock doors) and decreases (odd number of dock doors) the expected distance when the width constraint is satisfied, because the midmost dock door with an odd number of dock doors coincides with the middle-cross-aisle;
  - increasing the number of shipping dock doors has a greater impact on expected distance than does increasing the number of receiving dock doors because shipping dock doors are aligned parallel to S/R locations;
  - increasing the number of shipping dock doors may increase or decrease the optimal shape factor depending on the number of shipping dock doors being odd or even; and
  - the optimal shape factor decreases for an increasing number of receiving dock doors until the width constraint comes into play.
- For Scenario 2,
  - increasing the number of dock doors will always increase expected distance regardless of warehouse type because dock-door locations are specified;
  - expected distances are smaller than that with Scenario 1 because dock doors are clustered around the centerlines of walls; and
  - increasing the number of shipping or receiving dock doors increases the optimal shape factor when the width constraint is satisfied.
- For Scenarios 1 and 2, the expected distance for a single-command focused warehouse is less than that for the corresponding dual-command focused warehouse because returning S/R equipment to the receiving dock-door locations results in traveling an additional distance greater than the distance reduced by performing a dual-command operation.

- For Scenario 3,
  - the expected distance for the dual-command focused warehouse is smaller than the single-command focused warehouse because the two sets of dock doors are located closer together;
  - the expected distance for the dual-command focused warehouse is less than that for the single-command focused warehouse because travel-between distance plus the additional travel of S/R equipment returning to receiving dock-door locations is less than one-half of the single-command travel;
  - for a single-command focused warehouse, increasing the number of dock doors decreases expected distance because increasing the number of dock doors results in locating dock doors closer to the centerlines of walls;
  - increasing the number of dock doors increases expected distance for a dual-command focused warehouse because the additional travel to return S/R equipment to the receiving dock-door locations increases with an increasing number of dock doors; and
  - increasing the number of dock doors decreases the optimal shape factor for a large number of dock doors because some shipping dock doors are located above the middle-cross aisle and the warehouse tends to be narrower.

Our research can be extended by considering class-based and turnover-based storage policies. Other opportunities for further research include considering unequal flows across the dock doors and different ratios of operations for warehouse types.

Bibliography

AMPL (2013) AMPL: *A Modeling Language for Mathematical Programming*. AMPL Optimization, Inc.

Ashayeri, J. and Gelders, L. F. (1985) Warehouse design optimization. *European Journal of Operational Research*, 21, 285-294.

Bassan, Y., Roll, Y. and Rosenblatt, M. J. (1980) Internal layout design of a warehouse. *AIIE Transactions,* 12(4), 317-322.

Cormier, G. and Gunn, E. (1992) A review of warehouse Formulations. *European Journal of Operational Research*, 58, 3-13.

Couenne (2006) *Couenne: Convex Over and Under Envelopes for Nonlinear Estimation, an exact solver for nonconvex MINLPs,* IBM and Carnegie Mellon University, available at https://projects.coin-or.org/Couenne/, accessed on May 04, 2016.

de Koster, R., Le-Duc, T. and Roodbergen, K. J. (2007) Design and control of warehouse order picking: a literature review. *European Journal of Operational Research*, 182(2), 481-501.

Francis, R. L. (1967) On some problems of rectangular warehouse design and layout. *The Journal of Industrial Engineering*, 18(10), 595-604.

Gu, J., Goetschalckx, M. and McGinnis, L. F. (2007) Research on warehouse operation: a comprehensive review. *European Journal of Operational Research*, 177(1), 1-21.

Gu, J., Goetschalckx, M. and McGinnis, L. F. (2010) Research on warehouse design and performance evaluation: A comprehensive review. *European Journal of Operational Research*, 203(3), 539-549.

Karásek, J. (2013) An overview of warehouse optimization. *International Journal of Advances in Telecommunications, Electrotechnics, Signals and Systems*, 2(3), 111-117.

Mayer Jr., H. E. (1961) Storage and retrieval of material. *The Western Electric Engineer*, 5(1), 42-48.

Malmborg, C. J. and Krishnakumar, B. (1987). On the optimality of the cube per order index for conventional warehouses with dual command cycles. *Material Flow,* 4, 169-175.

Mathematica (2015) *Mathematica 10.2 for Students,* Microsoft Windows Version. Wolfram Research, Inc.

Pohl, L. M., Meller, R. D. and Gue, K. R. (2009) An analysis of dual command travel in common warehouse designs. *Transportation Research, Part E: Logistics and Transportation Review*, 45(3), 367-379.

Roodbergen, K. J. (2007) *Warehousing literature*, available at http://www.roodbergen.com/literature/wh_literature.pdf, accessed on January 05, 2016.

Roodbergen, K. J. and de Koster, R. (2001). Routing order pickers in a warehouse with a middle aisle. *European Journal of Operational Research*, 133(1), 32-43.

Roodbergen, K. J., Sharp, G. P. and Vis, I. F. (2008) Designing the layout structure of manual order picking areas in warehouses. *IIE Transactions*, 40(11), 1032-1045.

Rosenblatt, M. J. and Roll, Y. (1984) Warehouse design with storage policy considerations. *International Journal of Production Research*, 22(5), 809-821.

Rouwenhorst, B., Reuter, B. V. Stockrahm, Van Houtum, G. J., Mantel, R. J. and Zijm, W. H. M. (2000) Warehouse design and control: Framework and literature review. *European Journal of Operational Research*, 122(3), 515-533.

Thomas, L. M. and Meller, R. D. (2014) Analytical Formulations for warehouse configuration. *IIE Transactions*, 46(9), 928-947.

Tutam, M. and White, J. A. (2016) Configuring a Multi-Dock Unit-Load Warehouse with a Cross-Aisle, in *Proceedings of the 2016 Industrial and Systems Engineering Research Conference*, Anaheim, California.

Tutam, M. and White, J. A. (in press) A multi-dock unit-load warehouse design, *IISE Transactions*. doi: 10.1080/24725854.2018.1488307.

Vaughan, T. S. and Petersen, C. G. (1999). The effect of warehouse cross aisles on order-picking efficiency. *International Journal of Production Research*, 37 (4), 881-897.

Appendix

Proof of Proposition 3.1

Case 1: When the nearest back-to-back storage location is to the right of the dock door, the distance between dock door *i* and the nearest storage location to the left and right of dock door *i* are $d_i - t_i + w / 2$ and $t_i - d_i + w / 2$, respectively.

One-sided S/R aisle: Because there are t_i / w storage locations to the left of dock door *i*, the distance between dock door *i* and storage location *j* located to the left of dock door *i* is $(d_i - t_i) + (t_i / w - j)\, w + w / 2 = d_i - j\, w + w / 2$ for $j = 1, 2, \ldots, t_i / w$ (see storage locations 1 thru 5 in Figure 3.6). If storage locations are to the right of dock door *i*, we consider storage locations being located on either the left or the right sides of the warehouse. In the former, there are $m /2 - t_i / w$ storage locations to the right of dock door *i*. Therefore, the distance between dock door *i* and storage location *j* located to the right of dock door *i* (storage locations are on the left side of the warehouse) is $t_i - d_i + (j - t_i / w - 1)\, w + w / 2 = j\, w - d_i - w / 2$ for $j = t_i / w + 1, t_i / w + 2, \ldots, m / 2$ (see storage locations 6 thru 8 in Figure 3.6). In the latter, there are $m / 2$ storage locations to the right of dock door *i*. Therefore, the distance between dock door *i* and storage location *j* located to the right of dock door *i* (storage locations are on the right side of the warehouse) is $t_i - d_i + (j - t_i / w - 1)\, w + w / 2 + 2v = j\, w - d_i - w / 2 + 2v$ for $j = m / 2 + 1, m / 2 + 2, \ldots, m$ (see storage locations 9 thru 16 in Figure 3.6).

Two-sided S/R aisle: First, the shortest path between dock door *i* and storage location *j* is determined. The number of storage locations visited is obtained by traveling to either the left or right side of dock door *i*. There are $m /2 - t_i / w$ storage locations visited by traveling to the left of dock door *i*. The distance between dock door *i* and storage location *j* by traveling to the left of dock door *i* is $d_i - t_i + w / 2 + t_i - w / 2 + w / 2 + (j - 1)\, w + 2v = d_i + j\, w - w / 2 + 2v$ for

$j = 1, 2, \ldots, m/2 - t_i / w$. (see storage locations 1 thru 3 in Figure 3.6). If storage locations are visited by traveling to the right of dock door *i*, we consider storage locations being on either the left or the right side of the warehouse. If storage locations are located on the left side of the warehouse, there are t_i / w storage locations visited by traveling to the right of dock door *i*. The distance between dock door *i* and storage location *j* by traveling to the right of dock door *i* is $t_i - d_i + w / 2 + m\, w / 2 - t_i - w / 2 + (m / 2 - j + 1 / 2)\, w + 2v = m\, w - d_i - j\, w + w / 2 + 2v$ for $j = m / 2 - t_i / w + 1, m / 2 - t_i / w + 2, \ldots, m / 2$ (see storage locations 4 thru 8 in Figure 3.6). If storage locations are located on the right side of the warehouse, there are $m / 2$ storage locations visited by traveling to the right of dock door *i*. The distance between dock door *i* and storage location *j* located to the right of dock door *i* is $t_i - d_i + w / 2 + (j - t_i / w - 1)\, w + 2v = j\, w - w / 2 - d_i + 2v$ for $j = m / 2 + 1, m / 2 + 2, \ldots, m$ (see storage locations 9 thru 16 in Figure 3.6).

Case 2: When a back-to-back storage location coincides with the location of dock door *i*, the distance from dock door *i* to the storage location by traveling either to the left of dock door *i* or to the right of dock door *i* is equal to $w / 2$. Equations for Case 2 can be obtained easily by replacing $d_i - t_i$ with zero in equations for Case 1.

Case 3: If the nearest back-to-back storage location is to the left of the dock door *i*, Equations provided for Case 1 still hold.

Case 4: When a storage location coincides with the location of dock door *i*, the distance to reach the nearest location is zero. Replacing $d_i - t_i$ with $w / 2$, Equations derived for Case 1 apply for Case 4.

Proof of Proposition 3.2

Four different expressions are developed for expected Horizontal roundtrip-distance depending on the location of dock door *i*.

If dock door *i* is located in Region 1 ($d_i \le w\,m/2$)

✓ Expected horizontal roundtrip-distance to the one-sided aisle is

$$\frac{2}{(2n+1)m}\left[\sum_{j=1}^{t_i/w}(d_i - j\,w + w/2) + \sum_{j=t_i/w+1}^{m/2}(j\,w - d_i - w/2) + \sum_{j=m/2+1}^{m}(j\,w - d_i - w/2 + 2v)\right]$$

✓ Expected horizontal roundtrip-distance to the two-sided aisle is

$$\frac{4n}{(2n+1)\,m}\left[\sum_{j=1}^{m/2-t_i/w}(d_i + j\,w - w/2 + 2v) + \sum_{j=m/2-t_i/w+1}^{m/2}(mw - d_i - j\,w + w/2 + 2v)\right]$$
$$+\frac{4n}{(2n+1)\,m}\left[\sum_{j=m/2+1}^{m}(j\,w - d_i - w/2 + 2v)\right]$$

If dock door *i* is located in Region 2 ($w\,m/2 < d_i \le w\,m/2 + v$)

Expected horizontal roundtrip-distance to the one-sided aisle is

$$\frac{2}{(2n+1)}\left[\frac{d_i - mw/2 + mw/4}{2} + \frac{mw/2 - d_i + 2v + mw/4}{2}\right]$$

Expected horizontal roundtrip-distance to the two-sided aisle is

$$\frac{4n}{(2n+1)}\left[\frac{mw/2 - d_i + 2v + mw/4}{2} + \frac{mw/2 - d_i + 2v + mw/4}{2}\right]$$

If dock door *i* is located in Region 3 ($w\,m/2 + v < d_i \le w\,m/2 + 2v$)

Expected horizontal roundtrip-distance to the one-sided aisle is

$$\frac{2}{(2n+1)}\left[\frac{d_i - mw/2 + mw/4}{2} + \frac{mw/2 - d_i + 2v + mw/4}{2}\right]$$

Expected horizontal roundtrip-distance to the two-sided aisle is

$$\frac{4n}{(2n+1)}\left[\frac{d_i - mw/2 + mw/4}{2} + \frac{d_i - mw/2 + mw/4}{2}\right]$$

If dock door i is located in Region 3 ($w\,m / 2 + 2v < d_i$)

Expected horizontal roundtrip-distance to the one-sided aisle is

$$\frac{2}{(2n+1)\,m}\left[\sum_{j=1}^{m/2}\left(d_i - j\,w + w/2\right) + \sum_{j=m/2+1}^{(t_i-2v)/w}\left(d_i - j\,w + w/2 - 2v\right) + \sum_{j=(t_i-2v)/w}^{m}\left(j\,w - d_i - w/2 + 2v\right)\right]$$

Expected horizontal roundtrip-distance to the two-sided aisle is

$$\frac{4n}{(2n+1)\,m}\left[\sum_{j=1}^{m/2}\left(d_i - j\,w + w/2\right) + \sum_{j=m/2+1}^{3m/2-(t_i-2v)/w}\left(d_i + j\,w - mw - w/2\right)\right]$$
$$+\frac{4n}{(2n+1)\,m}\left[\sum_{j=3m/2-(t_i-2v)/w+1}^{m}\left(2mw - d_i - j\,w + w/2\right)\right]$$

Proof of Propositions 3.1, 3.2, 3.3, 3.4, 3.5 and 3.6

Using Lemma 2.1 and Corollary 2.1 in Chapter 2. the proof of Proposition 2.2 from Chapter 2 can be applied to Propositions 3.1, 3.2 and 3.3. However, using Lemma 2.2 and Corollary 2.2, the proof of Proposition 2.2 can be applied to Propositions 3.4, 3.5 and 3.6.

Equations for Layout D with Continuous Approximations

Scenario 1

The expected distance traveled in Layout D with an even number of dock doors is

$$E[SC] = \left(p_1 + 0.5p_2\right)\left\{\frac{(2k_1+1)W}{3(k_1+1)} + wm + 6v + 2\sum_{j=1}^{k_2/2}\left[\frac{L(L+4v) - 2d_{2j}^{\,2}}{k_2\,L}\right] + \frac{2\,n\,a(n+1)}{(2n+1)} + 2v\right\}$$
$$+\,p_2\left\{\frac{1}{n}\left[\frac{L}{3} + v + (n-1)\left(\frac{5L}{12} + 2v\right)\right] + \frac{2na(n+1)}{3+6n}\right\}$$
$$+\left(1 - p_1 - 0.5p_2\right)\left\{\frac{2}{k_1\,k_2}\sum_{i=1}^{k_1}\sum_{j=1}^{k_2}\left[d_{1i} + d_{2j} + 4v\right]\right\}$$

where $d_{1i} = [W\,(k_1 - i + 1)] / (k_1 + 1)$ and $d_{2j} = [\,j\,(L + 2v)] / (k_2 + 1)$.

The expected distance traveled in Layout D with an odd number of dock doors is

$$E[SC] = \left(p_1 + 0.5p_2\right)\left\{\frac{\left(2k_1+1\right)W}{3\left(k_1+1\right)} + wm + 6v\right\}$$
$$+\left(p_1 + 0.5p_2\right)\left\{2\sum_{j=1}^{(k_2-1)/2}\left[\frac{L\left(L+4v\right)-2d_{2j}^{\;2}}{k_2\,L}\right] + \frac{L+4v}{2k_2} + \frac{2\,n\,a\left(n+1\right)}{\left(2n+1\right)} + 2v\right\}$$
$$+\,p_2\left\{\frac{1}{n}\left[\frac{L}{3} + v + \left(n-1\right)\left(\frac{5L}{12} + 2v\right)\right] + \frac{2na\left(n+1\right)}{3+6n}\right\}$$
$$+\left(1 - p_1 - 0.5p_2\right)\left\{\frac{2}{k_1\,k_2}\sum_{i=1}^{k_1}\sum_{j=1}^{k_2}\left[d_{1i} + d_{2j} + 4v\right]\right\}$$

where $d_{1i} = [W(k_1 - i + 1)] / (k_1 + 1)$ and $d_{2j} = [j(L + 2v)] / (k_2 + 1)$

Scenario 2

The expected distance traveled in Layout D with an even number of dock doors is

$$E[SC] = \left(p_1 + 0.5p_2\right)\left\{\frac{W}{2} + \frac{\delta_1^{\,2}(k_1^{\,2}-1)}{6W} + wm + 6v\right\}$$
$$+\left(p_1 + 0.5p_2\right)\left\{2\sum_{j=1}^{k_2/2}\left[\frac{L\left(L+4v\right)-2d_{2j}^{\;2}}{k_2\,L}\right] + \frac{2\,n\,a\left(n+1\right)}{\left(2n+1\right)} + 2v\right\}$$
$$+\,p_2\left\{\frac{1}{n}\left[\frac{L}{3} + v + \left(n-1\right)\left(\frac{5L}{12} + 2v\right)\right] + \frac{2na\left(n+1\right)}{3+6n}\right\}$$
$$+\left(1 - p_1 - 0.5p_2\right)\left\{\frac{2}{k_1\,k_2}\sum_{i=1}^{k_1}\sum_{j=1}^{k_2}\left[d_{1i} + d_{2j} + 4v\right]\right\}$$

where $d_{1i} = [W + (k_1 - 1)\,\delta_1] / 2 - (i - 1)\,\delta_1$ and $d_{2j} = [(L + 2v) - (k_2 - 1)\,\delta_2] / 2 + (j - 1)\,\delta_2$

The expected distance traveled in Layout D with an odd number of dock doors is

$$E[SC] = \left(p_1 + 0.5p_2\right)\left\{\frac{W}{2} + \frac{\delta_1^2(k_1^2 - 1)}{6W} + wm + 6v\right\}$$
$$+\left(p_1 + 0.5p_2\right)\left\{2\sum_{j=1}^{(k_2-1)/2}\left[\frac{L\left(L+4v\right) - 2d_{2j}^{\ 2}}{k_2 L}\right] + \frac{L+4v}{2k_2} + \frac{2na\left(n+1\right)}{\left(2n+1\right)} + 2v\right\}$$
$$+\ p_2\left\{\frac{1}{n}\left[\frac{L}{3} + v + \left(n-1\right)\left(\frac{5L}{12} + 2v\right)\right] + \frac{2na\left(n+1\right)}{3+6n}\right\}$$
$$+\left(1 - p_1 - 0.5p_2\right)\left\{\frac{2}{k_1 k_2}\sum_{i=1}^{k_1}\sum_{j=1}^{k_2}\left[d_{1i} + d_{2j} + 4v\right]\right\}$$

where $d_{1i} = [W + (k_1 - 1)\ \delta_1] / 2 - (i - 1)\ \delta_1$ and $d_{2j} = [(L + 2v) - (k_2 - 1)\ \delta_2] / 2 + (j - 1)\ \delta_2$

Scenario 3

<u>Case 1</u> ($d_{2k_2} \leq L / 2$):

The expected distance traveled in Layout D is

$$E[SC] = \left(p_1 + 0.5p_2\right)\left\{W + \frac{6\varphi_1^2 + 6\left(k_1 - 1\right)\varphi_1\delta_1 + \left(2k_1^2 - 3k_1 + 1\right)\delta_1^2}{3W} - \left(2\varphi_1 + \left(k_1 - 1\right)\delta_1\right)\right\}$$
$$+\left(p_1 + 0.5p_2\right)\left\{wm + 6v + 2\sum_{j=1}^{k_2/2}\left[\frac{L\left(L+4v\right) - 2d_{2j}^{\ 2}}{k_2 L}\right] + \frac{2na\left(n+1\right)}{\left(2n+1\right)} + 2v\right\}$$
$$+\ p_2\left\{\frac{1}{n}\left[\frac{L}{3} + v + \left(n-1\right)\left(\frac{5L}{12} + 2v\right)\right] + \frac{2na\left(n+1\right)}{3+6n}\right\}$$
$$+\left(1 - p_1 - 0.5p_2\right)\left\{\frac{2}{k_1 k_2}\sum_{i=1}^{k_1}\sum_{j=1}^{k_2}\left[d_{1i} + d_{2j} + 4v\right]\right\}$$

where $d_{1i} = \phi_1 + (k_1 - i)\ \delta_1$ for Scenario 2 and $d_{2j} = \phi_2 + (k_2 - j)\ \delta_2$.

Case 2 ($L / 2 < d_{2k_2} \leq L / 2 + v$):

The expected distance traveled in Layout D is

$$E[SC] = \left(p_1 + 0.5p_2\right)\left\{W + \frac{6\varphi_1^{\,2} + 6\left(k_1 - 1\right)\varphi_1\,\delta_1 + \left(2k_1^{\,2} - 3k_1 + 1\right)\delta_1^2}{3W} - \left(2\varphi_1 + \left(k_1 - 1\right)\delta_1\right)\right\}$$
$$+\left(p_1 + 0.5p_2\right)\left\{wm + 6v + \sum_{j=1}^{\left\lceil \frac{L-2\varphi_2}{2\delta_2} \right\rceil}\left[\frac{L\left(L + 4v\right) - 2d_{2j}^{\ 2}}{k_2\,L}\right] + \sum_{j=\left\lceil \frac{L-2\varphi_2}{2\delta_2} \right\rceil + 1}^{k_2}\left[\frac{3L - 4d_{2j} + 8v}{2k_2}\right]\right\}$$
$$+\left(p_1 + 0.5p_2\right)\left\{\frac{2\,n\,a\left(n+1\right)}{\left(2n+1\right)} + 2v\right\} + p_2\left\{\frac{1}{n}\left[\frac{L}{3} + v + \left(n-1\right)\left(\frac{5L}{12} + 2v\right)\right] + \frac{2na\left(n+1\right)}{3+6n}\right\}$$
$$+\left(1 - p_1 - 0.5p_2\right)\left\{\frac{2}{k_1\,k_2}\sum_{i=1}^{k_1}\sum_{j=1}^{k_2}\left[d_{1i} + d_{2j} + 4v\right]\right\}$$

where $d_{1\text{i}} = \phi_1 + (k_1 - i)\,\delta_1$ for Scenario 2 and $d_{2\text{j}} = \phi_2 + (k_2 - j)\,\delta_2$.

Case 3 ($L / 2 + v < d_{2k_2} \leq L / 2 + 2v$):

The expected distance traveled in Layout D is

$$E[SC] = \left(p_1 + 0.5p_2\right)\left\{W + \frac{6\varphi_1^{\,2} + 6\left(k_1 - 1\right)\varphi_1\,\delta_1 + \left(2k_1^{\,2} - 3k_1 + 1\right)\delta_1^2}{3W} - \left(2\varphi_1 + \left(k_1 - 1\right)\delta_1\right)\right\}$$
$$+\left(p_1 + 0.5p_2\right)\left\{wm + 6v + \sum_{j=1}^{\left\lceil \frac{L-2\varphi_2}{2\delta_2} \right\rceil}\left[\frac{L\left(L + 4v\right) - 2d_{2j}^{\ 2}}{k_2\,L}\right] + \sum_{j=\left\lceil \frac{L-2\varphi_2}{2\delta_2} \right\rceil + 1}^{\left\lceil \frac{L-2\varphi_2+2v}{2\delta_2} \right\rceil}\left[\frac{3L - 4d_{2j} + 8v}{2k_2}\right]\right\}$$
$$+\left(p_1 + 0.5p_2\right)\left\{\sum_{j=\left\lceil \frac{L-2\varphi_2+2v}{2\delta_2} \right\rceil + 1}^{k_2}\left[\frac{4d_{2j} - L}{2k_2}\right] + \frac{2\,n\,a\left(n+1\right)}{\left(2n+1\right)} + 2v\right\}$$
$$+ p_2\left\{\frac{1}{n}\left[\frac{L}{3} + v + \left(n-1\right)\left(\frac{5L}{12} + 2v\right)\right] + \frac{2na\left(n+1\right)}{3+6n}\right\}$$
$$+\left(1 - p_1 - 0.5p_2\right)\left\{\frac{2}{k_1\,k_2}\sum_{i=1}^{k_1}\sum_{j=1}^{k_2}\left[d_{1i} + d_{2j} + 4v\right]\right\}$$

where $d_{1\text{i}} = \phi_1 + (k_1 - i)\,\delta_1$ for Scenario 2 and $d_{2\text{j}} = \phi_2 + (k_2 - j)\,\delta_2$.

Case 4 ($L / 2 + 2v < d_{2k_2}$):

The expected distance traveled in Layout D is

$$E[SC] = \left(p_1 + 0.5p_2\right)\left\{W + \frac{6\varphi_1^2 + 6\left(k_1 - 1\right)\varphi_1\,\delta_1 + \left(2k_1^2 - 3k_1 + 1\right)\delta_1^2}{3W} - \left(2\varphi_1 + \left(k_1 - 1\right)\delta_1\right)\right\}$$

$$+\left(p_1 + 0.5p_2\right)\left\{wm + 6v + \frac{2\,n\,a\left(n+1\right)}{\left(2n+1\right)} + 2v + \sum_{j=1}^{\left\lceil \frac{L-2\varphi_2}{2\delta_2} \right\rceil}\left[\frac{L\left(L+4v\right) - 2d_{2j}^2}{k_2\,L}\right]\right\}$$

$$+\left(p_1 + 0.5p_2\right)\left\{\sum_{j=\left\lceil \frac{L-2\varphi_2}{2\delta_2} \right\rceil+1}^{\left\lceil \frac{L-2\varphi_2+2v}{2\delta_2} \right\rceil}\left[\frac{3L - 4d_{2j} + 8v}{2k_2}\right] + \sum_{j=\left\lceil \frac{L-2\varphi_2+2v}{2\delta_2} \right\rceil+1}^{\left\lceil \frac{L-2\varphi_2+4v}{2\delta_2} \right\rceil}\left[\frac{4d_{2j} - L}{2k_2}\right] + \right\}$$

$$+\left(p_1 + 0.5p_2\right)\left\{\sum_{j=\left\lceil \frac{L-2\varphi_2+4v}{2\delta_2} \right\rceil+1}^{k_2}\left[\frac{4d_{2j}\left(L+2v\right) - L\left(L+4v\right) - 8v^2 - 2d_i^2}{k_2\,L}\right]\right\}$$

$$+\,p_2\left\{\frac{1}{n}\left[\frac{L}{3} + v + \left(n-1\right)\left(\frac{5L}{12} + 2v\right)\right] + \frac{2na\left(n+1\right)}{3+6n}\right\}$$

$$+\left(1 - p_1 - 0.5p_2\right)\left\{\frac{2}{k_1\,k_2}\sum_{i=1}^{k_1}\sum_{j=1}^{k_2}\left[d_{1i} + d_{2j} + 4v\right]\right\}$$

where $d_{1i} = \phi_1 + (k_1 - i)\,\delta_1$ for Scenario 2 and $d_{2j} = \phi_2 + (k_2 - j)\,\delta_2$.

Certification of Student Work

UNIVERSITY OF ARKANSAS

College of Engineering
Department of Industrial Engineering
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MEMORANDUM

TO: University of Arkansas Graduate School

FROM: John A. White, Distinguished Professor

DATE: June 27, 2018

SUBJECT: Certification of Student Effort and Contribution

I certify Mr. Mahmut Tutam contributed more than 51 percent of the work included in the chapter entitled, "Contribution 2: A Working Paper on, 'Multi-Dock Unit-Load Warehouse Designs with a Cross-Aisle'" contained in the doctoral dissertation entitled, "Configuring Traditional Multi-Dock, Unit-Load Warehouses".

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Chapter 4

Contribution 3: A Working Paper on, “Configuring Contour-Line-Shaped Storage Region(s) in a Multi-Dock, Unit-Load Warehouse”

Abstract

The performance of a unit-load warehouse having multiple dock doors is analyzed when a storage region or storage regions can be either rectangle-shaped or contour-line-shaped. Assuming a uniform distribution of unit loads over a storage region, a randomized storage policy is used. Moreover, considering the activity levels of unit-loads, an ABC class-based storage policy is used by assigning unit loads to three storage regions on a priority basis. Expected distances traveled in rectangle-shaped storage regions are compared with expected distances in their counterpart contour-line-based storage regions. With an objective of minimizing expected roundtrip rectilinear distance, the best rectangle-shaped and contour-line-shaped storage regions are determined for different numbers and locations of dock doors. Specifically, we consider dock doors to be either equally dispersed along an entire wall of the warehouse or centrally located with a specified distance between them; significantly, for the former scenario, a rectangle-shaped warehouse outperforms a corresponding contour-line-shaped warehouse for multiple dock doors. When dock doors are distributed with a specified distance between them, requiring the warehouse to be rectangle-shaped instead of contour-line shaped increases the expected roundtrip distance from approximately six percent to less than one percent, depending on the number of dock doors and skewness of the ABC curve.

Keywords: Multiple dock doors, Shape factor, Class-Based Storage Policy, Contour-Line-Shaped, Single-Command.

4.1. Introduction

We focus on developing expected-distance approximations for both rectangle-shaped and contour-line-shaped warehouse designs when either randomized or class-based storage policies are in use. Specifically, we determine the warehouse design that minimizes rectilinear roundtrip distance between dock doors and storage locations in a continuous region. The continuous formulations provide valuable insights regarding the effects of the number and location of dock doors on expected distance and the optimal storage configuration.

The following assumptions underlie the formulations obtained:

1. A randomized storage policy is used when S/R locations are distributed uniformly over a continuous region (not necessarily rectangle-shaped).
2. A dedicated storage policy is used among classes of products when S/R locations are divided into three classed and a random storage policy is used within each class.
3. Rectilinear roundtrip distance is measured.
4. Times to store/retrieve and travel vertically are ignored.
5. Acceleration and deceleration of S/R equipment are negligible; therefore, travel velocity is the same for both horizontal and vertical directions.
6. Each dock door is equally likely to be selected for travel to/from storage locations.
7. When we refer to the configuration of a warehouse, we actually mean the configuration of the storage region within the warehouse; we recognize many other functions are performed in the warehouse. Our focus is on the unit-load storage function.

As illustrated in Chapter 2, travel distance between two S/R locations in different S/R aisles underestimates the rectilinear travel distance by approximately 31.69% for a particular set of parameter values. Therefore, we consider only single-command operations in which S/R equipment transports a unit load from a dock door to a storage location and returns empty to the dock door or it travels empty from a dock door to a retrieval location and transports a unit load to the dock door.

In assigning unit loads to storage locations, a large number of storage assignment policies can be selected and implemented. Random and class-based storage are widely used storage assignment policies. With a random storage policy, a unit load can be stored in an equally-likely-selected location from among all empty storage locations in the warehouse. With a class-based storage policy, a specific unit load can be stored in an equally-likely-selected location from among all empty storage locations in a storage region assigned to the particular class of products. Dividing the storage region into three different classes (ABC class-based storage policy) and storing the most popular unit loads of products in the class "closest" to the dock door(s) has been widely studied and applied.

The shape factor for a warehouse is defined as the ratio of the width and depth of a rectangle-shaped warehouse, where the width designates the length of the wall containing dock doors. Optimizing the shape factor results in minimizing expected distance traveled in a warehouse. An objective of this study is to determine the optimal shape factor for each rectangle-shaped class within a multi-dock-door, unit-load warehouse.

A contour line encloses all storage locations having expected distance traveled between dock doors and storage locations less than or equal to the value of the contour line (Francis *et al*., 1992). Hence, the storage locations on a contour line have identical expected distances from/to the set of dock doors. Initially, we develop contour lines of a warehouse having multiple dock doors. Moreover, similar to other studies of class-based storage, we categorize products into *three classes* and calculate expected distance for the overall warehouse. Figure 4.1 illustrates rectangle-shaped (left) and contour-line-shaped (right) warehouses for 3 centrally located dock doors with a specified distance δ between adjacent dock doors.

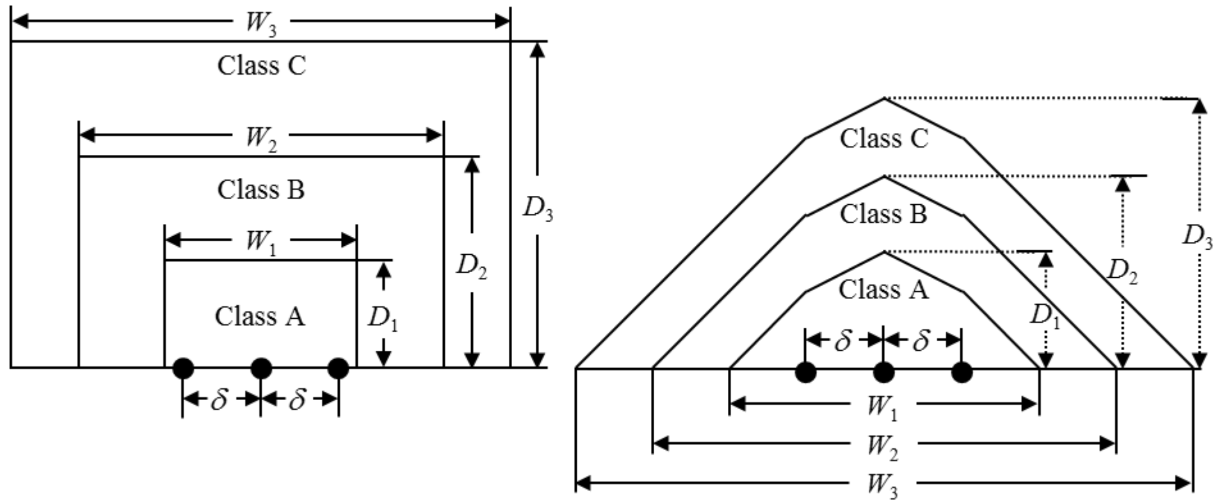

Figure 4.1: Rectangle-shaped and contour-line-shaped warehouses with 3 dock doors and ABC storage regions

Throughout the research, the first two scenarios from previous chapters are considered: Scenario 1 consists of equally spaced dock doors dispersed over the entire width of the wall and Scenario 2 consists of dock doors centrally located about the wall's centerline with a specified distance δ between adjacent dock doors. Comparing the results for contour-line-shaped warehouses with the results for corresponding rectangle-shaped warehouses, the expected-distance penalty incurred by requiring the warehouse configuration to be rectangle-shaped is determined for various numbers and locations of dock doors. Additionally, the effect of ABC curve shapes on the penalty is examined for different skewness levels.

The chapter is organized as follows. First, research literature related to rectangle-shaped and contour-line-shaped warehouse configurations is reviewed. Next, the notation used in subsequent sections is presented. The derivation of expected-distance expressions for each class is provided in Section 4.4. Sections 4.5 and 4.6 include expected-distance formulations for rectangle-shaped and contour-line-shaped warehouses, respectively. In Section 4.7, we examine the penalty, as measured by expected distance, by forcing a storage region to be rectangle-shaped instead of contour-line-shaped. Section 4.8 provides computational results based on specified values of parameters. Section 4.9 contains design conclusions and provides recommendations for future research.

4.2. Literature Review

The first to develop an analytical model for expected single-command distance in a single-dock-door warehouse was Francis (1967a). He solved facility layout problems in the context of warehouse design by considering the location of the dock door as the known point and storage locations as facilities. He concluded a width-to-depth ratio of 2:1 for a rectangle-shaped warehouse minimizes expected rectilinear distances between a centrally located dock door and storage locations. Using contour lines, he developed total cost models for single and multiple types of products and configured the areas of storage regions in a continuous space using contour lines. He provided solutions to the models and, based on his results, offered useful design benchmarks.

Francis (1967b) employed a procedure based on a special case of the Neyman-Pearson Lemma to calculate expected distance. Specifically, based on two mathematical properties, he cleverly transformed a two-dimensional spatial domain to a one-dimensional objective function domain. Instead of integrating over the two-dimensional storage region, he integrated over the objective function space contained within a contour line. Developing properties of facility designs and providing examples of optimal designs, he also presented sufficient conditions for optimal facility designs for warehouses having one or more dock doors. In addition to warehouse designs, he cited parking lot and stadium designs as possible applications.

For a given set of dock-door locations, Francis (1967b) proved there is no other shape for a storage region having an expected distance less than that for a contour-line-shaped storage region. Hence, a rectangle-shaped storage region will have an expected distance at least as great as that for a contour-line-shaped storage region. Importantly, as will be demonstrated in Section 4.8, Francis's properties apply to Scenario 2, but they do not apply to Scenario 1, because the dock-door locations will differ for the contour-line-shaped warehouse and the optimally configured rectangle-shaped warehouse.

Mallette and Francis (1972) represented the facility design problem as a generalized assignment problem by considering the plane to be composed of grid squares. Providing necessary and sufficient conditions, they evaluated the performance of a multi-dock-door rectangle-shaped warehouse under a class-based storage policy with rectilinear travel between dock doors and centroids of grid squares.

Developing continuous formulations for the warehouse layout problem, Francis and White (1974) provided expected-distance results when travel is based on rectilinear, Euclidean, Chebyshev, and squared-Euclidean metrics. Specifically, using the contour-line approach, they provided optimal warehouse designs. Illustrating the contour-line approach in calculating expected distances traveled, Francis *et al*. (1992) provided expressions and examples of optimum designs for up to three dock doors. Generalizing their studies, our research develops the expected-distance formulations for k dock doors considering class-based storage regions.

Whereas the previous research addressed the overall shape of the storage region, additional research has addressed the configuration of aisles within a warehouse. For example, in evaluating the effect on space utilization of aisle width and the angle of alignment of the pallets, Moder and Thornton (1965) appear to be the first researchers to consider non-traditional aisle designs. Developing formulations for total warehouse volume and material handling costs, Berry (1968) investigated two types of aisle design: rectangular and diagonal. He concluded a warehouse having a diagonal aisle configuration has lower total cost than a warehouse having a rectangular aisle configuration. He also concluded a warehouse layout that maximizes space utilization (area occupied) differs from one that minimizes expected distance.

White (1972) combined rectilinear travel with radial travel by considering the combination of a set of rectilinear aisles and a set of radial aisles in a continuous space warehouse. He showed expected distance shifts from rectilinear distance to Euclidian distance as the number of radial aisles increases.

Gue and Meller (2006) proposed fishbone aisle design and their results showed single-command distance in a traditional warehouse can be reduced up to 20.3% by using a fishbone design. Gue and Meller (2009) studied two non-traditional aisle configurations within a warehouse: flying-V and fishbone. Inserting a nonlinear cross-aisle in the warehouse layout, they showed expected distance can be reduced by 8-12% depending on the size of the warehouse. Having a diagonal and straight middle-cross-aisle, and arranging S/R aisles perpendicularly above the cross-aisle, they concluded expected distance can be reduced by as much as 20.3%.

Meller and Gue (2009) presented the first implementation of two non-traditional warehouse designs. Because having a single centrally located dock door is a disadvantage for the fishbone design, they introduced a new design, the chevron aisle design. They concluded the performance of the warehouse having chevron aisle design is very close to the warehouse having fishbone aisle design.

Using Monte Carlo simulation, Pohl *et al*. (2007) evaluated the performance of designs proposed by Gue and Meller (2009) for dual-command travel. Their results indicate the flying-V design reduces expected distance by approximately 12.5%; whereas, reduction in expected distance for fishbone design is approximately 15.9%. Because the reduction in expected distance for fishbone design is greater than for flying-V, Pohl *et al*. (2009) concentrated on developing analytical formulations for expected distance in a fishbone design. They noted expected distance with dual-command travel in a fishbone design can be approximately 10%-15% less than in a traditional warehouse of the same size. They concluded the fishbone design dominates other warehouse designs they considered when the half-warehouse shape is approximately square.

Based on a turnover-based storage policy and single-command and dual-command travel, Pohl *et al*. (2011) compared the expected distance for flying-V and fishbone designs. They concluded flying-V does not perform well compared to traditional warehouses, whereas fishbone design performs better. They concluded the reduction in dual-command travel distances is between 6% and 16% depending on the size of the warehouse.

Gue *et al*. (2012) extended the work in Gue and Meller (2009) and considered multiple dock doors. They proposed two new aisle designs: modified flying-V and inverted-V. The former design can reduce expected distances 3-6%; whereas the latter design results in either a reduction of less than 1% or an incremental increase in expected distance. They also showed that increasing the number of dock doors decreases the benefit of the flying-V design and the best location for pickup and deposit (P&D) points is the centerline of the warehouse.

Gálvez and Ting (2012) confirmed the results drawn by Gue *et al*. (2012) and proposed a rotated fishbone layout which performs better than other layouts when 2 dock doors are located in the upper corners of the warehouse. For a big warehouse, their experiment showed the rotated fishbone design performs better than a traditional aisle design up to 17% and 18% for single- and dual-command travel, respectively.

Using a continuous approach, Cardona *et al*. (2012) determined the slope of a cross-aisle in a fishbone design that minimizes expected distance. From their analytical study, they agreed with Gue and Meller (2009) that the savings on the expected distance for the fishbone design are greater than 18%.

Incorporating vertical travel distances into flying-V and fishbone designs, Clark and Meller (2013) concluded increasing the height of vertical travel decreases the improvement over traditional warehouses for both designs by between 3% and 5% with a 20/80 demand curve.

Inserting one, two and three cross-aisles in a unit-load warehouse, Ozturkoglu *et al*. (2012) proposed chevron, leaf and butterfly designs. Allowing cross-aisles and S/R aisles to be located at any angle with respect to the wall containing the dock door, they provided continuous space formulations for expected distance. They also developed discrete formulations to more accurately measure travel distances. Comparing the proposed aisle designs with a traditional aisle design, their results showed chevron is the best design for warehouses with 27 or fewer aisles and the reduction in expected distance is approximately 16%. For middle-size warehouses (more than 27 aisles and less than 65 aisles), the leaf aisle design occupies 6% more space than a traditional aisle design, but reduces expected distance by 19.3%. For warehouses with more than 65 aisles, the butterfly aisle design performs slightly better than the leaf aisle design and reduces expected distance by approximately 20% compared to an equivalent traditional aisle design.

Relaxing the assumption by Gue *et al*. (2012) of multiple dock doors located on one side of the warehouse, Ozturkoglu *et al*. (2014) considered multiple dock doors distributed on different sides of the warehouse. They developed a network-based formulation to obtain the expected distance in a given design. Determining the best angle for cross-aisles and S/R aisles for a given number of dock doors in a unit-load warehouse, they concluded the potential benefit of alternative aisle designs depends on the number and locations of dock doors.

An early study employing a class-based storage policy is credited to Heskitt (1963). Defining the cube-per-order (CPO) index as the ratio of required storage area to order frequency for a SKU, he proposed assigning SKUs with the lowest CPO index to locations with the smallest expected distance. Francis (1967a) proved the optimality of CPO index for single-command travel when the expected distance between dock doors and storage locations is not a function of the products assigned to the storage locations.

Hausman *et al*. (1976) introduced the problem of assigning classes of SKUs to storage locations in an AS/RS with the objective of minimizing travel time. Subsequent to their publication, numerous papers addressed class-based storage policies in the design of an AS/RS with the objective of maximizing throughput. A relatively recent review of literature on class-based storage policies can be found in de Koster *et al*. (2007) and Gu *et al*. (2007).

Bender (1981) studied approaches to represent the Pareto curve, as well as their limitations. He proposed a new approach to describe the Pareto curve mathematically. Moreover, he included three applications of his model to illustrate the concept behind his approach. Using his formulations, we examined the effect of ABC curve shapes on the penalty resulting from requiring a warehouse to be rectangle-shaped, rather than contour-line-shaped.

Recently, Thomas and Meller (2014) presented expected-distance models for put-away, order picking and replenishment operations for both random and class-based storage policies by using Bender's formulations to determine the percent of activity for each class in a traditional warehouse design. Moreover, they allowed dock doors to be uniformly distributed along either one side or two opposite sides of the warehouse. They determined the optimal shape factor of the warehouse design by incorporating horizontal travel distances for put-away, order picking and replenishment operations. Their numerical results demonstrated the optimal shape factor differs among the operations they considered. Extending their study for specified dock-door locations (Scenario 2) may provide useful rules of thumb for warehouse designers.

4.3. Notation

The notation used in developing expected-distance formulations is listed below and illustrated in Figure 4.1.

W_i = width of the union of storage areas 1 thru *i* (1 = A, 2 = A∪B, 3 = A∪B∪C)

D_i = depth of the union of storage areas 1 thru *i* (1 = A, 2 = A∪B, 3 = A∪B∪C)

S_i = shape factor ($S_i = W_i / D_i$) for the storage space containing classes 1 thru *i* (1 = A, 2 = A∪B, 3 = A∪B∪C)

A_i = the total storage area required by product class *i* (*i* = A, B, C)

$A_{i \cup j}$ = the total storage space required for classes *i* and *j* (*i* = A, B, C and *j* = A, B, C)

T_i = throughput rate, measured in number of roundtrips per unit time, for product class *i* (*i* = A, B, C)

t_i = percentage of the movement for class *i* (*i* = A, B, C and $t_i = T_i / \sum_{\forall i} T_i$)

p_i = percentage of the storage space required for class *i* (*i* = A, B, C and $p_i = A_i / \sum_{\forall i} A_i$)

k = number of dock doors

k_i = number of dock doors "covered" by storage area *i* (*i* = A, B, C)

$k_{i \cup j}$ = number of dock doors "covered" by the union of storage areas *i* and *j* (*i* = A, B, C and *j* = A, B, C)

ω = the width of a dock door

ψ = the clearance between adjacent dock doors

δ = the distance between centerlines of two adjacent dock doors (i.e. i^{th} and $(i+1)^{th}$ dock doors) ($\delta = \omega + \psi$)

SC_i = single-command roundtrip distance of storage area i (i = A, B, C)

$z_{k,i}$ = the objective function value of contour line i in a warehouse having k dock doors

$f^*_{k,i}$ = the minimum value of the objective function for contour line i in a warehouse having k dock doors

$h_{k,i}$ = the distance from dock door k to contour line i

$A_{k,i}$ = the area enclosed by contour line i in a warehouse having k dock doors

$q(z_{k,i})$ = the functional relationship between $A_{k,i}$ and $h_{k,i}$

$r(z_{k,i})$ = inverse function relating $A_{k,i}$ and $h_{k,i}$ (found by solving $q(z_{k,i})$ for k)

$E[D_{k,i}]$ = one-way expected distance of contour line i in a warehouse having k dock doors

4.4. Derivation of expected-distance formula for each class

Expected-distance expressions for storage area A, the union of storage areas A and B, and the union of storage areas A, B and C can be developed directly. However, it remains to develop an expected-distance expression for only storage area B or for only storage area C. To do so, we first use the relationship between expected distance for Class A and the expected distance for the union of storage areas A and B, which is $E\,[SC_{A\cup B}] = p_A\, E\,[SC_A] + p_B\, E\,[SC_B]$. Because $p_A = A_A / A_{A\cup B} = (W_1\, D_1) / (W_2\, D_2)$ and $p_B = (A_{A\cup B} - A_A) / A_{A\cup B} = (W_2\, D_2 - W_1\, D_1) / (W_2\, D_2)$, the expected distance for Class B is

$$E\,[SC_B] = (A_{A\cup B}\, E\,[SC_{A\cup B}] - A_A\, E\,[SC_A]) / (A_{A\cup B} - A_A) \text{ or}$$
$$E\,[SC_B] = (W_2\, D_2\, E\,[SC_{A\cup B}] - W_1\, D_1\, E\,[SC_A]) / (W_2\, D_2 - W_1\, D_1). \quad (4.1)$$

Now, obtaining the expected distance for Class C, we use the relationship between the expected distance for the union of storage areas A and B, and the expected distance for the union of storage areas A, B and C, $E\ [SC_{A\cup B\cup C}] = p_{A\cup B}\ E\ [SC_{A\cup B}] + p_C\ E\ [SC_C]$. Because $p_{A\cup B} = A_{A\cup B}\ /\ A_{A\cup B\cup C} = (W_2\ D_2)\ /\ (W_3\ D_3)$ and $p_C = (A_{A\cup B\cup C} - A_{A\cup B})\ /\ A_{A\cup B\cup C} = (W_3\ D_3 - W_2\ D_2)\ /\ (W_3\ D_3)$, the expected distance for Class C is

$$E\ [SC_C] = (A_{A\cup B\cup C}\ E\ [SC_{A\cup B\cup C}] - A_{A\cup B}\ E\ [SC_{A\cup B}])\ /\ (A_{A\cup B\cup C} - A_{A\cup B}) \text{ or}$$
$$E\ [SC_C] = (W_3\ D_3\ E\ [SC_{A\cup B\cup C}] - W_2\ D_2\ E\ [SC_{A\cup B}])\ /\ (W_3\ D_3 - W_2\ D_2). \tag{4.2}$$

4.5. Rectangle-shaped warehouse

In this section, we develop expected single-command distance formulas for a rectangle-shaped warehouse. Assuming storage/retrieval (S/R) locations are uniformly distributed over continuous storage regions, the optimal width and the optimal depth of each storage region is approximated. Specifically, two scenarios are considered regarding the number and locations of dock doors: 1) k dock doors are dispersed over an entire wall of the warehouse with an equal distance between adjacent dock doors and 2) k dock doors are located along one wall of the warehouse with a fixed distance (δ) between adjacent dock doors.

From Chapter 2, the expected single-command distance for k dock doors dispersed over the entire wall of the warehouse is

$$E\ [SC] = [(2k + 1)\ W]\ /\ [3(k + 1)] + D, \tag{4.3}$$

and the expected single-command distance for k centrally located dock doors with a specified distance (δ) between adjacent dock doors is

$$E\ [SC] = W / 2 + [(k^2 - 1)\ \delta^2]\ /\ 6W + D. \tag{4.4}$$

In deriving expected-distance formulas, three cases are taken into consideration for each scenario, as illustrated in Figure 4.2: 1) all dock doors are covered by the storage area A; 2) all dock doors are covered by the union of storage areas A and B, but some dock doors are not covered by storage area A; and 3) all dock doors are covered by the union of storage areas A, B and C, but some dock doors are not covered by the union of storage areas A and B. Although the number of dock doors shown differs among the cases, the formulations are valid for any number of dock doors.

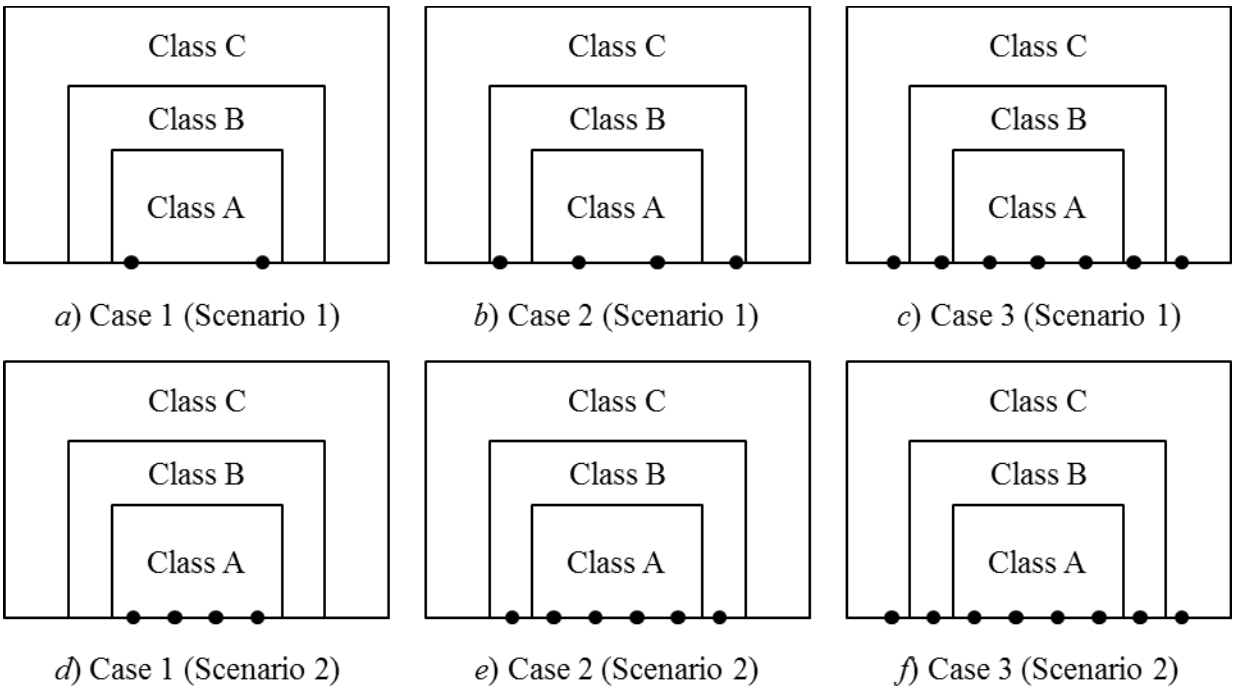

Figure 4.2: Cases for dock-door locations in a rectangle-shaped warehouse for each scenario

Employing expected-distance formulas, a general formulation of the nonlinear-programming optimization problem is used to determine the widths and depths of storage regions:

Minimize : $E\,[SC_{\text{Case1}}]\,x_1 + E\,[SC_{\text{Case2}}]\,x_2 + E\,[SC_{\text{Case3}}]\,x_3$

Subject to : $W_1 D_1 = A_{\text{A}}$, $W_2 D_2 = A_{\text{A}\cup\text{B}}$ and $W_3 D_3 = A_{\text{A}\cup\text{B}\cup\text{C}}$

1) $W_3 \geq (k + 1)\,(\omega + \psi)$ and 2) $W_3 \geq k\,\delta$

$W_3 \geq W_2 \geq W_1$ and $D_3 \geq D_2 \geq D_1$

$x_1 + x_2 + x_3 = 1$

$W_i > 0$ and $D_i > 0$.

x_1, x_2 and x_3 binary

The first constraint assures the space requirement is met for each storage region. Pre-determined areas are given as A_{A}, $A_{\text{A}\cup\text{B}}$ and $A_{\text{A}\cup\text{B}\cup\text{C}}$. The second constraint requires the width of the overall warehouse ($A_{\text{A}\cup\text{B}\cup\text{C}}$) to be sufficient for the location of k dock doors; the constraint differs, depending on the scenario. Alternatively, we could have relaxed the rectangularity assumption and based the width constraint on different storage regions (e.g. $(\omega + \psi) \leq W_1 (k + 1)$ or $(\omega + \psi) \leq W_3 (k + 1)$ for Scenario 1); we defer such considerations to future research. Satisfying the rectangularity assumption for the overall warehouse, the third constraint is added to our optimization model. The fourth constraint guarantees only one case is chosen in calculating expected single-command distance. The last two constraints define the set of constraints for the nonnegative and binary properties of decision variables, respectively.

Solving the nonlinear-programming optimization problem, we used an open source code, *Couenne* (2006), in *AMPL* (2013) software package. *Couenne* (2006) solves Mixed-Integer Nonlinear Programming (MINLP) formulations by using linearization, bound reduction and branching methods within a branch and bound algorithm (Belotti, 2009; Belotti *et al*. 2009). Notice, binary variables are used to incorporate conditional expressions for expected single-command distances. Section 4.8 includes results from *Couenne* (2006).

4.5.1. Dock doors dispersed over an entire wall

In this sub-section, we develop expected single-command distance formulations for a rectangle-shaped warehouse having k dock doors dispersed over the entire wall. Because the distance between adjacent dock doors, $W_3 / (k + 1)$, is a function of the width of the entire warehouse (W_3), the width of the storage areas for Class A (W_1) and the union of Classes A and

B (W_2) do not affect the spacing between adjacent dock doors. Therefore, Equation (4.3) is used for the union of Classes A, B and C; whereas, Equation (4.4) is used to develop expected-distance expressions for Class A and the union of Classes A and B.

By adjusting Equations (4.3) and (4.4), the expected distance for Class A and the union of Classes A and B, and the union of Classes A, B and C are obtained as follows

$$E\,[SC_{\mathrm{A}}] = W_1 / 2 + [(k-1)\, W_3^{\,2}] / [6(k+1)\, W_1] + D_1, \tag{4.5}$$

$$E\,[SC_{\mathrm{A}\cup\mathrm{B}}] = W_2 / 2 + [(k-1)\, W_3^{\,2}] / [6(k+1)\, W_2] + D_2, \tag{4.6}$$

$$E\,[SC_{\mathrm{A}\cup\mathrm{B}\cup\mathrm{C}}] = [(2k+1)\, W_3] / [3(k+1)] + D_3. \tag{4.7}$$

Equations (5), (6) and (7) hold for all cases.

Case 1: If all dock doors are covered by storage area A, $[(k-1)\, W_3] / (k+1) \leq W_1$. Substituting Equations (4.5) and (4.6) into Equation (4.1) and reducing the resulted equation, the expected distance for Class B is

$$E\left[SC_B\right] = \frac{3\left(k+1\right)\left(W_2^{\,2} D_2 - W_1^2 D_1\right) + \left(k-1\right)\left(D_2 - D_1\right) W_3^2}{6\left(k+1\right)\left(W_2 D_2 - W_1 D_1\right)}\,. \tag{4.8}$$

Similarly, substituting Equations (4.6) and (4.7) into Equation (4.2), the expected distance for Class C is

$$E\left[SC_C\right] = \frac{2W_3 D_3\left[\left(2k+1\right)W_3 + 3\left(k+1\right)D_3\right] - D_2\left[3\left(k+1\right)\left(2D_2 + W_2\right)W_2 + \left(k-1\right)W_3^{\,2}\right]}{6\left(k+1\right)\left(W_3 D_3 - W_2 D_2\right)} \tag{4.9}$$

Because the percentage of the movement for class i is t_i, the expected distance traveled for a rectangle-shaped warehouse having k dock doors for Case 1 is

$$E[SC] = t_A\left[\frac{W_3^2(k-1)+3(k+1)(2D_1+W_1)W_1}{6(k+1)W_1}\right]+t_B\left[\frac{3(k+1)\left(W_2^2 D_2-W_1^2 D_1\right)}{6(k+1)(W_2 D_2-W_1 D_1)}\right]$$
$$+t_B\left[\frac{(k-1)(D_2-D_1)W_3^2}{6(k+1)(W_2 D_2-W_1 D_1)}\right]+t_C\left[\frac{2W_3 D_3\left[(2k+1)W_3+3(k+1)D_3\right]}{6(k+1)(W_3 D_3-W_2 D_2)}\right] \tag{4.10}$$
$$-t_C\left[\frac{D_2\left[3(k+1)(2D_2+W_2)W_2+(k-1)W_3^2\right]}{6(k+1)(W_3 D_3-W_2 D_2)}\right].$$

Case 2: If some dock doors are not covered by storage area A, but all dock doors are covered by the union of storage area A and B, $W_1 \le [(k-1)\ W_3] / (k+1) \le W_2$. Developing expected-distance formulas, we first determine the number of dock doors covered by each class, $k_A = k - 2 \lceil \{(k-1)\ W_2 - (k+1)\ W_1\} / 2W_2 \rceil$ and $k_B = k - k_A$. Adjusting Equation (4.4) for k_A dock doors and storage area A, the expected distance from/to dock doors covered by Class A is

$$E\,[SC_{k_A}] = W_1 / 2 + [(k_A^2 - 1)\ W_3^{\,2}] / [6(k+1)^2\ W_1] + D_1. \tag{4.11}$$

A new formulation for the expected distance from/to dock doors <u>not</u> covered by Class A, but covered by Class B, is

$$E\,[SC_{k_B}] = (2k - k_B)\ W_3 / 2(k+1) + D_1. \tag{4.12}$$

Multiplying the two previous equations by the corresponding percent of usage for dock doors, then summing these equations and dividing by k, the expected distance for Class A is

$$E[SC_A] = \frac{k_A\left(k_A^2-1\right)W_3^2+3(k+1)\left[k_A(k+1)W_1+k_B(2k-k_B)W_3\right]W_1}{6k(k+1)^2 W_1}+D_1. \tag{4.13}$$

Because all dock doors are covered by the union of storage areas A and B, and the union of storage area A, B and C, Equations (4.6) and (4.7) can be used directly for Case 2. Therefore, substituting Equations (4.6) and (4.13) into Equation (4.1), the expected distance for Class B is

$$E[SC_B] = \frac{D_2\left[(k-1)W_3^2 + 3(k+1)(W_2+2D_2)W_2\right]}{6(k+1)(W_2 D_2 - W_1 D_1)} - \frac{k_A\left(k_A^2-1\right)W_3^2 D_1}{6k(k+1)^2(W_2 D_2 - W_1 D_1)} - \frac{3(k+1)\left[k_A(k+1)W_1 + k_B(2k-k_B)W_3 + 2k(k+1)D_1\right]W_1 D_1}{6k(k+1)^2(W_2 D_2 - W_1 D_1)}, \quad (4.14)$$

and substituting Equations (4.6) and (4.7) into Equation (4.2), the expected distance for Class C is obtained

$$E[SC_C] = \frac{2W_3 D_3\left[(2k+1)W_3 + 3(k+1)D_3\right] - D_2\left[3(k+1)(2D_2+W_2)W_2 + (k-1)W_3^2\right]}{6(k+1)(W_3 D_3 - W_2 D_2)}. \quad (4.15)$$

Multiplying the expected distance for each storage area by the percentage of the movement for storage area, the expected distance for a rectangle-shaped warehouse with Case 2 is

$$\begin{aligned} E[SC] = {} & t_A\left[\frac{k_A\left(k_A^2-1\right)W_3^2 + 3(k+1)\left[k_A(k+1)W_1 + k_B(2k-k_B)W_3\right]W_1}{6k(k+1)^2 W_1} + D_1\right] \\ & + t_B\left[\frac{D_2\left[(k-1)W_3^2 + 3(k+1)(W_2+2D_2)W_2\right]}{6(k+1)(W_2 D_2 - W_1 D_1)}\right] - t_B\left[\frac{k_A\left(k_A^2-1\right)W_3^2 D_1}{6k(k+1)^2(W_2 D_2 - W_1 D_1)}\right] \\ & - t_B\left[\frac{3(k+1)\left[k_A(k+1)W_1 + k_B(2k-k_B)W_3 + 2k(k+1)D_1\right]W_1 D_1}{6k(k+1)^2(W_2 D_2 - W_1 D_1)}\right] \\ & + t_C\left[\frac{2W_3 D_3\left[(2k+1)W_3 + 3(k+1)D_3\right] - D_2\left[3(k+1)(2D_2+W_2)W_2 + (k-1)W_3^2\right]}{6(k+1)(W_3 D_3 - W_2 D_2)}\right]. \end{aligned} \quad (4.16)$$

Case 3: If some dock doors are not covered by the union of storage areas A and B, $[(k-1)W_3]/(k+1) > W_2$. Calculating the number of dock doors covered by storage area A, the equation given in Case 2 can be used for Case 3 because some dock doors also are not covered by Class A, $k_A = k - 2\lceil \{(k-1)W_3 - (k+1)W_1\}/2W_3 \rceil$. Hence, $k_{B \cup C} = k - k_A$. Likewise, the number of dock doors covered by the union of storage areas A and B is $k_{A \cup B} = k - 2\lceil \{(k-1)W_3 - (k+1)W_2\}/2W_3 \rceil$. Therefore, $k_C = k - k_{A \cup B}$. Using Equation (4.11) for dock doors covered by Class A and adjusting Equation (4.12) for dock doors covered by the union of Classes B and C ($k_{B\cup C}$ instead of k_B), the expected distance for Class A is

$$E\left[SC_A\right]=\frac{k_A\left(k_A{}^2-1\right)W_3^2+3\left(k+1\right)W_1\left[k_A\left(k+1\right)W_1+k_{B\cup C}\left(2k-k_{B\cup C}\right)W_3\right]}{6k\left(k+1\right)^2W_1}+D_1. \qquad (4.17)$$

Similarly, the expected distance traveled for dock doors covered by the union of storage areas A and B is

$$E\ [SC_{k_{A\cup B}}] = W_2 / 2 + [(k_{A\cup B}{}^2 - 1)\ W_3{}^2] / [6(k + 1)^2\ W_2] + D_2, \qquad (4.18)$$

and the expected distance traveled for dock doors not covered by the union of storage areas A and B is

$$E\ [SC_{k_C}] = (2k - k_C)\ W_3 / 2(k + 1) + D_2. \qquad (4.19)$$

Therefore, the expected distance for the union of storage areas A and B is

$$E\left[SC_{A\cup B}\right]=\frac{k_{A\cup B}\left(k_{A\cup B}{}^2-1\right)W_3^2+3\left(k+1\right)W_2\left[k_{A\cup B}\left(k+1\right)W_2+k_C\left(2k-k_C\right)W_3\right]}{6k\left(k+1\right)^2W_2}+D_2. \qquad (4.20)$$

Substituting Equations (4.17) and (4.20) into Equation (4.1), the expected distance for Class B is obtained

$$\begin{aligned}E\left[SC_B\right]=&\frac{k_{A\cup B}\left(k_{A\cup B}{}^2-1\right)W_3{}^2D_2-k_A\left(k_A{}^2-1\right)W_3{}^2D_1}{6k\left(k+1\right)^2\left(W_2D_2-W_1D_1\right)}\\&+\frac{3\left(k+1\right)\left[k_{A\cup B}\left(k+1\right)W_2+k_C\left(2k-k_C\right)W_3+2k\left(k+1\right)D_2\right]W_2D_2}{6k\left(k+1\right)^2\left(W_2D_2-W_1D_1\right)}\\&-\frac{3\left(k+1\right)\left[k_A\left(k+1\right)W_1+k_{B\cup C}\left(2k-k_{B\cup C}\right)W_3+2k\left(k+1\right)D_1\right]W_1D_1}{6k\left(k+1\right)^2\left(W_2D_2-W_1D_1\right)}.\end{aligned} \qquad (4.21)$$

Likewise, substituting Equations (4.7) and (4.20) into Equation (2), the expected distance for Class C is

$$E\left[SC_C\right]=\frac{W_3 D_3\left[\left(2k+1\right)W_3+3\left(k+1\right)D_3\right]}{3\left(k+1\right)\left(W_3 D_3-W_2 D_2\right)}-\frac{k_{A\cup B}\left(k_{A\cup B}{}^2-1\right)W_3{}^2}{6k\left(k+1\right)^2\left(W_3 D_3-W_2 D_2\right)}$$
$$-\frac{3\left(k+1\right)\left[k_{A\cup B}\left(k+1\right)W_2+k_C\left(2k-k_C\right)W_3+2k\left(k+1\right)D_2\right]W_2 D_2}{6k\left(k+1\right)^2\left(W_3 D_3-W_2 D_2\right)}\ . \tag{4.22}$$

Multiplying the expected distance for each class by the percentage of the movement for each class and summing the resulting equations, the expected distance for Case 3 is

$$\begin{aligned}
E\left[SC\right]=\ & t_A\left[\frac{k_A\left(k_A{}^2-1\right)W_3{}^2+3\left(k+1\right)W_1\left[k_A\left(k+1\right)W_1+k_{B\cup C}\left(2k-k_{B\cup C}\right)W_3\right]}{6k\left(k+1\right)^2 W_1}+D_1\right]\\
&+t_B\left[\frac{k_{A\cup B}\left(k_{A\cup B}{}^2-1\right)W_3{}^2 D_2-k_A\left(k_A{}^2-1\right)W_3{}^2 D_1}{6k\left(k+1\right)^2\left(W_2 D_2-W_1 D_1\right)}\right]\\
&+t_B\left[\frac{3\left(k+1\right)\left[k_{A\cup B}\left(k+1\right)W_2+k_C\left(2k-k_C\right)W_3+2k\left(k+1\right)D_2\right]W_2 D_2}{6k\left(k+1\right)^2\left(W_2 D_2-W_1 D_1\right)}\right]\\
&-t_B\left[\frac{3\left(k+1\right)\left[k_A\left(k+1\right)W_1+k_{B\cup C}\left(2k-k_{B\cup C}\right)W_3+2k\left(k+1\right)D_1\right]W_1 D_1}{6k\left(k+1\right)^2\left(W_2 D_2-W_1 D_1\right)}\right]\\
&+t_C\left[\frac{W_3 D_3\left[\left(2k+1\right)W_3+3\left(k+1\right)D_3\right]}{3\left(k+1\right)\left(W_3 D_3-W_2 D_2\right)}\right]-t_C\left[\frac{k_{A\cup B}\left(k_{A\cup B}{}^2-1\right)W_3{}^2}{6k\left(k+1\right)^2\left(W_3 D_3-W_2 D_2\right)}\right]\\
&-t_C\left[\frac{3\left(k+1\right)\left[k_{A\cup B}\left(k+1\right)W_2+k_C\left(2k-k_C\right)W_3+2k\left(k+1\right)D_2\right]W_2 D_2}{6k\left(k+1\right)^2\left(W_3 D_3-W_2 D_2\right)}\right].
\end{aligned} \tag{4.23}$$

As stated, for Scenario 1, the distance between adjacent dock doors is a function of the width of the entire warehouse. Another approach is to allow the distance between adjacent dock doors to be a decision variable. From Chapter 2, locating dock doors as close as possible to the center of a wall minimizes expected distance traveled. Therefore, the optimal solution will be the smallest feasible distance between adjacent dock doors. Hence, finding the minimum distance between adjacent dock doors converts the problem to the next scenario.

4.5.2. Dock doors along one wall with δ separation between adjacent dock doors

When dock doors are centrally located with a fixed distance (δ) between adjacent dock doors, we employ a process similar to that employed in the previous sub-section to develop expected-distance expressions. However, only Equation (4.4) is used because the distance between adjacent dock doors does not depend on storage area widths.

Adjusting Equation (4.4), the expected distance for Class A and the union of Classes A and B, and the union of Classes A, B and C are

$$E\,[SC_{\mathrm{A}}] = W_1 / 2 + [(k^2 - 1)\,\delta^2] / 6W_1 + D_1, \tag{4.24}$$

$$E\,[SC_{\mathrm{A \cup B}}] = W_2 / 2 + [(k^2 - 1)\,\delta^2] / 6W_2 + D_2, \tag{4.25}$$

$$E\,[SC_{\mathrm{A \cup B \cup C}}] = W_3 / 2 + [(k^2 - 1)\,\delta^2] / 6W_3 + D_3. \tag{4.26}$$

As stated, three cases are considered. Equations (4.24), (4.25) and (4.26) hold for all cases.

Case 1: If all dock doors are covered by Class A, $(k - 1)\,\delta \le W_1$. Expected-distance expressions for Classes B and C are obtained by substituting Equations (4.24) and (4.25) into Equation (4.1), and Equations (4.25) and (4.26) into Equation (4.2), respectively. Therefore, the expected distance for Class B is

$$E\left[SC_B\right] = \frac{D_2\left[\delta^2\left(k^2-1\right)+3W_2\left(W_2+2D_2\right)\right]+D_1\left[\delta^2\left(k^2-1\right)+3W_1\left(W_1+2D_1\right)\right]}{6\left(W_2\,D_2-W_1\,D_1\right)}, \tag{4.27}$$

and the expected distance for Class C is

$$E\left[SC_C\right] = \frac{D_3\left[\delta^2\left(k^2-1\right)+3W_3\left(W_3+2D_3\right)\right]+D_2\left[\delta^2\left(k^2-1\right)+3W_2\left(W_2+2D_2\right)\right]}{6\left(W_3\,D_3-W_2\,D_2\right)}. \tag{4.28}$$

Consequently, the expected distance for a rectangle-shaped warehouse having k dock doors with Case 1 is

$$E[SC] = t_A \left[\frac{\delta^2 (k^2 - 1) + 3W_1 (W_1 + 2D_1)}{6W_1} \right]$$
$$+ t_B \left[\frac{D_2 \left[\delta^2 (k^2 - 1) + 3W_2 (W_2 + 2D_2) \right] + D_1 \left[\delta^2 (k^2 - 1) + 3W_1 (W_1 + 2D_1) \right]}{6(W_2 D_2 - W_1 D_1)} \right] \quad (4.29)$$
$$+ t_C \left[\frac{D_3 \left[\delta^2 (k^2 - 1) + 3W_3 (W_3 + 2D_3) \right] + D_2 \left[\delta^2 (k^2 - 1) + 3W_2 (W_2 + 2D_2) \right]}{6(W_3 D_3 - W_2 D_2)} \right].$$

Case 2: If some dock doors are not covered by storage area A, but are covered by storage area B, $W_1 \leq (k-1)\,\delta \leq W_2$. The number of dock doors covered by Class A is $k_A = k - 2 \lceil [(k-1)\,\delta - W_1] / 2\delta \rceil$. Hence, $k_B = k - k_A$. Adjusting Equation (4.24) for k_A dock doors, the expected distance for dock doors covered by Class A is

$$E\,[SC_{k_A}] = W_1 / 2 + [(k_A{}^2 - 1)\,\delta^2] / [6W_1] + D_1. \quad (4.30)$$

Developing a new formula for k_B dock doors, the expected distance for dock doors <u>not</u> covered by Class A, but covered by Class B, is

$$E\,[SC_{k_B}] = (k - k_B / 2)\,\delta + D_1. \quad (4.31)$$

Therefore, the expected-distance expression for Class A with Case 2 is

$$E[SC_A] = \left[\frac{k_A (k_A{}^2 - 1)\delta^2 + 3W_1 \left[k_A W_1 + 2k_B (k - k_B/2)\delta \right]}{6\,k\,W_1} \right] + D_1. \quad (4.32)$$

Obtaining the expected distance for Classes B and C, Equations (4.25) and (4.26) can be used directly because the union of storage areas A and B, and the union of storage areas A, B and C cover all dock doors. Therefore, substituting Equations (4.25) and (4.32) into Equation (4.1), the expected distance for Class B is

$$E[SC_B] = \frac{D_2 k\left[\delta^2\left(k^2-1\right)+3W_2\left(W_2+2D_1\right)\right]}{6k\left(W_2 D_2 - W_1 D_1\right)} - \frac{D_1\left\{k_A\left(k_A{}^2-1\right)\delta^2+3W_1\left[k_A W_1+2k_B\left(k-k_B/2\right)\delta+2k D_1\right]\right\}}{6k\left(W_2 D_2 - W_1 D_1\right)}, \tag{4.33}$$

and substituting Equations (4.25) and (4.26) into Equation (4.2), the expected distance for Class C is obtained

$$E[SC_C] = \frac{D_3\left[\delta^2\left(k^2-1\right)+3W_3\left(W_3+2D_3\right)\right]-D_2\left[\delta^2\left(k^2-1\right)+3W_2\left(W_2+2D_2\right)\right]}{6\left(W_3 D_3 - W_2 D_2\right)}. \tag{4.34}$$

Finally, the expected distance for a rectangle-shaped warehouse with Case 2 is

$$\begin{aligned} E[SC] = {} & t_A\left[\left[\frac{k_A\left(k_A{}^2-1\right)\delta^2+3W_1\left[k_A W_1+2k_B\left(k-k_B/2\right)\delta\right]}{6kW_1}\right]+D_1\right] \\ & + t_B\left[\frac{D_2\left[\delta^2\left(k^2-1\right)+3W_2\left(W_2+2D_1\right)\right]}{6\left(W_2 D_2 - W_1 D_1\right)}\right] \\ & - t_B\left[\frac{D_1\left\{k_A\left(k_A{}^2-1\right)\delta^2+3W_1\left[k_A W_1+2k_B\left(k-k_B/2\right)\delta+2k D_1\right]\right\}}{6k\left(W_2 D_2 - W_1 D_1\right)}\right] \\ & + t_C\left[\frac{D_3\left[\delta^2\left(k^2-1\right)+3W_3\left(W_3+2D_3\right)\right]-D_2\left[\delta^2\left(k^2-1\right)+3W_2\left(W_2+2D_2\right)\right]}{6\left(W_3 D_3 - W_2 D_2\right)}\right]. \end{aligned} \tag{4.35}$$

Case 3: If some dock doors are not covered by the union of storage areas A and B, $(k-1)\,\delta > W_2$. The number of dock doors covered by Class A and by the union of Classes A and B are $k_A = k - 2\lceil \{(k-1)\,\delta - W_1\} / 2\delta \rceil$ and $k_{A\cup B} = k - 2\lceil \{(k-1)\,\delta - W_2\} / 2\delta \rceil$, respectively. Hence, $k_{B\cup C} = k - k_A$ and $k_C = k - k_{A\cup B}$. Using Equation (4.30) for dock doors covered by Class A and adjusting Equation (4.31) for dock doors covered by the union of Classes B and C ($k_{B\cup C}$ instead of k_B), the expected distance for Class A is

$$E\left[SC_A\right]=\frac{k_A\left(k_A{}^2-1\right)\delta^2+3W_1\left[W_1k_A+2k_{B\cup C}\left(k-k_{B\cup C}/2\right)\delta\right]}{6\,k\,W_1}+D_1. \qquad (4.36)$$

Then, the expected distance traveled for dock doors covered by the union of Classes A and B is

$$E\,[SC_{k_{A\cup B}}] = W_2 / 2 + [(k_{A\cup B}{}^2 - 1)\, W_3{}^2] / [6(k + 1)^2\, W_2] + D_2, \qquad (4.37)$$

and the expected distance traveled for dock doors not covered by the union of Classes A and B is

$$E\,[SC_{k_C}] = (2k - k_C)\, W_3 / 2(k + 1) + D_2. \qquad (4.38)$$

Therefore, the expected distance for the union of Classes A and B is

$$E\left[SC_{A\cup B}\right]=\frac{k_{A\cup B}\delta^2\left(k_{A\cup B}{}^2-1\right)+3W_2\left[k_{A\cup B}W_2+2k_C\left(k-k_C/2\right)\delta\right]}{6kW_2}+D_2\,. \qquad (4.39)$$

Substituting Equations (4.36) and (4.39) into Equation (4.1), the expected distance for Class B is obtained

$$E\left[SC_B\right]=\frac{D_2\left\{k_{A\cup B}\delta^2\left(k_{A\cup B}{}^2-1\right)+3W_2\left[k_{A\cup B}W_2+2k_C\left(k-k_C/2\right)\delta+2kD_2\right]\right\}}{6k\left(W_2\,D_2-W_1\,D_1\right)} -\frac{D_1\left\{k_A\left(k_A{}^2-1\right)\delta^2+3W_1\left[k_AW_1+2k_{B\cup C}\left(k-k_{B\cup C}/2\right)\delta+2kD_1\right]\right\}}{6\,k\left(W_2\,D_2-W_1\,D_1\right)}\,. \qquad (4.40)$$

Likewise, substituting Equations (4.26) and (4.39) into Equation (4.2), the expected distance for Class C is

$$E\left[SC_C\right]=\frac{D_3\left[\left(k^2-1\right)\delta^2+3W_3\left(W_3+2D_3\right)\right]}{6\left(W_3\,D_3-W_2\,D_2\right)} -\frac{D_2\left\{k_{A\cup B}\delta^2\left(k_{A\cup B}{}^2-1\right)+3W_2\left[k_{A\cup B}W_2+2k_C\left(k-k_C/2\right)\delta+2kD_2\right]\right\}}{6k\left(W_3\,D_3-W_2\,D_2\right)}\,. \qquad (4.41)$$

The expected distance traveled for a rectangle-shaped warehouse having *k* dock doors for Case 3 is obtained

$$E[SC] = t_A\left[\frac{k_A\left(k_A{}^2-1\right)\delta^2+3W_1\left[W_1k_A+2k_{B\cup C}\left(k-k_{B\cup C}/2\right)\delta\right]}{6kW_1}+D_1\right]$$
$$+t_B\left[\frac{D_2\left\{k_{A\cup B}\delta^2\left(k_{A\cup B}{}^2-1\right)+3W_2\left[k_{A\cup B}W_2+2k_C\left(k-k_C/2\right)\delta+2kD_2\right]\right\}}{6k\left(W_2D_2-W_1D_1\right)}\right]$$
$$-t_B\left[\frac{D_1\left\{k_A\left(k_A{}^2-1\right)\delta^2+3W_1\left[k_AW_1+2k_{B\cup C}\left(k-k_{B\cup C}/2\right)\delta+2kD_1\right]\right\}}{6k\left(W_2D_2-W_1D_1\right)}\right] \qquad (4.42)$$
$$+t_C\left[\frac{D_3\left[\left(k^2-1\right)\delta^2+3W_3\left(W_3+2D_3\right)\right]}{6\left(W_3D_3-W_2D_2\right)}\right]$$
$$-t_C\left[\frac{D_2\left\{k_{A\cup B}\delta^2\left(k_{A\cup B}{}^2-1\right)+3W_2\left[k_{A\cup B}W_2+2k_C\left(k-k_C/2\right)\delta+2kD_2\right]\right\}}{6k\left(W_3D_3-W_2D_2\right)}\right].$$

4.6. Contour-line-shaped warehouse

In this section, the concept of contour sets and contour lines defined by Francis *et al*. (1992) is used to develop expected single-command distance formulas for contour-line shaped warehouses. As stated, a contour line includes all points with expected distances to/from dock doors that are less than or equal to the value of the contour line. Contour lines determine the shape of the storage regions or/and the overall shape of the warehouse. For a detailed procedure to construct contour lines, see Francis (1963). After constructing contour lines, expected single-distance formulations are developed. As stated, Francis (1967b) employed a special case of the Neyman-Pearson Lemma to calculate expected distance. For proofs of the properties underlying his procedure, see Francis (1967c). To illustrate the procedure, we consider a contour-line-shaped warehouse having three dock doors.

As illustrated in Figure 4.3 (left), consider a storage area, *A*, with three dock doors having a specified distance between adjacent dock doors, δ. Assuming travel to/from storage locations is equally likely for each dock door, contour lines are illustrated in Figure 4.3 (right). Notice the number of contour sets is k / 2 and (k + 1) / 2 for an even and an odd number of dock doors, respectively. All contour lines within a set have the same shape.

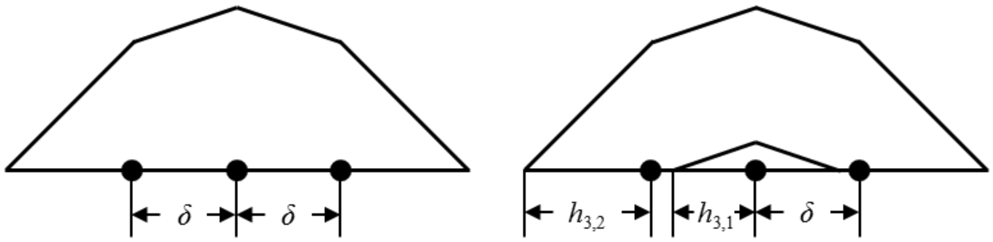

Figure 4.3: Contour line construction for a storage region having three dock doors with a specified distance between adjacent dock doors

Within the storage region shown in Figure 4.3, there are 2 contour sets. The first set is defined by the triangular-shaped set of points. The second set is defined by the points lying outside the first contour set and within the 5-sided contour set of points. The objective function value for the first contour set is

$$z_{3,1} = (1 / 3)\,[(\delta - h_{3,1}) + h_{3,1} + (\delta + h_{3,1})] = (2\delta + h_{3,1}) / 3. \quad (4.43)$$

The minimum objective function value (2δ / 3) occurs when $h_{3,1}$ equals zero; therefore, for the first contour set, $f^*_{3,1} = 2\delta / 3$. Solving Equation (4.43) for $h_{3,1}$ gives $h_{3,1} = 3z_{3,1} - 2\delta$. Solving for the area contained within a contour line having value $h_{3,1}$, we obtain $A_{3,1} = h_{3,1}^{\,2} / 3$. Therefore, solving for $h_{3,1}$ as a function of the area gives $h_{3,1} = (3A_{3,1})^{1/2}$. Furthermore, the functional relationship between $A_{3,1}$ and $z_{3,1}$ is

$$q(z_{3,1}) = A_{3,1} = h_{3,1}^{\,2} / 3 = (3z_{3,1} - 2\delta)^2 / 3 = 3z_{3,1}^{\,2} - 4z_{3,1}(\delta) + 4\delta^2 / 3. \quad (4.44)$$

Based on the assumption of uniformly distributed points over the storage region, Equation (4.44) can be treated as the cumulative distribution function for single-command travel distance. Taking the first derivative of Equation (4.44) with respect to $z_{3,1}$ yields the probability density function for the first contour set, $q'(z_{3,1}) = 6z_{3,1} - 4\delta$. Solving Equation (4.44) for $z_{3,1}$ yields the inverse function related to $z_{3,1}$ and $A_{3,1}$, which is the value of the objective function on the contour line enclosing $A_{3,1}$,

$$r(A_{3,1}) = z_{3,1} = (2\delta + h_{3,1}) / 3 = 2\delta / 3 + (3A_{3,1})^{1/2} / 3 \tag{4.45}$$

Because $h_{3,1} \leq \delta$ for the first contour set, the maximum objective function equals δ. Therefore, the expected one-way distance for the first contour set is

$$\begin{aligned} E\left[D_{3,1}\right] &= \frac{1}{A_{3,1}} \int_{A_{3,1}} f\left(X\right) dX = \frac{1}{A_{3,1}} \int_{f^*_{3,1}}^{r\left(A_{3,1}\right)} z_{3,1}\, q'\left(z_{3,1}\right) dz_{3,1} = \frac{1}{A_{3,1}} \int_{2\delta/3}^{\delta} z_{3,1}\left(6z_{3,1} - 4\delta\right) dz_{3,1} \\ &= \frac{2}{A_{3,1}} \left[\left(z_{3,1}{}^{3} - \delta\, z_{3,1}{}^{2}\right)\Big|_{2\delta/3}^{\delta}\right] = \frac{8\delta^3}{27 A_{3,1}} \end{aligned} \tag{4.46}$$

Similarly, the objective function value for the second contour set having 5 sides is

$$z_{3,2} = (1 / 3)\, [h_{3,2} + (\delta + h_{3,2}) + (2\delta + h_{3,2})] = \delta + h_{3,2}. \tag{4.47}$$

The minimum objective function value for the second contour set is equivalent to the maximum value of the first contour set, which is $f^*_{3,2} = \delta$. Solving Equation (4.47) for $h_{3,2}$ gives $h_{3,2} = z_{3,2} - \delta$. The storage area enclosed by any contour line having value $h_{3,2}$ is $A_{3,2} = h_{3,2}{}^2 + 2h_{3,2}(\delta) + \delta^2 / 3$; therefore, solving for $h_{3,2}$ as a function of the area gives $h_{3,2} = (A_{3,2} + \delta^2 / 3)^{1/2} - \delta$. As with the first contour set, the functional relationship between $A_{3,2}$ and $z_{3,2}$ is

$$q(z_{3,2}) = A_{3,2} = h_{3,2}{}^2 + 2h_{3,2}(\delta) + \delta^2 / 3 = (z_{3,2} - \delta)^2 + 2(z_{3,2} - \delta)\,\delta + \delta^2 / 3 = z_{3,2}{}^2 - 2\delta^2 / 3. \tag{4.48}$$

Taking the derivative of Equation (4.48) with respect to $z_{3,2}$ equals $2z_{3,2}$, which is the probability density function for the second contour set. Obtaining the value of the objective function for the second contour set enclosed by $A_{3,2}$, Equation (4.48) is solved for $z_{3,2}$. The value of the objective function is

$$r(A_{3,2}) = z_{3,2} = \delta + h_{3,2} = \delta + (A_{3,2} + 2\delta^2 / 3)^{1/2} - \delta = (A_{3,2} + 2\delta^2 / 3)^{1/2} \tag{4.49}$$

The expected one-way distance for the second contour set is

$$E\left[D_{3,2}\right]=\frac{1}{A_{3,2}-A_{3,1}}\int_{A_{3,2}-A_{3,1}} f\left(X\right)dX=\frac{1}{A_{3,2}-A_{3,1}}\int_{f^*_{3,2}}^{r\left(A_{3,2}\right)} z_{3,2}\,q'\left(z_{3,2}\right)dz_{3,2}$$
$$=\frac{1}{A_{3,2}-A_{3,1}}\int_{\delta}^{\sqrt{A_{3,2}+2\delta^2/3}} z_{3,2}\left(2z_{3,2}\right)dz_{3,2}=\frac{2\left[\left(A_{3,2}+2\delta^2/3\right)^{3/2}-\delta^3\right]}{3\left(A_{3,2}-A_{3,1}\right)}. \tag{4.50}$$

Equations (4.46) and (4.50) are conditional expected values. To calculate the overall expected value, we remove the conditions by multiplying the result in Equation (4.46) by the probability of traveling to a point within the first contour set ($A_{3,1}$ / $A_{3,2}$) and multiplying the result in Equation (4.50) by the probability of traveling to a point in the second contour set ($[A_{3,2} - A_{3,1}]$ / $A_{3,2}$). Summing the results obtained and multiplying by 2, the expected single-command distance in a contour-line-shaped warehouse having three dock doors is

$$E\left[SC\right]=\frac{4}{3A_{3,2}}\left(A_{3,2}+2\delta^2/3\right)^{3/2}-\frac{20\delta^3}{27A_{3,2}} \tag{4.51}$$

A continuation of the approach given above leads to the following expected-distance formulations for Scenarios 1 and 2, respectively.

$$E\left[SC\right]=\frac{\left(2A\right)^{1/2}\left(12k^2+9k-1\right)}{5k\left[3\left(k+2\right)\left(k+1\right)\right]^{1/2}} \tag{4.52}$$

$$E\left[SC\right]=\frac{\left(12A+\delta^2k^2-\delta^2\right)^{3/2}}{18\sqrt{3}A}-\frac{\delta^3\left(3k^4-5k^2+2\right)}{90kA} \tag{4.53}$$

Although the objective functions of contour sets differ for an even and an odd number of dock doors, Equations (4.52) and (4.53) are valid for any number of dock doors. A proof of the claim and the step-by-step derivations of Equations (4.52) and (4.53) are provided in the Appendix.

As with a rectangle-shaped warehouse, we consider three cases for the locations of dock doors for each scenario (see Figure 4.4).

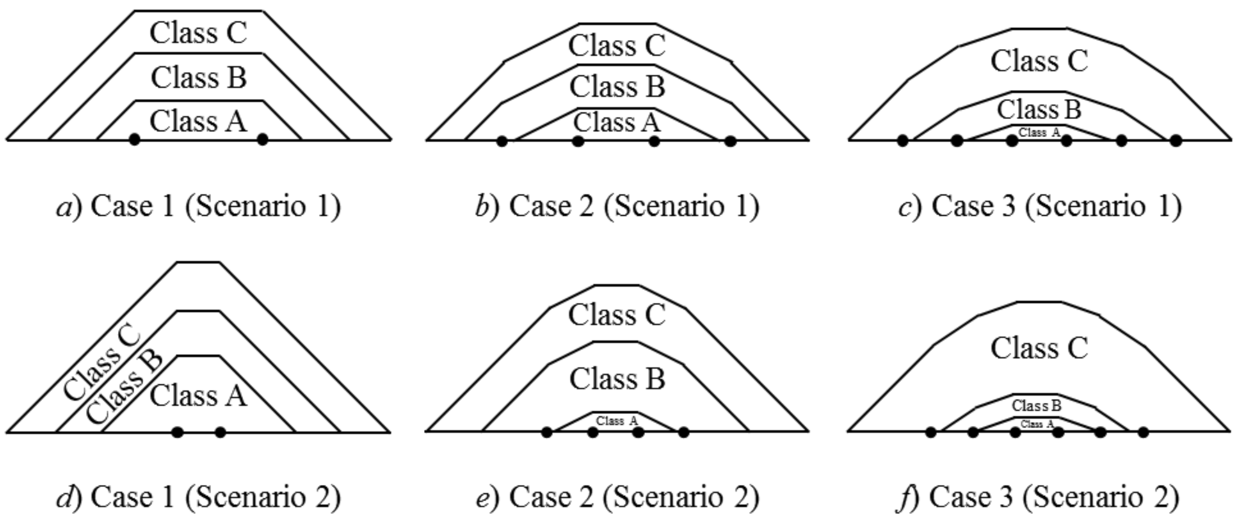

Figure 4.4: Cases for dock-door locations in contour-line-shaped storage regions for each scenario

4.6.1. Dock doors dispersed over an entire wall

In this sub-section, we develop a general formulation of expected single-command distance in a contour-line-shaped warehouse having k dock doors dispersed over an entire wall with a class-based storage policy. Notice the distance between adjacent dock doors, $h_{k,\ k/2} = h_{k,\ (k+1)/2} = [6A_{A \cup B \cup C} / (k^2 + 3k + 2)]^{1/2}$, is a function of the storage area of the entire warehouse ($A_{A \cup B \cup C}$); thus, the storage regions for Class A (A_A) and the union of Classes A and B ($A_{A \cup B}$) do not affect the spacing between adjacent dock doors. Hereafter, for the sake of simplicity, we use h_k instead of $h_{k,\ k/2}$ and $h_{k,\ (k+1)/2}$ in formulations.

Case 1: If all dock doors are covered by Class A, $A_{A \cup B \cup C}\ (k^2 - 3k + 2) / (k^2 + 3k + 2) \leq A_A$. Based on Case 1 of Scenario 1, the space between adjacent dock doors is determined by the union of Classes A, B and C. However, in calculating expected distances for Class A and the union of

Classes A and B, the expected-distance formulations for Case 1 in Scenario 2 apply. To calculate the expected distance for Class A, Equation (4.53) is used, with A replaced by A_A, the area for Class A. To calculate the expected distance for Class B, we employ Equation (4.1) after calculating the expected distance for the union of Classes A and B by replacing A in Equation (4.53) with the area for the union of Classes A and B. After making the appropriate substitutions in Equation (4.1), we obtain

$$E\left[SC_B\right]=\frac{\left(12A_{A\cup B}+h_k{}^2k^2-h_k{}^2\right)^{3/2}-\left(12A_A+h_k{}^2k^2-h_k{}^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B}-A_A\right)}. \tag{4.54}$$

Similarly, adjusting Equation (4.53) for the union of Classes A and B and Equation (4.52) for the union of Classes A, B and C and substituting the resulting equations into Equation (4.2), the expected distance for Class C is

$$\begin{aligned}E\left[SC_C\right]=&\frac{2^{1/2}A_{A\cup B\cup C}{}^{3/2}\left(12k^2+9k-1\right)}{5k\left[3\left(k+2\right)\left(k+1\right)\right]^{1/2}\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}\\&-\left[\frac{\left(12A_{A\cup B}+h_k{}^2k^2-h_k{}^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}-\frac{h_k{}^3\left(3k^4-5k^2+2\right)}{90k\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}\right].\end{aligned} \tag{4.55}$$

With the percentage of the movement for each class, the expected distance for a contour-line-shaped warehouse with Case 1 is

$$
\begin{aligned}
E[SC] = t_A & \left[\frac{\left(12A_A + h_k{}^2k^2 - h_k{}^2\right)^{3/2}}{18\sqrt{3}A_A} - \frac{h_k{}^3\left(3k^4 - 5k^2 + 2\right)}{90k\,A_A} \right] \\
& + t_B \left[\frac{\left(12A_{A\cup B} + h_k{}^2k^2 - h_k{}^2\right)^{3/2} - \left(12A_A + h_k{}^2k^2 - h_k{}^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B} - A_A\right)} \right] \\
& + t_C \left[\frac{2^{1/2} A_{A\cup B\cup C}{}^{3/2}\left(12k^2 + 9k - 1\right)}{5k\left[3(k+2)(k+1)\right]^{1/2}\left(A_{A\cup B\cup C} - A_{A\cup B}\right)} \right] \\
& - t_C \left[\frac{\left(12A_{A\cup B} + h_k{}^2k^2 - h_k{}^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B\cup C} - A_{A\cup B}\right)} - \frac{h_k{}^3\left(3k^4 - 5k^2 + 2\right)}{90k\left(A_{A\cup B\cup C} - A_{A\cup B}\right)} \right]
\end{aligned}
\tag{4.56}
$$

Case 2: If some dock doors are not covered by storage area A, but all dock doors are covered by the union of storage areas A and B, $A_A < A_{A\cup B\cup C}\,(k^2 - 3k + 2) / (k^2 + 3k + 2) \leq A_{A\cup B}$. Determining the number of dock doors covered by Class A, the functional relationship between the distance from/to dock doors to/from contour line *i* and the area enclosed by the contour line *i* is used. Because the resulted equation is a cubic function and solving the equation requires manipulation of complex numbers, Viète's trigonometric solution is used to obtain the number of dock doors. A simpler approach in finding the number of dock doors covered by storage area A is to use a mathematical software package, such as *Mathematica* (2015).

Equation (A.4) and Equation (A.13) relate the area enclosed by a particular contour line and the number of dock doors; with an even number of dock doors, from Equation (A.4), given the area covered by a contour line, we can determine the number of dock doors covered by a contour line. Using a similar approach with Equation (A.13) for an odd number of dock doors, we can determine the number of dock doors covered by a contour line. For Class A, with an even number of dock doors, the number of dock doors covered by Class A is $k_A = 2\lfloor 3^{-1/2} \cos \{\arccos [3^{3/2}\, k\,(3 + 3k + k^2)\, A_A / (2\, A_{A\cup B\cup C})] / 3\} + 0.5\rfloor$; for an odd number of dock doors,

$k_A = 2\lfloor 3^{-1/2} \cos \{\arccos [3^{3/2}\, k\,(3 + 3k\; + k^2)\, A_A\, /\, (2\, A_{A\cup B\cup C})] / 3\}\rfloor + 1$. Therefore, the expected distance traveled for Class A is

$$E\left[SC_A\right] = \frac{135A_A\, h_k k\left(k^2 - k_A{}^2\right) - h_k{}^3\left(9k_A{}^5 - 15k_A{}^3 + 6k_A\right)}{270A_A\, k^2} + \frac{5\left(3k_A\right)^{1/2}\left[12A_A\, k + h_k{}^2 k_A\left(k_A{}^2 - 1\right)\right]^{3/2}}{270A_A\, k^2}. \tag{4.57}$$

Because all dock doors are covered by the union of Classes A and B, adjusting Equation (4.53), the expected distance for the union of Classes A and B is

$$E\left[SC_{A\cup B}\right] = \frac{\left(12A_{A\cup B} + h_k{}^2 k^2 - h_k{}^2\right)^{3/2}}{18\sqrt{3}A_{A\cup B}} - \frac{h_k{}^3\left(3k^4 - 5k^2 + 2\right)}{90kA_{A\cup B}}. \tag{4.58}$$

Therefore, substituting Equations (4.57) and (4.58) into Equation (4.1), the expected distance for Class B is

$$\begin{aligned} E\left[SC_B\right] = {} & \frac{\left(12A_{A\cup B} + h_k{}^2 k^2 - h_k{}^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B} - A_A\right)} - \frac{h_k{}^3\left(3k^4 - 5k^2 + 2\right)}{90k\left(A_{A\cup B} - A_A\right)} \\ & - \frac{135A_A h_k k\left(k^2 - k_A{}^2\right) - h_k{}^3\left(9k_A{}^5 - 15k_A{}^3 + 6k_A\right)}{270k^2\left(A_{A\cup B} - A_A\right)} \\ & - \frac{5\left(3k_A\right)^{1/2}\left[12A_A\, k + h_k{}^2 k_A\left(k_A{}^2 - 1\right)\right]^{3/2}}{270k^2\left(A_{A\cup B} - A_A\right)}. \end{aligned} \tag{4.59}$$

Likewise, adjusting Equation (4.52) for the union of storage areas A, B and C, substituting the adjusted equation and Equation (4.58) into Equation (4.2), the expected distance for Class C is obtained

$$E\left[SC_C\right]=\frac{2^{1/2}\left(A_{A\cup B\cup C}\right)^{3/2}\left(12k^2+9k-1\right)}{5k\left[3\left(k+2\right)\left(k+1\right)\right]^{1/2}\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}$$
$$-\left[\frac{\left(12A_{A\cup B}+h_k{}^2k^2-h_k{}^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}-\frac{h_k{}^3\left(3k^4-5k^2+2\right)}{90k\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}\right]. \tag{4.60}$$

Including the percentage of the movement for each class, the expected distance for a contour-line-shaped warehouse with Case 2 is

$$E\left[SC\right]=t_A\left[\frac{135A_A h_k k\left(k^2-k_A{}^2\right)-h_k{}^3\left(9k_A{}^5-15k_A{}^3+6k_A\right)}{270A_A k^2}\right]$$
$$+t_A\left[\frac{5\left(3k_A\right)^{1/2}\left[12A_A k+h_k{}^2k_A\left(k_A{}^2-1\right)\right]^{3/2}}{270A_A k^2}\right]+t_B\left[\frac{\left(12A_{A\cup B}+h_k{}^2k^2-h_k{}^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B}-A_A\right)}\right]$$
$$-t_B\left[\frac{h_k{}^3\left(3k^4-5k^2+2\right)}{90k\left(A_{A\cup B}-A_A\right)}+\frac{135A_A h_k k\left(k^2-k_A{}^2\right)-h_k{}^3\left(9k_A{}^5-15k_A{}^3+6k_A\right)}{270k^2\left(A_{A\cup B}-A_A\right)}\right] \tag{4.61}$$
$$-t_B\left[\frac{5\left(3k_A\right)^{1/2}\left[12A_A k+h_k{}^2k_A\left(k_A{}^2-1\right)\right]^{3/2}}{270k^2\left(A_{A\cup B}-A_A\right)}\right]-t_C\left[\frac{\left(12A_{A\cup B}+h_k{}^2k^2-h_k{}^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}-\right]$$
$$+t_C\left[\frac{2^{1/2}\left(A_{A\cup B\cup C}\right)^{3/2}\left(12k^2+9k-1\right)}{5k\left[3\left(k+2\right)\left(k+1\right)\right]^{1/2}\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}+\frac{h_k{}^3\left(3k^4-5k^2+2\right)}{90k\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}\right].$$

Case 3: If some dock doors are not covered by the union of storage areas A and B, $A_{A\cup B} < A_{A\cup B\cup C}\,(k^2 - 3k + 2) / (k^2 + 3k + 2)$. The equations used in Case 2 to calculate the number of dock doors covered by the storage area A can be used for Class A with Case 3. Similarly, following the same steps for Case 2, the number of dock doors covered by the union of Classes A and B is $k_{A\cup B} = 2\lfloor 3^{-1/2} \cos \{\arccos [3^{3/2}\, k\, (3 + 3k + k^2)\, A_{A\cup B} / (2\, A_{A\cup B\cup C})] / 3\} + 0.5\rfloor$ for an even number of dock doors and $k_{A\cup B} = 2\lfloor 3^{-1/2} \cos \{\arccos [3^{3/2}\, k\, (3 + 3k + k^2)\, A_{A\cup B} / (2\, A_{A\cup B\cup C})] / 3\}\rfloor + 1$ for an odd number of dock doors. Therefore, the expected distance traveled for dock doors covered by Class A is

$$E[SC_A] = \frac{135A_A h_k k\left(k^2 - k_A{}^2\right) - h_k{}^3\left(9k_A{}^5 - 15k_A{}^3 + 6k_A\right)}{270A_A k^2} + \frac{5\left(3k_A\right)^{1/2}\left[12A_A k + h_k{}^2 k_A\left(k_A{}^2 - 1\right)\right]^{3/2}}{270A_A k^2}. \tag{4.62}$$

Because not all dock doors are covered by the union of Classes A and B for Case 3, adjusting Equation (4.62), the expected distance traveled for dock doors covered by the union of Classes A and B is obtained

$$E[SC_{A\cup B}] = \frac{135A_{A\cup B} h_k k\left(k^2 - k_{A\cup B}{}^2\right) - h_k{}^3\left(9k_{A\cup B}{}^5 - 15k_{A\cup B}{}^3 + 6k_{A\cup B}\right)}{270A_{A\cup B} k^2} + \frac{5\left(3k_{A\cup B}\right)^{1/2}\left[12A_{A\cup B} k + h_k{}^2 k_{A\cup B}\left(k_{A\cup B}{}^2 - 1\right)\right]^{3/2}}{270A_{A\cup B} k^2}. \tag{4.63}$$

Therefore, substituting Equations (4.62) and (4.63) into Equation (4.1), the expected distance for Class B is

$$\begin{aligned} E[SC_B] = {} & \frac{135A_{A\cup B} h_k k\left(k^2 - k_{A\cup B}{}^2\right) - h_k{}^3\left(9k_{A\cup B}{}^5 - 15k_{A\cup B}{}^3 + 6k_{A\cup B}\right)}{270k^2\left(A_{A\cup B} - A_A\right)} \\ & + \frac{5\left(3k_{A\cup B}\right)^{1/2}\left[12A_{A\cup B} k + h_k{}^2 k_{A\cup B}\left(k_{A\cup B}{}^2 - 1\right)\right]^{3/2}}{270k^2\left(A_{A\cup B} - A_A\right)} - \frac{135A_A h_k k\left(k^2 - k_A{}^2\right)}{270k^2\left(A_{A\cup B} - A_A\right)} \\ & - \frac{h_k{}^3\left(9k_A{}^5 - 15k_A{}^3 + 6k_A\right) + 5\left(3k_A\right)^{1/2}\left[12A_A k + h_k{}^2 k_A\left(k_A{}^2 - 1\right)\right]^{3/2}}{270k^2\left(A_{A\cup B} - A_A\right)}. \end{aligned} \tag{4.64}$$

As with Case 2, adjusting Equation (4.52) for the union of storage areas A, B and C, substituting the adjusted equation and Equation (4.63) into Equation (4.2), the expected distance for Class C is obtained

$$E\left[SC_C\right]=\frac{2^{1/2}\left(A_{A\cup B\cup C}\right)^{3/2}\left(12k^2+9k-1\right)}{5k\left[3\left(k+2\right)\left(k+1\right)\right]^{1/2}\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}-\frac{135A_{A\cup B}\,h_k k\left(k^2-k_{A\cup B}{}^2\right)}{270\,k^2\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}$$
$$-\frac{5\left(3k_{A\cup B}\right)^{1/2}\left[12A_{A\cup B}\,k+h_k{}^2k_{A\cup B}\left(k_{A\cup B}{}^2-1\right)\right]^{3/2}-h_k{}^3\left(9k_{A\cup B}{}^5-15k_{A\cup B}{}^3+6k_{A\cup B}\right)}{270\,k^2\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}. \tag{4.65}$$

Because of the length of the formula for the expected distance with Case 3, we do not include the overall formula. Multiplying the percentage of the movement of each storage area (t_A, t_B and t_C) by the expected distance for the corresponding storage area (E [SC_A], E [SC_B] and E [SC_C]) the expected distance for a contour-line-shaped warehouse can be obtained.

4.6.2. Centrally located dock doors with δ separation between adjacent dock doors

In this sub-section, we use a similar process to that employed in the previous sub-section. In contrast to Scenario 1, because the distance between dock doors is specified in Scenario 2, all formulations are based on Equation (4.53).

Case 1: If all dock doors are covered by Class A, $(k^2 - 3k + 2)\,\delta^2 / 6 \le A_A$. Adjusting Equation (4.53) for Class A and the union of Classes A and B and substituting adjusted equations into Equation (4.1), the expected distance for Class B is

$$E\left[SC_B\right]=\frac{\left(12A_{A\cup B}+\delta^2k^2-\delta^2\right)^{3/2}-\left(12A_A+\delta^2k^2-\delta^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B}-A_A\right)}. \tag{4.66}$$

Similarly, adjusting Equation (4.53) for the union of Classes A and B and the union of Classes A, B and C and substituting adjusted equations into Equation (4.2), the expected distance for Class C is

$$E\left[SC_C\right]=\frac{\left(12A_{A\cup B\cup C}+\delta^2k^2-\delta^2\right)^{3/2}-\left(12A_{A\cup B}+\delta^2k^2-\delta^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}. \tag{4.67}$$

Therefore, the expected distance for a contour-line-shaped warehouse with Case 1 is

$$E[SC] = t_A \left[\frac{\left(12A_A + \delta^2 k^2 - \delta^2\right)^{3/2}}{18\sqrt{3}A_A} - \frac{\delta^3\left(3k^4 - 5k^2 + 2\right)}{90kA_A} \right] + t_B \left[\frac{\left(12A_{A\cup B} + \delta^2 k^2 - \delta^2\right)^{3/2} - \left(12A_A + \delta^2 k^2 - \delta^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B} - A_A\right)} \right] + t_C \left[\frac{\left(12A_{A\cup B\cup C} + \delta^2 k^2 - \delta^2\right)^{3/2} - \left(12A_{A\cup B} + \delta^2 k^2 - \delta^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B\cup C} - A_{A\cup B}\right)} \right]. \tag{4.68}$$

Case 2: If some dock doors are not covered by storage area A, but all dock doors are covered by the union of storage areas A and B, $A_A < (k^2 - 3k + 2)\,\delta^2 / 6 \leq A_{A\cup B}$. As with Scenario 1, the number of dock doors covered by Class A is determined by using Viète's trigonometric solution, $k_A = 2\lfloor 3^{-1/2} \cos\{\arccos[3^{5/2}\, k\, A_A / \delta^2] / 3\} + 0.5\rfloor$ for an even number of dock doors and $k_A = 2\lfloor 3^{-1/2} \cos\{\arccos[3^{5/2}\, k\, A_A / \delta^2)] / 3\}\rfloor + 1$ for an odd number of dock doors. Therefore, the expected distance for Class A is obtained

$$E[SC_A] = \frac{135A_A\,\delta\, k\left(k^2 - k_A^{\;2}\right) - \delta^3\left(9k_A^{\;5} - 15k_A^{\;3} + 6k_A\right)}{270A_A\, k^2} + \frac{5\left(3k_A\right)^{1/2}\left[12A_A\, k + \delta^2 k_A\left(k_A^{\;2} - 1\right)\right]^{3/2}}{270A_A\, k^2}. \tag{4.69}$$

Adjusting Equation (4.53) for the union of Classes A and B, and substituting the resulted equation and Equation (4.69) into Equation (4.1), the expected distance for Class B is

$$E[SC_B] = \frac{A_{A\cup B}}{A_{A\cup B} - A_A}\left\{ \frac{\left(12A_{A\cup B} + \delta^2 k^2 - \delta^2\right)^{3/2}}{18\sqrt{3}A_{A\cup B}} - \frac{\delta^3\left(3k^4 - 5k^2 + 2\right)}{90kA_{A\cup B}} \right\} - \frac{135A_A\,\delta\, k\left(k^2 - k_A^{\;2}\right) - \delta^3\left(9k_A^{\;5} - 15k_A^{\;3} + 6k_A\right)}{270k^2\left(A_{A\cup B} - A_A\right)} - \frac{5\left(3k_A\right)^{1/2}\left[12A_A\, k + \delta^2 k_A\left(k_A^{\;2} - 1\right)\right]^{3/2}}{270k^2\left(A_{A\cup B} - A_A\right)}. \tag{4.70}$$

Likewise, adjusting Equation (4.53) for the union of Classes A and B, and the union of Classes A, B and C, the expected distance for Class C is

$$E\left[SC_C\right]=\frac{\left(12A_{A\cup B\cup C}+\delta^2k^2-\delta^2\right)^{3/2}-\left(12A_{A\cup B}+\delta^2k^2-\delta^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}. \tag{4.71}$$

Therefore, the expected distance traveled for a contour-line-shaped warehouse with Case 2 is

$$\begin{aligned}E\left[SC\right]=\;&t_A\left[\frac{135A_A\,\delta\,k\left(k^2-k_A{}^2\right)-\delta^3\left(9k_A{}^5-15k_A{}^3+6k_A\right)}{270A_A\,k^2}\right]\\&+t_A\left[\frac{5\left(3k_A\right)^{1/2}\left[12A_A\,k+\delta^2k_A\left(k_A{}^2-1\right)\right]^{3/2}}{270A_A\,k^2}\right]+t_B\left[\frac{\left(12A_{A\cup B}+\delta^2k^2-\delta^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B}-A_A\right)}\right]\\&-t_B\left[\frac{\delta^3\left(3k^4-5k^2+2\right)}{90k\left(A_{A\cup B}-A_A\right)}+\frac{135A_A\delta\,k\left(k^2-k_A{}^2\right)}{270k^2\left(A_{A\cup B}-A_A\right)}\right]\\&+t_B\left[\frac{\delta^3\left(9k_A{}^5-15k_A{}^3+6k_A\right)-5\left(3k_A\right)^{1/2}\left[12A_A\,k+\delta^2k_A\left(k_A{}^2-1\right)\right]^{3/2}}{270k^2\left(A_{A\cup B}-A_A\right)}\right]\\&+t_C\left[\frac{\left(12A_{A\cup B\cup C}+\delta^2k^2-\delta^2\right)^{3/2}-\left(12A_{A\cup B}+\delta^2k^2-\delta^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}\right].\end{aligned} \tag{4.72}$$

Case 3: If some dock doors are not covered by the union of storage areas A and B, $A_{A\cup B} < (k^2 - 3k + 2)\,\delta^2 / 6$. The equations used in Case 2 to calculate the number of dock doors covered by the storage area A can be used for Class A with Case 3. Similarly, following the same steps for Case 2, the number of dock doors covered by the union of Classes A and B is $k_{A\cup B} = 2\lfloor 3^{-1/2}\cos\{\arccos[3^{5/2}\,k\,A_{A\cup B} / \delta^2] / 3\} + 0.5\rfloor$ for an even number of dock doors and $k_{A\cup B} = 2\lfloor 3^{-1/2}\cos\{\arccos[3^{5/2}\,k\,A_{A\cup B} / \delta^2)] / 3\}\rfloor + 1$ for an odd number of dock doors. Therefore, the expected distance traveled for dock doors covered by Class A is

$$E\left[SC_A\right]=\frac{135A_A\,\delta\,k\left(k^2-k_A{}^2\right)-\delta^3\left(9k_A{}^5-15k_A{}^3+6k_A\right)}{270A_A\,k^2}+\frac{5\left(3k_A\right)^{1/2}\left[12A_A\,k+\delta^2k_A\left(k_A{}^2-1\right)\right]^{3/2}}{270A_A\,k^2}. \tag{4.73}$$

Because not all dock doors are covered by the union of Classes A and B, adjusting Equation (4.73), the expected distance traveled for dock doors covered by the union of Classes A and B is

$$E\left[SC_{A\cup B}\right]=\frac{135A_{A\cup B}\,\delta\,k\left(k^2-k_{A\cup B}{}^2\right)-\delta^3\left(9k_{A\cup B}{}^5-15k_{A\cup B}{}^3+6k_{A\cup B}\right)}{270A_{A\cup B}\,k^2}+\frac{5\left(3k_{A\cup B}\right)^{1/2}\left[12A_{A\cup B}\,k+\delta^2k_{A\cup B}\left(k_{A\cup B}{}^2-1\right)\right]^{3/2}}{270A_{A\cup B}\,k^2}. \tag{4.74}$$

Therefore, substituting Equations (4.73) and (4.74) into Equation (4.1), the expected distance for Class B is

$$E\left[SC_B\right]=\frac{135A_{A\cup B}\,\delta\,k\left(k^2-k_{A\cup B}{}^2\right)-\delta^3\left(9k_{A\cup B}{}^5-15k_{A\cup B}{}^3+6k_{A\cup B}\right)}{270\,k^2\left(A_{A\cup B}-A_A\right)}+\frac{5\left(3k_{A\cup B}\right)^{1/2}\left[12A_{A\cup B}\,k+\delta^2k_{A\cup B}\left(k_{A\cup B}{}^2-1\right)\right]^{3/2}}{270\,k^2\left(A_{A\cup B}-A_A\right)}-\frac{135A_A\delta\,k\left(k^2-k_A{}^2\right)}{270k^2\left(A_{A\cup B}-A_A\right)}+\frac{\delta^3\left(9k_A{}^5-15k_A{}^3+6k_A\right)-5\left(3k_A\right)^{1/2}\left[12A_A\,k+\delta^2k_A\left(k_A{}^2-1\right)\right]^{3/2}}{270k^2\left(A_{A\cup B}-A_A\right)}. \tag{4.75}$$

Adjusting Equation (4.53) and substituting it and Equation (4.74) into Equation (4.2), the expected distance for Class C is

$$E\left[SC_C\right]=\frac{\left(12A_{A\cup B\cup C}+\delta^2k^2-\delta^2\right)^{3/2}}{18\sqrt{3}\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}-\frac{\delta^3\left(3k^4-5k^2+2\right)}{90k\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}-\frac{135A_{A\cup B}\,\delta\,k\left(k^2-k_{A\cup B}{}^2\right)-\delta^3\left(9k_{A\cup B}{}^5-15k_{A\cup B}{}^3+6k_{A\cup B}\right)}{270\,k^2\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}-\frac{5\left(3k_{A\cup B}\right)^{1/2}\left[12A_{A\cup B}\,k+\delta^2k_{A\cup B}\left(k_{A\cup B}{}^2-1\right)\right]^{3/2}}{270\,k^2\left(A_{A\cup B\cup C}-A_{A\cup B}\right)}. \tag{4.76}$$

For the same reason as Case 2, we do not include the overall formula for the expected distance for Case 3. As stated, the expected distance for a contour-line-shaped warehouse can be obtained.by multiplying the percentage of the movement of each storage area (t_A, t_B and t_C) by the expected distance for the corresponding storage area (E [SC_A], E [SC_B] and E [SC_C]).

4.7. The penalty of forcing a storage region to be rectangle-shaped

In this section, we extend results contained in the two previous sections. Specifically, we are concerned with comparing the expected distance for a rectangle-shaped storage region with the expected distance for a corresponding contour-line-shaped storage region. In contrast to earlier sections, we limit our attention, initially, to one storage region. To calculate the penalty, we subtract the expected distance for a contour-line-shaped warehouse from that for a rectangle-shaped warehouse, and divide the result by the expected distance for a contour-line-shaped warehouse ({E [$SC_{\text{Rectangle}}$] – E [$SC_{\text{Contour-line}}$] / E [$SC_{\text{Contour-line}}$]}.

Let ξ_i denote the penalty of requiring a storage region to be rectangle-shaped for Scenario i (i = 1 and 2). Using Equations (4.3) and (4.52), ξ_1 is given

$$\xi_1 = \left[\frac{5k\left[3D(k+1)+W(2k+1)\right]\left(k^2+3k+2\right)^{1/2}}{(6A)^{1/2}(k+1)\left(12k^2+9k-1\right)} - 1 \right], \tag{4.77}$$

and using Equations (4.4) and (4.53), ξ_2 is obtained

$$\xi_2 = \left[\frac{45A\,k\left[3W^2+6W\,D\left(k^2-1\right)\delta^2\right]}{5\sqrt{3}\,k\,W\left(12A+k^2\delta^2-\delta^2\right)-3W\,\delta^3\left(3k^4-5k^2+2\right)} - 1 \right]. \tag{4.78}$$

For the case of a single-dock-door warehouse with the dock door centrally located along a wall, using either Equation (4.77) or Equation (4.78), an optimally shaped storage region (W = $(2A)^{1/2}$ and

$D = (A / 2)^{1/2}$) yields a penalty of 0.0607 or 6.07% when compared with a triangularly shaped storage region. The same result was obtained by Francis (1967a) for a single dock door. Extending his study, we obtain the penalty of requiring the storage region to be rectangle-shaped for any number of dock doors by using Equation (4.77) for Scenario 1 and Equation (4.78) for Scenario 2.

4.8. Computational Results

This section presents computational results from our research by solving the nonlinear-programming optimization problem provided in Section 4.5 and by applying formulations developed in Section 4.6. At the beginning, we provide the penalty of requiring a single-class warehouse of 250,000 ft^2 to be rectangle-shaped, instead of contour-line-shaped. Notice the distance between adjacent dock doors cannot be smaller than 12 ft for Scenario 1 ($\omega + \psi \geq 12$ ft); whereas it is a specified value of 12 ft for Scenario 2 ($\delta = 12$ ft). In Subsection 4.8.2, the penalty for our initial settings with three classes are presented and explained in detail. We assume the areas of the three storage regions for class-based storage are $A_A = 50{,}000$ ft^2, $A_B = 75{,}000$ ft^2, and $A_C = 125{,}000$ ft^2. We also assume the following throughput rates apply for each product class: $T_A = 300$ roundtrips / hour, $T_B = 130$ roundtrips / hour and $T_C = 70$ roundtrips / hour. Finally, we investigate the effect of ABC curve's skewness on the penalty. Tabulated computational results are provided in the Appendix.

4.8.1. Penalty calculations for a single-class warehouse

In this section, the expected single-command distance for a rectangle-shaped storage region is compared to that for a contour-line-shaped storage region. As illustrated in Figure 4.5, increasing the number of dock doors decreases the penalty when the width constraint is satisfied for both scenarios although the penalty is negative-valued for Scenario 1 (except for a single-dock-door). Significantly, the rectangle-shaped storage region performs better than the corresponding contour-line-shaped warehouse for Scenario 1 because dock-door locations change depending on the width of the overall warehouse. (This result demonstrates the Neyman Pearson Lemma requirement for the locations of dock doors to be fixed). Our results indicate the width of the rectangle-shaped warehouse for both scenarios is narrower than that for the contour-line-shaped warehouse.

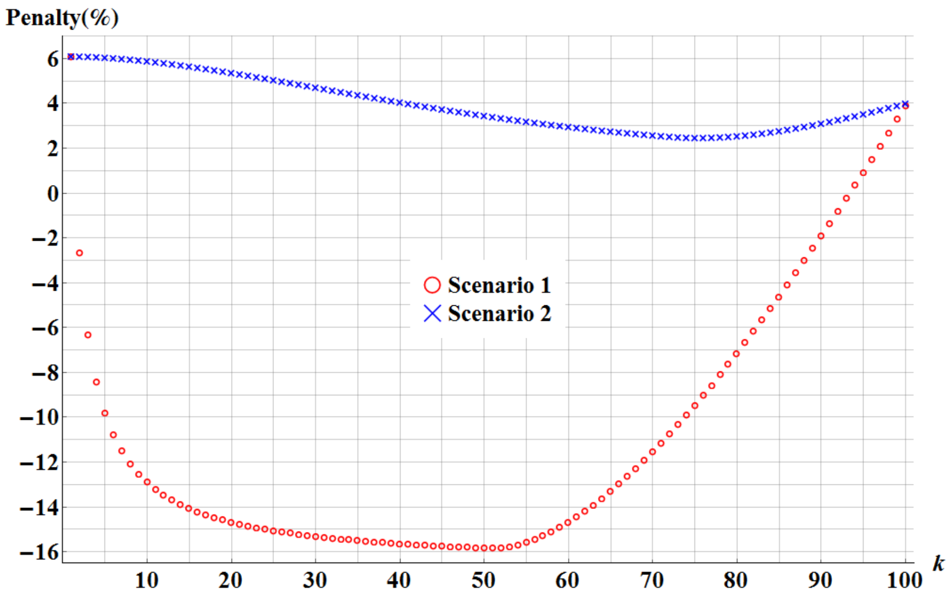

Figure 4.5: The penalty for Scenarios 1 and 2

When the width constraint is active for the rectangle-shaped storage region with both scenarios, the penalty increases with an increasing number of dock doors because it forces the storage region to be wider than it would be if it were optimally shaped.

As with Chapter 2, the optimal shape factor for Scenario 1 is between 1.50 and 2.00; also, increasing the number of dock doors decreases the optimal shape factor. For Scenario 2, the optimal shape factor is equal to or greater than 2.00; increasing the number of dock doors increases the optimal shape factor. In our example, the width constraint is active for 73 and 51 dock doors with Scenarios 1 and 2, respectively.

4.8.2. Penalty calculations for a warehouse having multiple classes

In this section, we extend the previous subsection by considering a class-based storage policy. From research results in Chapter 2, when dock-door locations are determined with Scenario 1 for the union of storage areas for Classes A, B and C, the optimal shape factor will be less than 2.0. However, once the dock-door locations are determined, then the calculation of expected distances for Class A and the union of Classes A and B will be based on Scenario 2; from Chapter 2, the optimal shape factor for Scenario 2 will be greater than 2.0. Therefore, depending on the values of the storage areas, the desired width of the storage areas for Class A and the union of Classes A and B might not be feasible, because the overall warehouse width was established by the union of Classes A, B and C.

Figure 4.6 displays the optimal width (left) and the optimal shape factor (right) of each rectangle-shaped storage region with Scenario 1 under a class-based storage policy. For a single dock door, the optimal shape factor for all storage regions are equal to 2.00, although each storage region has a different width. For our example, the width of a storage region for the union of Classes A and B is constrained by the width of the overall warehouse when k = 2 thru k = 51.

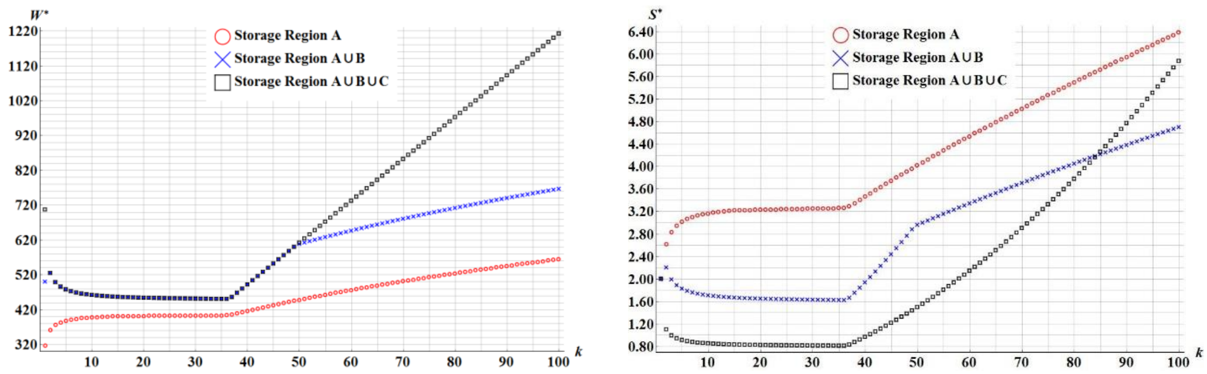

Figure 4.6: The optimal width and optimal shape factor for each storage region with Scenario 1 for a rectangle-shaped warehouse.

The width constraint for the overall warehouse is active for k = 37. Thereafter, increasing the number of dock doors will increase the width of the overall warehouse. After locating 51 dock doors, the width of storage region for the union of Classes A and B will not be constrained because the width of the overall warehouse will be large. Therefore, the special case of the Neyman-Pearson Lemma to calculate expected distance does not apply to Scenario 1.

As depicted in Figure 4.7, with k = 2, the width of the warehouse is less than the depth of the warehouse and the storage area for Class C is located behind the storage area for the union of the storage areas for Classes A and B.

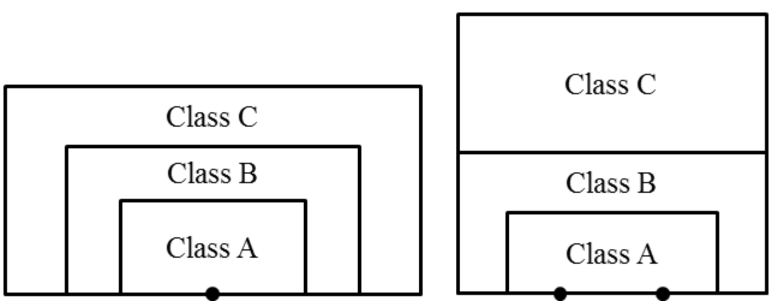

Figure 4.7: Rectangle-shaped warehouse design with 1 dock door (left) vs. 2 dock doors (right) for Scenario 1

An interesting observation is the optimal shape factor for the overall warehouse is smaller than 1 when k = 3 thru k = 40. The smallest value of the optimal shape factor equals 0.81 with 36 dock doors.

Similarly, Figure 4.8 illustrates the optimal width (left) and the optimal shape factor (right) of each rectangle-shaped storage region with Scenario 2 and a class-based storage policy. The optimal shape factor for each storage region is equal to or greater than 2.00.

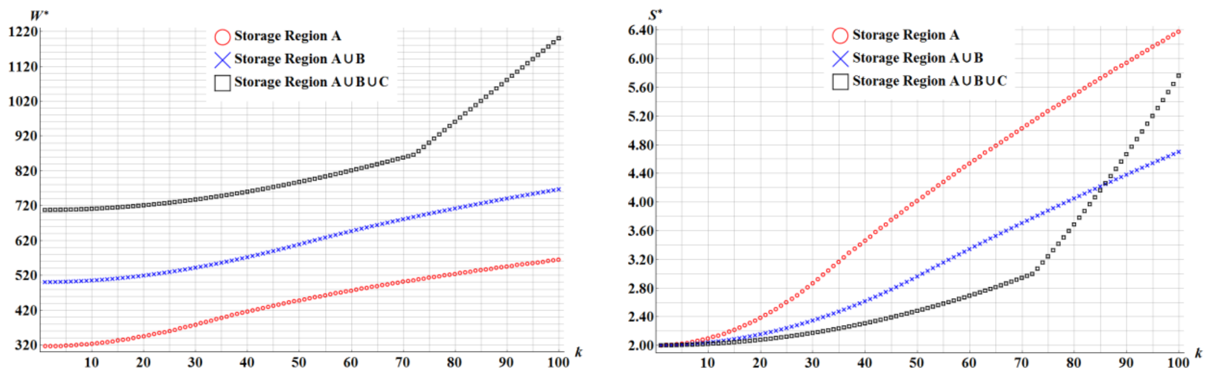

Figure 4.8: The optimal width and optimal shape factor for each storage region with Scenario 1

Because the width the overall warehouse is large, it is not a constraint for the storage region of the union of Classes A and B. Increasing the number of dock doors will always increase the optimal shape factor for each storage region.

Figure 4.9 demonstrates the penalty of requiring the storage regions to be rectangle-shaped. Similar to a single-class warehouse, increasing the number of dock doors decreases the penalty when the width constraint is satisfied. Except for a single-dock-door, the penalty is negative-valued for Scenario 1. As with a single-class warehouse, the width of the rectangle-shaped warehouse for both scenarios is narrower than that for the contour-line-shaped warehouse. With given parameter values, the penalty increases when the number of dock doors exceeds 37 for Scenario 1 and exceeds 81 for Scenario 2. When the width constraint is active, the penalty dramatically increases with an increasing number of dock doors for Scenario 1; however, for

Scenario 2, the penalty increases slightly with an increasing number of dock doors. As with one storage region, the warehouse with rectangle-shaped storage regions outperforms the corresponding warehouse with contour-line-shaped storage regions because of the flexibility of dock-door locations in Scenario 1.

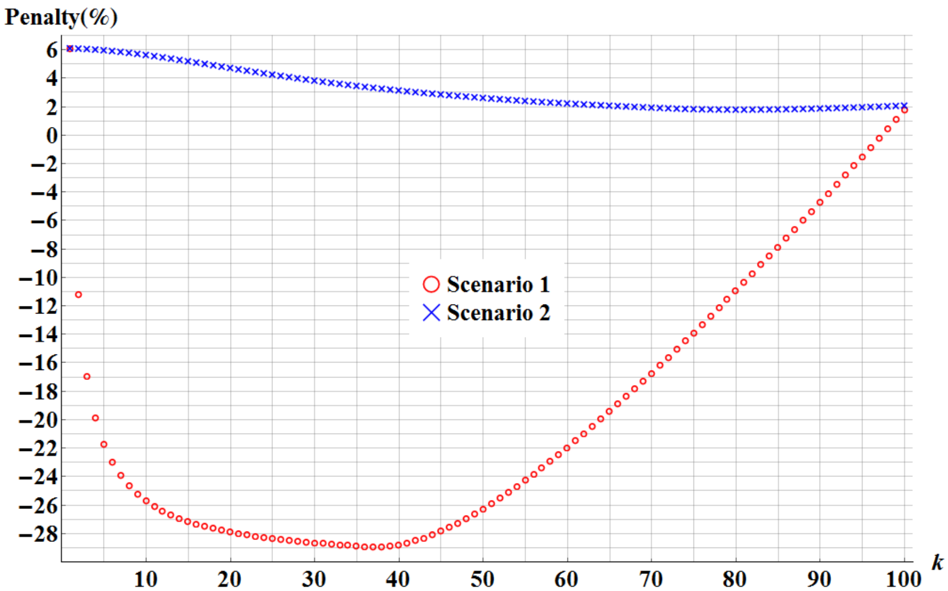

Figure 4.9: The penalty for Scenarios 1 and 2 under class-based storage policy

4.8.3. Penalty calculations for different shapes of the ABC curve

How dependent on the shape of the ABC curve is the penalty resulting from requiring a rectangle shape for the warehouse? To address the question, we calculate the skewness of the ABC curve using Bender's formulation (Bender, 1981), $Y = (1 + \beta) X / (\beta + X)$, where the β represents the skewness of the curve. The value of β must be calculated in such a way that the curve fits data points. Given the percentage of overall storage area within region i (X_i) and the percentage of overall activity in region i (Y_i), the value of parameter β is determined by using the least squares method, $\sum Y_i - (1 - \beta) \sum [X_i / (\beta - X_i)] = 0$. For a detailed procedure, see Bender (1981). Specifically, Table 4.1 contains values for β for 15 ABC curves.

Table 4.1: β parameter values and ABC curves

		Roundtrips (%)			Storage (%)		
Curve	**β parameter**	**Class A**	**Class B**	**Class C**	**Class A**	**Class B**	**Class C**
1	0.0001	0.9996	0.0003	0.0001	0.2	0.3	0.5
2	0.0005	0.9980	0.0015	0.0005	0.2	0.3	0.5
3	0.0010	0.9960	0.0030	0.0010	0.2	0.3	0.5
4	0.0050	0.9805	0.0146	0.0050	0.2	0.3	0.5
5	0.0100	0.9619	0.0283	0.0098	0.2	0.3	0.5
6	0.0500	0.8400	0.1145	0.0455	0.2	0.3	0.5
7	0.1000	0.7333	0.1833	0.0833	0.2	0.3	0.5
8	0.1988	0.6000	0.2600	0.1400	0.2	0.3	0.5
9	0.5000	0.4286	0.3214	0.2500	0.2	0.3	0.5
10	1.0000	0.3333	0.3333	0.3333	0.2	0.3	0.5
11	2.0000	0.2727	0.3273	0.4000	0.2	0.3	0.5
12	3.0000	0.2500	0.3214	0.4286	0.2	0.3	0.5
13	4.0000	0.2381	0.3175	0.4444	0.2	0.3	0.5
14	5.0000	0.2308	0.3147	0.4545	0.2	0.3	0.5
15	6.0000	0.2258	0.3127	0.4615	0.2	0.3	0.5
16	7.0000	0.2222	0.3111	0.4667	0.2	0.3	0.5

Based on the work of Francis (1967a), we know expected distance is minimized by ranking classes based on the ratio of throughput or number of roundtrips to the amount of storage space required for each class; it is not based on throughput ranking, alone. Table 4.2 includes several β parameter values and associated minimum, maximum and average penalty values for both scenarios. The average penalty is the numerical average of the penalty for the number of dock doors ranging from 1 to 100 with Scenario 2.

Based on the computational results, for Scenario 1, it appears requiring a storage region to be rectangle-shaped, rather than contour-line-shaped, results in a penalty ranging from -48.53% to 6.07% when the number of dock doors ranges from 1 to 100. However, increasing the value of parameter β decreases the minimum penalty percentage because the effect of Class C on the expected distance traveled for rectangle-shaped warehouse heavily increases.

Table 4.2: Minimum, maximum and average penalty values for 15 ABC curves

Curve	*β* parameter	Scenario 1 Min	Scenario 1 Max	Scenario 1 Avg	Scenario 2 Min	Scenario 2 Max	Scenario 2 Avg
1	0.0001	-48.53%	6.07%	-30.12%	0.87%	6.07%	2.34%
2	0.0005	-48.42%	6.07%	-30.07%	0.87%	6.07%	2.34%
3	0.0010	-48.28%	6.07%	-30.01%	0.88%	6.07%	2.35%
4	0.0050	-47.22%	6.07%	-29.51%	0.92%	6.07%	2.38%
5	0.0100	-46.01%	6.07%	-28.93%	0.97%	6.07%	2.43%
6	0.0500	-39.07%	6.07%	-25.39%	1.31%	6.07%	2.68%
7	0.1000	-34.07%	6.07%	-22.66%	1.53%	6.07%	2.89%
8	0.5000	-22.94%	6.07%	-15.85%	2.04%	6.07%	3.45%
9	1.0000	-20.01%	6.07%	-13.84%	2.20%	6.07%	3.63%
10	2.0000	-18.12%	6.07%	-12.54%	2.30%	6.07%	3.76%
11	3.0000	-17.41%	6.07%	-12.04%	2.34%	6.07%	3.80%
12	4.0000	-17.04%	6.07%	-11.78%	2.36%	6.07%	3.83%
13	5.0000	-16.81%	6.07%	-11.62%	2.37%	6.07%	3.85%
14	6.0000	-16.66%	6.07%	-11.52%	2.38%	6.07%	3.86%
15	7.0000	-16.55%	6.07%	-11.44%	2.39%	6.07%	3.86%

For Scenario 2, the penalty ranges from a high of 6.07% to a low of 0.87% when the number of dock doors ranges from 1 to 100. When the ABC curve is almost linear, the minimum penalty percentage is greater because the effect of Class C on the expected distance traveled is greater. The maximum penalty of approximately 6.07 percent is not significantly affected by the number of dock doors or skewness of the ABC curve for both scenarios.

4.9. Conclusion

Designing a unit-load warehouse is a challenging problem due to a large number of feasible warehouse designs and numerous design parameters. After developing expected single-command distance formulas, we studied the performance of a unit-load warehouse having multiple dock doors when a storage region or storage regions can be either rectangle-shaped or contour-line-shaped. Although designing a contour-line-shaped warehouse might be impractical and very expensive to construct, results obtained from the formulas we developed can be used as lower bounds for the expected single-command distance. Therefore, the penalty incurred by requiring the warehouse configuration to be the most common configuration (rectangular) can be calculated for various number and locations of dock doors.

For a single dock door, the expected-distance penalty for a rectangle-shaped warehouse is about 6.07% greater than the corresponding contour-line-shaped warehouse. Interestingly, for multiple dock doors, the rectangle-shaped warehouse outperforms the corresponding contour-line-shaped warehouse when dock doors are uniformly dispersed over the entire wall of the warehouse because the distance between adjacent dock doors is not the same for a contour-line-shaped warehouse and a rectangle-shaped warehouse (e.g. see Figure 4.10 for the case of three dock doors). Notice this is true when result depend on the parameters of the ABC curve. The penalty ranges from -48.53% to 6.07% depending the number of dock doors and skewness of the ABC curve. When dock doors are dispersed over an entire wall (Scenario 1) and the number of dock doors ranges from 1 to 100, the optimal shape factor for the overall rectangle-shaped warehouse ranges from 2.00 to 0.81. Similarly, the optimal shape factor for the union of storage regions A and B ranges from 2.20 to 1.62. However, the optimal shape factor for storage region A is equal to or greater than 2.00 for any number of dock doors.

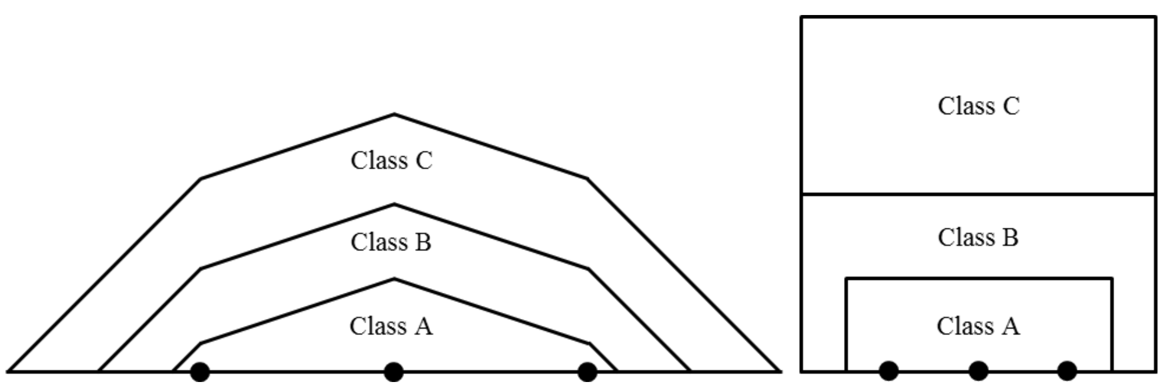

Figure 4.10: The distance between adjacent dock doors with Scenario 1 when $k = 3$

When dock doors are located with a fixed distance between adjacent dock doors (Scenario 2), the penalty ranges from 6.07 % to 0.87% as the number of dock doors increases, regardless of the skewness of the ABC curve or storage policy. The maximum penalty of requiring a rectangle-shaped warehouse is no greater than approximately 6.07 percent regardless of the locations of dock doors. When skewness of the ABC curve increases, the minimum percentage decreases because the effect of Class C on the expected distance traveled decreases. However, the optimal shape factor for all storage regions are equal to or greater than 2.00 for any number of dock doors.

This research can be extended to incorporate *n* classes. Using a similar approach to that employed in Section 4, the expected distance for the overall warehouse can be calculated by using formulas derived for three classes of storage. In addition, relaxing the nesting requirement for storage regions, the penalty for different shapes (not nested rectangles) can be calculated. Further, as noted in Section 5, consideration of a different width constraint for Scenario 1 might yield interesting results, particularly regarding the penalty of requiring a rectangle-shaped storage region. Finally, constructing contour lines in an existing warehouse (requiring the overall storage region to be rectangle-shaped with contour-line-shaped storage regions inside the rectangle-shaped storage region) might also prove beneficial for designers.

Bibliography

AMPL (2013) AMPL: A Modeling Language for Mathematical Programming. AMPL Optimization, Inc.

Bender, P. S. (1981) Mathematical modeling of the 20/80 rule: Theory and practice. *Journal of Business Logistics*, 2(2), 139-157.

Berry, J. (1968) Elements of warehouse layout. *The International Journal of Production Research*, 7(2), 105-121.

Clark, K. A. and Meller, R. D. (2013) Incorporating vertical travel into non-traditional cross-aisles for unit-load warehouse designs. *IIE Transactions*, 45(12), 1322-1331.

Cardona, L. F., Rivera, L. and Martínez, H. J. (2012) Analytical study of the fishbone warehouse layout. *International Journal of Logistics Research and Applications*, 15(6), 365-388.

Cormier, G. and Gunn, E. (1992) A review of warehouse Formulations. *European Journal of Operational Research*, 58, 3-13.

Couenne (2006) *Couenne: Convex Over and Under Envelopes for Nonlinear Estimation, an exact solver for nonconvex MINLPs*, IBM and Carnegie Mellon University, available at https://projects.coin-or.org/Couenne/, accessed on May 04, 2016.

de Koster, R., Le-Duc, T. and Roodbergen, K. J. (2007) Design and control of warehouse order picking: a literature review. *European Journal of Operational Research*, 182(2), 481-501.

Francis, R. L. (1967a) On some problems of rectangular warehouse design and layout. *The Journal of Industrial Engineering*, 18(10), 595-604.

Francis, R. L. (1967b) Sufficient conditions for some optimum property facility designs. *Operations Research*, 15(3), 448-466.

Francis, R. L. and White, J. A. (1974) *Facility Layout and Location: An Analytic Approach*. Prentice-Hall, New Jersey, NJ.

Francis, R.L., McGinnis Jr., L.F. and White, J.A. (1992) *Facility Layout and Location: An Analytical Approach*, second edition, Prentice Hall, Englewood Cliffs, NJ.

Gálvez, O.D. and Ting, C.J. (2012) Analysis of unit-load warehouses with nontraditional aisles and multiple P&D points, in *Proceedings of the 2012 Asia Pacific Industrial Engineering & Management Systems Conference*, Phuket, Thailand.

Gu, J., Goetschalckx, M. and McGinnis, L. F. (2007) Research on warehouse operation: a comprehensive review. *European Journal of Operational Research*, 177(1), 1-21.

Gue, K. R., Ivanovic`, G. and Meller, R. D. (2012) A unit-load warehouse with multiple pickup and deposit points and non-traditional aisles. *Transportation Research Part E: Logistics and Transportation Review*, 48(4), 795-806.

Gue, K. R. and Meller, R. D. (2006) Improving the unit-load warehouse, in *Material Handling Institute*, Charlotte, NC, pp. 187-194.

Gue, K. R. and Meller, R. D. (2009) Aisle configurations for unit-load warehouses. *IIE Transactions*, 41(3), 171-182.

Hausman, W. H., Schwarz, L. B. and Graves, S. C. (1976) Optimal storage assignment in automatic warehousing systems. *Management Science*, 22(6), 629-638.

Heskitt, J. L. (1963) Cube-per-order index—a key to warehouse stock location. *Transportation and Distribution Management*, 3, 27-31.

Mallette, A. J. and Francis, R. L. (1972) A generalized assignment approach to optimal facility layout. *AIIE Transactions*, 4 (2), 144-147.

Mathematica (2015) *Mathematica 10.2 for Students*, Microsoft Windows Version. Wolfram Research, Inc.

Meller, R.D. and Gue, K.R. (2009) The application of new aisle designs for unit-load warehouses, in *Proceedings of the 2009 NSF Engineering Research and Innovation Conference*, Honolulu, Hawaii.

Moder, J. and Thornton, H. (1965) Quantitative analysis of the factors affecting floor space utilization of palletized storage. *The Journal of Industrial Engineering*, 16, 8-18.

Ozturkoglu, O., Gue, K. R. and Meller, R. D. (2012) Optimal unit-load warehouse designs for single-command operations. *IIE Transactions*, 44(6), 459-475.

Ozturkoglu, O., Gue, K. R. and Meller, R. D. (2014) A constructive aisle design formulation for unit-load warehouses with multiple pickup and deposit points. *European Journal of Operational Research*, 236(1), 382-394.

Pohl, L. M., Meller, R. D. and Gue, K. R. (2007) An evaluation of two new warehouse aisle designs for dual-command travel, in *Proceedings of the 2007 Industrial Engineering Research Conference,* Nashville, Tennessee.

Pohl, L. M., Meller, R. D., and Gue, K. R. (2009) Optimizing fishbone aisles for dual-command operations in a warehouse. *Naval Research Logistics*, 56(5), 389-403.

Pohl, L. M., Meller, R. D. and Gue, K. R. (2011) Turnover-based storage in non-traditional unit-load warehouse designs. *IIE Transactions*, 43(10), 703-720.

Thomas, L. M. and Meller, R. D. (2014) Analytical Formulations for warehouse configuration. *IIE Transactions*, 46(9), 928-947.

White, J. A. (1972) Optimum design of warehouses having radial aisles. *AIIE Transactions*, 4(4), 333-336.

Appendix

The expected single-command distance formulas for contour-line shaped warehouse with Scenarios 1 and 2 are provided below. We begin with a detailed explanation of derivations for Scenario 1 and an even number of dock doors. Then, we present only equations for an odd number of dock doors with Scenario 1 and for both an even and an odd number of dock doors with Scenario 2.

Derivation of Equation 52 (even number of dock doors)

The objective function value for contour set *i* is

$$z_{k,i} = \frac{1}{k}\left[\sum_{j=1}^{k/2-i}\left(j\,h_{k,k/2} - h_{k,i}\right) + \sum_{j=1}^{k/2+i}\left\{\left(j-1\right)h_{k,k/2} + h_{k,i}\right\}\right] = \frac{8i\,h_{k,i} + h_{k,k/2}\left[k^2 + 4i\left(i-1\right)\right]}{4k} \qquad \text{(A.1)}$$

The minimum objective function value occurs when $h_{k,i}$ equals zero; therefore, the minimum objective function value for contour set *i* is obtained:

$$f^*_{k,i} = \frac{1}{k}\left[\sum_{j=1}^{k/2-i} j\,h_{k,k/2} + \sum_{j=1}^{k/2+i}\left(j-1\right)h_{k,k/2}\right] = \frac{h_{k,k/2}\left[k^2 + 4i\left(i-1\right)\right]}{4k} \qquad \text{(A.2)}$$

Solving Equation (A.2) for $h_{k,i}$ gives

$$h_{k,i} = \frac{4k\,z_{k,i} - h_{k,k/2}\left[k^2 + 4i\left(i-1\right)\right]}{8i} \qquad \text{(A.3)}$$

Solving for the area contained within a contour line having value $h_{k,i}$, we obtain

$$\begin{aligned} A_{k,i} &= \frac{1}{k}\left[2i\,h_{k,i}\left\{h_{k,i} + h_{k,k/2}\left(2i-1\right)\right\} + \sum_{j=1}^{i-1}\left(2j\,h_{k,k/2}\right)^2\right] \\ &= \frac{2i\left[3h_{k,i}^{\;2} + h_{k,k/2}\left\{3h_{k,i}\left(2i-1\right) + h_{k,k/2}\left(2i^2 - 3i + 1\right)\right\}\right]}{3k} \end{aligned} \qquad \text{(A.4)}$$

Therefore, solving for $h_{k,i}$ as a function of the area gives

$$h_{k,i}=\frac{3i\,h_{k,k/2}\left(1-2i\right)+\sqrt{3i\left[6k\,A_{k,i}+i\,h_{k,k/2}{}^{2}\left(4i^{2}-1\right)\right]}}{6i} \tag{A.5}$$

Furthermore, the functional relationship between $A_{k,i}$ and $z_{k,i}$ is

$$q\left(z_{k,i}\right)=A_{k,i}=\frac{24k\,z_{k,i}\left[2k\,z_{k,i}-h_{k,k/2}\left(k^{2}-4i^{2}\right)\right]+3k^{4}h_{k,k/2}{}^{2}-8i^{2}h_{k,k/2}{}^{2}\left(3k^{2}+2i^{2}-2\right)}{96\,i\,k} \tag{A.6}$$

Based on the assumption of uniformly distributed points over the storage region, Equation (A.6) can be treated as the cumulative distribution function for single-command travel distance. Taking the first derivative of Equation (A.6) with respect to $z_{k,i}$ yields the probability density function for contour set *i*:

$$q'\left(z_{k,i}\right)=\frac{h_{k,k/2}\left(4i^{2}-k^{2}\right)+4k\,z_{k,i}}{4i} \tag{A.7}$$

Solving Equation (A.6) for $z_{k,i}$ yields the inverse function related to $z_{k,i}$ and $A_{k,i}$, which is the value of the objective function on the contour line enclosing $A_{k,i}$,

$$r(A_{k,i})=z_{k,i}=\frac{3h_{k,k/2}\left(k^{2}-4i^{2}\right)+\sqrt{48i\left[6k\,A_{k,i}+i\,h_{k,k/2}{}^{2}\left(4i^{2}-1\right)\right]}}{12k} \tag{A.8}$$

Because $r\,(A_{k,i}) = f^*_{k,\,i+1}$ if $i < k\,/\,2$, the expected round-trip single-command distance for Scenario 1 with an even number of dock doors is

$$\begin{aligned} E\left[SC\right]&=\frac{2}{A}\left[\int_{f^*_{k,k/2}}^{r(A_{k,k/2})} z_{k,k/2}\left[q'\left(z_{k,k/2}\right)\right]dz_{k,k/2}+\sum_{i=1}^{k/2-1}\left(\int_{f^*_{k,i}}^{f^*_{k,i+1}} z_{k,i}\left[q'\left(z_{k,i}\right)\right]dz_{k,i}\right)\right]\\ &=\frac{\left(2A\right)^{1/2}\left(12k^{2}+9k-1\right)}{5k\left[3\left(k+2\right)\left(k+1\right)\right]^{1/2}} \end{aligned} \tag{A.9}$$

Derivation of Equation 52 (odd number of dock doors)

Following the steps for an even number of dock doors, equations for an odd number of dock doors with Scenario 1 are obtained:

$$z_{k,i} = \frac{1}{k}\left[\sum_{j=1}^{(k+1)/2-i}\left(j\,h_{k,(k+1)/2} - h_{k,i}\right) + \sum_{j=1}^{(k-1)/2+i}\left\{\left(j-1\right)h_{k,(k+1)/2} + h_{k,i}\right\}\right]$$
$$= \frac{4h_{k,i}\left(2i-1\right) + h_{k,(k+1)/2}\left[k^2+3+4i\left(i-2\right)\right]}{4k} \tag{A.10}$$

$$f*_{k,i} = \frac{1}{k}\left[\sum_{j=1}^{(k+1)/2-i} j\,h_{k,(k+1)/2} + \sum_{j=1}^{(k-1)/2+i}\left(j-1\right)h_{k,(k+1)/2}\right] = \frac{h_{k,(k+1)/2}\left[k^2+3+4i\left(i-2\right)\right]}{4k} \tag{A.11}$$

$$h_{k,i} = \frac{4k\,z_{k,i} - h_{k,(k+1)/2}\left[k^2+3+4i\left(i-2\right)\right]}{4\left(2i-1\right)} \tag{A.12}$$

$$A_{k,i} = \frac{1}{k}\left[\left(2i-1\right)\left\{h_{k,i} + 2\left(i-1\right)h_{k,(k+1)/2}\right\} + \sum_{j=1}^{i-1}\left\{\left(2j-1\right)h_{k,(k+1)/2}\right\}^2\right]$$
$$= \frac{\left(2i-1\right)\left[3h_{k,i}^{\;2} + h_{k,(k+1)/2}\left\{6h_{k,i}\left(i-1\right) + h_{k,(k+1)/2}\left(2i^2-5i+3\right)\right\}\right]}{3k} \tag{A.13}$$

$$h_{k,i} = \frac{3\left(3i-2i^2-1\right)h_{k,(k+1)/2} + \sqrt{3\left(2i-1\right)\left[3k\,A_{k,i} + i\,h_{k,(k+1)/2}^{\;2}\left(2i^2-3i+1\right)\right]}}{3\left(2i-1\right)} \tag{A.14}$$

$$q\left(z_{k,i}\right) = A_{k,i} = \frac{24k\,z_{k,i}\left[2k\,z_{k,i} - h_{k,(k+1)/2}\left(k^2-4i^2+4i-1\right)\right] + 3h_{k,(k+1)/2}^{\;2}\left(k^2-1\right)^2}{96\,i\,k}$$
$$- \frac{8i\,h_{k,(k+1)/2}^{\;2}\left[4i^2-2i^3-2-\left(i-1\right)\left(3k^2+1\right)\right]}{96\,i\,k} \tag{A.15}$$

$$q'\left(z_{k,i}\right) = \frac{h_{k,(k+1)/2}\left(4i^2-4i+1-k^2\right) + 4k\,z_{k,i}}{2\left(2i-1\right)} \tag{A.16}$$

$$r(A_{k,i}) = z_{k,i} = \frac{3h_{k,(k+1)/2}\left(k^2-4i^2+4i-1\right) + \sqrt{48\left(2i-1\right)\left(3k\,A_{k,i} + i\,h_{k,(k+1)/2}^{\;2}\left(2i^2-3i+1\right)\right)}}{12k} \tag{A.17}$$

Because $r\,(A_{k,i}) = f*_{k,\,i+1}$ if $i < (k+1)\,/\,2$, the expected round-trip single-command distance for Scenario 1 with an odd number of dock doors is

$$E\left[SC\right]=\frac{2}{A}\left[\int_{f^*_{k,(k+1)/2}}^{r(A_{k,(k+1)/2})} z_{k,(k+1)/2}\left[q'\left(z_{k,(k+1)/2}\right)\right]dz_{k,(k+1)/2}+\sum_{i=1}^{(k-1)/2}\left(\int_{f^*_{k,i}}^{f^*_{k,i+1}} z_{k,i}\left[q'\left(z_{k,i}\right)\right]dz_{k,i}\right)\right]$$
$$=\frac{(2A)^{1/2}\left(12k^2+9k-1\right)}{5k\left[3(k+2)(k+1)\right]^{1/2}} \quad \text{(A.18)}$$

Derivation of Equation 53 (even number of dock doors)

Following the steps for Scenario 1, equations for an even number of dock doors with Scenario 2 are:

$$z_{k,i}=\frac{1}{k}\left[\sum_{j=1}^{k/2-i}\left(j\delta-h_{k,i}\right)+\sum_{j=1}^{k/2+i}\left\{(j-1)\delta+h_{k,i}\right\}\right]=\frac{8i\,h_{k,i}+\delta\left[k^2+4i(i-1)\right]}{4k} \quad \text{(A.19)}$$

$$f^*_{k,i}=\frac{1}{k}\left[\sum_{j=1}^{k/2-i} j\delta+\sum_{j=1}^{k/2+i}(j-1)\delta\right]=\frac{\delta\left[k^2+4i(i-1)\right]}{4k} \quad \text{(A.20)}$$

$$h_{k,i}=\frac{4k\,z_{k,i}-\delta\left[k^2+4i(i-1)\right]}{8i} \quad \text{(A.21)}$$

$$A_{k,i}=\frac{1}{k}\left[2i\,h_{k,i}\left\{h_{k,i}+\delta(2i-1)\right\}+\sum_{j=1}^{i-1}(2j\delta)^2\right]=\frac{2i\left[3h_{k,i}^{\;2}+\delta\left\{3h_{k,i}(2i-1)+\delta\left(2i^2-3i+1\right)\right\}\right]}{3k} \quad \text{(A.22)}$$

$$h_{k,i}=\frac{3i\,\delta(1-2i)+\sqrt{3i\left[6k\,A_{k,i}+i\,\delta^2\left(4i^2-1\right)\right]}}{6i} \quad \text{(A.23)}$$

$$q\left(z_{k,i}\right)=A_{k,i}=\frac{24k\,z_{k,i}\left[2k\,z_{k,i}-\delta\left(k^2-4i^2\right)\right]+3k^4\delta^2-8i^2\delta^2\left(3k^2+2i^2-2\right)}{96i\,k} \quad \text{(A.24)}$$

$$q'\left(z_{k,i}\right)=\frac{\delta\left(4i^2-k^2\right)+4k\,z_{k,i}}{4i} \quad \text{(A.25)}$$

$$r(A_{k,i})=z_{k,i}=\frac{3\delta\left(k^2-4i^2\right)+\sqrt{48i\left[6k\,A_{k,i}+i\,\delta^2\left(4i^2-1\right)\right]}}{12k} \quad \text{(A.26)}$$

Because $r(A_{k,i}) = f^*_{k,\,i+1}$ if $i < (k+1)/2$, the expected round-trip single-command distance for Scenario 2 with an even number of dock doors is

$$E[SC] = \frac{2}{A}\left[\int_{f^*_{k,k/2}}^{r(A_{k,k/2})} z_{k,k/2}\left[q'\left(z_{k,k/2}\right)\right]dz_{k,k/2} + \sum_{i=1}^{k/2-1}\left(\int_{f^*_{k,i}}^{f^*_{k,i+1}} z_{k,i}\left[q'\left(z_{k,i}\right)\right]dz_{k,i}\right)\right]$$
$$= \frac{\left(12A+\delta^2k^2-\delta^2\right)^{3/2}}{18\sqrt{3}A} - \frac{\delta^3\left(3k^4-5k^2+2\right)}{90kA} \tag{A.27}$$

Derivation of Equation 53 (odd number of dock doors)

Following the steps for Scenario 1, equations for an odd number of dock doors with Scenario 2 are obtained:

$$z_{k,i} = \frac{1}{k}\left[\sum_{j=1}^{(k+1)/2-i}\left(j\delta - h_{k,i}\right) + \sum_{j=1}^{(k-1)/2+i}\left\{\left(j-1\right)\delta + h_{k,i}\right\}\right] = \frac{4h_{k,i}\left(2i-1\right)+\delta\left[k^2+3+4i\left(i-2\right)\right]}{4k} \tag{A.28}$$

$$f^*_{k,i} = \frac{1}{k}\left[\sum_{j=1}^{(k+1)/2-i} j\delta + \sum_{j=1}^{(k-1)/2+i}\left(j-1\right)\delta\right] = \frac{\delta\left[k^2+3+4i\left(i-2\right)\right]}{4k} \tag{A.29}$$

$$h_{k,i} = \frac{4k\,z_{k,i} - \delta\left[k^2+3+4i\left(i-2\right)\right]}{4\left(2i-1\right)} \tag{A.30}$$

$$A_{k,i} = \frac{1}{k}\left[\left(2i-1\right)\left\{h_{k,i}+2\left(i-1\right)\delta\right\} + \sum_{j=1}^{i-1}\left\{\left(2j-1\right)\delta\right\}^2\right]$$
$$= \frac{\left(2i-1\right)\left[3h_{k,i}{}^2+\delta\left\{6h_{k,i}\left(i-1\right)+\delta\left(2i^2-5i+3\right)\right\}\right]}{3k} \tag{A.31}$$

$$h_{k,i} = \frac{3\left(3i-2i^2-1\right)\delta + \sqrt{3\left(2i-1\right)\left[3k\,A_{k,i}+i\,\delta^2\left(2i^2-3i+1\right)\right]}}{3\left(2i-1\right)} \tag{A.32}$$

$$q\left(z_{k,i}\right)=A_{k,i}=\frac{24k\,z_{k,i}\left[2k\,z_{k,i}-\delta\left(k^2-4i^2+4i-1\right)\right]+3\delta^2\left(k^2-1\right)^2}{96\,i\,k}-\frac{8\,i\,\delta^2\left[4i^2-2i^3-2-\left(i-1\right)\left(3k^2+1\right)\right]}{96\,i\,k} \tag{A.33}$$

$$q'\left(z_{k,i}\right)=\frac{\delta\left(4i^2-4i+1-k^2\right)+4k\,z_{k,i}}{2\left(2i-1\right)} \tag{A.34}$$

$$r(A_{k,i})=z_{k,i}=\frac{3\delta\left(k^2-4\,i^2+4\,i-1\right)+\sqrt{48\left(2i-1\right)\left(3\,k\,A_{k,i}+i\,\delta^2\left(2i^2-3i+1\right)\right)}}{12\,k} \tag{A.35}$$

Because $r\,(A_{k,i}) = f\,*_{k,\,i+1}$ if $i < (k + 1) / 2$, the expected round-trip single-command distance for Scenario 2 with an odd number of dock doors is

$$\begin{aligned}E\left[SC\right]&=\frac{2}{A}\left[\int_{f*_{k,\,(k+1)/2}}^{r(A_{k,\,(k+1)/2})} z_{k,\,(k+1)/2}\left[q'\left(z_{k,\,(k+1)/2}\right)\right]dz_{k,\,(k+1)/2}+\sum_{i=1}^{(k-1)/2}\left(\int_{f*_{k,i}}^{f*_{k,i+1}} z_{k,i}\left[q'\left(z_{k,i}\right)\right]dz_{k,i}\right)\right]\\&=\frac{\left(12A+\delta^2k^2-\delta^2\right)^{3/2}}{18\sqrt{3}A}-\frac{\delta^3\left(3k^4-5k^2+2\right)}{90kA}\end{aligned} \tag{A.36}$$

Proof by induction

We prove by induction the general expression derived for the contour-line-shaped warehouse is valid for any number of dock doors.

$$E\left[SC_k\right]=\frac{\left(2A\right)^{1/2}\left(12k^2+9k-1\right)}{5k\left[3\left(k+2\right)\left(k+1\right)\right]^{1/2}} \tag{A.37}$$

Assume Equation (A.37) holds for k dock doors.

$$E\left[SC_{k+1}\right]=\frac{\left(2A\right)^{1/2}\left(12k^2+33k+20\right)}{5\left(k+1\right)\left[3\left(k+3\right)\left(k+2\right)\right]^{1/2}} \tag{A.38}$$

It is sufficient to show Equation (A.38) holds for k + 1 dock doors regardless k + 1 being either an even number of dock doors or an odd number of dock doors.

Expression for an even number of dock doors (k + 1 is even)

$$z_{k+1,i} = \frac{8i\,h_{k+1,i} + h_{k+1,(k+1)/2}\left[k^2 + 2k + 1 + 4i(i-1)\right]}{4(k+1)^2} \tag{A.39}$$

$$f*_{k+1,i} = \frac{h_{k+1,(k+1)/2}\left[k^2 + 2k + 1 + 4i(i-1)\right]}{4(k+1)} \tag{A.40}$$

$$h_{k+1,i} = \frac{4(k+1)z_{k,i} - h_{k+1,(k+1)/2}\left[k^2 + 2k + 1 + 4i(i-1)\right]}{8i} \tag{A.41}$$

$$A_{k+1,i} = \frac{2i\left[3h_{k+1,i}{}^2 + h_{k+1,(k+1)/2}\left\{3h_{k+1,i}(2i-1) + h_{k+1,(k+1)/2}(2i^2 - 3i + 1)\right\}\right]}{3(k+1)} \tag{A.42}$$

$$h_{k+1,i} = \frac{3i\,h_{k+1,(k+1)/2}(1-2i) + \sqrt{3i\left[6(k+1)A_{k+1,i} + i\,h_{k+1,(k+1)/2}{}^2(4i^2 - 1)\right]}}{6i} \tag{A.43}$$

$$q(z_{k+1,i}) = A_{k+1,i} = \frac{24z_{k+1,i}\left[2(k+1)z_{k+1,i} - h_{k+1,(k+1)/2}(k^2 + 2k + 1 - 4i^2)\right]}{96i} + \frac{3(k+1)^3 h_{k+1,(k+1)/2}{}^2}{96i} - \frac{8i^2 h_{k+1,(k+1)/2}{}^2(3k^2 + 6k + 2i^2 + 1)}{96i(k+1)} \tag{A.44}$$

$$q'(z_{k+1,i}) = \frac{h_{k+1,(k+1)/2}(4i^2 - k^2 - 2k - 1) + 4(k+1)z_{k,i}}{4i} \tag{A.45}$$

$$r(A_{k+1,i}) = z_{k+1,i} = \frac{3h_{k+1,(k+1)/2}(k^2 + 2k + 1 - 4i^2) + \sqrt{48i\left[6(k+1)A_{k+1,i} + i\,h_{k+1,(k+1)/2}{}^2(4i^2 - 1)\right]}}{12(k+1)} \tag{A.46}$$

Because $r(A_{k,i}) = f*_{k,i+1}$ if $i < (k+1)/2$, the expected round-trip single-command distance for Scenario 1 with $k + 1$ dock doors is

$$E\left[SC_{k+1}\right] = \frac{(2A)^{1/2}(12k^2 + 33k + 20)}{5(k+1)\left[3(k+3)(k+2)\right]^{1/2}} \tag{A.47}$$

Expression for an odd number of dock doors (k + 1 is odd)

$$z_{k+1,i} = \frac{4h_{k+1,i}(2i-1) + h_{k+1,(k+2)/2}\left[k^2 + 2k + 4 + 4i(i-2)\right]}{4(k+1)} \tag{A.48}$$

$$f*_{k+1,i} = \frac{h_{k+1,(k+2)/2}\left[k^2 + 2k + 4 + 4i(i-2)\right]}{4(k+1)} \tag{A.49}$$

$$h_{k+1,i} = \frac{4(k+1)z_{k,i} - h_{k+1,(k+2)/2}\left[k^2 + 2k + 4 + 4i(i-2)\right]}{4(2i-1)} \tag{A.50}$$

$$A_{k+1,i} = \frac{(2i-1)\left[3h_{k+1,i}{}^2 + h_{k+1,(k+2)/2}\left\{6h_{k+1,i}(i-1) + h_{k+1,(k+2)/2}\left(2i^2 - 5i + 3\right)\right\}\right]}{3(k+1)} \tag{A.51}$$

$$h_{k+1,i} = \frac{3\left(3i - 2i^2 - 1\right)h_{k+1,(k+2)/2} + \sqrt{3(2i-1)\left[3(k+1)A_{k+1,i} + i\,h_{k+1,(k+2)/2}{}^2\left(2i^2 - 3i + 1\right)\right]}}{3(2i-1)} \tag{A.52}$$

$$q\left(z_{k+1,i}\right) = A_{k+1,i} = \frac{24(k+1)z_{k+1,i}\left[2(k+1)z_{k+1,i} - h_{k+1,(k+2)/2}\left(k^2 + 2k - 4i^2 + 4i\right)\right]}{96i(k+1)} + \frac{3h_{k+1,(k+2)/2}{}^2\left(k^2+2k\right)^2 - 8i\,h_{k+1,(k+2)/2}{}^2\left[4i^2 - 2i^3 - 2 - (i-1)\left(3k^2 + 6k + 4\right)\right]}{96i(k+1)} \tag{A.53}$$

$$q'\left(z_{k+1,i}\right) = \frac{h_{k+1,(k+2)/2}\left(4i^2 - 4i - k^2 - 2k\right) + 4(k+1)z_{k,i}}{2(2i-1)} \tag{A.54}$$

$$r(A_{k+1,i}) = z_{k+1,i} = \frac{3h_{k+1,(k+2)/2}\left(k^2 + 2k - 4i^2 + 4i\right) + \sqrt{48(2i-1)\left(3(k+1)A_{k+1,i} + i\,h_{k+1,(k+2)/2}{}^2\left(2i^2 - 3i + 1\right)\right)}}{12(k+1)} \tag{A.55}$$

Because $r(A_{k,i}) = f*_{k,\,i+1}$ if $i < (k+2)/2$, the expected round-trip single-command distance for Scenario 1 with k + 1 dock doors is

$$E\left[SC_{k+1}\right] = \frac{(2A)^{1/2}\left(12k^2 + 33k + 20\right)}{5(k+1)\left[3(k+3)(k+2)\right]^{1/2}} \tag{A.56}$$

Therefore, Equation (A.38) holds for any number of dock doors regardless of the number of dock doors being even or odd.

Table 4.3: Penalty calculations for a storage region with Scenario 1

k	$\omega + \psi$	$E[SC_{Contour}]$	S^*	$E[SC_{Rectangle}]$	Penalty (%)
1	500.00	666.6667	2.0000	707.1068	6.066
2	353.55	766.0323	1.8000	745.3560	-2.699
3	273.86	815.4980	1.7143	763.7626	-6.344
4	223.61	845.9791	1.6667	774.5967	-8.438
5	188.98	866.7985	1.6364	781.7360	-9.813
6	163.66	881.9640	1.6154	786.7958	-10.790
7	144.34	893.5183	1.6000	790.5694	-11.522
8	129.10	902.6203	1.5882	793.4920	-12.090
9	116.77	909.9788	1.5789	795.8224	-12.545
10	106.60	916.0524	1.5714	797.7240	-12.917
11	98.06	921.1515	1.5652	799.3053	-13.228
12	90.78	925.4938	1.5600	800.6408	-13.490
13	84.52	929.2363	1.5556	801.7837	-13.716
14	79.06	932.4954	1.5517	802.7730	-13.911
15	74.26	935.3594	1.5484	803.6376	-14.082
16	70.01	937.8959	1.5455	804.3997	-14.234
17	66.23	940.1583	1.5429	805.0765	-14.368
18	62.83	942.1886	1.5405	805.6816	-14.488
19	59.76	944.0209	1.5385	806.2258	-14.597
20	56.98	945.6829	1.5366	806.7178	-14.695
21	54.45	947.1971	1.5349	807.1649	-14.784
22	52.13	948.5826	1.5333	807.5729	-14.865
23	50.00	949.8551	1.5319	807.9466	-14.940
24	48.04	951.0278	1.5306	808.2904	-15.009
25	46.23	952.1121	1.5294	808.6075	-15.072
26	44.54	953.1175	1.5283	808.9011	-15.131
27	42.98	954.0525	1.5273	809.1736	-15.186
28	41.52	954.9242	1.5263	809.4272	-15.236
29	40.16	955.7387	1.5254	809.6639	-15.284
30	38.89	956.5015	1.5246	809.8852	-15.328
31	37.69	957.2175	1.5238	810.0926	-15.370
32	36.56	957.8907	1.5231	810.2874	-15.409
33	35.50	958.5249	1.5224	810.4707	-15.446
34	34.50	959.1235	1.5217	810.6435	-15.481
35	33.56	959.6892	1.5211	810.8066	-15.514
36	32.66	960.2249	1.5205	810.9609	-15.545
37	31.81	960.7327	1.5200	811.1071	-15.574
38	31.01	961.2148	1.5195	811.2457	-15.602
39	30.24	961.6731	1.5190	811.3774	-15.629
40	29.51	962.1094	1.5185	811.5027	-15.654
41	28.82	962.5251	1.5181	811.6219	-15.678
42	28.16	962.9218	1.5176	811.7356	-15.701
43	27.52	963.3006	1.5172	811.8441	-15.723
44	26.92	963.6628	1.5169	811.9478	-15.744
45	26.34	964.0094	1.5165	812.0470	-15.764
46	25.79	964.3415	1.5161	812.1419	-15.783
47	25.25	964.6599	1.5158	812.2329	-15.801
48	24.74	964.9654	1.5155	812.3201	-15.819
49	24.25	965.2588	1.5152	812.4038	-15.836
50	23.78	965.5408	1.5149	812.4843	-15.852

Table 4.3: Penalty calculations for a storage region with Scenario 1 (Cont.)

k	$\omega + \psi$	$E\ [SC_{Contour}]$	***S****	$E\ [SC_{Rectangle}]$	**Penalty (%)**
51	23.33	965.8121	1.5575	812.6410	-15.859
52	22.89	966.0733	1.6180	813.0818	-15.836
53	22.47	966.3249	1.6796	813.8025	-15.784
54	22.07	966.5674	1.7424	814.7879	-15.703
55	21.68	966.8013	1.8063	816.0238	-15.596
56	21.30	967.0272	1.8714	817.4971	-15.463
57	20.94	967.2453	1.9377	819.1954	-15.306
58	20.58	967.4560	2.0051	821.1073	-15.127
59	20.24	967.6598	2.0736	823.2222	-14.926
60	19.92	967.8570	2.1433	825.5301	-14.705
61	19.60	968.0479	2.2141	828.0215	-14.465
62	19.29	968.2328	2.2861	830.6878	-14.206
63	18.99	968.4119	2.3593	833.5208	-13.929
64	18.70	968.5856	2.4336	836.5128	-13.636
65	18.42	968.7541	2.5091	839.6566	-13.326
66	18.14	968.9175	2.5857	842.9453	-13.001
67	17.88	969.0762	2.6634	846.3725	-12.662
68	17.62	969.2304	2.7423	849.9324	-12.309
69	17.37	969.3801	2.8224	853.6190	-11.942
70	17.13	969.5257	2.9036	857.4272	-11.562
71	16.89	969.6673	2.9860	861.3519	-11.170
72	16.66	969.8050	3.0695	865.3881	-10.767
73	16.44	969.9390	3.1542	869.5315	-10.352
74	16.22	970.0695	3.2400	873.7778	-9.926
75	16.01	970.1965	3.3270	878.1228	-9.490
76	15.80	970.3203	3.4151	882.5628	-9.044
77	15.60	970.4410	3.5044	887.0940	-8.589
78	15.41	970.5586	3.5948	891.7131	-8.124
79	15.21	970.6732	3.6864	896.4167	-7.650
80	15.03	970.7851	3.7791	901.2016	-7.168
81	14.85	970.8943	3.8730	906.0650	-6.677
82	14.67	971.0008	3.9681	911.0040	-6.179
83	14.49	971.1048	4.0643	916.0159	-5.673
84	14.32	971.2064	4.1616	921.0980	-5.159
85	14.16	971.3057	4.2601	926.2481	-4.639
86	14.00	971.4027	4.3597	931.4636	-4.111
87	13.84	971.4974	4.4605	936.7424	-3.577
88	13.68	971.5901	4.5625	942.0824	-3.037
89	13.53	971.6807	4.6656	947.4815	-2.490
90	13.39	971.7694	4.7699	952.9377	-1.938
91	13.24	971.8561	4.8753	958.4493	-1.380
92	13.10	971.9410	4.9818	964.0143	-0.816
93	12.96	972.0241	5.0895	969.6312	-0.246
94	12.82	972.1054	5.1984	975.2982	0.328
95	12.69	972.1850	5.3084	981.0139	0.908
96	12.56	972.2631	5.4196	986.7766	1.493
97	12.43	972.3395	5.5319	992.5850	2.082
98	12.31	972.4144	5.6454	998.4377	2.676
99	12.19	972.4878	5.7600	1004.3333	3.275
100	12.07	972.5598	5.8758	1010.2706	3.877

Table 4.4: Penalty calculations for a storage region with Scenario 2

k	***E* [*SC*$_{\text{Contour}}$]**	***S****	***E* [*SC*$_{\text{Rectangle}}$]**	**Penalty (%)**
1	666.6667	2.0000	707.1068	6.066
2	666.8095	2.0006	707.2086	6.059
3	667.0456	2.0015	707.3783	6.046
4	667.3735	2.0029	707.6157	6.030
5	667.7921	2.0046	707.9209	6.009
6	668.2999	2.0067	708.2937	5.984
7	668.8956	2.0092	708.7341	5.956
8	669.5780	2.0121	709.2418	5.924
9	670.3458	2.0154	709.8169	5.888
10	671.1977	2.0190	710.4590	5.849
11	672.1325	2.0230	711.1681	5.808
12	673.1489	2.0275	711.9438	5.763
13	674.2457	2.0323	712.7861	5.716
14	675.4217	2.0374	713.6946	5.667
15	676.6757	2.0430	714.6692	5.615
16	678.0065	2.0490	715.7094	5.561
17	679.4129	2.0553	716.8152	5.505
18	680.8938	2.0620	717.9861	5.448
19	682.4481	2.0691	719.2218	5.389
20	684.0745	2.0766	720.5220	5.328
21	685.7721	2.0845	721.8864	5.266
22	687.5397	2.0927	723.3146	5.203
23	689.3761	2.1014	724.8062	5.139
24	691.2804	2.1104	726.3608	5.075
25	693.2514	2.1198	727.9780	5.009
26	695.2882	2.1296	729.6575	4.943
27	697.3896	2.1398	731.3987	4.877
28	699.5546	2.1503	733.2012	4.810
29	701.7823	2.1613	735.0646	4.743
30	704.0716	2.1726	736.9885	4.675
31	706.4216	2.1843	738.9723	4.608
32	708.8312	2.1964	741.0155	4.540
33	711.2995	2.2089	743.1178	4.473
34	713.8255	2.2218	745.2785	4.406
35	716.4084	2.2350	747.4972	4.340
36	719.0471	2.2486	749.7733	4.273
37	721.7408	2.2627	752.1064	4.207
38	724.4886	2.2771	754.4959	4.142
39	727.2896	2.2918	756.9412	4.077
40	730.1428	2.3070	759.4419	4.013
41	733.0475	2.3226	761.9974	3.949
42	736.0027	2.3385	764.6071	3.886
43	739.0077	2.3548	767.2705	3.824
44	742.0616	2.3715	769.9870	3.763
45	745.1635	2.3886	772.7561	3.703
46	748.3127	2.4061	775.5772	3.643
47	751.5083	2.4239	778.4497	3.585
48	754.7496	2.4422	781.3732	3.527
49	758.0358	2.4608	784.3469	3.471
50	761.3661	2.4798	787.3703	3.415

Table 4.4: Penalty calculations for a storage region with Scenario 2 (Cont.)

k	$E\,[SC_{\text{Contour}}]$	S^*	$E\,[SC_{\text{Rectangle}}]$	Penalty (%)
51	764.7398	2.4992	790.4429	3.361
52	768.1561	2.5190	793.5641	3.308
53	771.6142	2.5391	796.7333	3.255
54	775.1136	2.5597	799.9500	3.204
55	778.6533	2.5806	803.2135	3.154
56	782.2328	2.6019	806.5234	3.105
57	785.8513	2.6236	809.8790	3.058
58	789.5081	2.6457	813.2798	3.011
59	793.2026	2.6682	816.7252	2.966
60	796.9341	2.6910	820.2146	2.921
61	800.7020	2.7142	823.7475	2.878
62	804.5055	2.7379	827.3234	2.836
63	808.3441	2.7619	830.9416	2.796
64	812.2171	2.7862	834.6017	2.756
65	816.1238	2.8110	838.3030	2.718
66	820.0638	2.8362	842.0451	2.680
67	824.0363	2.8617	845.8274	2.644
68	828.0409	2.8876	849.6493	2.610
69	832.0768	2.9139	853.5104	2.576
70	836.1435	2.9406	857.4101	2.543
71	840.2405	2.9677	861.3478	2.512
72	844.3671	2.9951	865.3231	2.482
73	848.5229	3.0695	869.3607	2.456
74	852.7072	3.1542	873.5045	2.439
75	856.9196	3.2400	877.7511	2.431
76	861.1595	3.3270	882.0965	2.431
77	865.4263	3.4151	886.5368	2.439
78	869.7197	3.5044	891.0684	2.455
79	874.0389	3.5948	895.6878	2.477
80	878.3837	3.6864	900.3917	2.506
81	882.7534	3.7791	905.1770	2.540
82	887.1476	3.8730	910.0407	2.581
83	891.5657	3.9681	914.9799	2.626
84	896.0074	4.0643	919.9921	2.677
85	900.4722	4.1616	925.0745	2.732
86	904.9595	4.2601	930.2248	2.792
87	909.4690	4.3597	935.4406	2.856
88	914.0002	4.4605	940.7197	2.923
89	918.5526	4.5625	946.0599	2.995
90	923.1258	4.6656	951.4593	3.069
91	927.7194	4.7699	956.9158	3.147
92	932.3330	4.8753	962.4275	3.228
93	936.9662	4.9818	967.9928	3.311
94	941.6184	5.0895	973.6099	3.398
95	946.2895	5.1984	979.2772	3.486
96	950.9788	5.3084	984.9931	3.577
97	955.6861	5.4196	990.7560	3.670
98	960.4110	5.5319	996.5646	3.764
99	965.1530	5.6454	1002.4175	3.861
100	969.9118	5.7600	1008.3133	3.959

Table 4.5: Computational Results for ABC storage regions with Scenario 1

k	h_k	k_A	$k_{A \cup B}$	Case ?	$E\ [SC_{Contour}]$	$E\ [SC_{Rectangle}]$	Penalty (%)
1	500.00	1	1	Case 1	452.15	479.58	6.066
2	353.55	2	2	Case 1	591.11	524.60	-11.251
3	273.86	3	3	Case 1	651.66	541.15	-16.959
4	223.61	4	4	Case 1	687.09	550.38	-19.896
5	188.98	3	5	Case 2	710.74	556.30	-21.729
6	163.66	4	6	Case 2	727.91	560.43	-23.009
7	144.34	5	7	Case 2	740.97	563.47	-23.956
8	129.10	6	8	Case 2	751.22	565.80	-24.683
9	116.77	5	7	Case 3	759.46	567.65	-25.256
10	106.60	6	8	Case 3	766.24	569.15	-25.721
11	98.06	7	9	Case 3	771.92	570.39	-26.107
12	90.78	8	10	Case 3	776.76	571.44	-26.433
13	84.52	9	11	Case 3	780.93	572.33	-26.711
14	79.06	8	12	Case 3	784.55	573.10	-26.951
15	74.26	9	13	Case 3	787.73	573.78	-27.161
16	70.01	10	14	Case 3	790.54	574.37	-27.345
17	66.23	11	15	Case 3	793.05	574.89	-27.509
18	62.83	12	16	Case 3	795.30	575.36	-27.655
19	59.76	11	15	Case 3	797.33	575.78	-27.787
20	56.98	12	16	Case 3	799.17	576.16	-27.905
21	54.45	13	17	Case 3	800.85	576.50	-28.013
22	52.13	14	18	Case 3	802.38	576.82	-28.112
23	50.00	15	19	Case 3	803.79	577.10	-28.202
24	48.04	14	20	Case 3	805.08	577.37	-28.285
25	46.23	15	21	Case 3	806.28	577.61	-28.361
26	44.54	16	22	Case 3	807.39	577.83	-28.432
27	42.98	17	23	Case 3	808.42	578.04	-28.498
28	41.52	16	24	Case 3	809.39	578.24	-28.559
29	40.16	17	23	Case 3	810.29	578.42	-28.616
30	38.89	18	24	Case 3	811.13	578.59	-28.669
31	37.69	19	25	Case 3	811.92	578.74	-28.719
32	36.56	20	26	Case 3	812.66	578.89	-28.766
33	35.50	19	27	Case 3	813.36	579.03	-28.810
34	34.50	20	28	Case 3	814.02	579.17	-28.851
35	33.56	21	29	Case 3	814.64	579.29	-28.890
36	32.66	22	30	Case 3	815.23	579.41	-28.928
37	31.81	23	31	Case 3	815.79	579.56	-28.958
38	31.01	22	30	Case 3	816.33	580.01	-28.949
39	30.24	23	31	Case 3	816.83	580.79	-28.898
40	29.51	24	32	Case 3	817.31	581.86	-28.808
41	28.82	25	33	Case 3	817.77	583.22	-28.682
42	28.16	26	34	Case 3	818.21	584.83	-28.523
43	27.52	25	35	Case 3	818.62	586.69	-28.332
44	26.92	26	36	Case 3	819.02	588.77	-28.113
45	26.34	27	37	Case 3	819.41	591.07	-27.866
46	25.79	28	38	Case 3	819.77	593.57	-27.594
47	25.25	29	39	Case 3	820.12	596.25	-27.298
48	24.74	28	38	Case 3	820.46	599.11	-26.979
49	24.25	29	39	Case 3	820.78	602.13	-26.640
50	23.78	30	40	Case 3	821.09	605.30	-26.281

Table 4.5: Computational Results for ABC storage regions with Scenario 1 (Cont.)

k	h_k	k_A	$k_{A\cup B}$	**Case ?**	***E*** **[*SC*Contour]**	***E*** **[*SC*Rectangle]**	**Penalty (%)**
51	23.33	31	41	Case 3	821.39	608.60	-25.906
52	22.89	32	42	Case 3	821.68	612.01	-25.517
53	22.47	31	43	Case 3	821.95	615.52	-25.115
54	22.07	32	44	Case 3	822.22	619.13	-24.700
55	21.68	33	45	Case 3	822.48	622.83	-24.274
56	21.30	34	46	Case 3	822.73	626.63	-23.835
57	20.94	33	47	Case 3	822.97	630.51	-23.386
58	20.58	34	46	Case 3	823.20	634.47	-22.927
59	20.24	35	47	Case 3	823.42	638.51	-22.457
60	19.92	36	48	Case 3	823.64	642.62	-21.979
61	19.60	37	49	Case 3	823.85	646.80	-21.491
62	19.29	36	50	Case 3	824.05	651.05	-20.995
63	18.99	37	51	Case 3	824.25	655.36	-20.490
64	18.70	38	52	Case 3	824.44	659.73	-19.978
65	18.42	39	53	Case 3	824.63	664.16	-19.459
66	18.14	40	54	Case 3	824.81	668.65	-18.932
67	17.88	39	53	Case 3	824.98	673.20	-18.399
68	17.62	40	54	Case 3	825.15	677.79	-17.859
69	17.37	41	55	Case 3	825.32	682.43	-17.313
70	17.13	42	56	Case 3	825.48	687.12	-16.760
71	16.89	43	57	Case 3	825.63	691.86	-16.202
72	16.66	42	58	Case 3	825.78	696.64	-15.639
73	16.44	43	59	Case 3	825.93	701.46	-15.070
74	16.22	44	60	Case 3	826.08	706.33	-14.496
75	16.01	45	61	Case 3	826.22	711.23	-13.917
76	15.80	46	62	Case 3	826.35	716.17	-13.333
77	15.60	45	61	Case 3	826.48	721.15	-12.745
78	15.41	46	62	Case 3	826.61	726.16	-12.152
79	15.21	47	63	Case 3	826.74	731.21	-11.555
80	15.03	48	64	Case 3	826.86	736.28	-10.954
81	14.85	47	65	Case 3	826.98	741.40	-10.349
82	14.67	48	66	Case 3	827.10	746.54	-9.741
83	14.49	49	67	Case 3	827.21	751.71	-9.128
84	14.32	50	68	Case 3	827.33	756.90	-8.512
85	14.16	51	69	Case 3	827.43	762.13	-7.892
86	14.00	50	70	Case 3	827.54	767.38	-7.269
87	13.84	51	69	Case 3	827.65	772.66	-6.643
88	13.68	52	70	Case 3	827.75	777.97	-6.014
89	13.53	53	71	Case 3	827.85	783.29	-5.382
90	13.39	54	72	Case 3	827.94	788.65	-4.747
91	13.24	53	73	Case 3	828.04	794.02	-4.108
92	13.10	54	74	Case 3	828.13	799.42	-3.468
93	12.96	55	75	Case 3	828.22	804.83	-2.824
94	12.82	56	76	Case 3	828.31	810.27	-2.178
95	12.69	57	77	Case 3	828.40	815.73	-1.529
96	12.56	56	76	Case 3	828.49	821.21	-0.878
97	12.43	57	77	Case 3	828.57	826.71	-0.225
98	12.31	58	78	Case 3	828.65	832.22	0.431
99	12.19	59	79	Case 3	828.73	837.76	1.089
100	12.07	60	80	Case 3	828.81	843.31	1.749

Table 4.6: Computational Results for ABC storage regions with Scenario 2

k	k_A	$k_{A\cup B}$	Case ?	$E[SC_{Contour}]$	$E[SC_{Rectangle}]$	Penalty (%)
1	1	1	Case 1	452.153	479.580	6.066
2	2	2	Case 1	452.387	479.748	6.048
3	3	3	Case 1	452.771	480.028	6.020
4	4	4	Case 1	453.301	480.419	5.982
5	5	5	Case 1	453.973	480.921	5.936
6	6	6	Case 1	454.785	481.533	5.881
7	7	7	Case 1	455.731	482.256	5.820
8	8	8	Case 1	456.808	483.087	5.753
9	9	9	Case 1	458.013	484.027	5.680
10	10	10	Case 1	459.342	485.075	5.602
11	11	11	Case 1	460.792	486.229	5.520
12	12	12	Case 1	462.359	487.488	5.435
13	13	13	Case 1	464.041	488.852	5.347
14	14	14	Case 1	465.834	490.319	5.256
15	15	15	Case 1	467.735	491.887	5.164
16	16	16	Case 1	469.741	493.556	5.070
17	17	17	Case 1	471.849	495.323	4.975
18	18	18	Case 1	474.056	497.189	4.880
19	19	19	Case 1	476.360	499.150	4.784
20	20	20	Case 1	478.758	501.206	4.689
21	21	21	Case 1	481.246	503.354	4.594
22	22	22	Case 1	483.823	505.594	4.500
23	23	23	Case 1	486.486	507.924	4.407
24	24	24	Case 1	489.233	510.342	4.315
25	25	25	Case 1	492.060	512.846	4.224
26	26	26	Case 1	494.967	515.436	4.135
27	27	27	Case 1	497.950	518.108	4.048
28	28	28	Case 1	501.007	520.862	3.963
29	29	29	Case 1	504.136	523.696	3.880
30	30	30	Case 1	507.335	526.609	3.799
31	31	31	Case 1	510.602	529.598	3.720
32	32	32	Case 1	513.935	532.663	3.644
33	33	33	Case 1	517.332	535.801	3.570
34	34	34	Case 1	520.791	539.010	3.498
35	35	35	Case 1	524.310	542.288	3.429
36	36	36	Case 1	527.887	545.631	3.361
37	37	37	Case 1	531.522	549.038	3.295
38	38	38	Case 1	535.211	552.506	3.231
39	39	39	Case 1	538.953	556.033	3.169
40	40	40	Case 1	542.748	559.618	3.108
41	41	41	Case 1	546.592	563.258	3.049
42	42	42	Case 1	550.485	566.952	2.991
43	43	43	Case 1	554.425	570.699	2.935
44	44	44	Case 1	558.410	574.496	2.881
45	45	45	Case 1	562.440	578.342	2.827
46	46	46	Case 1	566.513	582.237	2.776
47	47	47	Case 1	570.627	586.177	2.725
48	46	48	Case 2	574.781	590.164	2.676
49	47	49	Case 2	578.974	594.195	2.629
50	48	50	Case 2	583.205	598.269	2.583

Table 4.6: Computational Results for ABC storage regions with Scenario 2 (Cont.)

k	k_A	$k_{A \cup B}$	Case ?	$E[SC_{Contour}]$	$E[SC_{Rectangle}]$	Penalty (%)
51	47	51	Case 2	587.472	602.385	2.539
52	48	52	Case 2	591.775	606.542	2.495
53	47	53	Case 2	596.112	610.739	2.454
54	48	54	Case 2	600.483	614.975	2.413
55	49	55	Case 2	604.886	619.247	2.374
56	48	56	Case 2	609.321	623.556	2.336
57	49	57	Case 2	613.787	627.899	2.299
58	50	58	Case 2	618.283	632.276	2.263
59	49	59	Case 2	622.807	636.686	2.228
60	50	60	Case 2	627.360	641.128	2.195
61	51	61	Case 2	631.940	645.601	2.162
62	50	62	Case 2	636.547	650.104	2.130
63	51	63	Case 2	641.180	654.635	2.098
64	52	64	Case 2	645.839	659.196	2.068
65	51	65	Case 2	650.522	663.785	2.039
66	52	66	Case 2	655.229	668.400	2.010
67	51	67	Case 2	659.959	673.042	1.982
68	52	68	Case 2	664.712	677.709	1.955
69	53	69	Case 2	669.487	682.402	1.929
70	52	70	Case 2	674.283	687.118	1.904
71	53	71	Case 2	679.101	691.859	1.879
72	54	72	Case 2	683.939	696.622	1.854
73	53	73	Case 2	688.797	701.416	1.832
74	54	72	Case 3	693.674	706.251	1.813
75	53	73	Case 3	698.570	711.126	1.797
76	54	74	Case 3	703.484	716.040	1.785
77	55	73	Case 3	708.416	720.992	1.775
78	54	74	Case 3	713.365	725.979	1.768
79	55	75	Case 3	718.332	731.002	1.764
80	56	74	Case 3	723.315	736.058	1.762
81	55	75	Case 3	728.314	741.146	1.762
82	56	76	Case 3	733.329	746.266	1.764
83	55	75	Case 3	738.359	751.415	1.768
84	56	76	Case 3	743.404	756.595	1.774
85	57	77	Case 3	748.464	761.802	1.782
86	56	76	Case 3	753.538	767.037	1.791
87	57	77	Case 3	758.626	772.298	1.802
88	56	78	Case 3	763.728	777.584	1.814
89	57	77	Case 3	768.843	782.896	1.828
90	58	78	Case 3	773.971	788.231	1.842
91	57	77	Case 3	779.111	793.591	1.859
92	58	78	Case 3	784.264	798.972	1.875
93	57	79	Case 3	789.429	804.375	1.893
94	58	78	Case 3	794.606	809.800	1.912
95	59	79	Case 3	799.794	815.245	1.932
96	58	80	Case 3	804.994	820.711	1.952
97	59	79	Case 3	810.205	826.195	1.974
98	58	80	Case 3	815.426	831.699	1.996
99	59	81	Case 3	820.658	837.221	2.018
100	60	80	Case 3	825.900	842.761	2.042

Certification of Student Work

UNIVERSITY OF ARKANSAS

College of Engineering
Department of Industrial Engineering
308 John A. White, Jr. Engineering Hall

MEMORANDUM

TO: University of Arkansas Graduate School

FROM: John A. White, Distinguished Professor

DATE: June 27, 2018

SUBJECT: Certification of Student Effort and Contribution

I certify Mr. Mahmut Tutam contributed more than 51 percent of the work included in the chapter entitled, "Contribution 3: A Working Paper on, 'Configuring Contour-Line-Shaped Storage Region(s) in a Multi-Dock, Unit-Load Warehouse'" contained in the doctoral dissertation entitled, "Configuring Traditional Multi-Dock, Unit-Load Warehouses".

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Chapter 5

Conclusions and Future Research

In this research, we relaxed the single-dock-door assumption and developed expected-distance formulations for single- and dual command travel in traditional unit-load warehouse designs having multiple dock doors along one wall or two adjacent walls of the warehouse. From the formulas derived, the shape factors (width-to-depth ratios) minimizing expected distances were provided for three traditional layout configurations; as well as a new layout configuration. We also compared the performance of a rectangle-shaped warehouse with that of a contour-line shaped warehouse by considering randomized and class-based storage policies.

5.1. Conclusions from Chapter 2

Discrete and continuous expected-distance formulations of optimization problems were developed for a rectangle-shaped, unit-load warehouse having dock doors aligned perpendicular to the wall containing dock doors.

For three multi-dock-door scenarios involving different dock-door locations, the shape factor minimizing expected distance was determined from optimization models for both single- and dual-command travel.

> *For both single- and dual-command travel, increasing the number of dock doors will always increase expected distance when dock doors are centrally located; however, expected distance may increase or decrease depending the number of dock doors when they are not centrally located. Specifically, dock doors should be located as near as possible to the centerline of the warehouse.*

The optimal shape of a unit-load warehouse was obtained for any number of dock doors and three scenarios of dock-door locations along a single wall.

> *The optimal shape factor depends on the number and locations of dock doors. When dock doors are spread over an entire wall of the warehouse, the distance between adjacent dock doors is a function of the warehouse's width; the optimal shape factor is between 1.5 and 2.0. However, when dock doors are distributed about the centerline of a warehouse wall and distances between adjacent dock doors are specified, the optimal shape factor is equal to or greater than 2.0. When dock doors are clustered toward the end of a wall, the optimal shape factor can be less than 1.5, between 1.5 and 2.0, or greater than 2.0, depending on the number of dock doors and the distance from the leftmost end of the wall and the nearest dock door.*

Penalties based on the increase in expected distance traveled when using a non-optimal design versus an optimal design were calculated. According to our computational results, we inferred that designing a balanced warehouse (expected horizontal roundtrip-distance is equal to expected vertical roundtrip-distance) is a reasonable design goal.

> *Configuring a warehouse optimally results in a balanced warehouse when dock doors are equally distributed over an entire warehouse wall; whereas, it results in an unbalanced warehouse when the distance between adjacent dock doors is specified.*

The findings of this study supported the rule of thumb used by warehouse designers (the warehouse width being twice the warehouse depth) even when multiple dock doors are installed along one of the warehouse walls.

5.2. Conclusions from Chapter 3

Extending our research described in Chapter 2, discrete and continuous optimization problems were developed for three different layout configurations containing a middle-cross-aisle. Moreover, we allowed dock doors to be located along two adjacent aisles. The performance of all warehouse designs was tested for an equal number of S/R locations and the optimal shape factor values were provided for each design.

> *With multi-dock-doors, Layout A outperforms Layouts B and C for single-command travel. In contrast to single-command travel, Layout B always outperforms Layouts A and C for dual-command travel. Designing a warehouse having S/R aisles perpendicular to the wall containing dock doors performs the best.*

Our study showed Layout A performs best for single-command travel when either multiple dock doors are uniformly distributed along one warehouse wall or the distance between adjacent dock doors is specified; whereas, Layout B performs better than Layouts A and C for dual-command travel. Because having S/R aisles parallel to the wall containing dock doors will prevent S/R equipment access directly to the S/R locations, Layout C will always performs the worst for multi-dock-doors.

> *When the distance between adjacent dock doors is fixed, increasing the number of dock doors will always increase expected distance traveled regardless of warehouse or operation types.*

A unit-load warehouse performs the best when its dock doors are located as near as possible to the centerline of the warehouse. Increasing the number of dock doors results in locating dock doors farther from the centerline of the warehouse. Therefore, using more than the necessary number of dock doors increases operating costs.

When dock doors are centrally located (Scenarios 1 and 2), the expected distance for a single-command focused warehouse is less than that for the corresponding dual-command focused warehouse. When two sets of dock doors are clustered near one corner of the warehouse (Scenario 3), the expected distance for the cross-docking focused warehouse is smaller than both the single- and dual-command focused warehouses.

Locating dock doors centrally along two adjacent wall’s of a warehouse results in an additional distance to return S/R equipment to receiving dock doors for dual-command travel. Therefore, the additional travel diminishes the improvement gained by using travel-between. When dock doors are located near one corner of the warehouse, dual-command travel improves the performance of the warehouse.

5.3. Conclusions from Chapter 4

We developed expected single-command distance formulations for a contour-line-shaped warehouse to analyze the performance of a unit-load warehouse having multiple dock doors. Using two scenarios from previous chapters and equations developed in Chapter 2 for a rectangle-shaped warehouse, the penalty of requiring a warehouse to be rectangle-shaped was calculated under a randomized storage policy. Moreover, the penalty results were provided under an ABC class-based storage policy by assigning unit loads to three storage regions on a priority basis.

For a single dock door, the expected distance for a rectangle-shaped warehouse is about 6.07% greater than the corresponding contour-line-shaped warehouse.

When a single dock door is located on the centerline of a warehouse wall, the contour-line-shaped warehouse performs approximately 6% better than the corresponding rectangle-shaped warehouse, regardless of the storage policy or skewness of ABC curves.

When multiple dock doors are spaced uniformly over an entire warehouse wall (Scenario 1), the rectangle-shaped storage region performs better than the corresponding contour-line-shaped warehouse.

The Neyman-Pearson Lemma does not apply to Scenario 1 because dock-door locations change depending on the width of the overall warehouse.

When dock doors have a fixed distance between them (Scenario 2), the penalty of requiring a rectangular warehouse ranges from a high of 6.07 % to a low of 0.87% as the number of dock doors increases.

Depending on the number of dock doors and skewness of the ABC curve, the penalty of requiring storage regions to be rectangle-shaped for Scenario 2 can be found by using formulations developed in this research effort.

5.4. Practical application of the research

Generally speaking, warehouse designers use three rules of thumb: 1) install dock doors over an entire wall of the warehouse; 2) employ a warehouse shape factor (width-to-depth ratio) equal to 2.0, regardless of the number of dock doors located along the warehouse wall; and 3) design rectangle-shaped warehouses.

Researchers, on the other hand, tend to develop mathematical models of travel in a rectangle-shaped warehouse using an assumption of a single, centrally located dock door. In this research effort, the single-dock-door assumption for a unit-load warehouse is relaxed to more accurately represent reality. Likewise, in recognition constraints might exist which prevent dock doors being centrally located on a warehouse wall, we developed formulations for cases in which the dock doors must be off-set from the centerline of the warehouse. Finally, we developed expected-distance formulations for the case where the warehouse is not required to be rectangular. Having developed numerous expected-distance formulations, how might the research results be used by the warehouse designer?

In our research, we addressed two scenarios in which dock doors might be located over an entire wall of a warehouse: regardless of the number of dock doors required, space them equally over an entire wall; and install as many dock doors as possible over a wall, but provide a practical spacing between adjacent dock doors. Based on visits to numerous warehouses, we found the spacing between the centerlines of adjacent dock doors ranged from 10 feet to 16 feet. In our research, we used a spacing of 12 feet. Therefore, if the wall containing the dock doors is 300 feet in length, 25 dock doors would be located along the wall.

In Chapters 2, 3, and 4, results were provided for Scenario 1 (dock doors dispersed over an entire wall, regardless of the number required). We did so because Scenario 1 was used by other researchers, not because we deemed it a practical approach for warehouse design. Yet, as indicated, a rule of thumb employed by warehouse designers is space dock doors equally over an entire wall, regardless of the number required. Why is the rule of thumb used?

The rationale for spacing dock doors over an entire wall of a warehouse is it is cheaper to install them during initial construction than it is to add dock doors later to an existing warehouse. The argument is based on the uncertainty of the number of dock doors required over the life of the facility. Based on years of experience, during which the mission for the facility and the need for dock doors change, designers tend to include as many dock doors as possible along the warehouse wall. Although installing dock doors over an entire wall of the warehouse might be less expensive from a capital cost perspective, our results show doing so can increase expected distance significantly if all dock doors are used equally. Hence, a trade-off occurs between capital cost and operating cost.

Based on our results, if designers include more dock doors than needed, we recommend warehouse operators not use dock doors at both ends of the wall, but use only the required number of doors distributed about the centerline of the warehouse. The challenge, of course, is Parkinson’s Law, which (when applied to warehouse design) claims dock door usage will expand to include all available dock doors. We are familiar with firms that include the floor-level dock door equipment in the wall, but do not provide doors for all docking stations. (A temporary wall exists where a door would normally appear.)

When picking aisles are aligned perpendicular to the wall containing dock doors (Layouts A and B), our results support the approach of designing warehouse with a shape factor of 2.0 even when multiple dock doors are used. However, there will be a significant penalty in distance traveled if picking aisles are aligned parallel to the wall containing dock doors (Layout C) and a shape factor of 2.0 is used. For this reason, for a unit-load warehouse with dock doors on a single wall, we do not recommend using Layout C.

The results obtained in Chapters 2 and 3 are based on discrete formulations and continuous approximations. If continuous approximations are used to determine the optimal shape factor, adjustments will be required when developing detailed designs for storage rack. Given the discrete formulations, the designer can calculate the expected distance for a range of discrete values for the number of picking aisles. Given the number of storage positions to be included, an integer value is easily determined for the length of the picking aisles; for each combination of integer values for n (the number of picking aisles) and m (the length of the picking aisle, measured in storage locations) the expected distance can be calculated using the formulas provided.

For Layouts A, B, and C, regardless of the number of dock doors, if only single command operations are performed, Layout A is preferred; if only dual command operations are performed, Layout B is preferred.

When dock doors are located on two adjacent walls of the warehouse (Layout D), the preferred locations for the dock doors depend heavily on the level of cross-docking occurring in the warehouse. If very little cross-docking occurs, the dock doors should be centrally located on each wall; if significant cross-docking occurs, then the dock doors for receiving should be located as close as possible to the dock doors for shipping. As the level of cross-docking increases, the centroids of the dock-door locations shift from the center of the walls to the adjacent ends of the walls.

How might the warehouse designer apply the results in Chapter 4? Although designing a contour-line-shaped warehouse might be impractical and very expensive to construct, results obtained from the formulas we developed can be used to obtain the penalty incurred by requiring the warehouse configuration to be the most common configuration (rectangular). In addition, the contour line does not have to define the physical boundaries of the warehouse; it can, instead, be used to define the boundaries for the storage regions for product classes located within a rectangle-shaped warehouse, with space not used for product storage used for ancillary activities.

For the warehouse size we considered (250,000 square feet of storage area), even though expected distance can be reduced by as much as 6 percent by employing contour-line-shaped storage regions instead of rectangle-shaped storage regions, the magnitude of the reduction decreases as the number of dock doors increases. For large-sized warehouses (those with more than 70 dock doors) the savings in distance traveled is reduced to 2 percent. Based on the results provided, warehouse designers can use Equation 4.78 to calculate the savings potential for any number of dock doors and any storage area. In the end, a judgment is required regarding the tradeoff between reductions in distance traveled and increased cost of installing and managing non-rectangle-shaped storage regions.

5.5. Future Research

In developing expected-distance expressions for traditional layout configurations, we assumed a random storage policy is used. A class-based storage policy is only applied for Layout A. Therefore, consideration of class-based and turnover-based storage policies for Layouts B, C and D would be welcome. Another assumption made throughout the research is that dock doors are equally likely to be used. Having unequal flows across the dock doors could prove to be an interesting research topic. For Layout D, different mixtures of single-command, dual-command and cross-docking travel might yield greater insights regarding the design of the warehouse. Likewise, a consideration of dock doors located on non-adjacent walls and on more than two walls might yield new insights for warehouse designers.

For Chapter 4, relaxing the nesting requirement for class-based storage regions would be an interesting idea to explore. Requiring the overall storage region to be rectangle-shaped with contour-line-shaped storage regions inside the rectangle-shaped storage region might prove beneficial to designers.