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Economic Potash Fertilizer Rate Recommendations

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Abstract

This thesis is comprised of two studies that estimate profit-maximizing potassium (K) fertilizer application rates for various crops across different time periods. Estimation of profit-maximizing fertilizer-K rate (K^*) for both studies considered the initial soil test level of K (STK) and yield response information, as traditional recommendations do, and added crop price and the cost of fertilizer. Profit maximum occurs where the marginal revenue from additional yield is equal to the marginal cost of applying an additional unit of fertilizer-K. The first study calculated K^* for corn (*Zea mays*) and cotton (*Gossypium hirsutum*) and compared results to previous studies on rice (*Oryza sativa* L.) and soybean [*Glycine max* (L.) Merr.] without consideration of impact on STK over time. Corn results showed that current extension fertilizer-K rate recommendations could be profitably curtailed with cost savings from reduced fertilizer-K application greater than yield loss. Contrastingly, cotton results proved that K^* was greater than the current recommendations with estimated yield increases that were more than sufficient to afford additional fertilizer-K costs even in years when crop price was relatively low and fertilizer cost was relatively high. This was attributed to both greater yield response to fertilizer-K and crop value in cotton compared to soybean, rice, and corn. Hence, paying attention to both agronomic and economic factors for making fertilizer-K rate recommendations is important. Decision support software was developed to quantify effects of STK, yield response by crop, and user-specified crop price and fertilizer cost on fertilizer-K rate recommendations. The second study adds estimation of changes in STK using long-term K-rate trial information for fields in a rice/soybean rotation. Results proved that previous analyses, where tracking STK was not possible, had more moderate yield response to K and higher average yield in comparison to the crop-rotation study where STK changes were tracked. Hence yields, K^*, and producer profit were lower when simulating profit-maximizing fertilizer-K rates in the rotation. However, because the

short-run field tests were conducted across more sites, the results using the crop rotation yield response to K were deemed less representative of average Arkansas conditions. Using either yield response estimation method, regardless of initial STK, STK converged to the same final STK by the end of an 11-yr simulation period. With the more moderate, short-term K response the final STK was 80 ppm, 86 ppm when using the greater long-term K response, and 85 ppm when applying K at uniform extension rate recommendations (K_E) with the short-term yield response curves. Using either of the two yield response curve estimates led to different K^* across years and resulted in final STK values considered low by agronomic standards. Hence, using K^* from the short-term trials vs. K_E, or the long-term K^*, is likely to lead to less K runoff and leads to lesser STK reserves in the soil at minimal yield loss and the potential for minable K reserves to last longer. A philosophy of building STK to ensure higher yield and/or to rely on STK should fertilizer-K cost spike, was considered second best.

Acknowledgements

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Chapter I. Introduction

A. Problem Statement and Study Justification

Potassium (K) is an essential nutrient responsible for several key physiological functions in the production of field crops (Marschner, 2012), and it is commonly applied to agricultural fields as fertilizer. As there is a limited minable supply of K (USGS, 2019), efficient short- and long-term fertilizer-K recommendations that consider both agronomic and economic values are key. Various crops, including those commonly produced in the United States mid-South region, such as rice (*Oryza sativa* L.), soybean [*Glycine max* (L.) Merr.], corn (*Zea mays* L.), and cotton (*Gossypium hirsutum* L.), have different fertilizer-K rate recommendations as each crop's yield response to fertilizer-K is different. While applying at higher than necessary fertilizer-K levels in the current year may have a tendency to build the level of K in the soil (STK) in the long-run and serve as insurance against possible future price increases of fertilizer-K, this practice may come with negative externalities, such as paying for inputs earlier than needed, greater potential runoff, and ultimately decreased profits in the current year. Therefore, determining the short-run profit-maximizing fertilizer-K that uses annual price and yield data is important, as well as analyzing the effects of various K fertilization rates on yield response and STK in the long-run.

When rice is deficient in K, producers will see visual symptoms that include yellowing of leaf tips and margins that usually starts at the lower canopy, reduced growth, and depressed response to nitrogen (N) fertilization mid-season. The leaves continue changing from a yellow to red to brown and will eventually die off (Slaton et al., 1995). Similarly, a soybean crop will first show K deficiency symptoms on leaf tips beginning as chlorosis, followed by reduced pods plant^{-1}, seed pod^{-1}, and seed weight and increased seed abortion (Parvej et al., 2015, 2016). Potassium-deficient corn initially shows symptoms of chlorosis beginning on the tips of lower-level leaves. The yellowing will continue down the margin of the leaves turning from yellow to light tan to brown (Welch & Flannery, 1985). When K nutrition is low, corn plants can experience slower photosynthesis rates that eventually can lead to stalk diseases or problems such as corn lodging (Welch & Flannery, 1985). Lastly, cotton plants deficient in potassium first show symptom on older leaves of yellow-white mottling that eventually becomes necrotic causing the leaf to turn a rust color and drop (Kerby & Adams, 1985). Because of the premature leaf loss, the plant will cease boll development or produce a smaller, immature boll that is hard to open (Kerby & Adams, 1985). Therefore, proper soil testing to assess existing soil-K nutrient availability to determine proper K fertilization rates is vital to plant health, achieving maximum yield in a field, and eventually can impact producer profit.

Previous studies that analyzed the short-term effects of using agronomic and economic values to calculate fertilizer-K in rice and soybean have been conducted that used experimental field data under a complete randomized block design and applied at various fertilizer-K rates (Popp et al., 2020, 2021). The rice analysis used 91 site-years of data from 2001 through 2018 (Popp et al., 2020). Similarly, the soybean study used 86 site-years across the time period from 2004 through 2019 (Popp et al., 2021). Both rice and soybean yield response curves illustrated that yield response to additional fertilizer-K was greater when STK levels are lower, and hence the slope of the response curve becomes flatter as initial STK increases (Popp et al., 2020, 2021). Lastly, both studies concluded that current fertilizer-K recommendations are greater than the recommendation for that of annual profit-maximization, and thereby showed the value of economic analyses when calculating the fertilizer-K rate for application on rice and soybean fields (Popp et al., 2020, 2021) possibly entrenching a soil amendment philosophy routed in sufficiency rather than a philosophy of building or maintaining soil STK. A decision tool resulted from these analyses that enables users to calculate a profit-maximizing fertilizer-K rate (K^*) under their field conditions (Popp et al., 2020, 2021). However, these are annual K^* recommendations, and long-term effects of these K^* on STK were left for further study.

In addition to a comparison of effects of K^* on longer term STK values, calculating K^* that considers both agronomic and economic values for corn and cotton crops was also left for study. Irrigated-corn yield response to fertilizer-K was examined under a study conducted on 42 experimental and commercial production field trials in Arkansas (Drescher et al., 2021). Results from this study concluded that corn producers experience the greatest yield increases from K fertilization when initial STK from soil testing is low. In other words, as STK increases, the corn yield response to fertilizer-K decreases (Drescher et al., 2021). The lack of literature on cotton that presents this type of research further iterates the need for cotton specific K^* calculations.

B. Objectives

The objective of Chapter II is to calculate profit-maximizing fertilizer-K rate recommendations specifically for corn and cotton that are conditioned on both agronomic values – including initial STK values, expected yield, specific crop yield-response to fertilizer-K application – as well as economic values – including crop price and fertilizer costs (including application). Based on previous studies (Popp et al., 2020, 2021), the null hypothesis for this research was that current fertilizer-K rate recommendations for corn and cotton are higher than the profit-maximizing rate.

The objective of Chapter III is to estimate a long-term profit-maximizing fertilizer-K rate for a rice/soybean rotation that considers economic and agronomic impacts of applying fertilizer-K. The long-term profit-maximizing fertilizer-K rate is also compared to the short-term rate to determine if the annual profit-maximizing rate is different from the long-term rate that uses long term fertilizer K rate experimental data conducted on fields over twenty years. That study tracked STK on soybean and rice plots grown in a two-year rotation at varying initial STK and field plots where K rates included no K fertilizer and 4 treatment alternatives at 40, 80, 120, and 160 lbs of K_2O/acre. The null hypothesis of this study was that applying at higher rates would build STK levels whereas the zero-rate control would mine STK. A second null hypothesis for this study was that applying fertilizer-K at profit-maximizing rates as determined using the annual potash rate calculator would lead to similar long-term profit in comparison to using profit-maximizing rates calculated using the estimated yield response curves from the long-term field rotation data. For both the short-term and long-term rate recommendations, as well as applying at current extension rate recommendations, tracking STK was performed by using estimates of long-term changes in STK that were based on one-year lagged STK, fertilizer K application and yield. Results of this work would inform about long-term profitability estimates of using various rate recommendations and attendant STK and fertilizer-K use.

C. Overview of Methods

The statistical methods employed within used various functional forms and generalized least squares regression to calculate coefficients that estimate yield response to K using relative yield between zero-rate K controls in comparison to three to five alternative K rates subject to STK as observed across site-years for different crops. The profit-maximizing fertilizer-K rate in both Chapters II and III is calculated as the point where the marginal cost of fertilizer-K is equal to the diminishing marginal crop revenue received from additional yield as a result of marginal fertilizer-K use. The manuscript prepared as Chapter II will be submitted to the *Agronomy Journal* for publication.

Chapter II measures results in metric units as required by the *Agronomy Journal.* Table 1.1 is a comprehensive table for the reader to reference for making metric unit to English unit conversions. Chapter III is targeted for submission to the *Journal of Agricultural and Applied Economics* where English units are acceptable.

D. Overview of Chapters

As stated above, Chapter II provides an overview of the short-term profit-maximizing fertilizer-K rates specifically for corn and cotton conditioned on both agronomic and economic values. These results are then compared to previous studies on rice and soybean (Popp et al., 2020, 2021) to gain greater insight about how yield response to K and crop value impact profit-maximizing fertilizer-K rates differently across crops. Chapter III discusses the impacts of using a long-term profit-maximizing fertilizer-K rate that considers the agronomic and economic impacts of applying fertilizer-K over time in a rice-soybean rotation as well as a comparison to applying at the current agronomic extension-based fertilizer-K rates. The intent is to identify differences in K^* rates on long-term STK with attendant spillover effects on profitability and fate of applied K fertilizer. The final chapter, Chapter IV, concludes by discussing findings and needs for future research.

E. References

Drescher, G., Slaton, N., Roberts, T., & Smartt, A. (2021). Corn yield response to phosphorus and potassium fertilization in Arkansas. Crop, Forage, & Turfgrass Management, e20120. https://doi.org/10.1002/cft2.20120

Marschner, H. 2012. *Marschner's mineral nutrition of higher plants*. Academic Press, New York.

Parvej, M. R., Slaton, N. A., Purcell, L. C. & Roberts, T. L. (2015). Potassium fertility effects yield 555 components and seed potassium concentration of determinate and indeterminate soybean. 556 Agronomy Journal, 107, 943–950. https://doi:10.2134/agronj14.0464 557

Parvej, M. R., Slaton, N. A., Purcell, L. C. & Roberts, T. L. (2016). Soybean yield components and seed 558 potassium concentration responses among nodes to potassium fertility. Agronomy Journal, 108, 559 854-863. https://doi:10.2134/agronj2015.0353

Slaton, N.A., R.D. Cartwright, & C.E. Wilson, Jr. 1995. Potassium deficiency and plant diseases observed in rice fields. Better Crops, 79:12–14.

Popp, M., Slaton, N. A., Norsworthy, J. S., & Dixon, B. 2021. Rice yield response to potassium: an economic analysis. Agronomy Journal, 113, 287-297. https://doi:10.1002/agj2.20471

Popp, M., Slaton, N. A., & Roberts, T. L. 2020. Profit-maximizing potassium fertilizer recommendations for soybean. Agronomy Journal, 112, 5081-5095. https://doi:10.1002/agj2.20424.

United States Geological Survey (USGS). (2019). Mineral Commodity Summaries. As accessed 4 Jun 2020. http://prd-wret.s3-us-west-2.amazonaws.com/assets/palladium/production/atoms/files/mcs2019_all.pdf.

F. Tables and Figures

Table 1.1. Metric to English Unit Conversion table as a reader reference

Unit Conversions Table				
General				
hectare (ha)	x	2.4711	=	acre
kg	x	2.2046	=	lb
Mg	=	Megagram	=	Metric Tonne
Mg	x	1,000	=	kg
K	x	1.2046	=	K_2O
Fertilizer-K				
kg K	x	2.6557	=	lb K_2O
kg K/ha	x	1.0747	=	lb K_2O/acre
$/kg K	x	1,000	=	$/Mg K
$/Mg K	x	0.4519	=	$/ton muriate of potash
Crop Price				
$/kg rice	x	45.36	=	$/cwt rice
$/cwt rice	÷	2.2222	=	$/bu rice
$/kg soybean	x	27.216	=	$/bu soybean
$/kg corn	x	25.402	=	$/bu corn
$/kg cotton	÷	2.2046	=	$/lb cotton
Yield				
kg/ha rice	x	0.0198	=	bu/acre rice
kg/ha soybean	x	0.0149	=	bu/acre soybean
kg/ha corn	x	0.0159	=	bu/acre corn
kg/ha cotton	x	0.8921	=	lb/acre cotton

Chapter II. Profit-Maximizing Potash Recommendations for Corn and Cotton with Rice and Soybean Comparisons

A. Introduction

The worldwide demand for potassium (K) fertilizer in agriculture has been increasing (Dhillon et al., 2019). Efficient fertilizer-K rate recommendations are essential in light of the limited, minable supply of K and eventual price increases (USGS, 2019; Zörb et al., 2014) on a cost item that contributes as much as 5% of the total cost of crop production depending on crop and production year (Watkins, 2021). To maximize profit and increase input use efficiency, fertilizer-K rate recommendations need to account for the value of the crop being produced and the costs of K fertilization in addition to the crop's yield response and initial Mehlich-3 K soil availability (STK). The profit-maximizing rate of applying a nutrient from fertilizers occurs when the marginal yield from an additional unit of nutrient is equal to the cost of that added nutrient (Debertin, 1986). Previous studies have shown that profit-maximizing fertilizer-K rates for both rice (*Oryza sativa* L.) and soybean [*Glycine max* (L.) Merr.] crops were lower than current fertilizer-K recommendations (Popp et al., 2020, 2021). While applying high rates of K can protect against potential yield loss and has the potential to build soil K, which may offer protection to future fertilizer-K price increases, the practice comes at the cost of paying for inputs earlier than needed, leading to lower profit and perhaps greater nutrient loss.

Given the recent efforts of Popp et al. (2020, 2021) targeted at isolating the impact of using economic information in addition to agronomic information on a field-specific basis to make fertilizer recommendations in rice and soybean, fertilizer-K rate recommendations for both corn (*Zea mays* L.) and cotton (*Gossypium hirsutum* L.) crops should also consider the following: i) existing availability of soil K as measured by STK (Mallarino et al., 1991a); ii) the yield response to added K by the specific crop (Mallarino et al., 1991b; Heckman & Kamprath, 1992); iii) the specific crop output price; and iv) the cost of fertilizer-K and its application.

Potassium is an essential nutrient for growth and many other physiological functions of crops, such as photosynthesis, disease resistance, enzyme activation, water use efficiency, protein synthesis, and carbohydrate translocation (Marschner, 2012). Cost of production can also be reduced as an outcome of the efficient use of K (Dhillon et al., 2019). Over time, inadequate K fertilization can lead to crop K deficiency in both corn and cotton, which can result in yield loss (Welch & Flannery, 1985; Kerby & Adams, 1985) that translates to revenue loss. As of 2018, approximately 63% of the corn acres grown across the United States received K fertilizer, and as of 2017, 45% of the cotton acres received K fertilizer (USDA ERS, 2019). Soil sampling and testing for K are therefore critical in determining K-fertilization rates as existing soil nutrient availability is a factor for determining the efficiency of fertilizer-K and/or the need to ensure the continued productivity of the soil. However, fertilizer rate recommendations for corn and cotton do not typically reflect crop value or fertilizer and fertilizer application costs (Table 2.1).

Therefore, while producers know they might not be able to afford as much fertilizer when crop prices are relatively low or fertilizer cost is relatively high, it is of interest to quantify how much to reduce fertilizer use under those conditions and/or increase fertilizer use when crop price is relatively high or fertilizer cost is relatively low.

The goal of this work was to calculate profit-maximizing fertilizer-K application rates for corn and cotton crops that are conditioned on initial STK, expected yield, crop yield response to fertilizer-K as well as fertilizer cost, crop price, and fertilizer application charges. Using historical price and crop yield information, we estimate profit-maximizing rates at varying STK that change from year to year to current recommendations that vary based on STK but not by year, to assess the value of using crop price and fertilizer cost information. Finally, we compare conclusions drawn from these analyses specific to each crop to determine commonalities and differences across crops (rice, soybean, corn, and cotton). Due to the complexity of predicting the fate of fertilizer-K from year to year, especially in crop rotation, long-term implications of profit-maximizing fertilizer-K rate recommendations remain subject to further investigation as we only consider annual STK values, crop price, yield expectation, and cost of fertilizer and its application for the current year.

B. Materials and Methods

1. Background on Evaluating Cotton Data

Experimental cotton yield data for this study were reported as seed cotton values (lint and cotton seed). However, producers are primarily concerned with cotton lint values as they receive most of their revenue from lint sales with cotton seed revenue captured by cotton gins used to offset producer ginning, transportation, and storage costs. Since cotton gins are often cooperatively owned by crop producers, the net proceeds from cotton seed sales are paid to producers as a ginning rebate. The long-term average of these ginning rebates is \$0.11 kg^{-1} of lint cotton but could range from \$0.007 to \$0.22 kg^{-1} (R. Benson, personal communication, 27 May 2021). While a host of factors, including cultivar and harvest method impact the ratio of lint to seed cotton weight, a default, average lint turnout ratio of 38% was deemed appropriate in this analysis (F. Bourland, personal communication, 24 May 2021; R. Benson, personal communication, 27 May 2021). Multiplying seed cotton yield times the lint turnout ratio leads to lint yield. Adding the average ginning rebate to lint prices now allows estimates of sales changes as a function of seed cotton yield differences as impacted by fertility changes.

2. Experimental Data

Experimental plots for both corn and cotton crops were located in producer and experiment station fields in Arkansas. Soil series at the research sites for both crops were all common to the Mid-southern region of the U.S. and included a range of sandy, silt-loam, and clayey soils, with most of the soils in each study texturally classified as silt loam, which are typically less rich in K than clayey soils. All site-years were soil-tested before fertilizer application at or near the time of crop establishment. To ensure sufficient macronutrient availability to reach yield potential, each trial was supplemented with the necessary N and P fertilizers, whereas K was applied at varying levels to elicit a yield response to K under those conditions. The K rate treatments in each study were arranged in a randomized complete block design to examine the effect of fertilizer-K rate on corn or cotton yield. Fertilizer-K application rates for both corn and cotton varied from 0 to 186 kg K ha^{-1}.

Furrow-irrigated corn yield response data to fertilizer-K application rates was collected from 2010 through 2020 (excluding 2016) under a variety of environmental conditions. This data set included 218 individual treatment means from 39 site-years of response trials. This same data set has been previously analyzed to determine the agronomic yield response of corn to K fertilization (Drescher et al., 2021). Given our focus on only K, we estimate slightly different yield response curves given differences not only in the data used but also in the statistical procedure. Experimental corn trials utilized 3 to 5 K-rate treatments (incl. no-K control) where fertilizer-K rate increments were 37, 47, or 58 kg K ha^{-1} with a majority utilizing 37 and 47 kg K ha^{-1} increments. The 39 site-years had initial Mehlich-3 extractable (Zhang et al., 2014) soil-K availability values in the top 15-cm soil layer ranging from 48 to 172 mg K kg^{-1} (Figure 2.1B). Corn hybrids chosen for experiments mirrored the hybrids producers grew over the analyzed period and are included in the supplemental material. Furrow-irrigated corn yields varied between 7,532 kg ha^{-1} and 19,568 kg ha^{-1} (Figure 2.1C).

Similar to corn, data on irrigated cotton yield response to fertilizer-K rate were collected from 2006 to 2019, excluding years 2008-2014, and thus represents weather, soil, and production differences across 7 years. The cotton data set included 123 individual treatment means from 24 site-years of experimental K response trials. Trials conducted on cotton fields used 4 to 6 K-rate treatments (incl. zero K control) with fertilizer-K rate increments ranging from 28 to 56 kg K ha^{-1}. Trial sites for the 24 site-years had STK (0-to 15-cm depth) ranging from 68 to 204 mg K kg^{-1} (Figure 2.1B). Cotton cultivars chosen for experiments again mirrored those producers grew over the analyzed period. Seed cotton yields in these trials ranged from 1,257 kg ha^{-1} to 4,633 kg ha^{-1} (Figure 2.1C). The site-year data for the cotton analysis come from several publications (Mozaffari et al. 2007; Mozaffari et al. 2008; C.E. Wilson Jr., personal communication, 2018; Wilson et al. 2018; Mozaffari et al. 2020; Slaton et al. 2019; and Lewis et al. 2021). Additional information for each site-year has been reported by Mozaffari et al. (2007, 2008, 2020), Wilson Jr. et al. (2018), Slaton et al. (2019), and Lewis et al. (2021).

Average corn and seed cotton yields (Y) were converted to a relative yield index (*RY*) using:

Eq. [1] $RY_{cijt} = \frac{\mathrm{Y}_{cijt}}{\max\limits_{k} \mathrm{Y}_{cijt}} \cdot 100$

where *c* represents the crop under study (corn or cotton), *i* represents a specific site, *j* represents the fertilizer-K treatment including the no-K control (0 kg fertilizer-K ha^{-1}), *t* represents a year from the range included in the dataset of the particular crop, and *k* is the subset of treatments excluding the no-K control treatment. As such, *RY* should range from 0 to 100, where the maximum observed yield for a trial at a specific site year is equal to 100. However, because the no-K control was omitted from the denominator in the *RY* calculation, a *RY* greater than 100 is possible. This occurs when the no-K control resulted in the highest yield and additional fertilizer K numerically decreased yield. For the corn data, a trial in 2014 involving a silt loam soil with STK = 140 mg K kg^{-1} (Figure 2.1D) is the only instance where corn yield was numerically reduced by K fertilization in this dataset. As for cotton, this scenario arose in a trial in 2017 that involved a sandy soil with high STK = 130 mg K kg^{-1} (Figure 2.1D).

3. Economic Analysis

Since yield data for both crops were collected over a range of years and cultivars, an array of factors could affect yield response to K fertilization. For example, genetic improvement in crop cultivars may increase yield potential over time. Also, weather or other environmental factors such as weed or insect pressure could impact specific trials. Since, the goal of this study was to estimate a typical, long-term crop yield response curve to fertilizer-K for each crop, using the relative yield at each site, isolates the fertilizer-K rate effect, as the same environmental factors using the same cultivar, impact each K rate treatment and thereby eliminates yield trend effects. Thus, using STK and fertilizer-K rate as regressors to estimate RY, we essentially capture the average long-term effect of fertilizer-K on yield across a range of observed yields, cultivars, and initial STK.

Additionally, we argue that most producers for both corn and cotton can estimate a particular field's yield potential fairly well from historical production records (Shober & Taylor, 2015). Yield potential is representative of the maximum amount of grain/lint yield for corn/cotton that can be achieved without a nutrient deficiency in a field. Many producers fertilize to reach the crop's yield potential to maximize their profits. As such, we utilize a user-specified yield potential along with an estimate of *K* response to the *RY* index to enable us to determine the marginal yield that results when changing rates of fertilizer-K given that there are no other macronutrient deficiencies that could prevent a field reaching its yield potential. Therefore, RY response to K contingent on STK was estimated for corn using:

Eq. [2]

$$RY_{cijt} = \alpha_0 + \alpha_1 STK_{it} + \alpha_2 STK_{it}^2 + \alpha_3 K_{ijt} + \alpha_4 K_{ijt}^2 + \\ + \alpha_5 STK_{it} \cdot K_{ijt} + \alpha_6 STK_{it}^2 \cdot K_{ijt} + \alpha_7 STK_{it} \cdot K_{ijt}^2 + \alpha_8 STK_{it}^2 \cdot K_{ijt}^2 + \mu_t \\ + \varepsilon_{ijt} \ \forall \ c = \text{corn}$$

where α_0, is the constant and base *RY* value that did not change with location (*i*) and year (*t*); α_1 and α_2 represent the average linear and non-linear, location- and year-independent, effect of initial soil-test K (STK_{it}) on *RY* where *STK* differed by *i* and *t*; K_{ijt} where the *j* indicates one of up to five fertilizer-K application rates that varied by *i* and *t*; coefficients α_3 to α_8 estimated linear and non-linear effects of the interaction between fertilizer-K application and *STK*; μ_t captured the random year effect for this model, and; ε_{ijt} represented the error term for each observation of *RY*.

Similarly, RY for cotton was estimated using the following equation:

Eq. [3]

$$RY_{cijt} = \beta_0 + \beta_1 STK_{it} + \beta_2 STK_{it}^2 + \beta_3 K_{ijt} + \beta_4 \sqrt{K}_{ijt} + \\ + \beta_5 STK_{it} \cdot K_{ijt} + \beta_6 STK_{it}^2 \cdot K_{ijt} + \alpha_7 STK_{it} \cdot \sqrt{K}_{ijt} + \alpha_8 STK_{it}^2 \cdot \sqrt{K}_{ijt} + \xi_t \\ + \rho_{ijt} \ \forall \ c = \text{cotton}$$

with a similar interpretation of coefficient estimates as for Eq. 2, except that the yield response to *K* used a square root rather than quadratic functional form for optimal goodness of fit; ξ_t signified the random year effect for this model, and; ρ_{ijt} represented the error term for each observation of *RY*. Production year was treated as a random effect rather than a fixed effect on the basis of a Hausman test (Green, 2008) in RY calculations for both crops.

Thus, the marginal revenue generated by applying an additional kg ha^{-1} of fertilizer-K can be calculated for corn and cotton by the following general equation:

Eq. [4] $MR_{ci} = \frac{\partial RY_{ci}}{\partial K_i} \cdot YP_i/100 \cdot P_{c_t}$

where the partial derivative for Eq. 2 for corn is:

Eq. [5] $\frac{\partial RY_{ci}}{\partial K_i} = \alpha_3 + 2 \cdot \alpha_4 K_i + \alpha_5 STK_{it} + \alpha_6 STK_{it}^2 + 2 \cdot \alpha_7 STK_{it} \cdot K_i +$

$+ 2 \cdot \alpha_8 STK_{it}^2 \cdot K_i \; \forall \; c = \text{corn}$

and the partial derivative for Eq. 4 for cotton is:

Eq. [6] $\frac{\partial RY_{ci}}{\partial K_i} = \beta_3 + \frac{\beta_4}{2} K_i^{-0.5} + \beta_5 STK_{it} + \beta_6 STK_{it}^2 + \frac{\beta_7}{2} STK_{it} K_i^{-0.5} + \frac{\beta_8}{2} STK_{it}^2 K_i^{-0.5}$

$\forall \; c = \text{cotton}$

Therefore, MR_i represents the year-independent marginal effect of fertilizer-K application on *RY* that varies with a particular year's economic (price of crop - P_{c_t}) and agronomic values – STK_{it} and the field's yield potential (*YP*). Further, given the non-linear estimation of yield response to K, the revenue impact of added fertilizer-K can decrease as fertilizer-K application rate increases. To obtain the profit-maximizing fertilizer-K rate, K_{it}^*, we solve for the fertilizer-K rate at which the diminishing marginal revenue is equal to the cost of K, f_K, as follows:

Eq. [7] $K_{cit}^* = \left[\frac{f_{K_t}}{\frac{YP_i}{100} \cdot P_{c_t}} - (\alpha_3 + \alpha_5 STK_{it} + \alpha_6 STK_{it}^2)\right] / [2 \cdot (\alpha_4 + \alpha_7 STK_{it} + \alpha_8 STK_{it}^2)]$

$\forall \; c = \text{corn}$

Eq. [8] $K_{cit}^* = \left\{(\beta_4 + \beta_7 STK_{it} + \beta_8 STK_{it}^2)/2 \left[\frac{f_{K_t}}{\frac{YP_i}{100} \cdot P_{c_t}} - (\beta_3 + \beta_5 STK_{it} + \beta_6 STK_{it}^2)\right]\right\}^2$

$\forall \; c = \text{cotton}$

Eqs. 7 & 8 assume that fertilizer cost, f_{K_t}, per unit does not change with K_{cit}^* but will change from year to year. Thus, we expect that the costs associated with applying fertilizer (e.g., labor, equipment, and fuel charges per hectare) do not considerably change when modifying fertilizer-K rate in a particular year. However, the additional revenue produced by applying K_{cit}^* needs to be greater than the cost of fertilizer-K itself and the fertilizer application charges (labor, equipment, and fuel costs) for the producer to profit from applying fertilizer-K. For example, under a scenario where a producer is growing cotton, should the estimate of K_{cit}^{*} suggest applying 3 kg K ha^{-1} at f_{K_t} = \$1.00 kg^{-1} K and that rate of application yields an extra 20 kg ha^{-1} at P_{c_t} = \$1.20 kg^{-1}, the added revenue from the yield increase (\$24 ha^{-1}) would cover the cost of the added fertilizer ($f_{K_t} \cdot K_{cit}^{*}$= \$3.00 ha^{-1}) and the positive net impact of \$21 ha^{-1} would be sufficient to pay for approximately \$18 ha^{-1} for custom fertilizer application (Mississippi State University, 2021) and leave a positive profit margin of \$3 ha^{-1}.

In sum, we expect K_{cit}^{*} to vary directly with the price of the crop and more so the greater the yield potential of that crop in a particular field. However, we expect for K_{cit}^{*} to change indirectly with the cost of fertilizer-K, f_{K_t}. Further, we expect K_{cit}^{*} to be affected by the field's initial STK because a change in *STK* results in a change in the *RY* intercept when plotting K rate against *RY* (Eqs. 1 & 2), no matter the crop being produced. Also, by way of regression coefficient estimates on *K* and *STK* · *K* interaction terms, *STK* impacts the shape and slope of the yield response to K fertilization and thereby marginal revenue. That is, we know from past fertilizer-K studies that high initial STK soils are less likely to show an increase in yield response to the application of fertilizer-K than soils with low initial STK (Mallarino et al., 1991a; Drescher et al., 2021).

The yield response functions shown in Eq. 2 & 3, were a result of choosing among a combination of linear, square root, and quadratic response functional forms for *K*, *STK*, and their interactions for both corn and cotton. Visual evaluation of the goodness of fit across different specifications of the *RY* curve for a particular crop in relation to observed yields, adjusted coefficient of determination, and the number of coefficient estimates with t-statistics that furthered the explanatory power of the model (|t –stat| > 1.0) were used to select the final functional form used to estimate yield response. Econometrics software, EViews v. 9.5 (Lilien et al., 2015), was utilized to calculate generalized least squares coefficient estimates with period random effects and the Wallace and Hussain estimator option for component variances. Initial ordinary least squares estimation of Eqs. 2 & 3 showed heteroskedasticity to be an issue using the Breusch-Pagan-Godfrey heteroskedasticity test ($p < .001$). As such, the coefficient covariance matrix was adjusted using White's cross-section option. Multicollinearity between *K* and *STK* was minimal (Pearson correlation coefficient $\rho = .0301$ for corn and $\rho = -.0566$ for cotton).

To assess annual relative profitability changes between using K^* (we drop subscript *cit* for crop, STK, and year from this point forward to assist with readability) rather than the current extension-recommended rates (K_E), we use Eqs. 9 & 10 to calculate respective partial returns to the application of fertilizer-K. Equations 9 and 10 define $PR_{K_c^*}$ and $PR_{K_{E_c}}$, respectively, which are the revenue from respective crop sales less the cost of fertilizer at the respective fertilizer-K rates. We repeat this process over ten years and over a range of *STK* to capture relative profitability differences across fertilizer-K rate recommendations with changing f_K and P_c for both crops under study. For yield potential we use the annual full-season irrigated corn yield averages and annual irrigated upland cotton yield averages reported for the state of Arkansas (USDA NASS, 2021c), *YP*, depending on the crop under analysis to yield annual estimates of partial returns and return differences as follows:

Eq. [9] $PR_{K_c^*} = \mathrm{YP}_c \cdot \widehat{RY_{K^*}} \cdot P_c - K^* \cdot f_K$

Eq. [10] $PR_{K_{E_c}} = \mathrm{YP}_c \cdot \widehat{RY}_{K_E} \cdot P_c - K_E \cdot f_K$

Eq. [11] $\Delta_{PR_c} = PR_{K_c^*} - PR_{K_{E_c}}$

It is important to note that the same *STK* is used to determine revenue potential in Eqs. 9 and 10, but at different K application treatment rates in the studies for each crop. Because fertilizer-K is usually coupled with phosphorus (P) fertilizer, when applying, we consider equipment, labor, and fuel application charges as a sunk cost or the same whether applying K^* or K_E. Also, since P and/or N fertilizer application would occur regardless of the amount of K applied and since K may be applied jointly with P or N or both, we ignore the cost of fertilizer-K application (fuel, labor, and equipment charges) for the moment as they would not impact relative profitability across K^* and K_E.

To keep comparisons manageable, we chose three levels of initial STK at current boundaries and a mid-point to fertilizer rate recommendations based on the STK range (Table 2.1) for each corn and cotton. Further, the spreadsheet-based decision support software, attached to this work, enables a user to easily calculate a profit-maximizing fertilizer-K rate recommendation for their operation when providing field- and year-specific information. Also, the decision support software will allow for the user to add a charge for fertilizer application, such as a custom rate or the cost for labor, fuel, and equipment used to apply the fertilizer if they so choose. The latter provides the means to evaluate whether or not to use K^* as sufficient added net revenue may not cover fertilizer application charges as already discussed. This calculation is important in a scenario when K^* is small (perhaps because of high *STK*, high f_K, or low P_c) or when fertilizer application charges are high (for example, when K is applied solely in a special application rather than in combination with other fertilizer(s) where the cost of fertilizer application could be divided among the different nutrient needs the fertilizer application serves). The interested reader is directed to Popp et al. (2020, 2021), where steps needed to use the decision support software for K^*, based on user-specified input for a specific crop, are explained.

C. Results

1. Corn

Statistical results from Eq. 2 when $c = corn$ show most explanatory variables were statistically significant ($p < .05$) and that approximately 43% of the variation in relative yield was explained by changes in the explanatory variables (Table 2.2) using the quadratic functional form for both *STK* and fertilizer-K rate (*K*). Even though the quadratic functional form for both *STK* and *K* produced a lower adjusted R^2 when compared to the quadratic functional form for *STK* with the square root functional form for *K*, recommendations for the profit-maximizing fertilizer-K rate are expected to range between 0 and less than the amount required to produce the maximum yield. Thus, a constant term with greater statistical significance became a key focus for functional form decisions. The quadratic functional form for both *STK* and *K* had the constant term that was most statistically significant ($p = .078$), and hence that functional form was chosen. Visual assessment of predictions in relation to observed values (Figure 2.2) confirmed this choice.

Overall, Figure 2.3A shows a contour plot of agronomic yield responses to fertilizer-K rate at different *STK* and indicates that yield increases to K fertilization diminish with increasing *STK*. Soils that have low *STK* values tend to have predicted corn yield increases to low fertilizer-K rates that are greater (steeper) than the predicted corn yield responses for soils that have higher *STK* values (Figure 2.2). Table 2.3 provides a summary of the effects of *STK* on K^* and also compares profitability and yield implications of using profit-maximizing rather than current extension recommendations using historical Arkansas corn price and yield information. The last column on the right reports the estimated STK threshold where it is no longer economically feasible to apply fertilizer-K given different corn price and fertilizer cost assumptions with fertilizer application charges set to zero.

Over the period from 2010-2019, we estimate that at a *STK* of 60 mg K kg^{-1}, the profit-maximizing fertilizer-K rate, K^*, was 38 kg K ha^{-1} less than the current recommendation, K_E, resulting in an average predicted yield loss of 32 kg ha^{-1} (Table 2.3). The yield loss incurred was smaller than the fertilizer cost savings and thus would net a producer an average $34.24 ha^{-1} more each year than had they followed the current recommendation. At a *STK* value of 75 mg K kg^{-1}, the fertilizer use, yield implications, and profit differences of following K^* as opposed to K_E were smaller than at a *STK* value of 60 mg K kg^{-1} but follow the same trend. At a *STK* value of 90 mg K kg^{-1}, fertilizer-K savings were the largest, and thereby yield loss was also the highest. However, the net effect of yield loss vs. cost savings was between the profit changes shown for *STK* = 60 and STK = 75 as presented in Table 2.3. Finally, given this data, there was no occasion when fertilizer cost was relatively low enough or corn price was relatively high enough to result in a K^* that was higher than K_E. However, in 2021, with P_c = $0.26 kg^{-1}, f_K = 0.73 $ kg^{-1} K and yield potential set to the experimental average at 13,257 kg ha^{-1}, K^* exceeds K_E by 13 kg K ha^{-1} when STK = 75 mg K kg^{-1} when using the decision support software. In other words, following the long-term average difference between K^* and K_E would not be a good recommendation.

The last column in Table 2.3 suggested curtailing the application of fertilizer-K once *STK* has a value of 97 mg K kg^{-1} on average. This is a higher STK threshold value with fertilizer application charges set to zero than if fertilizer application cost was not considered a sunk cost. Profitable fertilizer application would be curtailed at a lower STK threshold value if application charges were considered since yield increases from fertilizer use would now also need to cover the equipment, labor, and fuel charges incurred to apply said fertilizer. In 2010, for example, if a producer paid \$18 ha^{-1} for fertilizer application, the adjusted threshold for applying fertilizer-K lowers from 95 to 87 mg K kg^{-1} as nearly 437 kg ha^{-1} of corn are needed to offset the costs associated with 59 kg ha^{-1} of fertilizer-K and the application charge of \$18 ha^{-1}. At *STK* > 87 mg K kg^{-1}, adequate nutrient availability in the soil raises the yield potential without fertilizer application and diminishes the predicted yield increase from fertilizer-K (Figure 2.2). Thus, Table 3 reveals that, overall, fertilizer-K rate could be reduced compared to the current recommendations and that corn price, fertilizer cost, and fertilizer application charges play an important role along with agronomic yield response to K fertilization and existing soil nutrient availability as shown in Figure 2.2 and Figure 2.3.

2. Cotton

When $c = cotton$, the statistical results from Eq. 3 illustrate that most explanatory variables were statistically significant ($p < .05$) with approximately 53% of the variation in relative cotton yield was explained by changes in the explanatory variables (Table 2.4). Using the quadratic functional form for *STK* and the square root functional form for fertilizer-K rate (*K*) resulted in the highest adj. R^2 and goodness of fit when analyzed visually (Figure 2.2).

The contour plot of agronomic cotton yield responses to fertilizer-K rate (Figure 2.3B) illustrates cotton yields respond to K fertilization at different STK. Like corn, cotton grown on soils with low *STK* values tend to have predicted yield increases to low fertilizer-K rates that are greater (steeper) than the predicted cotton yield responses for soils that have higher *STK* values (Figure 2.2). However, yield potential (95% RY) is estimated not to be attainable for any of the STK values observed in the study. As such, supplemental fertilizer-K plays an important role.

Table 2.5 provides a summary of comparisons between profit-maximizing and current fertilizer-K rate recommendations along with attendant yield and profitability implications similar to results shown in Table 2.3 for corn. Using historical Arkansas irrigated cotton price and yield information we estimate that at a *STK* of 75 mg K kg^{-1}, the profit-maximizing fertilizer-K rate, K^*, was on average 36 kg K ha^{-1} more than the current recommendation, K_E, resulting in an average predicted yield increase of 27 kg ha^{-1} (Table 2.5). The estimated yield increase was larger than the fertilizer cost outlay and therefore would improve the producer profitability by an average of $11.54 ha^{-1} each year in comparison to following the current rate recommendations. The same scenario unfolds for *STK* values of 90 and 110 mg K kg^{-1}. Profit-maximizing fertilizer rates are higher than current recommendations resulting in greater yield and more profit. The profit-maximizing fertilizer use increases relative to current recommendations the higher the STK but somewhat abruptly stops at STK > 118 mg K kg^{-1}. This is in part a function of the response equation estimated and was similar regardless of what functional form for K and STK was chosen for Eq. 3. The contour plot in Figure 2.3B, as well as Figure 2.2, indicates that yield responses to K are relatively minor for 120 < STK < 177 mg K kg^{-1}. Interestingly, the response curve becomes slightly U-shaped and may well be likened to being nearly flat although yield response near the higher end of the spectrum of fertilizer-K application rates does show a yield response. As can also be observed in Figure 2.2, the number of actual observations used at higher STK are fewer, and hence statistical methods for curve fitting were less influenced by observations for that STK range. It may well be that weather and/or cultivar differences contributed to these findings that could not be accounted for in this analysis given our relative yield approach. Hence, the user of the spreadsheet tool is warned to exercise caution about following profit-maximizing rate recommendations for STK > 120 mg K

kg^{-1}. Further, Figure 2.3B shows that reaching full yield potential is attainable at STK < 120 mg K kg^{-1}. As such, building STK beyond that level is likely counterintuitive.

Notable, overall, in Figure 2.2 is that the yield response curves are steeper for cotton than corn when making pairwise comparisons at identical STK. From an economic perspective, the steeper yield response curves suggest that cotton is more sensitive to K deficiency than corn and hence fertilizer-K recommendations for cotton should be higher than currently recommended. In stark contrast to Table 2.3, Table 2.5 reveals no instance where K^* was lower than K_E. In addition to yield response to K, the value of the crop thereby plays an important role. The value of a one percent increase in relative yield for corn assuming average yield and the past 10 yr-average price is 109.39 kg ha^{-1} × \$0.18 kg ha^{-1} = \$19.69 ha^{-1}. The same valuation for a 1% increase in relative yield amounts to \$20.92 ha^{-1} for cotton. With fertilizer cost the same regardless of what crop it is used for, the revenue change for added fertilizer-K for cotton is higher than for corn and along with the greater fertilizer-K crop response in cotton (Figure 2.2) we expect a producer to use greater K fertilizer rates in cotton than corn.

Much like for the results in corn, application charges (fuel, equipment, and labor) will lower the threshold STK value for cotton as well. Using 2010 as an example, as we did for corn, the STK threshold drops from 122 to 117 mg K kg^{-1}. Again, the more valuable the crop, the less increase in yield is required to pay for the application charges as the corn STK threshold dropped from 95 to 87 mg K kg^{-1}. As with corn, cotton price, fertilizer cost, and fertilizer application costs all play a crucial role along with agronomic yield response to applied fertilizer-K and existing soil K availability as shown in Figure 2.2 and Figure 2.3. Overall, fertilizer-K rate could be profitability increased in comparison to current recommendations up to STK = 120 mg K kg^{-1} simultaneously increasing yield and profits given the high value associated with cotton lint

(Table 2.5). Beyond that level, fertilizer-K is still estimated to increase yield but the results are based on too few observations to be considered reliable.

3. Four Crop Comparison – Rice, Soybean, Corn, and Cotton

Because growers typically produce more than one crop on their operation, it is of interest to compare the differences across four crops that have to date been analyzed for profit-maximizing application rates or K^*. Table 2.6 provides a summary of the 10-year averages of changes in the amount of fertilizer-K applied, yield, and relative profit changes for soybean, rice, corn, and cotton when applying the profit-maximizing rate versus the current extension recommendation at a uniform rate across the field. For most crops, K^* is lower than K_E which is a function of crop value as well as yield response. All crops display a diminishing yield response to K fertilizer when STK increases. At a relatively low STK value of 75 mg K kg^{-1} crop yield response differences as well as crop value point to results shown in Table 2.6. If crop value is relatively low and crop response is intermediate as observed for soybean (Figure 2.4), K fertilizer use can be curtailed profitably. When the crop value is relatively high but the yield response is intermediate to low, K fertilizer use can also be profitably lessened, as is the case for rice and corn (Figure 2.2 & 2.4). When the crop value is relatively high and K yield response is high, K fertilizer use is profitable as is the case for cotton (Figure 2.2).

D. Discussion

Without added fertilizer-K, reaching near maximum yield (95% RY) in corn production can be accomplished when STK is 109 mg K kg^{-1}, but the current soil-test-based fertilizer-K recommendations advise applying fertilizer-K until *STK* exceeds 175 mg K kg^{-1}, the upper boundary of the optimum STK level (Table 2.1). The current recommendation to apply 47 kg K ha^{-1} fertilizer-K when STK is 131-175 mg K kg^{-1} is a "grower option" recommendation approximating the K removed by a corn yield of 12,589 kg ha^{-1}, which is intended to maintain the soil-test K in the optimal range. Fertilizer-K rate recommendations based on soil tests are usually created using a combination of yield response to fertilization, soil nutrient management philosophy, and professional judgment. Agronomic-based fertilizer recommendations commonly reduce the risk of yield loss that may result from under-fertilization to try to replace the nutrients that will be removed by the harvested crop and to maintain *STK* values that are at or near an optimal level. However, the calculated profit-maximizing fertilizer-K application rate is lower than the current agronomic recommendation for corn crop production. In line with the results of this study, Mallarino et al. (1991b) reported positive returns to K fertilization of corn and soybean at low STK levels and negative returns to K fertilization at medium or high STK levels. Therefore, they concluded that fertilization practices for corn and soybean could be profitably curtailed by not applying fertilizer-K to soils with a medium-level STK.

Olson et al. (1982) presented that crop fertilizer rate decisions based on the "sufficiency approach" proved to be more profitable than an approach that uses fertilizer rates to "build and maintain" soil-test nutrient levels. However, farmers often believe that the lower fertilizer rate of the sufficiency-based recommendations are too conservative and will reduce the soil's fertility level, prevent the production of maximum crop yields over time, or the worst-case scenario, both. It may be economically logical to use profit-maximizing fertilizer-K rates with the sufficiency approach as part of the producer’s nutrient management strategy because the increased amount of fertilizer-K applied does not always build STK in the surface soil (Fulford & Culman, 2018). This is because i) some of the K may leach below the recommended soil-sample depth in sandy-textured soils while still being available for crop uptake (Rehm et al., 1984); ii) STK and the fertilizer-K rate may have a weak relationship (Fryer et al., 2019); iii) K loss from runoff and soil erosion are not accounted for in nutrient replacement equations (Jones & Hinesly, 1986); iv) STK varies throughout time (Oltmans & Mallarino, 2015), space (Mallarino & Wittrey, 2004) and soil moisture (Luebs et al., 1956), making consistent soil-test results difficult to obtain; and, v) the same yield potential can be obtained with supplemental fertilizer-K regardless of STK (Figure 2.2 and Figure 2.3) which ultimately results in a producer paying for fertilizer earlier than needed when the goal is to build STK in the case of producing corn.

Contrastingly, reaching near maximum (≥95% RY) cotton yield without added fertilizer-K was estimated to be unachievable across the range of STK evaluated (Figure 2.3B) indicating some level of supplementation is profitable for cotton. Long-term fertility plots have documented that cotton yield is more sensitive to K deficiency than corn and soybean (Mitchell et al., 2005). Cotton was a more valuable crop than corn, on average, over the period of 2010-19 and because we estimate a greater yield response to K for cotton than for corn, the profit-maximizing fertilizer-K rates were higher than current extension recommendations up to STK < 120 mg K kg^{-1}. We estimate a STK threshold near 120 mg K kg^{-1} at which point fertilizer-K recommendations are less reliably going to yield sufficient added revenue to pay for both the K fertilizer and application charges.

Paying attention to corn and cotton value and fertilizer cost will pay dividends when applying K fertilizer at uniform rates in comparison to following recommendations based on agronomic input alone. Thus, the results showcase the need for the decision support software to assist producers with selecting profit-maximizing fertilizer-K rates that consider yield response as well as input cost and output price, rather than selecting a rate based primarily on yield response. Although building STK by using increased fertilizer-K rates offers producers insurance to meet a yield target in the sense that they produce under a flatter yield response curve to fertilizer-K, this insurance comes at a cost.

Overall, the results for corn and cotton showcase the need for decision support software, as already available for soybean and rice, to aid in selecting profit-maximizing fertilizer-K rates that include both agronomic and economic information (crop prices and fertilizer K costs). Additional research is needed to examine the long-term economic and agronomic benefits of applying fertilizer to either obtain maximum profitability rather than building or maintaining STK as this study did not model carryover of K.

E. References

Debertin, D. L. (1986). *Agricultural production economics*. MacMillan Publishing Company. New York: New York. p. 52.

Dhillon, J. S., Eickhoff, E. M., Mullen, R. W., & Raun, W. R. (2019). World potassium use efficiency in cereal crops. *Agronomy Journal*, *111*, 889-896. https://doi.org/10.2134/agronj2018.07.0462

Drescher, G., Slaton, N., Roberts, T., & Smartt, A. (2021). Corn yield response to phosphorus and potassium fertilization in Arkansas. *Crop, Forage, & Turfgrass Management,* e20120. https://doi.org/10.1002/cft2.20120

Fryer, M. S., Slaton, N. A., Roberts, T. L., & Ross, W. J. (2019). Validation of soil-test-based phosphorus and potassium fertilizer recommendations for irrigated corn. *Soil Science Society America Journal, 83,* 825-837. https://doi:10.2136/sssaj2019.02.0032

Fulford, A. M. & Culman, S. W. (2018). Over-fertilization does not build soil test phosphorus and potassium in Ohio. *Agronomy Journal, 110,* 56-65. https://doi:10.2134/agronj2016.12.0701

Green, W. H. (2008). Econometric Analysis, 6th Ed. New York: New York. Pearson-Prentice Hall. p 208.

Heckman, J. R., & Kamprath, E. J. (1992). Potassium accumulation and corn yield related to potassium fertilizer rate and placement. Agronomy Journal, 56, 141-148. https://doi.org/10.2136/sssaj1992.03615995005600010022x

Jones, R. L., & Hinesly, T. D. (1986). Potassium losses in runoff and drainage waters from cropped, large-scale lysimeters. Journal Environmental Quality, 15, 137-140. https://doi.org/10.2134/jeq1986.00472425001500020010x

Kelley, J., & Capps, C. (2021). 2021 Arkansas Corn Quick Facts. University of Arkansas Cooperative Extension. https://www.uaex.edu/farm-ranch/crops-commercial-horticulture/corn/2021%20Arkansas%20Corn%20Quick%20Facts.pdf

Kerby, T. and Adams, F. (1985). Potassium Nutrition of Cotton. In Potassium in Agriculture, R.D. Munson (Ed.). https://doi.org/10.2134/1985.potassium.c36

Lewis, K., Morgan, G., Frame, W. H., Fromme, D., Dodds, D. M., Edmisten, K. L., Robertson, B., Boman, R., Cutts, T., Delaney, D. P., Burke, J. A., & Nichols, R. L. (2021). Cotton yield response to soil applied potassium across the U. S. cotton belt. Agronomy Journal, 113, 3600– 3614. https://doi.org/10.1002/agj2.20719

Lilien, D., Sueyoshi, G., Wilkins, C., Wong, J., Thomas, G., Yoo, S., … Noh, J. (2015). Eviews 9. Irvine, CA: IHS Global Inc.

Luebs, R. E., Stanford, G., & Scott, A. D. (1956). Relation of available potassium to soil moisture. Soil Science Society of America Journal, 20, 45-50. https://doi:10.2136/sssaj1956.03615995002000010011x

Mallarino, A. P., Webb, J. R., & Blackmer, A. M. (1991a). Corn and corn yields during 11 years of phosphorus and potassium fertilization on high testing soils. Journal of Production Agriculture, 4, 312-317.

Mallarino, A. P., Webb, J. R., & Blackmer, A. M. (1991b). Soil test values and grain yields during 14 years of potassium fertilization of corn and corn. Journal of Production Agriculture, 4, 562-566.

Mallarino, A. P. & Wittrey, D. J. (2004). Efficacy of grid and zone soil sampling approaches for site-specific assessment of phosphorus, potassium, pH, and organic matter. Precision Agriculture, 5, 131-144. https://doi.org/10.1023/B:PRAG.0000022358.24102.1b

Marschner, H. 2012. Marschner’s mineral nutrition of higher plants. Academic Press, New York.

Mozaffari, M., Slaton, N. A., Varvil, J., Bourland, F. M., & Kennedy, C. (2007). Seedcotton yield and petiole potassium concentrations as affected by potassium fertilization. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2006 (pp. 21–26). Research Series 548. Univ. of Arkansas Agricultural Experiment Station.

Mozaffari, M., Slaton, N. A., Long, J., Bourland, F. M., Hood, A. J., & Kennedy, C. (2008). Cotton response to potassium fertilization at multiple locations. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2007 (pp. 32–34). Research Series 558. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/558.pdf

Mozaffari, M., & Slaton, N. A. (2011). Potassium fertilization increases corn grain yield. In (N. A. Slaton Ed.), W. E. Sabbe Arkansas soil fertility studies 2010 (pp. 27–29). Research Series 588. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/588.pdf

Mozaffari, M., Slaton, N. A., Hayes, S., & Griffin, B. (2012). Corn response to soil applied phosphorus and potassium fertilizer in Arkansas. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2011 (pp. 30–33). Research Series 599. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/599.pdf

Mozaffari, M., Slaton, N. A., Apple, B., Baker, S., Chlapecka, R., Elkins, C., Griffin, B., Hayes, S., &Wimberley, R. (2013). Soil applied phosphorus and potassium increase corn yield in Arkansas. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2012 (pp. 29–34). Research Series 608. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/608.pdf

Mozaffari, M., Slaton, N. A., Hayes, S., & Hedge, J. (2014). Soil-applied phosphorus and potassium increase corn yield in Arkansas. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2013 (pp. 29–33). Research Series 616. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/616.pdf

Mozaffari, M., Slaton, N. A., Hedge, J., Hayes, S., Baker, R., Crow, M., Davis, A., & Hamilton, M. (2015). Corn response to soil-applied phosphorus and potassium at multiple locations in Arkansas. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2014 (pp. 45–49). Research Series 624. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/624.pdf

Mozaffari, M., Slaton, N. A., Davis, A., Liyew, Y. D., Hayes, S., & Hedge, J. (2016). Corn responds positively to soil-applied phosphorus and potassium at multiple locations in Arkansas. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2015 (pp. 33–36). Research Series 633. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/633.pdf

Mozaffari, M., Wilson, C. E., Jr., Hays, H. C., Liyew, Y. D., Runsick, S., Carroll, A. G., Horton, P., & Griffin, B. (2018). Corn response to soil-applied phosphorus and potassium at multiple locations in Arkansas. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2017 (pp. 25–28). Research Series 649. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/649.pdf

Mozaffari, M., Wilson C. E., Jr., Hays, Z. M., Hays, H. C., Hedge, J. M., Gibson, C. D., Perkins, K. J., & Runsick, S. K. (2019). Effect of soil-applied phosphorus and potassium on corn grain yield in Arkansas. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2018 (pp. 28–31). Research Series 657. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/657_Sabbe_Arkansas_Soil_Fertility_Studies_2018.pdf

Mozaffari, M., Wilson, C. E., Jr., Hays, Z. M., Hedge, J. M., Mann, M. G., Perkins, K. M., Wimberley, R. A., & Sayger, A. M. (2020). Corn grain yield response to soil-applied phosphorus and potassium in Arkansas. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2019 (pp. 51–55). Research Series 666. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/666_Sabbe_Arkansas_Soil_Fertility_Studies_2019.pdf

Mozaffari, M., Wilson Jr., C. E., Hays, Z. M., Beach, A. B., Brown, E. G., Martin, L. R., & Hayes, S. (2020). Effect of soil-applied phosphorus and potassium on seedcotton yield in Arkansas. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2019 (pp. 47–50). Research Series 666. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/666_Sabbe_Arkansas_Soil_Fertility_Studies_2019.pdf

Mississippi State University (MSU). (2021). Archived Budget Publications. Department of Agricultural and Applied Economics, Mississippi State University. As accessed 17 May 2021. https://www.agecon.msstate.edu/whatwedo/budgets/archive.php

Mitchell, C. C, Delaney, D., & Balkcom, K. S. (2005) Cullars Rotation: The South's oldest continuous soil fertility experiment. Better Crops with Plant Food, 89(4), 5-9.

Olson, R. A., Frank, K. D., Grabouski, P. H., & Rehm, G. W. (1982). Economic and agronomic impacts of varied philosophies of soil testing. Agronomy Journal, 74, 492–499. https://doi.org/10.2134/agronj1982.00021962007400030022x

Oltmans, R. R., & Mallarino, A. P. (2015), Potassium uptake by corn and corn, recycling to soil, and impact on soil test potassium. Soil Science Society of America Journal, 79, 314-327. https://doi:10.2136/sssaj2014.07.0272

Popp, M., Slaton, N. A., Norsworthy, J. S., & Dixon, B. 2021. Rice yield response to potassium: an economic analysis. Agronomy Journal, 113, 287-297. https://doi:10.1002/agj2.20471

Popp, M., Slaton, N. A., & Roberts, T. L. 2020. Profit-maximizing potassium fertilizer recommendations for soybean. Agronomy Journal, 112, 5081-5095. https://doi:10.1002/agj2.20424.

Rehm, G. W., Sorensen, R. C., & Wiese, R. A. (1984). Soil test values for phosphorus, potassium, and zinc as affected by rate applied to corn. Soil Science Society America Journal, 48, 814-818. https://doi.org/10.2136/sssaj1984.03615995004800040023x

Robertson, B., Barber, T., & Lorenz, G. (2021). 2021 Arkansas Cotton Quick Facts. University of Arkansas Cooperative Extension. https://www.uaex.edu/farm-ranch/crops-commercial-horticulture/docs/2021_Arkansas_Cotton_Quick_Facts_final_accessible.pdf

Shober, A. L., & Taylor, R. W. (2015). Estimating Yield Goal for Crops. University of Delaware Cooperative Extension. https://www.udel.edu/academics/colleges/canr/cooperative-extension/fact-sheets/estimating-yield-goal-crops/

Slaton, N. A., Roberts, T. L., Martin, L., Hayes, S., Treat, C., & Smartt, A. (2019). Cover crop and phosphorus and potassium effects on soil-test values and cotton yield. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2018 (pp. 52–56). Research Series 657. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/657_Sabbe_Arkansas_Soil_Fertility_Studies_2018.pdf

Smartt, A. D., Slaton, N. A., Roberts, T. L., Drescher, G. L., Martin, L., Hayes, S., & Treat, C. (2021). Cover crop and phosphorus and potassium application rate effects on soil-test values and corn yield. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2020 (pp. 43–50). Research Series 675. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/675_Sabbe_Arkansas_Soil_Fertility_Studies_2020.pdf

United States Geological Survey (USGS). (2019). Mineral Commodity Summaries. As accessed 4 Jun 2020. http://prd-wret.s3-us-west-2.amazonaws.com/assets/palladium/production/atoms/files/mcs2019_all.pdf.

United States Department of Agriculture (USDA) – Economic Research Service. (2019). All fertilizer use and price tables in a single workbook. As accessed 15 June 2021. https://www.ers.usda.gov/data-products/fertilizer-use-and-price.aspx

United States Department of Agriculture (USDA) – National Agricultural Statistics Service. (2021a). Annual Arkansas Marketing Year Corn Prices 2010-2020. As accessed 5 May 2021. https://quickstats.nass.usda.gov/results/1ED83457-BF89-3BBB-BD4B-6A24EADC09AC.

United States Department of Agriculture (USDA) – National Agricultural Statistics Service. (2021b). Arkansas Irrigated Corn Yield 2010-2020. As accessed 5 May 2021. https://quickstats.nass.usda.gov/results/D5098EB7-ACF3-3DB5-A803-9F9F6CC12EC5.

United States Department of Agriculture (USDA) – National Agricultural Statistics Service. (2021c). Annual Arkansas Marketing Year Upland Cotton Prices 1977-2020. As accessed 20 May 2021. https://quickstats.nass.usda.gov/results/72730B6F-97E7-3C78-AB81-EDB383BC3422.

United States Department of Agriculture (USDA) – National Agricultural Statistics Service. (2021d). Arkansas Upland Cotton Yield 1977-2020. As accessed 21 May 2021. https://quickstats.nass.usda.gov/results/BFA9252C-0A70-38D3-A1B2-94589A88F8DB

Watkins, B. (2021). 2021 Corn Crop Production Budgets for Farm. University of Arkansas Cooperative Extension Service. https://www.uaex.uada.edu/farm-ranch/economics-marketing/farm-planning/budgets/crop-budgets.aspx (accessed Jun 1, 2021)

Welch, L., & Flannery, R. (1985). Potassium Nutrition of Corn. In Potassium in Agriculture, R.D. Munson (Ed.). https://doi.org/10.2134/1985.potassium.c29

Wilson Jr., C.E., Mozaffari, M., & Hays, H. (2018). Cotton response to phosphorus and potassium fertilizer at multiple locations in Arkansas. In N. A. Slaton (Ed.), W. E. Sabbe Arkansas soil fertility studies 2017 (pp. 52–55). Research Series 649. Univ. of Arkansas Agricultural Experiment Station. https://agcomm.uark.edu/agnews/publications/649.pdf

Zhang, H., Hardy, D. H., Mylavarapu, R., & Wang, J. J. (2014). Mehlich-3. In F. J. Sikora & K. P. Moore (Eds.), Soil test methods from the southeastern United States (pp. 101–110). Southern Coop. Ser. Bull. 419. Univ. Georgia. Retrieved from http://aesl.ces.uga.edu/sera6/PUB/MethodsManualFinalSERA6.asp.

Zörb, C., Senbayram, M., & Peiter, E. (2014). Potassium in agriculture– status and perspectives. Journal of Plant Physiology, 171, 656–669. https://doi.org/10.1016/j.jplph.2013.08.008

F. Tables and Figures

Table 2.1. Observed soil-test K levels as defined by Mehlich-3 extractable soil K concentrations, and the related recommended fertilizer-K rates for full-season irrigated corn from 2010 to 2020 and cotton from 2006-2019 in Arkansas.

Soil-test K[a]		Corn Recommended Fertilizer-K Rate	# of observations from corn trials	Cotton Recommended Fertilizer-K Rate	# of observations from cotton trials
Level	mg K kg^{-1}	kg K ha^{-1}		kg K ha^{-1}	
Very Low	< 61	149	40	130	0
Low	61-90	107	101	88	40
Medium	91-130	74	65	56	33
Optimum	131-175	47	12	37	41
Above Optimum	> 175	0	0	0	9

[a] Recommendations from Kelley and Capps (2021) and Robertson et al. (2021) and represent Mehlich-3 extractable K for a soil depth of 0-15 cm for corn and cotton.

Table 2.2. Statistical results using various functional forms of initial Mehlich-3 soil-test K (*STK*) and fertilizer-K application rate (*K*) to explain relative corn yield (*RY* [a]) from 218 individual treatment observations of trials conducted from 2010 to 2020 in eastern Arkansas with irrigated corn using generalized least squares and treating production year as a random effect.

Model Specification	Square Root STK & Quadratic K	Quadratic STK & Quadratic K		Square Root STK & Square Root K		Quadratic STK & Square Root K	
Explanatory Variable[b]	Coefficient Estimate (SE[c])	Explanatory Variable	Coefficient Estimate (SE)	Explanatory Variable	Coefficient Estimate (SE)	Explanatory Variable	Coefficient Estimate (SE)
Constant	-45.42	Const.	34.07	Const.	-57.76	Const.	29.11
(a_0)	(78.64)		(19.22)		(75.19)		(18.84)
STK	-0.92	*STK*	0.93*	*STK*	-1.01	*STK*	1.00*
(a_1)	(0.93)		(0.45)		(0.89)		(0.44)
$STK^{0.5}$	23.00	STK^{2}	$-3.4\text{x}10^{-3}$	$STK^{0.5}$	25.10	STK^{2}	$-3.7\text{x}10^{-3}$
(a_2)	(17.27)		($2.5\text{x}10^{-3}$)		(16.52)		($2.4\text{x}10^{-3}$)
K	2.57*	*K*	1.16***	*K*	-0.95	*K*	-0.46**
(a_3)	(1.08)		(0.26)		(0.54)		(0.15)
K^{2}	-0.01*	K^{2}	$-4.5\text{x}10^{-3,***}$	$K^{0.5}$	25.39**	$K^{0.5}$	11.64***
(a_4)	($3.8\text{x}10^{-3}$)		($9.4\text{x}10^{-4}$)		(9.49)		(2.54)
$STK \cdot K$	0.02	$STK \cdot K$	-0.02**	$STK \cdot K$	-0.01	$STK \cdot K$	0.01*
(a_5)	(0.01)		(0.01)		(0.01)		($3.1\text{x}10^{-3}$)
$STK^{0.5} \cdot K$	-0.42	$STK^{2} \cdot K$	$6.8\text{x}10^{-5,*}$	$STK^{0.5} \cdot K$	0.15	$STK^{2} \cdot K$	$-2.7\text{x}10^{-5}$
(a_6)	(0.24)		($3.4\text{x}10^{-5}$)		(0.11)		($1.6\text{x}10^{-5}$)
$STK \cdot K^{2}$	$-6.3\text{x}10^{-5}$	$STK \cdot K^{2}$	$-2.7\text{x}10^{-7,*}$	$STK \cdot K^{0.5}$	0.16	$STK \cdot K^{0.5}$	-0.18**
(a_7)	($4.5\text{x}10^{-5}$)		($1.2\text{x}10^{-7}$)		(0.11)		(0.06)
$STK^{0.5} \cdot K^{2}$	$1.6\text{x}10^{-3}$	$STK^{2} \cdot K^{2}$	$-7.0\text{x}10^{-5,**}$	$STK^{0.5} \cdot K^{0.5}$	-4.12*	$STK^{2} \cdot K^{0.5}$	$6.7\text{x}10^{-4,*}$
(a_8)	($8.4\text{x}10^{-4}$)		($2.2\text{x}10^{-5}$)		(2.04)		($3.0\text{x}10^{-4}$)
Adj. R^2	.428		.427		.450		.449

[a] Relative Yield Index calculated using Eq. 1.

[b] Observed soil-test K concentrations as defined by Mehlich-3 extractable soil-K concentrations in mg K kg^{-1} (*STK*) and fertilizer-K application rate (*K*) in kg K ha^{-1}.

[c] The coefficient covariance matrix was adjusted using White's cross-section option. Statistical significance: * -- $p < .05$, ** -- $p < .01$, *** --- $p < .001$.

Table 2.3. Estimates of the differences in fertilizer-K rate (K, kg ha^{-1}), yield (Y, kg ha^{-1}), and partial returns (PR, \$ ha^{-1}) when applying at current agronomic fertilizer-K rate recommendations (Table 2.1) vs. profit-maximizing fertilizer-K rates using past corn prices, fertilizer-K cost, and average Arkansas irrigated corn yields at varying initial soil-test K values assuming the producer uses a single uniform rate.

Year	Corn price	Fertilizer-K cost[a]	Corn Grain Yield	Changes in K, Y, and Profit (PR)[b]	Soil-test K (mg K kg^{-1}) 60	75	90	Soil-test K where $K^* = 0$[c]
	\$ kg^{-1}	\$ kg^{-1}	kg ha^{-1}					mg K kg^{-1}
				K	-41	-15	-64	
2010	0.18	1.02	9,415	Y	-41	-70	-244	95
				PR	34.20	2.64	21.16	
				K	-41	-14	-62	
2011	0.25	1.29	8,851	Y	-36	-61	-219	96
				PR	43.21	3.03	25.73	
				K	-34	-3	-37	
2012	0.27	1.32	11,173	Y	-10	-15	-136	99
				PR	42.39	0.23	12.86	
				K	-34	-4	-38	
2013	0.20	1.05	11,675	Y	-12	-18	-147	99
				PR	33.81	0.23	10.67	
				K	-39	-11	-55	
2014	0.16	1.04	11,738	Y	-36	-60	-243	96
				PR	34.43	1.59	17.58	
				K	-37	-8	-49	
2015	0.16	0.94	11,361	Y	-26	-43	-201	97
				PR	30.72	0.91	13.58	
				K	-36	-6	-44	
2016	0.15	0.75	10,734	Y	-18	-29	-164	98
				PR	24.32	0.43	9.32	
				K	-37	-8	-49	
2017	0.14	0.84	11,487	Y	-26	-43	-201	97
				PR	27.40	0.79	12.02	
				K	-37	-9	-50	
2018	0.15	0.88	11,361	Y	-28	-47	-209	97
				PR	28.95	0.97	13.29	
				K	-46	-24	-84	
2019	0.15	1.22	10,985	Y	-86	-147	-434	93
				PR	42.95	6.68	36.45	
				K	-38	-10	-53	
Avg.[d]	0.18	1.04	10,878	Y	-32	-53	-220	97
				PR	34.24	1.75	17.27	

[a] Fertilizer cost is based on the price of muriate of potash fertilizer (Mississippi State University, 2021) converted to \$ kg^{-1} K. Annual average irrigated Arkansas corn yield and corn prices were obtained from USDA NASS (2021a, 2021b).

[b] See Eq. 7 for K^*, profit-maximizing fertilizer-K rate recommendation. K_E, current fertilizer-K rate recommendations, are summarized in Table 1. Changes in K, Y, and PR are calculated using Eqs. 9 to 11 once yields are estimated at the respective fertilizer-K rates using Eq. 2.

Table 2.3 (Cont.)

[c] Soil-test K values where K^*, the profit-maximizing fertilizer-K rate, is equal to 0 are lower when yield increases from K application need to cover an application charge. The reader can determine that *STK* threshold by using the accompanying tool by changing application charges and modifying *STK* until profit changes compared to no fertilizer use become positive.

[d] Changes in fertilizer-K rate recommendations, yield, and profit are simply averaged across 2010-2019 rather than calculated using the average corn price, fertilizer cost, and yield data.

Table 2.4. Statistical results explaining relative cotton yield (*RY*[a]) as a function of initial Mehlich-3 soil-test K (*STK*) and fertilizer-K application rate (*K*) from 123 individual treatment observations of fertilization trials conducted from 2006 to 2019 (excluding years 2008-2014) in eastern Arkansas with irrigated full-season upland cotton using generalized least squares treating production year as a random effect for alternative model specifications.

Model Specification	Square Root STK & Quadratic K	Quadratic STK & Quadratic K		Square Root STK & Square Root K		Quadratic STK & Square Root K	
Explanatory Variable[b]	Coefficient Estimate (SE[c])	Explanatory Variable	Coefficient Estimate (SE)	Explanatory Variable	Coefficient Estimate (SE)	Explanatory Variable	Coefficient Estimate (SE)
Constant (β_0)	-114.76 (134.91)	Const.	13.38 (37.87)	Const.	-158.07 (144.65)	Const.	-0.59 (40.72)
STK (β_1)	-1.23 (1.01)	STK	0.99 (0.58)	STK	-1.55 (1.08)	STK	1.20 (0.62)
$\sqrt{STK}$ (β_2)	32.10 (23.66)	STK^2	$-3.0x10^{-3}$ ($2.1x10^{-3}$)	$STK^{0.5}$	39.54 (25.33)	STK^2	$-3.8x10^{-3}$ ($2.2x10^{-3}$)
K (β_3)	3.70^{*} (1.66)	K	1.38^{**} (0.50)	K	-2.78^{***} (0.54)	K	-0.85^{***} (0.18)
K^2 (β_4)	-0.01^{**} ($5.0x10^{-3}$)	K^2	$-5.2x10^{-3,**}$ ($1.6x10^{-3}$)	$K^{0.5}$	54.48^{**} (16.32)	$K^{0.5}$	18.20^{***} (5.19)
$STK \cdot K$ (β_5)	0.02 (0.01)	$STK \cdot K$	-0.02^{*} (0.01)	$STK \cdot K$	-0.02^{***} ($4.2x10^{-3}$)	$STK \cdot K$	0.01^{***} ($2.6x10^{-3}$)
$\sqrt{STK} \cdot K$ (β_6)	-0.59^{*} (0.29)	$STK^2 \cdot K$	$5.7x10^{-5,*}$ ($2.7x10^{-5}$)	$STK^{0.5} \cdot K$	0.49^{***} (0.10)	$STK^2 \cdot K$	$-4.7x10^{-5,***}$ ($9.5x10^{-6}$)
$STK \cdot K^2$ (β_7)	$-9.4x10^{-5,*}$ ($3.7x10^{-5}$)	$STK \cdot K^2$	$7.0x10^{-5,**}$ ($2.5x10^{-5}$)	$STK \cdot K^{0.5}$	0.38^{**} (0.12)	$STK \cdot K^{0.5}$	-0.26^{***} (0.08)
$\sqrt{STK} \cdot K^2$ (β_8)	$2.3x10^{-3,**}$ ($8.7x10^{-4}$)	$STK^2 \cdot K^2$	$-2.3x10^{-7,**}$ ($8.7x10^{-8}$)	$STK^{0.5} \cdot K^{0.5}$	-9.13^{**} (2.80)	$STK^2 \cdot K^{0.5}$	$8.7x10^{-4,**}$ ($2.7x10^{-4}$)
Adj. R^2	.493		.495		.530		.532

[a] Relative Yield Index as calculated using Eq. 1.

[b] Observed soil-test K concentrations as defined by Mehlich-3 extractable soil K concentrations in mg K kg^{-1} (*STK*) and fertilizer-K application rate (*K*) in kg K ha^{-1}.

[c] The coefficient covariance matrix was adjusted using White's cross-section option. Statistical significance: * -- $p < .05$, ** -- $p < .01$, *** --- $p < .001$.

Table 2.5. Estimates of the differences in fertilizer-K rate (K, kg ha^{-1}), yield (Y, kg ha^{-1}), and partial returns (PR, \$ ha^{-1}) when applying at current agronomic fertilizer-K rate recommendations (Table 2.1) vs. profit-maximizing fertilizer-K rates using past cotton prices, fertilizer-K cost, and average Arkansas irrigated upland cotton yields at varying initial soil-test K values assuming the producer uses a single uniform rate.

Year	Cotton Lint Price	Fertilizer-K cost[a]	Cotton Yield	Changes in K, Y, and Profit (PR)[b]	Soil-test K (mg K kg^{-1}) 75	90	110	Soil-test K where $K^* = 0$[c]
	\$ kg^{-1}	\$ kg^{-1}	kg ha^{-1}					mg K kg^{-1}
				K	34	49	130	
2010	1.73	1.02	1,171	Y	26	35	89	122
				PR	9.80	9.84	20.96	
				K	26	32	76	
2011	2.20	1.29	1,041	Y	18	21	48	118
				PR	6.78	5.39	7.26	
				K	15	12	6	
2012	1.69	1.32	1,193	Y	13	10	5	118
				PR	2.22	0.80	0.09	
				K	43	68	130	
2013	1.86	1.05	1,270	Y	33	49	96	118
				PR	16.64	19.39	42.08	
				K	30	40	130	
2014	1.53	1.04	1,283	Y	26	32	97	118
				PR	7.60	6.92	12.69	
				K	36	52	130	
2015	1.56	0.94	1,224	Y	28	38	93	118
				PR	9.86	10.19	22.10	
				K	53	94	130	
2016	1.62	0.75	1,205	Y	37	59	91	118
				PR	19.38	25.48	50.07	
				K	54	96	130	
2017	1.68	0.84	1,319	Y	40	66	100	118
				PR	22.49	29.82	58.02	
				K	47	79	130	
2018	1.67	0.88	1,270	Y	36	55	96	118
				PR	17.61	21.76	45.46	
				K	18	17	20	
2019	1.46	1.22	1,328	Y	17	16	17	118
				PR	3.04	1.56	0.76	
				K	36	54	101	
Avg.[d]	1.70	1.04	1,231	Y	27	38	73	118
				PR	11.54	13.12	25.95	

[a] Fertilizer cost is based on the price of muriate of potash fertilizer (Mississippi State University, 2021) converted to \$ kg^{-1} K. Annual average irrigated Arkansas seed cotton yield and cotton prices were obtained from USDA NASS (2021c, 2021d). Seed cotton values were converted to cotton lint yield using a 38% lint turnout ratio and cotton lint price by adding a \$0.11 kg^{-1} gin rebate experienced by producers (R. Benson, personal communication, 27 May 2021).

Table 2.5 (Cont.)

[b] See Eq. 8 for K^*, profit-maximizing fertilizer-K rate recommendation. K_E, current fertilizer-K rate recommendations, are summarized in Table 1. Changes in K, Y, and PR are calculated using Eqs. 9 to 11 once yields are estimated at the respective fertilizer-K rates using Eq. 3.

[c] Soil-test K values where K*, the profit-maximizing fertilizer-K rate, is equal to 0 are lower when yield increases from K application need to cover an application charge. The reader can determine that *STK* threshold by using the accompanying tool by changing application charges and modifying *STK* until profit changes compared to no fertilizer use become positive.

[d] Changes in fertilizer-K rate recommendations, yield, and profit are simply averaged across 2010-2020 rather than calculated using the average cotton price, fertilizer cost, and yield data.

Table 2.6. Ten-year average inter-crop comparison of estimates of differences in fertilizer use, yield, and profitability when applying at current agronomic fertilizer-K rate recommendations vs. profit-maximizing fertilizer-K rates using past crop prices, fertilizer-K cost, and average Arkansas crop yields at varying initial soil-test K values assuming the producer uses a single uniform rate.

		Crop			
		Soybean	**Rice**	**Corn**	**Cotton**
Price ($ kg^{-1})		0.40	0.28	0.18	1.70
Yield (kg ha^{-1})		3,261	8,239	10,878	1,231
Fertilizer-K cost[a] ($ kg^{-1})		1.04			
STK		**Changes in Fertilizer Use, Yield, and Profit by STK[b]**			
60	K (kg K ha^{-1})	-47	-13	-38	NA
	Y (kg ha^{-1})	-52	-41	-32	
	PR ($ ha^{-1})	29.10	3.70	34.24	
75	K	-20	-0.35	-10	36
	Y	-44	-4	-53	27
	PR	5.15	0.88	1.75	11.54
90	K	-35	-41	-53	54
	Y	-67	-129	-220	38
	PR	11.61	8.87	17.27	13.12
110	K	-9	NA	NA	101
	Y	-25			73
	PR	1.77			25.95
Threshold STK where K^{*}= 0[c]		129	97	97	118
Value of 1% Relative Yield in $ ha^{-1}		$13.01	$23.07	$19.69	$20.92

[a] Fertilizer cost is based on the price of muriate of potash fertilizer (Mississippi State University, 2021) converted to $ kg^{-1} K. Annual average irrigated Arkansas seed cotton yield and cotton prices were obtained from USDA NASS (2021c, 2021d). Seed cotton values were converted to cotton lint yield using a 38% lint turnout ratio and cotton lint price by adding a $0.11 kg^{-1} gin rebate experienced by producers (R. Benson, personal communication, 27 May 2021).

[b] Profit-maximizing fertilizer-K rate recommendations are compared to K_E, current agronomic fertilizer-K rate recommendations in a similar manner across all crops. Changes in K, Y, and PR are calculated using Eqs. 9 to 11 once yields are estimated at the respective fertilizer-K rates using crop-specific yield response equations.

[c] Soil-test K values where K*, the profit-maximizing fertilizer-K rate, is equal to 0 are lower when yield increases from K application need to cover an application charge. The reader can determine that *STK* threshold by using the accompanying tool by changing application charges and modifying *STK* until profit changes compared to no fertilizer use become positive.

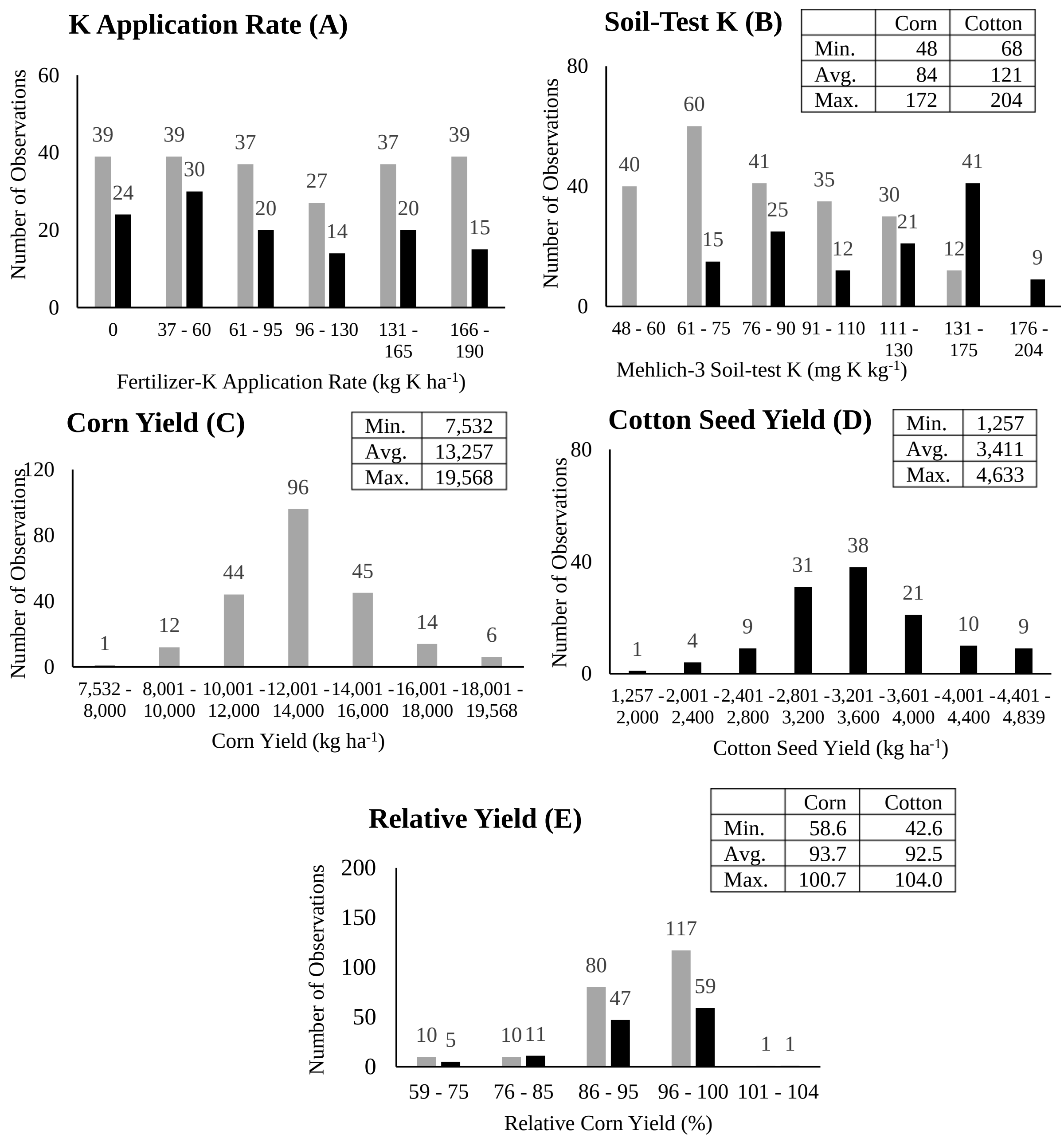

Figure 2.1. Frequency distributions for corn (gray) and cotton (black) of i) application rates for fertilizer-K (A); ii) concentrations of Mehlich-3 extractable soil-test K (STK) at the 0-15 cm soil depth (B); iii) corn grain yields (C); iv) cotton seed yield (D); and v) relative yield percentages (E) as used or observed in Arkansas for corn across 39 site-years with 218 individual fertilizer-K rate treatments from 2010 to 2020 and for cotton across 24 site-years and 123 individual fertilizer-K rate treatments from 2006-2019.

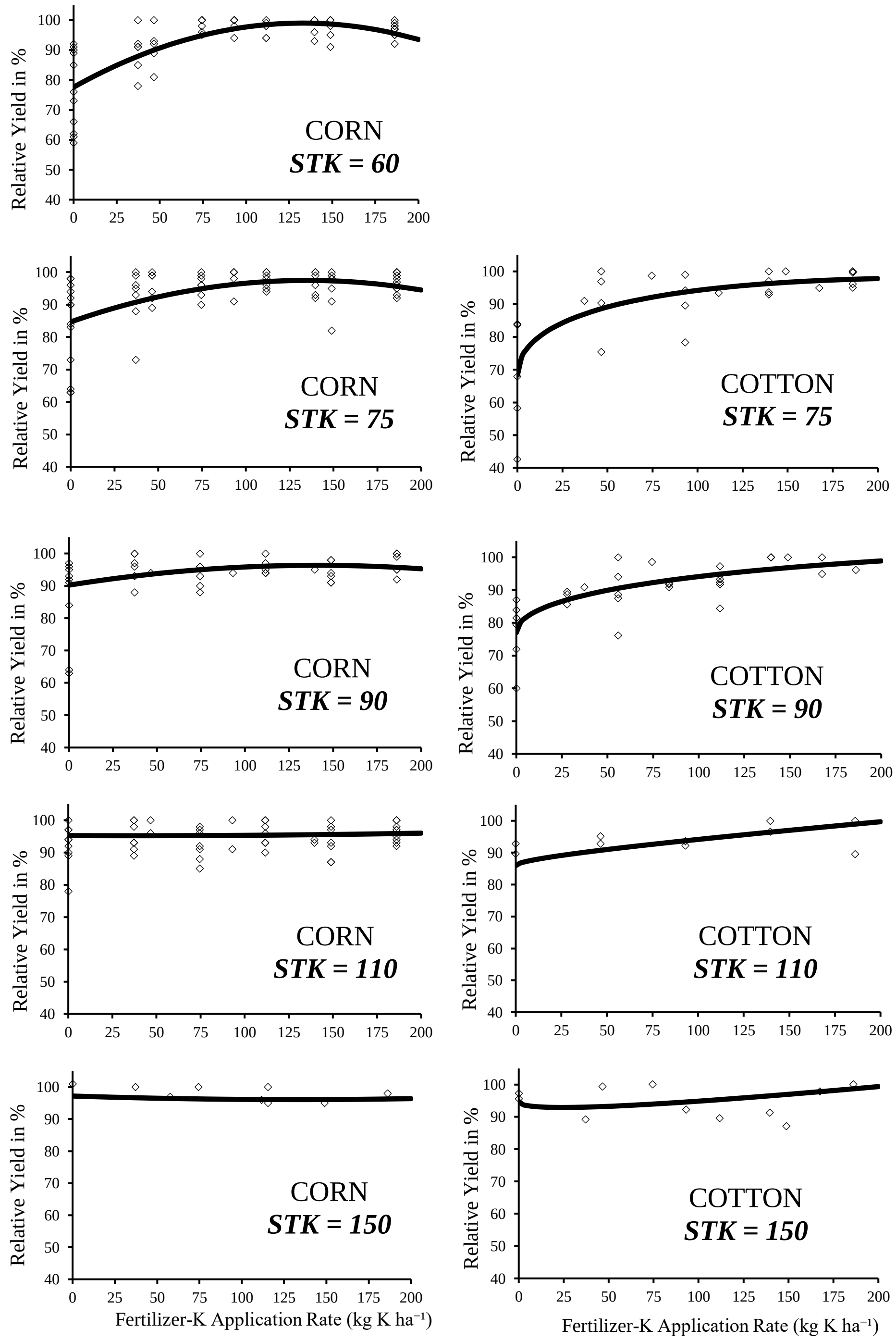

Figure 2.2. Estimated corn and cotton relative yield responses to fertilizer-K rate at 60, 75, 90, 110, and 150 mg K kg $^{-1}$ Mehlich-3 extractable soil-K (STK) concentrations (mg kg $^{-1}$) for the 0-15 cm depth. The observed relative yields (◊) are shown for observations that fell within a band of ± 10 mg K kg $^{-1}$ STK. Cotton observations were not available for STK < 68 mg K kg $^{-1}$.

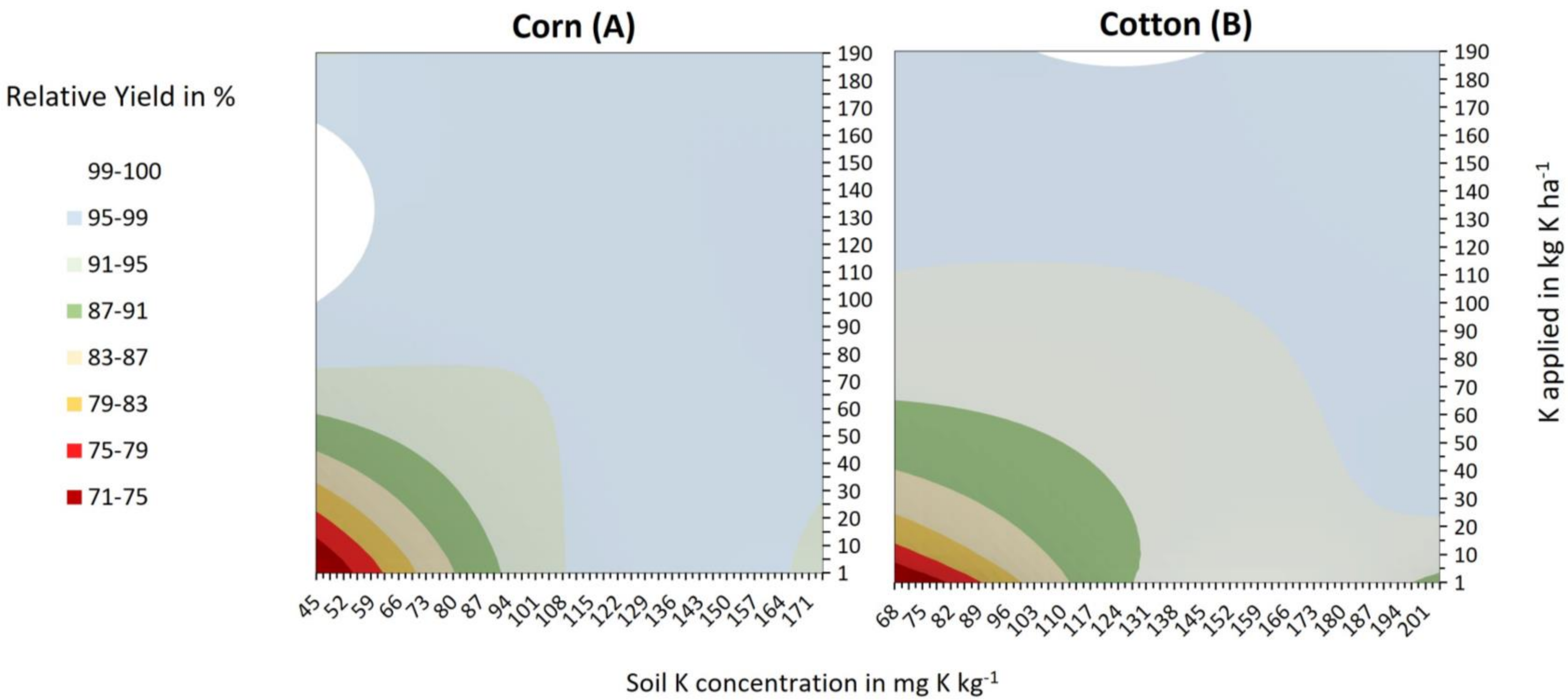

Figure 2.3. Estimated covariate relative yield response of fertilizer-K rate and initial Mehlich-3 soil-test K (STK) concentration on corn (left) and cotton (right).

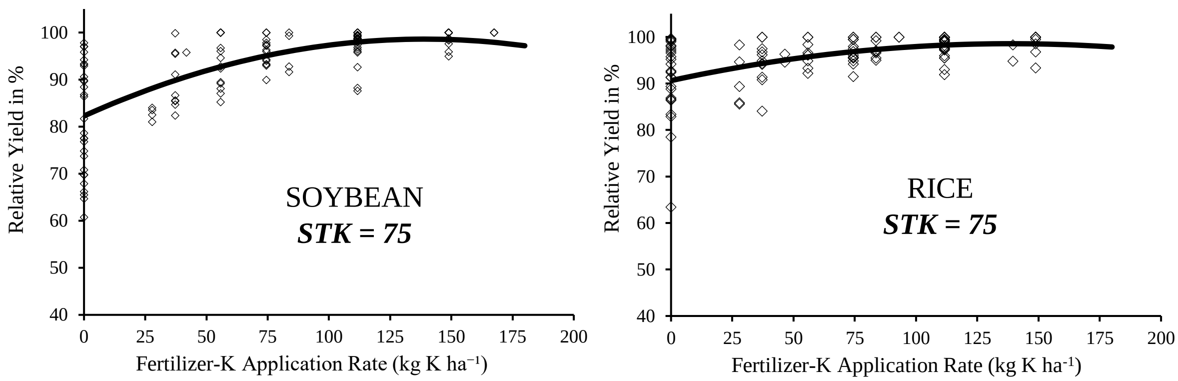

Figure 2.4. Estimated yield response to fertilizer-K for soybean and rice from prior research (Popp et al., 2020, 2021).

Chapter III. Profit-Maximizing Potash Fertilizer Rate Recommendations: Justifying the Use of Short-Term Studies

A. Introduction

An essential macronutrient in the production of most agricultural commodities grown in the U.S. Mid-South region, specifically in both rice and soybean, is potassium (K). K is responsible for growth, disease resistance abilities, and various functions, such as carbohydrate translocation, photosynthesis, and enzyme reactions (Marschner, 2012). However, as agricultural demand for K fertilizer sources have increased (Dhillon et al., 2019), it has become of more importance to efficiently use this depletable resource (USGS, 2019; Zörb et al., 2014). Producers typically choose to apply fertilizer-K at rates that are based on a "sufficiency approach" versus an approach to "build and maintain" (Leikam et al., 2003). The sufficiency approach method derives fertilizer recommendations based on soil test levels that generate a yield response from the crop where, past a certain point when there is no longer a sufficient yield response to warrant fertilizer application, fertilizer-K is no longer applied (Olson et al., 1982). Risks associated with the sufficiency approach are the potential depletion of STK levels in a field, yield loss from crop-K deficiency, and with potentially low STK levels, the need for supplemental K fertilizer every year even when fertilizer-cost is high. In contrast, the build and maintain approach applies more fertilizer, despite there no longer being a yield response at higher STK levels (Leikam et al., 2003), to maintain or increase the level of STK for future crops. The argument for doing so is to provide insurance to the producer as the next year crop prices could falter or fertilizer costs could spike (Leikam et al., 2003). In past studies, the sufficiency approach to fertilizer rates have proven more profitable than fertilizer strategies that build and/or maintain STK (Olson et al., 1982, Popp et al. 2020, 2021).

Current recommendations for rice (*Oryza sativa* L.) as well as soybean (*Glycine Max* L. Merr.) for K fertilization aim to maintain the STK levels within the optimal ranges (Table 3.1). These rates are considered "grower options" that aim to maximize yield via K fertilization while gradually building STK (Slaton et al. 2011). Also, current recommendations are mainly a function of the yield response of the crop and the STK level of a particular field, which excludes economic information such as crop price and input cost. Previous studies have been conducted to analyze the profit-maximizing fertilizer-K rates for both rice and soybean (Popp et al., 2020, 2021) and showed that these recommended rates could be profitably curtailed to lower levels. Popp et al. 2020 and 2021 estimate yield response subject to STK and add crop price, fertilizer K cost per unit, and fertilizer-K application costs in their rate recommendations in comparison to those shown in Table 3.1. However, these studies did not track STK over time as most of the field trials they used did not track systemic changes in STK when applying K at different rates, and hence these studies are termed 'short-term' from hereon. Quantifying the long-term effects on STK as a result of K fertilization is an additional factor to consider when making fertilizer-K rate recommendations.

The objective of this study is to examine the economic and agronomic impacts of applying fertilizer at profit-maximizing fertilizer-K rates as a function of STK, crop price and fertilizer cost using yield response to K from short term fertilizer K rate studies as in Popp et al., 2020 and 2021 vs. those where STK was tracked in a so-called 'long-term' K rate study on plots growing soybean and rice in rotation. Using time-lagged values, including STK, K application rate, and a yield index value representing K removed based on yield, we estimate changes in STK as a function of yield and K applied for fields that have different starting STK values. In essence, we can analyze and compare K rate recommendations using a long-term yield response to K study in a soybean-rice rotation to estimate the impact on STK when applying at profit-maximizing rates vs. short-term rate recommendations formed using the currently available crop-specific calculators that do not consider crop-rotation or provide an estimate of STK implications. We conduct this analysis over an 11-year period using historical crop prices and fertilization costs, to i) estimate the profitability implications of applying at the long- vs. short-term profit-maximizing fertilizer-K rates; ii) demonstrate ramifications on STK over time for fields that start at varying initial STK; and, iii) a comparison of STK, long-term profit and fertilizer-K use between short-term profit maximizing rates and extension-based, agronomic recommendations.

B. Materials and Methods

1. Experimental Data

Experimental data for this study was collected from two sites with a two year rice/soybean crop rotation over the period from 2000-2020 located at the Pine Tree Experiment Station in Arkansas. Both sites were located in fields with Calhoun silt loam soil, which tend to be less rich in K availability compared to clayey soils. This dataset included 840 individual treatment observation from 42 site-years of rice/soybean rotation response trials. Each plot was soil-tested annually near the time of planting the specific crop. Supplemental fertilizers of N and P were applied to nutrient deficient areas to ensure the crop could reach yield potential and to isolate the long-term effects of varying K fertilization rates on yield response over time. The K rate treatments for each site were arranged in a randomized complete block design to examine the effect of added fertilizer-K rate on rice and soybean yields. Fertilizer-K rate application rates were replicated four times per K rate treatment every year, and each treatment ranged from 0 to 160 lbs K_2O/acre and increased in increments of 40 lbs K_2O/acre. STK values within this study ranged from low to optimum levels by agronomic standards portrayed in Table 3.1 where the minimum observed STK was 22 ppm and the maximum was 172 ppm. Given two sites with different initial STK in each plot, each fertilizer-K treatment was thus replicated eight times within a growing season (i.e. both tracks grew rice and then soybean the following year).

Figure 3.1 summarizes yield and STK observations from the dataset over the period from 2000 through 2020 by K rate treatment. The replicate average and standard deviation of rice yield for even years is depicted in the left column using the left hand vertical axis. Using the right vertical axis, replicate average STK values are shown. Similarly, average and standard deviation for soybean yield for odd years from 2001 to 2019 are depicted in the right column to showcase trend in yield and STK when different levels of fertilizer-K were applied. Over time, a slight increase in yields is observable as the K rate applied for both crops increases (Figure 3.1). For rice, STK values trend downward at decreasing rates when the K rate increases, while yields are simultaneously trending upward. In comparison, STK values for years when a soybean crop was in rotation, STK values eventually flatten out across the years as the fertilizer-K treatment rate increases (Figure 3.1). Figure 3.2 showcases the same comparison but of relative yield values vs. STK over the full analysis time period from 2000-2020 for the no fertilizer-K rate control treatment and the K = 120 lbs K_2O/acre treatment. Even at the higher rate of fertilizer-K application, the STK still trends downward while RY values are near yield maximum. This suggests that producers are only able to maintain the level of STK, even at higher rates of fertilizer-K application but do not necessarily build STK. Also, the smaller downward trend in RY overtime in the no-fertilizer K application in Figure 3.2 coupled with the upward bushel/acre yield at the various K-rate treatments depicted in Figure 3.1 suggest that supplemental fertilizer-K allows producers to reach yield maximum at low STK.

2. Yield Indices

To compare the yield values each year under a specific crop rotation at a specific site across the five K rate treatments in this study, a relative yield index value was calculated. Rice and soybean yields (Y) were converted to this relative yield index (*RY*) using:

Eq. [1] $$RY_{csrjt} = \frac{Y_{csrjt}}{\max_{k} Y_{csrjt}} \cdot 100$$

where *c* represents the crop under study (rice or soybean), *s* represents a specific site for the five K-rates with 4 replicates per K-rate or 20 plots, *r* represents the replicate, *j* represents the j^{th} of *k* fertilizer-K rate treatments including the no-K control (0 lbs of K_20/acre), *t* represents a year from the range included in the dataset of the particular crop, and *k* is the subset of treatments excluding the no-K control treatment. Thus, *RY* should take on values from 0 to 100, where the maximum observed yield for a replication within a specific replicate of a K-rate treatment plot is equal to 100. However, the no-K control was excluded from the denominator in the *RY* calculation in Eq. 1, so *RY* values greater than 100 are possible in the case of a negative yield response to K (which did not happen in this study).

A yield index that compares yield values within each crop across time was also calculated to assess whether a particular crop year had relatively low or high yield in comparison to long term trend for a particular K rate treatment. Because rice and soybean yield values differ greatly, this yield index was calculated as follows:

Eq. [2] $$YI_{csrjt} = \frac{Y_{csrjt}}{Average\ Y_{cj}} \cdot 100$$

where *c* represents a specific crop (rice or soybean), *t* represents a year from the range included in the dataset of the particular crop, *s* represents the site where the crop was produced, *j* represents a subset of the *k* - K rate treatments, and *Y* represents the yield produced. Using a yield index is expected to obviate the need to account for differences in yield potential across crop when attempting to estimate STK changes over time.

3. Soil Test K and Relative Yield Regression Methodology

As a part of the long-term framework, we aim to offer insights that estimate the impacts of changing fertilizer-K application rates on STK over a period of time. The level of initial *STK* in the soil in the current year is a function of the previous year's *STK*, the rate of fertilizer-K applied, and the amount of K removed by the specific crop planted which is expected to vary by *YI*. Predicting STK will allow producers to better understand the impact of current fertilization decisions and was estimated by:

Eq. [3] $$STK_t = \alpha_0 + \alpha_1 STK_{t-1} + \alpha_2 K_{t-1} + \alpha_3 YI_{t-1} + \delta_t$$

where *t* represents a year from the range included in the dataset of the particular crop, *YI* represents the yield index across K rate treatments over time from Eq. 2, and δ_t is an error term. We drop the site, k-rate treatment, replicate and crop subscripts for ease of presentation.

As yield data for this rice/soybean rotation study was collected over a 21-year period from 2000-2020, a number of factors could affect one of the two sites' yield response to K fertilization. For example, genetic improvement of cultivars over time could increase yield potential, whereas detrimental weather conditions in a given year could hinder a field's yield potential. The goal, however, is to estimate a long-term yield response curve to fertilizer-K for both rice and soybean. Therefore, regressing *RY,* as calculated in Eq. 1, eliminates yield trend and weather effects thereby isolating the effect of fertilizer-K as the same cultivar and weather impacted a particular K rate treatment in a particular year. Hence, regressing RY on STK and fertilizer-K will essentially capture the long-term effect of fertilizer-K rate across a range of observed yields, initial STK information, and commonly used crop cultivars while still needing to control for crop specific differences in yield response to K. Therefore, long-term RY response to K contingent on STK was estimated for rice and soybean using:

Eq. [4] $$RY_t = \beta_0 + \beta_1 K_t + \beta_2 K_t^2 + \beta_3 STK_t + \beta_4 STK_t^2 + \beta_5 K_t \cdot STK_t + + \beta_6 K_t \cdot STK_t^2 + \beta_7 K_t^2 \cdot STK_t + \beta_8 K_t^2 \cdot STK_t^2 + \beta_9 Rice_t + + \beta_{10} Rice_t \cdot K_t + \beta_{11} Rice_t \cdot K_t^2 + \beta_{12} Rice_t \cdot STK_t + + \beta_{13} Rice_t \cdot STK_t^2 + \beta_{14} Rice_t \cdot K_t \cdot STK_t + \beta_{15} Rice_t \cdot K_t \cdot STK_t^2 + + \beta_{16} Rice_t \cdot K_t^2 \cdot STK_t + \beta_{17} Rice_t \cdot K_t^2 \cdot STK_t^2 + \mu_t$$

Again, we drop the site, k-rate treatment, replicate and crop subscripts for ease of presentation. The constant term β_0, is the base RY value that did not change with year, site, fertilizer-K rate applied, or replicate; β_1 and β_2 represent the average linear and non-linear, location- and year-independent, effect of *K*; β_3 and β_4 represent the average linear and non-linear, location- and year-independent, effect of initial soil-test K (*STK*) on *RY*; β_5 to β_8 represent the two-way interactions between *STK* and *K*; β_9 is the crop specific intercept shifter for RY when rice is grown (rice =1; soybean =0); β_{10} and β_{11} represent the crop dummy variable interaction with linear and non-linear forms of *K*; β_{12} and β_{13} represent the dummy variable interaction with linear and non-linear forms of *STK* ; β_{14} through β_{17} represent the three-way interactions between the dummy variable with linear and non-linear forms of both *K* and *STK*; μ_t captured the error term for each observation of *RY*.

We considered that solving for STK and RY should be accomplished by using a system of equations as the current year's *STK* value is a regressor in Eq. 4 and later report those results in Tables 3.2 and 3.3.

4. Statistical methods and goodness of fit

Eqs. 3 and 4 were a result of choosing among various effects to each model. Results for each equation were first analyzed to determine variables that increased the explanatory power of the model (|t –stat| > 1.0) as well as decreased the Akaike Information Criterion (AIC). AIC is preferred in this analysis over adjusted R^2 as this is time series data, which tends to overstate the adj. R^2 value as there tends to be a significant correlation with time in most models. AIC also provides a stronger correction for the number of variables used as it penalizes models with a high number of explanatory variables that overfit the model. Next, we chose between estimating Eqs. 3 and 4 simultaneously as an OLS system of equations or as separate panel data equations. Heteroskedasticity was an issue in initial coefficient estimation of Eqs. 3 & 4 using the Breusch-Pagan-Godfrey heteroskedasticity test ($p < .001$). Therefore, White's cross-section option was used to adjust the coefficient covariance matrix of each equation. Because the system model did not offer a correction for this issue, we continued with the separate panel estimations that estimated robust standard errors. Lastly, we ran period random effects, as the short-term models did in Chapter II as well as Popp et al. (2020 and 2021), but those effects were not statistically significant. This is most likely because crops were rotated each year.

STK coefficient estimates in Eq. 3 and *RY* coefficient estimates in Eq. 4 were determined using EViews v. 9.5 (Lilien et al., 2015). Eq. 4 uses both linear and non-linear forms of *K* and *STK* and includes two- and three-way interaction terms between the explanatory and dummy variables. Interaction terms included in the regression model were selected based on increased explanatory power of the model (|t –stat| > 1.0), decreased Akaike information criterion (AIC), and visual evaluation across different specifications. AIC was the main focus of model explanatory power as this is time series data, which can skew the adj. R^2 value. If interaction terms increased the AIC, then they were eliminated. Multicollinearity was not an issue between variables *K* and *YI* in Eq. 3 (Pearson correlation coefficient ρ =-0.0168), *K* and *STK* (Pearson correlation coefficient ρ =0.4716), or *YI* and *STK* (Pearson correlation coefficient ρ =-0.1243). There was also minimal multicollinearity between *K* and *STK* in Eq. 4 (Pearson correlation coefficient ρ = 0.4555), *K* and *Rice* (Pearson correlation coefficient ρ < .0001) and *STK* and *Rice K* (Pearson correlation coefficient ρ = -0.1303).

5. Economic Analysis

To calculate profit-maximizing fertilizer-K use, the economic benefit of applying an additional lb/acre of fertilizer-K in terms of marginal revenue can be computed by the following:

Eq. [5] $$MR_{cp} = \frac{\partial RY_c}{\partial K} \cdot YP_p/100 \cdot P_{c_t}$$

where the partial derivative for Eq. 4 is:

Eq. [6] $$\frac{\partial RY_c}{\partial K} = (\beta_1 + Rice \cdot \beta_{10}) + 2 \cdot (\beta_2 + Rice \cdot \beta_{11})K + (\beta_5 + Rice \cdot \beta_{14})STK_p + (\beta_6 + Rice \cdot \beta_{15})STK_p^2 + 2 \cdot (\beta_7 + Rice \cdot \beta_{16})K \cdot STK_p + 2 \cdot (\beta_8 + Rice \cdot \beta_{17})K \cdot STK_p^2$$

and varies by crop using the *Rice* dummy variable. Most producers (*p*) want to maximize yield potential (*YP*) each year, which is the amount of yield that can be achieved in a field without a nutrient deficiency. For this long-term analysis, we use the average rice and soybean yield from the K-rate treatment block that applied at 120 pounds of K_2O/acre each year for these calculations as yield response to K rate was found to be minimal above those rates in prior studies for both crops (Popp et al, 2020, 2021). Therefore, *YP* for rice and soybean producers in this analysis were 176.20 and 61.75 bu/acre, respectively. This yield potential along with a long-term estimate of the yield response to *K* enables us to calculate the marginal physical product of fertilizer-K under the assumption that there are no other macronutrient deficiencies preventing a field from reaching *YP*.

Thus, to obtain the long-term, profit-maximizing fertilizer-K rate, K^*, we solve for the rate at which the diminishing marginal revenue received from fertilizer-K is equal to the per unit cost of K, c_K, as follows:

Eq. [7]
$$K_{tp}^* = \{\frac{c_{K_t}}{\frac{YP_p}{100} \cdot P_{c_t}} - \left[(\beta_1 + Rice \cdot \beta_{10}) + (\beta_5 + Rice \cdot \beta_{14})STK_{tp} + (\beta_6 + Rice \cdot \beta_{15})STK_{tp}^2\right] / [2 \cdot \left(\beta_2 + Rice \cdot \beta_{11} + (\beta_7 + Rice \cdot \beta_{16})STK_{tp} + (\beta_8 + Rice \cdot \beta_{17})STK_{tp}^2\right)]\}$$

Eq. 7 assumes that the cost per unit of fertilizer does not change as K^* changes. That is application costs to apply fertilizer do not differ whether applying at low or high fertilizer-K rates. However, c_K, will vary over time as does *STK* in the p^{th} producer field.

In summary, we expect that higher levels of fertilizer-K application over time lead to increases in STK over time ($\alpha_2 > 0$) and that relatively high yields over time would negatively impact STK ($\alpha_3 > 0$) in Eq. 3. Because both K and STK affect the shape and slope of the yield response curve in Eq. 4, we use $STK \cdot K$ interaction terms to determine their relationship as well as the interaction by crop. We expect diminishing positive yield response to K with greater STK, as in previous studies. We also expect that crop price will positively impact the profit-maximizing K rate whereas fertilizer cost would have the opposite effect.

6. Changes in STK over time

The *STK* and *RY* values calculated using Eqs. 3 and 4, respectively, are both contingent on the K^* value calculated using Eq. 7 for the long-term framework results. Short-term results utilize the decision support software developed in Popp et al. (2020 and 2021) to calculate *RY* and K^* values. Therefore, to track the changes in STK over time under both frameworks of analysis, the resulting *RY* value must be converted to a *YI* value to estimate STK in Eq. 3. Recall that the *RY* value provides an index of yield values across the various K-rate treatments under study, whereas the *YI* value is an index value of rice and soybean yields over a span of time. To make this conversion, the following equation is used:

Eq. [8] $$YI_{K_c^*} = YE_{ct} \,/\, YP_c$$

where *YE* is the yield estimate in bu/acre of the crop (rice or soybean depending on rotation year) and *YP* represents the constant yield potential used for each crop. Since YP_c is the yield potential and equivalent to the yield when *RY* = 100, *YI* and *RY* resolve to the same value.

7. Profitability differences using Short-Term vs. Long-Term models

To assess profitability impacts across frameworks each year, the partial returns (*PR*) of applying K^* (subscripts for year and site are dropped from this point for readability) can be calculated as follows:

Eq. [9] $$PR_{K_c^*} = \mathrm{YP}_{cp} \cdot \widehat{RY_{K^*}} \cdot P_c - K^* \cdot c_K$$

where the cost of fertilizer and its application are deducted from the revenue generated by crop sales. We begin this analysis in 2010 and repeat until 2020 to capture annual profitability differences from applying K^* using the short-term model or the long-term model while accounting for changes in *STK* as a function of lagged *STK*, lagged K^* and lagged *YI.*

From there, each years' *PR* can be discounted to the same starting year using a net present value (NPV) calculation and summing across years to consider the time value of money of the total partial returns realized by producers. This calculation involves:

Eq. [10] $$\mathrm{NPV}_t = \sum_{t=1}^{n} \frac{PR_t}{(1+d)^t}$$

where *d* represents a selected discount rate. The NPV under the short-term vs. long-term framework is expected to differ as the short-term yield estimations use a larger, more comprehensive dataset of average Arkansas conditions vs. the long-term framework using two sites at one location in eastern Arkansas.

C. Results

Statistical results from Eq. 3 showed most variables statistically significant ($p < .05$) using various methodologies for solving for STK (Table 3.2). Tables 3.2 and 3.3 show that the system estimation was considered to calculate STK and RY and resulted in very similar coefficient estimates in comparison to the panel least squares estimation. However, the system values do not include a heteroskedasticity correction in the error terms, as previously discussed. Thus, we eliminated the system model methodology as robust standard error terms that produced more conservative statistical significance estimates were more important, and we had used site-specific STK values in the dataset. The model with period random effects for Eq. 3, while higher in R^2 led to a positive coefficient estimate on *YI*, which was counter to expectations (Table 3.2). We therefore continued with the panel least squares estimation for Eq. 3, without period random effects that provided White's heteroskedasticity consistent standard error estimates. We also considered the addition of an interaction term between YI_{t-1} and the *Rice* dummy variable (Supplemental Table 1); however, this interaction term was not statistically significant and lowered the explanatory power of the YI_{t-1} variable.

Supplemental Table 2 lists the coefficients and standard errors from Eq. 4. However, most variables were not statistically significant ($p < .05$) when all three-way interaction terms were included as regressors. Based on statistical results of analyzing different combinations of interaction terms in Eq. 4 and their impact to the *RY* curve, the resulting *RY* equation estimation was condensed to:

Eq. [11] $RY_t = \beta_0 + \beta_1 K_t + \beta_2 K_t^2 + \beta_3 STK_t + \beta_4 STK_t^2 + \beta_5 K_t \cdot STK_t +$

$$+ \beta_6 K_t \cdot STK_t^2 + \beta_7 K_t^2 \cdot STK_t + \beta_8 Rice_t + \beta_9 Rice_t \cdot K_t +$$

$$+ \beta_{10} Rice_t \cdot K_t \cdot STK_t + \mu_t$$

Similar to Eq. 3, most variables in Eq. 11 were statistically significant ($p < .05$) in a system estimation as well as a panel least squares estimation with and without random effects (Table 3.3). Because the estimation of a system did not allow for White's heteroskedasticity consistent standard error estimates for coefficients, it was no longer considered. Also, since *RY* is already an index value that controlled for cultivar, yield trend, weather effects, and the crop rotation design of this dataset controls for time, the addition of period random effects was deemed unnecessary. Therefore, while also staying consistent with the *STK* regression methodology, the panel least squares coefficient estimates of Eq. 4 were selected. Visual assessment of yield response functions confirmed this choice in the sense that curvature changed according to expectations at varying *STK*.

Contour plots in Figure 3.4 depict the relationship between agronomic yield responses to fertilizer-K rate at different STK for rice (left) and soybean (right) for the short-term estimation, and Figure 3.5 does the same for the long-term estimation. Rice and soybean yield potential (95% RY) can be reached at levels of STK considered low by agronomic standards with supplemental K fertilization. Also, yield benefits from added fertilizer-K for both crops diminish as the level of *STK* increases under both estimation strategies (Figures 3.3 and 3.4). Yield response curves confirm this assessment as steeper (greater) yield responses are experienced at lower levels of STK compared to soils with higher levels (Figure 3.3). Notably, the dashed-line yield response curves for the long-term estimation in Figure 3.3 are steeper compared to solid-line response curves that represent the short-term yield response as calculated for rice and soybean in previous work (Popp et al., 2020, 2021). We surmise these differences to be a function of calculating yield responses across more sites with the short-term model than the long-term model.

To analyze the impacts of applying at the long-term versus short-term K^*, we chose three levels of beginning STK based on the average observed STK in the dataset as well as very low and medium ranges (Table 3.1). We did not analyze the effects at higher levels of STK as the short-term recommendations require no supplemental fertilizer-K application above 96 ppm for rice (Popp et al., 2020) and 128 ppm for soybean (Popp et al., 2021), on average. Also contour plots of yield response to K shown in Figure 3.4 suggest that yield maximum is achievable at low STK. Hence, producers have no incentive to build STK if there is a chance for nutrient runoff and fertilizer purchases can be delayed. Also, field evidence suggests that applying at increased fertilizer-K rates when producing rice and soybean may not actually build the STK level (Figures 3.1 and 3.2). At best there is a possibility of maintaining STK levels when applying at high K rates when a soybean crop is in rotation. Factors, such as luxury consumption of K by the plant and its amount determined by the specific crop grown, can play a role explaining why STK levels are not building over time. Nutrient runoff may also play a role.

Tables 3.3 through 3.9 showcase comparisons of the long-term and short-term profit maximization strategies at various starting levels of *STK*. At all levels of *STK* under analysis, we begin the long-term estimations in 2010 using the short-term K^* and yield index values. For example, at STK = 45 ppm, the short-term profit-maximizing fertilizer-K application strategy recommends applying 116 lbs of K_2O/acre for the rice crop grown that year. The resulting $RY = YI$ = 98.14 translates to 172.92 bu/acre using average yield potential. These values then begin the long-term analysis in Table 3.4. The K^* values differ after 2010 under the two frameworks, and the long-term results recommend for producers to apply at higher fertilizer-K rates, especially in years when rice is in the rotation as it has a steeper yield response in the long-term vs. short-term (Figure 3.3) and higher crop value in comparison to soybean, with lower corresponding yield index values compared to the short-term yield response functions for both rice and soybean crops (Tables 3.4 – 3.9). The short-term framework, regardless of initial *STK* in 2010, also results in a higher NPV as compared to the long-term. This is a result of the decreased fertilizer-K costs at lower application rates and increased revenue from higher yields (Tables 3.4 – 3.9).

Tables 3.10 through 3.12 represent estimates of yield, profitability and STK implications using the short-term yield response curves with current extension-based fertilizer-K rate recommendations (K_E) and the three initial STK values of 45, 78, and 95 ppm. Table 3.1 outlines the K_E values over four STK ranges. Agronomic-based fertilizer recommendation rates, such as K_E, are set at values with efforts to reduce the risk of yield loss that may result from under-fertilization and to attempt to replace the nutrients that will be removed by the harvested crop to maintain STK values at or near an optimal level. In this analysis, the K_E value falls between the K^* values for the short-term and long-term frameworks, excluding 2010 where the long-term framework began with the same K^* as the short-term framework (Tables 3.3-3.12). Under the 11-yr analysis, the extension recommendation for a rice/soybean rotation is one of three values K_E = 60, 90, or 120 lbs K_2O/acre (Tables 3.10-3.12), whereas the short-term and long-term frameworks have more variation in K^*. This is because K_E is one value for a certain range of STK (Table 3.1), whereas the short-term and long-term K^* values are calculated using Eq. 7.

Beginning in 2010, we estimate that applying at the long-term K^* each year on a rice/soybean rotation with an initial *STK* = 45, 78, or 95 ppm, producers will experience a final *STK* value in 2020 of 86 ppm (Tables 3.4, 3.6, and 3.8). There were minimal differences in *STK* from 2010 to 2014 for each *STK* starting value in 2010. However, the values converged to the same *STK* value in 2015 (Table 3.13). The results regarding *STK* changes using the short-term K^* mirrored those of the long-term K^* application results in that all initial *STK* values resulted in minute yearly differences from 2010 to 2015 (Tables 3.5, 3.7, and 3.9) and converged to the same *STK* in 2016 with a resulting final *STK* value in 2020 of 80 ppm (Table 3.13). The same trend also occurs when applying at K_E, where an initial *STK* = 45, 78, or 95 ppm results in a final *STK* in 2020 of 85 ppm (Table 3.13). Minimal yearly differences are observed in *STK* from 2010-2014 under the K_E framework, and the values converge in 2015-2020 to be the same STK levels (Tables 3.10-3.12).

Similar to convergence for *STK*, K^* were the same across initial *STK* from 2012-2020 under the long-term framework and 2014-2020 under the short-term framework (Table 3.13). K_E values converge the quickest in 2011 under the various application strategies (Table 3.13), which is once again a result of the K_E rate being a single value for a given range of *STK* instead of a calculated rate using Eq. 7. This implies that the largest impact to the NPV is in the first two and four years for the long- and short-term analyses, respectively, and after the first year when following extension recommendations as those are the only years that have substantially different K^* application rate recommendations. The NPV is largest for the short-term framework, intermediate under extension recommendations, and smallest under the long-term framework (Tables 3.3-3.12). Although the NPV of the short-term framework is only slightly larger than when applying under the extension recommendations, the producer is still earning increased profits with reduced fertilizer-K usage as shown in Table 3.14.

Under conditions when crop price is relatively high or fertilizer cost is relatively low, K^* values tend to be higher as producers are able to afford more inputs to produce higher yields under both frameworks. In 2016, fertilizer costs were relatively low at $0.28/lb of potash, which led to K^* recommendations in both the long- and short-term application frameworks that are higher than years when fertilizer costs were relatively high. For example, at the initial 2010 *STK* = 45 ppm, the long-term application framework suggests to apply 120 lbs/K_2O per acre to the rice crop being grown in 2016 (Table 3.4). Comparing this K^* value to a year when fertilizer costs were relatively high, such as in 2012 when potash price was almost double that of 2016 at \$0.50/lb, the K^* was only 106 lbs/K_2O. The short-term framework mirrored these results in that the 2016 K^* recommendation was larger than that of the 2012 value (Table 3.5). The K^* values tend to be larger under the long-term application framework as its yield response curves are much steeper than the moderate response curves produced in the short-term framework (Figure 3.3), which implies that the short-term K^* is more responsive to changes in crop price and fertilizer cost. This is because under a strategy that uses a more moderate yield response, producers will be more conservative about applying at higher application rates compared to a more aggressive or steeper yield response curve using the long-term framework.

Lastly, Table 3.14 compares and summarizes the total amount of fertilizer-K used under each application framework at *STK* = 45, 78, and 95 ppm. Total usage was lowest under the short-term framework and highest under the long-term framework, with extension-based recommendations falling in between. If producers chose to apply at the short-term profit-maximizing rates vs. the extension-based recommendations, they would use 105 lbs of K_2O/acre less over the 11-yr period under the short-term strategy when the initial *STK* = 45 ppm in 2010 when compared to the extension recommendation. This 105 lbs of K_2O/acre savings out of the total usage of 1,065 lbs of K_2O/acre over the 11-yr simulation using the short-term rate recommendations amounts to an approximate 10% reduction of fertilizer-K use in comparison to K_E rates. The difference between the short-term and long-term profit-maximizing rates in terms of total fertilizer-K application at this same starting *STK* level of 45 ppm is even larger at 261 lbs of K_2O/acre or a 25% reduction from the long-term rate recommendations. The reduction in fertilizer-K usage in the short-term from both the extension-based recommendations and long-term profit-maximizing strategies is partially responsible for the higher NPV experienced when using the short-term framework opposed to applying at K_E.

D. Conclusions

Producers using the long-term framework for calculating profit-maximizing fertilizer-K application rates will experience higher *STK* values over time than when profit-maximizing rates are based on the short-term framework. The magnitude of this difference averages to approximately 6 ppm per acre over an 11-yr simulation analysis. This difference in final *STK* values between the two frameworks is a result of the differences in application rate and the yield response curves of each crop. Notably, the long-term framework offered insight about changes in *STK* values from year to year that could not be observed using short-term trial information. Higher *STK* values, as a result of greater fertilizer-K application using the long-term framework in comparison to the short-term framework, led to higher fertilizer cost. Also, *RY* value estimates were higher (larger Y-intercept) with the short-term framework in relation to long-term estimates leading to greater yield values with the short-term framework. The combination of more revenue and less fertilizer cost for the short-term than long-term framework thus led to larger NPV of cash flows over our 11-year simulation period for the short-term framework. Producers applying at the short-term profit-maximizing application strategy will also experience higher NPV with decreased fertilizer-K usage compared to those applying based on extension recommendations with 5 ppm less per acre in final *STK*. However, the producers applying at the extension recommendation will experience slightly lower fertilizer-K usage than those applying at the long-term profit-maximizing strategy and increased NPV with only a 1 ppm increase in final *STK* values. Therefore, the extension recommendations that are set to minimize the risk of yield loss and potentially build the level of *STK* in the soil will, at best, maintain STK as each application strategy converges to a final *STK* value at the higher end of the "low" *STK* range in Table 3.1 given the three initial *STK* values in this analysis.

These results, subject to the long-term framework considering only two sites, continue to align with previous recommendations that fertilizer-K application rates could be curtailed (Popp et al., 2020, 2021) and that applying fertilizer-K to build and maintain levels of *STK* is less profitable than applying at rates that follow the sufficiency approach (Olson et al., 1982). Applying additional fertilizer-K when using higher rates using the long-term framework resulted in final *STK* only slightly higher than the *STK* achieved using the short-term framework with lower K^* rates. This suggests that the additional K applied under the long-term strategy may lead to more run-off and requires greater fertilizer expense leading to lesser profit potential.

Because K is a nonrenewable resource that is mined, reducing the application rates today can extend the useful life of the supply of K in a mine. Therefore, the reduction in fertilizer-K under the short-term application framework opposed to the long-term K^* and K_E also has a resource conservation value that is not calculated in this paper. However, increasing the useful life of a nonrenewable resource while generating higher producer profits further reiterates the conclusion that producers could be urged to follow the short-term framework.

Since the short-term yield response curves are likely more reflective of average Arkansas conditions as they used experimental data from fertilizer trials that were conducted short term (2 to 3 years) and long-term (20 years) across a larger set of conditions and locations compared to the long-term yield responses that account for *STK* over time for two sites at the Pine Tree

Experiment Station only, the use of the short-term framework is encouraged. Further, the results lead to *STK* convergence over a similar period for both estimation methods, and the final *STK* values in 2020 do not greatly differ between the estimation methods and initial *STK* values (Table 3.13).

E. References

Green, W. H. (2008). Econometric Analysis, 6th Ed. New York: New York. Pearson-Prentice Hall. p 208.

Leikam, D. F., Lamond, R. E., & Mengel, D. B. (2003). Providing flexibility in phosphorus and potassium fertilizer recommendations. Better Crops, 87(3), 6-10.

Marschner, H. 2012. Marschner's mineral nutrition of higher plants. Academic Press, New York.

Mississippi State University (MSU). (2021). Archived Budget Publications. Department of Agricultural and Applied Economics, Mississippi State University. As accessed 17 May 2021. https://www.agecon.msstate.edu/whatwedo/budgets/archive.php

Olson, R. A., Frank, K. D., Grabouski, P. H., & Rehm, G. W. (1982). Economic and agronomic impacts of varied philosophies of soil testing. Agronomy Journal, 74, 492–499. https://doi.org/10.2134/agronj1982.00021962007400030022x

Popp, M., Slaton, N. A., Norsworthy, J. S., & Dixon, B. 2021. Rice yield response to potassium: an economic analysis. Agronomy Journal, 113, 287-297. https://doi:10.1002/agj2.20471

Popp, M., Slaton, N. A., & Roberts, T. L. 2020. Profit-maximizing potassium fertilizer recommendations for soybean. Agronomy Journal, 112, 5081-5095. https://doi:10.1002/agj2.20424.

United States Geological Survey (USGS). (2019). Mineral Commodity Summaries. As accessed 4 Jun 2020. http://prd-wret.s3-us-west-2.amazonaws.com/assets/palladium/production/atoms/files/mcs2019_all.pdf.

United States Department of Agriculture (USDA) – Economics, Statistics and Market Information System. (2021). Rice year book: Average Arkansas rough rice price received by farmers by marketing year and rough rice yield across long-, medium, and short-grain classes. Retrieved from https://www.ers.usda.gov/data-products/rice-yearbook/.

United States Department of Agriculture (USDA) – National Agricultural Statistics Service (NASS). (2021). Annual Arkansas Marketing Year Soybean Prices 2010-2020. As accessed 21 September 2021. https://quickstats.nass.usda.gov/results/63F4CF18-E855-3D5D-8673-9E67081B6D0F.

Zörb, C., Senbayram, M., & Peiter, E. (2014). Potassium in agriculture– status and perspectives. Journal of Plant Physiology, 171, 656–669. https://doi.org/10.1016/j.jplph.2013.08.008

F. Tables and Figures

Table 3.1. Soil-test K levels as defined by Mehlich-3 extractable soil K concentrations, and the corresponding recommended fertilizer-K rates for full-season irrigated rice and soybean from 2010-2020 in Arkansas.

Soil-test K[a]		Rice Recommended Fertilizer-K Rate	Soybean Recommended Fertilizer-K Rate
Level	mg K kg^{-1}	lb K_2O/acre	lb K_2O/acre
Very Low	< 61	120	160
Low	61-90	90	120
Medium	91-130	60	60
Optimum	131-175	0	51
Above Optimum	> 175	0	0

[a] Recommendations from Slaton et al. 2011 for rice and from Slaton et al. 2013 for soybean and represent Mehlich-3 extractable soil K for soil depth of 0-4.0 inches (0-10 cm) for rice and soybean.

Table 3.2. Statistical results comparison of using time-lagged Mehlich-3 soil-test K (STK_{t-1}), fertilizer-K application rate (K_{t-1}), and yield index ($YI_{t-1}{}^{a}$) to explain the current time period STK with and without period random effects from 840 individual treatment observations of trials conducted from 2000 to 2020 in eastern Arkansas under an irrigated rice and soybean rotation using panel least squares regression.

Model Specification	**Panel Least Squares (PLS)**	**PLS – Period Random Effects**	**System of Equations**
Explanatory Variable[b]	Coefficient Estimate (SE[c])	Coefficient Estimate (SE)	Coefficient Estimate (SE)
Constant	56.55***	43.39***	56.55***
(α_0)	(12.98)	(5.09)	(4.74)
STK_{t-1}	0.35**	0.20***	0.35***
(α_1)	(0.13)	(0.05)	(0.04)
K_{t-1}	0.14***	0.17***	0.14***
(α_2)	(0.04)	(0.02)	(0.01)
YI_{t-1}	-0.17	0.05	-0.17***
(α_3)	(0.11)	(0.04)	(0.04)
Adj. R^2	0.331	0.515	0.331

[a] Relative Yield Index calculated using Eq. 2.

[b] Lag of observed soil-test K concentrations as defined by Mehlich-3 extractable soil-K concentrations in ppm (STK_{t-1}) and lag of fertilizer-K application rate (K_{t-1}) in lbs K_2O/acre.

[c] The coefficient covariance matrix was adjusted using White's cross-section option. Statistical significance: * -- $p < 0.05$, ** -- $p < 0.01$, *** --- $p < 0.001$

Table 3.3. Statistical results of using various functional forms of initial Mehlich-3 soil-test K (*STK*) and fertilizer-K application rate (*K*) to explain relative yield (*RY* [a]) from 840 individual treatment observations of trials conducted from 2000 to 2020 in eastern Arkansas with irrigated rice and soybean using generalized least squares and treating production year as a random effect.

Model Specification	**Panel Least Squares (PLS)**	**PLS – Period Random Effects**	**System of Equations**
Explanatory Variable[b]	Coefficient Estimate (SE[c])	Coefficient Estimate (SE)	Coefficient Estimate (SE)
Constant	45.98***	44.57***	45.98***
(β_0)	(9.33)	(7.98)	(4.07)
K	0.80***	0.82***	0.80***
(β_1)	(0.09)	(0.08)	(0.07)
K^2	$-2.29\text{x}10^{-3,***}$	$-2.17\text{x}10^{-3,***}$	$-2.29\text{x}10^{-3,***}$
(β_2)	($3.76\text{x}10^{-4}$)	($3.70\text{x}10^{-4}$)	($4.60\text{x}10^{-4}$)
STK	0.57*	0.53**	0.57***
(β_3)	(0.22)	(0.19)	(0.11)
STK^2	$-2.40\text{x}10^{-3,*}$	$-1.57\text{x}10^{-3}$	$-2.40\text{x}10^{-3,**}$
(β_4)	($1.21\text{x}10^{-3}$)	($9.74\text{x}10^{-4}$)	($7.93\text{x}10^{-4}$)
$K \cdot STK$	$-8.48\text{x}10^{-3,***}$	$-9.22\text{x}10^{-3,***}$	$-8.48\text{x}10^{-3,***}$
(β_5)	($2.14\text{x}10^{-3}$)	($1.80\text{x}10^{-3}$)	($1.23\text{x}10^{-3}$)
$K \cdot STK^2$	$1.93\text{x}10^{-5}$	$2.26\text{x}10^{-5,**}$	$1.93\text{x}10^{-5,*}$
(β_6)	($1.00\text{x}10^{-5}$)	($8.31\text{x}10^{-6}$)	($8.01\text{x}10^{-6}$)
$K^2 \cdot STK$	$1.98\text{x}10^{-5,***}$	$1.88\text{x}10^{-5,***}$	$1.98\text{x}10^{-5,***}$
(β_7)	($3.52\text{x}10^{-6}$)	($3.92\text{x}10^{-6}$)	($5.52\text{x}10^{-6}$)
Rice	7.62*	7.91*	7.62***
(β_8)	(3.43)	(3.38)	(1.07)
$Rice \cdot K$	-0.14*	-0.13**	-0.14***
(β_9)	(0.06)	(0.05)	(0.03)
$Rice \cdot K \cdot STK$	$1.16\text{x}10^{-3}$	$1.08\text{x}10^{-3,*}$	$1.16\text{x}10^{-3,***}$
(β_{10})	($5.95\text{x}10^{-4}$)	($4.62\text{x}10^{-4}$)	($2.91\text{x}10^{-4}$)
Adj. R^2	0.519	0.537	0.511

[a] Relative Yield calculated using Eq. 1.

[b] Observed soil-test K concentrations as defined by Mehlich-3 extractable soil-K concentrations in mg K kg^{-1} (*STK*) and fertilizer-K application rate (*K*) in lbs K_2O/acre.

[c] The coefficient covariance matrix was adjusted using White's cross-section option. Statistical significance: * -- $p < 0.05$, ** -- $p < 0.01$, *** --- $p < 0.001$.

Table 3.4. Estimates of relative yield index, yield (bu/acre), STK (ppm), and partial returns ($/acre) when applying at the long-term profit maximizing fertilizer-K recommendation from 2010 through 2020 using average rice and soybean prices and fertilizer-K costs with initial STK = 45 ppm and starting relative yield value and profit-maximizing fertilizer-K rate value equal to those produced in the short-term analysis tool.

	Observed Prices[a]					**Long Term**						
							Est. RY		**Est. Yield**			
Year	**$/cwt P_r Rice**	**$/bu Ps Soybean**	**$/ton Potash**	**$/lb K**	**K^{*b}**	**Yield Index**	**Rice**	**Soybean**	**Rice**	**Soybean**	**Est. STK[c]**	**Partial Returns[e]**
2010	13.40	10.90	460.00	0.38	116	98.1	98.14		172.92		45	$990.78
2011	11.30	12.30	583.80	0.49	129	97.1		97.08		59.95	71	$667.12
2012	13.40	14.30	596.00	0.50	**106**	96.4	96.40		169.86		83	$964.09
2013	14.30	13.10	475.00	0.40	137	95.3		95.31		58.86	83	$709.28
2014	15.20	10.60	472.00	0.39	120	96.7	96.70		170.39		88	$1,110.75
2015	12.00	9.46	425.40	0.35	128	94.1		94.09		58.10	**87**	$496.93
2016	10.90	9.83	339.40	0.28	120	96.7	96.70		170.38		88	$794.24
2017	9.39	9.77	379.60	0.32	135	94.5		94.49		58.35	87	$519.89
2018	11.10	8.81	400.00	0.33	116	96.5	96.51		170.04		89	$803.23
2019	10.70	8.87	550.00	0.46	107	92.6		92.64		57.21	87	$450.79
2020	11.90	11.10	442.20	0.37	113	96.6	96.55		170.12		86	$861.90
											NPV_{2010}[e]	$6,420.94

[a] Annual average Arkansas rice and soybean prices were obtained from the *2021 Rice Yearbook* (USDA, 2021) and USDA NASS (2021), respectively. Fertilizer-K cost is based on the price of a muriate of potash fertilizer (Mississippi State University, 2021) converted to $ ton^{-1}.

[b] See Eq. 7 for K^*, profit-maximizing fertilizer-K recommendations, for rice and soybean, respectively. The initial K^* and corresponding yield index value in 2010 are a result of using short-term decision support software (Popp et al., 2020).

[c] See Eq. 3 for the calculation of STK, soil-test K. The yield index calculated in Eq. 2 is determined based on the estimated yield value produced at K^* and is then used to determine the level of STK in the next year.

[d] Net present value (NPV) discounts the partial returns (Eq. 10) received each year to the value of 2010 dollars when applying at the long-term estimated K^* at a discount rate of 5%.

[e] Partial return values are calculated using an average fertilizer-K application cost of $7.50/acre (Eq. 9).

Table 3.5. Estimates of relative yield index, yield (bu/acre), STK (ppm), and partial returns ($/acre) when applying at the short-term profit maximizing fertilizer-K recommendation from 2010 through 2020 using average rice and soybean prices and fertilizer-K costs with initial STK = 45 ppm.

	Observed Prices[a]				Short Term[b]							
							Est. RY		Est. Yield			
Year	**$/cwt P_r Rice**	**$/bu P_s Soybean**	**$/ton Potash**	**$/lb K**	**K^***	**Yield Index**	**Rice**	**Soybean**	**Rice**	**Soybean**	**Est. STK[c]**	**Partial Returns[e]**
2010	13.40	10.90	460.00	0.38	116	97.8	97.83		172.37		45	$987.51
2011	11.30	12.30	583.80	0.49	109	97.4		97.38		60.13	71	$678.94
2012	13.40	14.30	596.00	0.50	73	96.7	96.71		170.39		80	$983.71
2013	14.30	13.10	475.00	0.40	115	97.7		97.71		60.34	78	$737.32
2014	15.20	10.60	472.00	0.39	92	97.5	97.55		171.88		83	$1,131.93
2015	12.00	9.46	425.40	0.35	104	97.1		97.08		59.95	81	$522.91
2016	10.90	9.83	339.40	0.28	**92**	97.6	97.55		171.89		**82**	$809.48
2017	9.39	9.77	379.60	0.32	110	97.4		97.44		60.17	81	$545.60
2018	11.10	8.81	400.00	0.33	80	97.2	97.15		171.18		83	$820.72
2019	10.70	8.87	550.00	0.46	88	95.9		95.90		59.22	80	$477.64
2020	11.90	11.10	442.20	0.37	86	97.3	97.26		171.37		80	$878.58
											NPV_{2010}[d]	$6,570.29

[a] Annual average Arkansas rice and soybean prices were obtained from the *2021 Rice Yearbook* (USDA, 2021) and USDA NASS (2021), respectively. Fertilizer-K cost is based on the price of a muriate of potash fertilizer (Mississippi State University, 2021) converted to $ ton^{-1}.

[b] The K^* and corresponding yield values are a result of using short-term decision support software (Popp et al., 2020, 2021). See Eq. 8 for yield index value calculation.

[c] See Eq. 3 for the calculation of STK, soil-test K. The yield index calculated in Eq. 2 is determined based on the estimated yield value produced at K^* and is then used to determine the level of STK in the next year.

[d] Net present value (NPV) discounts the partial returns (Eq. 10) received each year to the value of 2010 dollars when applying at the long-term estimated K^* at a discount rate of 5%.

[e] Partial return values are calculated using an average fertilizer-K application cost of $7.50/acre (Eq. 9).

Table 3.6. Estimates of relative yield index, yield (bu/acre), STK (ppm), and partial returns ($/acre) when applying at the long-term profit maximizing fertilizer-K recommendation from 2010 through 2020 using average rice and soybean prices and fertilizer-K costs with initial STK = 78 ppm and starting relative yield value and profit-maximizing fertilizer-K rate value equal to those produced in the short-term analysis tool.

	Observed Prices[a]				**Long Term**							
							Est. RY		**Est. Yield**			
Year	**$/cwt P_r Rice**	**$/bu P_s Soybean**	**$/ton Potash**	**$/lb K**	**K^{*b}**	**Yield Index**	**Rice**	**Soybean**	**Rice**	**Soybean**	**Est. STK**[c]	**Partial Returns**[e]
2010	13.40	10.90	460.00	0.38	94	95.9	95.94		169.04		78	$975.70
2011	11.30	12.30	583.80	0.49	126	95.3		95.27		58.83	80	$654.76
2012	13.40	14.30	596.00	0.50	**106**	96.3	96.28		169.65		86	$962.63
2013	14.30	13.10	475.00	0.40	137	95.1		95.12		58.74	84	$707.63
2014	15.20	10.60	472.00	0.39	120	96.7	96.69		170.37		89	$1,110.53
2015	12.00	9.46	425.40	0.35	128	94.0		94.05		58.08	**87**	$496.71
2016	10.90	9.83	339.40	0.28	120	96.7	96.69		170.37		88	$794.21
2017	9.39	9.77	379.60	0.32	135	94.5		94.48		58.34	87	$519.85
2018	11.10	8.81	400.00	0.33	116	96.5	96.51		170.04		89	$803.23
2019	10.70	8.87	550.00	0.46	107	92.6		92.63		57.20	87	$450.79
2020	11.90	11.10	442.20	0.37	113	96.6	96.55		170.12		86	$861.90
											NPV_{2010}[d]	$6,392.36

[a] Annual average Arkansas rice and soybean prices were obtained from the *2021 Rice Yearbook* (USDA, 2021) and USDA NASS (2021), respectively. Fertilizer-K cost is based on the price of a muriate of potash fertilizer (Mississippi State University, 2021) converted to $ ton^{-1}.

[b] See Eq. 7 for K^*, profit-maximizing fertilizer-K recommendations, for rice and soybean, respectively. The initial K^* and corresponding yield index value in 2010 are a result of using short-term decision support software (Popp et al., 2020).

[c] See Eq. 3 for the calculation of STK, soil-test K. The yield index calculated in Eq. 2 is determined based on the estimated yield value produced at K^* and is then used to determine the level of STK in the next year.

[d] Net present value (NPV) discounts the partial returns (Eq. 10) received each year to the value of 2010 dollars when applying at the long-term estimated K^* at a discount rate of 5%.

[e] Partial return values are calculated using an average fertilizer-K application cost of $7.50/acre (Eq. 9).

Table 3.7. Estimates of relative yield index, yield (bu/acre), STK (ppm), and partial returns ($/acre) when applying at the long-term profit maximizing fertilizer-K recommendation from 2010 through 2020 using average rice and soybean prices and fertilizer-K costs with initial STK = 78 ppm.

	Observed Prices[a]				Short Term[b]							
	$/cwt P_r	$/bu P_s	$/ton			Yield	Est. RY		Est. Yield		Est.	Partial
Year	**Rice**	**Soybean**	**Potash**	**$/lb K**	**K***	**Index**	**Rice**	**Soybean**	**Rice**	**Soybean**	**STK[c]**	**Returns[e]**
2010	13.40	10.90	460.00	0.38	94	97.5	97.53		171.85		78	$992.65
2011	11.30	12.30	583.80	0.49	102	97.0		96.99		59.89	80	$679.40
2012	13.40	14.30	596.00	0.50	67	96.5	96.53		170.09		82	$984.76
2013	14.30	13.10	475.00	0.40	115	97.7		97.71		60.34	78	$737.32
2014	15.20	10.60	472.00	0.39	**92**	97.5	97.55		171.88		83	$1,131.92
2015	12.00	9.46	425.40	0.35	104	97.1		97.08		59.95	81	$522.91
2016	10.90	9.83	339.40	0.28	92	97.6	97.55		171.89		**82**	$809.48
2017	9.39	9.77	379.60	0.32	110	97.4		97.44		60.17	81	$545.60
2018	11.10	8.81	400.00	0.33	80	97.2	97.15		171.18		83	$820.72
2019	10.70	8.87	550.00	0.46	88	95.9		95.90		59.22	80	$477.64
2020	11.90	11.10	442.20	0.37	86	97.3	97.26		171.37		80	$878.58
											NPV_{2010}[d]	$6,576.51

[a] Annual average Arkansas rice and soybean prices were obtained from the *2021 Rice Yearbook* (USDA, 2021) and USDA NASS (2021), respectively. Fertilizer-K cost is based on the price of a muriate of potash fertilizer (Mississippi State University, 2021) converted to $ ton^{-1}.

[b] The K^* and corresponding yield values are a result of using short-term decision support software (Popp et al., 2020, 2021). See Eq. 8 for yield index value calculation.

[c] See Eq. 3 for the calculation of STK, soil-test K. The yield index calculated in Eq. 2 is determined based on the estimated yield value produced at K^* and is then used to determine the level of STK in the next year.

[d] Net present value (NPV) discounts the partial returns (Eq. 10) received each year to the value of 2010 dollars when applying at the long-term estimated K^* at a discount rate of 5%.

[e] Partial return values are calculated using an average fertilizer-K application cost of $7.50/acre (Eq. 9).

Table 3.8. Estimates of relative yield index, yield (bu/acre), STK (ppm), and partial returns ($/acre) when applying at the long-term profit maximizing fertilizer-K recommendation from 2010 through 2020 using average rice and soybean prices and fertilizer-K costs with initial STK = 95 ppm and starting relative yield value and profit-maximizing fertilizer-K rate value equal to those produced in the short-term analysis tool.

	Observed Prices[a]				Long Term							
	$/cwt P_r	$/bu P_s	$/ton			Yield	Est. RY		Est. Yield		Est.	Partial
Year	**Rice**	**Soybean**	**Potash**	**$/lb K**	**K^{*b}**	**Index**	**Rice**	**Soybean**	**Rice**	**Soybean**	**STK[c]**	**Returns[e]**
2010	13.40	10.90	460.00	0.38	37	90.3	90.31		159.12		95	$938.00
2011	11.30	12.30	583.80	0.49	126	95.5		95.49		58.97	79	$656.27
2012	13.40	14.30	596.00	0.50	**106**	96.3	96.30		169.67		85	$962.82
2013	14.30	13.10	475.00	0.40	137	95.1		95.14		58.75	84	$707.84
2014	15.20	10.60	472.00	0.39	120	96.7	96.70		170.38		89	$1,110.56
2015	12.00	9.46	425.40	0.35	128	94.1		94.05		58.08	**87**	$496.74
2016	10.90	9.83	339.40	0.28	120	96.7	96.69		170.37		88	$794.21
2017	9.39	9.77	379.60	0.32	135	94.5		94.48		58.34	87	$519.85
2018	11.10	8.81	400.00	0.33	116	96.5	96.51		170.04		89	$803.23
2019	10.70	8.87	550.00	0.46	107	92.6		92.63		57.20	87	$450.79
2020	11.90	11.10	442.20	0.37	113	96.6	96.55		170.12		86	$861.90
											NPV_{2010}[d]	$6,358.21

[a] Annual average Arkansas rice and soybean prices were obtained from the *2021 Rice Yearbook* (USDA, 2021) and USDA NASS (2021), respectively. Fertilizer-K cost is based on the price of a muriate of potash fertilizer (Mississippi State University, 2021) converted to $ ton^{-1}.

[b] See Eq. 7 for K^*, profit-maximizing fertilizer-K recommendations, for rice and soybean, respectively. The initial K^* and corresponding yield index value in 2010 are a result of using short-term decision support software (Popp et al., 2020).

[c] See Eq. 3 for the calculation of STK, soil-test K. The yield index calculated in Eq. 2 is determined based on the estimated yield value produced at K^* and is then used to determine the level of STK in the next year.

[d] Net present value (NPV) discounts the partial returns (Eq. 10) received each year to the value of 2010 dollars when applying at the long-term estimated K^* at a discount rate of 5%.

[e] Partial return values are calculated using an average fertilizer-K application cost of $7.50/acre (Eq. 9).

Table 3.9. Estimates of relative yield index, yield (bu/acre), STK (ppm), and partial returns ($/acre) when applying at the long-term profit maximizing fertilizer-K recommendation from 2010 through 2020 using average rice and soybean prices and fertilizer-K costs with initial STK = 95 ppm.

	Observed Prices[a]				**Short Term**[b]							
							Est. RY		**Est. Yield**			
Year	**$/cwt P_r Rice**	**$/bu P_s Soybean**	**$/ton Potash**	**$/lb K**	**K^***	**Yield Index**	**Rice**	**Soybean**	**Rice**	**Soybean**	**Est. STK**[c]	**Partial Returns**[e]
2010	13.40	10.90	460.00	0.38	37	96.1	96.14		169.40		95	$999.97
2011	11.30	12.30	583.80	0.49	104	97.1		97.07		59.94	78	$679.26
2012	13.40	14.30	596.00	0.50	68	96.6	96.57		170.15		81	$984.53
2013	14.30	13.10	475.00	0.40	115	97.7		97.71		60.34	78	$749.27
2014	15.20	10.60	472.00	0.39	**92**	97.5	97.55		171.88		79	$1,131.92
2015	12.00	9.46	425.40	0.35	104	97.1		97.08		59.95	80	$522.91
2016	10.90	9.83	339.40	0.28	92	97.6	97.55		171.89		**82**	$809.48
2017	9.39	9.77	379.60	0.32	110	97.4		97.44		60.17	81	$545.60
2018	11.10	8.81	400.00	0.33	80	97.2	97.15		171.18		83	$820.72
2019	10.70	8.87	550.00	0.46	88	95.9		95.90		59.22	80	$477.64
2020	11.90	11.10	442.20	0.37	86	97.3	97.26		171.37		80	$878.58
											NPV_{2010}[d]	$6,592.98

[a] Annual average Arkansas rice and soybean prices were obtained from the *2021 Rice Yearbook* (USDA, 2021) and USDA NASS (2021), respectively. Fertilizer-K cost is based on the price of a muriate of potash fertilizer (Mississippi State University, 2021) converted to $ ton^{-1}.

[b] The K^* and corresponding yield values are a result of using short-term decision support software (Popp et al., 2020, 2021). See Eq. 8 for yield index value calculation.

[c] See Eq. 3 for the calculation of STK, soil-test K. The yield index calculated in Eq. 2 is determined based on the estimated yield value produced at K^* and is then used to determine the level of STK in the next year.

[d] Net present value (NPV) discounts the partial returns (Eq. 10) received each year to the value of 2010 dollars when applying at the long-term estimated K^* at a discount rate of 5%.

[e] Partial return values are calculated using an average fertilizer-K application cost of $7.50/acre (Eq. 9).

Table 3.10. Estimates of relative yield index, yield (bu/acre), STK (ppm), and partial returns ($/acre) when applying at the current extension recommendations (Table 3.1) from 2010 through 2020 using average rice and soybean prices and fertilizer-K costs with initial STK = 45 ppm.

	Obs. Prices[a]				Extension Recommendation							
	$/cwt	$/bu Ps	$/ton	$/lb		Yield	Est. RY		Est. Yield		Est.	Partial
Year	**P_r Rice**	**Soybean**	**Potash**	**K**	**K_E**	**Index**	**Rice**	**Soybean**	**Rice**	**Soybean**	**STK**	**Returns**
2010	13.40	10.90	460.00	0.38	120	98.0	97.96		172.61		45	$987.33
2011	11.30	12.30	583.80	0.49	120	98.0		97.97		60.50	72	$678.26
2012	13.40	14.30	596.00	0.50	90	97.4	97.45		171.70		81	$983.18
2013	14.30	13.10	475.00	0.40	120	97.9		97.91		60.46	81	$737.03
2014	15.20	10.60	472.00	0.39	90	97.5	97.51		171.81		84	$1,132.31
2015	12.00	9.46	425.40	0.35	120	97.9		97.90		60.46	82	$521.87
2016	10.90	9.83	339.40	0.28	90	97.5	97.52		171.83		85	$809.86
2017	9.39	9.77	379.60	0.32	120	97.9		97.90		60.45	82	$545.18
2018	11.10	8.81	400.00	0.33	90	97.5	97.52		171.83		85	$820.78
2019	10.70	8.87	550.00	0.46	120	97.9		97.90		60.45	82	$473.73
2020	11.90	11.10	442.20	0.37	90	97.5	97.52		171.83		85	$879.48
											NPV_{2010}	$6,566.47

[a] Annual average Arkansas rice and soybean prices were obtained from the *2021 Rice Yearbook* (USDA, 2021) and USDA NASS (2021), respectively. Fertilizer-K cost is based on the price of a muriate of potash fertilizer (Mississippi State University, 2021) converted to $ ton^{-1}.

[b] Extension recommendations and corresponding yield values (bu/ac) calculated using the decision support software from Popp et al., 2020, 2021. See Eq. 8 for yield index calculation.

[c] See Eq. 3 for the calculation of STK, soil-test K. The yield index calculated in Eq. 2 is determined based on the estimated yield value produced at K^* and is then used to determine the level of STK in the next year.

[d] Net present value (NPV) discounts the partial returns (Eq. 10) received each year to the value of 2010 dollars when applying at the long-term estimated K^* at a discount rate of 5%.

[e] Partial return values are calculated using an average fertilizer-K application cost of $7.50/acre (Eq. 9).

Table 3.11. Estimates of relative yield index, yield (bu/acre), STK (ppm), and partial returns ($/acre) when applying at the current extension recommendations (Table 3.1) from 2010 through 2020 using average rice and soybean prices and fertilizer-K costs with initial STK = 78 ppm.

	Obs. Prices				Extension Recommendation							
							Est. RY		Est. Yield			
Year	**$/cwt P_r Rice**	**$/bu Ps Soybean**	**$/ton Potash**	**$/lb K**	**K_E**	**Yield Index**	**Rice**	**Soybean**	**Rice**	**Soybean**	**Est. STK**	**Partial Returns**
2010	13.40	10.90	460.00	0.38	90	97.4	97.38		171.57		78	$992.59
2011	11.30	12.30	583.80	0.49	120	97.9		97.91		60.47	79	$677.84
2012	13.40	14.30	596.00	0.50	90	97.5	97.50		171.80		84	$983.75
2013	14.30	13.10	475.00	0.40	120	97.9		97.90		60.46	81	$736.97
2014	15.20	10.60	472.00	0.39	90	97.5	97.52		171.83		85	$1,132.38
2015	12.00	9.46	425.40	0.35	120	97.9		97.90		60.45	82	$521.86
2016	10.90	9.83	339.40	0.28	90	97.5	97.48		171.76		85	$809.51
2017	9.39	9.77	379.60	0.32	120	97.9		97.90		60.45	82	$545.18
2018	11.10	8.81	400.00	0.33	90	97.5	97.52		171.83		85	$820.78
2019	10.70	8.87	550.00	0.46	120	97.9		97.90		60.45	82	$473.73
2020	11.90	11.10	442.20	0.37	90	97.5	97.52		171.83		85	$879.48
											NPV_{2010}	$6,571.36

[a] Annual average Arkansas rice and soybean prices were obtained from the *2021 Rice Yearbook* (USDA, 2021) and USDA NASS (2021), respectively. Fertilizer-K cost is based on the price of a muriate of potash fertilizer (Mississippi State University, 2021) converted to $ ton^{-1}.

[b] Extension recommendations and corresponding yield values (bu/ac) calculated using the decision support software from Popp et al., 2020, 2021. See Eq. 8 for yield index calculation.

[c] See Eq. 3 for the calculation of STK, soil-test K. The yield index calculated in Eq. 2 is determined based on the estimated yield value produced at K^* and is then used to determine the level of STK in the next year.

[d] Net present value (NPV) discounts the partial returns (Eq. 10) received each year to the value of 2010 dollars when applying at the long-term estimated K^* at a discount rate of 5%.

[e] Partial return values are calculated using an average fertilizer-K application cost of $7.50/acre (Eq. 9).

Table 3.12. Estimates of relative yield index, yield (bu/acre), STK (ppm), and partial returns ($/acre) when applying at the current extension recommendations (Table 3.1) from 2010 through 2020 using average rice and soybean prices and fertilizer-K costs with initial STK = 95 ppm.

	Obs. Prices				Extension Recommendation							
	$/cwt	$/bu Ps	$/ton	$/lb		Yield	Est. RY		Est. Yield		Est.	Partial
Year	**P_r Rice**	**Soybean**	**Potash**	**K**	**K_E**	**Index**	**Rice**	**Soybean**	**Rice**	**Soybean**	**STK**	**Returns**
2010	13.40	10.90	460.00	0.38	60	96.9	96.92		170.76		95	$999.21
2011	11.30	12.30	583.80	0.49	120	97.9		97.90		60.46	81	$677.74
2012	13.40	14.30	596.00	0.50	90	97.5	97.52		171.82		85	$983.89
2013	14.30	13.10	475.00	0.40	120	97.9		97.90		60.46	82	$736.96
2014	15.20	10.60	472.00	0.39	90	97.5	97.52		171.83		85	$1,132.40
2015	12.00	9.46	425.40	0.35	120	97.9		97.90		60.45	82	$521.86
2016	10.90	9.83	339.40	0.28	90	97.5	97.52		171.83		85	$809.86
2017	9.39	9.77	379.60	0.32	120	97.9		97.90		60.45	82	$545.18
2018	11.10	8.81	400.00	0.33	90	97.5	97.52		171.83		85	$820.78
2019	10.70	8.87	550.00	0.46	120	97.9		97.90		60.45	82	$473.73
2020	11.90	11.10	442.20	0.37	90	97.5	97.52		171.83		85	$879.48
											NPV_{2010}	$6,577.95

[a] Annual average Arkansas rice and soybean prices were obtained from the *2021 Rice Yearbook* (USDA, 2021) and USDA NASS (2021), respectively. Fertilizer-K cost is based on the price of a muriate of potash fertilizer (Mississippi State University, 2021) converted to $ ton^{-1}.

[b] Extension recommendations and corresponding yield values (bu/ac) calculated using the decision support software from Popp et al., 2020, 2021. See Eq. 8 for yield index calculation.

[c] See Eq. 3 for the calculation of STK, soil-test K. The yield index calculated in Eq. 2 is determined based on the estimated yield value produced at K^* and is then used to determine the level of STK in the next year.

[d] Net present value (NPV) discounts the partial returns (Eq. 10) received each year to the value of 2010 dollars when applying at the long-term estimated K^* at a discount rate of 5%.

[e] Partial return values are calculated using an average fertilizer-K application cost of $7.50/acre (Eq. 9).

Table 3.13. Convergence points comparison of soil-test K (*STK*) and the fertilizer-K rates (K^* and K_E) where the respective values become the same each year regardless of the 2010 initial *STK* between the short-term vs. long-term frameworks.

	Short-Term Analysis			**Long-Term Analysis**			**Extension Rates**		
	2010 Initial STK			**2010 Initial STK**			**2010 Initial STK**		
Year	**45 ppm**	**78 ppm**	**95 ppm**	**45 ppm**	**78 ppm**	**95 ppm**	**45 ppm**	**78 ppm**	**95 ppm**
	STK[a]			**STK**			**STK**		
2014	83	83	79	88	89	88	84	85	85
2015	81	81	80	**87**	**87**	**87**	**82**	**82**	**82**
2016	**82**	**82**	**82**	88	88	88	85	85	85
2017	81	81	81	87	87	87	82	82	82
	K^* **Rate**[b]			K^* **Rate**			K_E **Rate**		
2010	116	94	37	116	94	37	120	90	60
2011	109	102	104	129	126	126	**120**	**120**	**120**
2012	73	67	68	**106**	**106**	**106**	90	90	90
2013	**115**	**115**	**115**	137	137	137	120	120	120
2014	92	92	92	120	120	120	90	90	90

[a] STK values measured in ppm.
[b] K^* and K_E values measured in lbs K_2O/acre.

Table 3.14. Total use of fertilizer-K under over the 11-yr simulation when applying at the various fertilizer-K rate application frameworks at each of the starting STK levels.

	ST *K**			**LT *K****			*Ke*		
	2010 Initial STK			**2010 Initial STK**			**2010 Initial STK**		
Year	**45 ppm**	**78 ppm**	**95 ppm**	**45 ppm**	**78 ppm**	**95 ppm**	**45 ppm**	**78 ppm**	**95 ppm**
2010	116	94	37	116	94	37	120	90	60
2011	109	102	104	129	126	126	120	120	120
2012	73	67	68	106	106	106	90	90	90
2013	115	115	115	137	137	137	120	120	120
2014	92	92	92	120	120	120	90	90	90
2015	104	104	104	128	128	128	120	120	120
2016	92	92	92	120	120	120	90	90	90
2017	110	110	110	135	135	135	120	120	120
2018	80	80	80	116	116	116	90	90	90
2019	88	88	88	107	107	107	120	120	120
2020	86	86	86	113	113	113	90	90	90
Sum	**1065**	**1031**	**976**	**1326**	**1303**	**1245**	**1170**	**1140**	**1110**

[a] STK values measured in ppm.
[b] K^* and K_E values measured in lbs K_2O/acre.
[c] Sum is the total of application between 2010-2020 at each starting level of STK in each respective strategy.

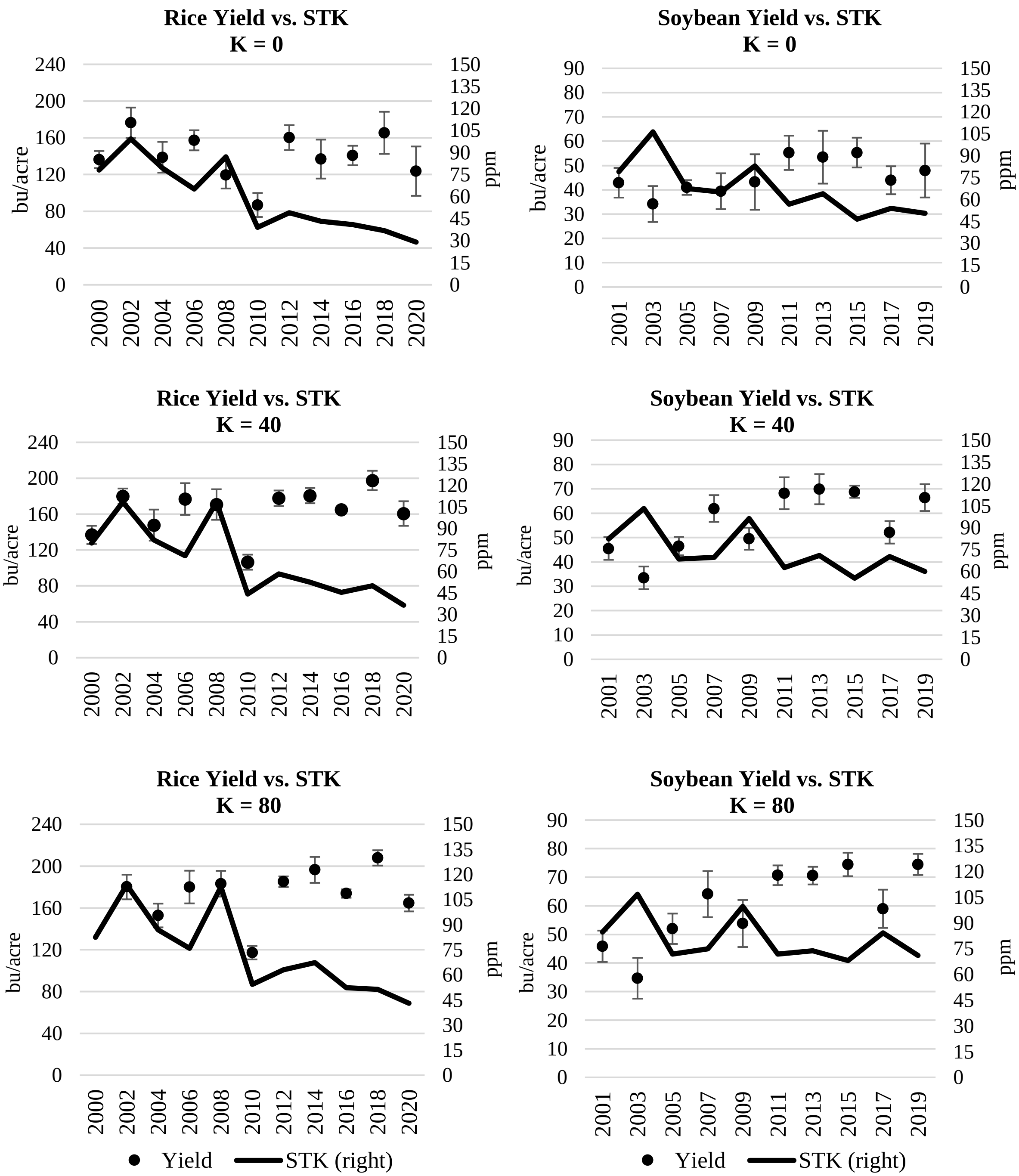

Figure 3.1. Comparison of average yield for rice (left) and soybean (right) in bu/acre of the eight replications within a year and a one standard deviation confidence interval around annual average yield to STK values at the five different K rate treatment applications conducted in Arkansas for 42 site-years and 840 individual observations where rice is grown in even years from 2000 to 2020 and soybean grown in odd years from 2001-2019. Data were provided by Dr. Nathan Slaton and Dr. Trent Roberts from field trials conducted over time.

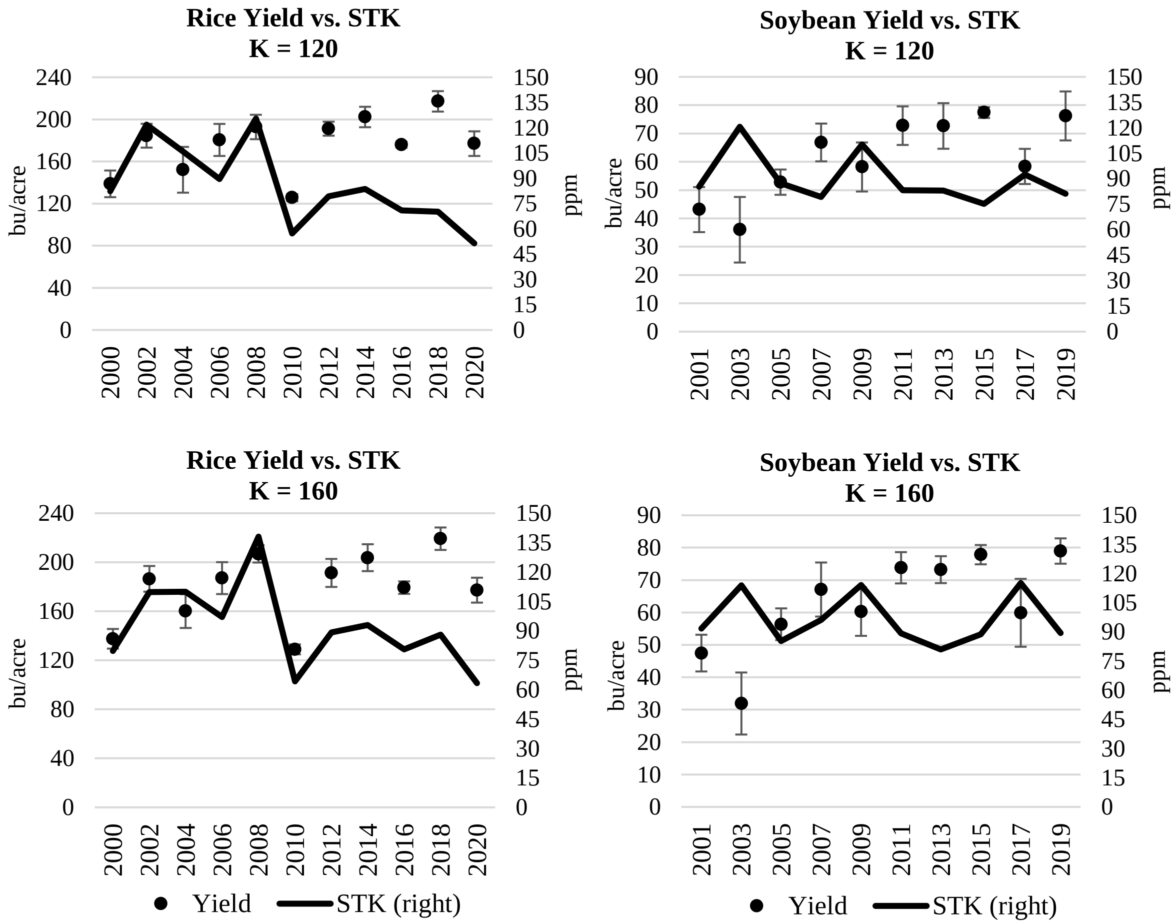

Figure 3.1. (Cont.) Comparison of average yield for rice (left) and soybean (right) in bu/acre of the eight replications within a year and a one standard deviation confidence interval around annual average yield to STK values at the five different K rate treatment applications conducted in Arkansas for 42 site-years and 840 individual observations where rice is grown in even years from 2000 to 2020 and soybean grown in odd years from 2001-2019. Data were provided by Dr. Nathan Slaton and Dr. Trent Roberts from field trials conducted over time.

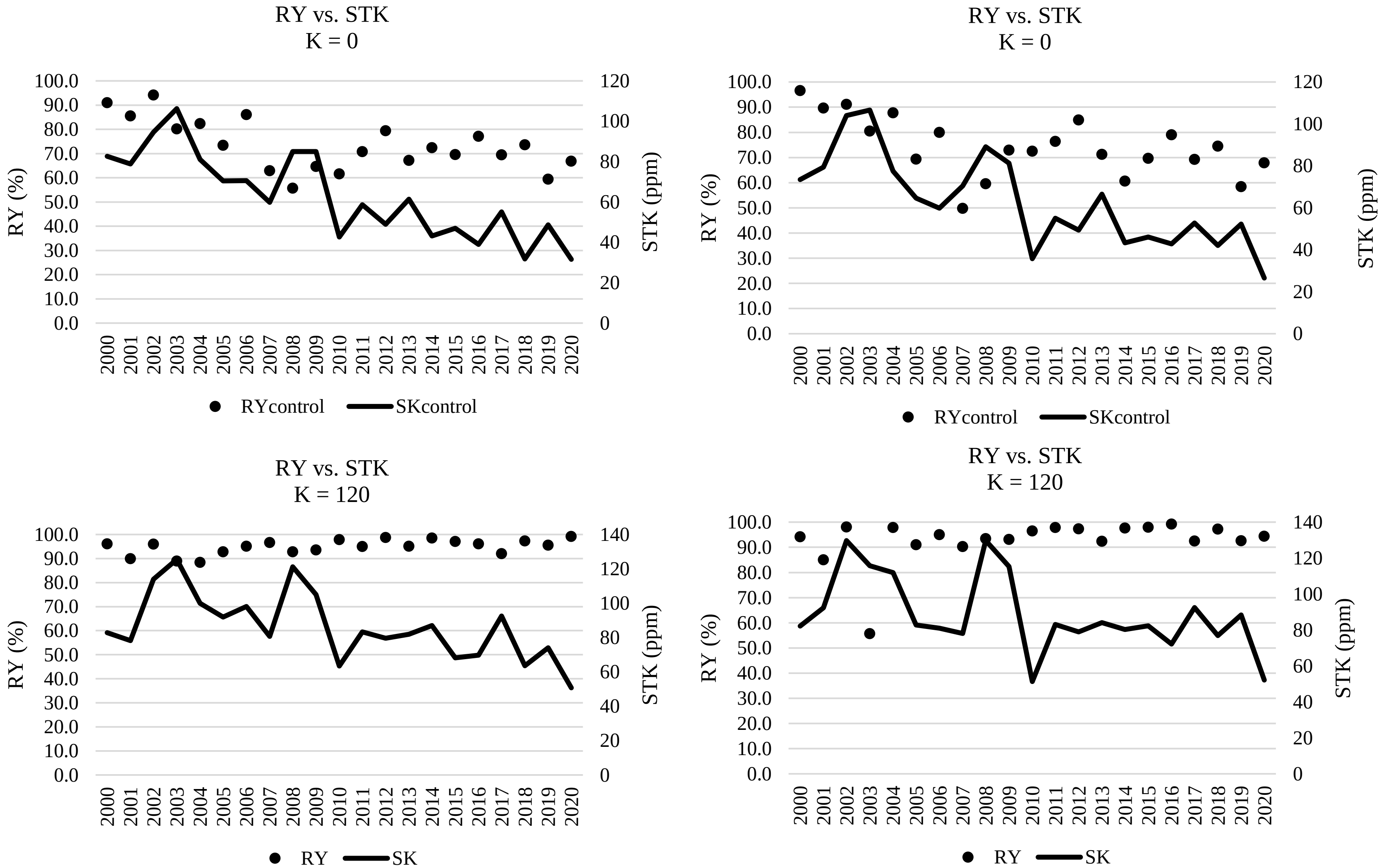

Figure 3.2. Average relative yield values (RY, data points) vs. soil-test K (STK, solid line) from 2000-2020 for the K= 0 lbs K_2O/acre and K = 120 lbs K_2O/acre application rates for the sites near Pine Tree, AR.

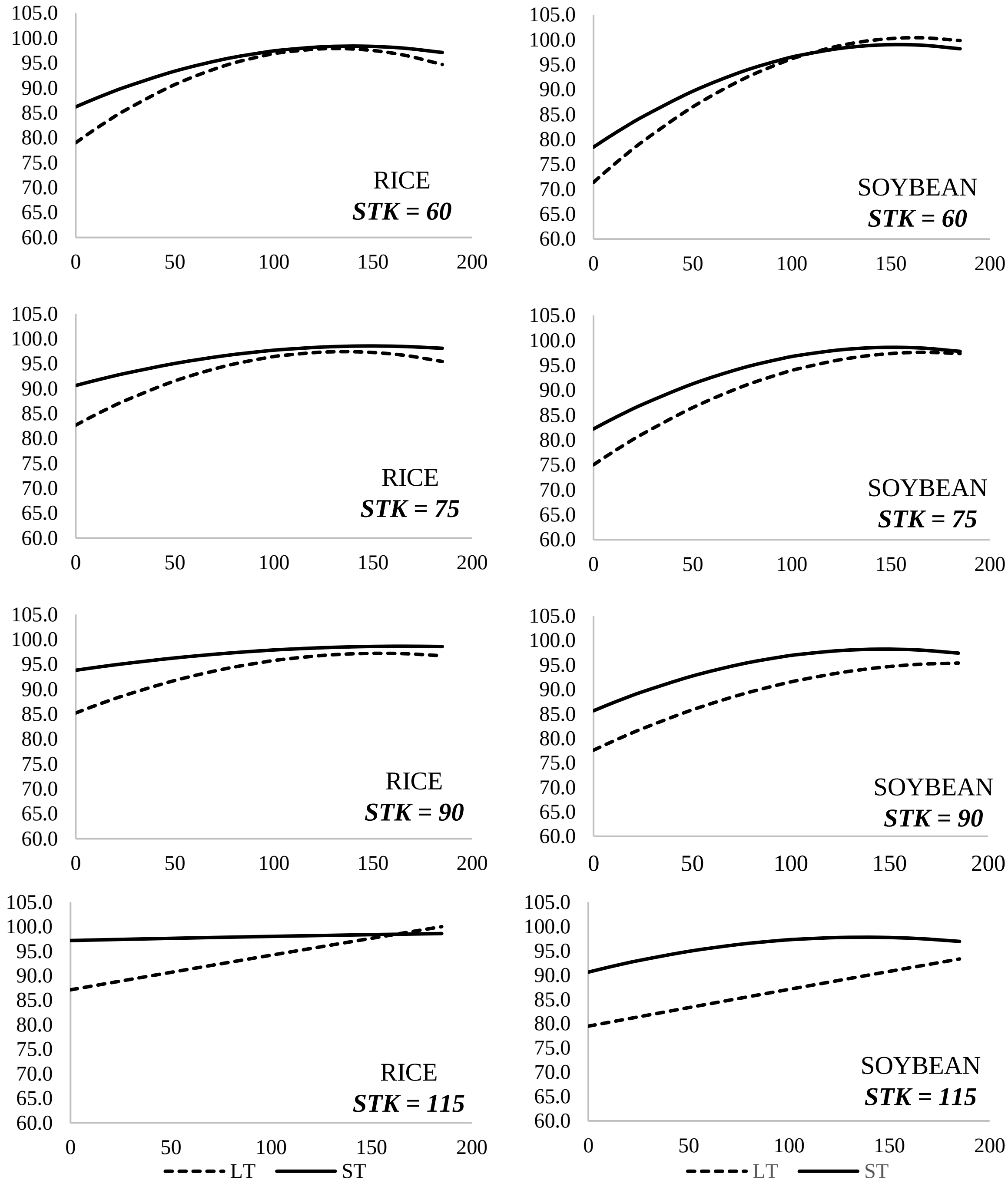

Figure 3.3. Comparison of rice (left) and soybean (right) relative yield responses to fertilizer-K rate at 60, 75, 90, and 115 ppm Mehlich-3 extractable soil-K (STK) concentrations (ppm) for the 0-10 cm depth under long-term (LT, dashed-line) and short-term (ST, solid-line) estimations.

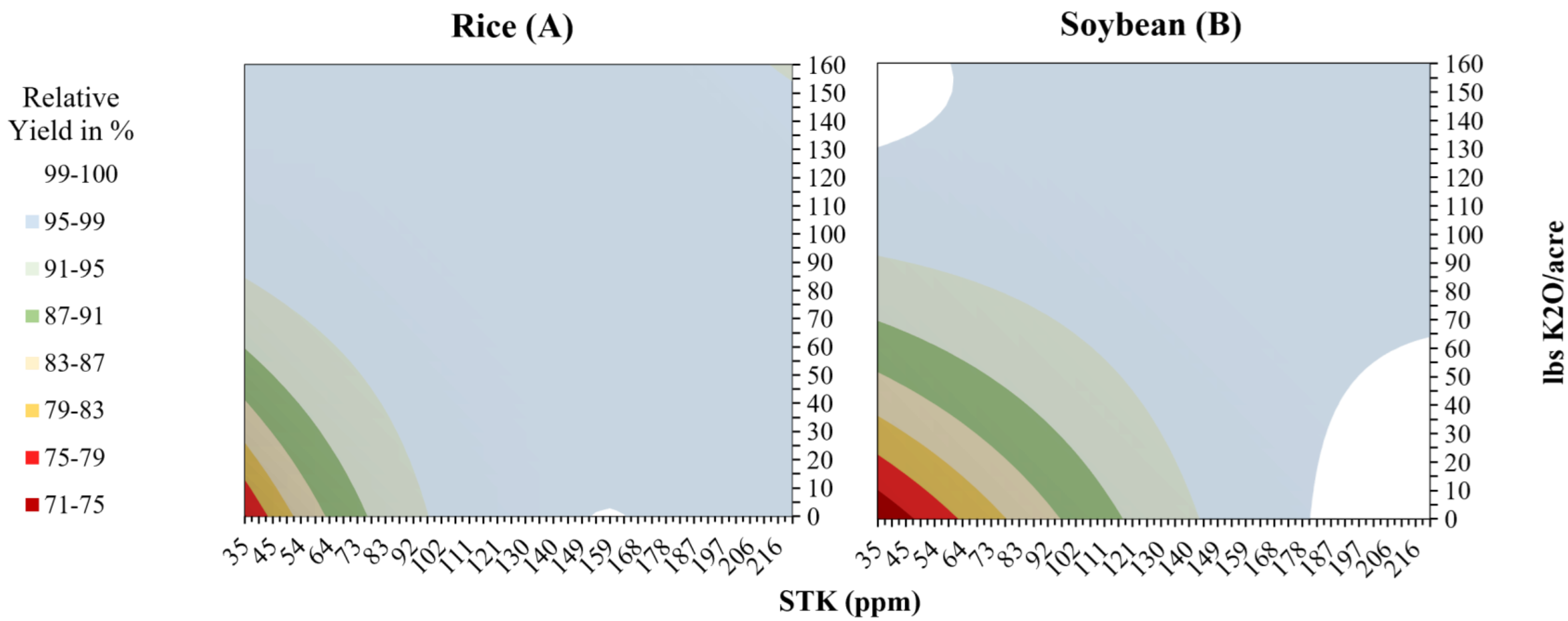

Figure 3.4. Estimated covariate relative yield response of fertilizer-K rate and initial Mehlich-3 soil-test K (STK) concentration on rice (A) and soybean (B) under the short-term framework.

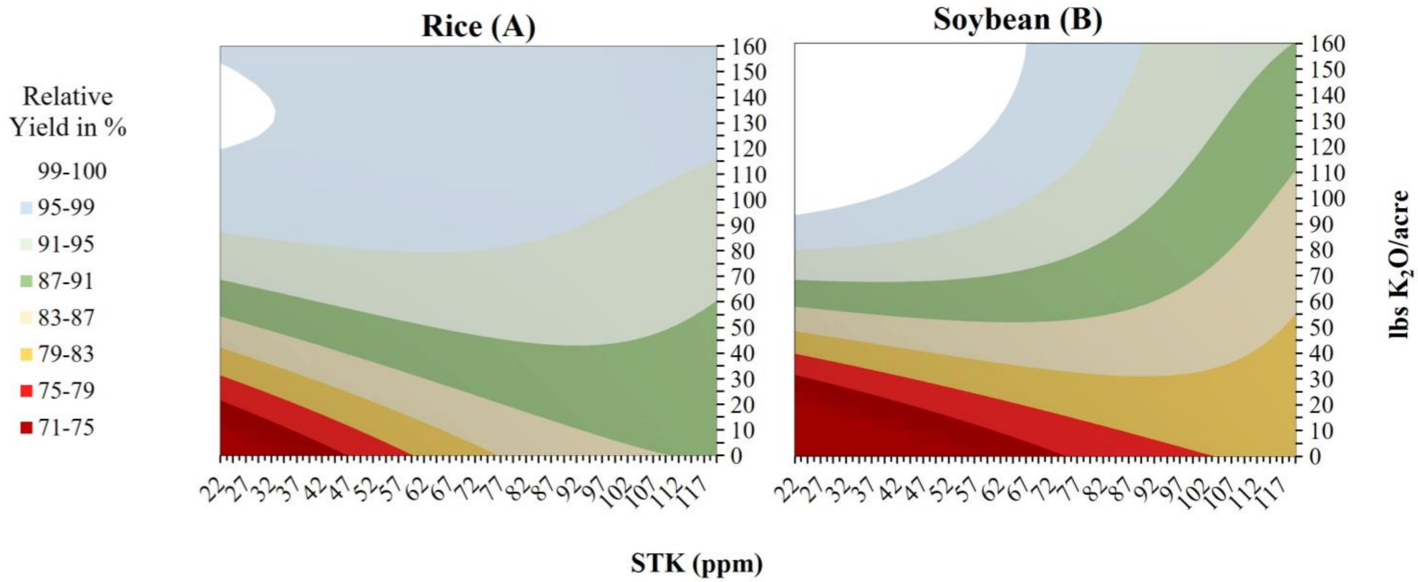

Figure 3.5. Estimated covariate relative yield response of fertilizer-K rate and initial Mehlich-3 soil-test K (STK) concentration on rice (A) and soybean (B) under the long-term framework.

Chapter IV. Summary of Results and Conclusions with Future Research Opportunities

A. Summary of Results and Conclusions

Chapter II calculated the profit-maximizing fertilizer-K rate for corn and cotton that considered both economic and agronomic values. While applying at higher fertilizer rates can serve as insurance against potential yield loss and can build STK, which protects against future fertilizer-K price increases, this practice results in paying for inputs earlier than needed, decreased profits, and increased potential for nutrient loss due to runoff. The profit-maximizing fertilizer-K rate considered the traditional initial STK and yield response information as well as crop price and the cost of fertilizer and its application. The null hypothesis for this research was that current fertilizer-K rate recommendations could be profitably curtailed for both corn and cotton, mirroring results found for rice and soybean (Popp et al., 2020, 2021). Results from this analysis on corn showed that fertilizer-K rate recommendations could, in fact, be profitably lowered with cost savings greater than yield loss. However, cotton results proved that a K^* greater than current recommendations enhances producers' profitability with estimated yield increases that are more than sufficient to afford the additional fertilizer-K costs. This was the case for cotton even in years when crop price was relatively low and fertilizer cost was relatively high and can be attributed to a greater yield response to K fertilization and relatively high cotton crop value vs. the other three crops analyzed. Therefore, cotton is the only crop from this analysis that repeatedly experienced K^* greater than current recommendations, while rice, soybean, and corn all could profitably reduce application. User-friendly decision support software, which will be available online to producers, can assist producers in selecting their crop specific K^* under varying STK and yield response, to estimate a yield response to K that is valued considering crop value and compared to attendant fertilizer costs.

Following Chapter II, Chapter III discussed a long-term approach to profit maximizing fertilizer-K rates on a rice/soybean rotation from 2010-2020 and compared results of long-term model yield response curve estimates to short-term model yield responses while also estimating changes in STK associated with different K rate applications. A comparison was also made between the two-profit maximizing strategies and applying at current extension rate recommendations. Results suggest that the short-term analyses from Popp et al. (2020 and 2021) are more profitable regardless of the initial STK level in the soil compared to both the long-term profit-maximizing strategy and extension recommendations. Further, since the short-term estimation technique is based on a more comprehensive set of sites across Arkansas, estimated yield responses are considered more representative than those reported for the two sites using the long-term framework. Under each profit-maximizing framework, regardless of initial STK in 2010, STK converged to the same STK in 2020 of 86 ppm using long-term yield response curves and 80 ppm when using short-term yield response curves. STK converged in 2020 to 85 ppm at each starting level of STK when following the extension-based recommendations and the short-term yield response curves. The null hypothesis of this study was that applying at higher rates would build STK levels over time whereas the zero rate control would remove STK. The second null hypothesis was that applying fertilizer-K at K^* rates as determined using the decision support software from Popp et al. (2020 and 2021) would lead to similar long-term profit in comparison to using profit-maximizing rates calculated using the estimated yield response curves from the long term field rotation data. However, the higher application rates in the long-term profit-maximizing strategy as well as the extension recommendations did not build STK levels over time to a significantly higher final STK in 2020 as compared to the short-term. Because findings using the short-term framework are considered more representative of average Arkansas conditions, producers are encouraged to use the decision tool from Popp et al. 2020 and 2021, which will soon be available for all crops discussed in this thesis. The tool allows entry of specific field information, yield potential and STK, to generate a profit-maximizing fertilizer-K application rate that is based not only on average yield response to K as it varies by field but also the value of the crop and the cost of fertilizer.

B. Study Limitations and Future Research Opportunities

Cotton results presented in Chapter II generate a U-shaped yield response curve when 120 ppm < STK < 177 ppm. Therefore, users are warned to use caution when following recommendations in that range. The U-shape of the yield response curve is counterintuitive to the typical crop response to fertilizer-K within a field. Under normal field conditions, fertilizer K increases the yield generated to a yield maximum point where applying past the yield maximum makes the additional fertilizer-K counterproductive. Chapter III presents results based on a dataset that included two sites over a 21-year period. Results and conclusions could be subject to change if more sites across Arkansas are used to represent a comprehensive state average yield response to then analyze the difference between the long-term and short-term profit maximizing fertilizer-K rate frameworks.

Future research may assess whether spatial variation in initial soil test K offers an opportunity to use spatially different, profit-maximizing fertilizer-K rates and thereby examine the economic feasibility of utilizing variable-rate technology (VRT) application. Additionally, because Chapter III was a preliminary analysis, additional rice/soybean rotation data as well as other crop rotations could be assessed to simulate and estimate the long-run implications of applying at various fertilizer-K rates and the specific impacts on STK by those particular crops. As the short-term profit-maximizing fertilizer-K rate in Chapter III used less fertilizer-K over time than both the current extension-based and long-term profit-maximizing rate recommendations, the value of extending the life of a potassium mine could also be incorporated as well as the impact of the decreased rate in the rise of fertilizer-K prices throughout time from the life extension of the non-renewable resource.

C. References

Popp, M., Slaton, N. A., Norsworthy, J. S., & Dixon, B. 2021. Rice yield response to potassium: an economic analysis. Agronomy Journal, 113, 287-297. https://doi:10.1002/agj2.20471

Popp, M., Slaton, N. A., & Roberts, T. L. 2020. Profit-maximizing potassium fertilizer recommendations for soybean. Agronomy Journal, 112, 5081-5095. https://doi:10.1002/agj2.20424.

Appendix

Supplemental Table 1. Statistical results comparison of using time-lagged Mehlich-3 soil-test K (STK_{t-1}), fertilizer-K application rate (K_{t-1}), yield index ($YI_{t-1}{}^{a}$), and an interaction term between YI_{t-1} and the *Rice* dummy variable to explain the current time period STK from 840 individual treatment observations of trials conducted from 2000 to 2020 in eastern Arkansas under an irrigated rice and soybean rotation using panel least squares regression.

Explanatory Variable[b]	Coefficient Estimate (SE[c])
Constant	55.00***
(α_0)	(12.43)
STK_{t-1}	0.35**
(α_1)	(0.11)
K_{t-1}	0.14***
(α_2)	(0.03)
YI_{t-1}	-0.12
(α_3)	(0.12)
YI_{t-1}	-0.09
(α_3)	(0.07)
Adj. R^2	0.331

[a] Relative Yield Index calculated using Eq. 2 in Chapter III.

[b] Lag of observed soil-test K concentrations as defined by Mehlich-3 extractable soil-K concentrations in ppm (STK_{t-1}) and lag of fertilizer-K application rate (K_{t-1}) in lbs K_2O/acre.

[c] The coefficient covariance matrix was adjusted using White's cross-section option. Statistical significance: * -- $p < 0.05$, ** -- $p < 0.01$, *** --- $p < 0.001$

Supplemental Table 2. Statistical results of using all interaction terms between initial Mehlich-3 soil-test K (*STK*) and fertilizer-K application rate (*K*) to explain relative yield (*RY* [a]) from 840 individual treatment observations of trials conducted from 2000 to 2020 in eastern Arkansas under a rice/soybean rotation using panel least squares and treating production year as a random effect.

Explanatory Variable[b]	Coefficient Estimate (SE[c])	Explanatory Variable[b]	Coefficient Estimate (SE[c])
Constant	30.75	*Rice*	20.60
(β_0)	(18.90)	(β_9)	(20.88)
K	1.26***	*Rice · K*	-0.49
(β_1)	(0.35)	(β_{10})	(0.38)
K^2	$-4.52\text{x}10^{-3,*}$	$Rice \cdot K^2$	$1.49\text{x}10^{-3}$
(β_2)	($2.28\text{x}10^{-3}$)	(β_{11})	($2.42\text{x}10^{-3}$)
STK	0.91	*Rice · STK*	-0.21
(β_3)	(0.48)	(β_{12})	(0.57)
STK^2	$-4.15\text{x}10^{-3}$	$Rice \cdot STK^2$	$4.89\text{x}10^{-4}$
(β_4)	($2.64\text{x}10^{-3}$)	(β_{13})	($3.44\text{x}10^{-3}$)
K · STK	-0.02*	*Rice · K · STK*	$6.08\text{x}10^{-3}$
(β_5)	(0.01)	(β_{14})	($9.71\text{x}10^{-3}$)
$K \cdot STK^2$	$6.60\text{x}10^{-5}$	$Rice \cdot K \cdot STK^2$	$-7.53\text{x}10^{-6}$
(β_6)	($4.71\text{x}10^{-5}$)	(β_{15})	($5.59\text{x}10^{-5}$)
$K^2 \cdot STK$	$6.44\text{x}10^{-5}$	$Rice \cdot K^2 \cdot STK$	$-1.53\text{x}10^{-5}$
(β_7)	($5.37\text{x}10^{-5}$)	(β_{16})	($5.76\text{x}10^{-5}$)
$K^2 \cdot STK^2$	$-2.04\text{x}10^{-7}$	$Rice \cdot K^2 \cdot STK^2$	$-1.78\text{x}10^{-8}$
(β_8)	($3.03\text{x}10^{-7}$)	(β_{17})	($3.26\text{x}10^{-7}$)
Adj. R^2	0.513		

[a] Relative Yield calculated using Eq. 1 in Chapter III.

[b] Observed soil-test K concentrations as defined by Mehlich-3 extractable soil-K concentrations in mg K kg^{-1} (*STK*) and fertilizer-K application rate (*K*) in kg K ha^{-1}.

[c] The coefficient covariance matrix was adjusted using White's cross-section option. Statistical significance: * -- $p < 0.05$, ** -- $p < 0.01$, *** --- $p < 0.001$.