A Study of the Impact of Prognostic Errors on System Performance

Mauricio Carrasco

University of Arkansas, Fayetteville
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By Mauricio Carrasco
Department of Industrial Engineering
Faculty Mentor: C. Richard Cassady, Ph.D., P.E.
Department of Industrial Engineering

Abstract:
Condition-based maintenance is growing in popularity as a means of improving equipment maintenance efficiency. Whether it be the maintenance of an airplane, a computer system, or any type of physical system, the prognostic tools associated with condition-based maintenance are subject to statistical error. These errors can lead to unnecessary preventive maintenance due to underestimation of system remaining life and unnecessary system failures due to overestimation of system remaining life. What is not clear is if these statistical errors outweigh the benefits of a condition-based maintenance policy. This study attempts to address this concern through the evaluation and comparison of three maintenance policies for a simple system. The maintenance policies are run-to-failure, scheduled preventive maintenance and condition-based maintenance. A discrete-event simulation model is used to estimate the average time between successful missions for the system under each of these policies. An extensive set of numerical experiments is used to analyze system performance under a wide variety of operating conditions. The results suggest that condition-based maintenance can improve system performance as much as 10% to 15% beyond that achieved using scheduled preventive maintenance. However, the results also suggest that moderate statistical error can render condition-based maintenance inferior to scheduled maintenance and severe statistical error can render condition-based maintenance inferior to run-to-failure. In addition to the results obtained by this study, the methodology used herein can aid maintenance managers in moving from a scheduled maintenance philosophy to a just-in-time maintenance philosophy; thereby increasing the availability of affected systems. Increasing the availability of any system is given considerable importance especially by industries that serve people. For example, in the airline and health industries the availability of a system is vital since any associated downtime results in large profit losses and customer dissatisfaction. Overall, the method presented herein can help any kind of industry in developing a way for assessing their maintenance policies which could help them improve the availability of their systems in the future.

Introduction:
The use of prognostics and condition-based maintenance has recently received an increased amount of interest from many industries. These methods use some physical assessment of a system to predict its remaining life and take maintenance action if appropriate. Ideally, such an action will take place instantaneously before failure so that no failures occur and no system uptime is lost unnecessarily. However, the challenge associated with prognostics is developing a system assessment mechanism that is both economically feasible and statistically valid as a means of predicting the remaining system life. Herein, the focus is on the second aspect of this challenge – statistical errors. The main objective of this research is to demonstrate a potential method for evaluating the impact of prognostic errors on system performance. To achieve this objective, a discrete-event simulation model is used to assess the performance of a system under three maintenance policies: (1) run-to-failure maintenance, (2) scheduled preventive maintenance, (3) condition-based maintenance. Various levels of prognostic error, including the ideal case in which prognostics are perfect, are modeled. The results of this experimentation are used to address three questions: (1) How much can perfect prognostics improve system performance beyond scheduled preventive maintenance? (2) How bad do prognostics have to be to make things worse than scheduled preventive maintenance? (3) How bad do prognostics have to be to make things worse than run-to-failure maintenance?

Model Development:
The goal of this research is to demonstrate a potential method for evaluating the impact of prognostic errors on system performance. To achieve this objective, a discrete-event simulation model was built to assess the performance of a simple system under three maintenance policies. The remainder of this section introduces the system being considered, describes the three maintenance policies, and explains the logic and assumptions behind the simulation model.

Consider a system that can be represented by a single, "black box" component. A new copy of this component has a
Weibull time to failure $X$ with cumulative distribution function $F(x)$ having shape parameter $\beta > 1$ and scale parameter $\eta > 0$, i.e.,

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\eta}\right)^\beta\right]$$

(1)

Note that the fact that $\beta > 1$ implies that the component has an increasing failure rate.

The system is required to perform a sequence of missions each having length $m$. If the system fails during a mission, then the mission is aborted and maintenance is performed. The time required to perform maintenance is $t_m$, and maintenance restores the system to an “as good as new” condition. The performance of this system is measured using the average time between successful missions $\mu$. If the system never experiences failure, then $\mu = m$. However, this ideal case never occurs. Therefore, we study, using discrete-event simulation, the performance of the system under three maintenance policies.

The first system maintenance policy considered is “run-to-failure” (RTF) maintenance. Under this policy, the system is maintained only upon failure. Let $\mu_{RTF}$ denote the average time between successful missions under this maintenance policy. Note that $\mu_{RTF} > m$ because time is “wasted” on unsuccessful missions and system maintenance. Simulation of system performance under the RTF policy requires the manipulation of three variables: (1) the time until failure of the system $(X)$, (2) the number of missions successfully completed $(N)$, and (3) the cumulative elapsed time required to reach $N_{\text{max}}$ successfully completed missions $(T_{\text{sum}})$. The input parameters for the simulation model are $\beta, \eta, m, t_m$ and $N_{\text{max}}$.

The second system maintenance policy considered is scheduled preventive maintenance (PM). Under this policy, an optimal, scheduled, preventive maintenance policy is applied to the system. This policy is summarized by the parameter $\tau$. Specifically, if a system successfully completes $\tau$ consecutive missions, then maintenance is performed prior to the next mission. The value of $\tau$ is determined using an embedded simulation-based optimization algorithm. Let $\tau_{\text{opt}}$ denote the average time between successful missions under this maintenance policy. Note that, since the PM policy is optimized, $\mu_{PM} < \mu_{RTF}$. Simulation of system performance under this policy requires the manipulation of five variables: (1) the time until failure of the system $(X)$, (2) the number of missions successfully completed $(N)$, (3) the current number of consecutive successfully completed missions $(N_{\text{con}})$, (4) the cumulative elapsed time required to reach $N_{\text{max}}$ successfully completed missions $(T_{\text{sum}})$, and (5) the upper limit on the number of consecutive successful missions $\tau$, which triggers the initiation of preventive maintenance. The input parameters for the simulation model are $\beta, \eta, m, t_m, N_{\text{max}}$ and $\tau$.

The third system maintenance policy considered is condition-based maintenance (CBM). Under this policy, scheduled preventive maintenance is replaced with a prognostic tool. The remaining life of the system is estimated at the end of each successful mission. If this estimate is less than the mission length $m$, then maintenance is performed prior to the next mission. We first consider “perfect prognostics”, i.e. the case in which the estimate of remaining life is exactly equal to the actual remaining life $X$. However, a perfect prognostic is an unrealistic standard. Therefore, we also consider cases in which the prognostic test is subject to error. Under imperfect prognostics, the estimate of the remaining life is equal to $X_{\text{est}}$, where

$$X_{\text{est}} = X + \varepsilon$$

(2)

and the prognostic error $\varepsilon$ is a normal random variable having a mean of zero and a standard deviation of $\alpha$ (note that $\alpha=0$ corresponds to perfect prognostics). This error creates the possibility of unnecessarily early maintenance due to overestimation of remaining life and system failure due to overestimation of remaining life. Let $\mu_{\text{CBM}}(\alpha)$ denote the average time between successful missions under this maintenance policy. Note that $\mu_{\text{CBM}}(0) < \mu_{PM}$. Furthermore, note that if $\alpha_1 < \alpha_2$, then $\mu_{\text{CBM}}(\alpha_1) < \mu_{\text{CBM}}(\alpha_2)$. Simulation of system performance under this policy requires the manipulation of four variables: (1) the time until failure of the system $(X)$, (2) the number of missions successfully completed $(N)$, (3) the cumulative elapsed time required to reach $N_{\text{max}}$ successfully completed missions $(T_{\text{sum}})$, and (4) the estimated time until failure of the system $X_{\text{est}}$. The input parameters for the simulation model are $\beta, \eta, m, t_m, N_{\text{max}}$ and $\alpha$.

Experimental Design:

The next step in achieving the main objective of this research and answering the associated questions was to design an experiment for evaluating system performance over a range of choices for the system reliability and maintainability characteristics. This section details how this experiment was designed to obtain the statistics of interest.

Without loss of generality, the characteristic life of a new system $\eta$ was set to 100. Then, four experimental factors were selected: $\beta, m, t_m, \tau$ and $\alpha$. Ten levels of $\beta, m$ and $t_m$ were considered (Table I). All combinations of these three factors were simulated, resulting in a total of 1,000 experiments to be simulated under each maintenance policy. For the CBM policy, 16 levels of $\alpha$ were considered for each experiment (Table I). The various levels for all these experimental factors were chosen in such a way as to envelop a wide range of operating circumstances for the system.

For each experiment 18 simulations were required: one for RTF, one for PM, and 16 for CBM. The statistics of interest collected from the simulations were the point estimates of $\mu_{\text{RTF}}, \mu_{\text{PM}}$ and 16 point estimates of $\mu_{\text{CBM}}(\alpha)$ (once for each value of
In order to ensure statistical validity of these estimates, each simulation was replicated 60 times with each replication having a run length of $N_{\text{max}} = 12,000$.

**Numerical Analysis:**

For all 1,000 experiments, the output of the simulation model can be used to assess the potential benefit of using CBM as opposed to PM. For each experiment, the maximum benefit resulting from the use of CBM as an alternative to PM can be estimated by

$$\frac{\hat{\mu}_{PM} - \hat{\mu}_{CBM}(0)}{\hat{\mu}_{PM}} \times 100\%.$$  

This value is referred to as the perfect prognostics improvement estimate and captures the percent improvement in system performance (average time between successful missions) resulting from the use of perfect prognostics ($\alpha = 0$) instead of PM. Table II contains summary statistics for the perfect prognostics improvement estimate across the 1,000 experiments, and Figure 1 contains a histogram of these 1,000 estimates.

### Table II. Perfect Prognostics Improvement Statistics

<table>
<thead>
<tr>
<th></th>
<th>Minimum</th>
<th>Median</th>
<th>Average</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.23%</td>
<td>8.71%</td>
<td>8.63%</td>
<td>14.55%</td>
</tr>
</tbody>
</table>

For all 1,000 experiments, the output of the simulation model can also be used to make a more formal comparison of the three maintenance policies. These comparisons are made through the use of statistical hypothesis testing. The first set of tests attempt to prove that CBM (for each of the 15 imperfect levels of $\alpha$) is superior to PM. In other words, the statistical hypothesis test is given by:

$$H_0: \mu_{CBM}(\alpha) \geq \mu_{PM}$$
$$H_1: \mu_{CBM}(\alpha) < \mu_{PM}$$

Using the output of the simulation model, these tests (15 tests for each of the 1,000 experiments) are evaluated using a two-sample $t$-test (variances not assumed to be equal) with a level of significance of 0.025. The second set of tests attempt to prove that CBM (for each of the 15 imperfect levels of $\alpha$) is inferior to PM. In other words, the statistical hypothesis test is given by:

$$H_0: \mu_{CBM}(\alpha) \leq \mu_{PM}$$
$$H_1: \mu_{CBM}(\alpha) > \mu_{PM}$$

Using the output of the simulation model, these tests (15 tests for each of the 1,000 experiments) are evaluated using a two-sample $t$-test (variances not assumed to be equal) with a level of significance of 0.025. The third set of tests attempt to prove that CBM (for each of the 15 imperfect levels of $\alpha$) is superior to RTF. In other words, the statistical hypothesis test is given by:

$$H_0: \mu_{CBM}(\alpha) \leq \mu_{RTF}$$
$$H_1: \mu_{CBM}(\alpha) > \mu_{RTF}$$

Using the output of the simulation model, these tests (15 tests for each of the 1,000 experiments) are evaluated using a two-sample $t$-test (variances not assumed to be equal) with a level of significance of 0.025. The fourth set of tests attempt to prove that CBM (for each of the 15 imperfect levels of $\alpha$) is inferior to RTF. In other words, the statistical hypothesis test is given by:

$$H_0: \mu_{CBM}(\alpha) \geq \mu_{RTF}$$
$$H_1: \mu_{CBM}(\alpha) < \mu_{RTF}$$

Using the output of the simulation model, these tests (15 tests for each of the 1,000 experiments) are evaluated using a two-sample $t$-test (variances not assumed to be equal) with a level of significance of 0.025.

The results of these four sets of tests are summarized in Table III and Table IV. For example, when $\alpha = 60$, the statistical testing suggests that CBM is superior to PM for 651 of the 1,000 experiments, inferior to PM for 322 experiments, and equivalent to PM for 27 experiments. Furthermore, when $\alpha = 100$, the
statistical testing suggests that CBM is superior to RTF for 913 of the 1,000 experiments, inferior to RTF for 77 experiments, and equivalent to RTF for 10 experiments. Note that as $\alpha$ increases (decreases), CBM is more often inferior (superior) to PM.

In comparing CBM to RTF, it should be noted that RTF is superior (inferior) to CBM increases (decreases), CBM is more often inferior (superior) to PM.

### Table III. Comparing CBM to PM

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Cases for which CBM is superior to PM</th>
<th>Cases for which CBM is inferior to PM</th>
<th>Cases for which CBM is equivalent to PM</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>971</td>
<td>22</td>
<td>7</td>
</tr>
<tr>
<td>20</td>
<td>910</td>
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</tr>
<tr>
<td>150</td>
<td>208</td>
<td>753</td>
<td>39</td>
</tr>
</tbody>
</table>

Analysis of the results of the hypothesis testing for each value of $\pm$ reveals apparent patterns in the individual test results. Specifically, three commonalities were observed. First, as $t_{im}$ increases (decreases), the number of tests concluding that CBM is superior (inferior) to PM increases. Longer PM breaks have a more negative effect on system performance, therefore, it is more desirable to avoid them through the use of prognostics. Second, as $t_{im}$ increases (decreases), the number of tests concluding that CBM is superior (inferior) to RTF decreases. This can be attributed to the fact that the only difference when comparing CBM to RTF is that under prognostics one could perform maintenance unnecessarily due to underestimated remaining life; however, if the remaining life is overestimated, the system will just fail which is what happens with RTF. As a result, longer maintenance breaks due to underestimated remaining life should be avoided or else RTF will become more effective. Third, as $\beta$ increases (decreases) so do the number of tests concluding that CBM is inferior (superior) to PM. This can be explained in general terms by the fact that as $\beta$ increases, the short-term reliability of the system improves whereas the long-term reliability worsens. This characteristic leads to much unnecessarily early maintenance under CBM because the effect of prognostic error intensifies as the long-term reliability worsens.

### Concluding Remarks:

This study is based on the assumption that the failure of the system under consideration is governed by a known Weibull distribution. Therefore, this study is somewhat biased in favor of PM. Therefore, future work should consider the case in which the parameters of the Weibull distribution are subject to statistical error. In this case, the PM policy will not necessarily be optimal and CBM will appear more effective. Furthermore, future work should consider the case in which system failure is governed by a physics-based model. In this case, CBM will be even more attractive as a maintenance policy.

This study provides a great deal of insight into how prognostic errors can impact and perhaps worsen the performance of a system. However, this study considers a system with a single-component or “black box” structure, a straightforward mission profile, a basic measure of performance, and a simple prognostic tool. Therefore, four obvious areas for further study are systems with: (1) more complex component structures, (2) more complex mission profiles, (3) more elaborate measures of performance, and (4) more realistic prognostic tools.

### Mentor comments:

Richard Cassidy, Mr. Carrasco's faculty mentor, had the following things to say about his student's work:

Although Mauricio has been involved with several research efforts during the past three years, the majority of his research activities were focused on the completion of his undergraduate honors thesis, A Study of the Impact of Prognostic Errors on System Performance. In this effort, Mauricio used discrete-event simulation to compare the performance of a simple system under three types of maintenance policies: (1) run-to-failure maintenance, (2) optimal, scheduled maintenance, (3) condition-based maintenance (real-time prognostics). In the case of prognostics, he considered the case of perfect prognostic information and various degrees of imperfect prognostic information.

Mauricio's work was recognized in several ways. First, he received a State Undergraduate Research Fellowship from the SILO Advisory Council. In my time at the University of Arkansas, only two of our students have obtained this fellowship. Second, he competed in two undergraduate student technical paper competitions. In the IEEE Region V competition, he placed third. At the National Technical and Career Conference of the Society of Hispanic Professional Engineers, he placed first. Third, he presented a paper (in a regular session) at the 2006 Reliability and Maintainability Symposium in Newport Beach, California. For this paper, he and I received the Stan Olthson Award for the outstanding paper presented at RAMS, authored or co-authored by a member of the Society of Reliability Engineers. Finally, he received the 2005 Undergraduate Research Award from the Department of Industrial Engineering.