ELEG 3124 Signal and Systems

Homework Manual

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ELEG 3124 Assignment # 1

1. Perform even and odd decomposition of the following signals.

   (a) $s(t) = e^{\Omega_0 t}$

   (b) $s(t) = \begin{cases} \sin(2t+3), & t > 0, \\ 0, & \text{otherwise.} \end{cases}$

2. Are the following signals periodic? If so, find their periods.

   (a) $x(t) = \sin(\pi t/3) + 2\cos(8\pi t/3)$

   (b) $x(t) = \exp\left(j\frac{7\pi}{6}t\right) + \exp\left(\frac{5\pi}{6}t\right)$

   (c) $x(t) = 2\sin\left(\frac{3\pi}{8}t\right) + \cos\left(\frac{3}{4}t\right)$

3. Determine whether the following signals are power or energy signals or neither. Justify your answers.

   (a) $x(t) = A\sin(t), -\infty < t < \infty$

   (b) $x(t) = \exp[-at], a > 0, t > 0$

   (c) $x(t) = A\exp[bt], b > 0$

   (d) $x(t) = \exp[-(a+jb)t], a > 0, t > 0$

ELEG 3124 Assignment # 2

1. Let

$$x(t) = \begin{cases} 2t+2, & -1 \le t < 0 \\ 2t-2, & 0 \le t < 1 \end{cases}$$

   (a) sketch $x(t)$

   (b) sketch $x(t-2), x(t+3), x(-3t-2)$ and $x\left(\frac{2}{3}t + \frac{1}{2}\right)$ and find the analytical expressions for these functions.

2. The rectangular signal $x(t) = p_2(t) = \begin{cases} 1/2, & -1 < t < 1 \\ 0, & \text{o.w.} \end{cases}$ is transmitted through the atmosphere and is reflected by different objects located at different distances. The received signal is

$$y(t) = x(t) + 0.5x(t - \frac{T}{2}) + 0.25x(t-T), T \gg 2 \tag{1}$$

   Sketch $y(t)$ for $T = 10$.

3. Sketch the following signals

   (a) $x_1(t) = u(t) + 5u(t-1) - 2u(t-2)$

   (b) $x_2(t) = r(t) - r(t-1) - u(t-2)$

   (c) $x_3(t) = x_1(2t+4)$

4. Evaluate the following integrals:

   (a) $\int_{-\infty}^{\infty} \left(\frac{2}{3}t - \frac{3}{2}\right) \delta(t-1)dt$

   (b) $\int_{-\infty}^{\infty} (t-1)\delta\left(\frac{2}{3}t - \frac{3}{2}\right) dt$

   (c) $\int_{-3}^{2} [\exp(-t+1) + \sin(2\pi t/3)]\delta(t-3/2)dt$

   (d) $\int_{-3}^{-2} [\exp(-t+1) + \sin(2\pi t/3)]\delta(t-3/2)dt$

ELEG 3124 Assignment # 3

1. Determine whether the following systems are linear or non-linear, causal or non-causal, time invariant or time variant, and memoryless or with memory

   (a) $y(t) = 2x(t) + 3$

   (b) $y(t) = 2x^2(t) + 3$

   (c) $y(t) = Atx(t)$

   (d) $y(t) = x(t)u(t) - x(t)u(-t)$

   (e) $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$

   (f) $y(t) = \int_{0}^{t} x(\tau)d\tau$

   (g) $y(t) = x(t)x(t-2)$

   (h) $y(t) = \frac{1}{T}\int_{t-T/2}^{t+T/2} x(\tau)d\tau$

ELEG 3124 Assignment # 4

Define a rectangular pulse $p(t) = u(t+1) - u(t-1) = \begin{cases} 1, & -1 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$.

1. Evaluate the following convolutions ($a > 0, b > 0$).

   (a) $u(t) \otimes u(t)$

   (b) $p\left(\frac{t-a}{a}\right) \otimes \delta(t-b)$

   (c) $p\left(\frac{t}{a}\right) \otimes p\left(\frac{t}{a}\right)$

   (d) $p\left(\frac{t}{a}\right) \otimes u(t)$.

   (e) $tu(t) \otimes p\left(\frac{t}{a}\right)$

2. An LTI system has an impulse response as $h(t) = \exp(-2t)u(t)$. If the input is $x(t) = \exp(-t)u(t) + u(t)$, find the output of the system.

3. **Graphically** evaluate the convolution: $p(t) \otimes p(t)$.

ELEG 3124 Assignment # 5

Define a rectangular pulse $p(t) = u(t+1) - u(t-1) = \begin{cases} 1, & -1 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$.

1. An LTI system has an impulse response $h(t) = tu(t-5)$. If the input is $x(t) = t^2[u(t-1) - u(t-3)]$, find the output.

2. Determine whether the contintuous-time LTI systems characterized by the following impulse responses are causal or non-causal, stable or non-stable.

   (a) $h(t) = e^{4t}u(-t)$

   (b) $h(t) = (-t)e^{-t}u(-t)$

   (c) $h(t) = e^{-|2t|}$

   (d) $h(t) = p(t/2)$.

   (e) $h(t) = \delta(t) + e^{-3t}u(t)$

3. Are the LTI systems with the following impulse responses invertible? If invertible, find the inverse system.

   (a) $h(t) = 3\delta(t+3)$

   (b) $h(t) = \delta(t-3) + \delta(t-5)$.

4. Consider a circuit with a voltage source, $v(t)$, a resistor with resistance R, and a capacitor with capacitance C connected in serise. If the input of the system is the voltage source $v(t)$, and the output of the system is the voltage across the capacitor, $v_c(t)$. Write the system equation in the form of a differential equation.

ELEG 3124 Assignment # 6

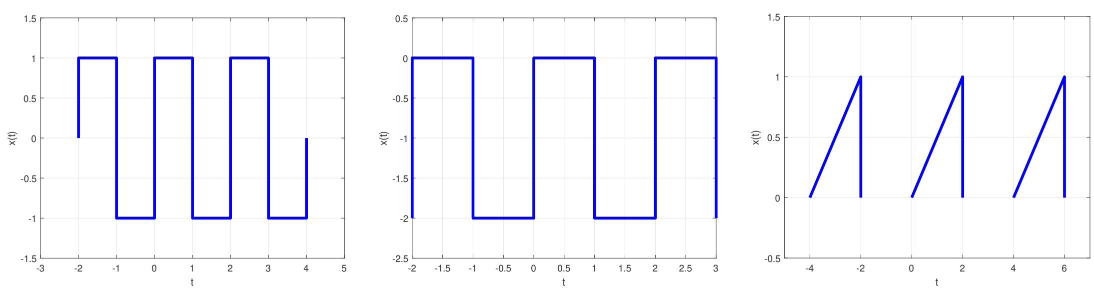
1. For the periodic signal

$$x(t) = 2 + \frac{1}{3}\cos\left(t + \frac{\pi}{6}\right) + 2\cos(3t) - 2\sin\left(5t + \frac{\pi}{6}\right) \tag{1}$$

   (a) Find the Fourier series

   (b) Use Matlab to sketch the magnitude and phase spectra as a function of the angular frequency Ω.

2. Find the Fourier series representation of the signals shown in Fig. 1. Plot the magnitude and phase spectrum for each case.



(a) (b) (c)

Figure 1: Question 2

3. The signal shown in Figure 2 (next page) is created with a sine voltage is rectified by a circuit with two diodes, a process known as full-wave rectification. Find the Fourier series of the signal. (Hint: $x(t) = \sin(t), 0 < t < \pi$. $x(t)$ can be expressed as complex exponentials with Euler's formula).

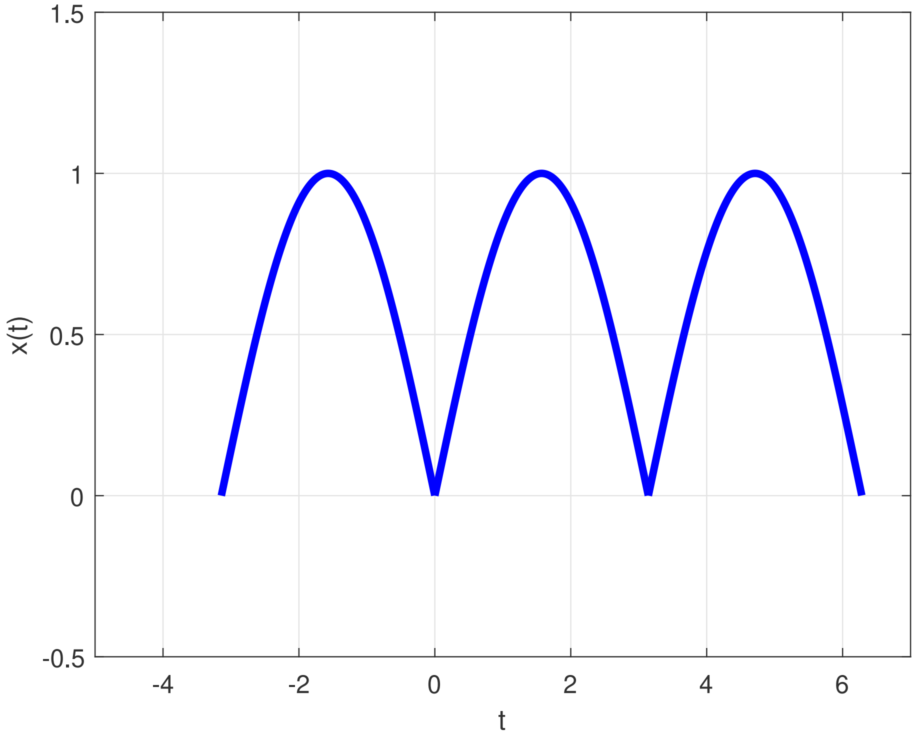


Figure 2: Question 3

ELEG 3124 Assignment # 7

1. Find the periods and Fourier series coefficents of the following signals
   (a) $s(t) = \sum_{n=-\infty}^{\infty} \delta(t-n)$.
   (b) $s(t) = \sum_{n=-\infty}^{\infty} (-1)^n \delta(t-n)$

2. A voltage $x(t)$ is applied to the circuit shown in Fig. 2. If the Fourier coefficients of $x(t)$ are given by

$$c_n = \frac{1}{n^2+1} e^{jn\frac{\pi}{3}} \quad (1)$$

   (a) Express the system in the form of a differential equation
   (b) Find the transfer function of the system
   (c) Plot the amplitude and phase of the transfer function with Matlab
   (d) Find the first three non-zero harmonics of y(t)

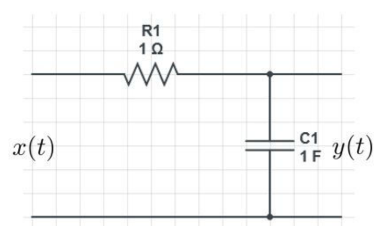


Figure 1: Questions 2, 3, and 4.

3. Repeat Question 3 if $y(t)$ is the voltage across the resistor instead.

4. For the RC circuit shown in Figure 2, find the voltage $y(t)$ across the capacitor if the input is

$$x(t) = 1 + 3\cos\left(t + \frac{\pi}{6}\right) + \cos(2t) \quad (2)$$

ELEG 3124 Assignment # 8

1. Find the Fourier transform of the following signals

   (a) $s(t) = \text{rect}\left(\frac{t-1}{2}\right)$.

   (b) $s(t) = e^{-2t}u(t)$

   (c) $s(t) = e^{2t}u(-t)$

   (d) $s(t) = 2e^{-3|t|} + 2\delta(t-3)$

2. Let $X(\omega) = \text{rect}\left(\frac{\omega-1}{2}\right)$. Find the Fourier transform of the following functions by using the properties of the Fourier transform

   (a) $x(-t)$

   (b) $x(-3t+6)$

   (c) $\frac{dx(t)}{dt}$

ELEG 3124 Assignment # 9

1. Let $X(\omega) = \frac{1}{2+j\omega}$. Find the Fourier transform of the following functions by using the properties of the Fourier transform
   (a) $tx(t)$
   (b) $x\left(\frac{t}{2} - 1\right)$
   (c) $t\frac{dx(t)}{dt}$
   (d) $(t-1)x(t+1)$
   (e) $x(2t-1)\exp(-j2t)$
   (f) $x(t)\cos(\omega_0 t)$

2. Use the properties of Fourier transform, find the Fourier transform of the following signals.
   (a) $\text{sinc}(t)$
   (b) $\exp(j\omega_0 t)$
   (c) $\sin(\omega_0 t)$

3. The impulse response of an LTI system is $\exp(-t)u(t)$. If the input is $\exp(-2t)u(t)$, find the output of the system by using Fourier transform.

4. Using Parseval's theorem, find the energy of the signal $\text{sinc}(t)$.

ELEG 3124 Assignment # 10

1. Consider a system with transfer function $H(\omega) = \text{rect}\left(\frac{\omega}{2\omega_f}\right)$. The input signal is $x(t) = \frac{\sin(\omega_1 t)}{t} + \frac{\sin(\omega_2 t)}{t}$, where $0 < \omega_1 < \omega_2$.
   (a) Find the impuse response $h(t)$.
   (b) Find $X(\omega)$.
   (c) Find $y(t)$ is $0 < \omega_f < \omega_1$.
   (d) Find $y(t)$ is $\omega_1 < \omega_f < \omega_2$.
   (e) Find $y(t)$ is $\omega_2 < \omega_f$.

2. The Fourier transform of $x(t)$ is $X(\omega)$. The pulse train is $p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$. Define $x_s(t) = x(t)p(t)$ as the sampled signal of $x(t)$ with a sampling period of T_s.
   (a) Find the Fourier transform of $x_s(t)$.
   (b) Assume the highest frequency of $x(t)$ is ω_0 and it satisfies $2\omega_0 \leq \omega_s = \frac{2\pi}{T_s}$. Pass $x_s(t)$ through a low pass filter with transfer function $H(\omega) = \text{rect}\left(\frac{\omega}{\omega_s}\right)$, what is the time domain signal at the output of the filter?

3. The amplitude modulation can be represented as $s(t) = m(t)\cos(\omega_c t)$, where $m(t)$ is the message signal with the highest frequency ω_0 and $\cos(\omega_c t)$ is the carrier signal. The carrier frequency is ω_c and $\omega_c >> \omega_0$. The Fourier transform of $m(t)$ is $M(\omega)$.
   (a) Find the Fourier transform of $s(t)$.
   (b) At the receiver, the coherent demodulator will perform $r(t) = s(t)\cos(\omega_c t)$, then pass the signal through a low pass filter with transfer function $H(\omega) = \text{rect}\left(\frac{\omega}{2\omega_0}\right)$. Find the Fourier transform of $r(t)$. Find the output of the low pass filter.

ELEG 3124 Assignment # 11

1. Find the bilateral Laplace transform of the following signals.

   (a) $\exp(t+1)$

   (b) $|t|$.

   (c) $\exp(-2|t|)$.

   (d) $\cos(at)u(-t)$.

2. Find the unilateral Laplace transform of the following signals.

   (a) $x(t) = t \cdot \text{rect}\left(\frac{t-1}{2}\right)$

   (b) $x(t) = Au(t) + 2\delta(t)$.

   (c) $x(t) = A\cos(\omega_0 t + \theta)$.

   (d) $x(t) = A\sin(\omega_0 t + \theta)$.

ELEG 3124 Assignment # 12

1. The Laplace transoform of a causal signal $x(t)$ is

$$X(s) = \frac{s+5}{s^2+3s+2}, \qquad \mathrm{Re}(s) > -1 \tag{1}$$

   Using the properties of Laplace transform, find the Laplace transform of the following signals

   (a) $3x(t/3)$
   (b) $x(t-2)$
   (c) $(t-1)x(t)$.
   (d) $\frac{dx(t)}{dt}$.
   (e) $x(t)\exp(-2t)$.
   (f) $x(t)\cos(2t)$ (don't need to simplify)

2. Determine the initial value and final value of the signal whose Laplace transform is $X(s) = \frac{s+2}{s^2-2s-3}$.

3. Solve the following differential equation:

$$y''(t) + 4y'(t) + 3y(t) = \exp(-2t)u(t), y(0^-) = 0, y'(0^-) = 1 \tag{2}$$

ELEG 3124 Assignment # 13

1. Find the inverse Laplace transform

   (a) $\frac{s+2}{s^2-s-2}$

   (b) $\frac{s^2}{s^2+3s+2}$

   (c) $\frac{s}{s^2+2s+7}$.

   (d) $\frac{1}{(s+2)^2}$

2. Consider a system with input $x(t)$ and output $y(t)$ described in the following equations. Find the impulse response $h(t)$.

$$x(t) = \exp(-2t)u(t) \tag{1}$$

$$y(t) = [\exp(-t) - 3\exp(-2t)]u(t) \tag{2}$$

3. Consider an LTI system described by the following equation (the system is initially relaxed)

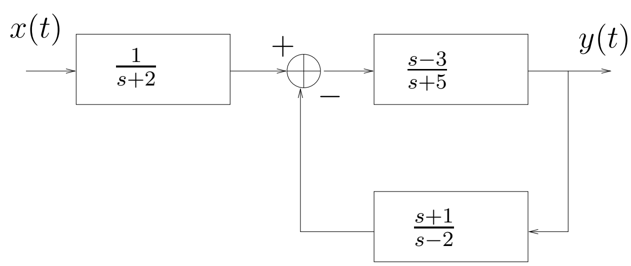
$$y''(t) + 4y'(t) + 3y(t) = 2x(t) - 3x'(t) \tag{3}$$

   (a) Find the transfer function $H(s)$

   (b) Draw the first canonical form representation of the system

   (c) Is the system BIBO stable?

4. The block diagram of a system is represented in the figure shown in the next page. Find the transfer function of the system. Is the system BIBO stable?


$x(t)$
$\frac{1}{s+2}$
+
−
$\frac{s-3}{s+5}$
$y(t)$
$\frac{s+1}{s-2}$

ELEG 3124 Assignment # 14

1. Find the Convolution Sum between $x(n)$ and $h(n)$.

   (a) $x(n) = \left(\frac{1}{3}\right)^n u(n)$, $h(n) = u(-n)$.

   (b) $x(n) = [\underset{\uparrow}{1}, 2, 3]$, $h(n) = [\underset{\uparrow}{1}, 2]$

2. Find the Bilateral Z-transform of the following signals

   (a) $x(n) = 3^n u(-n-2)$

   (b) $x(n) = [2, \underset{\uparrow}{1}, 3, 5, 6]$

3. Consider a system described by the following difference equation

$$y(n) - 0.5y(n-2) = 2x(n) - x(n-1) - 0.5x(n-2) \tag{1}$$

   (a) Find the transfer function in the Z-domain

   (b) Is the system BIBO-stable?